Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.4-d+e-x-^m-f+g-x-^n-a+b-x+c-x^2-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [958]. This is test number [35].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (958)	% 0. (0)
Mathematica	% 97.81 (937)	% 2.19 (21)
Maple	% 76.1 (729)	% 23.9 (229)
Maxima	% 18.68 (179)	% 81.32 (779)
Fricas	% 59.39 (569)	% 40.61 (389)
Sympy	% 27.24 (261)	% 72.76 (697)
Giac	% 27.97 (268)	% 72.03 (690)

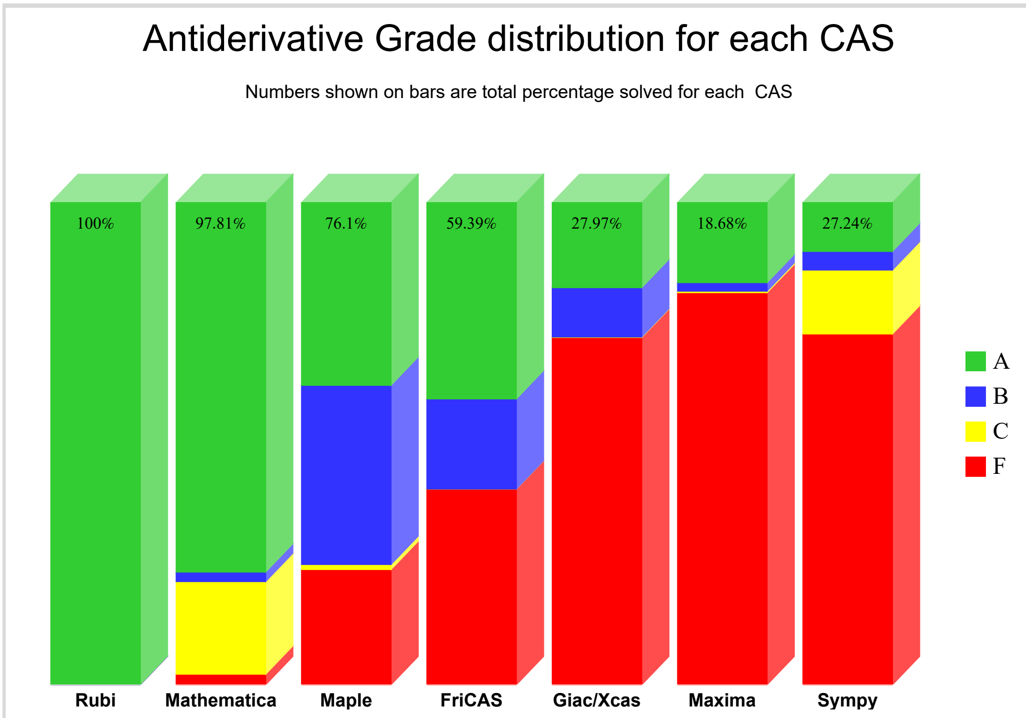
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

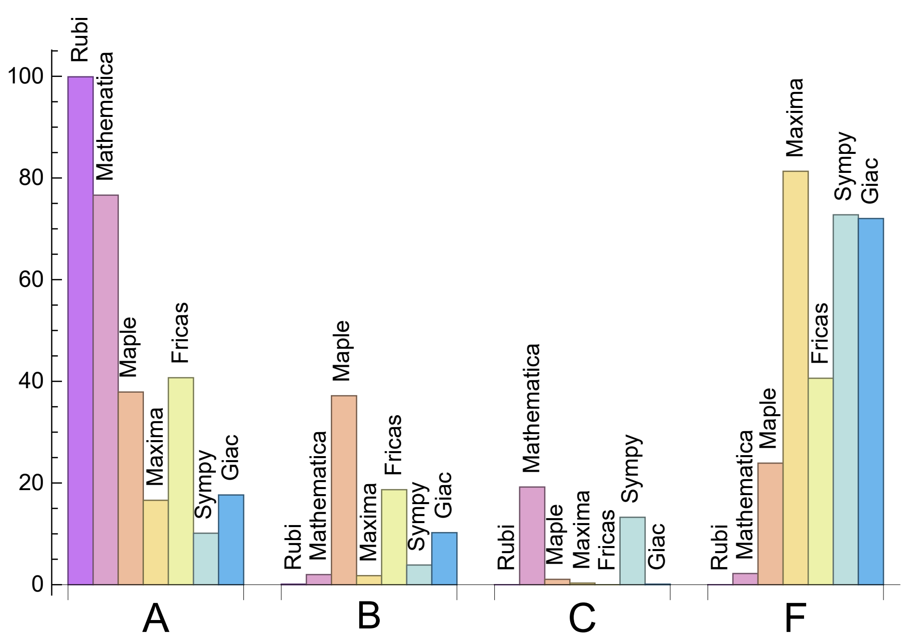
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.9	0.1	0.	0.
Mathematica	76.62	1.98	19.21	2.19
Maple	37.89	37.16	1.04	23.9
Maxima	16.6	1.77	0.31	81.32
Fricas	40.71	18.68	0.	40.61
Sympy	10.13	3.86	13.26	72.76
Giac	17.64	10.23	0.1	72.03

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.48	234.01	1.	175.	1.
Mathematica	1.02	654.33	1.51	145.	0.9
Maple	0.22	1938.59	3.96	288.	1.77
Maxima	1.22	256.45	1.7	186.	1.52
Fricas	5.3	1275.48	5.98	493.	3.74
Sympy	23.4	865.2	5.55	393.	3.33
Giac	1.64	640.89	2.93	231.	1.89

1.4 list of integrals that has no closed form antiderivative

{948, 952, 957}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {420}

Mathematica {267, 275, 276, 282, 286, 287, 291, 292, 296, 297, 301, 302, 314, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 425, 426, 427, 477, 479, 498, 501, 507, 514, 521, 604, 656, 802, 833, 886, 887, 890, 891, 892, 893, 894, 897, 898, 899, 900, 905, 906, 907, 908, 914, 915, 917, 918, 947, 951, 956}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

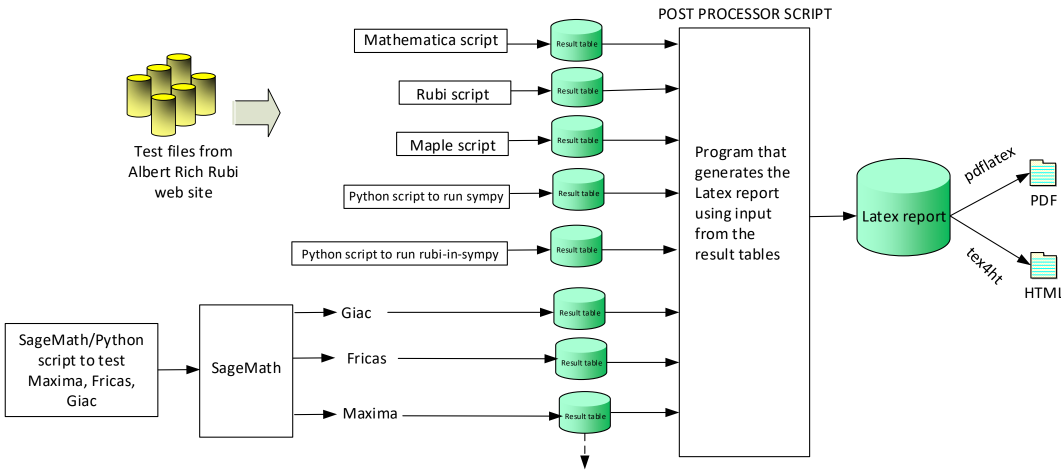
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958 }

B grade: { 833 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 269, 270, 271, 272, 273, 278, 279, 280, 281, 283, 288, 289, 290, 293, 298, 299, 300, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 425, 426, 427, 430, 431, 432, 433, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 497, 504, 511, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 589, 590, 591, 592, 593, 594, 596, 597, 598, 599, 603, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 621, 657, 658, 659, 660, 661, 662, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 684, 685, 689, 690, 691, 692, 693, 694, 696, 700, 701, 702, 703, 704, 705, 708, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 726, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 749, 750, 752, 753, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 919, 920, 921, 922, 923, 924, 925, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 947, 948, 951, 952, 953, 956, 957 }

B grade: { 263, 268, 274, 277, 284, 285, 294, 295, 304, 305, 360, 458, 605, 656, 751, 802, 907, 918, 926 }

C grade: { 8, 9, 10, 11, 13, 14, 15, 42, 43, 50, 51, 52, 53, 62, 63, 64, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 91, 267, 275, 276, 282, 286, 287, 291, 292, 296, 297, 301, 302, 315, 325, 339, 340, 341, 477, 479, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 587, 588, 595, 600, 601, 602, 604, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 663, 664, 669, 670, 671, 676, 677, 678, 686, 687, 688, 695, 697, 698, 699, 706, 707, 709, 710, 711, 720, 721, 727, 728, 745, 754, 755, 791, 838, 839, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 958 }

F grade: { 379, 380, 408, 416, 422, 423, 424, 428, 429, 434, 435, 436, 541, 803, 804, 945, 946, 949, 950, 954, 955}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 96, 103, 107, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 198, 200, 201, 211, 212, 213, 214, 215, 216, 218, 219, 224, 315, 326, 327, 328, 332, 333, 341, 350, 352, 353, 440, 470, 471, 472, 473, 474, 475, 476, 480, 481, 482, 487, 489, 490, 491, 493, 494, 496, 497, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 511, 512, 514, 515, 516, 518, 521, 524, 548, 549, 550, 551, 552, 553, 557, 558, 559, 560, 561, 562, 563, 566, 569, 570, 571, 572, 573, 577, 578, 582, 583, 584, 588, 589, 590, 591, 592, 593, 596, 597, 598, 599, 600, 604, 619, 640, 648, 649, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 679, 680, 681, 682, 683, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 696, 697, 700, 701, 702, 703, 704, 708, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 735, 736, 737, 738, 739, 740, 741, 744, 746, 747, 748, 749, 750, 753, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 788, 808, 814, 820, 821, 822, 823, 827, 828, 829, 840, 841, 848, 849, 875, 912, 913, 918, 919, 948, 952, 957, 958 }

B grade: { 9, 10, 11, 45, 78, 79, 84, 85, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 136, 137, 138, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 191, 194, 195, 196, 197, 199, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 220, 221, 222, 223, 225, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 351, 355, 356, 357, 359, 360, 361, 437, 438, 439, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 477, 478, 479, 483, 484, 485, 486, 488, 492, 495, 503, 509, 510, 513, 517, 519, 520, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 554, 555, 556, 564, 565, 567, 568, 574, 575, 576, 579, 580, 581, 585, 586, 587, 594, 595, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 641, 642, 643, 644, 645, 646, 647, 650, 651, 652, 653, 654, 678, 688, 698, 699, 705, 706, 707, 709, 710, 711, 720, 721, 727, 733, 734, 742, 743, 745, 751, 752, 754, 755, 756, 789, 790, 791, 805, 806, 807, 815, 816, 817, 818, 819, 824, 825, 826, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 914, 915, 916, 917, 920, 921, 925, 926 }

C grade: { 792, 793, 794, 795, 796, 797, 798, 799, 800, 801 }

F grade: { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 541, 542, 543, 544, 545, 546, 547, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 802, 803, 804, 809, 810, 811, 812, 813, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 16, 17, 18, 23, 24, 25, 26, 30, 31, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 86, 87, 88, 117, 118, 150, 151, 152, 153, 351, 352, 355, 356, 490, 497, 504, 511, 518, 524, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 589, 590, 591, 592, 596, 597, 598, 599, 604, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 768, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 792, 793, 794, 797, 798, 799, 814, 815, 816, 819, 820, 821, 822, 826, 827, 828, 829, 948, 952, 957

B grade: { 19, 20, 21, 22, 44, 45, 83, 84, 85, 211, 359, 360, 579, 580, 581, 582, 583 }

C grade: { 107, 161, 162 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 27, 28, 29, 38, 39, 40, 41, 42, 43, 50, 51, 52, 53, 64, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 353, 354, 357, 358, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 585, 586, 587, 588, 593, 594, 595, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 661, 662, 663, 664, 669, 670, 671, 676, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 788, 789, 790, 791, 795, 796, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 817, 818, 823, 824, 825, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 222, 224, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 341, 348, 349, 350, 437, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 457, 458, 459, 460, 462, 463, 464, 470, 471, 472, 473, 474, 475, 476, 490, 493, 497, 500, 504, 507, 511, 514, 518, 521, 524, 548, 549, 550, 551, 552, 557, 558, 559, 560, 562, 569, 570, 572, 573, 583, 584, 589, 590, 591, 592, 593, 596, 597, 598, 599, 603, 604, 620, 621, 657, 658, 659, 660, 661, 665, 666, 667, 668, 672, 673, 674, 679, 680, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 700, 701, 702, 703, 705, 706, 707, 712, 713, 714, 715, 720, 721, 722, 727, 728, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 788, 792, 793, 794, 797, 798, 814, 815, 816, 819, 820, 821, 822, 823, 826, 827, 828, 829, 834, 835, 836, 837, 840, 842, 843, 844, 845, 857, 948, 952, 957 }

B grade: { 18, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 132, 138, 139, 140, 141, 142, 143, 144, 148, 149, 216, 221, 223, 225, 330, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 347, 351, 352, 353, 355, 356, 357, 359, 360, 361, 477, 478, 479, 480, 481, 482, 483, 484, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 553, 554, 555, 556, 561, 563, 564, 565, 566, 567, 568, 571, 574, 575, 576, 577, 578, 579, 580, 581, 582, 585, 586, 587, 588, 594, 595, 600, 601, 602, 607, 611, 612, 618, 619, 662, 663, 664, 669, 670, 671, 675, 676, 677, 678, 685, 686, 687, 688, 696, 697, 698, 699, 704, 708, 709, 710, 711, 716, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 789, 790, 791, 795, 799, 805, 806, 807, 808, 824, 825, 830, 831, 832, 833, 838, 839, 846, 847, 848, 849, 874, 880, 881, 882, 920, 921, 925, 926 }

C grade: { }

F grade: { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 342, 343, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 445, 454, 455, 456, 461, 465, 466, 467, 468, 469, 485, 486, 487, 488, 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 532, 539, 540, 541, 542, 543, 544, 545, 546, 547, 605, 606, 608, 609, 610, 613, 614, 615, 616, 617, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 817, 818, 841, 850, 851, 852, 853, 854, 855, 856, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 875, 876, 877, 878, 879, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

2.1.6 Sympy

A grade: { 3, 5, 6, 25, 32, 34, 36, 37, 54, 55, 56, 57, 58, 62, 65, 67, 69, 104, 106, 117, 157, 159, 161, 220, 222, 224, 244, 245, 253, 254, 255, 262, 264, 265, 315, 351, 352, 353, 356, 357, 385, 386, 394, 395, 396, 403, 405, 406, 524, 529, 548, 549, 550, 551, 557, 558, 559, 560, 561, 562, 566, 569, 570, 571, 572, 573, 576, 577, 578, 588, 589, 590, 591, 592, 593, 596, 597, 598, 599, 600, 806, 807, 808, 819, 820, 821, 822, 823, 826, 827, 828, 829, 830, 920, 921, 948, 952 }

B grade: { 19, 21, 23, 240, 241, 242, 243, 249, 250, 251, 252, 258, 259, 260, 261, 263, 354, 358, 362, 527, 528, 530, 531, 552, 553, 554, 555, 556, 563, 564, 565, 567, 568, 574, 575, 814, 815 }

C grade: { 1, 2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 33, 35, 38, 39, 40, 41, 42, 43, 59, 60, 61, 63, 64, 66, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 102, 103, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 226, 227, 228, 229, 236, 246, 247, 248, 256, 257, 266, 267, 268, 269, 270, 271, 272, 273, 274, 306, 307, 308, 309, 310, 313, 377, 378, 381, 382, 383, 384, 387, 388, 389, 390, 391, 392, 393, 397, 398, 400, 401, 402, 404, 407, 433, 604, 793, 794, 799 }

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2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 117, 130, 134, 150, 151, 152, 153, 154, 186, 187, 195, 196, 197, 210, 218, 219, 316, 317, 318, 319, 320, 322, 323, 324, 326, 327, 328, 329, 330, 332, 333, 335, 336, 338, 340, 341, 490, 504, 524, 548, 552, 557, 558, 563, 564, 575, 576, 581, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 604, 618, 621, 771, 780, 792, 793, 794, 817, 820, 821, 822, 823, 824, 828, 829, 830, 831, 833, 834, 835, 836, 837, 838, 842, 845, 874, 948, 952, 957 }

B grade: { 9, 10, 11, 12, 13, 14, 15, 40, 41, 42, 43, 60, 61, 62, 63, 64, 75, 76, 77, 78, 79, 80, 81, 82, 98, 118, 155, 156, 325, 334, 337, 347, 351, 352, 353, 355, 356, 357, 359, 360, 361, 497, 549, 550, 551, 559, 560, 561, 562, 569, 570, 571, 572, 573, 574, 579, 580, 585, 587, 596, 605, 606, 619, 620, 768, 769, 770, 797, 798, 799, 805, 806, 807, 808, 819, 825, 826, 827, 832, 839, 840, 841, 843, 844, 846, 847, 848, 849, 860, 877, 880, 881, 882, 885, 920, 921, 925, 926 }

C grade: { 211 }

F grade: { 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192,

193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 321, 331, 339, 342, 343, 344, 345, 346, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 553, 554, 555, 556, 565, 566, 567, 568, 577, 578, 586, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 795, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 814, 815, 816, 818, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 875, 876, 878, 879, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958
}

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	112	125	158	205	280	100
normalized size	1	1.	0.85	0.95	1.2	1.55	2.12	0.76
time (sec)	N/A	0.076	0.136	0.058	1.609	1.821	6.229	1.263

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	157	198	257	323	833	158
normalized size	1	1.	0.78	0.99	1.28	1.61	4.14	0.79
time (sec)	N/A	0.149	0.24	0.067	1.759	1.83	22.206	1.3

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	146	173	223	289	779	143
normalized size	1	1.	0.85	1.01	1.3	1.68	4.53	0.83
time (sec)	N/A	0.101	0.204	0.06	1.563	1.762	19.548	1.312

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	135	148	189	262	656	130
normalized size	1	1.	0.85	0.93	1.19	1.65	4.13	0.82
time (sec)	N/A	0.108	0.191	0.059	1.577	1.914	14.952	1.302

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	124	123	155	230	583	113
normalized size	1	1.	1.07	1.06	1.34	1.98	5.03	0.97
time (sec)	N/A	0.034	0.162	0.055	1.616	1.886	17.026	1.292

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	124	123	155	230	583	113
normalized size	1	1.	1.07	1.06	1.34	1.98	5.03	0.97
time (sec)	N/A	0.033	0.047	0.052	1.537	1.932	12.828	1.283

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	124	151	0	228	476	134
normalized size	1	1.	1.1	1.34	0.	2.02	4.21	1.19
time (sec)	N/A	0.098	0.202	0.054	0.	1.826	23.338	1.312

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	124	182	0	257	393	212
normalized size	1	1.	1.06	1.56	0.	2.2	3.36	1.81
time (sec)	N/A	0.092	0.19	0.056	0.	1.79	9.187	1.267

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	110	212	0	269	471	293
normalized size	1	1.	0.91	1.75	0.	2.22	3.89	2.42
time (sec)	N/A	0.094	0.086	0.059	0.	1.811	13.997	1.236

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	111	235	0	269	469	352
normalized size	1	1.	0.92	1.96	0.	2.24	3.91	2.93
time (sec)	N/A	0.092	0.069	0.064	0.	2.003	7.679	1.253

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	133	260	0	248	552	401
normalized size	1	1.	1.13	2.2	0.	2.1	4.68	3.4
time (sec)	N/A	0.092	0.102	0.064	0.	1.939	10.104	1.307

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	133	158	0	203	785	497
normalized size	1	1.	1.23	1.46	0.	1.88	7.27	4.6
time (sec)	N/A	0.063	0.067	0.074	0.	1.88	9.538	1.319

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	59	186	0	232	930	582
normalized size	1	1.	0.41	1.3	0.	1.62	6.5	4.07
time (sec)	N/A	0.095	0.025	0.079	0.	1.77	13.967	1.277

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	72	211	0	265	1049	667
normalized size	1	1.	0.42	1.23	0.	1.54	6.1	3.88
time (sec)	N/A	0.125	0.022	0.095	0.	2.016	15.007	1.282

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	73	236	0	292	1171	582
normalized size	1	1.	0.36	1.17	0.	1.45	5.83	2.9
time (sec)	N/A	0.156	0.03	0.122	0.	2.094	30.682	1.234

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	70	102	127	153	178	73
normalized size	1	1.	0.68	0.99	1.23	1.49	1.73	0.71
time (sec)	N/A	0.053	0.046	0.053	1.58	1.853	5.423	1.196

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	77	99	123	176	185	89
normalized size	1	1.	1.05	1.36	1.68	2.41	2.53	1.22
time (sec)	N/A	0.042	0.036	0.053	1.633	2.087	7.274	1.177

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	52	55	119	204	233	69
normalized size	1	1.	0.9	0.95	2.05	3.52	4.02	1.19
time (sec)	N/A	0.026	0.025	0.047	1.	2.146	9.637	1.175

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	155	227	439	575	2006	162
normalized size	1	1.	0.96	1.41	2.73	3.57	12.46	1.01
time (sec)	N/A	0.139	0.109	0.117	1.489	2.678	48.822	1.188

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	142	195	393	541	1822	147
normalized size	1	1.	0.97	1.33	2.67	3.68	12.39	1.
time (sec)	N/A	0.12	0.098	0.091	1.531	2.418	40.3	1.175

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	130	166	355	497	1741	131
normalized size	1	1.	1.07	1.36	2.91	4.07	14.27	1.07
time (sec)	N/A	0.081	0.087	0.076	1.512	2.091	43.494	1.181

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	77	215	343	420	86
normalized size	1	1.	0.98	0.92	2.56	4.08	5.	1.02
time (sec)	N/A	0.052	0.027	0.048	0.962	1.961	38.03	1.216

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	82	77	181	340	338	78
normalized size	1	1.	0.91	0.86	2.01	3.78	3.76	0.87
time (sec)	N/A	0.042	0.026	0.046	0.986	1.964	13.452	1.207

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	82	77	151	340	515	86
normalized size	1	1.	0.87	0.82	1.61	3.62	5.48	0.91
time (sec)	N/A	0.046	0.028	0.048	0.993	1.907	13.119	1.185

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	77	117	339	434	77
normalized size	1	1.	0.99	0.93	1.41	4.08	5.23	0.93
time (sec)	N/A	0.023	0.039	0.049	1.006	2.01	11.474	1.188

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	77	108	340	605	88
normalized size	1	1.	1.02	0.96	1.35	4.25	7.56	1.1
time (sec)	N/A	0.021	0.028	0.049	0.995	1.952	12.423	1.173

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	131	163	0	493	2382	165
normalized size	1	1.	1.12	1.39	0.	4.21	20.36	1.41
time (sec)	N/A	0.103	0.068	0.055	0.	1.977	22.322	1.187

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	147	195	0	548	2409	255
normalized size	1	1.	0.96	1.27	0.	3.58	15.75	1.67
time (sec)	N/A	0.127	0.078	0.058	0.	2.156	20.113	1.187

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	183	227	0	595	2696	351
normalized size	1	1.	0.99	1.23	0.	3.23	14.65	1.91
time (sec)	N/A	0.16	0.148	0.061	0.	2.473	22.257	1.184

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	104	99	182	486	904	104
normalized size	1	1.	0.86	0.82	1.5	4.02	7.47	0.86
time (sec)	N/A	0.053	0.045	0.053	0.995	2.315	17.931	1.194

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	126	121	213	640	1402	122
normalized size	1	1.	0.85	0.82	1.44	4.32	9.47	0.82
time (sec)	N/A	0.062	0.054	0.052	1.018	4.29	38.653	1.195

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	85	101	151	102	95
normalized size	1	1.	0.93	1.57	1.87	2.8	1.89	1.76
time (sec)	N/A	0.034	0.031	0.053	1.478	1.845	5.721	1.154

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	103	174	224	236	561	113
normalized size	1	1.	0.6	1.01	1.29	1.36	3.24	0.65
time (sec)	N/A	0.227	0.111	0.07	1.467	1.829	13.518	1.19

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	92	149	190	204	359	99
normalized size	1	1.	0.64	1.03	1.32	1.42	2.49	0.69
time (sec)	N/A	0.186	0.096	0.061	1.471	1.783	7.011	1.133

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	124	157	181	389	85
normalized size	1	1.	0.7	1.08	1.37	1.57	3.38	0.74
time (sec)	N/A	0.145	0.076	0.057	1.472	1.809	7.753	1.192

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	69	98	122	150	219	66
normalized size	1	1.	0.83	1.18	1.47	1.81	2.64	0.8
time (sec)	N/A	0.085	0.062	0.057	1.47	1.816	4.576	1.169

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	71	85	126	270	54
normalized size	1	1.	0.7	0.86	1.02	1.52	3.25	0.65
time (sec)	N/A	0.028	0.039	0.053	1.458	1.792	4.088	1.18

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	91	0	149	187	88
normalized size	1	1.	1.	1.38	0.	2.26	2.83	1.33
time (sec)	N/A	0.111	0.029	0.051	0.	1.848	5.502	1.176

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	93	0	162	214	144
normalized size	1	1.	1.	1.37	0.	2.38	3.15	2.12
time (sec)	N/A	0.115	0.037	0.055	0.	1.867	3.741	1.183

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	122	86	0	128	224	230
normalized size	1	1.	1.52	1.08	0.	1.6	2.8	2.88
time (sec)	N/A	0.109	0.272	0.055	0.	1.838	5.455	1.164

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	87	114	0	153	313	323
normalized size	1	1.	0.81	1.07	0.	1.43	2.93	3.02
time (sec)	N/A	0.137	0.161	0.058	0.	1.886	5.232	1.205

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	155	139	0	184	459	412
normalized size	1	1.	1.11	0.99	0.	1.31	3.28	2.94
time (sec)	N/A	0.172	0.161	0.059	0.	1.837	9.215	1.154

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	79	164	0	207	520	493
normalized size	1	1.	0.47	0.97	0.	1.22	3.08	2.92
time (sec)	N/A	0.195	0.044	0.075	0.	1.866	8.226	1.165

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	111	193	390	397	0	143
normalized size	1	1.	0.78	1.35	2.73	2.78	0.	1.
time (sec)	N/A	0.271	0.234	0.103	1.558	2.015	0.	1.165

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	96	236	420	359	0	128
normalized size	1	1.	0.79	1.95	3.47	2.97	0.	1.06
time (sec)	N/A	0.208	0.214	0.083	1.544	1.813	0.	1.163

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	63	65	209	230	0	85
normalized size	1	1.	0.65	0.67	2.15	2.37	0.	0.88
time (sec)	N/A	0.173	0.064	0.049	1.012	1.868	0.	1.151

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	63	66	177	235	0	82
normalized size	1	1.	0.72	0.76	2.03	2.7	0.	0.94
time (sec)	N/A	0.12	0.06	0.05	1.004	1.79	0.	1.157

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	62	64	147	228	0	86
normalized size	1	1.	0.7	0.72	1.65	2.56	0.	0.97
time (sec)	N/A	0.033	0.056	0.05	1.011	1.873	0.	1.175

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	65	105	230	0	82
normalized size	1	1.	0.82	0.84	1.36	2.99	0.	1.06
time (sec)	N/A	0.02	0.044	0.049	0.987	1.934	0.	1.177

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	81	160	0	350	0	159
normalized size	1	1.	0.69	1.37	0.	2.99	0.	1.36
time (sec)	N/A	0.159	0.047	0.058	0.	2.027	0.	1.159

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	90	193	0	401	0	254
normalized size	1	1.	0.62	1.33	0.	2.77	0.	1.75
time (sec)	N/A	0.275	0.054	0.064	0.	1.954	0.	1.164

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	117	224	0	440	0	351
normalized size	1	1.	0.64	1.23	0.	2.42	0.	1.93
time (sec)	N/A	0.358	0.061	0.069	0.	2.033	0.	1.17

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	105	249	0	471	0	439
normalized size	1	1.	0.5	1.19	0.	2.25	0.	2.1
time (sec)	N/A	0.474	0.06	0.069	0.	2.274	0.	1.178

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	42	71	95	131	73	46
normalized size	1	1.	0.52	0.88	1.17	1.62	0.9	0.57
time (sec)	N/A	0.093	0.037	0.051	1.492	1.767	1.439	1.125

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	37	57	76	119	60	41
normalized size	1	1.	0.59	0.9	1.21	1.89	0.95	0.65
time (sec)	N/A	0.081	0.03	0.048	1.487	1.854	0.762	1.112

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	26	41	54	97	37	28
normalized size	1	1.	0.63	1.	1.32	2.37	0.9	0.68
time (sec)	N/A	0.048	0.019	0.047	1.469	1.852	0.427	1.147

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	25	29	38	86	27	26
normalized size	1	1.	0.62	0.72	0.95	2.15	0.68	0.65
time (sec)	N/A	0.012	0.015	0.057	1.503	2.029	0.208	1.106

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	55	111	31	46
normalized size	1	1.	1.	0.91	1.72	3.47	0.97	1.44
time (sec)	N/A	0.059	0.011	0.048	1.488	2.136	4.914	1.149

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	57	124	51	74
normalized size	1	1.	1.	0.91	1.73	3.76	1.55	2.24
time (sec)	N/A	0.063	0.015	0.054	1.491	2.165	3.968	1.123

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	42	73	97	116	123
normalized size	1	1.	0.78	0.82	1.43	1.9	2.27	2.41
time (sec)	N/A	0.061	0.02	0.052	1.484	1.985	6.602	1.139

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	56	92	108	128	169
normalized size	1	1.	0.64	0.84	1.37	1.61	1.91	2.52
time (sec)	N/A	0.074	0.023	0.051	1.483	1.845	7.731	1.118

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	73	70	111	126	223	220
normalized size	1	1.	0.82	0.79	1.25	1.42	2.51	2.47
time (sec)	N/A	0.09	0.04	0.053	1.492	1.815	12.45	1.104

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	50	84	130	138	201	269
normalized size	1	1.	0.47	0.79	1.21	1.29	1.88	2.51
time (sec)	N/A	0.103	0.018	0.05	1.708	1.854	14.071	1.131

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	196	212	0	230	556	398
normalized size	1	1.	1.46	1.58	0.	1.72	4.15	2.97
time (sec)	N/A	0.215	0.262	0.064	0.	1.763	9.606	1.173

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	212	291	382	529	2280	230
normalized size	1	1.	0.68	0.94	1.23	1.71	7.35	0.74
time (sec)	N/A	0.487	0.379	0.154	1.445	1.888	139.604	1.136

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	200	266	348	487	2035	216
normalized size	1	1.	0.71	0.95	1.24	1.73	7.24	0.77
time (sec)	N/A	0.406	0.361	0.112	1.525	1.924	83.595	1.131

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	189	241	315	455	1926	201
normalized size	1	1.	0.75	0.96	1.25	1.81	7.64	0.8
time (sec)	N/A	0.364	0.318	0.092	1.487	2.028	66.983	1.124

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	178	216	281	412	1688	188
normalized size	1	1.	0.8	0.97	1.26	1.85	7.57	0.84
time (sec)	N/A	0.306	0.284	0.079	1.479	1.938	51.338	1.131

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	167	191	247	360	1561	173
normalized size	1	1.	0.73	0.83	1.07	1.57	6.79	0.75
time (sec)	N/A	0.121	0.411	0.067	1.494	1.87	53.33	1.143

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	156	154	197	324	1290	158
normalized size	1	1.	0.83	0.82	1.05	1.72	6.86	0.84
time (sec)	N/A	0.082	0.351	0.06	1.485	2.017	40.916	1.17

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	168	231	0	358	1273	193
normalized size	1	1.	0.88	1.22	0.	1.88	6.7	1.02
time (sec)	N/A	0.307	0.38	0.056	0.	1.983	55.4	1.145

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	221	243	0	377	1068	269
normalized size	1	1.	1.15	1.26	0.	1.95	5.53	1.39
time (sec)	N/A	0.305	0.576	0.061	0.	1.959	22.339	1.174

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	259	252	0	392	1073	354
normalized size	1	1.	1.25	1.22	0.	1.89	5.18	1.71
time (sec)	N/A	0.313	0.687	0.063	0.	2.026	25.174	1.186

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	251	277	0	386	926	429
normalized size	1	1.	1.2	1.32	0.	1.84	4.41	2.04
time (sec)	N/A	0.314	0.293	0.066	0.	1.994	19.066	1.155

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	195	302	0	369	1047	505
normalized size	1	1.	0.93	1.44	0.	1.77	5.01	2.42
time (sec)	N/A	0.319	0.109	0.074	0.	1.895	20.546	1.258

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	199	327	0	387	1197	581
normalized size	1	1.	0.92	1.51	0.	1.79	5.54	2.69
time (sec)	N/A	0.313	0.105	0.086	0.	1.906	17.522	1.256

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	286	352	0	392	1420	655
normalized size	1	1.	1.34	1.64	0.	1.83	6.64	3.06
time (sec)	N/A	0.312	0.242	0.106	0.	2.041	23.537	1.324

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	247	377	0	385	1537	689
normalized size	1	1.	1.2	1.83	0.	1.87	7.46	3.34
time (sec)	N/A	0.311	0.169	0.131	0.	1.836	24.05	1.228

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	245	402	0	382	1742	726
normalized size	1	1.	1.2	1.97	0.	1.87	8.54	3.56
time (sec)	N/A	0.304	0.166	0.176	0.	2.27	34.532	1.272

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	218	250	0	328	1914	837
normalized size	1	1.	1.17	1.34	0.	1.75	10.24	4.48
time (sec)	N/A	0.26	0.183	0.253	0.	2.413	34.807	1.263

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	102	278	0	367	2184	922
normalized size	1	1.	0.45	1.24	0.	1.63	9.71	4.1
time (sec)	N/A	0.298	0.074	0.373	0.	3.09	54.191	1.283

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	112	303	0	417	2422	1007
normalized size	1	1.	0.44	1.19	0.	1.64	9.54	3.96
time (sec)	N/A	0.329	0.068	0.569	0.	3.168	54.722	1.289

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	131	222	429	409	0	159
normalized size	1	1.	0.75	1.28	2.47	2.35	0.	0.91
time (sec)	N/A	0.405	0.262	0.137	1.504	1.789	0.	1.204

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	119	262	455	367	0	144
normalized size	1	1.	0.84	1.85	3.2	2.58	0.	1.01
time (sec)	N/A	0.325	0.212	0.114	1.526	1.666	0.	1.165

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	112	234	417	336	0	128
normalized size	1	1.	0.95	1.98	3.53	2.85	0.	1.08
time (sec)	N/A	0.216	0.167	0.093	1.562	1.695	0.	1.185

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	58	55	208	212	0	97
normalized size	1	1.	0.62	0.59	2.24	2.28	0.	1.04
time (sec)	N/A	0.126	0.085	0.056	1.014	1.543	0.	1.228

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	55	52	173	204	0	81
normalized size	1	1.	0.64	0.6	2.01	2.37	0.	0.94
time (sec)	N/A	0.036	0.19	0.049	1.	1.651	0.	1.237

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	58	55	136	215	0	95
normalized size	1	1.	0.56	0.53	1.32	2.09	0.	0.92
time (sec)	N/A	0.048	0.067	0.048	0.991	1.563	0.	1.189

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	81	158	0	327	0	158
normalized size	1	1.	0.71	1.39	0.	2.87	0.	1.39
time (sec)	N/A	0.159	0.062	0.066	0.	1.609	0.	1.231

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	96	190	0	374	0	250
normalized size	1	1.	0.66	1.31	0.	2.58	0.	1.72
time (sec)	N/A	0.287	0.06	0.064	0.	1.69	0.	1.222

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	119	222	0	424	0	350
normalized size	1	1.	0.65	1.22	0.	2.33	0.	1.92
time (sec)	N/A	0.362	0.076	0.065	0.	1.739	0.	1.222

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	91	208	0	207	0	104
normalized size	1	1.	0.62	1.41	0.	1.41	0.	0.71
time (sec)	N/A	0.142	0.151	0.064	0.	1.578	0.	1.196

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	80	185	0	177	0	89
normalized size	1	1.	0.68	1.57	0.	1.5	0.	0.75
time (sec)	N/A	0.099	0.115	0.064	0.	1.544	0.	1.153

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	69	160	0	153	0	73
normalized size	1	1.	0.8	1.86	0.	1.78	0.	0.85
time (sec)	N/A	0.11	0.085	0.058	0.	1.555	0.	1.185

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	140	0	127	0	58
normalized size	1	1.	0.92	2.26	0.	2.05	0.	0.94
time (sec)	N/A	0.041	0.067	0.054	0.	1.57	0.	1.235

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	77	0	101	0	42
normalized size	1	1.	0.93	1.67	0.	2.2	0.	0.91
time (sec)	N/A	0.016	0.025	0.047	0.	1.657	0.	1.216

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	137	0	111	0	65
normalized size	1	1.	1.	2.98	0.	2.41	0.	1.41
time (sec)	N/A	0.059	0.043	0.063	0.	1.556	0.	1.236

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	222	0	97	0	138
normalized size	1	1.	1.04	4.35	0.	1.9	0.	2.71
time (sec)	N/A	0.057	0.069	0.062	0.	1.593	0.	1.235

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	70	254	0	128	0	0
normalized size	1	1.	0.85	3.1	0.	1.56	0.	0.
time (sec)	N/A	0.078	0.115	0.066	0.	1.85	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	84	280	0	157	0	0
normalized size	1	1.	0.74	2.46	0.	1.38	0.	0.
time (sec)	N/A	0.11	0.116	0.069	0.	1.925	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	95	304	0	180	0	0
normalized size	1	1.	0.66	2.13	0.	1.26	0.	0.
time (sec)	N/A	0.135	0.136	0.077	0.	1.755	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	112	222	0	205	280	0
normalized size	1	1.	0.99	1.96	0.	1.81	2.48	0.
time (sec)	N/A	0.131	0.132	0.056	0.	1.952	6.758	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	135	330	0	323	833	0
normalized size	1	1.	0.67	1.64	0.	1.61	4.14	0.
time (sec)	N/A	0.16	0.191	0.066	0.	1.797	22.281	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	124	305	0	288	779	0
normalized size	1	1.	0.72	1.77	0.	1.67	4.53	0.
time (sec)	N/A	0.123	0.139	0.059	0.	1.658	20.617	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	113	282	0	262	656	0
normalized size	1	1.	0.81	2.01	0.	1.87	4.69	0.
time (sec)	N/A	0.15	0.109	0.058	0.	1.586	13.911	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	102	260	0	228	583	0
normalized size	1	1.	0.88	2.24	0.	1.97	5.03	0.
time (sec)	N/A	0.059	0.094	0.054	0.	1.649	17.399	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	91	147	147	200	439	0
normalized size	1	1.	0.91	1.47	1.47	2.	4.39	0.
time (sec)	N/A	0.031	0.061	0.051	1.492	1.609	10.601	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	108	245	0	227	476	0
normalized size	1	1.	0.96	2.17	0.	2.01	4.21	0.
time (sec)	N/A	0.116	0.094	0.062	0.	1.673	24.104	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	114	380	0	257	393	0
normalized size	1	1.	0.99	3.3	0.	2.23	3.42	0.
time (sec)	N/A	0.118	0.135	0.064	0.	1.606	13.258	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	119	411	0	270	471	0
normalized size	1	1.	0.98	3.4	0.	2.23	3.89	0.
time (sec)	N/A	0.113	0.169	0.067	0.	1.65	11.39	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	116	439	0	269	469	0
normalized size	1	1.	0.97	3.66	0.	2.24	3.91	0.
time (sec)	N/A	0.114	0.169	0.069	0.	1.659	12.788	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	111	463	0	247	552	0
normalized size	1	1.	0.93	3.89	0.	2.08	4.64	0.
time (sec)	N/A	0.115	0.203	0.081	0.	1.636	15.738	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	106	493	0	204	785	0
normalized size	1	1.	0.98	4.56	0.	1.89	7.27	0.
time (sec)	N/A	0.089	0.159	0.097	0.	1.637	14.107	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	117	521	0	232	930	0
normalized size	1	1.	0.82	3.64	0.	1.62	6.5	0.
time (sec)	N/A	0.12	0.201	0.124	0.	1.725	20.139	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	128	546	0	266	1049	0
normalized size	1	1.	0.74	3.17	0.	1.55	6.1	0.
time (sec)	N/A	0.154	0.216	0.149	0.	1.763	26.665	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	139	571	0	292	1171	0
normalized size	1	1.	0.69	2.84	0.	1.45	5.83	0.
time (sec)	N/A	0.19	0.237	0.195	0.	1.854	30.168	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	34	38	82	29	26
normalized size	1	1.	0.96	1.26	1.41	3.04	1.07	0.96
time (sec)	N/A	0.017	0.034	0.046	1.483	1.613	2.862	1.29

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	238	111	169	170	169
normalized size	1	1.	0.96	4.67	2.18	3.31	3.33	3.31
time (sec)	N/A	0.072	0.036	0.067	1.458	1.63	6.298	1.38

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	91	147	0	236	0	0
normalized size	1	1.	0.77	1.25	0.	2.	0.	0.
time (sec)	N/A	0.096	0.096	0.063	0.	1.693	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	80	120	0	205	0	0
normalized size	1	1.	0.88	1.32	0.	2.25	0.	0.
time (sec)	N/A	0.064	0.067	0.058	0.	1.567	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	97	0	177	0	0
normalized size	1	1.	0.77	1.26	0.	2.3	0.	0.
time (sec)	N/A	0.097	0.08	0.053	0.	1.599	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	74	0	143	0	0
normalized size	1	1.	0.94	1.42	0.	2.75	0.	0.
time (sec)	N/A	0.022	0.032	0.053	0.	1.631	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	29	0	72	0	0
normalized size	1	1.	1.03	0.94	0.	2.32	0.	0.
time (sec)	N/A	0.011	0.006	0.047	0.	1.559	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	88	0	131	0	0
normalized size	1	1.	0.96	1.63	0.	2.43	0.	0.
time (sec)	N/A	0.044	0.039	0.056	0.	1.647	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	62	108	0	176	0	0
normalized size	1	1.	0.77	1.33	0.	2.17	0.	0.
time (sec)	N/A	0.064	0.056	0.059	0.	1.567	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	127	133	0	220	0	0
normalized size	1	1.	1.12	1.18	0.	1.95	0.	0.
time (sec)	N/A	0.091	0.338	0.068	0.	1.6	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	106	208	0	381	0	0
normalized size	1	1.	0.83	1.62	0.	2.98	0.	0.
time (sec)	N/A	0.105	0.186	0.066	0.	1.664	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	93	179	0	346	0	0
normalized size	1	1.	0.82	1.58	0.	3.06	0.	0.
time (sec)	N/A	0.094	0.151	0.062	0.	1.714	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	80	153	0	312	0	0
normalized size	1	1.	0.9	1.72	0.	3.51	0.	0.
time (sec)	N/A	0.07	0.136	0.056	0.	1.599	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	48	0	203	0	1
normalized size	1	1.	1.	0.8	0.	3.38	0.	0.02
time (sec)	N/A	0.035	0.057	0.048	0.	1.629	0.	1.259

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	44	0	189	0	0
normalized size	1	1.	0.97	0.76	0.	3.26	0.	0.
time (sec)	N/A	0.02	0.054	0.048	0.	1.562	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	46	0	193	0	0
normalized size	1	1.	1.	0.79	0.	3.33	0.	0.
time (sec)	N/A	0.015	0.038	0.048	0.	1.529	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	83	142	0	301	0	0
normalized size	1	1.	0.94	1.61	0.	3.42	0.	0.
time (sec)	N/A	0.075	0.109	0.057	0.	1.6	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	101	188	0	354	0	1
normalized size	1	1.	0.84	1.57	0.	2.95	0.	0.01
time (sec)	N/A	0.103	0.126	0.062	0.	1.609	0.	1.221

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	115	216	0	394	0	0
normalized size	1	1.	0.76	1.42	0.	2.59	0.	0.
time (sec)	N/A	0.129	0.115	0.063	0.	1.702	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	128	318	0	575	0	0
normalized size	1	1.	0.79	1.96	0.	3.55	0.	0.
time (sec)	N/A	0.16	0.262	0.087	0.	2.001	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	115	288	0	543	0	0
normalized size	1	1.	0.78	1.95	0.	3.67	0.	0.
time (sec)	N/A	0.137	0.207	0.073	0.	1.803	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	103	259	0	497	0	0
normalized size	1	1.	0.84	2.12	0.	4.07	0.	0.
time (sec)	N/A	0.102	0.151	0.066	0.	1.717	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	70	0	344	0	0
normalized size	1	1.	0.96	0.82	0.	4.05	0.	0.
time (sec)	N/A	0.074	0.095	0.055	0.	1.613	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	0	340	0	0
normalized size	1	1.	0.9	0.77	0.	3.74	0.	0.
time (sec)	N/A	0.069	0.078	0.048	0.	1.658	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	82	70	0	339	0	0
normalized size	1	1.	0.86	0.74	0.	3.57	0.	0.
time (sec)	N/A	0.053	0.069	0.048	0.	1.746	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	70	0	339	0	0
normalized size	1	1.	0.96	0.82	0.	3.99	0.	0.
time (sec)	N/A	0.029	0.058	0.05	0.	1.743	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	70	0	342	0	0
normalized size	1	1.	1.	0.85	0.	4.17	0.	0.
time (sec)	N/A	0.022	0.043	0.048	0.	1.727	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	106	196	0	493	0	0
normalized size	1	1.	0.89	1.65	0.	4.14	0.	0.
time (sec)	N/A	0.107	0.096	0.059	0.	1.702	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	122	268	0	549	0	0
normalized size	1	1.	0.79	1.74	0.	3.56	0.	0.
time (sec)	N/A	0.136	0.134	0.059	0.	1.943	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	137	298	0	595	0	0
normalized size	1	1.	0.74	1.6	0.	3.2	0.	0.
time (sec)	N/A	0.168	0.144	0.065	0.	2.17	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	148	326	0	626	0	0
normalized size	1	1.	0.69	1.52	0.	2.91	0.	0.
time (sec)	N/A	0.21	0.182	0.069	0.	2.465	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	104	92	0	475	0	0
normalized size	1	1.	0.88	0.78	0.	4.03	0.	0.
time (sec)	N/A	0.077	0.138	0.049	0.	2.134	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	104	92	0	485	0	0
normalized size	1	1.	0.85	0.75	0.	3.94	0.	0.
time (sec)	N/A	0.058	0.084	0.049	0.	2.068	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	54	100	92	169	0	105
normalized size	1	1.	0.82	1.52	1.39	2.56	0.	1.59
time (sec)	N/A	0.054	0.057	0.056	1.467	1.671	0.	1.257

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	37	84	70	151	0	95
normalized size	1	1.	0.67	1.53	1.27	2.75	0.	1.73
time (sec)	N/A	0.086	0.055	0.048	1.476	1.622	0.	1.248

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	65	45	134	0	70
normalized size	1	1.	0.91	1.91	1.32	3.94	0.	2.06
time (sec)	N/A	0.017	0.025	0.053	1.46	1.633	0.	1.252

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	22	31	61	0	46
normalized size	1	1.	0.96	0.85	1.19	2.35	0.	1.77
time (sec)	N/A	0.01	0.006	0.044	1.473	1.543	0.	1.254

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	58	0	116	0	100
normalized size	1	1.	1.	1.41	0.	2.83	0.	2.44
time (sec)	N/A	0.04	0.03	0.053	0.	1.581	0.	1.292

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	50	73	0	151	0	203
normalized size	1	1.	0.78	1.14	0.	2.36	0.	3.17
time (sec)	N/A	0.053	0.045	0.058	0.	1.556	0.	1.32

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	63	94	0	192	0	288
normalized size	1	1.	0.7	1.04	0.	2.13	0.	3.2
time (sec)	N/A	0.079	0.059	0.057	0.	1.64	0.	1.268

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	135	375	0	313	573	0
normalized size	1	1.	0.59	1.64	0.	1.37	2.5	0.
time (sec)	N/A	0.315	0.23	0.076	0.	1.611	21.445	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	124	350	0	300	694	0
normalized size	1	1.	0.62	1.75	0.	1.5	3.47	0.
time (sec)	N/A	0.27	0.171	0.068	0.	1.606	27.256	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	113	327	0	259	452	0
normalized size	1	1.	0.66	1.91	0.	1.51	2.64	0.
time (sec)	N/A	0.215	0.135	0.065	0.	1.521	19.365	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	102	303	0	230	544	0
normalized size	1	1.	0.72	2.13	0.	1.62	3.83	0.
time (sec)	N/A	0.177	0.121	0.061	0.	1.583	22.11	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	91	198	225	204	323	0
normalized size	1	1.	0.67	1.46	1.65	1.5	2.38	0.
time (sec)	N/A	0.057	0.081	0.059	1.472	1.603	11.17	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	80	194	161	177	354	0
normalized size	1	1.	0.74	1.8	1.49	1.64	3.28	0.
time (sec)	N/A	0.042	0.058	0.054	1.797	1.559	8.565	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	290	0	196	272	0
normalized size	1	1.	1.	3.02	0.	2.04	2.83	0.
time (sec)	N/A	0.162	0.109	0.067	0.	1.703	12.045	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	100	425	0	231	354	0
normalized size	1	1.	0.95	4.05	0.	2.2	3.37	0.
time (sec)	N/A	0.161	0.143	0.073	0.	1.6	9.247	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	102	456	0	240	355	0
normalized size	1	1.	0.93	4.15	0.	2.18	3.23	0.
time (sec)	N/A	0.163	0.153	0.07	0.	1.627	8.925	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	96	479	0	217	347	0
normalized size	1	1.	0.94	4.7	0.	2.13	3.4	0.
time (sec)	N/A	0.163	0.188	0.076	0.	1.602	8.458	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	95	513	0	181	432	0
normalized size	1	1.	0.88	4.75	0.	1.68	4.	0.
time (sec)	N/A	0.147	0.188	0.115	0.	1.603	11.874	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	106	541	0	208	668	0
normalized size	1	1.	0.76	3.86	0.	1.49	4.77	0.
time (sec)	N/A	0.177	0.191	0.1	0.	1.652	13.082	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	117	566	0	234	816	0
normalized size	1	1.	0.69	3.35	0.	1.38	4.83	0.
time (sec)	N/A	0.208	0.265	0.12	0.	1.704	18.444	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	128	591	0	263	843	0
normalized size	1	1.	0.65	2.98	0.	1.33	4.26	0.
time (sec)	N/A	0.236	0.262	0.154	0.	1.703	18.205	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	106	198	0	360	0	0
normalized size	1	1.	0.86	1.61	0.	2.93	0.	0.
time (sec)	N/A	0.238	0.172	0.069	0.	1.59	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	70	65	0	230	0	0
normalized size	1	1.	0.71	0.66	0.	2.32	0.	0.
time (sec)	N/A	0.204	0.087	0.047	0.	1.586	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	70	65	0	234	0	0
normalized size	1	1.	0.79	0.73	0.	2.63	0.	0.
time (sec)	N/A	0.142	0.071	0.05	0.	1.684	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	69	64	0	228	0	0
normalized size	1	1.	0.76	0.7	0.	2.51	0.	0.
time (sec)	N/A	0.036	0.063	0.049	0.	1.596	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	70	66	0	231	0	0
normalized size	1	1.	0.77	0.73	0.	2.54	0.	0.
time (sec)	N/A	0.031	0.04	0.05	0.	1.547	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	95	187	0	350	0	0
normalized size	1	1.	0.81	1.58	0.	2.97	0.	0.
time (sec)	N/A	0.178	0.098	0.062	0.	1.69	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	112	234	0	402	0	0
normalized size	1	1.	0.77	1.6	0.	2.75	0.	0.
time (sec)	N/A	0.3	0.118	0.067	0.	1.69	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	127	259	0	440	0	0
normalized size	1	1.	0.69	1.42	0.	2.4	0.	0.
time (sec)	N/A	0.375	0.148	0.068	0.	1.758	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	98	212	0	409	0	0
normalized size	1	1.	0.55	1.2	0.	2.31	0.	0.
time (sec)	N/A	0.439	0.199	0.069	0.	1.754	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	85	187	0	369	0	0
normalized size	1	1.	0.58	1.28	0.	2.53	0.	0.
time (sec)	N/A	0.365	0.159	0.061	0.	1.703	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	73	163	0	336	0	0
normalized size	1	1.	0.61	1.36	0.	2.8	0.	0.
time (sec)	N/A	0.261	0.117	0.065	0.	1.573	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	52	55	0	213	0	0
normalized size	1	1.	0.55	0.58	0.	2.24	0.	0.
time (sec)	N/A	0.128	0.067	0.05	0.	1.518	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	49	52	0	204	0	0
normalized size	1	1.	0.51	0.54	0.	2.1	0.	0.
time (sec)	N/A	0.045	0.055	0.048	0.	1.624	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	52	55	0	216	0	0
normalized size	1	1.	0.52	0.55	0.	2.16	0.	0.
time (sec)	N/A	0.037	0.031	0.049	0.	1.566	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	76	179	0	327	0	0
normalized size	1	1.	0.66	1.56	0.	2.84	0.	0.
time (sec)	N/A	0.179	0.127	0.063	0.	1.618	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	92	199	0	375	0	1
normalized size	1	1.	0.63	1.36	0.	2.57	0.	0.01
time (sec)	N/A	0.305	0.192	0.069	0.	1.555	0.	1.165

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	107	222	0	424	0	1
normalized size	1	1.	0.58	1.21	0.	2.32	0.	0.01
time (sec)	N/A	0.38	0.191	0.066	0.	1.824	0.	1.164

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	109	297	0	435	0	0
normalized size	1	1.	0.53	1.46	0.	2.13	0.	0.
time (sec)	N/A	0.593	0.206	0.069	0.	1.895	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	98	273	0	414	0	0
normalized size	1	1.	0.61	1.71	0.	2.59	0.	0.
time (sec)	N/A	0.415	0.203	0.1	0.	1.751	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	85	212	0	377	0	0
normalized size	1	1.	0.57	1.43	0.	2.55	0.	0.
time (sec)	N/A	0.246	0.145	0.063	0.	1.78	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	73	214	0	332	0	0
normalized size	1	1.	0.63	1.86	0.	2.89	0.	0.
time (sec)	N/A	0.147	0.129	0.066	0.	1.779	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	50	42	0	205	0	0
normalized size	1	1.	0.78	0.66	0.	3.2	0.	0.
time (sec)	N/A	0.027	0.051	0.046	0.	1.63	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	43	0	213	0	0
normalized size	1	1.	0.76	0.64	0.	3.18	0.	0.
time (sec)	N/A	0.024	0.03	0.045	0.	1.691	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	76	196	0	323	0	0
normalized size	1	1.	0.69	1.78	0.	2.94	0.	0.
time (sec)	N/A	0.22	0.149	0.067	0.	1.758	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	92	361	0	386	0	1
normalized size	1	1.	0.64	2.52	0.	2.7	0.	0.01
time (sec)	N/A	0.306	0.226	0.068	0.	1.719	0.	1.169

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	107	389	0	428	0	1
normalized size	1	1.	0.58	2.13	0.	2.34	0.	0.01
time (sec)	N/A	0.392	0.242	0.073	0.	1.659	0.	1.199

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	118	412	0	448	0	1
normalized size	1	1.	0.56	1.96	0.	2.13	0.	0.
time (sec)	N/A	0.494	0.298	0.076	0.	1.926	0.	1.241

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	131	416	0	351	0	0
normalized size	1	1.	0.52	1.65	0.	1.39	0.	0.
time (sec)	N/A	0.663	0.264	0.073	0.	1.666	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	125	393	0	333	0	0
normalized size	1	1.	0.56	1.75	0.	1.49	0.	0.
time (sec)	N/A	0.534	0.189	0.071	0.	1.677	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	109	285	0	292	0	0
normalized size	1	1.	0.57	1.48	0.	1.52	0.	0.
time (sec)	N/A	0.435	0.158	0.066	0.	1.689	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	103	288	0	273	0	0
normalized size	1	1.	0.57	1.58	0.	1.5	0.	0.
time (sec)	N/A	0.219	0.135	0.065	0.	1.597	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	83	290	0	236	0	0
normalized size	1	1.	0.64	2.23	0.	1.82	0.	0.
time (sec)	N/A	0.064	0.112	0.06	0.	1.674	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	75	284	0	212	0	0
normalized size	1	1.	0.66	2.51	0.	1.88	0.	0.
time (sec)	N/A	0.048	0.076	0.056	0.	1.64	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	79	378	0	236	0	0
normalized size	1	1.	0.89	4.25	0.	2.65	0.	0.
time (sec)	N/A	0.211	0.17	0.075	0.	1.65	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	515	0	262	0	0
normalized size	1	1.	0.89	5.48	0.	2.79	0.	0.
time (sec)	N/A	0.216	0.213	0.072	0.	1.599	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	85	504	0	225	0	0
normalized size	1	1.	0.77	4.58	0.	2.05	0.	0.
time (sec)	N/A	0.215	0.247	0.084	0.	1.626	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	94	575	0	252	0	0
normalized size	1	1.	0.69	4.2	0.	1.84	0.	0.
time (sec)	N/A	0.297	0.275	0.087	0.	1.573	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	107	600	0	286	0	0
normalized size	1	1.	0.63	3.53	0.	1.68	0.	0.
time (sec)	N/A	0.394	0.305	0.093	0.	1.643	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	118	628	0	305	0	0
normalized size	1	1.	0.6	3.2	0.	1.56	0.	0.
time (sec)	N/A	0.517	0.372	0.112	0.	1.625	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	50	200	0	278	0	217
normalized size	1	1.	0.53	2.11	0.	2.93	0.	2.28
time (sec)	N/A	0.132	0.126	0.064	0.	1.6	0.	1.11

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	50	44	207	223	0	258
normalized size	1	1.	0.57	0.5	2.35	2.53	0.	2.93
time (sec)	N/A	0.123	0.077	0.047	1.124	1.604	0.	1.1

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	137	132	0	703	0	0
normalized size	1	1.	0.66	0.63	0.	3.36	0.	0.
time (sec)	N/A	0.315	0.198	0.055	0.	4.571	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	137	132	0	713	0	0
normalized size	1	1.	0.66	0.63	0.	3.41	0.	0.
time (sec)	N/A	0.207	0.112	0.056	0.	4.999	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	137	132	0	697	0	0
normalized size	1	1.	0.65	0.63	0.	3.3	0.	0.
time (sec)	N/A	0.105	0.094	0.055	0.	5.409	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	137	132	0	709	0	0
normalized size	1	1.	0.67	0.64	0.	3.46	0.	0.
time (sec)	N/A	0.091	0.073	0.052	0.	5.902	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	161	385	0	995	0	0
normalized size	1	1.	0.69	1.65	0.	4.25	0.	0.
time (sec)	N/A	0.385	0.201	0.075	0.	5.725	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	183	484	0	1116	0	0
normalized size	1	1.	0.68	1.79	0.	4.12	0.	0.
time (sec)	N/A	0.68	0.231	0.071	0.	8.266	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	93	95	0	473	0	149
normalized size	1	1.	0.91	0.93	0.	4.64	0.	1.46
time (sec)	N/A	0.112	0.101	0.198	0.	1.594	0.	1.149

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	67	58	0	252	0	69
normalized size	1	1.	1.72	1.49	0.	6.46	0.	1.77
time (sec)	N/A	0.04	0.034	0.141	0.	1.599	0.	1.134

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	62	0	248	83	0
normalized size	1	1.	1.	1.77	0.	7.09	2.37	0.
time (sec)	N/A	0.01	0.016	0.159	0.	1.578	1.96	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	76	0	486	0	0
normalized size	1	1.	1.	2.17	0.	13.89	0.	0.
time (sec)	N/A	0.029	0.01	0.156	0.	1.807	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	57	0	244	29	0
normalized size	1	1.	1.	1.68	0.	7.18	0.85	0.
time (sec)	N/A	0.009	0.013	0.167	0.	1.593	1.927	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	86	0	494	0	0
normalized size	1	1.	1.	2.53	0.	14.53	0.	0.
time (sec)	N/A	0.028	0.013	0.139	0.	1.795	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	79	0	292	148	0
normalized size	1	1.	0.78	1.25	0.	4.63	2.35	0.
time (sec)	N/A	0.015	0.021	0.135	0.	1.579	3.547	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	92	0	525	0	0
normalized size	1	1.	0.78	1.46	0.	8.33	0.	0.
time (sec)	N/A	0.033	0.008	0.135	0.	1.756	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	199	0	0	0	513	0
normalized size	1	1.	0.8	0.	0.	0.	2.05	0.
time (sec)	N/A	0.386	0.214	0.572	0.	0.	134.441	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	174	0	0	0	442	0
normalized size	1	1.	0.84	0.	0.	0.	2.15	0.
time (sec)	N/A	0.209	0.124	0.527	0.	0.	101.046	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	121	0	0	0	374	0
normalized size	1	1.	0.75	0.	0.	0.	2.31	0.
time (sec)	N/A	0.084	0.058	0.364	0.	0.	62.439	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	61	0
normalized size	1	1.	0.98	0.	0.	0.	0.76	0.
time (sec)	N/A	0.025	0.019	0.365	0.	0.	47.742	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.059	0.574	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	173	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	0.122	0.606	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	245	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.381	0.204	0.548	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	199	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	0.204	0.522	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	174	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.122	0.507	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	162	121	0	0	0	0	0
normalized size	1	1.31	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.057	0.352	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	60	0
normalized size	1	1.	0.98	0.	0.	0.	0.75	0.
time (sec)	N/A	0.025	0.019	0.365	0.	0.	63.938	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.06	0.608	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	176	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	0.134	0.59	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	200	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	0.224	0.645	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	967	0
normalized size	1	1.	0.89	0.	0.	0.	6.53	0.
time (sec)	N/A	0.094	0.099	0.472	0.	0.	8.67	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	129	0	0	0	967	0
normalized size	1	1.	0.88	0.	0.	0.	6.58	0.
time (sec)	N/A	0.091	0.091	0.458	0.	0.	7.93	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	106	0	0	0	382	0
normalized size	1	1.	0.88	0.	0.	0.	3.18	0.
time (sec)	N/A	0.071	0.085	0.442	0.	0.	5.721	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	103	0	0	0	382	0
normalized size	1	1.	0.87	0.	0.	0.	3.21	0.
time (sec)	N/A	0.067	0.081	0.454	0.	0.	5.019	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	85	0
normalized size	1	1.	1.	0.	0.	0.	0.96	0.
time (sec)	N/A	0.034	0.047	0.436	0.	0.	3.842	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	82	0
normalized size	1	1.	1.	0.	0.	0.	0.99	0.
time (sec)	N/A	0.023	0.052	0.421	0.	0.	3.625	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	78	0
normalized size	1	1.	1.	0.	0.	0.	0.75	0.
time (sec)	N/A	0.055	0.04	0.45	0.	0.	7.75	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	0	0	0	82	0
normalized size	1	1.	1.	0.	0.	0.	0.76	0.
time (sec)	N/A	0.057	0.055	0.461	0.	0.	4.991	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	106	0	0	0	85	0
normalized size	1	1.	0.96	0.	0.	0.	0.77	0.
time (sec)	N/A	0.059	0.056	0.451	0.	0.	5.136	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	159	0	0	0	2916	0
normalized size	1	1.	0.89	0.	0.	0.	16.38	0.
time (sec)	N/A	0.145	0.141	0.67	0.	0.	15.009	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	186	0	0	0	1010	0
normalized size	1	1.	1.01	0.	0.	0.	5.46	0.
time (sec)	N/A	0.173	0.139	0.782	0.	0.	9.589	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	138	0	0	0	1323	0
normalized size	1	1.	0.93	0.	0.	0.	8.88	0.
time (sec)	N/A	0.13	0.136	0.688	0.	0.	9.376	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	168	0	0	0	425	0
normalized size	1	1.	1.08	0.	0.	0.	2.74	0.
time (sec)	N/A	0.139	0.117	0.653	0.	0.	6.485	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	110	0	0	0	440	0
normalized size	1	1.	0.93	0.	0.	0.	3.73	0.
time (sec)	N/A	0.091	0.081	0.666	0.	0.	5.594	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	134	0	0	0	124	0
normalized size	1	1.	1.89	0.	0.	0.	1.75	0.
time (sec)	N/A	0.031	0.061	0.642	0.	0.	3.826	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	103	0	0	0	136	0
normalized size	1	1.	0.8	0.	0.	0.	1.06	0.
time (sec)	N/A	0.095	0.071	0.592	0.	0.	7.831	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	153	0	0	0	116	0
normalized size	1	1.	1.2	0.	0.	0.	0.91	0.
time (sec)	N/A	0.116	0.077	0.587	0.	0.	6.277	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	131	0	0	0	139	0
normalized size	1	1.	0.94	0.	0.	0.	1.	0.
time (sec)	N/A	0.125	0.088	0.632	0.	0.	7.08	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	205	0	0	0	2958	0
normalized size	1	1.	0.92	0.	0.	0.	13.32	0.
time (sec)	N/A	0.191	0.268	0.587	0.	0.	19.131	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	219	0	0	0	2958	0
normalized size	1	1.	1.	0.	0.	0.	13.57	0.
time (sec)	N/A	0.177	0.269	0.586	0.	0.	17.992	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	187	0	0	0	1365	0
normalized size	1	1.	0.97	0.	0.	0.	7.07	0.
time (sec)	N/A	0.182	0.196	0.588	0.	0.	12.729	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	187	0	0	0	1365	0
normalized size	1	1.	0.99	0.	0.	0.	7.22	0.
time (sec)	N/A	0.169	0.206	0.583	0.	0.	10.482	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	159	0	0	0	479	0
normalized size	1	1.	1.37	0.	0.	0.	4.13	0.
time (sec)	N/A	0.066	0.29	0.593	0.	0.	7.562	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	155	0	0	0	476	0
normalized size	1	1.	2.12	0.	0.	0.	6.52	0.
time (sec)	N/A	0.026	0.176	0.618	0.	0.	7.051	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	169	0	0	0	178	0
normalized size	1	1.	0.99	0.	0.	0.	1.04	0.
time (sec)	N/A	0.129	0.152	0.59	0.	0.	11.851	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	158	0	0	0	177	0
normalized size	1	1.	0.99	0.	0.	0.	1.11	0.
time (sec)	N/A	0.185	0.097	0.61	0.	0.	7.366	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	182	0	0	0	177	0
normalized size	1	1.	1.1	0.	0.	0.	1.07	0.
time (sec)	N/A	0.215	0.114	0.614	0.	0.	8.37	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	66	0	0	0	4446	0
normalized size	1	1.	0.45	0.	0.	0.	30.04	0.
time (sec)	N/A	0.109	0.114	0.656	0.	0.	22.999	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	245	0	0	0	5100	0
normalized size	1	1.	2.02	0.	0.	0.	42.15	0.
time (sec)	N/A	0.093	0.317	0.648	0.	0.	15.952	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	198	0	0	0	4134	0
normalized size	1	1.	1.66	0.	0.	0.	34.74	0.
time (sec)	N/A	0.085	0.261	0.67	0.	0.	10.622	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	147	0	0	0	430	0
normalized size	1	1.	1.63	0.	0.	0.	4.78	0.
time (sec)	N/A	0.058	0.112	0.662	0.	0.	6.053	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	323	0
normalized size	1	1.	1.03	0.	0.	0.	4.42	0.
time (sec)	N/A	0.033	0.039	0.702	0.	0.	6.8	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	151	0	0	0	359	0
normalized size	1	1.	1.45	0.	0.	0.	3.45	0.
time (sec)	N/A	0.068	0.118	0.671	0.	0.	7.512	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	167	0	0	0	452	0
normalized size	1	1.	1.58	0.	0.	0.	4.26	0.
time (sec)	N/A	0.074	0.191	0.671	0.	0.	8.348	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	219	0	0	0	500	0
normalized size	1	1.	2.03	0.	0.	0.	4.63	0.
time (sec)	N/A	0.08	0.615	0.678	0.	0.	9.288	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	66	0	0	0	0	0
normalized size	1	1.	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	0.168	0.705	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	66	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.205	0.125	0.694	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	332	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	0.314	0.705	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	177	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.176	0.662	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	102	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.096	0.683	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.045	0.66	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	201	0	0	0	0	0
normalized size	1	1.	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.168	0.645	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	82	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	0.113	0.656	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	283	0	0	0	0	0
normalized size	1	1.	1.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	0.539	0.668	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	334	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	0.451	0.683	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	389	0	0	0	0	0
normalized size	1	1.	2.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	0.551	0.695	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	66	0	0	0	0	0
normalized size	1	1.	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	0.146	0.698	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	66	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	0.122	0.671	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	130	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.133	0.668	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.088	0.711	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.045	0.665	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	82	0	0	0	0	0
normalized size	1	1.	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.097	0.65	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	83	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.136	0.661	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	341	0	0	0	0	0
normalized size	1	1.	1.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	0.681	0.68	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	393	0	0	0	0	0
normalized size	1	1.	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.275	0.545	0.711	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	446	0	0	0	0	0
normalized size	1	1.	2.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.277	0.624	0.693	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	66	0	0	0	0	0
normalized size	1	1.	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.306	0.157	0.722	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	66	0	0	0	0	0
normalized size	1	1.	0.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.375	0.121	0.732	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	130	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	0.135	0.742	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.09	0.706	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.045	0.688	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	83	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.115	0.674	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	82	0	0	0	0	0
normalized size	1	1.	0.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.191	0.679	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	399	0	0	0	0	0
normalized size	1	1.	1.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.368	0.856	0.704	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	452	0	0	0	0	0
normalized size	1	1.	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.391	0.674	0.677	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	505	0	0	0	0	0
normalized size	1	1.	2.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.415	0.834	0.681	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	194	0	0	0	262	0
normalized size	1	1.	0.73	0.	0.	0.	0.99	0.
time (sec)	N/A	0.373	0.175	0.682	0.	0.	61.798	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	169	0	0	0	192	0
normalized size	1	1.	0.82	0.	0.	0.	0.93	0.
time (sec)	N/A	0.168	0.094	0.639	0.	0.	33.385	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	116	0	0	0	122	0
normalized size	1	1.	0.76	0.	0.	0.	0.8	0.
time (sec)	N/A	0.068	0.036	0.465	0.	0.	19.365	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	61	0
normalized size	1	1.	0.97	0.	0.	0.	0.81	0.
time (sec)	N/A	0.02	0.01	0.466	0.	0.	5.765	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	124	0	0	0	337	0
normalized size	1	1.	0.76	0.	0.	0.	2.07	0.
time (sec)	N/A	0.136	0.054	0.674	0.	0.	11.852	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	180	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.223	0.111	0.694	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	206	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.442	0.203	0.714	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	308	0
normalized size	1	1.	0.87	0.	0.	0.	3.46	0.
time (sec)	N/A	0.056	0.046	0.674	0.	0.	10.214	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	90	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.111	0.729	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	60	240	0	1048	68	0
normalized size	1	1.	0.28	1.12	0.	4.9	0.32	0.
time (sec)	N/A	0.247	0.043	0.077	0.	1.675	8.442	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	259	560	0	2407	0	340
normalized size	1	1.	1.02	2.2	0.	9.44	0.	1.33
time (sec)	N/A	0.629	0.668	0.278	0.	40.748	0.	1.197

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	225	515	0	2076	0	271
normalized size	1	1.	1.07	2.44	0.	9.84	0.	1.28
time (sec)	N/A	0.39	0.465	0.237	0.	37.417	0.	1.255

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	193	448	0	1710	0	212
normalized size	1	1.	1.26	2.93	0.	11.18	0.	1.39
time (sec)	N/A	0.211	0.348	0.237	0.	3.848	0.	1.195

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	175	423	0	1507	0	182
normalized size	1	1.	1.38	3.33	0.	11.87	0.	1.43
time (sec)	N/A	0.105	0.303	0.24	0.	3.469	0.	1.188

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	99	381	0	1274	0	147
normalized size	1	1.	0.96	3.7	0.	12.37	0.	1.43
time (sec)	N/A	0.071	0.03	0.233	0.	2.31	0.	1.143

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	113	420	0	2957	0	0
normalized size	1	1.	0.97	3.62	0.	25.49	0.	0.
time (sec)	N/A	0.1	0.056	0.243	0.	6.195	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	178	486	0	1326	0	196
normalized size	1	1.	1.7	4.63	0.	12.63	0.	1.87
time (sec)	N/A	0.167	0.249	0.24	0.	2.137	0.	1.197

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	283	567	0	1577	0	311
normalized size	1	1.	1.77	3.54	0.	9.86	0.	1.94
time (sec)	N/A	0.211	0.425	0.239	0.	2.253	0.	1.189

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	301	600	0	1791	0	417
normalized size	1	1.	1.58	3.14	0.	9.38	0.	2.18
time (sec)	N/A	0.232	1.173	0.272	0.	2.225	0.	1.212

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	344	703	0	2152	0	805
normalized size	1	1.	1.26	2.57	0.	7.85	0.	2.94
time (sec)	N/A	0.297	1.162	0.24	0.	2.709	0.	1.236

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	149	260	0	2218	0	220
normalized size	1	1.	0.76	1.33	0.	11.37	0.	1.13
time (sec)	N/A	0.482	0.266	0.237	0.	16.292	0.	1.238

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	131	217	0	1917	0	174
normalized size	1	1.	0.86	1.43	0.	12.61	0.	1.14
time (sec)	N/A	0.274	0.237	0.261	0.	16.405	0.	1.227

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	105	172	0	1590	0	142
normalized size	1	1.	0.96	1.58	0.	14.59	0.	1.3
time (sec)	N/A	0.128	0.085	0.264	0.	2.921	0.	1.159

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	151	0	1353	0	119
normalized size	1	1.	1.	1.76	0.	15.73	0.	1.38
time (sec)	N/A	0.044	0.027	0.232	0.	2.682	0.	1.228

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	127	0	435	0	80
normalized size	1	1.	1.	2.35	0.	8.06	0.	1.48
time (sec)	N/A	0.017	0.007	0.223	0.	2.092	0.	1.198

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	158	0	1358	0	0
normalized size	1	1.	1.	1.84	0.	15.79	0.	0.
time (sec)	N/A	0.083	0.048	0.24	0.	2.243	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	107	180	0	1639	0	192
normalized size	1	1.	0.96	1.62	0.	14.77	0.	1.73
time (sec)	N/A	0.095	0.09	0.236	0.	2.421	0.	1.141

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	163	236	0	1987	0	323
normalized size	1	1.	0.97	1.4	0.	11.83	0.	1.92
time (sec)	N/A	0.14	0.717	0.246	0.	2.802	0.	1.173

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	179	396	0	3092	0	404
normalized size	1	1.	1.23	2.71	0.	21.18	0.	2.77
time (sec)	N/A	0.312	0.515	0.239	0.	31.802	0.	1.227

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	153	354	0	2691	0	296
normalized size	1	1.	1.24	2.88	0.	21.88	0.	2.41
time (sec)	N/A	0.165	0.317	0.242	0.	32.701	0.	1.197

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	311	0	913	0	235
normalized size	1	1.	1.	3.27	0.	9.61	0.	2.47
time (sec)	N/A	0.111	0.097	0.234	0.	2.47	0.	1.143

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	283	0	863	0	219
normalized size	1	1.	1.	3.22	0.	9.81	0.	2.49
time (sec)	N/A	0.052	0.058	0.23	0.	2.593	0.	1.142

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	260	0	913	0	232
normalized size	1	1.	1.	2.77	0.	9.71	0.	2.47
time (sec)	N/A	0.046	0.05	0.225	0.	2.67	0.	1.173

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	132	318	0	2695	0	0
normalized size	1	1.	0.9	2.16	0.	18.33	0.	0.
time (sec)	N/A	0.135	0.191	0.236	0.	4.001	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	163	363	0	3152	0	359
normalized size	1	1.	0.84	1.87	0.	16.25	0.	1.85
time (sec)	N/A	0.167	0.447	0.24	0.	4.434	0.	1.223

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	275	203	439	0	3868	0	483
normalized size	1	1.	0.74	1.59	0.	14.01	0.	1.75
time (sec)	N/A	0.236	0.382	0.247	0.	6.968	0.	1.254

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	230	474	0	0	0	0
normalized size	1	1.	0.94	1.94	0.	0.	0.	0.
time (sec)	N/A	0.89	0.572	0.253	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	208	435	0	0	0	0
normalized size	1	1.	1.02	2.13	0.	0.	0.	0.
time (sec)	N/A	0.523	0.43	0.255	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	184	386	0	2989	0	0
normalized size	1	1.	1.15	2.41	0.	18.68	0.	0.
time (sec)	N/A	0.327	0.317	0.247	0.	51.124	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	172	368	0	2615	0	0
normalized size	1	1.	1.26	2.69	0.	19.09	0.	0.
time (sec)	N/A	0.168	0.372	0.245	0.	46.145	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	340	0	783	0	0
normalized size	1	1.	1.	3.78	0.	8.7	0.	0.
time (sec)	N/A	0.038	0.057	0.247	0.	2.594	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	115	210	0	783	0	439
normalized size	1	1.	1.26	2.31	0.	8.6	0.	4.82
time (sec)	N/A	0.034	0.082	0.303	0.	2.565	0.	4.841

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	178	364	0	2619	0	0
normalized size	1	1.	0.99	2.03	0.	14.63	0.	0.
time (sec)	N/A	0.137	0.252	0.256	0.	4.891	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	197	395	0	3093	0	0
normalized size	1	1.	0.93	1.86	0.	14.59	0.	0.
time (sec)	N/A	0.168	0.39	0.238	0.	4.687	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	229	452	0	3771	0	0
normalized size	1	1.	0.85	1.69	0.	14.07	0.	0.
time (sec)	N/A	0.222	0.531	0.249	0.	9.008	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	114	328	284	786	4078	842
normalized size	1	1.	0.84	2.43	2.1	5.82	30.21	6.24
time (sec)	N/A	0.082	0.101	0.049	1.025	1.933	10.467	1.232

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	109	195	197	516	2179	554
normalized size	1	1.	1.07	1.91	1.93	5.06	21.36	5.43
time (sec)	N/A	0.052	0.119	0.046	1.028	1.889	4.734	1.13

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	65	100	0	308	978	320
normalized size	1	1.	0.93	1.43	0.	4.4	13.97	4.57
time (sec)	N/A	0.031	0.047	0.049	0.	1.902	2.313	1.205

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	64	0	0	0	347	0
normalized size	1	1.	0.83	0.	0.	0.	4.51	0.
time (sec)	N/A	0.051	0.045	0.365	0.	0.	6.484	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	199	1000	603	2246	0	2363
normalized size	1	1.	0.86	4.31	2.6	9.68	0.	10.19
time (sec)	N/A	0.139	0.198	0.054	1.059	1.994	0.	1.158

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	323	677	452	1612	8803	1709
normalized size	1	1.	1.75	3.66	2.44	8.71	47.58	9.24
time (sec)	N/A	0.1	0.556	0.052	1.042	2.316	17.464	1.137

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	160	420	0	1092	5049	1149
normalized size	1	1.	1.14	3.	0.	7.8	36.06	8.21
time (sec)	N/A	0.068	0.194	0.054	0.	2.152	33.486	1.137

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	1678	0
normalized size	1	1.	0.89	0.	0.	0.	11.34	0.
time (sec)	N/A	0.21	0.142	0.53	0.	0.	11.224	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	302	2232	1073	4895	0	5013
normalized size	1	1.	0.88	6.51	3.13	14.27	0.	14.62
time (sec)	N/A	0.211	0.272	0.065	1.082	2.227	0.	1.256

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	709	1639	844	3702	0	3849
normalized size	1	1.	2.51	5.81	2.99	13.13	0.	13.65
time (sec)	N/A	0.168	1.652	0.059	1.084	2.245	0.	1.266

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	347	1140	0	2692	0	2815
normalized size	1	1.	1.56	5.11	0.	12.07	0.	12.62
time (sec)	N/A	0.126	0.529	0.058	0.	2.159	0.	1.211

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	226	0	0	0	5690	0
normalized size	1	1.	0.92	0.	0.	0.	23.13	0.
time (sec)	N/A	0.345	0.213	0.526	0.	0.	20.609	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	217	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.403	0.52	0.692	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	168	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	0.231	0.724	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	170	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.146	0.737	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	151	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.067	0.768	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	145	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.074	0.739	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	189	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.131	0.742	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	167	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.221	0.29	0.752	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	413	0	0	0	0	0
normalized size	1	1.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.451	0.788	0.716	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	247	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.41	0.845	0.769	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	403	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.53	0.648	0.764	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	230	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.297	0.37	0.727	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	253	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.417	0.323	0.738	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	489	489	391	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.605	0.934	0.73	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	437	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.698	0.76	0.753	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	377	275	0	0	0	131	0
normalized size	1	0.94	0.69	0.	0.	0.	0.33	0.
time (sec)	N/A	0.765	0.147	0.628	0.	0.	116.433	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	150	113	0	0	0	82	0
normalized size	1	0.91	0.69	0.	0.	0.	0.5	0.
time (sec)	N/A	0.133	0.069	0.428	0.	0.	18.911	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	0.116	0.752	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.418	0.223	0.73	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	112	0	0	0	1010	0
normalized size	1	1.	0.9	0.	0.	0.	8.08	0.
time (sec)	N/A	0.084	0.088	0.439	0.	0.	59.926	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	112	0	0	0	1010	0
normalized size	1	1.	0.9	0.	0.	0.	8.08	0.
time (sec)	N/A	0.085	0.071	0.418	0.	0.	37.912	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	0	394	0
normalized size	1	1.	0.87	0.	0.	0.	3.94	0.
time (sec)	N/A	0.065	0.075	0.411	0.	0.	28.613	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	0	394	0
normalized size	1	1.	0.87	0.	0.	0.	3.94	0.
time (sec)	N/A	0.059	0.071	0.414	0.	0.	18.649	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	65	0
normalized size	1	1.	0.95	0.	0.	0.	0.87	0.
time (sec)	N/A	0.033	0.034	0.405	0.	0.	14.341	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	98	0	0	0	61	0
normalized size	1	1.	1.4	0.	0.	0.	0.87	0.
time (sec)	N/A	0.02	0.074	0.406	0.	0.	9.821	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	65	0
normalized size	1	1.	1.	0.	0.	0.	0.74	0.
time (sec)	N/A	0.053	0.055	0.399	0.	0.	15.988	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	68	0
normalized size	1	1.	1.	0.	0.	0.	0.75	0.
time (sec)	N/A	0.053	0.051	0.42	0.	0.	19.373	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	71	0
normalized size	1	1.	0.97	0.	0.	0.	0.77	0.
time (sec)	N/A	0.053	0.047	0.428	0.	0.	27.983	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	205	0	0	0	3043	0
normalized size	1	1.	1.09	0.	0.	0.	16.19	0.
time (sec)	N/A	0.18	0.215	0.614	0.	0.	82.476	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	169	156	0	0	0	1046	0
normalized size	1	0.95	0.88	0.	0.	0.	5.91	0.
time (sec)	N/A	0.163	0.17	0.598	0.	0.	84.849	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	152	0	0	0	1384	0
normalized size	1	1.	1.02	0.	0.	0.	9.29	0.
time (sec)	N/A	0.138	0.151	0.62	0.	0.	44.162	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	144	139	0	0	0	430	0
normalized size	1	0.95	0.91	0.	0.	0.	2.83	0.
time (sec)	N/A	0.137	0.159	0.519	0.	0.	39.55	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	184	0	0	0	439	0
normalized size	1	1.	1.63	0.	0.	0.	3.88	0.
time (sec)	N/A	0.097	0.192	0.498	0.	0.	21.208	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	125	133	0	0	0	97	0
normalized size	1	0.94	1.	0.	0.	0.	0.73	0.
time (sec)	N/A	0.077	0.111	0.502	0.	0.	18.859	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	101	0	0	0	109	0
normalized size	1	1.	0.86	0.	0.	0.	0.92	0.
time (sec)	N/A	0.093	0.101	0.492	0.	0.	19.203	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	134	0	0	0	95	0
normalized size	1	1.	1.06	0.	0.	0.	0.75	0.
time (sec)	N/A	0.123	0.08	0.516	0.	0.	25.362	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	119	0	0	0	119	0
normalized size	1	1.	0.94	0.	0.	0.	0.94	0.
time (sec)	N/A	0.125	0.096	0.509	0.	0.	32.826	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	241	249	0	0	0	0	0
normalized size	1	0.98	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	0.233	0.539	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	241	249	0	0	0	3079	0
normalized size	1	0.97	1.	0.	0.	0.	12.37	0.
time (sec)	N/A	0.235	0.216	0.516	0.	0.	87.261	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	201	196	0	0	0	1420	0
normalized size	1	0.97	0.95	0.	0.	0.	6.86	0.
time (sec)	N/A	0.201	0.162	0.579	0.	0.	76.063	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	202	196	0	0	0	1420	0
normalized size	1	0.96	0.93	0.	0.	0.	6.76	0.
time (sec)	N/A	0.204	0.153	0.555	0.	0.	44.436	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	159	228	0	0	0	471	0
normalized size	1	0.95	1.37	0.	0.	0.	2.82	0.
time (sec)	N/A	0.15	0.244	0.536	0.	0.	33.897	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	169	223	0	0	0	468	0
normalized size	1	0.96	1.27	0.	0.	0.	2.66	0.
time (sec)	N/A	0.149	0.219	0.535	0.	0.	21.155	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	165	170	0	0	0	144	0
normalized size	1	0.96	0.99	0.	0.	0.	0.84	0.
time (sec)	N/A	0.133	0.131	0.513	0.	0.	28.281	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	154	0	0	0	143	0
normalized size	1	1.	0.97	0.	0.	0.	0.9	0.
time (sec)	N/A	0.189	0.124	0.515	0.	0.	29.659	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	174	0	0	0	150	0
normalized size	1	1.	1.04	0.	0.	0.	0.89	0.
time (sec)	N/A	0.221	0.13	0.526	0.	0.	36.813	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.674	0.658	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	260	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.355	0.648	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	227	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.254	0.652	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	172	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.212	0.635	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	131	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.053	0.63	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	170	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.203	0.645	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	214	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.379	0.652	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	256	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	0.298	0.746	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	392	392	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.888	0.726	0.664	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	321	343	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.545	0.735	0.65	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	277	300	0	0	0	0	0
normalized size	1	0.99	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	0.332	0.648	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	223	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	0.173	0.652	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	244	191	141	0	0	0	0	0
normalized size	1	0.78	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	0.087	0.658	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	368	368	303	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.427	0.372	0.646	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	421	421	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.446	0.13	0.64	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	449	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.938	0.901	0.688	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	416	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.592	0.634	0.695	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	290	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.57	0.26	0.69	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	229	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.4	0.224	0.687	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	142	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.329	0.12	0.679	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	700	700	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.818	0.12	0.645	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	754	754	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.86	0.175	0.793	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	254	182	0	0	0	0	0
normalized size	1	0.92	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.468	0.203	0.557	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	194	158	0	0	0	0	0
normalized size	1	0.95	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	0.117	0.548	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	106	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.037	0.381	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0
normalized size	1	1.	0.97	0.	0.	0.	0.82	0.
time (sec)	N/A	0.019	0.009	0.397	0.	0.	34.847	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.097	0.677	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	238	238	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.279	0.094	0.638	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	321	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.374	0.202	0.699	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	304	946	0	1438	0	0
normalized size	1	1.	0.88	2.74	0.	4.17	0.	0.
time (sec)	N/A	0.506	1.77	0.07	0.	2.606	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	245	713	0	1126	0	0
normalized size	1	1.	0.98	2.84	0.	4.49	0.	0.
time (sec)	N/A	0.26	0.896	0.062	0.	2.054	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	197	516	0	902	0	0
normalized size	1	1.	0.95	2.49	0.	4.36	0.	0.
time (sec)	N/A	0.192	0.7	0.056	0.	2.088	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	155	205	0	725	0	0
normalized size	1	1.	1.18	1.56	0.	5.53	0.	0.
time (sec)	N/A	0.07	0.809	0.057	0.	1.947	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	210	439	0	2012	0	0
normalized size	1	1.	1.25	2.61	0.	11.98	0.	0.
time (sec)	N/A	0.167	0.188	0.059	0.	3.205	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	594	0	755	0	0
normalized size	1	1.	0.85	4.34	0.	5.51	0.	0.
time (sec)	N/A	0.142	0.144	0.061	0.	2.567	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	162	882	0	945	0	0
normalized size	1	1.	0.8	4.37	0.	4.68	0.	0.
time (sec)	N/A	0.276	0.173	0.063	0.	5.466	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	210	1165	0	1169	0	0
normalized size	1	1.	0.73	4.07	0.	4.09	0.	0.
time (sec)	N/A	0.404	0.257	0.069	0.	15.702	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	273	1494	0	0	0	0
normalized size	1	1.	0.7	3.84	0.	0.	0.	0.
time (sec)	N/A	0.594	0.378	0.073	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	425	1883	0	2233	0	0
normalized size	1	1.	0.95	4.19	0.	4.97	0.	0.
time (sec)	N/A	0.57	2.605	0.065	0.	2.331	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	497	1560	0	1797	0	0
normalized size	1	1.	1.41	4.43	0.	5.11	0.	0.
time (sec)	N/A	0.328	3.001	0.061	0.	1.924	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	276	1279	0	1426	0	0
normalized size	1	1.	0.94	4.34	0.	4.83	0.	0.
time (sec)	N/A	0.282	1.3	0.064	0.	1.86	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	264	566	0	1125	0	0
normalized size	1	1.	1.31	2.82	0.	5.6	0.	0.
time (sec)	N/A	0.117	0.708	0.054	0.	1.865	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	275	1130	0	2855	0	0
normalized size	1	1.	1.1	4.5	0.	11.37	0.	0.
time (sec)	N/A	0.276	0.89	0.065	0.	26.25	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	263	1310	0	2589	0	0
normalized size	1	1.	1.1	5.46	0.	10.79	0.	0.
time (sec)	N/A	0.273	1.376	0.068	0.	10.475	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	285	1604	0	2934	0	0
normalized size	1	1.	1.11	6.27	0.	11.46	0.	0.
time (sec)	N/A	0.283	2.555	0.073	0.	13.764	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	188	1945	0	1168	0	0
normalized size	1	1.	0.89	9.22	0.	5.54	0.	0.
time (sec)	N/A	0.236	0.283	0.077	0.	10.369	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	253	2427	0	0	0	0
normalized size	1	1.	0.86	8.23	0.	0.	0.	0.
time (sec)	N/A	0.386	0.314	0.09	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	310	2888	0	0	0	0
normalized size	1	1.	0.78	7.31	0.	0.	0.	0.
time (sec)	N/A	0.514	0.524	0.102	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	380	3387	0	0	0	0
normalized size	1	1.	0.76	6.8	0.	0.	0.	0.
time (sec)	N/A	0.725	0.885	0.108	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	574	681	3178	0	3413	0	0
normalized size	1	1.	1.19	5.54	0.	5.95	0.	0.
time (sec)	N/A	0.695	4.297	0.074	0.	3.529	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	1221	2731	0	2803	0	0
normalized size	1	1.	2.7	6.04	0.	6.2	0.	0.
time (sec)	N/A	0.41	6.142	0.072	0.	3.173	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	506	2411	0	2264	0	0
normalized size	1	1.	1.33	6.33	0.	5.94	0.	0.
time (sec)	N/A	0.386	3.614	0.06	0.	2.962	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	384	1123	0	1789	0	0
normalized size	1	1.	1.4	4.1	0.	6.53	0.	0.
time (sec)	N/A	0.185	1.259	0.056	0.	2.472	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	390	2180	0	0	0	0
normalized size	1	1.	0.99	5.53	0.	0.	0.	0.
time (sec)	N/A	0.45	2.04	0.063	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	350	2364	0	3652	0	0
normalized size	1	1.	0.99	6.72	0.	10.38	0.	0.
time (sec)	N/A	0.432	2.214	0.066	0.	104.892	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	334	2688	0	3301	0	0
normalized size	1	1.	0.99	7.93	0.	9.74	0.	0.
time (sec)	N/A	0.388	2.372	0.075	0.	46.721	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	357	3144	0	3684	0	0
normalized size	1	1.	0.96	8.47	0.	9.93	0.	0.
time (sec)	N/A	0.47	3.326	0.082	0.	59.465	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	404	3646	0	0	0	0
normalized size	1	1.	1.	9.02	0.	0.	0.	0.
time (sec)	N/A	0.456	3.728	0.087	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	295	3991	0	0	0	0
normalized size	1	1.	1.02	13.81	0.	0.	0.	0.
time (sec)	N/A	0.328	0.961	0.102	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	344	4735	0	0	0	0
normalized size	1	1.	0.89	12.27	0.	0.	0.	0.
time (sec)	N/A	0.495	1.027	0.121	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	500	500	408	5353	0	0	0	0
normalized size	1	1.	0.82	10.71	0.	0.	0.	0.
time (sec)	N/A	0.639	0.781	0.138	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	512	6030	0	0	0	0
normalized size	1	1.	0.82	9.6	0.	0.	0.	0.
time (sec)	N/A	0.885	1.5	0.201	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	298	331	391	0	1530	0	0
normalized size	1	1.1	1.22	1.44	0.	5.65	0.	0.
time (sec)	N/A	0.341	0.537	0.06	0.	7.756	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	255	241	0	1195	0	0
normalized size	1	1.	1.31	1.24	0.	6.13	0.	0.
time (sec)	N/A	0.346	0.378	0.059	0.	3.642	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	189	131	0	922	0	0
normalized size	1	1.	1.36	0.94	0.	6.63	0.	0.
time (sec)	N/A	0.109	0.419	0.055	0.	3.515	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	51	0	117	0	0
normalized size	1	1.	0.81	0.98	0.	2.25	0.	0.
time (sec)	N/A	0.024	0.016	0.048	0.	2.482	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	131	136	0	938	0	0
normalized size	1	1.	0.92	0.95	0.	6.56	0.	0.
time (sec)	N/A	0.175	0.139	0.061	0.	4.64	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	201	253	0	1245	0	0
normalized size	1	1.	0.88	1.1	0.	5.44	0.	0.
time (sec)	N/A	0.292	0.129	0.064	0.	11.05	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	283	414	0	1593	0	0
normalized size	1	1.	0.86	1.26	0.	4.84	0.	0.
time (sec)	N/A	0.508	0.205	0.066	0.	27.726	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	515	515	2132	1680	0	4361	0	0
normalized size	1	1.	4.14	3.26	0.	8.47	0.	0.
time (sec)	N/A	0.619	8.664	0.069	0.	50.962	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	387	1266	0	3615	0	0
normalized size	1	1.	0.88	2.89	0.	8.25	0.	0.
time (sec)	N/A	0.541	1.43	0.063	0.	21.341	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	1443	977	0	2908	0	0
normalized size	1	1.	4.86	3.29	0.	9.79	0.	0.
time (sec)	N/A	0.293	4.994	0.06	0.	23.41	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	99	145	0	602	0	0
normalized size	1	1.	0.79	1.15	0.	4.78	0.	0.
time (sec)	N/A	0.108	0.077	0.072	0.	19.284	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	100	149	0	621	0	0
normalized size	1	1.	0.72	1.08	0.	4.5	0.	0.
time (sec)	N/A	0.094	0.037	0.052	0.	20.406	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	95	138	0	599	0	0
normalized size	1	1.	0.79	1.14	0.	4.95	0.	0.
time (sec)	N/A	0.042	0.034	0.053	0.	14.16	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	262	682	0	2923	0	0
normalized size	1	1.	0.97	2.52	0.	10.79	0.	0.
time (sec)	N/A	0.338	0.444	0.059	0.	39.112	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	370	912	0	3671	0	0
normalized size	1	1.	0.94	2.31	0.	9.32	0.	0.
time (sec)	N/A	0.59	0.589	0.065	0.	128.576	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	467	1319	0	0	0	0
normalized size	1	1.	0.89	2.53	0.	0.	0.	0.
time (sec)	N/A	0.8	0.972	0.067	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	664	664	593	1705	0	0	0	0
normalized size	1	1.	0.89	2.57	0.	0.	0.	0.
time (sec)	N/A	1.17	1.44	0.072	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	235	366	0	0	0	0
normalized size	1	1.	0.91	1.41	0.	0.	0.	0.
time (sec)	N/A	0.236	0.142	0.054	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	433	663	0	0	0	0
normalized size	1	1.	1.27	1.94	0.	0.	0.	0.
time (sec)	N/A	0.289	0.213	0.06	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	221	257	0	0	0	0
normalized size	1	1.	1.3	1.51	0.	0.	0.	0.
time (sec)	N/A	0.066	0.927	0.589	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	30	61	0	36
normalized size	1	1.	1.	0.78	1.3	2.65	0.	1.57
time (sec)	N/A	0.018	0.034	0.043	1.472	2.192	0.	1.14

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	347	361	0	0	0	0
normalized size	1	1.	1.18	1.23	0.	0.	0.	0.
time (sec)	N/A	0.095	0.515	0.569	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	169	252	0	0	0	0
normalized size	1	1.	1.17	1.75	0.	0.	0.	0.
time (sec)	N/A	0.032	0.506	0.565	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	197	43	0	169	0	0
normalized size	1	1.	2.98	0.65	0.	2.56	0.	0.
time (sec)	N/A	0.033	0.411	0.091	0.	1.706	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	349	363	0	0	0	0
normalized size	1	1.	1.22	1.26	0.	0.	0.	0.
time (sec)	N/A	0.097	0.403	1.33	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	185	259	0	0	0	0
normalized size	1	1.	1.27	1.77	0.	0.	0.	0.
time (sec)	N/A	0.044	0.41	1.089	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	235	262	0	0	0	0
normalized size	1	1.	1.17	1.3	0.	0.	0.	0.
time (sec)	N/A	0.076	0.89	1.13	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	36	73	0	100
normalized size	1	1.	1.	0.78	1.57	3.17	0.	4.35
time (sec)	N/A	0.021	0.043	0.042	1.446	1.705	0.	1.199

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	244	366	0	0	0	0
normalized size	1	1.	0.75	1.13	0.	0.	0.	0.
time (sec)	N/A	0.107	0.488	0.577	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	176	257	0	0	0	0
normalized size	1	1.	1.02	1.49	0.	0.	0.	0.
time (sec)	N/A	0.046	0.645	0.575	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	201	57	0	182	0	0
normalized size	1	1.	2.14	0.61	0.	1.94	0.	0.
time (sec)	N/A	0.041	0.298	0.09	0.	1.747	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	323	244	368	0	0	0	0
normalized size	1	1.	0.76	1.14	0.	0.	0.	0.
time (sec)	N/A	0.111	0.507	0.6	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	192	264	0	0	0	0
normalized size	1	1.	1.1	1.51	0.	0.	0.	0.
time (sec)	N/A	0.058	0.354	0.592	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	169	248	0	0	0	0
normalized size	1	1.	1.19	1.75	0.	0.	0.	0.
time (sec)	N/A	0.047	0.618	1.324	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	30	47	0	24
normalized size	1	1.	1.	0.78	1.3	2.04	0.	1.04
time (sec)	N/A	0.019	0.026	0.043	1.472	1.684	0.	1.17

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	375	275	0	0	0	0
normalized size	1	1.	1.48	1.09	0.	0.	0.	0.
time (sec)	N/A	0.063	1.014	1.325	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	148	137	0	0	0	0
normalized size	1	1.	1.35	1.25	0.	0.	0.	0.
time (sec)	N/A	0.022	0.145	0.651	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	2463	33	0	122	0	0
normalized size	1	1.	58.64	0.79	0.	2.9	0.	0.
time (sec)	N/A	0.026	12.793	0.872	0.	1.763	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	400	363	0	0	0	0
normalized size	1	1.	1.42	1.29	0.	0.	0.	0.
time (sec)	N/A	0.097	0.871	0.349	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	171	259	0	0	0	0
normalized size	1	1.	1.19	1.8	0.	0.	0.	0.
time (sec)	N/A	0.045	0.668	0.347	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	161	245	0	0	0	0
normalized size	1	1.	1.18	1.79	0.	0.	0.	0.
time (sec)	N/A	0.048	0.581	1.175	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	62	0	0
normalized size	1	1.	1.	0.78	1.	2.7	0.	0.
time (sec)	N/A	0.021	0.03	0.044	1.476	1.745	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	402	356	0	0	0	0
normalized size	1	1.	1.43	1.26	0.	0.	0.	0.
time (sec)	N/A	0.084	0.756	1.463	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	216	247	0	0	0	0
normalized size	1	1.	1.58	1.8	0.	0.	0.	0.
time (sec)	N/A	0.033	0.373	1.325	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	2511	43	0	205	0	0
normalized size	1	1.	38.05	0.65	0.	3.11	0.	0.
time (sec)	N/A	0.034	6.082	0.937	0.	1.746	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	409	363	0	0	0	0
normalized size	1	1.	1.29	1.15	0.	0.	0.	0.
time (sec)	N/A	0.109	0.879	1.331	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	170	259	0	0	0	0
normalized size	1	1.	1.	1.52	0.	0.	0.	0.
time (sec)	N/A	0.065	0.403	1.35	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	178	467	0	0	0	0
normalized size	1	1.	1.06	2.78	0.	0.	0.	0.
time (sec)	N/A	0.057	0.537	1.419	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	73	0	0
normalized size	1	1.	1.	0.78	1.39	3.17	0.	0.
time (sec)	N/A	0.021	0.041	0.042	1.491	1.715	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	409	688	0	0	0	0
normalized size	1	1.	1.29	2.16	0.	0.	0.	0.
time (sec)	N/A	0.105	0.885	1.191	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	178	469	0	0	0	0
normalized size	1	1.	1.06	2.79	0.	0.	0.	0.
time (sec)	N/A	0.044	0.492	1.175	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	2539	69	0	258	0	0
normalized size	1	1.	26.45	0.72	0.	2.69	0.	0.
time (sec)	N/A	0.037	6.088	1.008	0.	1.738	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	414	695	0	0	0	0
normalized size	1	1.	1.19	1.99	0.	0.	0.	0.
time (sec)	N/A	0.14	0.966	1.474	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	183	481	0	0	0	0
normalized size	1	1.	0.9	2.37	0.	0.	0.	0.
time (sec)	N/A	0.072	0.649	1.411	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	101	377	88	96
normalized size	1	1.	0.8	0.7	1.04	3.89	0.91	0.99
time (sec)	N/A	0.071	0.047	0.052	1.464	1.75	0.212	1.155

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	568	2218	0	11800	0	0
normalized size	1	1.	1.16	4.53	0.	24.08	0.	0.
time (sec)	N/A	14.847	0.766	0.301	0.	6.287	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	397	466	1764	0	8779	0	0
normalized size	1	1.22	1.43	5.41	0.	26.93	0.	0.
time (sec)	N/A	7.466	0.553	0.283	0.	4.157	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	375	1329	0	5998	1630	0
normalized size	1	1.	1.19	4.21	0.	18.98	5.16	0.
time (sec)	N/A	3.158	0.468	0.295	0.	2.683	120.946	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	301	926	0	3465	1435	0
normalized size	1	1.	1.05	3.23	0.	12.07	5.	0.
time (sec)	N/A	3.2	0.427	0.278	0.	2.136	74.743	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	175	545	0	1466	155	0
normalized size	1	1.	0.88	2.75	0.	7.4	0.78	0.
time (sec)	N/A	0.271	0.407	0.273	0.	2.003	10.837	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	267	581	0	5018	1295	0
normalized size	1	1.	0.97	2.11	0.	18.25	4.71	0.
time (sec)	N/A	1.129	0.923	0.283	0.	5.131	60.515	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	356	364	999	0	9783	1588	0
normalized size	1	0.97	0.99	2.71	0.	26.58	4.32	0.
time (sec)	N/A	3.664	1.599	0.29	0.	72.07	91.294	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	531	516	1486	0	0	0	0
normalized size	1	1.	0.97	2.8	0.	0.	0.	0.
time (sec)	N/A	3.569	2.388	0.294	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	650	808	3685	0	31622	0	0
normalized size	1	1.	1.24	5.67	0.	48.65	0.	0.
time (sec)	N/A	2.679	1.164	0.318	0.	90.312	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	680	2988	0	24287	0	0
normalized size	1	1.	1.17	5.14	0.	41.8	0.	0.
time (sec)	N/A	15.247	0.949	0.317	0.	43.733	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	538	2358	0	17508	0	0
normalized size	1	1.	1.22	5.35	0.	39.7	0.	0.
time (sec)	N/A	2.15	0.76	0.308	0.	17.223	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	779	1714	0	11256	0	0
normalized size	1	1.	1.72	3.78	0.	24.85	0.	0.
time (sec)	N/A	4.529	1.713	0.286	0.	7.543	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	317	1138	0	5581	0	0
normalized size	1	1.	0.98	3.53	0.	17.33	0.	0.
time (sec)	N/A	1.244	0.736	0.28	0.	3.398	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	331	944	0	10272	0	0
normalized size	1	1.	0.97	2.78	0.	30.21	0.	0.
time (sec)	N/A	1.578	1.148	0.291	0.	54.426	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	402	393	1215	0	0	0	0
normalized size	1	1.	0.98	3.01	0.	0.	0.	0.
time (sec)	N/A	3.075	1.587	0.293	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	607	587	1880	0	0	0	0
normalized size	1	1.	0.97	3.1	0.	0.	0.	0.
time (sec)	N/A	3.931	2.873	0.304	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	201	201	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.363	0.309	1.344	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	261	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.77	0.772	1.278	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	202	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.382	0.364	1.309	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	183	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	0.275	1.308	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	163	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.26	0.247	1.303	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	207	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.385	0.414	1.257	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	246	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.477	0.438	1.307	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	134	186	236	371	156	336
normalized size	1	1.	0.95	1.32	1.67	2.63	1.11	2.38
time (sec)	N/A	0.183	0.077	0.043	0.984	1.696	0.698	1.175

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	103	145	186	292	116	285
normalized size	1	1.	0.94	1.33	1.71	2.68	1.06	2.61
time (sec)	N/A	0.136	0.05	0.046	0.961	1.7	0.707	1.164

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	73	110	131	204	75	232
normalized size	1	1.	1.12	1.69	2.02	3.14	1.15	3.57
time (sec)	N/A	0.06	0.035	0.045	0.955	1.805	0.541	1.156

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	82	85	136	46	181
normalized size	1	1.	0.86	1.64	1.7	2.72	0.92	3.62
time (sec)	N/A	0.034	0.019	0.044	0.947	1.782	0.402	1.19

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	107	111	165	112	109
normalized size	1	1.	0.89	1.73	1.79	2.66	1.81	1.76
time (sec)	N/A	0.08	0.022	0.049	0.992	1.742	0.733	1.125

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	149	153	343	182	0
normalized size	1	1.	0.95	1.73	1.78	3.99	2.12	0.
time (sec)	N/A	0.086	0.049	0.054	0.986	1.837	1.148	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	206	201	548	185	0
normalized size	1	1.	1.	2.37	2.31	6.3	2.13	0.
time (sec)	N/A	0.097	0.077	0.055	0.972	1.822	1.115	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	122	259	278	807	248	0
normalized size	1	1.	1.08	2.29	2.46	7.14	2.19	0.
time (sec)	N/A	0.116	0.058	0.054	0.992	1.804	1.407	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	142	312	319	1031	282	0
normalized size	1	1.	1.02	2.24	2.29	7.42	2.03	0.
time (sec)	N/A	0.133	0.094	0.056	1.043	1.752	1.694	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	226	286	348	716	255	495
normalized size	1	1.	1.04	1.31	1.6	3.28	1.17	2.27
time (sec)	N/A	0.284	0.126	0.053	0.99	1.71	1.204	1.165

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	185	245	294	612	204	441
normalized size	1	1.	1.05	1.38	1.66	3.46	1.15	2.49
time (sec)	N/A	0.232	0.12	0.049	0.981	1.763	1.083	1.186

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	154	204	246	529	167	393
normalized size	1	1.	1.05	1.4	1.68	3.62	1.14	2.69
time (sec)	N/A	0.181	0.092	0.052	0.97	1.722	0.953	1.234

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	115	167	190	421	122	338
normalized size	1	1.	1.07	1.56	1.78	3.93	1.14	3.16
time (sec)	N/A	0.136	0.088	0.05	0.976	1.751	0.829	1.188

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	83	138	140	321	92	286
normalized size	1	1.	1.06	1.77	1.79	4.12	1.18	3.67
time (sec)	N/A	0.096	0.063	0.05	1.007	1.717	0.723	1.243

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	96	93	184	60	216
normalized size	1	1.	0.92	1.92	1.86	3.68	1.2	4.32
time (sec)	N/A	0.057	0.044	0.048	0.985	1.702	0.545	1.192

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	91	156	154	343	180	215
normalized size	1	1.	1.06	1.81	1.79	3.99	2.09	2.5
time (sec)	N/A	0.082	0.048	0.09	0.976	1.717	1.143	1.144

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	85	180	150	286	155	136
normalized size	1	1.	1.15	2.43	2.03	3.86	2.09	1.84
time (sec)	N/A	0.029	0.038	0.055	0.984	1.729	0.814	1.152

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	139	253	286	810	279	0
normalized size	1	1.	1.15	2.09	2.36	6.69	2.31	0.
time (sec)	N/A	0.136	0.105	0.055	0.983	1.8	1.507	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	171	270	266	674	236	0
normalized size	1	1.	1.17	1.85	1.82	4.62	1.62	0.
time (sec)	N/A	0.155	0.099	0.056	1.015	1.744	1.627	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	195	341	402	1303	371	0
normalized size	1	1.	1.1	1.92	2.26	7.32	2.08	0.
time (sec)	N/A	0.2	0.149	0.059	1.036	2.03	2.157	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	229	394	462	1413	420	0
normalized size	1	1.	1.09	1.88	2.2	6.73	2.	0.
time (sec)	N/A	0.242	0.183	0.062	1.088	2.132	2.582	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	193	263	306	722	223	491
normalized size	1	1.	1.08	1.47	1.71	4.03	1.25	2.74
time (sec)	N/A	0.241	0.098	0.052	0.991	1.772	1.738	1.162

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	157	228	254	620	180	437
normalized size	1	1.	1.05	1.53	1.7	4.16	1.21	2.93
time (sec)	N/A	0.197	0.088	0.052	0.974	1.739	1.5	1.162

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	118	198	201	504	148	369
normalized size	1	1.	1.	1.68	1.7	4.27	1.25	3.13
time (sec)	N/A	0.143	0.093	0.051	0.976	1.738	1.343	1.146

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	93	151	142	320	99	306
normalized size	1	1.	1.15	1.86	1.75	3.95	1.22	3.78
time (sec)	N/A	0.101	0.04	0.05	0.984	1.774	1.047	1.164

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	105	109	205	80	263
normalized size	1	1.	0.8	1.72	1.79	3.36	1.31	4.31
time (sec)	N/A	0.06	0.026	0.05	1.001	1.723	0.709	1.176

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	90	218	203	547	185	266
normalized size	1	1.	1.02	2.48	2.31	6.22	2.1	3.02
time (sec)	N/A	0.099	0.081	0.051	1.062	1.802	1.213	1.159

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	140	257	285	810	277	258
normalized size	1	1.	1.15	2.11	2.34	6.64	2.27	2.11
time (sec)	N/A	0.115	0.104	0.055	1.033	1.718	1.459	1.143

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	110	298	205	475	143	171
normalized size	1	1.	0.87	2.35	1.61	3.74	1.13	1.35
time (sec)	N/A	0.061	0.044	0.057	0.988	1.664	1.133	1.139

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	197	348	416	1307	320	0
normalized size	1	1.	1.05	1.85	2.21	6.95	1.7	0.
time (sec)	N/A	0.21	0.158	0.058	1.073	1.792	2.116	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	244	421	485	1604	371	0
normalized size	1	1.	1.04	1.79	2.06	6.83	1.58	0.
time (sec)	N/A	0.273	0.174	0.059	1.092	1.85	2.408	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	193	1308	2167	1693	0	725
normalized size	1	1.	0.72	4.86	8.06	6.29	0.	2.7
time (sec)	N/A	0.975	0.998	0.18	1.581	2.511	0.	1.258

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	168	1030	1608	1277	0	555
normalized size	1	1.	0.78	4.79	7.48	5.94	0.	2.58
time (sec)	N/A	0.666	0.76	0.121	1.566	3.227	0.	1.212

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	182	713	1220	917	0	417
normalized size	1	1.	0.99	3.9	6.67	5.01	0.	2.28
time (sec)	N/A	0.403	0.809	0.093	1.577	2.217	0.	1.203

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	110	131	787	562	0	267
normalized size	1	1.	0.76	0.9	5.43	3.88	0.	1.84
time (sec)	N/A	0.224	0.42	0.048	1.009	1.87	0.	1.18

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	83	85	504	378	0	188
normalized size	1	1.	0.71	0.73	4.31	3.23	0.	1.61
time (sec)	N/A	0.059	0.235	0.05	1.024	1.896	0.	1.23

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	58	55	136	215	0	95
normalized size	1	1.	0.56	0.53	1.32	2.09	0.	0.92
time (sec)	N/A	0.049	0.062	0.047	1.008	1.773	0.	1.193

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	225	3961	0	3537	0	4004
normalized size	1	1.	0.93	16.37	0.	14.62	0.	16.55
time (sec)	N/A	0.618	0.411	0.259	0.	2.264	0.	1.284

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	341	6760	0	6650	0	0
normalized size	1	1.	1.1	21.74	0.	21.38	0.	0.
time (sec)	N/A	1.264	0.649	0.211	0.	5.7	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	387	9593	0	11237	0	8123
normalized size	1	1.	0.97	24.1	0.	28.23	0.	20.41
time (sec)	N/A	2.568	1.261	0.258	0.	25.165	0.	2.716

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	91	114	0	1030	107	157
normalized size	1	1.	0.81	1.02	0.	9.2	0.96	1.4
time (sec)	N/A	0.215	0.094	0.239	0.	1.895	26.087	1.143

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	207	365	440	747	1040	510
normalized size	1	1.	0.86	1.52	1.83	3.11	4.33	2.12
time (sec)	N/A	0.345	0.255	0.047	0.986	1.8	97.777	1.171

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	149	215	266	452	673	328
normalized size	1	1.	0.85	1.23	1.52	2.58	3.85	1.87
time (sec)	N/A	0.237	0.155	0.048	0.985	1.799	61.377	1.127

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	94	101	140	243	374	181
normalized size	1	1.	0.83	0.89	1.24	2.15	3.31	1.6
time (sec)	N/A	0.074	0.091	0.046	1.006	1.66	35.19	1.136

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	41	72	96	150	72
normalized size	1	1.	0.72	0.67	1.18	1.57	2.46	1.18
time (sec)	N/A	0.026	0.027	0.045	0.986	1.705	6.908	1.152

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	132	0	628	100	144
normalized size	1	1.	0.88	1.27	0.	6.04	0.96	1.38
time (sec)	N/A	0.124	0.17	0.077	0.	1.83	20.354	1.183

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	171	237	0	1116	0	200
normalized size	1	1.	1.4	1.94	0.	9.15	0.	1.64
time (sec)	N/A	0.203	0.321	0.217	0.	1.866	0.	1.147

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	207	384	0	1835	0	375
normalized size	1	1.	1.16	2.16	0.	10.31	0.	2.11
time (sec)	N/A	0.303	0.886	0.219	0.	2.008	0.	1.15

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	207	365	451	749	328	612
normalized size	1	1.	0.87	1.53	1.89	3.15	1.38	2.57
time (sec)	N/A	0.268	0.246	0.049	1.031	1.758	56.07	1.172

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	149	215	277	466	204	371
normalized size	1	1.	0.86	1.24	1.6	2.69	1.18	2.14
time (sec)	N/A	0.204	0.16	0.049	0.996	1.725	30.823	1.147

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	92	101	151	252	112	193
normalized size	1	1.	0.83	0.91	1.36	2.27	1.01	1.74
time (sec)	N/A	0.066	0.086	0.046	0.999	1.728	14.724	1.124

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	43	41	73	107	58	76
normalized size	1	1.	0.73	0.69	1.24	1.81	0.98	1.29
time (sec)	N/A	0.026	0.028	0.043	1.014	1.684	5.669	1.144

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	90	165	0	1015	104	136
normalized size	1	1.	0.8	1.47	0.	9.06	0.93	1.21
time (sec)	N/A	0.174	0.071	0.214	0.	1.867	20.389	1.169

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	118	269	0	1817	0	304
normalized size	1	1.	0.82	1.87	0.	12.62	0.	2.11
time (sec)	N/A	0.271	0.088	0.245	0.	2.281	0.	1.16

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	140	546	0	3135	0	487
normalized size	1	1.	0.65	2.55	0.	14.65	0.	2.28
time (sec)	N/A	0.505	0.107	0.224	0.	2.147	0.	1.18

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	155	306	0	787	0	209
normalized size	1	1.	1.05	2.08	0.	5.35	0.	1.42
time (sec)	N/A	0.141	0.576	0.331	0.	2.355	0.	1.206

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	16	16	66	13	12	36	129	16
normalized size	1	1.	4.12	0.81	0.75	2.25	8.06	1.
time (sec)	N/A	0.009	0.091	0.042	0.966	1.673	12.567	1.17

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	1587	2497	0	0	0	4543
normalized size	1	1.	3.86	6.08	0.	0.	0.	11.05
time (sec)	N/A	2.511	6.064	0.493	0.	0.	0.	48.711

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	381	1569	0	0	0	4077
normalized size	1	1.	1.11	4.59	0.	0.	0.	11.92
time (sec)	N/A	2.086	1.342	0.378	0.	0.	0.	36.914

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	233	1383	0	3802	0	0
normalized size	1	1.	0.97	5.76	0.	15.84	0.	0.
time (sec)	N/A	0.341	0.33	0.367	0.	79.148	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	267	5383	0	0	0	0
normalized size	1	1.	0.76	15.34	0.	0.	0.	0.
time (sec)	N/A	2.153	0.671	0.441	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	613	613	355	14861	0	0	0	0
normalized size	1	1.	0.58	24.24	0.	0.	0.	0.
time (sec)	N/A	3.16	4.221	0.458	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	343	2336	0	0	0	0
normalized size	1	1.	1.02	6.93	0.	0.	0.	0.
time (sec)	N/A	2.463	1.036	0.385	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	233	1383	0	3812	0	0
normalized size	1	1.	0.97	5.76	0.	15.88	0.	0.
time (sec)	N/A	0.334	0.31	0.4	0.	69.837	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	229	1415	0	8408	0	0
normalized size	1	1.	1.	6.15	0.	36.56	0.	0.
time (sec)	N/A	0.204	0.194	0.413	0.	141.759	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	291	10977	0	0	0	0
normalized size	1	1.	0.82	31.01	0.	0.	0.	0.
time (sec)	N/A	0.614	0.862	0.451	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	625	340	8264	0	0	0	0
normalized size	1	1.	0.54	13.22	0.	0.	0.	0.
time (sec)	N/A	2.531	0.892	0.508	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	267	5383	0	0	0	0
normalized size	1	1.	0.76	15.34	0.	0.	0.	0.
time (sec)	N/A	1.806	0.597	0.426	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	291	10977	0	0	0	0
normalized size	1	1.	0.82	31.01	0.	0.	0.	0.
time (sec)	N/A	0.763	0.843	0.572	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	543	525	30648	0	0	0	0
normalized size	1	0.99	0.96	55.83	0.	0.	0.	0.
time (sec)	N/A	1.324	3.245	0.652	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	63	305	0	2338	0	1
normalized size	1	1.	0.97	4.69	0.	35.97	0.	0.02
time (sec)	N/A	0.049	0.061	0.231	0.	2.767	0.	1.503

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	91	125	0	432	0	359
normalized size	1	1.	1.14	1.56	0.	5.4	0.	4.49
time (sec)	N/A	0.143	0.152	0.064	0.	2.058	0.	1.304

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	148	1178	0	679	0	281
normalized size	1	1.	1.38	11.01	0.	6.35	0.	2.63
time (sec)	N/A	0.241	0.307	0.289	0.	3.103	0.	1.467

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	120	524	0	679	0	177
normalized size	1	1.	1.12	4.9	0.	6.35	0.	1.65
time (sec)	N/A	0.179	0.109	0.23	0.	3.193	0.	1.346

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	851	851	1034	6457	0	0	0	0
normalized size	1	1.	1.22	7.59	0.	0.	0.	0.
time (sec)	N/A	2.697	10.585	0.479	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	635	635	809	4351	0	0	0	0
normalized size	1	1.	1.27	6.85	0.	0.	0.	0.
time (sec)	N/A	1.622	7.417	0.28	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	434	434	610	2551	0	0	0	0
normalized size	1	1.	1.41	5.88	0.	0.	0.	0.
time (sec)	N/A	0.488	4.644	0.447	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	536	1162	0	0	0	0
normalized size	1	1.	1.48	3.21	0.	0.	0.	0.
time (sec)	N/A	0.317	2.968	0.29	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	683	683	1216	2496	0	0	0	0
normalized size	1	1.	1.78	3.65	0.	0.	0.	0.
time (sec)	N/A	2.122	9.133	0.325	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	650	1331	6044	0	0	0	0
normalized size	1	1.	2.05	9.3	0.	0.	0.	0.
time (sec)	N/A	1.661	7.171	0.293	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1205	1205	2703	19180	0	0	0	0
normalized size	1	1.	2.24	15.92	0.	0.	0.	0.
time (sec)	N/A	4.199	11.446	0.317	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	666	864	5079	0	0	0	0
normalized size	1	1.	1.3	7.63	0.	0.	0.	0.
time (sec)	N/A	1.571	8.103	0.287	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	503	712	3278	0	0	0	0
normalized size	1	0.99	1.4	6.45	0.	0.	0.	0.
time (sec)	N/A	0.977	5.36	0.259	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	545	1828	0	0	0	0
normalized size	1	1.	1.5	5.02	0.	0.	0.	0.
time (sec)	N/A	0.377	4.934	0.302	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	456	688	0	0	0	0
normalized size	1	1.	1.42	2.14	0.	0.	0.	0.
time (sec)	N/A	0.204	2.027	0.299	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	1096	1216	0	0	0	0
normalized size	1	1.	2.32	2.57	0.	0.	0.	0.
time (sec)	N/A	0.639	4.423	0.325	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	1336	6034	0	0	0	0
normalized size	1	1.	1.93	8.69	0.	0.	0.	0.
time (sec)	N/A	1.722	6.837	0.313	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1241	1241	2197	19170	0	0	0	0
normalized size	1	1.	1.77	15.45	0.	0.	0.	0.
time (sec)	N/A	4.237	11.04	0.357	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	527	747	3922	0	0	0	0
normalized size	1	0.99	1.41	7.39	0.	0.	0.	0.
time (sec)	N/A	1.122	6.343	0.32	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	596	2470	0	0	0	0
normalized size	1	1.	1.45	6.02	0.	0.	0.	0.
time (sec)	N/A	0.564	4.593	0.297	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	464	1286	0	0	0	0
normalized size	1	1.	1.4	3.89	0.	0.	0.	0.
time (sec)	N/A	0.264	3.433	0.265	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	294	396	0	0	0	0
normalized size	1	1.	2.16	2.91	0.	0.	0.	0.
time (sec)	N/A	0.057	0.465	0.261	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	300	439	0	0	0	0
normalized size	1	1.	0.94	1.38	0.	0.	0.	0.
time (sec)	N/A	0.488	1.216	0.249	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	698	698	1330	5743	0	0	0	0
normalized size	1	1.	1.91	8.23	0.	0.	0.	0.
time (sec)	N/A	1.933	6.172	0.263	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1246	1246	2450	20364	0	0	0	0
normalized size	1	1.	1.97	16.34	0.	0.	0.	0.
time (sec)	N/A	4.402	11.397	0.301	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	600	808	1440	3164	0	0	0	0
normalized size	1	1.35	2.4	5.27	0.	0.	0.	0.
time (sec)	N/A	0.946	9.726	0.273	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	927	959	0	0	0	0
normalized size	1	1.	1.98	2.04	0.	0.	0.	0.
time (sec)	N/A	0.637	1.096	0.257	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	625	2949	0	0	0	0
normalized size	1	1.	1.37	6.45	0.	0.	0.	0.
time (sec)	N/A	0.613	4.566	0.265	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	473	1769	0	0	0	0
normalized size	1	1.	1.33	4.97	0.	0.	0.	0.
time (sec)	N/A	0.405	3.634	0.263	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	439	520	0	0	0	0
normalized size	1	1.	1.52	1.81	0.	0.	0.	0.
time (sec)	N/A	0.163	1.721	0.253	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	186	200	0	0	0	0
normalized size	1	1.	1.37	1.47	0.	0.	0.	0.
time (sec)	N/A	0.059	0.226	0.328	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	311	235	0	0	0	0
normalized size	1	1.	1.86	1.41	0.	0.	0.	0.
time (sec)	N/A	0.336	0.965	0.296	0.	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	1349	5738	0	0	0	0
normalized size	1	1.	1.81	7.69	0.	0.	0.	0.
time (sec)	N/A	2.109	6.992	0.281	0.	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1257	1257	2990	20365	0	0	0	0
normalized size	1	1.	2.38	16.2	0.	0.	0.	0.
time (sec)	N/A	4.333	12.344	0.332	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	468	2011	0	0	0	0
normalized size	1	1.	1.21	5.2	0.	0.	0.	0.
time (sec)	N/A	0.574	3.349	0.345	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	818	818	1917	9409	0	0	0	0
normalized size	1	1.	2.34	11.5	0.	0.	0.	0.
time (sec)	N/A	0.996	9.238	0.334	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	261	215	0	0	0	0
normalized size	1	1.	2.37	1.95	0.	0.	0.	0.
time (sec)	N/A	0.304	0.902	0.342	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	344	433	0	0	0	0
normalized size	1	1.	0.76	0.95	0.	0.	0.	0.
time (sec)	N/A	0.629	1.39	0.624	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	52	52	107	58	0	0	0	0
normalized size	1	1.	2.06	1.12	0.	0.	0.	0.
time (sec)	N/A	0.046	0.238	0.115	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	136	188	294	405	0	0
normalized size	1	1.	0.51	0.7	1.09	1.51	0.	0.
time (sec)	N/A	0.423	0.135	0.051	1.219	1.791	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	89	116	180	265	0	0
normalized size	1	1.	0.44	0.58	0.9	1.32	0.	0.
time (sec)	N/A	0.233	0.083	0.051	1.195	1.944	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	53	67	88	158	0	0
normalized size	1	1.	0.42	0.54	0.7	1.26	0.	0.
time (sec)	N/A	0.092	0.047	0.047	1.114	2.01	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	50	24	107	0	0
normalized size	1	1.	0.76	1.09	0.52	2.33	0.	0.
time (sec)	N/A	0.021	0.017	0.048	1.051	1.578	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	93	87	0	556	0	0
normalized size	1	1.	1.16	1.09	0.	6.95	0.	0.
time (sec)	N/A	0.131	0.044	0.319	0.	1.651	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	136	168	0	1478	0	0
normalized size	1	1.	0.97	1.2	0.	10.56	0.	0.
time (sec)	N/A	0.191	0.112	0.297	0.	1.72	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	77	285	0	2587	0	0
normalized size	1	1.	0.36	1.34	0.	12.15	0.	0.
time (sec)	N/A	0.314	0.048	0.329	0.	1.888	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	77	450	0	4030	0	0
normalized size	1	1.	0.28	1.61	0.	14.39	0.	0.
time (sec)	N/A	0.424	0.046	0.378	0.	2.009	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	134	187	223	446	0	0
normalized size	1	1.	0.52	0.73	0.87	1.74	0.	0.
time (sec)	N/A	0.331	0.111	0.049	1.219	1.616	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	88	116	132	308	0	0
normalized size	1	1.	0.49	0.64	0.73	1.7	0.	0.
time (sec)	N/A	0.185	0.074	0.052	1.178	1.591	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	51	66	65	201	0	0
normalized size	1	1.	0.34	0.44	0.43	1.34	0.	0.
time (sec)	N/A	0.144	0.044	0.046	1.423	1.593	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	50	24	157	0	0
normalized size	1	1.	0.76	1.09	0.52	3.41	0.	0.
time (sec)	N/A	0.022	0.012	0.044	1.257	1.544	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	71	128	0	1170	0	0
normalized size	1	1.	0.53	0.96	0.	8.8	0.	0.
time (sec)	N/A	0.176	0.029	0.278	0.	1.746	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	73	225	0	2176	0	0
normalized size	1	1.	0.36	1.11	0.	10.77	0.	0.
time (sec)	N/A	0.257	0.036	0.342	0.	1.83	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	77	379	0	3673	0	0
normalized size	1	1.	0.28	1.38	0.	13.41	0.	0.
time (sec)	N/A	0.352	0.039	0.3	0.	1.995	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	131	187	296	508	0	0
normalized size	1	1.	0.55	0.78	1.24	2.13	0.	0.
time (sec)	N/A	0.28	0.113	0.048	1.296	1.655	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	87	116	186	366	0	0
normalized size	1	1.	0.41	0.55	0.88	1.73	0.	0.
time (sec)	N/A	0.22	0.075	0.05	1.244	1.556	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	52	66	99	270	0	0
normalized size	1	1.	0.34	0.43	0.64	1.75	0.	0.
time (sec)	N/A	0.134	0.05	0.046	1.177	1.567	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	38	221	0	0
normalized size	1	1.	0.77	1.04	0.79	4.6	0.	0.
time (sec)	N/A	0.022	0.031	0.045	1.095	1.51	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	73	219	0	2083	0	0
normalized size	1	1.	0.39	1.16	0.	11.08	0.	0.
time (sec)	N/A	0.27	0.04	0.419	0.	1.803	0.	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	75	424	0	3753	0	0
normalized size	1	1.	0.28	1.58	0.	14.	0.	0.
time (sec)	N/A	0.34	0.049	0.31	0.	1.978	0.	0.

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	79	670	0	5742	0	0
normalized size	1	1.	0.23	1.96	0.	16.79	0.	0.
time (sec)	N/A	0.538	0.054	0.348	0.	2.146	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	195	283	432	798	0	0
normalized size	1	1.	0.58	0.84	1.29	2.38	0.	0.
time (sec)	N/A	0.607	0.185	0.054	1.264	1.618	0.	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	136	188	294	554	0	0
normalized size	1	1.	0.51	0.7	1.09	2.06	0.	0.
time (sec)	N/A	0.39	0.124	0.052	1.234	1.62	0.	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	90	116	180	369	0	0
normalized size	1	1.	0.45	0.58	0.9	1.84	0.	0.
time (sec)	N/A	0.228	0.086	0.049	1.133	1.627	0.	0.

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	54	67	88	221	0	0
normalized size	1	1.	0.43	0.54	0.7	1.77	0.	0.
time (sec)	N/A	0.095	0.053	0.047	1.104	1.613	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	24	128	0	0
normalized size	1	1.	0.77	1.04	0.5	2.67	0.	0.
time (sec)	N/A	0.021	0.024	0.044	1.049	1.594	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	101	153	0	702	0	0
normalized size	1	1.	0.81	1.23	0.	5.66	0.	0.
time (sec)	N/A	0.186	0.135	0.38	0.	1.703	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	161	0	1204	0	0
normalized size	1	1.	0.83	1.22	0.	9.12	0.	0.
time (sec)	N/A	0.161	0.198	0.349	0.	1.769	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	79	285	0	2148	0	0
normalized size	1	1.	0.38	1.38	0.	10.38	0.	0.
time (sec)	N/A	0.273	0.042	0.329	0.	1.752	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	79	453	0	3432	0	0
normalized size	1	1.	0.29	1.64	0.	12.39	0.	0.
time (sec)	N/A	0.35	0.048	0.331	0.	1.922	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	79	696	0	5183	0	0
normalized size	1	1.	0.23	2.01	0.	14.94	0.	0.
time (sec)	N/A	0.454	0.048	0.349	0.	2.104	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	195	283	558	1007	0	0
normalized size	1	1.	0.58	0.84	1.66	3.	0.	0.
time (sec)	N/A	0.607	0.246	0.054	1.308	1.691	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	137	188	397	711	0	0
normalized size	1	1.	0.51	0.7	1.48	2.64	0.	0.
time (sec)	N/A	0.407	0.179	0.058	1.176	1.637	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	90	116	259	481	0	0
normalized size	1	1.	0.45	0.58	1.3	2.4	0.	0.
time (sec)	N/A	0.233	0.125	0.05	1.188	1.618	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	54	67	144	290	0	0
normalized size	1	1.	0.43	0.54	1.15	2.32	0.	0.
time (sec)	N/A	0.101	0.078	0.052	1.147	1.593	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	58	161	0	0
normalized size	1	1.	0.77	1.04	1.21	3.35	0.	0.
time (sec)	N/A	0.023	0.033	0.045	1.065	1.671	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	132	263	0	902	0	0
normalized size	1	1.	0.74	1.47	0.	5.04	0.	0.
time (sec)	N/A	0.304	0.269	0.332	0.	1.716	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	75	306	0	975	0	0
normalized size	1	1.	0.42	1.72	0.	5.48	0.	0.
time (sec)	N/A	0.251	0.062	0.341	0.	1.954	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	135	276	0	1750	0	0
normalized size	1	1.	0.69	1.42	0.	8.97	0.	0.
time (sec)	N/A	0.256	0.34	0.339	0.	1.793	0.	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	79	453	0	2890	0	0
normalized size	1	1.	0.3	1.71	0.	10.91	0.	0.
time (sec)	N/A	0.349	0.065	0.333	0.	2.023	0.	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	79	665	0	4467	0	0
normalized size	1	1.	0.24	1.99	0.	13.33	0.	0.
time (sec)	N/A	0.452	0.064	0.35	0.	2.313	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	79	955	0	6377	0	0
normalized size	1	1.	0.2	2.36	0.	15.75	0.	0.
time (sec)	N/A	0.563	0.075	0.366	0.	2.33	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	205	283	672	1214	0	0
normalized size	1	1.	0.61	0.84	2.	3.61	0.	0.
time (sec)	N/A	0.618	0.224	0.051	1.255	1.695	0.	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	147	188	489	876	0	0
normalized size	1	1.	0.55	0.7	1.82	3.26	0.	0.
time (sec)	N/A	0.397	0.168	0.055	1.246	1.641	0.	0.

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	100	116	328	595	0	0
normalized size	1	1.	0.5	0.58	1.64	2.98	0.	0.
time (sec)	N/A	0.235	0.121	0.052	1.164	1.635	0.	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	64	67	190	362	0	0
normalized size	1	1.	0.51	0.54	1.52	2.9	0.	0.
time (sec)	N/A	0.1	0.08	0.048	1.106	1.599	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	81	193	0	0
normalized size	1	1.	0.77	1.04	1.69	4.02	0.	0.
time (sec)	N/A	0.021	0.044	0.048	1.058	1.727	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	145	431	0	1266	0	0
normalized size	1	1.	0.61	1.83	0.	5.36	0.	0.
time (sec)	N/A	0.467	0.399	0.333	0.	1.961	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	75	523	0	1423	0	0
normalized size	1	1.	0.32	2.23	0.	6.06	0.	0.
time (sec)	N/A	0.378	0.076	0.336	0.	2.306	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	79	526	0	1442	0	0
normalized size	1	1.	0.32	2.14	0.	5.86	0.	0.
time (sec)	N/A	0.342	0.075	0.334	0.	4.343	0.	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	171	441	0	2349	0	0
normalized size	1	1.	0.68	1.74	0.	9.28	0.	0.
time (sec)	N/A	0.336	0.39	0.328	0.	1.902	0.	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	79	665	0	3784	0	0
normalized size	1	1.	0.24	2.06	0.	11.72	0.	0.
time (sec)	N/A	0.473	0.086	0.345	0.	2.059	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	79	924	0	5499	0	0
normalized size	1	1.	0.2	2.35	0.	13.99	0.	0.
time (sec)	N/A	0.572	0.086	0.389	0.	2.239	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	79	1261	0	7769	0	0
normalized size	1	1.	0.17	2.72	0.	16.78	0.	0.
time (sec)	N/A	0.716	0.104	0.351	0.	2.398	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	269	511	0	1798	0	0
normalized size	1	1.	0.86	1.63	0.	5.74	0.	0.
time (sec)	N/A	0.562	0.665	0.404	0.	9.655	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	234	328	0	1432	0	0
normalized size	1	1.	0.96	1.34	0.	5.87	0.	0.
time (sec)	N/A	0.366	0.508	0.381	0.	7.394	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	213	201	0	1156	0	0
normalized size	1	1.	1.26	1.19	0.	6.84	0.	0.
time (sec)	N/A	0.222	0.201	0.368	0.	6.943	0.	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	160	120	0	752	0	0
normalized size	1	1.	1.52	1.14	0.	7.16	0.	0.
time (sec)	N/A	0.118	0.113	0.378	0.	6.607	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	63	0	242	0	0
normalized size	1	1.	0.82	1.03	0.	3.97	0.	0.
time (sec)	N/A	0.065	0.034	0.057	0.	1.707	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	98	0	587	0	0
normalized size	1	1.	0.53	0.76	0.	4.55	0.	0.
time (sec)	N/A	0.143	0.06	0.051	0.	1.739	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	1123	0	0
normalized size	1	1.	0.53	0.85	0.	5.67	0.	0.
time (sec)	N/A	0.219	0.092	0.053	0.	1.783	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	0	1871	0	0
normalized size	1	1.	0.57	0.97	0.	7.01	0.	0.
time (sec)	N/A	0.313	0.129	0.057	0.	1.85	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	100	648	0	2034	0	0
normalized size	1	1.	0.33	2.15	0.	6.76	0.	0.
time (sec)	N/A	0.469	0.124	0.386	0.	7.51	0.	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	100	396	0	1544	0	0
normalized size	1	1.	0.44	1.74	0.	6.8	0.	0.
time (sec)	N/A	0.314	0.087	0.379	0.	7.005	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	176	210	0	1227	0	0
normalized size	1	1.	1.09	1.3	0.	7.62	0.	0.
time (sec)	N/A	0.196	0.395	0.389	0.	6.67	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	63	0	262	0	0
normalized size	1	1.	0.82	1.03	0.	4.3	0.	0.
time (sec)	N/A	0.069	0.032	0.052	0.	1.671	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	64	97	0	648	0	0
normalized size	1	1.	0.52	0.78	0.	5.23	0.	0.
time (sec)	N/A	0.154	0.052	0.049	0.	1.75	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	105	168	0	1256	0	0
normalized size	1	1.	0.55	0.88	0.	6.54	0.	0.
time (sec)	N/A	0.247	0.076	0.053	0.	1.866	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	150	259	0	2063	0	0
normalized size	1	1.	0.57	0.99	0.	7.87	0.	0.
time (sec)	N/A	0.329	0.104	0.055	0.	2.173	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	102	652	0	2183	0	0
normalized size	1	1.	0.35	2.26	0.	7.55	0.	0.
time (sec)	N/A	0.432	0.151	0.383	0.	7.15	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	102	343	0	1607	0	0
normalized size	1	1.	0.47	1.57	0.	7.34	0.	0.
time (sec)	N/A	0.289	0.11	0.388	0.	6.898	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	394	0	0
normalized size	1	1.	0.83	1.	0.	6.25	0.	0.
time (sec)	N/A	0.066	0.033	0.053	0.	1.723	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	68	99	0	641	0	0
normalized size	1	1.	0.53	0.77	0.	5.01	0.	0.
time (sec)	N/A	0.141	0.06	0.049	0.	1.771	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	103	169	0	1281	0	0
normalized size	1	1.	0.53	0.87	0.	6.6	0.	0.
time (sec)	N/A	0.22	0.078	0.052	0.	1.916	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	152	258	0	2051	0	0
normalized size	1	1.	0.58	0.99	0.	7.89	0.	0.
time (sec)	N/A	0.313	0.113	0.054	0.	2.079	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	300	870	0	2244	0	0
normalized size	1	1.	0.78	2.26	0.	5.83	0.	0.
time (sec)	N/A	0.718	1.209	0.338	0.	15.916	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	255	602	0	1790	0	0
normalized size	1	1.	0.81	1.92	0.	5.72	0.	0.
time (sec)	N/A	0.518	0.874	0.328	0.	9.437	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	215	385	0	1427	0	0
normalized size	1	1.	0.89	1.6	0.	5.92	0.	0.
time (sec)	N/A	0.352	0.609	0.323	0.	7.312	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	173	198	0	1150	0	0
normalized size	1	1.	1.04	1.19	0.	6.89	0.	0.
time (sec)	N/A	0.209	0.836	0.338	0.	6.834	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	169	197	0	1146	0	0
normalized size	1	1.	1.07	1.25	0.	7.25	0.	0.
time (sec)	N/A	0.188	0.801	0.348	0.	6.668	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	360	0	0
normalized size	1	1.	0.83	1.	0.	5.71	0.	0.
time (sec)	N/A	0.065	0.034	0.05	0.	1.915	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	813	0	0
normalized size	1	1.	0.53	0.77	0.	6.3	0.	0.
time (sec)	N/A	0.139	0.06	0.052	0.	2.084	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	1490	0	0
normalized size	1	1.	0.53	0.85	0.	7.53	0.	0.
time (sec)	N/A	0.22	0.105	0.053	0.	2.091	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	0	2323	0	0
normalized size	1	1.	0.57	0.97	0.	8.7	0.	0.
time (sec)	N/A	0.308	0.155	0.054	0.	1.995	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	302	870	0	2217	0	0
normalized size	1	1.	0.79	2.28	0.	5.8	0.	0.
time (sec)	N/A	0.711	1.231	0.338	0.	17.3	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	254	602	0	1790	0	0
normalized size	1	1.	0.82	1.94	0.	5.77	0.	0.
time (sec)	N/A	0.533	0.881	0.327	0.	9.617	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	193	325	0	1427	0	0
normalized size	1	1.	0.81	1.37	0.	6.	0.	0.
time (sec)	N/A	0.345	0.803	0.345	0.	6.256	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	102	383	0	1442	0	0
normalized size	1	1.	0.46	1.73	0.	6.5	0.	0.
time (sec)	N/A	0.303	0.167	0.351	0.	5.849	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	188	331	0	1499	0	0
normalized size	1	1.	0.88	1.55	0.	7.	0.	0.
time (sec)	N/A	0.278	1.093	0.405	0.	5.607	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	482	0	0
normalized size	1	1.	0.83	1.	0.	7.65	0.	0.
time (sec)	N/A	0.07	0.054	0.049	0.	1.475	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	1061	0	0
normalized size	1	1.	0.53	0.77	0.	8.22	0.	0.
time (sec)	N/A	0.148	0.09	0.053	0.	1.53	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	1808	0	0
normalized size	1	1.	0.53	0.85	0.	9.13	0.	0.
time (sec)	N/A	0.228	0.135	0.052	0.	1.756	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	0	2820	0	0
normalized size	1	1.	0.57	0.97	0.	10.56	0.	0.
time (sec)	N/A	0.324	0.186	0.061	0.	1.798	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	974	1191	0	2781	0	0
normalized size	1	1.	2.17	2.66	0.	6.21	0.	0.
time (sec)	N/A	0.889	6.129	0.41	0.	28.578	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	299	870	0	2244	0	0
normalized size	1	1.	0.8	2.31	0.	5.97	0.	0.
time (sec)	N/A	0.678	1.189	0.358	0.	13.457	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	229	508	0	1793	0	0
normalized size	1	1.	0.75	1.67	0.	5.9	0.	0.
time (sec)	N/A	0.487	1.044	0.368	0.	8.05	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	112	635	0	1937	0	0
normalized size	1	1.	0.38	2.16	0.	6.59	0.	0.
time (sec)	N/A	0.428	0.106	0.348	0.	6.413	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	112	638	0	2059	0	0
normalized size	1	1.	0.39	2.25	0.	7.25	0.	0.
time (sec)	N/A	0.404	0.127	0.385	0.	5.881	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	224	511	0	1974	0	0
normalized size	1	1.	0.82	1.86	0.	7.2	0.	0.
time (sec)	N/A	0.366	1.429	0.415	0.	5.702	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	625	0	0
normalized size	1	1.	0.83	1.	0.	9.92	0.	0.
time (sec)	N/A	0.069	0.083	0.049	0.	1.52	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	79	99	0	1276	0	0
normalized size	1	1.	0.61	0.77	0.	9.89	0.	0.
time (sec)	N/A	0.149	0.087	0.053	0.	1.701	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	115	169	0	2198	0	0
normalized size	1	1.	0.58	0.85	0.	11.1	0.	0.
time (sec)	N/A	0.23	0.118	0.053	0.	1.753	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	162	260	0	3291	0	0
normalized size	1	1.	0.61	0.97	0.	12.33	0.	0.
time (sec)	N/A	0.324	0.159	0.052	0.	1.959	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0
normalized size	1	1.17	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.086	1.763	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	110	98	0	0	0	0	0
normalized size	1	1.06	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.047	1.721	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	118	98	0	0	0	0	0
normalized size	1	1.13	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.047	1.765	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	120	100	0	0	0	0	0
normalized size	1	1.15	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.072	1.722	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0
normalized size	1	1.17	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.108	1.656	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	122	110	0	0	0	0	0
normalized size	1	1.17	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.072	1.705	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	107	95	0	0	0	0	0
normalized size	1	1.04	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.055	1.717	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	134	527	447	1396	0	2732
normalized size	1	1.	0.39	1.54	1.3	4.07	0.	7.97
time (sec)	N/A	0.449	0.167	0.051	1.157	1.456	0.	1.422

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	131	235	261	695	0	1324
normalized size	1	1.	0.53	0.96	1.06	2.83	0.	5.38
time (sec)	N/A	0.205	0.115	0.055	1.105	1.35	0.	1.321

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	67	89	127	292	0	498
normalized size	1	1.	0.45	0.59	0.85	1.95	0.	3.32
time (sec)	N/A	0.082	0.063	0.048	1.065	1.363	0.	1.271

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	45	116	0	117
normalized size	1	1.	0.78	1.06	0.83	2.15	0.	2.17
time (sec)	N/A	0.015	0.021	0.046	1.019	1.402	0.	1.317

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	82	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.028	1.767	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	84	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.034	1.73	0.	0.	0.	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.037	1.732	0.	0.	0.	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.058	1.638	0.	0.	0.	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.039	1.662	0.	0.	0.	0.

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.033	1.671	0.	0.	0.	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.034	1.753	0.	0.	0.	0.

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.037	1.655	0.	0.	0.	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	64	66	144	0	154
normalized size	1	1.	0.82	0.98	1.02	2.22	0.	2.37
time (sec)	N/A	0.044	0.03	0.049	1.47	1.369	0.	1.183

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	43	62	0	0
normalized size	1	1.	0.82	0.	0.55	0.79	0.	0.
time (sec)	N/A	0.15	0.036	1.881	1.264	1.345	0.	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	222	145	0	0	0	0	0
normalized size	1	1.04	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.135	1.633	0.	0.	0.	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	246	641	936	1246	0	0
normalized size	1	1.	0.49	1.28	1.87	2.49	0.	0.
time (sec)	N/A	0.894	0.437	0.055	1.719	1.467	0.	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	264	425	653	837	0	0
normalized size	1	1.	0.64	1.03	1.58	2.03	0.	0.
time (sec)	N/A	0.627	0.279	0.051	1.673	1.536	0.	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	169	255	417	533	0	0
normalized size	1	1.	0.53	0.79	1.3	1.66	0.	0.
time (sec)	N/A	0.42	0.19	0.053	1.452	1.65	0.	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	96	131	227	300	0	0
normalized size	1	1.	0.46	0.63	1.09	1.44	0.	0.
time (sec)	N/A	0.198	0.099	0.05	1.544	1.541	0.	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	54	69	88	158	0	0
normalized size	1	1.	0.5	0.63	0.81	1.45	0.	0.
time (sec)	N/A	0.059	0.044	0.049	1.427	1.552	0.	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	140	163	0	1085	0	0
normalized size	1	1.	1.01	1.17	0.	7.81	0.	0.
time (sec)	N/A	0.192	0.111	0.329	0.	1.628	0.	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	347	0	1863	0	0
normalized size	1	1.	0.91	2.04	0.	10.96	0.	0.
time (sec)	N/A	0.234	0.158	0.331	0.	1.582	0.	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	189	673	0	3411	0	0
normalized size	1	1.	0.72	2.58	0.	13.07	0.	0.
time (sec)	N/A	0.355	0.464	0.345	0.	1.647	0.	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	132	1142	0	5411	0	0
normalized size	1	1.	0.38	3.25	0.	15.42	0.	0.
time (sec)	N/A	0.558	0.118	0.35	0.	1.789	0.	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	229	602	554	570	0	518
normalized size	1	1.	0.71	1.86	1.71	1.76	0.	1.6
time (sec)	N/A	0.934	0.262	0.183	1.808	1.398	0.	1.583

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	114	291	275	313	631	243
normalized size	1	1.	0.69	1.75	1.66	1.89	3.8	1.46
time (sec)	N/A	0.319	0.119	0.171	1.99	1.39	141.49	1.708

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	117	105	167	282	97
normalized size	1	1.	0.71	1.86	1.67	2.65	4.48	1.54
time (sec)	N/A	0.063	0.034	0.156	1.904	1.462	26.415	1.56

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	260	1759	0	8660	0	0
normalized size	1	1.	0.92	6.24	0.	30.71	0.	0.
time (sec)	N/A	0.522	0.589	0.51	0.	2.523	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	571	571	508	41837	0	0	0	0
normalized size	1	1.	0.89	73.27	0.	0.	0.	0.
time (sec)	N/A	5.235	1.409	1.205	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	239	755	549	802	0	961
normalized size	1	1.	0.87	2.74	1.99	2.91	0.	3.48
time (sec)	N/A	0.598	0.244	0.225	1.679	1.716	0.	1.577

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	127	380	270	443	0	509
normalized size	1	1.	0.94	2.81	2.	3.28	0.	3.77
time (sec)	N/A	0.195	0.125	0.174	1.518	1.672	0.	1.351

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	39	151	99	215	255	246
normalized size	1	1.	0.98	3.78	2.48	5.38	6.38	6.15
time (sec)	N/A	0.051	0.049	0.23	1.5	1.606	159.678	1.183

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	335	11142	0	0	0	0
normalized size	1	1.	0.76	25.15	0.	0.	0.	0.
time (sec)	N/A	1.443	2.508	0.477	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	939	938	890	108969	0	0	0	0
normalized size	1	1.	0.95	116.05	0.	0.	0.	0.
time (sec)	N/A	11.844	9.331	4.968	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	167	0	0	0	0	0
normalized size	1	1.	3.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.206	0.937	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.101	0.987	0.	0.	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.08	1.041	0.	0.	0.	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	249	2017	0	4350	0	5076
normalized size	1	1.	0.91	7.33	0.	15.82	0.	18.46
time (sec)	N/A	0.263	0.331	0.06	0.	2.102	0.	1.282

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	187	1048	0	2392	11628	2854
normalized size	1	1.	0.9	5.04	0.	11.5	55.9	13.72
time (sec)	N/A	0.188	0.213	0.055	0.	1.99	25.99	1.509

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	141	449	0	1173	4908	1374
normalized size	1	1.	0.97	3.08	0.	8.03	33.62	9.41
time (sec)	N/A	0.112	0.292	0.05	0.	1.645	9.549	1.48

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	147	0	462	1532	504
normalized size	1	1.	0.87	1.75	0.	5.5	18.24	6.
time (sec)	N/A	0.063	0.106	0.048	0.	1.571	3.986	1.105

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	93	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	0.158	0.669	0.	0.	0.	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	83	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.093	0.692	0.	0.	0.	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	106	0	0	0	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.102	0.723	0.	0.	0.	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	106	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.111	0.767	0.	0.	0.	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	227	179	0	0	0	0	0
normalized size	1	0.98	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.262	0.176	0.683	0.	0.	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	85	142	117	198	420	0
normalized size	1	1.	1.02	1.71	1.41	2.39	5.06	0.
time (sec)	N/A	0.1	0.054	0.051	1.013	1.302	11.337	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	177	444	344	639	989	0
normalized size	1	1.	0.96	2.41	1.87	3.47	5.38	0.
time (sec)	N/A	0.312	0.16	0.056	0.973	2.995	98.611	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	531	476	1232	973	1465	0	0
normalized size	1	1.	0.9	2.32	1.83	2.76	0.	0.
time (sec)	N/A	0.988	0.466	0.068	1.036	20.518	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	246	606	0	0	0	529
normalized size	1	1.	1.	2.46	0.	0.	0.	2.15
time (sec)	N/A	0.468	0.377	0.166	0.	0.	0.	1.131

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	644	644	710	9103	0	0	0	0
normalized size	1	1.	1.1	14.14	0.	0.	0.	0.
time (sec)	N/A	2.052	3.236	0.256	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	249	540	579	971	1544	763
normalized size	1	1.	0.87	1.88	2.02	3.38	5.38	2.66
time (sec)	N/A	0.495	0.433	0.053	0.997	2.133	169.264	1.148

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	184	315	352	586	1001	490
normalized size	1	1.	0.87	1.49	1.66	2.76	4.72	2.31
time (sec)	N/A	0.339	0.362	0.059	0.983	1.761	96.045	1.123

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	144	174	297	549	269
normalized size	1	1.	0.96	1.05	1.27	2.17	4.01	1.96
time (sec)	N/A	0.109	0.209	0.05	0.968	1.742	67.712	1.14

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	53	104	127	223	104
normalized size	1	1.	0.74	0.73	1.42	1.74	3.05	1.42
time (sec)	N/A	0.042	0.049	0.046	0.957	1.715	10.504	1.167

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	118	189	0	714	112	173
normalized size	1	1.	1.02	1.63	0.	6.16	0.97	1.49
time (sec)	N/A	0.168	0.2	0.217	0.	1.858	31.455	1.124

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	150	371	0	1300	0	236
normalized size	1	1.	1.07	2.65	0.	9.29	0.	1.69
time (sec)	N/A	0.29	0.648	0.263	0.	1.883	0.	1.114

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	297	538	0	2240	0	504
normalized size	1	1.	1.44	2.61	0.	10.87	0.	2.45
time (sec)	N/A	0.386	0.726	0.245	0.	1.781	0.	1.189

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	249	540	590	971	452	903
normalized size	1	1.	0.87	1.89	2.07	3.41	1.59	3.17
time (sec)	N/A	0.405	0.747	0.051	0.978	1.551	159.296	1.181

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	184	315	363	603	272	545
normalized size	1	1.	0.88	1.5	1.73	2.87	1.3	2.6
time (sec)	N/A	0.289	0.376	0.051	0.97	1.481	62.753	1.128

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	128	144	185	306	141	275
normalized size	1	1.	0.95	1.07	1.37	2.27	1.04	2.04
time (sec)	N/A	0.098	0.18	0.047	0.961	1.505	26.879	1.129

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	53	89	136	70	100
normalized size	1	1.	0.76	0.75	1.25	1.92	0.99	1.41
time (sec)	N/A	0.042	0.057	0.048	0.96	1.455	10.088	1.174

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	124	237	0	1112	116	151
normalized size	1	1.	1.02	1.94	0.	9.11	0.95	1.24
time (sec)	N/A	0.221	0.319	0.236	0.	1.772	34.847	1.142

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	176	418	0	2184	0	381
normalized size	1	1.	1.07	2.53	0.	13.24	0.	2.31
time (sec)	N/A	0.367	0.466	0.244	0.	1.79	0.	1.161

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	290	847	0	3864	0	624
normalized size	1	1.	1.17	3.42	0.	15.58	0.	2.52
time (sec)	N/A	0.629	1.235	0.246	0.	1.884	0.	1.186

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	191	113	231	0	671	0	22
normalized size	1	2.1	1.24	2.54	0.	7.37	0.	0.24
time (sec)	N/A	0.141	0.208	0.125	0.	1.686	0.	1.166

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	173	425	0	879	0	242
normalized size	1	1.	1.05	2.59	0.	5.36	0.	1.48
time (sec)	N/A	0.179	0.822	0.307	0.	1.995	0.	1.21

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	313	1207	0	1912	0	605
normalized size	1	1.	0.94	3.62	0.	5.74	0.	1.82
time (sec)	N/A	0.353	1.671	0.286	0.	3.152	0.	1.29

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	225	763	0	1277	0	393
normalized size	1	1.	0.91	3.1	0.	5.19	0.	1.6
time (sec)	N/A	0.256	1.086	0.301	0.	2.126	0.	1.237

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	173	425	0	879	0	242
normalized size	1	1.	1.05	2.59	0.	5.36	0.	1.48
time (sec)	N/A	0.143	0.718	0.336	0.	1.998	0.	1.183

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	222	697	0	1256	0	271
normalized size	1	1.	1.72	5.4	0.	9.74	0.	2.1
time (sec)	N/A	0.135	0.617	0.347	0.	9.919	0.	1.273

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	173	773	0	1671	0	680
normalized size	1	1.	1.08	4.83	0.	10.44	0.	4.25
time (sec)	N/A	0.18	0.244	0.327	0.	26.921	0.	1.447

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	178	238	0	740	0	1458
normalized size	1	1.	0.9	1.2	0.	3.74	0.	7.36
time (sec)	N/A	0.212	0.239	0.057	0.	76.255	0.	1.584

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	332	468	0	0	0	2522
normalized size	1	1.	1.18	1.67	0.	0.	0.	8.98
time (sec)	N/A	0.291	0.365	0.056	0.	0.	0.	2.016

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	196	834	0	1268	0	320
normalized size	1	1.	0.79	3.35	0.	5.09	0.	1.29
time (sec)	N/A	0.28	1.171	0.333	0.	6.999	0.	1.245

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	204	571	0	1250	0	975
normalized size	1	1.	0.85	2.38	0.	5.21	0.	4.06
time (sec)	N/A	0.225	0.91	0.375	0.	2.126	0.	1.463

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	163	392	0	948	0	602
normalized size	1	1.	0.93	2.23	0.	5.39	0.	3.42
time (sec)	N/A	0.171	0.472	0.318	0.	1.762	0.	1.321

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	135	247	0	710	0	196
normalized size	1	1.	1.11	2.02	0.	5.82	0.	1.61
time (sec)	N/A	0.124	0.421	0.318	0.	1.979	0.	1.192

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	134	438	0	1000	0	261
normalized size	1	1.	1.24	4.06	0.	9.26	0.	2.42
time (sec)	N/A	0.12	0.358	0.388	0.	2.357	0.	1.212

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	128	601	0	1407	0	294
normalized size	1	1.	1.1	5.18	0.	12.13	0.	2.53
time (sec)	N/A	0.121	0.305	0.345	0.	3.358	0.	1.3

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	110	150	0	608	0	455
normalized size	1	1.	0.83	1.13	0.	4.57	0.	3.42
time (sec)	N/A	0.13	0.088	0.055	0.	5.959	0.	1.298

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	173	248	0	1021	0	707
normalized size	1	1.	0.92	1.31	0.	5.4	0.	3.74
time (sec)	N/A	0.192	0.125	0.057	0.	14.242	0.	1.453

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	404	11688	0	0	0	0
normalized size	1	1.	0.97	28.03	0.	0.	0.	0.
time (sec)	N/A	3.141	1.768	0.695	0.	0.	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	268	5482	0	0	0	0
normalized size	1	1.	0.94	19.24	0.	0.	0.	0.
time (sec)	N/A	0.526	1.04	0.474	0.	0.	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	269	5507	0	0	0	0
normalized size	1	1.	0.94	19.19	0.	0.	0.	0.
time (sec)	N/A	0.415	0.804	0.553	0.	0.	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	340	47351	0	0	0	0
normalized size	1	1.	0.79	110.38	0.	0.	0.	0.
time (sec)	N/A	1.356	2.119	0.916	0.	0.	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	559	3941	0	0	0	0
normalized size	1	1.	1.05	7.41	0.	0.	0.	0.
time (sec)	N/A	1.704	1.031	0.321	0.	0.	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	372	2602	0	0	0	0
normalized size	1	1.	1.14	8.01	0.	0.	0.	0.
time (sec)	N/A	0.708	0.426	0.325	0.	0.	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	216	1559	0	0	0	0
normalized size	1	1.	0.99	7.12	0.	0.	0.	0.
time (sec)	N/A	0.324	0.369	0.299	0.	0.	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	145	715	0	2244	0	0
normalized size	1	1.	0.95	4.7	0.	14.76	0.	0.
time (sec)	N/A	0.151	0.153	0.312	0.	9.523	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	218	1529	0	0	0	0
normalized size	1	1.	0.96	6.71	0.	0.	0.	0.
time (sec)	N/A	0.331	0.347	0.517	0.	0.	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	222	3162	0	0	0	0
normalized size	1	1.	0.45	6.45	0.	0.	0.	0.
time (sec)	N/A	0.695	0.527	0.428	0.	0.	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	609	6714	0	0	0	2489
normalized size	1	1.	0.9	9.98	0.	0.	0.	3.7
time (sec)	N/A	0.862	1.469	0.355	0.	0.	0.	5.477

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	933	933	858	11995	0	0	0	0
normalized size	1	1.	0.92	12.86	0.	0.	0.	0.
time (sec)	N/A	1.225	4.871	0.329	0.	0.	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1098	1098	743	10058	0	0	0	0
normalized size	1	1.	0.68	9.16	0.	0.	0.	0.
time (sec)	N/A	3.864	2.485	0.291	0.	0.	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	662	662	536	6860	0	0	0	0
normalized size	1	1.	0.81	10.36	0.	0.	0.	0.
time (sec)	N/A	1.554	1.323	0.287	0.	0.	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	420	4188	0	0	0	0
normalized size	1	1.	0.95	9.5	0.	0.	0.	0.
time (sec)	N/A	0.853	1.176	0.276	0.	0.	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	236	1946	0	0	0	0
normalized size	1	1.	0.94	7.72	0.	0.	0.	0.
time (sec)	N/A	0.346	0.444	0.27	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	491	323	4226	0	0	0	0
normalized size	1	1.	0.66	8.61	0.	0.	0.	0.
time (sec)	N/A	0.841	1.14	0.385	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	787	787	357	7959	0	0	0	0
normalized size	1	1.	0.45	10.11	0.	0.	0.	0.
time (sec)	N/A	1.376	1.568	0.408	0.	0.	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1066	1066	1036	15927	0	0	0	0
normalized size	1	1.	0.97	14.94	0.	0.	0.	0.
time (sec)	N/A	1.706	3.766	0.356	0.	0.	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	886	886	647	9052	0	0	0	0
normalized size	1	1.	0.73	10.22	0.	0.	0.	0.
time (sec)	N/A	1.8	2.695	0.372	0.	0.	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	553	1597	0	0	0	0
normalized size	1	1.	1.28	3.71	0.	0.	0.	0.
time (sec)	N/A	1.37	0.893	0.304	0.	0.	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	358	1007	0	0	0	0
normalized size	1	1.	1.33	3.73	0.	0.	0.	0.
time (sec)	N/A	0.708	0.401	0.301	0.	0.	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	170	613	0	0	0	0
normalized size	1	1.	0.97	3.48	0.	0.	0.	0.
time (sec)	N/A	0.305	0.537	0.296	0.	0.	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	126	349	0	0	0	0
normalized size	1	1.	0.96	2.66	0.	0.	0.	0.
time (sec)	N/A	0.093	0.152	0.271	0.	0.	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	157	0	743	0	97
normalized size	1	1.	0.99	1.99	0.	9.41	0.	1.23
time (sec)	N/A	0.038	0.017	0.273	0.	2.496	0.	1.185

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	169	327	0	0	0	0
normalized size	1	1.	0.93	1.8	0.	0.	0.	0.
time (sec)	N/A	0.219	0.261	0.387	0.	0.	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	256	788	0	0	0	0
normalized size	1	1.	0.75	2.32	0.	0.	0.	0.
time (sec)	N/A	0.394	1.059	0.419	0.	0.	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	587	549	1817	0	0	0	3046
normalized size	1	1.	0.94	3.1	0.	0.	0.	5.19
time (sec)	N/A	0.81	2.736	0.379	0.	0.	0.	3.676

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	587	4453	0	0	0	0
normalized size	1	1.	1.18	8.98	0.	0.	0.	0.
time (sec)	N/A	1.199	2.95	0.277	0.	0.	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	373	3127	0	0	0	0
normalized size	1	1.	1.04	8.76	0.	0.	0.	0.
time (sec)	N/A	0.528	1.174	0.317	0.	0.	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	265	2123	0	4150	0	1022
normalized size	1	1.	1.1	8.85	0.	17.29	0.	4.26
time (sec)	N/A	0.305	0.693	0.263	0.	44.006	0.	1.18

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	183	1261	0	3411	0	767
normalized size	1	1.	0.98	6.74	0.	18.24	0.	4.1
time (sec)	N/A	0.135	0.186	0.258	0.	37.097	0.	1.2

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	177	603	0	2768	0	603
normalized size	1	1.	1.14	3.89	0.	17.86	0.	3.89
time (sec)	N/A	0.101	0.162	0.272	0.	6.185	0.	1.233

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	317	1343	0	0	0	0
normalized size	1	1.	0.9	3.82	0.	0.	0.	0.
time (sec)	N/A	0.443	1.617	0.453	0.	0.	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	868	2807	0	0	0	0
normalized size	1	1.	1.35	4.37	0.	0.	0.	0.
time (sec)	N/A	0.911	6.219	0.416	0.	0.	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1064	1064	1013	5459	0	0	0	19887
normalized size	1	1.	0.95	5.13	0.	0.	0.	18.69
time (sec)	N/A	1.898	5.981	0.484	0.	0.	0.	22.539

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1551	1551	26600	32647	0	0	0	0
normalized size	1	1.	17.15	21.05	0.	0.	0.	0.
time (sec)	N/A	8.093	18.264	0.599	0.	0.	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1015	1015	15781	20224	0	0	0	0
normalized size	1	1.	15.55	19.93	0.	0.	0.	0.
time (sec)	N/A	3.775	15.866	0.393	0.	0.	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	652	8432	10711	0	0	0	0
normalized size	1	1.	12.93	16.43	0.	0.	0.	0.
time (sec)	N/A	1.112	13.913	0.371	0.	0.	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	697	4356	0	0	0	0
normalized size	1	1.	1.36	8.49	0.	0.	0.	0.
time (sec)	N/A	0.536	10.854	0.352	0.	0.	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	764	969	35245	6812	0	0	0	0
normalized size	1	1.27	46.13	8.92	0.	0.	0.	0.
time (sec)	N/A	4.099	15.368	0.398	0.	0.	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	743	934	16573	16696	0	0	0	0
normalized size	1	1.26	22.31	22.47	0.	0.	0.	0.
time (sec)	N/A	3.316	13.988	0.402	0.	0.	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1034	1705	33765	55360	0	0	0	0
normalized size	1	1.65	32.65	53.54	0.	0.	0.	0.
time (sec)	N/A	8.091	16.874	0.503	0.	0.	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1098	1098	17771	22215	0	0	0	0
normalized size	1	1.	16.18	20.23	0.	0.	0.	0.
time (sec)	N/A	5.764	16.375	0.395	0.	0.	0.	0.

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	755	755	10030	12923	0	0	0	0
normalized size	1	1.	13.28	17.12	0.	0.	0.	0.
time (sec)	N/A	1.931	14.363	0.426	0.	0.	0.	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	911	6207	0	0	0	0
normalized size	1	1.	1.76	11.96	0.	0.	0.	0.
time (sec)	N/A	0.535	10.608	0.37	0.	0.	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	936	1854	0	0	0	0
normalized size	1	1.	2.11	4.18	0.	0.	0.	0.
time (sec)	N/A	0.322	8.707	0.347	0.	0.	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	700	700	16471	3126	0	0	0	0
normalized size	1	1.	23.53	4.47	0.	0.	0.	0.
time (sec)	N/A	1.953	13.987	0.4	0.	0.	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	736	957	6911	13874	0	0	0	0
normalized size	1	1.3	9.39	18.85	0.	0.	0.	0.
time (sec)	N/A	3.23	13.476	0.423	0.	0.	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1049	1747	36617	57841	0	0	0	0
normalized size	1	1.67	34.91	55.14	0.	0.	0.	0.
time (sec)	N/A	8.184	16.914	0.538	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	774	774	10649	14978	0	0	0	0
normalized size	1	1.	13.76	19.35	0.	0.	0.	0.
time (sec)	N/A	2.106	14.923	0.395	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	567	567	1002	8248	0	0	0	0
normalized size	1	1.	1.77	14.55	0.	0.	0.	0.
time (sec)	N/A	0.931	11.583	0.348	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	638	3805	0	0	0	0
normalized size	1	1.	1.41	8.42	0.	0.	0.	0.
time (sec)	N/A	0.437	7.508	0.324	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	365	747	0	0	0	0
normalized size	1	1.	1.94	3.97	0.	0.	0.	0.
time (sec)	N/A	0.065	0.816	0.311	0.	0.	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	467	467	379	834	0	0	0	0
normalized size	1	1.	0.81	1.79	0.	0.	0.	0.
time (sec)	N/A	1.602	1.646	0.329	0.	0.	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	994	994	18563	13017	0	0	0	0
normalized size	1	1.	18.68	13.1	0.	0.	0.	0.
time (sec)	N/A	3.701	13.866	0.37	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1786	1786	36634	59522	0	0	0	0
normalized size	1	1.	20.51	33.33	0.	0.	0.	0.
time (sec)	N/A	8.396	17.304	0.514	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	675	675	1385	1879	0	0	0	0
normalized size	1	1.	2.05	2.78	0.	0.	0.	0.
time (sec)	N/A	1.655	3.922	0.325	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1138	1138	37137	7464	0	0	0	0
normalized size	1	1.	32.63	6.56	0.	0.	0.	0.
time (sec)	N/A	2.138	16.213	0.326	0.	0.	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	631	12746	8755	0	0	0	0
normalized size	1	1.	20.2	13.87	0.	0.	0.	0.
time (sec)	N/A	1.09	13.771	0.351	0.	0.	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	1080	4295	0	0	0	0
normalized size	1	1.	2.25	8.97	0.	0.	0.	0.
time (sec)	N/A	0.553	12.774	0.329	0.	0.	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	814	1014	0	0	0	0
normalized size	1	1.	2.07	2.58	0.	0.	0.	0.
time (sec)	N/A	0.21	5.697	0.317	0.	0.	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	308	287	0	0	0	0
normalized size	1	1.	1.63	1.52	0.	0.	0.	0.
time (sec)	N/A	0.072	0.684	0.311	0.	0.	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	499	330	0	0	0	0
normalized size	1	1.	1.78	1.18	0.	0.	0.	0.
time (sec)	N/A	1.247	1.828	0.386	0.	0.	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1037	1037	10881	14048	0	0	0	0
normalized size	1	1.	10.49	13.55	0.	0.	0.	0.
time (sec)	N/A	3.527	14.106	0.374	0.	0.	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1114	1762	40396	64947	0	0	0	0
normalized size	1	1.58	36.26	58.3	0.	0.	0.	0.
time (sec)	N/A	8.012	18.226	0.616	0.	0.	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	950	4757	0	0	0	0
normalized size	1	1.	1.72	8.6	0.	0.	0.	0.
time (sec)	N/A	1.613	6.88	0.444	0.	0.	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1125	1125	14762	27597	0	0	0	0
normalized size	1	1.	13.12	24.53	0.	0.	0.	0.
time (sec)	N/A	2.31	15.871	0.46	0.	0.	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	475	475	1118	645	0	0	0	0
normalized size	1	1.	2.35	1.36	0.	0.	0.	0.
time (sec)	N/A	0.419	9.951	0.637	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	588	588	375	605	0	0	0	0
normalized size	1	1.	0.64	1.03	0.	0.	0.	0.
time (sec)	N/A	1.167	3.517	0.471	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	198	1347	0	2974	15144	3699
normalized size	1	1.	0.9	6.12	0.	13.52	68.84	16.81
time (sec)	N/A	0.215	0.31	0.062	0.	2.358	17.642	1.239

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	180	503	0	1297	5795	1569
normalized size	1	1.	1.25	3.49	0.	9.01	40.24	10.9
time (sec)	N/A	0.113	0.353	0.052	0.	2.012	6.691	1.164

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.159	0.147	0.662	0.	0.	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.161	0.737	0.	0.	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	243	157	0	0	0	0	0
normalized size	1	0.99	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.317	0.168	0.691	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	492	5890	0	10647	0	14160
normalized size	1	1.	0.94	11.22	0.	20.28	0.	26.97
time (sec)	N/A	0.615	0.847	0.06	0.	2.416	0.	1.402

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	655	2563	0	5231	0	6669
normalized size	1	1.	2.11	8.24	0.	16.82	0.	21.44
time (sec)	N/A	0.392	1.54	0.054	0.	1.957	0.	1.325

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	265	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.865	0.448	1.641	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	261	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	1.141	0.395	1.626	0.	0.	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	257	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	1.487	0.412	1.459	0.	0.	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	151	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.285	1.43	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	117	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	0.116	1.305	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	91	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	0.09	1.404	0.	0.	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.081	1.129	0.	0.	0.	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	94	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.107	1.245	0.	0.	0.	0.

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	110	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	0.167	1.312	0.	0.	0.	0.

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	152	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.221	0.134	1.34	0.	0.	0.	0.

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	251	0	0	0	0	0
normalized size	1	1.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.295	0.309	1.332	0.	0.	0.	0.

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	252	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.281	0.3	1.35	0.	0.	0.	0.

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	156	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	0.294	1.329	0.	0.	0.	0.

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	149	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.178	1.311	0.	0.	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	150	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.162	1.174	0.	0.	0.	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	274	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.504	0.548	1.341	0.	0.	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	287	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	0.575	1.35	0.	0.	0.	0.

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	230	171	0	0	0	0	0
normalized size	1	0.97	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.349	0.292	0.628	0.	0.	0.	0.

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	509	506	0	0	0	0	0	0
normalized size	1	0.99	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.842	1.456	1.539	0.	0.	0.	0.

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.36	0.749	1.436	0.	0.	0.	0.

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.02	0.043	0.	0.	0.	0.

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.109	1.922	0.	0.	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	502	500	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.681	1.271	1.701	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.347	0.742	1.243	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.023	0.043	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.122	1.486	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	263	187	0	0	0	0	0
normalized size	1	0.99	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	0.208	0.623	0.	0.	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	525	523	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.702	2.204	1.473	0.	0.	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.337	1.226	1.454	0.	0.	0.	0.

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	205	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.104	0.047	0.	0.	0.	0.

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.173	1.63	0.	0.	0.	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	188	148	0	0	0	0
normalized size	1	1.	2.11	1.66	0.	0.	0.	0.
time (sec)	N/A	0.227	0.636	0.706	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [429] had the largest ratio of [0.7]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	5	1.	25	0.2
2	A	8	5	1.	25	0.2
3	A	7	5	1.	25	0.2
4	A	12	5	1.	25	0.2
5	A	5	4	1.	23	0.174
6	A	5	4	1.	23	0.174
7	A	8	7	1.	25	0.28

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	8	8	1.	25	0.32
9	A	8	7	1.	25	0.28
10	A	8	8	1.	25	0.32
11	A	8	7	1.	25	0.28
12	A	6	5	1.	25	0.2
13	A	7	6	1.	25	0.24
14	A	8	6	1.	25	0.24
15	A	9	6	1.	25	0.24
16	A	8	5	1.	25	0.2
17	A	5	5	1.	25	0.2
18	A	3	3	1.	25	0.12
19	A	6	4	1.	25	0.16
20	A	6	4	1.	25	0.16
21	A	5	4	1.	25	0.16
22	A	4	3	1.	25	0.12
23	A	3	3	1.	25	0.12
24	A	3	3	1.	25	0.12
25	A	3	3	1.	23	0.13
26	A	3	3	1.	22	0.136
27	A	7	5	1.	25	0.2
28	A	7	5	1.	25	0.2
29	A	8	6	1.	25	0.24
30	A	4	4	1.	25	0.16
31	A	5	4	1.	25	0.16
32	A	4	4	1.	24	0.167
33	A	7	5	1.	27	0.185
34	A	6	5	1.	27	0.185
35	A	5	5	1.	27	0.185
36	A	4	4	1.	25	0.16
37	A	4	4	1.	24	0.167
38	A	7	7	1.	27	0.259
39	A	7	7	1.	27	0.259
40	A	5	5	1.	27	0.185
41	A	6	6	1.	27	0.222
42	A	7	6	1.	27	0.222
43	A	8	6	1.	27	0.222
44	A	6	5	1.	27	0.185
45	A	6	5	1.	27	0.185
46	A	3	2	1.	27	0.074
47	A	3	3	1.	27	0.111
48	A	3	3	1.	25	0.12
49	A	3	3	1.	24	0.125
50	A	7	6	1.	27	0.222
51	A	7	5	1.	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
52	A	8	6	1.	27	0.222
53	A	9	6	1.	27	0.222
54	A	5	4	1.	20	0.2
55	A	4	4	1.	20	0.2
56	A	3	3	1.	18	0.167
57	A	3	3	1.	17	0.176
58	A	6	6	1.	20	0.3
59	A	6	6	1.	20	0.3
60	A	5	5	1.	20	0.25
61	A	6	6	1.	20	0.3
62	A	7	6	1.	20	0.3
63	A	8	6	1.	20	0.3
64	A	9	8	1.	27	0.296
65	A	12	6	1.	27	0.222
66	A	11	6	1.	27	0.222
67	A	10	6	1.	27	0.222
68	A	9	6	1.	27	0.222
69	A	9	6	1.	25	0.24
70	A	8	5	1.	24	0.208
71	A	11	8	1.	27	0.296
72	A	11	9	1.	27	0.333
73	A	11	8	1.	27	0.296
74	A	11	9	1.	27	0.333
75	A	11	9	1.	27	0.333
76	A	11	8	1.	27	0.296
77	A	11	9	1.	27	0.333
78	A	11	9	1.	27	0.333
79	A	11	8	1.	27	0.296
80	A	9	6	1.	27	0.222
81	A	10	7	1.	27	0.259
82	A	11	7	1.	27	0.259
83	A	7	5	1.	27	0.185
84	A	6	4	1.	27	0.148
85	A	5	4	1.	27	0.148
86	A	3	3	1.	27	0.111
87	A	3	3	1.	25	0.12
88	A	4	3	1.	24	0.125
89	A	7	6	1.	27	0.222
90	A	7	5	1.	27	0.185
91	A	8	6	1.	27	0.222
92	A	7	5	1.	27	0.185
93	A	6	5	1.	27	0.185
94	A	6	6	1.	27	0.222
95	A	4	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.	24	0.125
97	A	7	7	1.	27	0.259
98	A	5	5	1.	27	0.185
99	A	6	6	1.	27	0.222
100	A	7	6	1.	27	0.222
101	A	8	6	1.	27	0.222
102	A	7	7	1.	27	0.259
103	A	9	6	1.	27	0.222
104	A	8	6	1.	27	0.222
105	A	8	7	1.	27	0.259
106	A	6	5	1.	25	0.2
107	A	5	4	1.	24	0.167
108	A	9	8	1.	27	0.296
109	A	9	9	1.	27	0.333
110	A	9	8	1.	27	0.296
111	A	9	9	1.	27	0.333
112	A	9	8	1.	27	0.296
113	A	7	6	1.	27	0.222
114	A	8	7	1.	27	0.259
115	A	9	7	1.	27	0.259
116	A	10	7	1.	27	0.259
117	A	3	3	1.	18	0.167
118	A	7	7	1.	26	0.269
119	A	6	6	1.	27	0.222
120	A	5	5	1.	27	0.185
121	A	5	5	1.	27	0.185
122	A	3	3	1.	25	0.12
123	A	1	1	1.	24	0.042
124	A	5	5	1.	27	0.185
125	A	5	5	1.	27	0.185
126	A	6	6	1.	27	0.222
127	A	6	5	1.	27	0.185
128	A	6	5	1.	27	0.185
129	A	5	5	1.	27	0.185
130	A	3	3	1.	27	0.111
131	A	2	2	1.	25	0.08
132	A	2	2	1.	24	0.083
133	A	6	6	1.	27	0.222
134	A	6	6	1.	27	0.222
135	A	7	7	1.	27	0.259
136	A	7	5	1.	27	0.185
137	A	7	5	1.	27	0.185
138	A	6	5	1.	27	0.185
139	A	5	4	1.	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	4	4	1.	27	0.148
141	A	3	3	1.	27	0.111
142	A	3	3	1.	25	0.12
143	A	3	3	1.	24	0.125
144	A	7	6	1.	27	0.222
145	A	7	6	1.	27	0.222
146	A	8	7	1.	27	0.259
147	A	9	7	1.	27	0.259
148	A	5	5	1.	27	0.185
149	A	4	4	1.	27	0.148
150	A	4	4	1.	25	0.16
151	A	4	4	1.	25	0.16
152	A	2	2	1.	23	0.087
153	A	1	1	1.	22	0.045
154	A	5	5	1.	26	0.192
155	A	5	5	1.	26	0.192
156	A	6	6	1.	26	0.231
157	A	10	7	1.	27	0.259
158	A	9	7	1.	27	0.259
159	A	8	7	1.	27	0.259
160	A	7	7	1.	27	0.259
161	A	6	5	1.	25	0.2
162	A	6	6	1.	24	0.25
163	A	9	9	1.	27	0.333
164	A	9	9	1.	27	0.333
165	A	9	9	1.	27	0.333
166	A	9	9	1.	27	0.333
167	A	7	7	1.	27	0.259
168	A	8	8	1.	27	0.296
169	A	9	8	1.	27	0.296
170	A	10	8	1.	27	0.296
171	A	7	6	1.	27	0.222
172	A	4	3	1.	27	0.111
173	A	4	4	1.	27	0.148
174	A	3	3	1.	25	0.12
175	A	3	2	1.	24	0.083
176	A	8	7	1.	27	0.259
177	A	8	6	1.	27	0.222
178	A	9	7	1.	27	0.259
179	A	8	6	1.	27	0.222
180	A	7	5	1.	27	0.185
181	A	6	5	1.	27	0.185
182	A	4	4	1.	27	0.148
183	A	3	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	3	2	1.	24	0.083
185	A	8	7	1.	27	0.259
186	A	8	6	1.	27	0.222
187	A	9	7	1.	27	0.259
188	A	9	6	1.	27	0.222
189	A	7	5	1.	27	0.185
190	A	9	7	1.	27	0.259
191	A	8	6	1.	27	0.222
192	A	2	2	1.	25	0.08
193	A	2	2	1.	24	0.083
194	A	8	7	1.	27	0.259
195	A	8	6	1.	27	0.222
196	A	9	7	1.	27	0.259
197	A	10	7	1.	27	0.259
198	A	11	6	1.	27	0.222
199	A	10	6	1.	27	0.222
200	A	9	6	1.	27	0.222
201	A	8	7	1.	27	0.259
202	A	6	6	1.	25	0.24
203	A	5	4	1.	24	0.167
204	A	9	9	1.	27	0.333
205	A	9	9	1.	27	0.333
206	A	7	7	1.	27	0.259
207	A	8	7	1.	27	0.259
208	A	9	7	1.	27	0.259
209	A	10	7	1.	27	0.259
210	A	7	5	1.	26	0.192
211	A	4	4	1.	26	0.154
212	A	9	5	1.	27	0.185
213	A	8	5	1.	27	0.185
214	A	7	4	1.	25	0.16
215	A	7	3	1.	24	0.125
216	A	12	7	1.	27	0.259
217	A	12	6	1.	27	0.222
218	A	4	4	1.	29	0.138
219	A	2	2	1.	29	0.069
220	A	3	3	1.	16	0.188
221	A	4	4	1.	29	0.138
222	A	3	3	1.	15	0.2
223	A	4	4	1.	30	0.133
224	A	4	3	1.	16	0.188
225	A	5	4	1.	29	0.138
226	A	7	4	1.	29	0.138
227	A	6	4	1.	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
228	A	5	3	1.	27	0.111
229	A	2	2	1.	22	0.091
230	A	8	5	1.	29	0.172
231	A	7	5	1.	29	0.172
232	A	8	5	1.	29	0.172
233	A	6	4	1.	29	0.138
234	A	6	4	1.	29	0.138
235	A	5	3	1.31	27	0.111
236	A	2	2	1.	22	0.091
237	A	8	5	1.	29	0.172
238	A	7	5	1.	29	0.172
239	A	7	5	1.	29	0.172
240	A	6	5	1.	23	0.217
241	A	6	5	1.	23	0.217
242	A	6	5	1.	23	0.217
243	A	6	5	1.	23	0.217
244	A	4	4	1.	21	0.19
245	A	3	3	1.	20	0.15
246	A	5	5	1.	23	0.217
247	A	5	5	1.	23	0.217
248	A	5	5	1.	23	0.217
249	A	7	6	1.	25	0.24
250	A	8	7	1.	25	0.28
251	A	7	6	1.	25	0.24
252	A	8	7	1.	25	0.28
253	A	7	6	1.	23	0.261
254	A	2	2	1.	22	0.091
255	A	7	7	1.	25	0.28
256	A	6	6	1.	25	0.24
257	A	6	6	1.	25	0.24
258	A	7	6	1.	25	0.24
259	A	7	6	1.	25	0.24
260	A	7	6	1.	25	0.24
261	A	7	6	1.	25	0.24
262	A	3	3	1.	23	0.13
263	A	2	2	1.	22	0.091
264	A	7	7	1.	25	0.28
265	A	8	8	1.	25	0.32
266	A	7	6	1.	25	0.24
267	A	7	6	1.	25	0.24
268	A	7	6	1.	25	0.24
269	A	7	6	1.	25	0.24
270	A	5	5	1.	23	0.217
271	A	2	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	6	6	1.	25	0.24
273	A	6	6	1.	25	0.24
274	A	6	6	1.	25	0.24
275	A	8	7	1.	25	0.28
276	A	9	8	1.	25	0.32
277	A	8	7	1.	25	0.28
278	A	9	8	1.	25	0.32
279	A	3	3	1.	23	0.13
280	A	2	2	1.	22	0.091
281	A	8	8	1.	25	0.32
282	A	7	7	1.	25	0.28
283	A	7	7	1.	25	0.28
284	A	7	7	1.	25	0.28
285	A	7	7	1.	25	0.28
286	A	8	7	1.	25	0.28
287	A	8	7	1.	25	0.28
288	A	4	4	1.	25	0.16
289	A	3	3	1.	23	0.13
290	A	2	2	1.	22	0.091
291	A	8	8	1.	25	0.32
292	A	9	9	1.	25	0.36
293	A	8	7	1.	25	0.28
294	A	8	7	1.	25	0.28
295	A	8	7	1.	25	0.28
296	A	9	8	1.	25	0.32
297	A	5	4	1.	25	0.16
298	A	4	4	1.	25	0.16
299	A	3	3	1.	23	0.13
300	A	2	2	1.	22	0.091
301	A	9	9	1.	25	0.36
302	A	9	9	1.	25	0.36
303	A	10	9	1.	25	0.36
304	A	9	7	1.	25	0.28
305	A	9	7	1.	25	0.28
306	A	7	4	1.	27	0.148
307	A	6	4	1.	27	0.148
308	A	5	3	1.	25	0.12
309	A	2	2	1.	20	0.1
310	A	8	5	1.	27	0.185
311	A	7	5	1.	27	0.185
312	A	8	5	1.	27	0.185
313	A	6	4	1.	25	0.16
314	A	4	3	1.	27	0.111
315	A	11	7	1.	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	9	6	1.	22	0.273
317	A	8	6	1.	22	0.273
318	A	8	7	1.	22	0.318
319	A	6	5	1.	20	0.25
320	A	6	5	1.	19	0.263
321	A	9	8	1.	22	0.364
322	A	15	11	1.	22	0.5
323	A	19	12	1.	22	0.546
324	A	20	13	1.	22	0.591
325	A	25	14	1.	22	0.636
326	A	8	5	1.	22	0.227
327	A	7	5	1.	22	0.227
328	A	7	6	1.	22	0.273
329	A	5	4	1.	20	0.2
330	A	2	2	1.	19	0.105
331	A	7	6	1.	22	0.273
332	A	8	7	1.	22	0.318
333	A	12	8	1.	22	0.364
334	A	7	6	1.	22	0.273
335	A	6	5	1.	22	0.227
336	A	4	4	1.	22	0.182
337	A	4	4	1.	20	0.2
338	A	4	4	1.	19	0.21
339	A	10	9	1.	22	0.409
340	A	12	11	1.	22	0.5
341	A	17	11	1.	22	0.5
342	A	9	6	1.	22	0.273
343	A	8	6	1.	22	0.273
344	A	7	6	1.	22	0.273
345	A	6	5	1.	22	0.227
346	A	3	3	1.	20	0.15
347	A	3	3	1.	19	0.158
348	A	10	7	1.	22	0.318
349	A	11	8	1.	22	0.364
350	A	15	9	1.	22	0.409
351	A	2	1	1.	18	0.056
352	A	2	1	1.	16	0.062
353	A	2	1	1.	15	0.067
354	A	3	3	1.	18	0.167
355	A	2	1	1.	20	0.05
356	A	2	1	1.	18	0.056
357	A	2	1	1.	17	0.059
358	A	4	3	1.	20	0.15
359	A	2	1	1.	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	2	1	1.	18	0.056
361	A	2	1	1.	17	0.059
362	A	4	3	1.	20	0.15
363	A	6	3	1.	20	0.15
364	A	6	3	1.	20	0.15
365	A	6	3	1.	20	0.15
366	A	4	2	1.	18	0.111
367	A	4	2	1.	17	0.118
368	A	7	4	1.	20	0.2
369	A	7	4	1.	20	0.2
370	A	5	3	1.	20	0.15
371	A	5	3	1.	20	0.15
372	A	5	3	1.	20	0.15
373	A	5	3	1.	18	0.167
374	A	5	3	1.	17	0.176
375	A	12	5	1.	20	0.25
376	A	12	6	1.	20	0.3
377	A	6	5	0.94	22	0.227
378	A	4	4	0.91	20	0.2
379	A	6	3	1.	22	0.136
380	A	12	4	1.	22	0.182
381	A	6	5	1.	18	0.278
382	A	6	5	1.	18	0.278
383	A	6	5	1.	18	0.278
384	A	6	5	1.	18	0.278
385	A	4	4	1.	16	0.25
386	A	3	3	1.	15	0.2
387	A	5	5	1.	18	0.278
388	A	5	5	1.	18	0.278
389	A	5	5	1.	18	0.278
390	A	7	6	1.	20	0.3
391	A	8	7	0.95	20	0.35
392	A	7	6	1.	20	0.3
393	A	8	7	0.95	20	0.35
394	A	7	6	1.	18	0.333
395	A	4	4	0.94	17	0.235
396	A	7	7	1.	20	0.35
397	A	6	6	1.	20	0.3
398	A	6	6	1.	20	0.3
399	A	7	6	0.98	20	0.3
400	A	7	6	0.97	20	0.3
401	A	7	6	0.97	20	0.3
402	A	7	6	0.96	20	0.3
403	A	7	6	0.95	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
404	A	4	4	0.96	17	0.235
405	A	7	7	0.96	20	0.35
406	A	8	8	1.	20	0.4
407	A	7	6	1.	20	0.3
408	A	7	6	1.	20	0.3
409	A	6	6	1.	20	0.3
410	A	6	6	1.	20	0.3
411	A	9	8	1.	18	0.444
412	A	6	5	1.	17	0.294
413	A	7	7	1.	20	0.35
414	A	7	7	1.	20	0.35
415	A	8	8	1.	20	0.4
416	A	12	10	1.	20	0.5
417	A	11	10	1.	20	0.5
418	A	10	9	0.99	20	0.45
419	A	10	9	1.	18	0.5
420	A	8	7	0.78	17	0.412
421	A	18	10	1.	20	0.5
422	A	20	12	1.	20	0.6
423	A	12	10	1.	20	0.5
424	A	11	9	1.	20	0.45
425	A	11	10	1.	20	0.5
426	A	11	9	1.	18	0.5
427	A	11	9	1.	17	0.529
428	A	29	12	1.	20	0.6
429	A	31	14	1.	20	0.7
430	A	7	4	0.92	22	0.182
431	A	6	4	0.95	22	0.182
432	A	5	3	1.	20	0.15
433	A	2	2	1.	15	0.133
434	A	5	3	1.	22	0.136
435	A	8	3	1.	22	0.136
436	A	10	3	1.	22	0.136
437	A	6	5	1.	40	0.125
438	A	5	5	1.	40	0.125
439	A	4	4	1.	38	0.105
440	A	3	3	1.	37	0.081
441	A	6	5	1.	40	0.125
442	A	4	4	1.	40	0.1
443	A	5	5	1.	40	0.125
444	A	6	5	1.	40	0.125
445	A	7	5	1.	40	0.125
446	A	7	6	1.	40	0.15
447	A	6	6	1.	40	0.15

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
448	A	5	5	1.	38	0.132
449	A	4	4	1.	37	0.108
450	A	7	6	1.	40	0.15
451	A	7	6	1.	40	0.15
452	A	7	6	1.	40	0.15
453	A	5	5	1.	40	0.125
454	A	6	6	1.	40	0.15
455	A	7	6	1.	40	0.15
456	A	8	6	1.	40	0.15
457	A	8	6	1.	40	0.15
458	A	7	6	1.	40	0.15
459	A	6	5	1.	38	0.132
460	A	5	4	1.	37	0.108
461	A	8	6	1.	40	0.15
462	A	8	7	1.	40	0.175
463	A	8	6	1.	40	0.15
464	A	8	7	1.	40	0.175
465	A	8	6	1.	40	0.15
466	A	6	5	1.	40	0.125
467	A	7	6	1.	40	0.15
468	A	8	6	1.	40	0.15
469	A	9	6	1.	40	0.15
470	A	5	5	1.1	40	0.125
471	A	4	4	1.	40	0.1
472	A	3	3	1.	38	0.079
473	A	1	1	1.	37	0.027
474	A	5	5	1.	40	0.125
475	A	5	5	1.	40	0.125
476	A	6	6	1.	40	0.15
477	A	6	5	1.	40	0.125
478	A	6	5	1.	40	0.125
479	A	5	5	1.	40	0.125
480	A	3	3	1.	40	0.075
481	A	2	2	1.	38	0.053
482	A	2	2	1.	37	0.054
483	A	6	5	1.	40	0.125
484	A	6	5	1.	40	0.125
485	A	7	6	1.	40	0.15
486	A	8	6	1.	40	0.15
487	A	3	3	1.	40	0.075
488	A	4	4	1.	40	0.1
489	A	4	4	1.	23	0.174
490	A	1	1	1.	23	0.043
491	A	5	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
492	A	3	3	1.	20	0.15
493	A	5	5	1.	23	0.217
494	A	5	5	1.	23	0.217
495	A	3	3	1.	23	0.13
496	A	5	4	1.	23	0.174
497	A	1	1	1.	23	0.043
498	A	6	5	1.	21	0.238
499	A	4	3	1.	20	0.15
500	A	6	5	1.	23	0.217
501	A	6	6	1.	23	0.261
502	A	4	4	1.	23	0.174
503	A	3	3	1.	23	0.13
504	A	1	1	1.	23	0.043
505	A	4	4	1.	21	0.19
506	A	2	2	1.	20	0.1
507	A	4	4	1.	23	0.174
508	A	5	5	1.	23	0.217
509	A	3	3	1.	23	0.13
510	A	3	3	1.	23	0.13
511	A	1	1	1.	23	0.043
512	A	5	5	1.	21	0.238
513	A	3	3	1.	20	0.15
514	A	5	5	1.	23	0.217
515	A	6	6	1.	23	0.261
516	A	4	4	1.	23	0.174
517	A	4	4	1.	23	0.174
518	A	1	1	1.	23	0.043
519	A	6	5	1.	21	0.238
520	A	4	3	1.	20	0.15
521	A	6	5	1.	23	0.217
522	A	7	6	1.	23	0.261
523	A	5	4	1.	23	0.174
524	A	7	6	1.	19	0.316
525	A	6	4	1.	25	0.16
526	A	6	4	1.22	25	0.16
527	A	6	4	1.	25	0.16
528	A	5	4	1.	23	0.174
529	A	4	3	1.	22	0.136
530	A	7	5	1.	25	0.2
531	A	9	6	0.97	25	0.24
532	A	12	6	1.	25	0.24
533	A	6	4	1.	25	0.16
534	A	6	4	1.	25	0.16
535	A	6	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
536	A	6	4	1.	23	0.174
537	A	5	4	1.	22	0.182
538	A	7	5	1.	25	0.2
539	A	9	6	1.	25	0.24
540	A	12	6	1.	25	0.24
541	A	6	3	1.	23	0.13
542	A	4	2	1.	23	0.087
543	A	4	2	1.	23	0.087
544	A	4	2	1.	21	0.095
545	A	4	2	1.	20	0.1
546	A	7	4	1.	23	0.174
547	A	8	4	1.	23	0.174
548	A	3	2	1.	29	0.069
549	A	3	2	1.	29	0.069
550	A	3	2	1.	29	0.069
551	A	3	2	1.	27	0.074
552	A	5	3	1.	22	0.136
553	A	3	2	1.	29	0.069
554	A	4	3	1.	29	0.103
555	A	4	3	1.	29	0.103
556	A	4	3	1.	29	0.103
557	A	3	2	1.	29	0.069
558	A	3	2	1.	29	0.069
559	A	3	2	1.	29	0.069
560	A	3	2	1.	29	0.069
561	A	3	2	1.	29	0.069
562	A	3	2	1.	29	0.069
563	A	3	2	1.	27	0.074
564	A	2	2	1.	22	0.091
565	A	4	3	1.	29	0.103
566	A	4	3	1.	29	0.103
567	A	4	3	1.	29	0.103
568	A	4	3	1.	29	0.103
569	A	3	2	1.	29	0.069
570	A	3	2	1.	29	0.069
571	A	3	2	1.	29	0.069
572	A	3	2	1.	29	0.069
573	A	3	2	1.	29	0.069
574	A	4	3	1.	29	0.103
575	A	4	3	1.	27	0.111
576	A	3	3	1.	22	0.136
577	A	4	3	1.	29	0.103
578	A	4	3	1.	29	0.103
579	A	7	5	1.	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
580	A	6	4	1.	31	0.129
581	A	5	4	1.	31	0.129
582	A	3	3	1.	31	0.097
583	A	3	3	1.	29	0.103
584	A	4	3	1.	24	0.125
585	A	6	5	1.	31	0.161
586	A	6	4	1.	31	0.129
587	A	7	5	1.	31	0.161
588	A	4	3	1.	24	0.125
589	A	3	2	1.	24	0.083
590	A	3	2	1.	24	0.083
591	A	2	1	1.	22	0.045
592	A	2	1	1.	17	0.059
593	A	4	3	1.	24	0.125
594	A	4	4	1.	24	0.167
595	A	4	4	1.	24	0.167
596	A	3	2	1.	24	0.083
597	A	3	2	1.	24	0.083
598	A	2	1	1.	22	0.045
599	A	2	1	1.	17	0.059
600	A	4	3	1.	24	0.125
601	A	4	4	1.	24	0.167
602	A	5	5	1.	24	0.208
603	A	5	5	1.	26	0.192
604	A	1	1	1.	22	0.045
605	A	11	8	1.	28	0.286
606	A	10	7	1.	28	0.25
607	A	6	3	1.	28	0.107
608	A	8	5	1.	28	0.179
609	A	11	7	1.	28	0.25
610	A	11	7	1.	28	0.25
611	A	6	3	1.	28	0.107
612	A	6	3	1.	28	0.107
613	A	8	4	1.	28	0.143
614	A	19	9	1.	28	0.321
615	A	8	5	1.	28	0.179
616	A	8	4	1.	28	0.143
617	A	12	6	0.99	28	0.214
618	A	6	3	1.	20	0.15
619	A	5	5	1.	26	0.192
620	A	6	6	1.	30	0.2
621	A	5	5	1.	29	0.172
622	A	10	6	1.	28	0.214
623	A	9	6	1.	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
624	A	7	6	1.	26	0.231
625	A	7	6	1.	21	0.286
626	A	14	10	1.	28	0.357
627	A	14	10	1.	28	0.357
628	A	23	11	1.	28	0.393
629	A	9	6	1.	28	0.214
630	A	8	6	0.99	28	0.214
631	A	6	5	1.	26	0.192
632	A	6	5	1.	21	0.238
633	A	10	9	1.	28	0.321
634	A	14	10	1.	28	0.357
635	A	23	11	1.	28	0.393
636	A	8	6	0.99	28	0.214
637	A	7	6	1.	28	0.214
638	A	6	5	1.	26	0.192
639	A	2	2	1.	21	0.095
640	A	7	7	1.	28	0.25
641	A	14	10	1.	28	0.357
642	A	23	11	1.	28	0.393
643	A	16	10	1.35	28	0.357
644	A	10	8	1.	28	0.286
645	A	7	6	1.	28	0.214
646	A	7	6	1.	28	0.214
647	A	5	4	1.	26	0.154
648	A	2	2	1.	21	0.095
649	A	4	4	1.	28	0.143
650	A	14	10	1.	28	0.357
651	A	23	10	1.	28	0.357
652	A	10	9	1.	28	0.321
653	A	17	12	1.	28	0.429
654	A	4	4	1.	28	0.143
655	A	2	2	1.	30	0.067
656	A	4	3	1.	26	0.115
657	A	4	3	1.	46	0.065
658	A	3	3	1.	46	0.065
659	A	2	2	1.	44	0.045
660	A	1	1	1.	39	0.026
661	A	2	2	1.	46	0.043
662	A	3	3	1.	46	0.065
663	A	4	3	1.	46	0.065
664	A	5	3	1.	46	0.065
665	A	4	4	1.	46	0.087
666	A	3	3	1.	46	0.065
667	A	2	2	1.	44	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
668	A	1	1	1.	39	0.026
669	A	3	3	1.	46	0.065
670	A	4	4	1.	46	0.087
671	A	5	4	1.	46	0.087
672	A	4	3	1.	46	0.065
673	A	3	3	1.	46	0.065
674	A	2	2	1.	44	0.045
675	A	1	1	1.	39	0.026
676	A	4	3	1.	46	0.065
677	A	5	4	1.	46	0.087
678	A	6	4	1.	46	0.087
679	A	5	3	1.	46	0.065
680	A	4	3	1.	46	0.065
681	A	3	3	1.	46	0.065
682	A	2	2	1.	44	0.045
683	A	1	1	1.	39	0.026
684	A	3	3	1.	46	0.065
685	A	3	3	1.	46	0.065
686	A	4	4	1.	46	0.087
687	A	5	4	1.	46	0.087
688	A	6	4	1.	46	0.087
689	A	5	3	1.	46	0.065
690	A	4	3	1.	46	0.065
691	A	3	3	1.	46	0.065
692	A	2	2	1.	44	0.045
693	A	1	1	1.	39	0.026
694	A	4	3	1.	46	0.065
695	A	4	4	1.	46	0.087
696	A	4	3	1.	46	0.065
697	A	5	4	1.	46	0.087
698	A	6	4	1.	46	0.087
699	A	7	4	1.	46	0.087
700	A	5	3	1.	46	0.065
701	A	4	3	1.	46	0.065
702	A	3	3	1.	46	0.065
703	A	2	2	1.	44	0.045
704	A	1	1	1.	39	0.026
705	A	5	3	1.	46	0.065
706	A	5	4	1.	46	0.087
707	A	5	4	1.	46	0.087
708	A	5	3	1.	46	0.065
709	A	6	4	1.	46	0.087
710	A	7	4	1.	46	0.087
711	A	8	4	1.	46	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
712	A	7	5	1.	48	0.104
713	A	6	5	1.	48	0.104
714	A	5	5	1.	48	0.104
715	A	4	4	1.	48	0.083
716	A	1	1	1.	48	0.021
717	A	2	2	1.	48	0.042
718	A	3	2	1.	48	0.042
719	A	4	2	1.	48	0.042
720	A	7	6	1.	48	0.125
721	A	6	6	1.	48	0.125
722	A	5	5	1.	48	0.104
723	A	1	1	1.	48	0.021
724	A	2	2	1.	48	0.042
725	A	3	3	1.	48	0.062
726	A	4	3	1.	48	0.062
727	A	7	6	1.	48	0.125
728	A	6	5	1.	48	0.104
729	A	1	1	1.	48	0.021
730	A	2	2	1.	48	0.042
731	A	3	2	1.	48	0.042
732	A	4	3	1.	48	0.062
733	A	8	6	1.	48	0.125
734	A	7	6	1.	48	0.125
735	A	6	6	1.	48	0.125
736	A	5	5	1.	48	0.104
737	A	5	5	1.	48	0.104
738	A	1	1	1.	48	0.021
739	A	2	2	1.	48	0.042
740	A	3	2	1.	48	0.042
741	A	4	2	1.	48	0.042
742	A	8	6	1.	48	0.125
743	A	7	6	1.	48	0.125
744	A	6	5	1.	48	0.104
745	A	6	6	1.	48	0.125
746	A	6	5	1.	48	0.104
747	A	1	1	1.	48	0.021
748	A	2	2	1.	48	0.042
749	A	3	2	1.	48	0.042
750	A	4	2	1.	48	0.042
751	A	9	6	1.	48	0.125
752	A	8	6	1.	48	0.125
753	A	7	5	1.	48	0.104
754	A	7	6	1.	48	0.125
755	A	7	6	1.	48	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
756	A	7	5	1.	48	0.104
757	A	1	1	1.	48	0.021
758	A	2	2	1.	48	0.042
759	A	3	2	1.	48	0.042
760	A	4	2	1.	48	0.042
761	A	3	3	1.17	46	0.065
762	A	3	3	1.06	46	0.065
763	A	3	3	1.13	46	0.065
764	A	3	3	1.15	46	0.065
765	A	3	3	1.17	46	0.065
766	A	3	3	1.17	46	0.065
767	A	3	3	1.04	44	0.068
768	A	4	3	1.	44	0.068
769	A	3	3	1.	44	0.068
770	A	2	2	1.	42	0.048
771	A	1	1	1.	37	0.027
772	A	2	2	1.	44	0.045
773	A	2	2	1.	44	0.045
774	A	2	2	1.	44	0.045
775	A	3	3	1.	46	0.065
776	A	3	3	1.	46	0.065
777	A	3	3	1.	46	0.065
778	A	3	3	1.	46	0.065
779	A	3	3	1.	46	0.065
780	A	1	1	1.	47	0.021
781	A	3	3	1.	73	0.041
782	A	4	4	1.04	46	0.087
783	A	6	4	1.	46	0.087
784	A	5	4	1.	46	0.087
785	A	4	4	1.	46	0.087
786	A	3	3	1.	44	0.068
787	A	2	2	1.	39	0.051
788	A	3	3	1.	46	0.065
789	A	3	3	1.	46	0.065
790	A	4	4	1.	46	0.087
791	A	5	4	1.	46	0.087
792	A	8	4	1.	32	0.125
793	A	6	4	1.	32	0.125
794	A	4	4	1.	30	0.133
795	A	6	4	1.	32	0.125
796	A	7	5	1.	32	0.156
797	A	7	5	1.	32	0.156
798	A	5	5	1.	32	0.156
799	A	4	4	1.	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
800	A	7	5	1.	32	0.156
801	A	8	6	1.	32	0.188
802	A	3	3	1.	25	0.12
803	A	4	3	1.	25	0.12
804	A	4	3	1.	28	0.107
805	A	2	1	1.	28	0.036
806	A	2	1	1.	28	0.036
807	A	2	1	1.	26	0.038
808	A	2	1	1.	21	0.048
809	A	3	3	1.	28	0.107
810	A	3	2	1.	28	0.071
811	A	3	3	1.	28	0.107
812	A	3	3	1.	28	0.107
813	A	4	4	0.98	28	0.143
814	A	2	1	1.	25	0.04
815	A	2	1	1.	27	0.037
816	A	2	1	1.	27	0.037
817	A	6	5	1.	27	0.185
818	A	9	6	1.	27	0.222
819	A	3	2	1.	27	0.074
820	A	3	2	1.	27	0.074
821	A	2	1	1.	25	0.04
822	A	2	1	1.	20	0.05
823	A	4	3	1.	27	0.111
824	A	4	4	1.	27	0.148
825	A	4	4	1.	27	0.148
826	A	3	2	1.	27	0.074
827	A	3	2	1.	27	0.074
828	A	2	1	1.	25	0.04
829	A	2	1	1.	20	0.05
830	A	4	3	1.	27	0.111
831	A	4	4	1.	27	0.148
832	A	5	5	1.	27	0.185
833	B	9	7	2.1	25	0.28
834	A	5	5	1.	29	0.172
835	A	7	6	1.	29	0.207
836	A	6	6	1.	29	0.207
837	A	5	5	1.	29	0.172
838	A	5	5	1.	29	0.172
839	A	5	5	1.	29	0.172
840	A	3	3	1.	29	0.103
841	A	4	4	1.	29	0.138
842	A	6	6	1.	29	0.207
843	A	7	6	1.	38	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
844	A	6	6	1.	38	0.158
845	A	5	5	1.	38	0.132
846	A	5	5	1.	38	0.132
847	A	5	5	1.	38	0.132
848	A	3	3	1.	38	0.079
849	A	4	4	1.	38	0.105
850	A	11	7	1.	31	0.226
851	A	6	3	1.	31	0.097
852	A	6	3	1.	31	0.097
853	A	8	4	1.	31	0.129
854	A	8	6	1.	29	0.207
855	A	7	6	1.	29	0.207
856	A	6	5	1.	27	0.185
857	A	6	5	1.	22	0.227
858	A	8	5	1.	29	0.172
859	A	20	7	1.	29	0.241
860	A	23	8	1.	29	0.276
861	A	27	9	1.	29	0.31
862	A	9	6	1.	29	0.207
863	A	8	6	1.	29	0.207
864	A	7	5	1.	27	0.185
865	A	7	6	1.	22	0.273
866	A	13	7	1.	29	0.241
867	A	23	8	1.	29	0.276
868	A	30	9	1.	29	0.31
869	A	15	7	1.	29	0.241
870	A	8	5	1.	29	0.172
871	A	7	5	1.	29	0.172
872	A	6	5	1.	29	0.172
873	A	5	4	1.	27	0.148
874	A	2	2	1.	22	0.091
875	A	6	3	1.	29	0.103
876	A	9	4	1.	29	0.138
877	A	13	6	1.	29	0.207
878	A	7	6	1.	29	0.207
879	A	6	5	1.	29	0.172
880	A	4	4	1.	29	0.138
881	A	4	4	1.	27	0.148
882	A	4	4	1.	22	0.182
883	A	10	5	1.	29	0.172
884	A	14	6	1.	29	0.207
885	A	19	7	1.	29	0.241
886	A	10	6	1.	31	0.194
887	A	9	6	1.	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
888	A	7	6	1.	29	0.207
889	A	7	6	1.	24	0.25
890	A	15	10	1.27	31	0.323
891	A	15	10	1.26	31	0.323
892	A	25	11	1.65	31	0.355
893	A	9	6	1.	31	0.194
894	A	8	6	1.	31	0.194
895	A	6	5	1.	29	0.172
896	A	6	5	1.	24	0.208
897	A	11	9	1.	31	0.29
898	A	15	10	1.3	31	0.323
899	A	25	11	1.67	31	0.355
900	A	8	6	1.	31	0.194
901	A	7	6	1.	31	0.194
902	A	6	5	1.	29	0.172
903	A	2	2	1.	24	0.083
904	A	8	7	1.	31	0.226
905	A	15	10	1.	31	0.323
906	A	25	11	1.	31	0.355
907	A	11	8	1.	31	0.258
908	A	17	10	1.	31	0.323
909	A	7	6	1.	31	0.194
910	A	7	6	1.	31	0.194
911	A	5	4	1.	29	0.138
912	A	2	2	1.	24	0.083
913	A	5	4	1.	31	0.129
914	A	15	10	1.	31	0.323
915	A	25	10	1.58	31	0.323
916	A	11	9	1.	31	0.29
917	A	18	12	1.	31	0.387
918	A	1	1	1.	33	0.03
919	A	2	2	1.	33	0.061
920	A	2	1	1.	25	0.04
921	A	2	1	1.	23	0.043
922	A	3	3	1.	25	0.12
923	A	3	3	1.	25	0.12
924	A	3	3	0.99	25	0.12
925	A	2	1	1.	27	0.037
926	A	2	1	1.	25	0.04
927	A	4	3	1.	27	0.111
928	A	4	3	1.	27	0.111
929	A	5	5	1.	27	0.185
930	A	4	2	1.	27	0.074
931	A	4	2	1.	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
932	A	4	2	1.	27	0.074
933	A	4	2	1.	25	0.08
934	A	4	2	1.	20	0.1
935	A	7	3	1.	27	0.111
936	A	8	3	1.	27	0.111
937	A	5	3	1.	27	0.111
938	A	5	3	1.	27	0.111
939	A	5	3	1.	27	0.111
940	A	5	3	1.	25	0.12
941	A	5	3	1.	20	0.15
942	A	12	4	1.	27	0.148
943	A	13	4	1.	27	0.148
944	A	4	4	0.97	27	0.148
945	A	6	4	0.99	29	0.138
946	A	5	3	1.	27	0.111
947	A	2	2	1.	22	0.091
948	A	0	0	0.	0	0.
949	A	6	4	1.	29	0.138
950	A	5	3	1.	27	0.111
951	A	2	2	1.	22	0.091
952	A	0	0	0.	0	0.
953	A	4	4	0.99	25	0.16
954	A	6	4	1.	27	0.148
955	A	5	3	1.	25	0.12
956	A	2	2	1.	20	0.1
957	A	0	0	0.	0	0.
958	A	5	5	1.	27	0.185

Chapter 3

Listing of integrals

3.1 $\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=132

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - (d^2*(d^2 - e^2*x^2)^(3/2))/(3*e^3) -
 (d*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rubi [A] time = 0.0757274, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {797, 641, 195, 217, 203}

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - (d^2*(d^2 - e^2*x^2)^(3/2))/(3*e^3) -
 (d*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx &= -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}dx}{e^2}+\frac{d^2\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2}\\ &= -\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{d\int \left(d^2-e^2x^2\right)^{3/2}dx}{e^2}+\frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{e^2}\\ &= \frac{d^3x\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\left(3d^3\right)\int \sqrt{d^2-e^2}\\ &= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\left(3d^5\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}}{8e^2}\\ &= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x}}\right.}{2e^3}\\ &= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x}}\right.}{8e^3}\end{aligned}$$

Mathematica [A] time = 0.136492, size = 112, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(-8d^2e^2x^2-15d^3ex-16d^4+30de^3x^3+24e^4x^4\right)+15d^4\sin^{-1}\left(\frac{ex}{d}\right)\right)}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-16*d^4 - 15*d^3*e*x - 8*d^2
*e^2*x^2 + 30*d*e^3*x^3 + 24*e^4*x^4) + 15*d^4*ArcSin[(e*x)/d]))/(120*e^3*S
qrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.058, size = 125, normalized size = 1.

$$-\frac{x^2}{5e}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{2d^2}{15e^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{dx}{4e^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{d^3x}{8e^2}\sqrt{-x^2e^2+d^2}+\frac{d^5}{8e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*x^2*(-e^2*x^2+d^2)^(3/2)/e-2/15*d^2*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*d*x*(
-e^2*x^2+d^2)^(3/2)/e^2+1/8*d^3*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/8*d^5/e^2/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.60867, size = 158, normalized size = 1.2

$$\frac{d^5 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{8\sqrt{e^2}e^2} + \frac{\sqrt{-e^2x^2 + d^2}d^3 x}{8\,e^2} - \frac{\left(-e^2x^2 + d^2\right)^{\frac{3}{2}} x^2}{5\,e} - \frac{\left(-e^2x^2 + d^2\right)^{\frac{3}{2}} dx}{4\,e^2} - \frac{2\left(-e^2x^2 + d^2\right)^{\frac{3}{2}} d^2}{15\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*d^5*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) + 1/8*sqrt(-e^2*x^2 + d
^2)*d^3*x/e^2 - 1/5*(-e^2*x^2 + d^2)^(3/2)*x^2/e - 1/4*(-e^2*x^2 + d^2)^(3/
2)*d*x/e^2 - 2/15*(-e^2*x^2 + d^2)^(3/2)*d^2/e^3
```

Fricas [A] time = 1.8215, size = 205, normalized size = 1.55

$$-\frac{30\,d^5 \arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) - \left(24\,e^4x^4 + 30\,de^3x^3 - 8\,d^2e^2x^2 - 15\,d^3ex - 16\,d^4\right)\sqrt{-e^2x^2 + d^2}}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 + 30
*d*e^3*x^3 - 8*d^2*e^2*x^2 - 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [C] time = 6.22928, size = 280, normalized size = 2.12

$$d\left(\begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2x^2|}{|d^2|} > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2}}{5} \\ \frac{x^4\sqrt{d^2}}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/
d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1
 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e*
```

```
Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
 e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqr
t(d**2)/4, True))
```

Giac [A] time = 1.26262, size = 100, normalized size = 0.76

$$\frac{1}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\,(d)-\frac{1}{120}\left(16\,d^4e^{(-3)}+\left(15\,d^3e^{(-2)}+2\left(4\,d^2e^{(-1)}-3\,(4\,xe+5\,d)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^5*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/120*(16*d^4*e^(-3) + (15*d^3*e^(-2)
 + 2*(4*d^2*e^(-1) - 3*(4*x*e + 5*d)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.2 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=201

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{d^3(128d+315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-$$

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) - (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) - (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d^3*(128*d + 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rubi [A] time = 0.149388, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {833, 780, 195, 217, 203}

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}-\frac{d^3(128d+315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}-\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```


```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) - (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) - (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d^3*(128*d + 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```


Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx = -\frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{\int x^3\left(-4d^2e-9de^2x\right)\left(d^2-e^2x^2\right)^{3/2}dx}{9e^2}$$

$$= -\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} + \frac{\int x^2\left(27d^3e^2+32d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}dx}{72e^4}$$

$$= -\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{\int x\left(-64d^4e^3-189d^3e\right.}{504}$$

$$= -\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{d^3(128d+315ex)\left(d^2-\right.}{5040e^5}$$

$$= \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} - \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{d^3(1}{}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} - \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(\right.}{}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} - \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(\right.}{}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} - \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} - \frac{x^4\left(\right.}{}$$

Mathematica [A] time = 0.239684, size = 157, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}\left(945d^8\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(512d^6e^2x^2+630d^5e^3x^3+384d^4e^4x^4-7560d^3e^5x^5-6400d^2e^6x^6+945d^7ex+\right.\right.}{40320e^5\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(1024*d^8 + 945*d^7*e*x + 5
12*d^6*e^2*x^2 + 630*d^5*e^3*x^3 + 384*d^4*e^4*x^4 - 7560*d^3*e^5*x^5 - 640
0*d^2*e^6*x^6 + 5040*d*e^7*x^7 + 4480*e^8*x^8)) + 945*d^8*ArcSin[(e*x)/d]))
/(40320*e^5*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.067, size = 198, normalized size = 1.

$$-\frac{x^4}{9e}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{4d^2x^2}{63e^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{8d^4}{315e^5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{dx^3}{8e^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{d^3x}{16e^4}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} + \frac{d^5x}{64e^4}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e-4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-8/315
*d^4/e^5*(-e^2*x^2+d^2)^(5/2)-1/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2-1/16*d^3/e
^4*x*(-e^2*x^2+d^2)^(5/2)+1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4+3/128*d^7*x*(
-e^2*x^2+d^2)^(1/2)/e^4+3/128*d^9/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^
2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.75911, size = 257, normalized size = 1.28

$$-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^4}{9\,e}+\frac{3\,d^9\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{128\,\sqrt{e^2}e^4}+\frac{3\,\sqrt{-e^2x^2+d^2}d^7x}{128\,e^4}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^3}{8\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x}{64\,e^4}-\frac{4\left(-e^2x^2+\cdots\right.}{63\,e\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*(-e^2*x^2 + d^2)^(5/2)*x^4/e + 3/128*d^9*arcsin(e^2*x/sqrt(d^2*e^2))/(
sqrt(e^2)*e^4) + 3/128*sqrt(-e^2*x^2 + d^2)*d^7*x/e^4 - 1/8*(-e^2*x^2 + d^2
)^(5/2)*d*x^3/e^2 + 1/64*(-e^2*x^2 + d^2)^(3/2)*d^5*x/e^4 - 4/63*(-e^2*x^2
+ d^2)^(5/2)*d^2*x^2/e^3 - 1/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 8/315*(-
e^2*x^2 + d^2)^(5/2)*d^4/e^5
```

Fricas [A] time = 1.82994, size = 323, normalized size = 1.61

$$-\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4480\,e^8x^8+5040\,de^7x^7-6400\,d^2e^6x^6-7560\,d^3e^5x^5+384\,d^4e^4x^4+630\,d^5e^3x^3+\cdots\right.}{40320\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4480*e^8*x^
8 + 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 - 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^4
+ 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 + 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*x^2
 + d^2))/e^5
```

Sympy [C] time = 22.206, size = 833, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
```

```
2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5
*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x*
*2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1
- e**2*x**2/d**2)), True)) + d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**
2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*s
qrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)),
 (x**6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128
*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(3
84*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x
**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/
d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/
(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**
2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 -
 e**2*x**2/d**2)), True)) - e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)
/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*s
qrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**
2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [A] time = 1.29974, size = 158, normalized size = 0.79

$$\frac{3}{128} d^9 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \mathrm{sgn}\,(d) - \frac{1}{40320} \left(1024\, d^8 e^{(-5)} + \left(945\, d^7 e^{(-4)} + 2\left(256\, d^6 e^{(-3)} + \left(315\, d^5 e^{(-2)} + 4\left(48\, d^4 e^{(-1)} - 5\left(18\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^9*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/40320*(1024*d^8*e^(-5) + (945*d^7
*e^(-4) + 2*(256*d^6*e^(-3) + (315*d^5*e^(-2) + 4*(48*d^4*e^(-1) - 5*(189*d
^3 + 2*(80*d^2*e - 7*(8*x*e^3 + 9*d*e^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

3.3 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}+\frac{3d^8\,\mathrm{t}}{}$$

```
[Out] (3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) + (d^4*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) - (x^3*(d^2 - e^2*x^2)^(5/2))
/(8*e) - (d^2*(32*d + 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) + (3*d^8*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rubi [A] time = 0.100889, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {833, 780, 195, 217, 203}

$$\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}+\frac{3d^8\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) + (d^4*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) - (x^3*(d^2 - e^2*x^2)^(5/2))
/(8*e) - (d^2*(32*d + 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) + (3*d^8*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{\int x^2\left(-3d^2e-8de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$
$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}+\frac{\int x\left(16d^3e^2+21d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{56e^4}$$
$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}+\frac{d^4\int\left(d^2-e^2x\right.}{16e^3}$$
$$= \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)}{560e^4}$$
$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d}{}$$
$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d}{}$$
$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}-\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{d^2(32d}{}$$

Mathematica [A] time = 0.203894, size = 146, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(105d^7\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(128d^5e^2x^2+70d^4e^3x^3-1024d^3e^4x^4-840d^2e^5x^5+105d^6ex+256d^7+640de^6\right.\right.}{4480e^4\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(256*d^7 + 105*d^6*e*x + 12
8*d^5*e^2*x^2 + 70*d^4*e^3*x^3 - 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 + 640*d
*e^6*x^6 + 560*e^7*x^7)) + 105*d^7*ArcSin[(e*x)/d]))/(4480*e^4*Sqrt[1 - (e^
2*x^2)/d^2])
```

Maple [A] time = 0.06, size = 173, normalized size = 1.

$$-\frac{x^3}{8e}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{d^2x}{16e^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{d^4x}{64e^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{3d^6x}{128e^3}\sqrt{-x^2e^2+d^2}+\frac{3d^8}{128e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/8*x^3*(-e^2*x^2+d^2)^(5/2)/e-1/16*d^2/e^3*x*(-e^2*x^2+d^2)^(5/2)+1/64*d^
4*x*(-e^2*x^2+d^2)^(3/2)/e^3+3/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^3+3/128*d^8
/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/7*d*x^2*(-e^2
*x^2+d^2)^(5/2)/e^2-2/35*d^3/e^4*(-e^2*x^2+d^2)^(5/2)
```

Maxima [A] time = 1.5631, size = 223, normalized size = 1.3

$$\frac{3\,d^8 \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{128\,\sqrt{e^2}e^3} + \frac{3\,\sqrt{-e^2x^2+d^2}d^6x}{128\,e^3} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^3}{8\,e} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^4x}{64\,e^3} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx^2}{7\,e^2} - \frac{\left(-e^2x^2+d^2\right)}{16\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*d^8*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^3) + 3/128*sqrt(-e^2*x^2
 + d^2)*d^6*x/e^3 - 1/8*(-e^2*x^2 + d^2)^(5/2)*x^3/e + 1/64*(-e^2*x^2 + d^2
)^(3/2)*d^4*x/e^3 - 1/7*(-e^2*x^2 + d^2)^(5/2)*d*x^2/e^2 - 1/16*(-e^2*x^2 +
 d^2)^(5/2)*d^2*x/e^3 - 2/35*(-e^2*x^2 + d^2)^(5/2)*d^3/e^4
```

Fricas [A] time = 1.76217, size = 289, normalized size = 1.68

$$-\frac{210\,d^8 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(560\,e^7x^7 + 640\,de^6x^6 - 840\,d^2e^5x^5 - 1024\,d^3e^4x^4 + 70\,d^4e^3x^3 + 128\,d^5e^2x^2 + 105\,d^6\right.}{4480\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 +
 640*d*e^6*x^6 - 840*d^2*e^5*x^5 - 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 + 128*
d^5*e^2*x^2 + 105*d^6*e*x + 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] time = 19.5484, size = 779, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + d**2*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*a
sin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**
3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2
)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((-
8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**
```

```
2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2
 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - e**3*Piecewise((-5
*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/
d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(
192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d*
*2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2
) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x
**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(1
92*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2))
- e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.31202, size = 143, normalized size = 0.83

$$\frac{3}{128}\,d^8\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}\left(d\right)-\frac{1}{4480}\left(256\,d^7e^{(-4)}+\left(105\,d^6e^{(-3)}+2\left(64\,d^5e^{(-2)}+\left(35\,d^4e^{(-1)}-4\left(128\,d^3+5\left(21\,d^2e-2\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^8*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/4480*(256*d^7*e^(-4) + (105*d^6*e
^(-3) + 2*(64*d^5*e^(-2) + (35*d^4*e^(-1) - 4*(128*d^3 + 5*(21*d^2*e - 2*(7
*x*e^3 + 8*d*e^2)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.4 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) - (d^2*(d^2 - e^2*x^2)^(5/2))/(5*e^3) - (d*x*(d^2 - e^2*x^2)^(5/2))/(6*e
^2) + (d^2 - e^2*x^2)^(7/2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(16*e^3)
```

Rubi [A] time = 0.10801, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {797, 641, 195, 217, 203}

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) - (d^2*(d^2 - e^2*x^2)^(5/2))/(5*e^3) - (d*x*(d^2 - e^2*x^2)^(5/2))/(6*e
^2) + (d^2 - e^2*x^2)^(7/2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(16*e^3)
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx = -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}dx}{e^2}+\frac{d^2\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}dx}{e^2}$$
$$= -\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{d\int\left(d^2-e^2x^2\right)^{5/2}dx}{e^2}+\frac{d^3\int\left(d^2-e^2x^2\right)^{3/2}dx}{e^2}$$
$$= \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{\left(5d^3\right)\int\left(d\right.}{}$$
$$= \frac{3d^5x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2\right.}{7e^3}$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x\right.}{7e^3}$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x\right.}{7e^3}$$
$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}-\frac{d^2\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{dx\left(d^2-e^2x^2\right)^{5/2}}{6e^2}+\frac{\left(d^2-e^2x\right.}{7e^3}$$

Mathematica [A] time = 0.191435, size = 135, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(105d^6\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(48d^4e^2x^2-490d^3e^3x^3-384d^2e^4x^4+105d^5ex+96d^6+280de^5x^5+240e^6x^6\right)\right)}{1680e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(96*d^6 + 105*d^5*e*x + 48*
d^4*e^2*x^2 - 490*d^3*e^3*x^3 - 384*d^2*e^4*x^4 + 280*d*e^5*x^5 + 240*e^6*x
^6)) + 105*d^6*ArcSin[(e*x)/d]))/(1680*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.059, size = 148, normalized size = 0.9

$$-\frac{x^2}{7e}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{2d^2}{35e^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{dx}{6e^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{d^3x}{24e^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{d^5x}{16e^2}\sqrt{-x^2e^2+d^2}+\frac{d^7}{16e^2}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/7*x^2*(-e^2*x^2+d^2)^(5/2)/e-2/35*d^2*(-e^2*x^2+d^2)^(5/2)/e^3-1/6*d*x*(
-e^2*x^2+d^2)^(5/2)/e^2+1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/16*d^5*x*(-e^
2*x^2+d^2)^(1/2)/e^2+1/16*d^7/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))
```

Maxima [A] time = 1.57686, size = 189, normalized size = 1.19

$$\frac{d^7 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{16\sqrt{e^2}e^2} + \frac{\sqrt{-e^2x^2+d^2}d^5x}{16e^2} + \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x}{24e^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x^2}{7e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx}{6e^2} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^2}{35e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^7*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) + 1/16*sqrt(-e^2*x^2 +
 d^2)*d^5*x/e^2 + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^3*x/e^2 - 1/7*(-e^2*x^2 + d
^2)^(5/2)*x^2/e - 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x/e^2 - 2/35*(-e^2*x^2 + d^2
)^(5/2)*d^2/e^3
```

Fricas [A] time = 1.91371, size = 262, normalized size = 1.65

$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(240\,e^6x^6 + 280\,de^5x^5 - 384\,d^2e^4x^4 - 490\,d^3e^3x^3 + 48\,d^4e^2x^2 + 105\,d^5ex + 96\,d^6\right)\sqrt{-e^2x^2+d^2}}{1680\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (240*e^6*x^6 +
 280*d*e^5*x^5 - 384*d^2*e^4*x^4 - 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 + 105*d
^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [C] time = 14.9523, size = 656, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e
*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqr
t(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) +
 d**2*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqr
t(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (
x**4*sqrt(d**2)/4, True)) - d*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5
) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqr
t(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**
6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*
x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d
**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - e**3*Piecewise((
-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x*
*2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**
```

```
2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

Giac [A] time = 1.30189, size = 130, normalized size = 0.82

$$\frac{1}{16}\, d^7 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \mathrm{sgn}\left(d\right) - \frac{1}{1680} \left(96\, d^6 e^{(-3)} + \left(105\, d^5 e^{(-2)} + 2\left(24\, d^4 e^{(-1)} - \left(245\, d^3 + 4\left(48\, d^2 e - 5\left(6\, xe^3 + 7\, de^2\right)x\right)\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^7*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/1680*(96*d^6*e^(-3) + (105*d^5*e^(
-2) + 2*(24*d^4*e^(-1) - (245*d^3 + 4*(48*d^2*e - 5*(6*x*e^3 + 7*d*e^2)*x)*
x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.5 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rubi [A] time = 0.0337452, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {780, 195, 217, 203}

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}$$
$$= \frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int \sqrt{d^2-e^2x^2}\,dx}{8e}$$
$$= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e}$$
$$= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,\right.}{16e}$$
$$= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Mathematica [A] time = 0.162085, size = 124, normalized size = 1.07

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^5\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(-96d^3e^2x^2-70d^2e^3x^3+15d^4ex+48d^5+48de^4x^4+40e^5x^5\right)\right)}{240e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(48*d^5 + 15*d^4*e*x - 96*d
^3*e^2*x^2 - 70*d^2*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5)) + 15*d^5*ArcSin[(
e*x)/d]))/(240*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.055, size = 123, normalized size = 1.1

$$-\frac{x}{6e}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{d^2x}{24e}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{d^4x}{16e}\sqrt{-x^2e^2+d^2}+\frac{d^6}{16e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{d}{5e^2}\left(-x^2e^2+d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/6*x*(-e^2*x^2+d^2)^(5/2)/e+1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e+1/16*d^4*x*
(-e^2*x^2+d^2)^(1/2)/e+1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))-1/5*d*(-e^2*x^2+d^2)^(5/2)/e^2
```

Maxima [A] time = 1.61585, size = 155, normalized size = 1.34

$$\frac{d^6\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{16\sqrt{e^2}e}+\frac{\sqrt{-e^2x^2+d^2}d^4x}{16e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e) + 1/16*sqrt(-e^2*x^2 + d
^2)*d^4*x/e + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^(5
/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

Fricas [A] time = 1.88607, size = 230, normalized size = 1.98

$$-\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(-e
^2*x^2 + d^2))/e^2
```

Sympy [A] time = 17.0263, size = 583, normalized size = 5.03

$$d^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d
**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**
2*x**2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(1
5*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**
2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**3*Piecewise((-I*d**6*
acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d
**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**
2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)
/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**
2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24
*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True
))
```

Giac [A] time = 1.2919, size = 113, normalized size = 0.97

$$\frac{1}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\operatorname{sgn}\left(d\right)-\frac{1}{240}\left(48\,d^5e^{(-2)}+\left(15\,d^4e^{(-1)}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,xe^3+6\,de^2\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) + (15*d^4*e^(-1
) - 2*(48*d^3 + (35*d^2*e - 4*(5*x*e^3 + 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

3.6 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rubi [A] time = 0.0333767, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {780, 195, 217, 203}

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}dx &= -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}dx}{6e}\\ &= \frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}\\ &= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int\frac{1}{\sqrt{d^2-e^2x^2}}dx}{16e}\\ &= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\,\mathrm{Subst}\left(\int\frac{1}{1+e^2x^2}dx,\right.}{16e}\\ &= \frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}\end{aligned}$$

Mathematica [A] time = 0.0473632, size = 124, normalized size = 1.07

$$\frac{\sqrt{d^2-e^2x^2}\left(15d^5\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(-96d^3e^2x^2-70d^2e^3x^3+15d^4ex+48d^5+48de^4x^4+40e^5x^5\right)\right)}{240e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(48*d^5 + 15*d^4*e*x - 96*d
^3*e^2*x^2 - 70*d^2*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5)) + 15*d^5*ArcSin[(
e*x)/d]))/(240*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.052, size = 123, normalized size = 1.1

$$-\frac{x}{6e}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{d^2x}{24e}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{d^4x}{16e}\sqrt{-x^2e^2+d^2}+\frac{d^6}{16e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{d}{5e^2}\left(-x^2e^2+d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/6*x*(-e^2*x^2+d^2)^(5/2)/e+1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e+1/16*d^4*x*
(-e^2*x^2+d^2)^(1/2)/e+1/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))-1/5*d*(-e^2*x^2+d^2)^(5/2)/e^2
```

Maxima [A] time = 1.53727, size = 155, normalized size = 1.34

$$\frac{d^6\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{16\sqrt{e^2}e}+\frac{\sqrt{-e^2x^2+d^2}d^4x}{16e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x}{24e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}x}{6e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e) + 1/16*sqrt(-e^2*x^2 + d
^2)*d^4*x/e + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^2*x/e - 1/6*(-e^2*x^2 + d^2)^(5
/2)*x/e - 1/5*(-e^2*x^2 + d^2)^(5/2)*d/e^2
```

Fricas [A] time = 1.93184, size = 230, normalized size = 1.98

$$-\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+48\,de^4x^4-70\,d^2e^3x^3-96\,d^3e^2x^2+15\,d^4ex+48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 48
*d*e^4*x^4 - 70*d^2*e^3*x^3 - 96*d^3*e^2*x^2 + 15*d^4*e*x + 48*d^5)*sqrt(-e
^2*x^2 + d^2))/e^2
```

Sympy [A] time = 12.8277, size = 583, normalized size = 5.03

$$d^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d
**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**
2*x**2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(1
5*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**
2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**3*Piecewise((-I*d**6*
acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d
**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**
2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)
/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**
2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24
*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True
))
```

Giac [A] time = 1.283, size = 113, normalized size = 0.97

$$\frac{1}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}\left(d\right)-\frac{1}{240}\left(48\,d^5e^{(-2)}+\left(15\,d^4e^{(-1)}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,xe^3+6\,de^2\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) + (15*d^4*e^(-1
) - 2*(48*d^3 + (35*d^2*e - 4*(5*x*e^3 + 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

3.7 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=113

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (d^2*(8*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d + 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 + (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.0983893, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (d^2*(8*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d + 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 + (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2-3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4+3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{8}\left(3d^4e\right.$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^5\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,\right.$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^5\,\mathrm{S}}{}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\,\mathrm{ta}$$

Mathematica [A] time = 0.202308, size = 124, normalized size = 1.1

$$\frac{1}{24}\sqrt{d^2-e^2x^2}\left(15d^2ex+32d^3-8de^2x^2-6e^3x^3\right) + \frac{3d^3\sqrt{d^2-e^2x^2}\sin^{-1}\left(\frac{ex}{d}\right)}{8\sqrt{1-\frac{e^2x^2}{d^2}}} + d^4\left(-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 + 15*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))/24 +
(3*d^3*Sqrt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(8*Sqrt[1 - (e^2*x^2)/d^2]) - d
^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Maple [A] time = 0.054, size = 151, normalized size = 1.3

$$\frac{ex}{4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{3\,ed^2x}{8}\sqrt{-x^2e^2+d^2}+\frac{3\,ed^4}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+\frac{d}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+d^3\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x)
```

```
[Out] 1/4*e*x*(-e^2*x^2+d^2)^(3/2)+3/8*e*d^2*x*(-e^2*x^2+d^2)^(1/2)+3/8*e*d^4/(e^
2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/3*d*(-e^2*x^2+d^2)^(3
/2)+d^3*(-e^2*x^2+d^2)^(1/2)-d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82572, size = 228, normalized size = 2.02

$$-\frac{3}{4}d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\frac{1}{24}\left(6\,e^3x^3+8\,de^2x^2-15\,d^2ex-32\,d^3\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) - 1/24*(6*e^3*x^3 + 8*d*e^2*x^2 - 15*d^2*e*x - 32*d^3)*sqr
t(-e^2*x^2 + d^2)
```

Sympy [C] time = 23.338, size = 476, normalized size = 4.21

$$d^3\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\,\mathrm{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\,\mathrm{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^2\,\mathrm{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2|}{|d}\\ \frac{d^2\,\mathrm{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{other}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x,x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d
```

```
**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**
2/(e**2*x**2) + 1), True)) + d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) -
 I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2
/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(
1 - e**2*x**2/d**2)/2, True)) - d*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**
2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) - e**3*Piecewise((-I*d
**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3
*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x
**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**
3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2
)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.31211, size = 134, normalized size = 1.19

$$\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d) - d^4\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2\,|x|}\right) + \frac{1}{24}\left(32\,d^3 + \left(15\,d^2e - 2\left(3\,xe^3 + 4\,de^2\right)x\right)x\right)\sqrt{-x^2e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 3/8*d^4*arcsin(x*e/d)*sgn(d) - d^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d
^2)*e)*e^(-2)/abs(x)) + 1/24*(32*d^3 + (15*d^2*e - 2*(3*x*e^3 + 4*d*e^2)*x)
*x)*sqrt(-x^2*e^2 + d^2)
```

3.8 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=117

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (d*e*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d - e*x)*(d^2 - e^2*x^2)^(3
/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.0916725, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {813, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] (d*e*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d - e*x)*(d^2 - e^2*x^2)^(3
/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = -\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{\left(-2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \frac{\int \frac{4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \left(d^4e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}\left(3d^3e^2\right.$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \frac{1}{2}\left(d^4e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x\right.$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^4\,\text{Subs}}{}$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3e\tan$$

Mathematica [C] time = 0.189624, size = 124, normalized size = 1.06

$$-\frac{d^5\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{d^2-e^2x^2}}-\frac{1}{3}e\left(\sqrt{d^2-e^2x^2}\left(e^2x^2-4d^2\right)+3d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] -(e*(Sqrt[d^2 - e^2*x^2]*(-4*d^2 + e^2*x^2) + 3*d^3*ArcTanh[Sqrt[d^2 - e^2*
x^2]/d]))/3 - (d^5*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[-3/2, -1/2, 1/
2, (e^2*x^2)/d^2])/(x*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.056, size = 182, normalized size = 1.6

$$\frac{e}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+ed^2\sqrt{-x^2e^2+d^2}-ed^4\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{1}{dx}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{e^2x}{d}\left(-x^2e^2+d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x)
```

```
[Out] 1/3*e*(-e^2*x^2+d^2)^(3/2)+e*d^2*(-e^2*x^2+d^2)^(1/2)-e*d^4/(d^2)^(1/2)*ln(
(2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/d/x*(-e^2*x^2+d^2)^(5/2)-e^
2/d*x*(-e^2*x^2+d^2)^(3/2)-3/2*d*e^2*x*(-e^2*x^2+d^2)^(1/2)-3/2*e^2*d^3/(e^
2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79018, size = 257, normalized size = 2.2

$$\frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+8\,d^3ex-\left(2\,e^3x^3+3\,de^2x^2-8\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 6*d^3*e*x*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) + 8*d^3*e*x - (2*e^3*x^3 + 3*d*e^2*x^2 - 8*d^
2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

Sympy [C] time = 9.18669, size = 393, normalized size = 3.36

$$d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2x^2|}{|d^2|}>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \frac{|d^2|}{|e^2||x^2|}>\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**2,x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*s
qrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**
2)), True)) + d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*a
cosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x*
*2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) +
 I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((-I*d**2*acos
h(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqr
t(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(
2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - e**3*Piecewise((x**2*sqrt(d
**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

Giac [A] time = 1.26675, size = 212, normalized size = 1.81

$$-\frac{3}{2}d^3\arcsin\left(\frac{xe}{d}\right)\operatorname{esgn}(d)-d^3e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)+\frac{d^3xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)d^3e}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/2*d^3*arcsin(x*e/d)*e*sgn(d) - d^3*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^
2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^3*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) -
 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^(-1)/x + 1/6*sqrt(-x^2*e^2 + d^2)
*(8*d^2*e - (2*x*e^3 + 3*d*e^2)*x)
```

3.9 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3} dx$

Optimal. Leaf size=121

$$-\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (-3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d - e*x)*(d^2 - e^2*x^2)^(
3/2))/(2*x^2) - (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^
2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rubi [A] time = 0.0935944, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {813, 844, 217, 203, 266, 63, 208}

$$-\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d - e*x)*(d^2 - e^2*x^2)^(
3/2))/(2*x^2) - (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^
2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx &= -\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{8}\int \frac{\left(-4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx\\
&= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{16}\int \frac{-8d^3e^2-8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-\frac{1}{2}\left(3d^2\right.\\
&= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{1}{4}\left(3d^3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x\right.\\
&= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}\left(3d^3\right)\\
&= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\text{ta}
\end{aligned}$$

Mathematica [C] time = 0.0858168, size = 110, normalized size = 0.91

$$-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)}{5d^3}-\frac{d^2e\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] -((d^2*e*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (e^2*x^2)/d
^2])/(x*Sqrt[1 - (e^2*x^2)/d^2])) - (e^2*(d^2 - e^2*x^2)^(5/2)*Hypergeometr
ic2F1[2, 5/2, 7/2, 1 - (e^2*x^2)/d^2])/(5*d^3)
```

Maple [B] time = 0.059, size = 212, normalized size = 1.8

$$-\frac{1}{2dx^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{e^2}{2d}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{3de^2}{2}\sqrt{-x^2e^2+d^2}+\frac{3e^2d^3}{2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{e}{d^2 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x)
```

```
[Out] -1/2/d/x^2*(-e^2*x^2+d^2)^(5/2)-1/2*e^2/d*(-e^2*x^2+d^2)^(3/2)-3/2*d*e^2*(-
e^2*x^2+d^2)^(1/2)+3/2*e^2*d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^
2+d^2)^(1/2))/x)-e/d^2/x*(-e^2*x^2+d^2)^(5/2)-e^3/d^2*x*(-e^2*x^2+d^2)^(3/2
)-3/2*e^3*x*(-e^2*x^2+d^2)^(1/2)-3/2*e^3*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81149, size = 269, normalized size = 2.22

$$\frac{6d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-3d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2d^2e^2x^2-\left(e^3x^3+2de^2x^2+2d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 3*d^2*e^2*x^
2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d^2*e^2*x^2 - (e^3*x^3 + 2*d*e^2*x
^2 + 2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

Sympy [C] time = 13.9967, size = 471, normalized size = 3.89

$$d^3\left(\begin{cases}-\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \frac{|d^2|}{|e^2||x^2|}>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2 \cdots}{|d \cdots}\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{other}\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**3,x)
```

```
[Out] d**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs
(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x)
)/(2*d), True)) + d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e
*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(
d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sq
rt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e*
*2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**
2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - e**3*Piece
wise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*
e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (
d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
```

Giac [B] time = 1.23643, size = 293, normalized size = 2.42

$$-\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^2\mathrm{sgn}\left(d\right)+\frac{3}{2}\,d^2e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)-\frac{1}{8}\left(\frac{4\left(de+\sqrt{-x^2e^2+d^2}e\right)d^2e^8}{x}+\frac{\left(de+\sqrt{-x^2 \cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/2*d^2*arcsin(x*e/d)*e^2*sgn(d) + 3/2*d^2*e^2*log(1/2*abs(-2*d*e - 2*sqrt
(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d^2*e^8/x + (d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^6/x^2)*e^(-8) - 1/2*sqrt
(-x^2*e^2 + d^2)*(x*e^3 + 2*d*e^2) + 1/8*(d^2*e^6 + 4*(d*e + sqrt(-x^2*e^2
+ d^2)*e)*d^2*e^4/x)*x^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^2
```

3.10 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=120

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (e^2*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d + 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rubi [A] time = 0.0921883, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {811, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] (e^2*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d + 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = -\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{\int \frac{\left(4d^3e^2+6d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{\int \frac{-12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \left(de^4\right.$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{1}{4}\left(3d^2e^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx\right.$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}\left(3d^2e\right)$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}de^3\tan$$

Mathematica [C] time = 0.0685181, size = 111, normalized size = 0.92

$$-\frac{d^3\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^3\left(d^2-e^2x^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] -(d^3*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (e^2*x^2)/d^2
])/(3*x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (e^3*(d^2 - e^2*x^2)^(5/2)*Hypergeomet
ric2F1[2, 5/2, 7/2, 1 - (e^2*x^2)/d^2])/(5*d^4)
```

Maple [B] time = 0.064, size = 235, normalized size = 2.

$$-\frac{1}{3\,dx^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{2\,e^2}{3\,d^3x}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{2\,e^4x}{3\,d^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{e^4x}{d}\sqrt{-x^2e^2+d^2}+de^4\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x)
```

```
[Out] -1/3/d/x^3*(-e^2*x^2+d^2)^(5/2)+2/3*e^2/d^3/x*(-e^2*x^2+d^2)^(5/2)+2/3*e^4/
d^3*x*(-e^2*x^2+d^2)^(3/2)+e^4/d*x*(-e^2*x^2+d^2)^(1/2)+d*e^4/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*e/d^2/x^2*(-e^2*x^2+d^2)^(5/2
)-1/2*e^3/d^2*(-e^2*x^2+d^2)^(3/2)-3/2*e^3*(-e^2*x^2+d^2)^(1/2)+3/2*e^3*d^2
/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00324, size = 269, normalized size = 2.24

$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+6\,de^3x^3+\left(6\,e^3x^3-8\,de^2x^2+3\,d^2ex+2\,d^3\right)\sqrt{-e^2x^2+}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*d*e^3*x^3*
log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 6*d*e^3*x^3 + (6*e^3*x^3 - 8*d*e^2*x^2
```

```
+ 3*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

Sympy [C] time = 7.67882, size = 469, normalized size = 3.91

$$d^3\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \frac{|d^2|}{|e^2||x^2|}>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)+d^2e\left(\begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \frac{|d^2|}{|e^2||x^2|}>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)-de^2\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**4,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d
**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2)
, True)) + d**2*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) +
e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(
Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2
*asin(d/(e*x))/(2*d), True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2
/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e*
*2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) +
 e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) - e**3*Piecewise((d**2/(e*x*sq
rt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) -
1), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x*
*2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Giac [B] time = 1.25321, size = 352, normalized size = 2.93

$$d\arcsin\left(\frac{xe}{d}\right)e^3\operatorname{sgn}(d)+\frac{3}{2}de^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(de^8+\frac{3\left(de+\sqrt{-x^2e^2+d^2}e\right)de^6}{x}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}e\right)^2de^4}{x^2}\right)x^3}{24\left(de+\sqrt{-x^2e^2+d^2}e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] d*arcsin(x*e/d)*e^3*sgn(d) + 3/2*d*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/24*(d*e^8 + 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d*e^6/x - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^4/x^2)*x^3*e/(d*e + sqrt(
-x^2*e^2 + d^2)*e)^3 + 1/24*(15*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^16/x - 3
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^14/x^2 - (d*e + sqrt(-x^2*e^2 + d^2)*
e)^3*d*e^12/x^3)*e^(-15) - sqrt(-x^2*e^2 + d^2)*e^3
```

3.11 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=118

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (e^2*(3*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d + 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) + e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rubi [A] time = 0.0922914, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {811, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] (e^2*(3*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d + 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) + e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx &= -\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}-\frac{\int \frac{\left(6d^3e^2+8d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+\frac{\int \frac{12d^5e^4+32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+\frac{1}{8}\left(3de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+e^5\int
\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+\frac{1}{16}\left(3de^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx
\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{8}\left(3de^2\right)\text{S}
\\
&= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-}
\end{aligned}$$

Mathematica [C] time = 0.101737, size = 133, normalized size = 1.13

$$-\frac{\sqrt{d^2-e^2x^2}\left(8d^3ex\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)+3d^2\left(2d^2-5e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}+9e^4x^4\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\right)}{24dx^4\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(3*d^2*(2*d^2 - 5*e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^2] +
9*e^4*x^4*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]] + 8*d^3*e*x*Hypergeometric2F1[-3
/2, -3/2, -1/2, (e^2*x^2)/d^2]))/(24*d*x^4*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [B] time = 0.064, size = 260, normalized size = 2.2

$$-\frac{e}{3\,d^{2}x^{3}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}+\frac{2\,e^{3}}{3\,d^{4}x}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}+\frac{2\,e^{5}x}{3\,d^{4}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}+\frac{e^{5}x}{d^{2}}\sqrt{-x^{2}e^{2}+d^{2}}+e^{5}\arctan\left(x\sqrt{e^{2}}\frac{1}{\sqrt{-x^{2}e^{2}+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x)
```

```
[Out] -1/3*e/d^2/x^3*(-e^2*x^2+d^2)^(5/2)+2/3*e^3/d^4/x*(-e^2*x^2+d^2)^(5/2)+2/3*
e^5/d^4*x*(-e^2*x^2+d^2)^(3/2)+e^5/d^2*x*(-e^2*x^2+d^2)^(1/2)+e^5/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/4/d/x^4*(-e^2*x^2+d^2)^(5/2
)+1/8*e^2/d^3/x^2*(-e^2*x^2+d^2)^(5/2)+1/8*e^4/d^3*(-e^2*x^2+d^2)^(3/2)+3/8
*e^4/d*(-e^2*x^2+d^2)^(1/2)-3/8*d*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93857, size = 248, normalized size = 2.1

$$-\frac{48\,e^{4}x^{4}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-9\,e^{4}x^{4}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(32\,e^{3}x^{3}+15\,de^{2}x^{2}-8\,d^{2}ex-6\,d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{24\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*e^4*x^4*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 15*d*e^2*x^2 - 8*d^2*e*x -
6*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

Sympy [C] time = 10.1043, size = 552, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
```

```
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/
(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e
*x))/(8*d**3), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*
x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(
x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2
/(e**2*x**2) + 1)/(3*d**2), True)) - d*e**2*Piecewise((-d**2/(2*e*x**3*sqrt
(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d
/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2
*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) - e**3*Piecewise((I*
d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 +
e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/
d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [B] time = 1.30651, size = 401, normalized size = 3.4

$$\arcsin\left(\frac{xe}{d}\right)e^4\mathrm{sgn}\left(d\right) + \frac{x^4\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)e^8}{x} - \frac{24\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^6}{x^2} - \frac{120\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^4}{x^3} + 3\,e^{10}\right)e^2}{192\left(de + \sqrt{-x^2e^2 + d^2}e\right)^4} + \frac{1}{192}\left(\frac{120\left(de + \sqrt{-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e^4*sgn(d) + 1/192*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/
x - 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^6/x^2 - 120*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^3*e^4/x^3 + 3*e^10)*e^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 + 1/192*
(120*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^26/x + 24*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^2*e^24/x^2 - 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^22/x^3 - 3*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^4*e^20/x^4)*e^(-24) - 3/8*e^4*log(1/2*abs(-2*d*e - 2*s
qrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))
```

3.12 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=108

$$\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] (3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) - (
d^2 - e^2*x^2)^(5/2)/(5*d*x^5) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*
d)
```

Rubi [A] time = 0.0629703, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {807, 266, 47, 63, 208}

$$\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] (3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) - (
d^2 - e^2*x^2)^(5/2)/(5*d*x^5) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*
d)
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{1}{2}e\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}dx,x,x^2\right) \\
&= -\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}dx,x,x^2\right) \\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{1}{16}\left(3e^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}dx,x,\right. \\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}dx,x,\sqrt{d^2}\right. \\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.0671693, size = 133, normalized size = 1.23

$$-\frac{-24d^4e^2x^2-35d^3e^3x^3+24d^2e^4x^4+15de^5x^5\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)+10d^5ex+8d^6+25de^5x^5-8e^6x^6}{40dx^5\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] -(8*d^6 + 10*d^5*e*x - 24*d^4*e^2*x^2 - 35*d^3*e^3*x^3 + 24*d^2*e^4*x^4 + 2
5*d*e^5*x^5 - 8*e^6*x^6 + 15*d*e^5*x^5*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt
[1 - (e^2*x^2)/d^2]])/(40*d*x^5*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.074, size = 158, normalized size = 1.5

$$-\frac{1}{5\,dx^5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{e}{4\,d^2x^4}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{e^3}{8\,d^4x^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{e^5}{8\,d^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{3\,e^5}{8\,d^2}\sqrt{-x^2e^2+d^2}-\frac{3\,e^5}{8}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x)
```

```
[Out] -1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5-1/4*e/d^2/x^4*(-e^2*x^2+d^2)^(5/2)+1/8*e^3/
d^4/x^2*(-e^2*x^2+d^2)^(5/2)+1/8*e^5/d^4*(-e^2*x^2+d^2)^(3/2)+3/8*e^5/d^2*(
```

```
-e^2*x^2+d^2)^(1/2)-3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d
^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88028, size = 203, normalized size = 1.88

$$\frac{15\,e^{5}x^{5}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(8\,e^{4}x^{4}-25\,de^{3}x^{3}-16\,d^{2}e^{2}x^{2}+10\,d^{3}ex+8\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{40\,dx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 - 25*d*e^3
*x^3 - 16*d^2*e^2*x^2 + 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

Sympy [C] time = 9.53821, size = 785, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**6,x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-
15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*
d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x*
*5 + 15*d*e**2*x**7), True)) + d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/
(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*
x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Ab
s(e**2)*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3
*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e*
*2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((
-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3
*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - e**3*
Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(
```

```
e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2
)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*
d), True))
```

Giac [B] time = 1.31877, size = 497, normalized size = 4.6

$$\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{10}}{x}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^8}{x^2}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^6}{x^3}+\frac{20\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^4}{x^4}+2\,e^{12}\right)e^3}{320\left(de+\sqrt{-x^2e^2+d^2}e\right)^5d}-\frac{3\,e^5\log\left(\frac{\left|-2\,de-2\sqrt{-x^2e^2}\right.}{2\,|x|}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/320*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^10/x - 10*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^8/x^2 - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^6/x^3 + 20*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^4/x^4 + 2*e^12)*e^3/((d*e + sqrt(-x^2*e^2
+ d^2)*e)^5*d) - 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(
-2)/abs(x))/d - 1/320*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^38/x - 40*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^36/x^2 - 10*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^3*d^4*e^34/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^32/x^4 + 2*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^4*e^30/x^5)*e^(-35)/d^5
```

3.13 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7} dx$

Optimal. Leaf size=143

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rubi [A] time = 0.0948237, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{\int \frac{\left(-6d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{6d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^2\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^2\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)}{12d}\\
&= -\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^4\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)}{16d}\\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^6\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2
\end{aligned}$$

$$\begin{aligned}
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^4\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right.}{16}\\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right.}{16d^2}
\end{aligned}$$

Mathematica [C] time = 0.024719, size = 59, normalized size = 0.41

$$-\frac{e\left(d^2-e^2x^2\right)^{5/2}\left(e^5x^5\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)+d^5\right)}{5d^7x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] -(e*(d^2 - e^2*x^2)^(5/2)*(d^5 + e^5*x^5*Hypergeometric2F1[5/2, 4, 7/2, 1 -
 (e^2*x^2)/d^2]))/(5*d^7*x^5)
```

Maple [A] time = 0.079, size = 186, normalized size = 1.3

$$-\frac{e}{5\,d^{2}x^{5}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}-\frac{1}{6\,dx^{6}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}-\frac{e^{2}}{24\,d^{3}x^{4}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}+\frac{e^{4}}{48\,d^{5}x^{2}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}+\frac{e^{6}}{48\,d^{5}}\left(-x^{2}e^{2}+d^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x)
```

```
[Out] -1/5*e*(-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6-1/24*e^2
/d^3/x^4*(-e^2*x^2+d^2)^(5/2)+1/48*e^4/d^5/x^2*(-e^2*x^2+d^2)^(5/2)+1/48*e^
6/d^5*(-e^2*x^2+d^2)^(3/2)+1/16*e^6/d^3*(-e^2*x^2+d^2)^(1/2)-1/16*e^6/d/(d^
2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7701, size = 232, normalized size = 1.62

$$\frac{15\,e^{6}x^{6}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(48\,e^{5}x^{5}+15\,de^{4}x^{4}-96\,d^{2}e^{3}x^{3}-70\,d^{3}e^{2}x^{2}+48\,d^{4}ex+40\,d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{240\,d^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x^5 + 15*d*e
^4*x^4 - 96*d^2*e^3*x^3 - 70*d^3*e^2*x^2 + 48*d^4*e*x + 40*d^5)*sqrt(-e^2*x
^2 + d^2))/(d^2*x^6)
```

Sympy [C] time = 13.967, size = 930, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**7,x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e
**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*
```

```
d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2
*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**2*e*Piecewise((3
*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6
*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2
*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15
*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e*
*2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
*7), True)) - d*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1))
 + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2
*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)
) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) -
I*e**4*asin(d/(e*x))/(8*d**3), True)) - e**3*Piecewise((-e*sqrt(d**2/(e**2*
x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2)/(
Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e
**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

Giac [B] time = 1.27681, size = 582, normalized size = 4.07

$$\frac{x^6\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{12}}{x}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{10}}{x^2}-\frac{60\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^8}{x^3}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^6}{x^4}+\frac{120\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^4}{x^5}+5e^{14}\right)e^4}{1920\left(de+\sqrt{-x^2e^2+d^2}e\right)^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*x^6*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^12/x - 15*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^2*e^10/x^2 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^8/x^3 - 15
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^6/x^4 + 120*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^5*e^4/x^5 + 5*e^14)*e^4/((d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2) - 1/16*e
^6*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 1/19
20*(120*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^10*e^52/x - 15*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*d^10*e^50/x^2 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^10*e^48
/x^3 - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^10*e^46/x^4 + 12*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^5*d^10*e^44/x^5 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^10
*e^42/x^6)*e^(-48)/d^12
```

3.14 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=172

$$\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d^
2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) - (e*(d^2 - e^2*x^2)^(5/2))/(6*d^2
*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) - (e^7*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/(16*d^3)
```

Rubi [A] time = 0.125398, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d^
2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) - (e*(d^2 - e^2*x^2)^(5/2))/(6*d^2
*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) - (e^7*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/(16*d^3)
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{\int \frac{\left(-7d^2e-2de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}+\frac{\int \frac{\left(12d^3e^2+7d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}+\frac{e^3\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}+\frac{e^3\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)}{12d^2}$$

$$= -\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\frac{e^5\,\text{Subst}\left(\int \ldots\right.}{1\ldots}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}+\ldots$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\ldots$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}-\ldots$$

Mathematica [C] time = 0.0219998, size = 72, normalized size = 0.42

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}\left(7e^7x^7\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)+2d^5e^2x^2+5d^7\right)}{35d^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] -((d^2 - e^2*x^2)^(5/2)*(5*d^7 + 2*d^5*e^2*x^2 + 7*e^7*x^7*Hypergeometric2F
1[5/2, 4, 7/2, 1 - (e^2*x^2)/d^2]))/(35*d^8*x^7)
```

Maple [A] time = 0.095, size = 211, normalized size = 1.2

$$-\frac{e}{6\,d^{2}x^{6}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}-\frac{e^{3}}{24\,d^{4}x^{4}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}+\frac{e^{5}}{48\,d^{6}x^{2}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}+\frac{e^{7}}{48\,d^{6}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}+\frac{e^{7}}{16\,d^{4}}\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x)
```

```
[Out] -1/6*e*(-e^2*x^2+d^2)^(5/2)/d^2/x^6-1/24*e^3/d^4/x^4*(-e^2*x^2+d^2)^(5/2)+1
/48*e^5/d^6/x^2*(-e^2*x^2+d^2)^(5/2)+1/48*e^7/d^6*(-e^2*x^2+d^2)^(3/2)+1/16
*e^7/d^4*(-e^2*x^2+d^2)^(1/2)-1/16*e^7/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1
/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7-2/35*e^2*(-e^2*
x^2+d^2)^(5/2)/d^3/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01573, size = 265, normalized size = 1.54

$$\frac{105\,e^{7}x^{7}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(96\,e^{6}x^{6}+105\,de^{5}x^{5}+48\,d^{2}e^{4}x^{4}-490\,d^{3}e^{3}x^{3}-384\,d^{4}e^{2}x^{2}+280\,d^{5}ex+240\,d^{6}\right)\sqrt{-e^{2}x}}{1680\,d^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (96*e^6*x^6 + 105*
d*e^5*x^5 + 48*d^2*e^4*x^4 - 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 + 280*d^5*e*
x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

Sympy [C] time = 15.0072, size = 1049, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**8,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2)/(Abs(e**2)*
Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-
d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)
/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True))
+ d**2*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x
**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2)
- 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(
16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**
2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/
(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(
e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewis
e((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*
d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*
e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I
*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(
e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
+ 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 1
5*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**
3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), True)) - e**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e*
*2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True))
```

Giac [B] time = 1.28193, size = 667, normalized size = 3.88

$$\frac{x^7\left(\frac{35\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{14}}{x}-\frac{21\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{12}}{x^2}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{10}}{x^3}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^8}{x^4}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^6}{x^5}+\frac{315\left(de\cdots\right.}{\cdots}\right.}{13440\left(de+\sqrt{-x^2e^2+d^2}e\right)^7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/13440*x^7*(35*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^14/x - 21*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^2*e^12/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^10/x^3 -
 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^8/x^4 - 105*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^5*e^6/x^5 + 315*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^4/x^6 + 15*e^16)
*e^5/((d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^3) - 1/16*e^7*log(1/2*abs(-2*d*e -
 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/13440*(315*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^18*e^68/x - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^18*e^
66/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e^64/x^3 - 105*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^4*d^18*e^62/x^4 - 21*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5
*d^18*e^60/x^5 + 35*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^18*e^58/x^6 + 15*(d*
e + sqrt(-x^2*e^2 + d^2)*e)^7*d^18*e^56/x^7)*e^(-63)/d^21
```

3.15 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=201

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}-\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{3e^8\tan}{}$$

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) - (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) - (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rubi [A] time = 0.156395, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {835, 807, 266, 47, 63, 208}

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}-\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{3e^8\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```


```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) - (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) - (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```


Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(-8d^2e-3de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} + \frac{\int \frac{\left(21d^3e^2+16d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{\int \frac{\left(-96d^4e^3-21d^3e^4x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{336d^6}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\int \frac{\left(d^2-e^2x^2\right)^{3/}}{x^5}}{16d^3}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5} + \frac{e^4\,\mathrm{Subst}\left(\int \right.}{}$$

$$= -\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)}{35d^4x^5}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} -$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} -$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} -$$

Mathematica [C] time = 0.0296739, size = 73, normalized size = 0.36

$$-\frac{e\left(d^2-e^2x^2\right)^{5/2}\left(7e^7x^7\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};1-\frac{e^2x^2}{d^2}\right)+2d^5e^2x^2+5d^7\right)}{35d^9x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] -(e*(d^2 - e^2*x^2)^(5/2)*(5*d^7 + 2*d^5*e^2*x^2 + 7*e^7*x^7*Hypergeometric
2F1[5/2, 5, 7/2, 1 - (e^2*x^2)/d^2]))/(35*d^9*x^7)
```

Maple [A] time = 0.122, size = 236, normalized size = 1.2

$$-\frac{1}{8\,d x^8}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{e^2}{16\,d^3x^6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{e^4}{64\,d^5x^4}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{e^6}{128\,d^7x^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{e^8}{128\,d^7}\left(-x^2e^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x)
```

```
[Out] -1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6-1/64*
e^4/d^5/x^4*(-e^2*x^2+d^2)^(5/2)+1/128*e^6/d^7/x^2*(-e^2*x^2+d^2)^(5/2)+1/1
28*e^8/d^7*(-e^2*x^2+d^2)^(3/2)+3/128*e^8/d^5*(-e^2*x^2+d^2)^(1/2)-3/128*e^
8/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/7*e*(-
e^2*x^2+d^2)^(5/2)/d^2/x^7-2/35*e^3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09431, size = 292, normalized size = 1.45

$$\frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(256\,e^7x^7+105\,de^6x^6+128\,d^2e^5x^5+70\,d^3e^4x^4-1024\,d^4e^3x^3-840\,d^5e^2x^2+640\,d^6ex\right.}{4480\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (256*e^7*x^7 + 105
*d*e^6*x^6 + 128*d^2*e^5*x^5 + 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 - 840*d^5*
e^2*x^2 + 640*d^6*e*x + 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

Sympy [C] time = 30.6822, size = 1171, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**9,x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2)/
(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1))
- 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(
-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) +
1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(
e*x))/(128*d**7), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/
(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**
2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105
*d**6), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**
5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**6), True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/
(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**
2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2)
- 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1)
, (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**
2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I
*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d*
*5), True)) - e**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x*
*5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x
**5 + 15*d*e**2*x**7), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2
*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x*
*2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d*
*2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2
)/(-15*d**3*x**5 + 15*d*e**2*x**7), True))
```

Giac [B] time = 1.23435, size = 582, normalized size = 2.9

$$\frac{x^8\left(\frac{80\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{16}}{x}-\frac{112\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{12}}{x^3}-\frac{280\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^{10}}{x^4}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^8}{x^5}+\frac{1680\left(de+\sqrt{-x^2e^2+d^2}e\right)^7e^4}{x^7}+35\,e\right.}{71680\left(de+\sqrt{-x^2e^2+d^2}e\right)^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/71680*x^8*(80*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^16/x - 112*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^3*e^12/x^3 - 280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^10/x^4
- 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^8/x^5 + 1680*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^7*e^4/x^7 + 35*e^18)*e^6/((d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^4) -
 3/128*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^
4 - 1/71680*(1680*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^28*e^86/x - 560*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^3*d^28*e^82/x^3 - 280*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^4*d^28*e^80/x^4 - 112*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^28*e^78/x^5 + 80*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^28*e^74/x^7 + 35*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^8*d^28*e^72/x^8)*e^(-80)/d^32
```

3.16 $\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=103

$$-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] -((d^2*Sqrt[d^2 - e^2*x^2])/e^3) - (d*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (d^2
 - e^2*x^2)^(3/2)/(3*e^3) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rubi [A] time = 0.0534872, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {797, 641, 195, 217, 203}

$$-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((d^2*Sqrt[d^2 - e^2*x^2])/e^3) - (d*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (d^2
 - e^2*x^2)^(3/2)/(3*e^3) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2} + \frac{d^2\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} - \frac{d\int \sqrt{d^2-e^2x^2}\,dx}{e^2} + \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} - \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2} + \frac{d^3\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-\cdots}}\right.}{e^2}$$

$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3} - \frac{d^3\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, -\cdots\right.}{2e^2}$$

$$= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Mathematica [A] time = 0.045859, size = 70, normalized size = 0.68

$$\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(4d^2+3dex+2e^2x^2\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(4*d^2 + 3*d*e*x + 2*e^2*x^2)) + 3*d^3*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(6*e^3)
```

Maple [A] time = 0.053, size = 102, normalized size = 1.

$$-\frac{x^2}{3\,e}\sqrt{-x^2e^2+d^2} - \frac{2\,d^2}{3\,e^3}\sqrt{-x^2e^2+d^2} - \frac{dx}{2\,e^2}\sqrt{-x^2e^2+d^2} + \frac{d^3}{2\,e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2/e*(-e^2*x^2+d^2)^(1/2)-2/3*d^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*d*x*(-
e^2*x^2+d^2)^(1/2)/e^2+1/2*d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))
```

Maxima [A] time = 1.58034, size = 127, normalized size = 1.23

$$-\frac{\sqrt{-e^2x^2+d^2}\,x^2}{3\,e} + \frac{d^3\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\,\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}\,dx}{2\,e^2} - \frac{2\,\sqrt{-e^2x^2+d^2}\,d^2}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*x^2/e + 1/2*d^3*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt
(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*d*x/e^2 - 2/3*sqrt(-e^2*x^2 + d^2)*d^
2/e^3
```

Fricas [A] time = 1.8527, size = 153, normalized size = 1.49

$$-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,e^2x^2+3\,dex+4\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2*e^2*x^2 + 3*d*e*
x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [C] time = 5.4229, size = 178, normalized size = 1.73

$$d\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2
)/(2*e**2), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x
/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), T
rue)) + e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d*
*2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

Giac [A] time = 1.19594, size = 73, normalized size = 0.71

$$\frac{1}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\left(d\right)-\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\left(4\,d^2e^{(-3)}+\left(2\,xe^{(-1)}+3\,de^{(-2)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3
) + (2*x*e^(-1) + 3*d*e^(-2))*x)
```

3.17 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}} dx$

Optimal. Leaf size=73

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] (d*(d + e*x))/(e^3*Sqrt[d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e^3 - (d*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rubi [A] time = 0.0418788, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {797, 641, 217, 203, 637}

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d*(d + e*x))/(e^3*Sqrt[d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e^3 - (d*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{d^2\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{e^2}$$

$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}$$

$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

$$= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Mathematica [A] time = 0.0359309, size = 77, normalized size = 1.05

$$\frac{-d\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2 + dex - e^2x^2}{e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (2*d^2 + d*e*x - e^2*x^2 - d*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]])/(e^3*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.053, size = 99, normalized size = 1.4

$$-\frac{x^2}{e}\frac{1}{\sqrt{-x^2e^2+d^2}} + 2\frac{d^2}{e^3\sqrt{-x^2e^2+d^2}} + \frac{dx}{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}} - \frac{d}{e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -x^2/e/(-e^2*x^2+d^2)^(1/2)+2*d^2/e^3/(-e^2*x^2+d^2)^(1/2)+d*x/e^2/(-e^2*x^
2+d^2)^(1/2)-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.63252, size = 123, normalized size = 1.68

$$-\frac{x^2}{\sqrt{-e^2x^2+d^2}e} + \frac{dx}{\sqrt{-e^2x^2+d^2}e^2} - \frac{d\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}e^2} + \frac{2\,d^2}{\sqrt{-e^2x^2+d^2}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/(sqrt(-e^2*x^2 + d^2)*e) + d*x/(sqrt(-e^2*x^2 + d^2)*e^2) - d*arcsin(e
^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) + 2*d^2/(sqrt(-e^2*x^2 + d^2)*e^3)
```

Fricas [A] time = 2.08667, size = 176, normalized size = 2.41

$$\frac{2\,dex-2\,d^2+2\left(dex-d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex-2\,d)}{e^4x-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*d*e*x - 2*d^2 + 2*(d*e*x - d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + sqrt(-e^2*x^2 + d^2)*(e*x - 2*d))/(e^4*x - d*e^3)
```

Sympy [C] time = 7.27369, size = 185, normalized size = 2.53

$$d\left(\begin{cases}\frac{i\operatorname{acosh}\left(\frac{ex}{d}\right)}{e^3}-\frac{ix}{de^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ -\frac{\operatorname{asin}\left(\frac{ex}{d}\right)}{e^3}+\frac{x}{de^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\tilde{\infty}x^4 & \text{for}\ \left(d=0\vee d=-\sqrt{e^2x^2}\vee d=\sqrt{e^2x^2}\right)\wedge\left(d\right.\\ \frac{x^4}{4\left(d^2\right)^{\frac{3}{2}}} & \text{for}\ e=0\\ \frac{2d^2}{e^4\sqrt{d^2-e^2x^2}}-\frac{x^2}{e^2\sqrt{d^2-e^2x^2}} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d*Piecewise((I*acosh(e*x/d)/e**3 - I*x/(d*e**2*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2)/Abs(d**2) > 1), (-asin(e*x/d)/e**3 + x/(d*e**2*sqrt(1 - e**2
*x**2/d**2)), True)) + e*Piecewise((zoo*x**4, (Eq(d, 0) | Eq(d, sqrt(e**2*x
**2)) | Eq(d, -sqrt(e**2*x**2))) & (Eq(e, 0) | Eq(d, sqrt(e**2*x**2)) | Eq(
d, -sqrt(e**2*x**2)))), (x**4/(4*(d**2)**(3/2)), Eq(e, 0)), (2*d**2/(e**4*s
qrt(d**2 - e**2*x**2)) - x**2/(e**2*sqrt(d**2 - e**2*x**2)), True))
```

Giac [A] time = 1.17666, size = 89, normalized size = 1.22

$$-d\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d)-\frac{\sqrt{-x^2e^2+d^2}\left(2\,d^2e^{(-3)}-\left(xe^{(-1)}-de^{(-2)}\right)x\right)}{x^2e^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -d*arcsin(x*e/d)*e^(-3)*sgn(d) - sqrt(-x^2*e^2 + d^2)*(2*d^2*e^(-3) - (x*e^
(-1) - d*e^(-2))*x)/(x^2*e^2 - d^2)
```

3.18 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] (x^2*(d + e*x))/(3*d*e*(d^2 - e^2*x^2)^(3/2)) - 2/(3*e^3*Sqrt[d^2 - e^2*x^2
])
```

Rubi [A] time = 0.0262578, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {796, 12, 261}

$$\frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (x^2*(d + e*x))/(3*d*e*(d^2 - e^2*x^2)^(3/2)) - 2/(3*e^3*Sqrt[d^2 - e^2*x^2
])
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{2d^2ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{x}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}$$

$$= \frac{x^2(d+ex)}{3de\left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0246986, size = 52, normalized size = 0.9

$$\frac{-2d^2+2dex+e^2x^2}{3de^3(d-ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (-2*d^2 + 2*d*e*x + e^2*x^2)/(3*d*e^3*(d - e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.047, size = 55, normalized size = 1.

$$-\frac{(-ex+d)(ex+d)^2\left(-x^2e^2-2\,dex+2\,d^2\right)}{3\,de^3}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/3*(-e*x+d)*(e*x+d)^2*(-e^2*x^2-2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(5/2)
```

Maxima [A] time = 1.0003, size = 119, normalized size = 2.05

$$\frac{x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e} + \frac{dx}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} - \frac{x}{3\sqrt{-e^2x^2+d^2}de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] x^2/((-e^2*x^2 + d^2)^(3/2)*e) + 1/3*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2/3
*d^2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

Fricas [B] time = 2.14648, size = 204, normalized size = 3.52

$$-\frac{2\,e^3x^3-2\,de^2x^2-2\,d^2ex+2\,d^3-\left(e^2x^2+2\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(de^6x^3-d^2e^5x^2-d^3e^4x+d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*e^3*x^3 - 2*d*e^2*x^2 - 2*d^2*e*x + 2*d^3 - (e^2*x^2 + 2*d*e*x - 2*
d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - d^2*e^5*x^2 - d^3*e^4*x + d^4*e^3)
```

Sympy [C] time = 9.63715, size = 233, normalized size = 4.02

$$d\left(\begin{cases} \frac{ix^3}{-3d^5\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2x^2|}{|d^2|}>1 \\ -\frac{x^3}{-3d^5\sqrt{1-\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)+e\left(\begin{cases} \frac{2d^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}}-\frac{3e^2x^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}} & \text{for } e \\ \frac{x^4}{4\left(d^2\right)^{\frac{5}{2}}} & \text{othe} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d*Piecewise((I*x**3/(-3*d**5*sqrt(-1 + e**2*x**2/d**2) + 3*d**3*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-x**3/(-3*d**5*s
qrt(1 - e**2*x**2/d**2) + 3*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), True)
) + e*Piecewise((2*d**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e**6*x**2*
sqrt(d**2 - e**2*x**2)) - 3*e**2*x**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2)
+ 3*e**6*x**2*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(5/2)),
True))
```

Giac [A] time = 1.17522, size = 69, normalized size = 1.19

$$\frac{\left(x^2\left(\frac{x}{d}+3\,e^{(-1)}\right)-2\,d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{3\left(x^2e^2-d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2*(x/d + 3*e^(-1)) - 2*d^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d
^2)^2
```

3.19 $\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=161

$$\frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}-\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

```
[Out] (x^6*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d + 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d + 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d + 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) - (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rubi [A] time = 0.139297, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {819, 780, 217, 203}

$$\frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}-\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^6*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d + 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d + 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d + 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) - (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^5\left(6d^3+7d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^3\left(24d^5+35d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(48d^7+105d^6ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{\left(7d^2\right)\int}{}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{\left(7d^2\right)S}{}$$

$$= \frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{7d^2\tan}{}$$

Mathematica [A] time = 0.109221, size = 155, normalized size = 0.96

$$\frac{-249d^4e^2x^2+4d^3e^3x^3+176d^2e^4x^4-105d^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+9d^5ex+96d^6-15de^5x^5-}{30e^8(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (96*d^6 + 9*d^5*e*x - 249*d^4*e^2*x^2 + 4*d^3*e^3*x^3 + 176*d^2*e^4*x^4 - 1
5*d*e^5*x^5 - 15*e^6*x^6 - 105*d^2*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2
]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(30*e^8*(d - e*x)^2*(d + e*x)*Sqrt[d^2
 - e^2*x^2])
```

Maple [A] time = 0.117, size = 227, normalized size = 1.4

$$-\frac{x^7}{2e}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} + \frac{7d^2x^5}{10e^3}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} - \frac{7d^2x^3}{6e^5}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}} + \frac{7d^2x}{2e^7}\frac{1}{\sqrt{-x^2e^2+d^2}} - \frac{7d^2}{2e^7}\arctan\left(x\sqrt{e^2}\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

[Out] -1/2*x^7/e/(-e^2*x^2+d^2)^(5/2)+7/10*d^2/e^3*x^5/(-e^2*x^2+d^2)^(5/2)-7/6*d
^2/e^5*x^3/(-e^2*x^2+d^2)^(3/2)+7/2*d^2/e^7*x/(-e^2*x^2+d^2)^(1/2)-7/2*d^2/
e^7/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-d*x^6/e^2/(-e^2*
x^2+d^2)^(5/2)+6*d^3/e^4*x^4/(-e^2*x^2+d^2)^(5/2)-8*d^5/e^6*x^2/(-e^2*x^2+d
^2)^(5/2)+16/5*d^7/e^8/(-e^2*x^2+d^2)^(5/2)

Maxima [B] time = 1.48889, size = 439, normalized size = 2.73

$$-\frac{x^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{7\,d^2x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)}{30\,e}-\frac{dx^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{7\,d^2x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,\cdots}{\left(-e^2x^2+\cdots\right.}\right.}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")

[Out] -1/2*x^7/((-e^2*x^2 + d^2)^(5/2)*e) + 7/30*d^2*x*(15*x^4/((-e^2*x^2 + d^2)^
(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d
^2)^(5/2)*e^6))/e - d*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/6*d^2*x*(3*x^2/(
(-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^3 + 6*d
^3*x^4/((-e^2*x^2 + d^2)^(5/2)*e^4) - 8*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*e^6
) + 16/5*d^7/((-e^2*x^2 + d^2)^(5/2)*e^8) + 14/15*d^4*x/((-e^2*x^2 + d^2)^(
3/2)*e^7) - 49/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^7) - 7/2*d^2*arcsin(e^2*x/s
qrt(d^2*e^2))/(sqrt(e^2)*e^7)

Fricas [A] time = 2.67781, size = 575, normalized size = 3.57

$$\frac{96\,d^2e^5x^5-96\,d^3e^4x^4-192\,d^4e^3x^3+192\,d^5e^2x^2+96\,d^6ex-96\,d^7+210\left(d^2e^5x^5-d^3e^4x^4-2\,d^4e^3x^3+2\,d^5e^2x^2+d^6ex-\cdots\right.}{30\left(e^{13}x^5-de^{12}x^4-2\,d^2e^{11}x^3+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] 1/30*(96*d^2*e^5*x^5 - 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 + 192*d^5*e^2*x^2 +
 96*d^6*e*x - 96*d^7 + 210*(d^2*e^5*x^5 - d^3*e^4*x^4 - 2*d^4*e^3*x^3 + 2*d
^5*e^2*x^2 + d^6*e*x - d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15
*e^6*x^6 + 15*d*e^5*x^5 - 176*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 249*d^4*e^2*x^2
 - 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 - d*e^12*x^4 - 2*d^2
*e^11*x^3 + 2*d^3*e^10*x^2 + d^4*e^9*x - d^5*e^8)

Sympy [B] time = 48.8218, size = 2006, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)

```
[Out] d*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x*
*2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 40*d**4*
e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**
2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*d**2*e**4*x**4/(
5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x*
*2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5*d**4*e**8*sqrt(
d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**
4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2)), True)) + e*P
iecewise((210*I*d**7*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9*s
qrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) +
60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 105*pi*d**7*sqrt(-1 + e**2*x**
2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(
-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 210*I*d
**6*e*x/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(
-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 420*I*d
**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9*sqrt(-1
+ e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e*
*13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 210*pi*d**5*e**2*x**2*sqrt(-1 + e**2*
x**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sq
rt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 490*
I*d**4*e**3*x**3/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x
**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2))
+ 210*I*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9
*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2)
+ 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 105*pi*d**3*e**4*x**4*sqrt(-
1 + e**2*x**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**1
1*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2
)) - 322*I*d**2*e**5*x**5/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**
3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**
2/d**2)) + 30*I*e**7*x**7/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**
3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-105*d**7*sqrt(1 - e**2*x**2/d**2
)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*s
qrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 105*d
**6*e*x/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1
- e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 210*d**5*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x*
*2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sq
rt(1 - e**2*x**2/d**2)) - 245*d**4*e**3*x**3/(30*d**5*e**9*sqrt(1 - e**2*x*
*2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sq
rt(1 - e**2*x**2/d**2)) - 105*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2)*asin(
e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 -
 e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 161*d**2*e**
5*x**5/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 -
 e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*e**7*x**7
/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*
x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.18818, size = 162, normalized size = 1.01

$$-\frac{7}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-8)}\mathrm{sgn}\left(d\right) - \frac{\left(96\, d^7 e^{(-8)} + \left(105\, d^6 e^{(-7)} - \left(240\, d^5 e^{(-6)} + \left(245\, d^4 e^{(-5)} - \left(180\, d^3 e^{(-4)} + \left(161\, d^2 e^{(-3)} - \right.\right.\right.\right.\right.\right.}{30\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -7/2*d^2*arcsin(x*e/d)*e^(-8)*sgn(d) - 1/30*(96*d^7*e^(-8) + (105*d^6*e^(-7
) - (240*d^5*e^(-6) + (245*d^4*e^(-5) - (180*d^3*e^(-4) + (161*d^2*e^(-3) -
 15*(x*e^(-1) + 2*d*e^(-2))*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2
 - d^2)^3
```

3.20 $\int \frac{x^6(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=147

$$\frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

```
[Out] (x^5*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d + 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d + 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) + (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rubi [A] time = 0.119666, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {819, 641, 217, 203}

$$\frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^5*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d + 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d + 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) + (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{x^4\left(5d^3+6d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{x^2\left(15d^5+24d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{15d^7+48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^6}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,\right.}{e^6}\\
&= \frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}
\end{aligned}
$$

Mathematica [A] time = 0.0983305, size = 142, normalized size = 0.97

$$
\frac{-87d^3e^2x^2+52d^2e^3x^3-15d(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-33d^4ex+48d^5+38de^4x^4-15e^5x^5}{15e^7(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (48*d^5 - 33*d^4*e*x - 87*d^3*e^2*x^2 + 52*d^2*e^3*x^3 + 38*d*e^4*x^4 - 15*
e^5*x^5 - 15*d*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(15*e^7*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.091, size = 195, normalized size = 1.3

$$
-\frac{x^6}{e}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+6\frac{d^2x^4}{e^3\left(-x^2e^2+d^2\right)^{5/2}}-8\frac{d^4x^2}{e^5\left(-x^2e^2+d^2\right)^{5/2}}+\frac{16\,d^6}{5\,e^7}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{dx^5}{5\,e^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}-\frac{dx^3}{3\,e^4}\left(-\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -x^6/e/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^3*x^4/(-e^2*x^2+d^2)^(5/2)-8*d^4/e^5*x^
2/(-e^2*x^2+d^2)^(5/2)+16/5*d^6/e^7/(-e^2*x^2+d^2)^(5/2)+1/5*d*x^5/e^2/(-e^
2*x^2+d^2)^(5/2)-1/3*d/e^4*x^3/(-e^2*x^2+d^2)^(3/2)+d/e^6*x/(-e^2*x^2+d^2)^
```

```
(1/2)-d/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [B] time = 1.53098, size = 393, normalized size = 2.67

$$\frac{1}{15}dx\left(\frac{15x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{dx\left(\frac{3x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)}{3e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*d*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/((-e^2*x^2 + d^2)^
(5/2)*e) - 1/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 +
 d^2)^(3/2)*e^4))/e^2 + 6*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^3) - 8*d^4*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^5) + 16/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^7) + 4/15
*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^6) - 7/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^6) -
 d*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^6)
```

Fricas [B] time = 2.41813, size = 541, normalized size = 3.68

$$\frac{48de^5x^5-48d^2e^4x^4-96d^3e^3x^3+96d^4e^2x^2+48d^5ex-48d^6+30\left(de^5x^5-d^2e^4x^4-2d^3e^3x^3+2d^4e^2x^2+d^5ex-d^6\right.}{15\left(e^{12}x^5-de^{11}x^4-2d^2e^{10}x^3+2d^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(48*d*e^5*x^5 - 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 + 96*d^4*e^2*x^2 + 48*
d^5*e*x - 48*d^6 + 30*(d*e^5*x^5 - d^2*e^4*x^4 - 2*d^3*e^3*x^3 + 2*d^4*e^2*
x^2 + d^5*e*x - d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5*x^
5 - 38*d*e^4*x^4 - 52*d^2*e^3*x^3 + 87*d^3*e^2*x^2 + 33*d^4*e*x - 48*d^5)*s
qrt(-e^2*x^2 + d^2))/(e^12*x^5 - d*e^11*x^4 - 2*d^2*e^10*x^3 + 2*d^3*e^9*x^
2 + d^4*e^8*x - d^5*e^7)
```

Sympy [C] time = 40.3004, size = 1822, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7
*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) +
30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d**5*sqrt(-1 + e**2*x**2
/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1
+ e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*I*d**4*
```

```
e*x/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 60*I*d**3*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1 + e**2*
x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*
sqrt(-1 + e**2*x**2/d**2)) + 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)
/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2
*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 70*I*d**2*e**3*x
**3/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*I*d*e**4*x
**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1 + e**2*x**
2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqr
t(-1 + e**2*x**2/d**2)) - 15*pi*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(30*d
**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/
d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 46*I*e**5*x**5/(30*d**
5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d*
*2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2)
> 1), (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 -
 e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*
x**4*sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*sqrt(1 - e**2*x*
*2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqr
t(1 - e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*asin(e*
x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e*
*2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 35*d**2*e**3*x*
*3/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2
*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*d*e**4*x**4*sq
rt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) -
 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2
*x**2/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**
3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d
**2)), True)) + e*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) -
10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x
**2)) - 40*d**4*e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**
10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*
d**2*e**4*x**4/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqr
t(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5
*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**
2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2
)), True))
```

Giac [A] time = 1.17469, size = 147, normalized size = 1.

$$-d \arcsin\left(\frac{xe}{d}\right) e^{(-7)} \mathrm{sgn}\left(d\right) - \frac{\left(48\, d^{6} e^{(-7)} + \left(15\, d^{5} e^{(-6)} - \left(120\, d^{4} e^{(-5)} + \left(35\, d^{3} e^{(-4)} - \left(90\, d^{2} e^{(-3)} - \left(15\, x e^{(-1)} - 23\, d e^{(-2)}\right) x\right) x}{15 \left(x^{2} e^{2} - d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -d*arcsin(x*e/d)*e^(-7)*sgn(d) - 1/15*(48*d^6*e^(-7) + (15*d^5*e^(-6) - (12
0*d^4*e^(-5) + (35*d^3*e^(-4) - (90*d^2*e^(-3) - (15*x*e^(-1) - 23*d*e^(-2)
)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.21 $\int \frac{x^5(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=122

$$\frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] (x^4*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d + 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d + 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) - Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rubi [A] time = 0.080827, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {819, 778, 217, 203}

$$\frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d + 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d + 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) - Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^3\left(4d^3+5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x\left(8d^5+15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

$$= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Mathematica [A] time = 0.0869623, size = 130, normalized size = 1.07

$$\frac{-27d^2e^2x^2-15(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+7d^3ex+8d^4-8de^3x^3+23e^4x^4}{15e^6(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (8*d^4 + 7*d^3*e*x - 27*d^2*e^2*x^2 - 8*d*e^3*x^3 + 23*e^4*x^4 - 15*(d - e*
x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e
^6*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.076, size = 166, normalized size = 1.4

$$\frac{x^5}{5e}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} - \frac{x^3}{3e^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}} + \frac{x}{e^5}\frac{1}{\sqrt{-x^2e^2+d^2}} - \frac{1}{e^5}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + \frac{dx^4}{e^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*x^5/e/(-e^2*x^2+d^2)^(5/2)-1/3/e^3*x^3/(-e^2*x^2+d^2)^(3/2)+1/e^5*x/(-e
^2*x^2+d^2)^(1/2)-1/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/
2))+d*x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4/3*d^3/e^4*x^2/(-e^2*x^2+d^2)^(5/2)+8/1
5*d^5/e^6/(-e^2*x^2+d^2)^(5/2)
```

Maxima [B] time = 1.51158, size = 355, normalized size = 2.91

$$\frac{1}{15}\,ex\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)}{3\,e}+\frac{dx^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2
)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*x^
2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + d*x^4/((-e^2*
x^2 + d^2)^(5/2)*e^2) - 4/3*d^3*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8/15*d^5
/((-e^2*x^2 + d^2)^(5/2)*e^6) + 4/15*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 7
/15*x/(sqrt(-e^2*x^2 + d^2)*e^5) - arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e
^5)
```

Fricas [B] time = 2.09142, size = 497, normalized size = 4.07

$$\frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5+30\left(e^5x^5-de^4x^4-2\,d^2e^3x^3+2\,d^3e^2x^2+d^4ex-d^5\right)\arctan\left(\right.}{15\left(e^{11}x^5-de^{10}x^4-2\,d^2e^9x^3+2\,d^3e^8x^2+d^4e^7x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*x
 - 8*d^5 + 30*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^4*e*
x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (23*e^4*x^4 - 8*d*e^3*
x^3 - 27*d^2*e^2*x^2 + 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x^5 -
 d*e^10*x^4 - 2*d^2*e^9*x^3 + 2*d^3*e^8*x^2 + d^4*e^7*x - d^5*e^6)
```

Sympy [B] time = 43.4936, size = 1741, normalized size = 14.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**
2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) - 20*d**2*
e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**
2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e**4*x**4/(15*d
**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2 - e**2*x**2)
+ 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(d**2)**(7/2))
, True)) + e*Piecewise((30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(3
0*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x*
*2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d**5*sqrt(-1
+ e**2*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x*
```

```
*2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) -
 30*I*d**4*e*x/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*
sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 60
*I*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt
(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*
e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2
*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqr
t(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 70*I*
d**2*e**3*x**3/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*
sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30
*I*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1
 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**
11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d*e**4*x**4*sqrt(-1 + e**2*x**2/
d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 +
 e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 46*I*e**5*x
**5/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)
/Abs(d**2) > 1), (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e*
*7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) +
15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*sqrt(
1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**
11*x**4*sqrt(1 - e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d*
*2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*
sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 35*d
**2*e**3*x**3/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sq
rt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*d*e
**4*x**4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x
**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sq
rt(1 - e**2*x**2/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**
2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 -
e**2*x**2/d**2)), True))
```

Giac [A] time = 1.18063, size = 131, normalized size = 1.07

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}\left(d\right)-\frac{\left(8\,d^5e^{(-6)}+\left(15\,d^4e^{(-5)}-\left(20\,d^3e^{(-4)}+\left(35\,d^2e^{(-3)}-\left(23\,xe^{(-1)}+15\,de^{(-2)}\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-6)*sgn(d) - 1/15*(8*d^5*e^(-6) + (15*d^4*e^(-5) - (20*d^
3*e^(-4) + (35*d^2*e^(-3) - (23*x*e^(-1) + 15*d*e^(-2))*x)*x)*x)*x)*sqrt(-x
^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.22 $\int \frac{x^4(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}-\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (x^4*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (4*d^2)/(15*e^5*(d^2 - e^2*
x^2)^(3/2)) + 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0522609, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {805, 266, 43}

$$\frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}-\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (4*d^2)/(15*e^5*(d^2 - e^2*
x^2)^(3/2)) + 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{x^3}{(d^2-e^2x^2)^{5/2}}\,dx}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x}{(d^2-e^2x)^{5/2}}\,dx, x, x^2\right)}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\,\mathrm{Subst}\left(\int \left(\frac{d^2}{e^2(d^2-e^2x)^{5/2}} - \frac{1}{e^2(d^2-e^2x)^{3/2}}\right)dx, x, x^2\right)}{5e}$$
$$= \frac{x^4(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0274661, size = 82, normalized size = 0.98

$$\frac{-12d^2e^2x^2-8d^3ex+8d^4+12de^3x^3+3e^4x^4}{15de^5(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (8*d^4 - 8*d^3*e*x - 12*d^2*e^2*x^2 + 12*d*e^3*x^3 + 3*e^4*x^4)/(15*d*e^5*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.048, size = 77, normalized size = 0.9

$$\frac{(-ex+d)(ex+d)^2\left(3x^4e^4+12x^3de^3-12d^2x^2e^2-8d^3xe+8d^4\right)}{15de^5}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(3*e^4*x^4+12*d*e^3*x^3-12*d^2*e^2*x^2-8*d^3*e*x+8*
d^4)/d/e^5/(-e^2*x^2+d^2)^(7/2)
```

Maxima [B] time = 0.962339, size = 215, normalized size = 2.56

$$\frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{dx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{4d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{3d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{8d^4}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^5} + \frac{dx}{10\left(-e^2x^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4
/3*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^3*x/((-e^2*x^2 + d^2)^(5/2
```

```
)*e^4) + 8/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 1/10*d*x/((-e^2*x^2 + d^2)
^(3/2)*e^4) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^4)
```

Fricas [B] time = 1.96115, size = 343, normalized size = 4.08

$$\frac{8\,e^5x^5-8\,de^4x^4-16\,d^2e^3x^3+16\,d^3e^2x^2+8\,d^4ex-8\,d^5-\left(3\,e^4x^4+12\,de^3x^3-12\,d^2e^2x^2-8\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\,\left(de^{10}x^5-d^2e^9x^4-2\,d^3e^8x^3+2\,d^4e^7x^2+d^5e^6x-d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 - 8*d*e^4*x^4 - 16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 + 8*d^4*e*x
 - 8*d^5 - (3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 - 8*d^3*e*x + 8*d^4)*
sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 - d^2*e^9*x^4 - 2*d^3*e^8*x^3 + 2*d^4*e^7
*x^2 + d^5*e^6*x - d^6*e^5)
```

Sympy [C] time = 38.0299, size = 420, normalized size = 5.

$$d\left(\begin{cases} -\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)+e\left(\begin{cases} \frac{8d^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} - \\ \frac{x^6}{6\left(d^2\right)^{\frac{7}{2}}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) - 10*d**5*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2)/Abs(d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2/d**2) - 10*d*
*5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(1 - e**2*x**2
/d**2)), True)) + e*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2)
- 30*d**2*e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2
*x**2)) - 20*d**2*e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*
e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) +
15*e**4*x**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(
d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6
/(6*(d**2)**(7/2)), True))
```

Giac [A] time = 1.2162, size = 86, normalized size = 1.02

$$-\frac{\left(8\,d^4e^{(-5)}+\left(3\,x^2\left(\frac{x}{d}+5\,e^{(-1)}\right)-20\,d^2e^{(-3)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\,\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(8*d^4*e^(-5) + (3*x^2*(x/d + 5*e^(-1)) - 20*d^2*e^(-3))*x^2)*sqrt(-x
^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.23 $\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=90

$$\frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (x^2*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d + 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) + x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0420948, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {819, 778, 191}

$$\frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d + 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) + x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x(2d^3+3d^2ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3}$$

$$= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0257207, size = 82, normalized size = 0.91

$$\frac{3d^2e^2x^2+2d^3ex-2d^4-3de^3x^3+3e^4x^4}{15d^2e^4(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*e^4*x^4)/(15*d^2*e^4*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.046, size = 77, normalized size = 0.9

$$-\frac{(-ex+d)(ex+d)^2\left(-3x^4e^4+3x^3de^3-3x^2d^2e^2-2d^3xe+2d^4\right)}{15d^2e^4}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^2*(-3*e^4*x^4+3*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*
d^4)/d^2/e^4/(-e^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 0.985996, size = 181, normalized size = 2.01

$$\frac{x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{dx^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{3d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{2d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4} + \frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3} + \frac{x}{5\sqrt{-e^2x^2+d^2}\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) + 1/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2)
 - 3/10*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 2/15*d^3/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 1/10*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 1/5*x/(sqrt(-e^2*x^2 + d^2)
*d^2*e^3)
```

Fricas [B] time = 1.96432, size = 340, normalized size = 3.78

$$-\frac{2 e^5 x^5 - 2 d e^4 x^4 - 4 d^2 e^3 x^3 + 4 d^3 e^2 x^2 + 2 d^4 e x - 2 d^5 + \left(3 e^4 x^4 - 3 d e^3 x^3 + 3 d^2 e^2 x^2 + 2 d^3 e x - 2 d^4\right)\sqrt{-e^2 x^2 + d^2}}{15 \left(d^2 e^9 x^5 - d^3 e^8 x^4 - 2 d^4 e^7 x^3 + 2 d^5 e^6 x^2 + d^6 e^5 x - d^7 e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
- 2*d^5 + (3*e^4*x^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2*d^3*e*x - 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^2*e^9*x^5 - d^3*e^8*x^4 - 2*d^4*e^7*x^3 + 2*d^5*e^6*x
^2 + d^6*e^5*x - d^7*e^4)
```

Sympy [B] time = 13.4521, size = 338, normalized size = 3.76

$$d\left(\begin{cases} -\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} + \frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for } e \neq 0 \\ \frac{x^4}{4\left(d^2\right)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} -\frac{\quad}{5d^7\sqrt{\quad}} \\ \frac{\quad}{5d^7\sqrt{1-}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x*
*2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)) + 5*e**2*x
**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*sqrt(d**2 - e*
*2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)
**(7/2)), True)) + e*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) -
 10*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2
*x**2/d**2) - 10*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4
*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.20696, size = 78, normalized size = 0.87

$$\frac{\left(2 d^3 e^{(-4)} - \left(\frac{3 x^3 e}{d^2} + 5 d e^{(-2)}\right) x^2\right) \sqrt{-x^2 e^2 + d^2}}{15 \left(x^2 e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*d^3*e^(-4) - (3*x^3*e/d^2 + 5*d*e^(-2))*x^2)*sqrt(-x^2*e^2 + d^2)/(
x^2*e^2 - d^2)^3
```

3.24 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (x^2*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d - e*x))/(15*d*e^3*(d^
2 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.045912, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {796, 778, 191}

$$\frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d - e*x))/(15*d*e^3*(d^
2 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x(2d^2e-2de^2x)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$
$$= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15de^2}$$
$$= \frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0284838, size = 82, normalized size = 0.87

$$\frac{3d^2e^2x^2+2d^3ex-2d^4+2de^3x^3-2e^4x^4}{15d^3e^3(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e^4*x^4)/(15*d^3*e^3*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.048, size = 77, normalized size = 0.8

$$-\frac{(-ex+d)(ex+d)^2\left(2e^4x^4-2x^3de^3-3x^2d^2e^2-2xd^3e+2d^4\right)}{15d^3e^3}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^2*(2*e^4*x^4-2*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*d
^4)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 0.992948, size = 151, normalized size = 1.61

$$\frac{x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{2d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} - \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2} - \frac{2x}{15\sqrt{-e^2x^2+d^2}d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e^2) -
 2/15*d^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e
^2) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e^2)
```

Fricas [B] time = 1.9073, size = 340, normalized size = 3.62

$$-\frac{2 e^5x^5 - 2 de^4x^4 - 4 d^2e^3x^3 + 4 d^3e^2x^2 + 2 d^4ex - 2 d^5 - \left(2 e^4x^4 - 2 de^3x^3 - 3 d^2e^2x^2 - 2 d^3ex + 2 d^4\right)\sqrt{-e^2x^2 + d^2}}{15 \left(d^3e^8x^5 - d^4e^7x^4 - 2 d^5e^6x^3 + 2 d^6e^5x^2 + d^7e^4x - d^8e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 - 2*d*e^4*x^4 - 4*d^2*e^3*x^3 + 4*d^3*e^2*x^2 + 2*d^4*e*x
- 2*d^5 - (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 2*d^3*e*x + 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^3*e^8*x^5 - d^4*e^7*x^4 - 2*d^5*e^6*x^3 + 2*d^6*e^5*x
^2 + d^7*e^4*x - d^8*e^3)
```

Sympy [C] time = 13.1192, size = 515, normalized size = 5.48

$$d\left(\begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2x^2|}{|d^2|} > 1 \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e**2*x**
2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**2*x**3/(15*d**9*sqrt(1 - e**2*x**2/d*
*2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1
 - e**2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d*
*7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**
2/d**2)), True)) + e*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2
) - 30*d**2*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**
2*x**2)) + 5*e**2*x**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*
x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0
)), (x**4/(4*(d**2)**(7/2)), True))
```

Giac [A] time = 1.18525, size = 86, normalized size = 0.91

$$\frac{\left(\left(x\left(\frac{2 x^2e^2}{d^3} - \frac{5}{d}\right) - 5 e^{(-1)}\right)x^2 + 2 d^2e^{(-3)}\right)\sqrt{-x^2e^2 + d^2}}{15 \left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((x*(2*x^2*e^2/d^3 - 5/d) - 5*e^(-1))*x^2 + 2*d^2*e^(-3))*sqrt(-x^2*e^
2 + d^2)/(x^2*e^2 - d^2)^3
```

3.25 $\int \frac{x(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (d + e*x)/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)
) - (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.022972, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {778, 192, 191}

$$-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)
) - (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{1}{(d^2-e^2x^2)^{5/2}}\,dx}{5e}$$

$$= \frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} - \frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2e}$$

$$= \frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0389166, size = 82, normalized size = 0.99

$$\frac{3d^2e^2x^2-3d^3ex+3d^4+2de^3x^3-2e^4x^4}{15d^4e^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (3*d^4 - 3*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e^4*x^4)/(15*d^4*e^2*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.049, size = 77, normalized size = 0.9

$$\frac{(-ex+d)(ex+d)^2(-2e^4x^4+2e^3x^3d+3x^2d^2e^2-3xd^3e+3d^4)}{15d^4e^2}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(-2*e^4*x^4+2*d*e^3*x^3+3*d^2*e^2*x^2-3*d^3*e*x+3*d
^4)/d^4/e^2/(-e^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 1.00599, size = 117, normalized size = 1.41

$$\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}+\frac{d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e}-\frac{2x}{15\sqrt{-e^2x^2+d^2}d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/1
5*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

Fricas [B] time = 2.01013, size = 339, normalized size = 4.08

$$\frac{3\,e^{5}x^{5}-3\,de^{4}x^{4}-6\,d^{2}e^{3}x^{3}+6\,d^{3}e^{2}x^{2}+3\,d^{4}ex-3\,d^{5}+\left(2\,e^{4}x^{4}-2\,de^{3}x^{3}-3\,d^{2}e^{2}x^{2}+3\,d^{3}ex-3\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\left(d^{4}e^{7}x^{5}-d^{5}e^{6}x^{4}-2\,d^{6}e^{5}x^{3}+2\,d^{7}e^{4}x^{2}+d^{8}e^{3}x-d^{9}e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x -
 3*d^5 + (2*e^4*x^4 - 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 3*d^3*e*x - 3*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^4*e^7*x^5 - d^5*e^6*x^4 - 2*d^6*e^5*x^3 + 2*d^7*e^4*x^
2 + d^8*e^3*x - d^9*e^2)
```

Sympy [A] time = 11.4744, size = 434, normalized size = 5.23

$$d\left(\begin{cases}\frac{1}{5d^{4}e^{2}\sqrt{d^{2}-e^{2}x^{2}}-10d^{2}e^{4}x^{2}\sqrt{d^{2}-e^{2}x^{2}}+5e^{6}x^{4}\sqrt{d^{2}-e^{2}x^{2}}} & \text{for}\ e\neq 0\\ \frac{x^{2}}{2\left(d^{2}\right)^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{5id^{2}x^{3}}{15d^{9}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}-30d^{7}e^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}+15d^{5}e^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\cdots\\ \frac{5d^{2}x^{3}}{15d^{9}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}-30d^{7}e^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}+15d^{5}e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**4*x**2*sqrt
(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**2/
(2*(d**2)**(7/2)), True)) + e*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 +
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x
**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**2*x**3/(1
5*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2
) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt
(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5
*e**4*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.18807, size = 77, normalized size = 0.93

$$\frac{\left(x^{3}\left(\frac{2\,x^{2}e^{3}}{d^{4}}-\frac{5\,e}{d^{2}}\right)-3\,de^{(-2)}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{15\left(x^{2}e^{2}-d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^3*(2*x^2*e^3/d^4 - 5*e/d^2) - 3*d*e^(-2))*sqrt(-x^2*e^2 + d^2)/(x^2
*e^2 - d^2)^3
```

3.26 $\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=80

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/
2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0209599, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {639, 192, 191}

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/
2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$
$$= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}$$
$$= \frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0281041, size = 82, normalized size = 1.02

$$\frac{-12d^2e^2x^2+12d^3ex+3d^4-8de^3x^3+8e^4x^4}{15d^5e(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 + 8*e^4*x^4)/(15*d^5*e*(
d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.049, size = 77, normalized size = 1.

$$\frac{(-ex+d)(ex+d)^2\left(8e^4x^4-8e^3x^3d-12e^2x^2d^2+12xd^3e+3d^4\right)}{15d^5e}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(8*e^4*x^4-8*d*e^3*x^3-12*d^2*e^2*x^2+12*d^3*e*x+3*
d^4)/d^5/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 0.995395, size = 108, normalized size = 1.35

$$\frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d} + \frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{4x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3} + \frac{8x}{15\sqrt{-e^2x^2+d^2}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*d) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e) + 4/15*x/
((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Fricas [B] time = 1.95169, size = 340, normalized size = 4.25

$$\frac{3\,e^{5}x^{5}-3\,de^{4}x^{4}-6\,d^{2}e^{3}x^{3}+6\,d^{3}e^{2}x^{2}+3\,d^{4}ex-3\,d^{5}-\left(8\,e^{4}x^{4}-8\,de^{3}x^{3}-12\,d^{2}e^{2}x^{2}+12\,d^{3}ex+3\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\,\left(d^{5}e^{6}x^{5}-d^{6}e^{5}x^{4}-2\,d^{7}e^{4}x^{3}+2\,d^{8}e^{3}x^{2}+d^{9}e^{2}x-d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x -
 3*d^5 - (8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 + 12*d^3*e*x + 3*d^4)*sq
rt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 - d^6*e^5*x^4 - 2*d^7*e^4*x^3 + 2*d^8*e^3*
x^2 + d^9*e^2*x - d^10*e)
```

Sympy [C] time = 12.4227, size = 605, normalized size = 7.56

$$d\left(\begin{cases}-\frac{15id^{4}x}{15d^{11}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}+15d^{7}e^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\frac{20id^{2}e^{2}x^{3}}{15d^{11}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}+15d^{7}e^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{}{15d^{11}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}-} \\ \frac{15d^{4}x}{15d^{11}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}+15d^{7}e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-\frac{20d^{2}e^{2}x^{3}}{15d^{11}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}+15d^{7}e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{8e^{4}x}{15d^{11}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}-30d^{9}e^{2}x^{2}\sqrt{1}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2))
, Abs(e**2*x**2)/Abs(d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d*
*2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1
 - e**2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d**2)
- 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e
**2*x**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*
e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d
**2)), True)) + e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**
2*e**4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), N
e(e, 0)), (x**2/(2*(d**2)**(7/2)), True))
```

Giac [A] time = 1.17335, size = 88, normalized size = 1.1

$$-\frac{\sqrt{-x^{2}e^{2}+d^{2}}\left(\left(4\,x^{2}\left(\frac{2\,x^{2}e^{4}}{d^{5}}-\frac{5\,e^{2}}{d^{3}}\right)+\frac{15}{d}\right)x+3\,e^{(-1)}\right)}{15\,\left(x^{2}e^{2}-d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((4*x^2*(2*x^2*e^4/d^5 - 5*e^2/d^3) + 15/d)*x +
3*e^(-1))/(x^2*e^2 - d^2)^3
```

3.27 $\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] (d + e*x)/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (5*d + 4*e*x)/(15*d^4*(d^2 - e^2*
x^2)^(3/2)) + (15*d + 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^
2 - e^2*x^2]/d]/d^6
```

Rubi [A] time = 0.103197, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {823, 12, 266, 63, 208}

$$\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (d + e*x)/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (5*d + 4*e*x)/(15*d^4*(d^2 - e^2*
x^2)^(3/2)) + (15*d + 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^
2 - e^2*x^2]/d]/d^6
```


Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{5d^3e^2+4d^2e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}\\
&= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{15d^5e^4+8d^4e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}\\
&= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{\int \frac{15d^7e^6}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}\\
&= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}\\
&= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d^5}\\
&= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d^5e^2}\\
&= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}
\end{aligned}$$

Mathematica [A] time = 0.0679326, size = 131, normalized size = 1.12

$$\frac{-27d^2e^2x^2-15(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-8d^3ex+23d^4+7de^3x^3+8e^4x^4}{15d^6(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (23*d^4 - 8*d^3*e*x - 27*d^2*e^2*x^2 + 7*d*e^3*x^3 + 8*e^4*x^4 - 15*(d - e*
x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(15*d^6*
(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.055, size = 163, normalized size = 1.4

$$\frac{ex}{5d^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{4ex}{15d^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{8ex}{15d^6}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{1}{5d}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{1}{3d^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{1}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/15*e/d^4*x/(-e^2*x^2+d^2)^(3/2)+8/15*e/d
^6*x/(-e^2*x^2+d^2)^(1/2)+1/5/d/(-e^2*x^2+d^2)^(5/2)+1/3/d^3/(-e^2*x^2+d^2)
^(3/2)+1/d^5/(-e^2*x^2+d^2)^(1/2)-1/d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)
*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97722, size = 493, normalized size = 4.21

$$\frac{23 e^5x^5 - 23 de^4x^4 - 46 d^2e^3x^3 + 46 d^3e^2x^2 + 23 d^4ex - 23 d^5 + 15 \left(e^5x^5 - de^4x^4 - 2 d^2e^3x^3 + 2 d^3e^2x^2 + d^4ex - d^5\right) \log \left(-\right.}{15 \left(d^6e^5x^5 - d^7e^4x^4 - 2 d^8e^3x^3 + 2 d^9e^2x^2 + d^{10}ex -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^5*x^5 - 23*d*e^4*x^4 - 46*d^2*e^3*x^3 + 46*d^3*e^2*x^2 + 23*d^4*
e*x - 23*d^5 + 15*(e^5*x^5 - d*e^4*x^4 - 2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 + d^
4*e*x - d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (8*e^4*x^4 + 7*d*e^3*x^3
- 27*d^2*e^2*x^2 - 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x^5 -
 d^7*e^4*x^4 - 2*d^8*e^3*x^3 + 2*d^9*e^2*x^2 + d^10*e*x - d^11)
```

Sympy [C] time = 22.3219, size = 2382, normalized size = 20.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d**
2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6)
 + 30*d**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) - 30*I*d**6*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x*
*2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*I*d**4*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d
**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**1
```

```
1*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**2*x**2*lo
g(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6
*x**6) + 90*I*d**4*e**2*x**2*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x**2
- 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**2*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7
*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e
**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e**4*x**4*log(e
*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x*
*6) - 90*I*d**2*e**4*x**4*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x**2 - 9
0*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*log(e**2*x**2/d**2)/(-
30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30
*e**6*x**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) + 30*I*e**6*x**6*asin(d/(e*x))/(-30*d**13 + 90*d**11*e*
*2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6), Abs(e**2*x**2)/Abs(d**2)
> 1), (-46*d**6*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 -
90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d**2)/(-30*d
**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 30*d**
6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d*
*9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*I*pi*d**6/(-30*d**13 + 90*d**11*e**2
*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*d**4*e**2*x**2*sqrt(1 -
 e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d
**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**1
1*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**2*x**2*lo
g(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e
**4*x**4 + 30*d**7*e**6*x**6) + 45*I*pi*d**4*e**2*x**2/(-30*d**13 + 90*d**1
1*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*d**2*e**4*x**4*sq
rt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4
+ 30*d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**13 + 9
0*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e**4*x
**4*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*
d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 45*I*pi*d**2*e**4*x**4/(-30*d**13 + 9
0*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*l
og(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30
*d**7*e**6*x**6) - 30*e**6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**1
3 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*I*pi*e
**6*x**6/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6
*x**6), True)) + e*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d*
*2) - 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(
-1 + e**2*x**2/d**2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d
**2) - 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2)
- 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (15*d**4*x/(15*d**11*sqrt
(1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7
*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 -
 e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**
4*x**4*sqrt(1 - e**2*x**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2
/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqr
t(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.1866, size = 165, normalized size = 1.41

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{8\,xe^5}{d^6}+\frac{15\,e^4}{d^5}\right)-\frac{20\,e^3}{d^4}\right)x-\frac{35\,e^2}{d^3}\right)x+\frac{15\,e}{d^2}\right)x+\frac{23}{d}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(8*x*e^5/d^6 + 15*e^4/d^5) - 20*e^3/d^4)*x
 - 35*e^2/d^3)*x + 15*e/d^2)*x + 23/d)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d
*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6
```

3.28 $\int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=153

$$\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

```
[Out] (d + e*x)/(5*d^2*x*(d^2 - e^2*x^2)^(5/2)) + (6*d + 5*e*x)/(15*d^4*x*(d^2 -
e^2*x^2)^(3/2)) + (8*d + 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[d^
2 - e^2*x^2])/(5*d^7*x) - (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rubi [A] time = 0.126677, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {823, 807, 266, 63, 208}

$$\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x*(d^2 - e^2*x^2)^(5/2)) + (6*d + 5*e*x)/(15*d^4*x*(d^2 -
e^2*x^2)^(3/2)) + (8*d + 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[d^
2 - e^2*x^2])/(5*d^7*x) - (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{6d^3e^2+5d^2e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{24d^5e^4+15d^4e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} + \frac{\int \frac{48d^7e^6+15d^6e^7x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} + \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,d}{d^6}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} + \frac{e\,\mathrm{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{2}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x}{e}}\right.}{}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} - \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-}}{d}\right.}{d^7}$$

Mathematica [A] time = 0.0778692, size = 147, normalized size = 0.96

$$\frac{52d^3e^2x^2-87d^2e^3x^3-15ex(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+38d^4ex-15d^5-33de^4x^4+48e^5x^5}{15d^7x(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-15*d^5 + 38*d^4*e*x + 52*d^3*e^2*x^2 - 87*d^2*e^3*x^3 - 33*d*e^4*x^4 + 48
*e^5*x^5 - 15*e*x*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^
2 - e^2*x^2]/d])/(15*d^7*x*(d - e*x)^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.058, size = 195, normalized size = 1.3

$$\frac{e}{5d^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{e}{3d^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{e}{d^6}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{e}{d^6}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{1}{dx}\left(-x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e/d^2/(-e^2*x^2+d^2)^(5/2)+1/3*e/d^4/(-e^2*x^2+d^2)^(3/2)+e/d^6/(-e^2*x
^2+d^2)^(1/2)-e/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2
))/x)-1/d/x/(-e^2*x^2+d^2)^(5/2)+6/5*e^2/d^3*x/(-e^2*x^2+d^2)^(5/2)+8/5*e^2
/d^5*x/(-e^2*x^2+d^2)^(3/2)+16/5*e^2/d^7*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15593, size = 548, normalized size = 3.58

$$\frac{23e^6x^6-23de^5x^5-46d^2e^4x^4+46d^3e^3x^3+23d^4e^2x^2-23d^5ex+15\left(e^6x^6-de^5x^5-2d^2e^4x^4+2d^3e^3x^3+d^4e^2x^2-\right.}{15\left(d^7e^5x^6-d^8e^4x^5-2d^9e^3x^4+2d^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^6*x^6 - 23*d*e^5*x^5 - 46*d^2*e^4*x^4 + 46*d^3*e^3*x^3 + 23*d^4*
e^2*x^2 - 23*d^5*e*x + 15*(e^6*x^6 - d*e^5*x^5 - 2*d^2*e^4*x^4 + 2*d^3*e^3*
x^3 + d^4*e^2*x^2 - d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (48*e^5*x
^5 - 33*d*e^4*x^4 - 87*d^2*e^3*x^3 + 52*d^3*e^2*x^2 + 38*d^4*e*x - 15*d^5)*
sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 - d^8*e^4*x^5 - 2*d^9*e^3*x^4 + 2*d^10*e
^2*x^3 + d^11*e*x^2 - d^12*x)
```

Sympy [C] time = 20.1131, size = 2409, normalized size = 15.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*
x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2
```

```
/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d
**8*e**6*x**6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 1
5*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*s
qrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x*
*4 + 5*d**8*e**6*x**6), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (5*I*d**6*e*s
qrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x
**4 + 5*d**8*e**6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(
-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40
*I*d**2*e**5*x**4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**
2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**
2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e
**6*x**6), True)) + e*Piecewise((-46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(-30*
d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d*
*6*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4
+ 30*d**7*e**6*x**6) + 30*d**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 -
 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**6*asin(d/(e*x))/(-30*d**1
3 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*I*d**4
*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d
**9*e**4*x**4 + 30*d**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/
(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) -
90*d**4*e**2*x**2*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4
*x**4 + 30*d**7*e**6*x**6) + 90*I*d**4*e**2*x**2*asin(d/(e*x))/(-30*d**13 +
 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**2*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9
*e**4*x**4 + 30*d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-3
0*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*
d**2*e**4*x**4*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x*
*4 + 30*d**7*e**6*x**6) - 90*I*d**2*e**4*x**4*asin(d/(e*x))/(-30*d**13 + 90
*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*lo
g(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*
d**7*e**6*x**6) - 30*e**6*x**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 -
 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 30*I*e**6*x**6*asin(d/(e*x))/(-30
*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6), Abs(e
**2*x**2)/Abs(d**2) > 1), (-46*d**6*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 9
0*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e*
*2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7
*e**6*x**6) + 30*d**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**
11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*I*pi*d**6/(-30*d
**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*d**
4*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d
**9*e**4*x**4 + 30*d**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/
(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) -
90*d**4*e**2*x**2*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e
**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 45*I*pi*d**4*e**2*x**2/
(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) -
30*d**2*e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2
- 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/
d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x*
*6) + 90*d**2*e**4*x**4*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d
**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 45*I*pi*d**2*e**4
*x**4/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x*
*6) + 15*e**6*x**6*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90
*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*e**6*x**6*log(sqrt(1 - e**2*x**2/
d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e*
*6*x**6) + 15*I*pi*e**6*x**6/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4
*x**4 + 30*d**7*e**6*x**6), True))
```

Giac [A] time = 1.18744, size = 255, normalized size = 1.67

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(3\left(x\left(\frac{11\,xe^6}{d^7}+\frac{5\,e^5}{d^6}\right)-\frac{25\,e^4}{d^5}\right)x-\frac{35\,e^3}{d^4}\right)x+\frac{45\,e^2}{d^3}\right)x+\frac{23\,e}{d^2}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{d^7}+\frac{xe^3}{2\left(de+\sqrt{-x^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(((3*(x*(11*x*e^6/d^7 + 5*e^5/d^6) - 25*e^4/d^5)
*x - 35*e^3/d^4)*x + 45*e^2/d^3)*x + 23*e/d^2)/(x^2*e^2 - d^2)^3 - e*log(1/
2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^7 + 1/2*x*e^3/((d
*e + sqrt(-x^2*e^2 + d^2)*e)*d^7) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-
1)/(d^7*x)
```

3.29 $\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

```
[Out] (d + e*x)/(5*d^2*x^2*(d^2 - e^2*x^2)^(5/2)) + (7*d + 6*e*x)/(15*d^4*x^2*(d^
2 - e^2*x^2)^(3/2)) + (35*d + 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) - (7
*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) - (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x) -
(7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rubi [A] time = 0.15962, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {823, 835, 807, 266, 63, 208}

$$-\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (d + e*x)/(5*d^2*x^2*(d^2 - e^2*x^2)^(5/2)) + (7*d + 6*e*x)/(15*d^4*x^2*(d^
2 - e^2*x^2)^(3/2)) + (35*d + 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) - (7
*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) - (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x) -
(7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```


Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{7d^3e^2+6d^2e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$=\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{35d^5e^4+24d^4e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$=\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{\int \frac{105d^7e^6+48d^6e^7x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$=\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-\frac{\int \frac{-96d^8e^7 \cdots}{x^2\sqrt{\cdots}}}{30\cdots}$$

$$=\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-\frac{16e\sqrt{d^2 \cdots}}{5d^8 \cdots}$$

$$=\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-\frac{16e\sqrt{d^2 \cdots}}{5d^8 \cdots}$$

$$=\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-\frac{16e\sqrt{d^2 \cdots}}{5d^8 \cdots}$$

$$=\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-\frac{16e\sqrt{d^2 \cdots}}{5d^8 \cdots}$$

Mathematica [A] time = 0.147898, size = 183, normalized size = 0.99

$$\frac{d\sqrt{1-\frac{e^2x^2}{d^2}}\left(176d^4e^2x^2+4d^3e^3x^3-249d^2e^4x^4-15d^5ex-15d^6+9de^5x^5+96e^6x^6\right)+105e^2x^2(d+ex)^2(ex-d)^3\tanh^{-1}\left(\right.}{30d^9x^2(d-ex)^2(d+ex)\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d*Sqrt[1 - (e^2*x^2)/d^2]*(-15*d^6 - 15*d^5*e*x + 176*d^4*e^2*x^2 + 4*d^3*
e^3*x^3 - 249*d^2*e^4*x^4 + 9*d*e^5*x^5 + 96*e^6*x^6) + 105*e^2*x^2*(-d + e
*x)^3*(d + e*x)^2*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(30*d^9*x^2*(d - e*x)^2
*(d + e*x)*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.061, size = 227, normalized size = 1.2

$$-\frac{1}{2\,dx^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{7\,e^2}{10\,d^3}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{7\,e^2}{6\,d^5}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{7\,e^2}{2\,d^7}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{7\,e^2}{2\,d^7}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2/d/x^2/(-e^2*x^2+d^2)^(5/2)+7/10*e^2/d^3/(-e^2*x^2+d^2)^(5/2)+7/6*e^2/d
^5/(-e^2*x^2+d^2)^(3/2)+7/2*e^2/d^7/(-e^2*x^2+d^2)^(1/2)-7/2*e^2/d^7/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-e/d^2/x/(-e^2*x^2+d^
2)^(5/2)+6/5*e^3/d^4*x/(-e^2*x^2+d^2)^(5/2)+8/5*e^3/d^6*x/(-e^2*x^2+d^2)^(3
/2)+16/5*e^3/d^8*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47298, size = 595, normalized size = 3.23

$$\frac{116\,e^7x^7-116\,de^6x^6-232\,d^2e^5x^5+232\,d^3e^4x^4+116\,d^4e^3x^3-116\,d^5e^2x^2+105\left(e^7x^7-de^6x^6-2\,d^2e^5x^5+2\,d^3e^4x^4+d^4e\right.}{30\left(d^8e^5x^7-d^9e^4x^6-2\,d^{10}e^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*e^7*x^7 - 116*d*e^6*x^6 - 232*d^2*e^5*x^5 + 232*d^3*e^4*x^4 + 116
*d^4*e^3*x^3 - 116*d^5*e^2*x^2 + 105*(e^7*x^7 - d*e^6*x^6 - 2*d^2*e^5*x^5 +
 2*d^3*e^4*x^4 + d^4*e^3*x^3 - d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) - (96*e^6*x^6 + 9*d*e^5*x^5 - 249*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 176*d^4
*e^2*x^2 - 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 - d^9*e^
4*x^6 - 2*d^10*e^3*x^5 + 2*d^11*e^2*x^4 + d^12*e*x^3 - d^13*x^2)
```

Sympy [C] time = 22.2567, size = 2696, normalized size = 14.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((30*I*d**8*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**1
3*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 322*I*d**6*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d*
*11*e**4*x**6 + 60*d**9*e**6*x**8) - 105*d**6*e**2*x**2*log(e**2*x**2/d**2)
/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6
*x**8) + 210*d**6*e**2*x**2*log(e*x/d)/(-60*d**15*x**2 + 180*d**13*e**2*x**
4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*I*d**6*e**2*x**2*asin(d/
(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**
9*e**6*x**8) + 490*I*d**4*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x*
*2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 315*d
**4*e**4*x**4*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 1
80*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 630*d**4*e**4*x**4*log(e*x/d)/(-6
0*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**
8) + 630*I*d**4*e**4*x**4*asin(d/(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x*
*4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*I*d**2*e**6*x**6*sqrt(-
1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*
x**6 + 60*d**9*e**6*x**8) - 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(-60*d**
15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) +
630*d**2*e**6*x**6*log(e*x/d)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d
**11*e**4*x**6 + 60*d**9*e**6*x**8) - 630*I*d**2*e**6*x**6*asin(d/(e*x))/(-
60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x*
*8) + 105*e**8*x**8*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x*
*4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*e**8*x**8*log(e*x/d)/(-
60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x*
*8) + 210*I*e**8*x**8*asin(d/(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x**4 -
 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), Abs(e**2*x**2)/Abs(d**2) > 1), (
30*d**8*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 18
0*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 322*d**6*e**2*x**2*sqrt(1 - e**2*x
**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*
d**9*e**6*x**8) - 105*d**6*e**2*x**2*log(e**2*x**2/d**2)/(-60*d**15*x**2 +
180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 210*d**6*e
**2*x**2*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2
*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 105*I*pi*d**6*e**2*x**2/
(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*
x**8) + 490*d**4*e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d
**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 315*d**4*e**4*x
**4*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e
**4*x**6 + 60*d**9*e**6*x**8) - 630*d**4*e**4*x**4*log(sqrt(1 - e**2*x**2/d
**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*
d**9*e**6*x**8) + 315*I*pi*d**4*e**4*x**4/(-60*d**15*x**2 + 180*d**13*e**2*
x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*d**2*e**6*x**6*sqrt(1
 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x
```

```
**6 + 60*d**9*e**6*x**8) - 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(-60*d**1
5*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 6
30*d**2*e**6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d
**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 315*I*pi*d**2*e
**6*x**6/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d
**9*e**6*x**8) + 105*e**8*x**8*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d*
*13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*e**8*x**8*lo
g(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180
*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 105*I*pi*e**8*x**8/(-60*d**15*x**2
+ 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), True)) +
e*Piecewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*
x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2
/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d
**8*e**6*x**6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 1
5*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*s
qrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x*
*4 + 5*d**8*e**6*x**6), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (5*I*d**6*e*s
qrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x
**4 + 5*d**8*e**6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(
-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40
*I*d**2*e**5*x**4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**
2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**
2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e
**6*x**6), True))
```

Giac [A] time = 1.18364, size = 351, normalized size = 1.91

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(3\left(x\left(\frac{11\,xe^7}{d^8}+\frac{15\,e^6}{d^7}\right)-\frac{25\,e^5}{d^6}\right)x-\frac{100\,e^4}{d^5}\right)x+\frac{45\,e^3}{d^4}\right)x+\frac{58\,e^2}{d^3}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{7\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^8}+\frac{x^2\left(\frac{4\left(de+\sqrt{-x\cdots}\right.}{\cdots}\right.}{8\left(de+\sqrt{-\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(((3*(x*(11*x*e^7/d^8 + 15*e^6/d^7) - 25*e^5/d^6
)*x - 100*e^4/d^5)*x + 45*e^3/d^4)*x + 58*e^2/d^3)/(x^2*e^2 - d^2)^3 - 7/2*
e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^8 + 1/8
*x^2*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*d^8) - 1/8*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*e^8/x + (d*e +
sqrt(-x^2*e^2 + d^2)*e)^2*d^8*e^6/x^2)*e^(-8)/d^16
```

3.30 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=121

$$\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (x^2*(d + e*x))/(7*d*e*(d^2 - e^2*x^2)^(7/2)) - (2*(d - 2*e*x))/(35*d*e^3*(
d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(
105*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0525345, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {796, 778, 192, 191}

$$\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (x^2*(d + e*x))/(7*d*e*(d^2 - e^2*x^2)^(7/2)) - (2*(d - 2*e*x))/(35*d*e^3*(
d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(
105*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx = \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{\int \frac{x\left(2d^2e-4de^2x\right)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7d^2e^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35de^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{105d^3e^2}$$

$$= \frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}} - \frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0453019, size = 104, normalized size = 0.86

$$\frac{15d^4e^2x^2+20d^3e^3x^3-20d^2e^4x^4+6d^5ex-6d^6-8de^5x^5+8e^6x^6}{105d^5e^3(d-ex)^3(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (-6*d^6 + 6*d^5*e*x + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 - 20*d^2*e^4*x^4 - 8*
d*e^5*x^5 + 8*e^6*x^6)/(105*d^5*e^3*(d - e*x)^3*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2])
```

Maple [A] time = 0.053, size = 99, normalized size = 0.8

$$-\frac{(-ex+d)\,(ex+d)^2\left(-8\,e^6x^6+8\,e^5x^5d+20\,e^4x^4d^2-20\,x^3d^3e^3-15\,x^2d^4e^2-6\,xd^5e+6\,d^6\right)}{105\,d^5e^3}\left(-x^2e^2+d^2\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x)
```

```
[Out] -1/105*(-e*x+d)*(e*x+d)^2*(-8*e^6*x^6+8*d*e^5*x^5+20*d^2*e^4*x^4-20*d^3*e^3
*x^3-15*d^4*e^2*x^2-6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(9/2)
```

Maxima [A] time = 0.995122, size = 182, normalized size = 1.5

$$\frac{x^2}{5\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e}+\frac{dx}{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^2}-\frac{2\,d^2}{35\left(-e^2x^2+d^2\right)^{\frac{7}{2}}e^3}-\frac{x}{35\left(-e^2x^2+d^2\right)^{\frac{5}{2}}de^2}-\frac{4\,x}{105\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3e^2}-\frac{}{105\sqrt{-e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^2/((-e^2*x^2 + d^2)^(7/2)*e) + 1/7*d*x/((-e^2*x^2 + d^2)^(7/2)*e^2) -
 2/35*d^2/((-e^2*x^2 + d^2)^(7/2)*e^3) - 1/35*x/((-e^2*x^2 + d^2)^(5/2)*d*e
```

```
^2) - 4/105*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d
^2)*d^5*e^2)
```

Fricas [B] time = 2.31488, size = 486, normalized size = 4.02

$$-\frac{6\,e^{7}x^{7}-6\,de^{6}x^{6}-18\,d^{2}e^{5}x^{5}+18\,d^{3}e^{4}x^{4}+18\,d^{4}e^{3}x^{3}-18\,d^{5}e^{2}x^{2}-6\,d^{6}ex+6\,d^{7}-\left(8\,e^{6}x^{6}-8\,de^{5}x^{5}-20\,d^{2}e^{4}x^{4}+20}{105\left(d^{5}e^{10}x^{7}-d^{6}e^{9}x^{6}-3\,d^{7}e^{8}x^{5}+3\,d^{8}e^{7}x^{4}+3\,d^{9}e^{6}x^{3}-3\,d^{10}e^{5}x^{2}-d^{11}e^{4}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(6*e^7*x^7 - 6*d*e^6*x^6 - 18*d^2*e^5*x^5 + 18*d^3*e^4*x^4 + 18*d^4*
e^3*x^3 - 18*d^5*e^2*x^2 - 6*d^6*e*x + 6*d^7 - (8*e^6*x^6 - 8*d*e^5*x^5 - 2
0*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x - 6*d^6)*sqrt(-
e^2*x^2 + d^2))/(d^5*e^10*x^7 - d^6*e^9*x^6 - 3*d^7*e^8*x^5 + 3*d^8*e^7*x^4
 + 3*d^9*e^6*x^3 - 3*d^10*e^5*x^2 - d^11*e^4*x + d^12*e^3)
```

Sympy [C] time = 17.9308, size = 904, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(9/2),x)
```

```
[Out] d*Piecewise((35*I*d**4*x**3/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**
11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*
x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) - 28*I*d**2*e**2
*x**5/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*
e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) + 8*I*e**4*x**7/(-105*d**13*sqrt(-1 +
e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*
e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-35*d**4*x**3/(-105*d**13*sqrt(1
- e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9
*e**4*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2
/d**2)) + 28*d**2*e**2*x**5/(-105*d**13*sqrt(1 - e**2*x**2/d**2) + 315*d**1
1*e**2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(1 - e**2*x**
2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2)) - 8*e**4*x**7/(-105*
d**13*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**
2) - 315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(
1 - e**2*x**2/d**2)), True)) + e*Piecewise((2*d**2/(-35*d**6*e**4*sqrt(d**2
 - e**2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**2*x**2) - 105*d**2*e**8*x
**4*sqrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d**2 - e**2*x**2)) - 7*e**2
*x**2/(-35*d**6*e**4*sqrt(d**2 - e**2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2
- e**2*x**2) - 105*d**2*e**8*x**4*sqrt(d**2 - e**2*x**2) + 35*e**10*x**6*sq
rt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(9/2)), True))
```

Giac [A] time = 1.19429, size = 104, normalized size = 0.86

$$\frac{\left(\left(\left(4\,x^2\left(\frac{2\,x^2e^4}{d^5}-\frac{7\,e^2}{d^3}\right)+\frac{35}{d}\right)x+21\,e^{(-1)}\right)x^2-6\,d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{105\left(x^2e^2-d^2\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*x^2*e^4/d^5 - 7*e^2/d^3) + 35/d)*x + 21*e^(-1))*x^2 - 6*d
^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^4
```

3.31 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}-\frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}$$

```
[Out] (x^2*(d + e*x))/(9*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d - 3*e*x))/(63*d*e^3*(
d^2 - e^2*x^2)^(7/2)) - (2*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(5/2)) - (8*x)/(
315*d^5*e^2*(d^2 - e^2*x^2)^(3/2)) - (16*x)/(315*d^7*e^2*Sqrt[d^2 - e^2*x^2
])
```

Rubi [A] time = 0.0617767, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {796, 778, 192, 191}

$$\frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}-\frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```


```
[Out] (x^2*(d + e*x))/(9*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d - 3*e*x))/(63*d*e^3*(
d^2 - e^2*x^2)^(7/2)) - (2*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(5/2)) - (8*x)/(
315*d^5*e^2*(d^2 - e^2*x^2)^(3/2)) - (16*x)/(315*d^7*e^2*Sqrt[d^2 - e^2*x^2
])
```


Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx = \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{x\left(2d^2e-6de^2x\right)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9d^2e^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{21de^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{105d^3e^2}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{16 \ldots}{\ldots}$$

$$= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}} - \frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}} - \frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\ldots}{315 \ldots}$$

Mathematica [A] time = 0.0543511, size = 126, normalized size = 0.85

$$\frac{35d^6e^2x^2+70d^5e^3x^3-70d^4e^4x^4-56d^3e^5x^5+56d^2e^6x^6+10d^7ex-10d^8+16de^7x^7-16e^8x^8}{315d^7e^3(d-ex)^4(d+ex)^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (-10*d^8 + 10*d^7*e*x + 35*d^6*e^2*x^2 + 70*d^5*e^3*x^3 - 70*d^4*e^4*x^4 -
56*d^3*e^5*x^5 + 56*d^2*e^6*x^6 + 16*d*e^7*x^7 - 16*e^8*x^8)/(315*d^7*e^3*(
d - e*x)^4*(d + e*x)^3*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.052, size = 121, normalized size = 0.8

$$-\frac{(-ex+d)\,(ex+d)^2\left(16\,e^8x^8-16\,e^7x^7d-56\,e^6x^6d^2+56\,e^5x^5d^3+70\,e^4x^4d^4-70\,x^3d^5e^3-35\,x^2d^6e^2-10\,xd^7e+10\,d^8\right)}{315\,d^7e^3}\,(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x)
```

```
[Out] -1/315*(-e*x+d)*(e*x+d)^2*(16*e^8*x^8-16*d*e^7*x^7-56*d^2*e^6*x^6+56*d^3*e^
5*x^5+70*d^4*e^4*x^4-70*d^5*e^3*x^3-35*d^6*e^2*x^2-10*d^7*e*x+10*d^8)/d^7/e
^3/(-e^2*x^2+d^2)^(11/2)
```

Maxima [A] time = 1.01844, size = 213, normalized size = 1.44

$$\frac{x^2}{7\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e} + \frac{dx}{9\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e^2} - \frac{2\,d^2}{63\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e^3} - \frac{x}{63\left(-e^2x^2+d^2\right)^{\frac{7}{2}}de^2} - \frac{2\,x}{105\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3e^2} - \frac{\ldots}{315\left(-e^2 \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/7*x^2/((-e^2*x^2 + d^2)^(9/2)*e) + 1/9*d*x/((-e^2*x^2 + d^2)^(9/2)*e^2) -
 2/63*d^2/((-e^2*x^2 + d^2)^(9/2)*e^3) - 1/63*x/((-e^2*x^2 + d^2)^(7/2)*d*e
^2) - 2/105*x/((-e^2*x^2 + d^2)^(5/2)*d^3*e^2) - 8/315*x/((-e^2*x^2 + d^2)^
(3/2)*d^5*e^2) - 16/315*x/(sqrt(-e^2*x^2 + d^2)*d^7*e^2)
```

Fricas [B] time = 4.28955, size = 640, normalized size = 4.32

$$-\frac{10\,e^9x^9 - 10\,de^8x^8 - 40\,d^2e^7x^7 + 40\,d^3e^6x^6 + 60\,d^4e^5x^5 - 60\,d^5e^4x^4 - 40\,d^6e^3x^3 + 40\,d^7e^2x^2 + 10\,d^8ex - 10\,d^9 - \left(16\,e^{8}\right.}{315\left(d^7e^{12}x^9 - d^8e^{11}x^8 - 4\,d^9e^{10}x^7 + 4\,d^{10}e^9x^6 + 6\,d^{11}e^8x^5 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/315*(10*e^9*x^9 - 10*d*e^8*x^8 - 40*d^2*e^7*x^7 + 40*d^3*e^6*x^6 + 60*d^
4*e^5*x^5 - 60*d^5*e^4*x^4 - 40*d^6*e^3*x^3 + 40*d^7*e^2*x^2 + 10*d^8*e*x -
 10*d^9 - (16*e^8*x^8 - 16*d*e^7*x^7 - 56*d^2*e^6*x^6 + 56*d^3*e^5*x^5 + 70
*d^4*e^4*x^4 - 70*d^5*e^3*x^3 - 35*d^6*e^2*x^2 - 10*d^7*e*x + 10*d^8)*sqrt(
-e^2*x^2 + d^2))/(d^7*e^12*x^9 - d^8*e^11*x^8 - 4*d^9*e^10*x^7 + 4*d^10*e^9
*x^6 + 6*d^11*e^8*x^5 - 6*d^12*e^7*x^4 - 4*d^13*e^6*x^3 + 4*d^14*e^5*x^2 +
d^15*e^4*x - d^16*e^3)
```

Sympy [C] time = 38.6534, size = 1402, normalized size = 9.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(11/2),x)
```

```
[Out] d*Piecewise((-105*I*d**6*x**3/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d
**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*
e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) + 126*I*d**4*e**2*x**5/(315*d**17*sqrt
(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 18
90*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1
 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) - 72*I*d
**2*e**4*x**7/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) -
 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-
1 + e**2*x**2/d**2)) + 16*I*e**6*x**9/(315*d**17*sqrt(-1 + e**2*x**2/d**2)
- 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 3
15*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1)
, (105*d**6*x**3/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2
*sqrt(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) -
 1260*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1
- e**2*x**2/d**2)) - 126*d**4*e**2*x**5/(315*d**17*sqrt(1 - e**2*x**2/d**2)
 - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqr
```

```
t(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315
*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) + 72*d**2*e**4*x**7/(315*d**17*sq
rt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 18
90*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 -
 e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) - 16*e**6*x
**9/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**
2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e
**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d
**2)), True)) + e*Piecewise((-2*d**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) -
 252*d**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 -
 e**2*x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sq
rt(d**2 - e**2*x**2)) + 9*e**2*x**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) -
252*d**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 -
e**2*x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqr
t(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(11/2)), True))
```

Giac [A] time = 1.19468, size = 122, normalized size = 0.82

$$\frac{\left(\left(\left(2\left(4x^2\left(\frac{2x^2e^6}{d^7}-\frac{9e^4}{d^5}\right)+\frac{63e^2}{d^3}\right)x^2-\frac{105}{d}\right)x-45e^{(-1)}\right)x^2+10d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{315\left(x^2e^2-d^2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((2*(4*x^2*(2*x^2*e^6/d^7 - 9*e^4/d^5) + 63*e^2/d^3)*x^2 - 105/d)*x
- 45*e^(-1))*x^2 + 10*d^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^5
```

3.32 $\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

```
[Out] -((1 - a*x)/(a^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/a^3 - ArcSin[a*x]/
a^3
```

Rubi [A] time = 0.0343437, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {797, 641, 216, 637}

$$-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -((1 - a*x)/(a^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/a^3 - ArcSin[a*x]/
a^3
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2} - \frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -\frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -\frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sin^{-1}(ax)}{a^3}$$

Mathematica [A] time = 0.0313935, size = 50, normalized size = 0.93

$$\frac{a^2x^2 - \sqrt{1-a^2x^2}\sin^{-1}(ax) + ax - 2}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-2 + a*x + a^2*x^2 - Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a^3*Sqrt[1 - a^2*x^2]
)
```

Maple [A] time = 0.053, size = 85, normalized size = 1.6

$$\frac{x^2}{a}\frac{1}{\sqrt{-a^2x^2+1}} - 2\frac{1}{a^3\sqrt{-a^2x^2+1}} + \frac{x}{a^2}\frac{1}{\sqrt{-a^2x^2+1}} - \frac{1}{a^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] x^2/a/(-a^2*x^2+1)^(1/2)-2/a^3/(-a^2*x^2+1)^(1/2)+x/a^2/(-a^2*x^2+1)^(1/2)-
1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.47832, size = 101, normalized size = 1.87

$$\frac{x^2}{\sqrt{-a^2x^2+1}a} + \frac{x}{\sqrt{-a^2x^2+1}a^2} - \frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2} - \frac{2}{\sqrt{-a^2x^2+1}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(-a^2*x^2 + 1)*a) + x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt
(a^2))/(sqrt(a^2)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^3)
```

Fricas [A] time = 1.84537, size = 151, normalized size = 2.8

$$-\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax+2)+2}{a^4x+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

Sympy [A] time = 5.72089, size = 102, normalized size = 1.89

$$-a\left(\begin{cases}-\frac{x^2}{a^2\sqrt{-a^2x^2+1}}+\frac{2}{a^4\sqrt{-a^2x^2+1}} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ix}{a^2\sqrt{a^2x^2-1}}+\frac{i\operatorname{acosh}(ax)}{a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x}{a^2\sqrt{-a^2x^2+1}}-\frac{\operatorname{asin}(ax)}{a^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*x+1)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -a*Piecewise((-x**2/(a**2*sqrt(-a**2*x**2 + 1)) + 2/(a**4*sqrt(-a**2*x**2 +
 1)), Ne(a, 0)), (x**4/4, True)) + Piecewise((-I*x/(a**2*sqrt(a**2*x**2 - 1
)) + I*acosh(a*x)/a**3, Abs(a**2*x**2) > 1), (x/(a**2*sqrt(-a**2*x**2 + 1))
 - asin(a*x)/a**3, True))
```

Giac [A] time = 1.15445, size = 95, normalized size = 1.76

$$-\frac{\arcsin(ax)\operatorname{sgn}(a)}{a^2|a|}-\frac{\sqrt{-a^2x^2+1}}{a^3}+\frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.33 $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=173

$$-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}-\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}+\frac{11d^6\tan^{-}}{1}$$

```
[Out] (-8*d^3*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (11*d^2*x^3*Sqrt[d^2 - e^2*x^2]
)/(24*e^2) - (2*d*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) - (x^5*Sqrt[d^2 - e^2*x^2]
)/6 - (d^4*(256*d + 165*e*x)*Sqrt[d^2 - e^2*x^2])/(240*e^5) + (11*d^6*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rubi [A] time = 0.226878, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}-\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}+\frac{11d^6\tan^{-}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```


```
[Out] (-8*d^3*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (11*d^2*x^3*Sqrt[d^2 - e^2*x^2]
)/(24*e^2) - (2*d*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) - (x^5*Sqrt[d^2 - e^2*x^2]
)/6 - (d^4*(256*d + 165*e*x)*Sqrt[d^2 - e^2*x^2])/(240*e^5) + (11*d^6*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```


Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^4\left(-11d^2e^2-12de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{6e^2}$$

$$= -\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x^3\left(48d^3e^3+55d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{30e^4}$$

$$= -\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^2\left(-165d^4e^4-192d^3e^5x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{120e^6}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x\left(384d^5e^5+495d^4e^6x\right)}{\sqrt{d^2-e^2x^2}}}{360e^8}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+165ex)}{240e^5}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+165ex)}{240e^5}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{d^4(256d+165ex)}{240e^5}$$

Mathematica [A] time = 0.111076, size = 103, normalized size = 0.6

$$\frac{165d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(128d^3e^2x^2+110d^2e^3x^3+165d^4ex+256d^5+96de^4x^4+40e^5x^5\right)}{240e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(256*d^5 + 165*d^4*e*x + 128*d^3*e^2*x^2 + 110*d^2*e
^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5)) + 165*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*
x^2]])/(240*e^5)
```

Maple [A] time = 0.07, size = 174, normalized size = 1.

$$-\frac{x^5}{6}\sqrt{-x^2e^2+d^2} - \frac{11\,d^2x^3}{24\,e^2}\sqrt{-x^2e^2+d^2} - \frac{11\,d^4x}{16\,e^4}\sqrt{-x^2e^2+d^2} + \frac{11\,d^6}{16\,e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{2\,dx^4}{5\,e}\sqrt{-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/6*x^5*(-e^2*x^2+d^2)^(1/2)-11/24*d^2*x^3*(-e^2*x^2+d^2)^(1/2)/e^2-11/16*
d^4*x*(-e^2*x^2+d^2)^(1/2)/e^4+11/16/e^4*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))-2/5*d*x^4*(-e^2*x^2+d^2)^(1/2)/e-8/15*d^3*x^2*(-e^
2*x^2+d^2)^(1/2)/e^3-16/15*d^5*(-e^2*x^2+d^2)^(1/2)/e^5
```

Maxima [A] time = 1.46733, size = 224, normalized size = 1.29

$$-\frac{1}{6}\sqrt{-e^2x^2+d^2}x^5-\frac{2\sqrt{-e^2x^2+d^2}dx^4}{5e}-\frac{11\sqrt{-e^2x^2+d^2}d^2x^3}{24e^2}-\frac{8\sqrt{-e^2x^2+d^2}d^3x^2}{15e^3}+\frac{11d^6\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{16\sqrt{e^2}e^4}-\frac{11\sqrt{-e^2x^2}}{16e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*x^5 - 2/5*sqrt(-e^2*x^2 + d^2)*d*x^4/e - 11/24*sq
rt(-e^2*x^2 + d^2)*d^2*x^3/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*d^3*x^2/e^3 + 11
/16*d^6*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^4) - 11/16*sqrt(-e^2*x^2 +
 d^2)*d^4*x/e^4 - 16/15*sqrt(-e^2*x^2 + d^2)*d^5/e^5
```

Fricas [A] time = 1.82931, size = 236, normalized size = 1.36

$$-\frac{330\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5+96\,de^4x^4+110\,d^2e^3x^3+128\,d^3e^2x^2+165\,d^4ex+256\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/240*(330*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 + 9
6*d*e^4*x^4 + 110*d^2*e^3*x^3 + 128*d^3*e^2*x^2 + 165*d^4*e*x + 256*d^5)*sq
rt(-e^2*x^2 + d^2))/e^5
```

Sympy [C] time = 13.5183, size = 561, normalized size = 3.24

$$d^2\left(\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{8d^4\sqrt{d^2-e^2x^2}}{15e^6}-\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^4}-\frac{x^4\sqrt{d}}{5} \\ \frac{x^6}{6\sqrt{d^2}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5
```

```
/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**4*as
in(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8
*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True
)) + 2*d*e*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**
2*sqrt(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2),
Ne(e, 0)), (x**6/(6*sqrt(d**2)), True)) + e**2*Piecewise((-5*I*d**6*acosh(e
*x/d)/(16*e**7) + 5*I*d**5*x/(16*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**3
*x**3/(48*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**5/(24*e**2*sqrt(-1 + e**
2*x**2/d**2)) - I*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(
d**2) > 1), (5*d**6*asin(e*x/d)/(16*e**7) - 5*d**5*x/(16*e**6*sqrt(1 - e**2
*x**2/d**2)) + 5*d**3*x**3/(48*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**5/(24*
e**2*sqrt(1 - e**2*x**2/d**2)) + x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)
)
```

Giac [A] time = 1.18962, size = 113, normalized size = 0.65

$$\frac{11}{16} d^6 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \operatorname{sgn}(d) - \frac{1}{240}\left(256\, d^5 e^{(-5)} + \left(165\, d^4 e^{(-4)} + 2\left(64\, d^3 e^{(-3)} + \left(55\, d^2 e^{(-2)} + 4\left(12\, de^{(-1)} + 5\, x\right)x\right)x\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 11/16*d^6*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/240*(256*d^5*e^(-5) + (165*d^4*e^
(-4) + 2*(64*d^3*e^(-3) + (55*d^2*e^(-2) + 4*(12*d*e^(-1) + 5*x)*x)*x)*x)*x
)*sqrt(-x^2*e^2 + d^2)
```

3.34 $\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}$$

```
[Out] (-3*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(2*e
) - (x^4*Sqrt[d^2 - e^2*x^2])/5 - (3*d^3*(8*d + 5*e*x)*Sqrt[d^2 - e^2*x^2])
/(20*e^4) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^4)
```

Rubi [A] time = 0.186042, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```


```
[Out] (-3*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(2*e
) - (x^4*Sqrt[d^2 - e^2*x^2])/5 - (3*d^3*(8*d + 5*e*x)*Sqrt[d^2 - e^2*x^2])
/(20*e^4) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^4)
```


Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^3\left(-9d^2e^2-10de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2}$$

$$= -\frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{x^2\left(30d^3e^3+36d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{x\left(-72d^4e^4-90d^3e^5x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{\left(3d^5\right)\int \frac{\cdots}{\sqrt{d^2\cdots}}}{4e^3}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{\left(3d^5\right)\text{Subst}\cdots}{\cdots}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4} + \frac{3d^5\tan^{-1}\left(\cdots\right.}{4e^4}$$

Mathematica [A] time = 0.0963739, size = 92, normalized size = 0.64

$$\frac{15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(12d^2e^2x^2+15d^3ex+24d^4+10de^3x^3+4e^4x^4\right)}{20e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(24*d^4 + 15*d^3*e*x + 12*d^2*e^2*x^2 + 10*d*e^3*x^3
 + 4*e^4*x^4)) + 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(20*e^4)
```

Maple [A] time = 0.061, size = 149, normalized size = 1.

$$-\frac{x^4}{5}\sqrt{-x^2e^2+d^2} - \frac{3\,d^2x^2}{5\,e^2}\sqrt{-x^2e^2+d^2} - \frac{6\,d^4}{5\,e^4}\sqrt{-x^2e^2+d^2} - \frac{dx^3}{2\,e}\sqrt{-x^2e^2+d^2} - \frac{3\,d^3x}{4\,e^3}\sqrt{-x^2e^2+d^2} + \frac{3\,d^5}{4\,e^3}\arctan\left(x\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*x^4*(-e^2*x^2+d^2)^(1/2)-3/5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)/e^2-6/5*d^4*
(-e^2*x^2+d^2)^(1/2)/e^4-1/2*d*x^3*(-e^2*x^2+d^2)^(1/2)/e-3/4*d^3*x*(-e^2*x
^2+d^2)^(1/2)/e^3+3/4*d^5/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^
```

```
2)^(1/2))
```

Maxima [A] time = 1.4709, size = 190, normalized size = 1.32

$$-\frac{1}{5}\sqrt{-e^2x^2+d^2}x^4-\frac{\sqrt{-e^2x^2+d^2}dx^3}{2e}-\frac{3\sqrt{-e^2x^2+d^2}d^2x^2}{5e^2}+\frac{3d^5\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{4\sqrt{e^2}e^3}-\frac{3\sqrt{-e^2x^2+d^2}d^3x}{4e^3}-\frac{6\sqrt{-e^2x^2+d^2}d^4}{5e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*x^4 - 1/2*sqrt(-e^2*x^2 + d^2)*d*x^3/e - 3/5*sqrt
(-e^2*x^2 + d^2)*d^2*x^2/e^2 + 3/4*d^5*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^
2)*e^3) - 3/4*sqrt(-e^2*x^2 + d^2)*d^3*x/e^3 - 6/5*sqrt(-e^2*x^2 + d^2)*d^4
/e^4
```

Fricas [A] time = 1.7826, size = 204, normalized size = 1.42

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4\,e^4x^4+10\,de^3x^3+12\,d^2e^2x^2+15\,d^3ex+24\,d^4\right)\sqrt{-e^2x^2+d^2}}{20\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^4*x^4 + 10*d
*e^3*x^3 + 12*d^2*e^2*x^2 + 15*d^3*e*x + 24*d^4)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] time = 7.01119, size = 359, normalized size = 2.49

$$d^2\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwi}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 -
e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + 2*d*e*Piecew
ise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1 + e**2*x*
*2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(4*d*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**4*asin(e*x/d)/(
8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*sqrt(
1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*P
iecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*sqrt(d**2
- e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(e, 0)), (
x**6/(6*sqrt(d**2)), True))
```

Giac [A] time = 1.13264, size = 99, normalized size = 0.69

$$\frac{3}{4}d^5\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}\,(d)-\frac{1}{20}\left(24\,d^4e^{(-4)}+\left(15\,d^3e^{(-3)}+2\left(6\,d^2e^{(-2)}+\left(5\,de^{(-1)}+2\,x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^5*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/20*(24*d^4*e^(-4) + (15*d^3*e^(-3)
+ 2*(6*d^2*e^(-2) + (5*d*e^(-1) + 2*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.35 $\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] (-2*d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) - (x^3*Sqrt[d^2 - e^2*x^2])/4 - (d^2*(
32*d + 21*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) + (7*d^4*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(8*e^3)
```

Rubi [A] time = 0.145128, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1809, 833, 780, 217, 203}

$$-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-2*d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) - (x^3*Sqrt[d^2 - e^2*x^2])/4 - (d^2*(
32*d + 21*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) + (7*d^4*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(8*e^3)
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{\int \frac{x^2\left(-7d^2e^2-8de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}\\ &= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}+\frac{\int \frac{x\left(16d^3e^3+21d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4}\\ &= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{\left(7d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}\\ &= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{\left(7d^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,-\right.}{8e^2}\\ &= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}\end{aligned}$$

Mathematica [A] time = 0.075886, size = 81, normalized size = 0.7

$$\frac{21d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\sqrt{d^2-e^2x^2}\left(21d^2ex+32d^3+16de^2x^2+6e^3x^3\right)}{24e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3)) +
21*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^3)
```

Maple [A] time = 0.057, size = 124, normalized size = 1.1

$$-\frac{x^3}{4}\sqrt{-x^2e^2+d^2}-\frac{7\,d^2x}{8\,e^2}\sqrt{-x^2e^2+d^2}+\frac{7\,d^4}{8\,e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{2\,dx^2}{3\,e}\sqrt{-x^2e^2+d^2}-\frac{4\,d^3}{3\,e^3}\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*x^3*(-e^2*x^2+d^2)^(1/2)-7/8/e^2*d^2*x*(-e^2*x^2+d^2)^(1/2)+7/8/e^2*d^
4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-2/3*d*x^2*(-e^2*x^
2+d^2)^(1/2)/e-4/3*d^3*(-e^2*x^2+d^2)^(1/2)/e^3
```

Maxima [A] time = 1.47184, size = 157, normalized size = 1.37

$$-\frac{1}{4}\sqrt{-e^2x^2+d^2}x^3-\frac{2\sqrt{-e^2x^2+d^2}dx^2}{3e}+\frac{7d^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}e^2}-\frac{7\sqrt{-e^2x^2+d^2}d^2x}{8e^2}-\frac{4\sqrt{-e^2x^2+d^2}d^3}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*x^3 - 2/3*sqrt(-e^2*x^2 + d^2)*d*x^2/e + 7/8*d^4*
arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 7/8*sqrt(-e^2*x^2 + d^2)*d^2*
x/e^2 - 4/3*sqrt(-e^2*x^2 + d^2)*d^3/e^3
```

Fricas [A] time = 1.80938, size = 181, normalized size = 1.57

$$-\frac{42\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3+16\,de^2x^2+21\,d^2ex+32\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(42*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 + 16*d
*e^2*x^2 + 21*d^2*e*x + 32*d^3)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [C] time = 7.75333, size = 389, normalized size = 3.38

$$d^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{3id^4\operatorname{acc}}{8\ldots}\\ \frac{3d^4\operatorname{asin}(}{8e^5}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d
**2)/(2*e**2), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) -
d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2))
, True)) + 2*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*
sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) +
e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5
/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**4*as
in(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8
*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True
))
```

Giac [A] time = 1.192, size = 85, normalized size = 0.74

$$\frac{7}{8}d^4 \arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\,(d) - \frac{1}{24}\left(32\,d^3e^{(-3)} + \left(21\,d^2e^{(-2)} + 2\left(8\,de^{(-1)} + 3\,x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 7/8*d^4*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24*(32*d^3*e^(-3) + (21*d^2*e^(-2)
+ 2*(8*d*e^(-1) + 3*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.36 $\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] -(x^2*Sqrt[d^2 - e^2*x^2])/3 - (d*(5*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(3*e^2
) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rubi [A] time = 0.084863, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1809, 780, 217, 203}

$$-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(x^2*Sqrt[d^2 - e^2*x^2])/3 - (d*(5*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(3*e^2
) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{\int \frac{x(-5d^2e^2-6de^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2}$$
$$= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}$$
$$= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}$$
$$= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2} - \frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} + \frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Mathematica [A] time = 0.0621549, size = 69, normalized size = 0.83

$$\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}\left(5d^2+3dex+e^2x^2\right)}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(5*d^2 + 3*d*e*x + e^2*x^2)) + 3*d^3*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(3*e^2)
```

Maple [A] time = 0.057, size = 98, normalized size = 1.2

$$-\frac{x^2}{3}\sqrt{-x^2e^2+d^2} - \frac{5\,d^2}{3\,e^2}\sqrt{-x^2e^2+d^2} - \frac{dx}{e}\sqrt{-x^2e^2+d^2} + \frac{d^3}{e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2*(-e^2*x^2+d^2)^(1/2)-5/3*d^2/e^2*(-e^2*x^2+d^2)^(1/2)-d*x*(-e^2*x^
2+d^2)^(1/2)/e+d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.47, size = 122, normalized size = 1.47

$$-\frac{1}{3}\sqrt{-e^2x^2+d^2}x^2 + \frac{d^3\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}e} - \frac{\sqrt{-e^2x^2+d^2}dx}{e} - \frac{5\sqrt{-e^2x^2+d^2}d^2}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*x^2 + d^3*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*
e) - sqrt(-e^2*x^2 + d^2)*d*x/e - 5/3*sqrt(-e^2*x^2 + d^2)*d^2/e^2
```

Fricas [A] time = 1.81565, size = 150, normalized size = 1.81

$$-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(e^2x^2+3\,dex+5\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (e^2*x^2 + 3*d*e*x
+ 5*d^2)*sqrt(-e^2*x^2 + d^2))/e^2
```

Sympy [A] time = 4.57588, size = 219, normalized size = 2.64

$$d^2\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\; e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for}\;\frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \\ \frac{x^4}{4\sqrt{d^2}} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)
/e**2, True)) + 2*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt
(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e
*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 -
 e**2*x**2/d**2)), True)) + e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/
(3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d
**2)), True))
```

Giac [A] time = 1.1695, size = 66, normalized size = 0.8

$$d^3\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\operatorname{sgn}(d)-\frac{1}{3}\sqrt{-x^2e^2+d^2}\left(5\,d^2e^{(-2)}+\left(3\,de^{(-1)}+x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] d^3*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/3*sqrt(-x^2*e^2 + d^2)*(5*d^2*e^(-2) +
(3*d*e^(-1) + x)*x)
```

3.37 $\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}} dx$

Optimal. Leaf size=83

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] (-3*d*Sqrt[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) +
(3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.0277766, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {671, 641, 217, 203}

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d*Sqrt[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) +
(3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{1}{2}(3d)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx\\ &= -\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{1}{2}\left(3d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\ &= -\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{1}{2}\left(3d^2\right)\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)\\ &= -\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}\end{aligned}$$

Mathematica [A] time = 0.0393503, size = 58, normalized size = 0.7

$$\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-(4d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-((4*d + e*x)*Sqrt[d^2 - e^2*x^2]) + 3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(2*e)
```

Maple [A] time = 0.053, size = 71, normalized size = 0.9

$$-\frac{x}{2}\sqrt{-x^2e^2+d^2}+\frac{3\,d^2}{2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-2\,\frac{d\sqrt{-x^2e^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*x*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*
x^2+d^2)^(1/2))-2*d*(-e^2*x^2+d^2)^(1/2)/e
```

Maxima [A] time = 1.45803, size = 85, normalized size = 1.02

$$\frac{3\,d^2\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\,\sqrt{e^2}}-\frac{1}{2}\,\sqrt{-e^2x^2+d^2}x-\frac{2\,\sqrt{-e^2x^2+d^2}d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*x
- 2*sqrt(-e^2*x^2 + d^2)*d/e
```

Fricas [A] time = 1.79197, size = 126, normalized size = 1.52

$$-\frac{6\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex+4\,d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2
)*(e*x + 4*d))/e
```

Sympy [A] time = 4.08795, size = 270, normalized size = 3.25

$$d^2\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+2de\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{}{2d\sqrt{}} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2),
(d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d
**2), (d**2 < 0) & (e**2 < 0))) + 2*d*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(
e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + e**2*Piecewise((-I*d**2*
acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*
x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**
2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.1802, size = 54, normalized size = 0.65

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\left(4\,de^{(-1)}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2)*(4*d*e^(-1)
+ x)
```

3.38 $\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.111386, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1809, 844, 217, 203, 266, 63, 208}

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx &= -\sqrt{d^2-e^2x^2}-\frac{\int \frac{-d^2e^2-2de^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\sqrt{d^2-e^2x^2}+d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+(2de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\sqrt{d^2-e^2x^2}+\frac{1}{2}d^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)+(2de)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x}}\right.\\
&= -\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d^2\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2}\\
&= -\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.0288008, size = 66, normalized size = 1.

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

Maple [A] time = 0.051, size = 91, normalized size = 1.4

$$-\sqrt{-x^2e^2+d^2}+2\,\frac{de}{\sqrt{e^2}}\arctan\left(\frac{\sqrt{e^2}x}{\sqrt{-x^2e^2+d^2}}\right)-d^2\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)+2*d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.848, size = 149, normalized size = 2.26

$$-4\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d*log(-(d - sqrt(-e^2*x^2
+ d^2))/x) - sqrt(-e^2*x^2 + d^2)
```

Sympy [C] time = 5.50238, size = 187, normalized size = 2.83

$$d^2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2<0\wedge e^2<0\end{cases}\right)+e^2\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\ e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I
*asin(d/(e*x))/d, True)) + 2*d*e*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**
2/d**2))/sqrt(d**2), (d**2 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sq
rt(-e**2/d**2))/sqrt(d**2), (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acos
h(x*sqrt(e**2/d**2))/sqrt(-d**2), (d**2 < 0) & (e**2 < 0))) + e**2*Piecewis
e((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True))
```

Giac [A] time = 1.17565, size = 88, normalized size = 1.33

$$2\,d\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d)-d\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)-\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d*arcsin(x*e/d)*sgn(d) - d*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x)) - sqrt(-x^2*e^2 + d^2)
```

3.39 $\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.115041, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 844, 217, 203, 266, 63, 208}

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{x}-\frac{\int \frac{-2d^3e-d^2e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+(2de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+(de)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)+e^2\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.0371478, size = 68, normalized size = 1.

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

Maple [A] time = 0.055, size = 93, normalized size = 1.4

$$e^2\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-2\frac{de}{\sqrt{d^2}}\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}}{x}\right)-\frac{1}{x}\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-2*d*e/(d^2)^(1/2
)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-(-e^2*x^2+d^2)^(1/2)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86666, size = 162, normalized size = 2.38

$$-\frac{2\,ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-2\,ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 2*e*x*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/x
```

Sympy [C] time = 3.74134, size = 214, normalized size = 3.15

$$d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2<0\wedge e^2<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, Abs(d**2)/(Abs(e**2)*Ab
s(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) + 2*d*e*Piece
wise((-acosh(d/(e*x))/d, Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*asin(d/(e
*x))/d, True)) + e**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sq
rt(d**2), (d**2 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d*
*2))/sqrt(d**2), (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e*
*2/d**2))/sqrt(-d**2), (d**2 < 0) & (e**2 < 0)))
```

Giac [A] time = 1.18257, size = 144, normalized size = 2.12

$$\arcsin\left(\frac{xe}{d}\right)e\mathrm{sgn}\left(d\right) - 2\,e\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2\,|x|}\right) + \frac{xe^3}{2\left(de + \sqrt{-x^2e^2 + d^2}e\right)} - \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)e^{(-1)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e*sgn(d) - 2*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x)) + 1/2*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) - 1/2*(d*e + sqr
t(-x^2*e^2 + d^2)*e)*e^(-1)/x
```

3.40 $\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(2*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d)
```

Rubi [A] time = 0.108883, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1807, 807, 266, 63, 208}

$$-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(2*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d)
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{2x^2} - \frac{\int \frac{-4d^3e-3d^2e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{2}\left(3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{4}\left(3e^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{dx} - \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right) \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{dx} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.271579, size = 122, normalized size = 1.52

$$
\frac{e\left(-\frac{4d\sqrt{d^2-e^2x^2}}{x} - 2de\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - e\sqrt{d^2-e^2x^2}\left(\frac{d^2}{e^2x^2} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}\right)\right)}{2d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (e*((-4*d*Sqrt[d^2 - e^2*x^2])/x - 2*d*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d] - e
*Sqrt[d^2 - e^2*x^2]*(d^2/(e^2*x^2) + ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]]/Sqrt
[1 - (e^2*x^2)/d^2])))/(2*d^2)
```

Maple [A] time = 0.055, size = 86, normalized size = 1.1

$$
-\frac{3e^2}{2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{1}{2x^2}\sqrt{-x^2e^2+d^2} - 2\frac{e\sqrt{-x^2e^2+d^2}}{dx}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -3/2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/2*(
-e^2*x^2+d^2)^(1/2)/x^2-2*e*(-e^2*x^2+d^2)^(1/2)/d/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83799, size = 128, normalized size = 1.6

$$\frac{3\,e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\sqrt{-e^2x^2+d^2}(4\,ex+d)}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - sqrt(-e^2*x^2 + d^2)*(4
*e*x + d))/(d*x^2)
```

Sympy [C] time = 5.45542, size = 224, normalized size = 2.8

$$d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x}-\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d}\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) - e**2*acosh(d/(e*
x))/(2*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I/(2*e*x**3*sqrt(-d**2
/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**2*as
in(d/(e*x))/(2*d**3), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) -
1)/d**2, Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
 + 1)/d**2, True)) + e**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2)/(Abs(e**2
)*Abs(x**2)) > 1), (I*asin(d/(e*x))/d, True))
```

Giac [B] time = 1.16385, size = 230, normalized size = 2.88

$$-\frac{3\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d}+\frac{x^2\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)e^4}{x}+e^6\right)}{8\left(de+\sqrt{-x^2e^2+d^2}e\right)^2d}-\frac{\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)de^8}{x}+\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)^2de^6}{x^2}\right)e^{(-8)}}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -3/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d +
1/8*x^2*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*d) - 1/8*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^8/x + (d*e + s
qrt(-x^2*e^2 + d^2)*e)^2*d*e^6/x^2)*e^(-8)/d^2
```

3.41 $\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (5*e^2*Sqr
t[d^2 - e^2*x^2])/(3*d^2*x) - (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rubi [A] time = 0.137031, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (5*e^2*Sqr
t[d^2 - e^2*x^2])/(3*d^2*x) - (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{\int \frac{-6d^3e-5d^2e^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}+\frac{\int \frac{10d^4e^2+6d^3e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{e^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.16066, size = 87, normalized size = 0.81

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{d\left(d^2+3dex+5e^2x^2\right)}{x^3}-\frac{3e^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-((d*(d^2 + 3*d*e*x + 5*e^2*x^2))/x^3) - (3*e^3*ArcTa
nh[Sqrt[1 - (e^2*x^2)/d^2]])/Sqrt[1 - (e^2*x^2)/d^2]))/(3*d^3)
```

Maple [A] time = 0.058, size = 114, normalized size = 1.1

$$-\frac{1}{3\,x^3}\sqrt{-x^2e^2+d^2}-\frac{5\,e^2}{3\,d^2x}\sqrt{-x^2e^2+d^2}-\frac{e}{dx^2}\sqrt{-x^2e^2+d^2}-\frac{e^3}{d}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*(-e^2*x^2+d^2)^(1/2)/x^3-5/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x-e*(-e^2*x^
2+d^2)^(1/2)/d/x^2-1/d*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^
2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88616, size = 153, normalized size = 1.43

$$\frac{3 e^3 x^3 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \left(5 e^2 x^2 + 3 dex + d^2\right)\sqrt{-e^2x^2 + d^2}}{3 d^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (5*e^2*x^2 + 3*d*e*x +
d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*x^3)
```

Sympy [C] time = 5.23226, size = 313, normalized size = 2.93

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for}\ \frac{|d^2|}{|e^2||x^2|} > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for}\ \frac{|d^2|}{|e^2||x^2|} > 1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases}\right) + e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*
sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2)
 + 1)/(3*d**4), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*
d**2*x) - e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1
), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e*
*2*x**2) + 1)) + I*e**2*asin(d/(e*x))/(2*d**3), True)) + e**2*Piecewise((-e
*sqrt(d**2/(e**2*x**2) - 1)/d**2, Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I
```

```
*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True))
```

Giac [B] time = 1.20519, size = 323, normalized size = 3.02

$$\frac{x^3\left(\frac{6\left(de+\sqrt{-x^2e^2+d^2}e\right)e^6}{x}+\frac{21\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^4}{x^2}+e^8\right)e}{24\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^2}-\frac{e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{d^2}-\frac{\left(\frac{21\left(de+\sqrt{-x^2e^2+d^2}e\right)d^4e^{16}}{x}+\frac{6\left(de+\sqrt{-x^2\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*x^3*(6*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^6/x + 21*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^2*e^4/x^2 + e^8)*e/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2) - e^3*l
og(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 1/24*(21
*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^16/x + 6*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^2*d^4*e^14/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^12/x^3)*e^(-15)/
d^6
```

3.42 $\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=140

$$-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(4*x^4) - (2*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (7*e^2
*Sqrt[d^2 - e^2*x^2])/(8*d^2*x^2) - (4*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) -
 (7*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

Rubi [A] time = 0.171534, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(4*x^4) - (2*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (7*e^2
*Sqrt[d^2 - e^2*x^2])/(8*d^2*x^2) - (4*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) -
 (7*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{\int \frac{-8d^3e-7d^2e^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{\int \frac{21d^4e^2+16d^3e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{\int \frac{-32d^5e^3-21d^4e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{\left(7e^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{\left(7e^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,\right.}{16d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{\left(7e^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\right.}{8d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}
\end{aligned}$$

Mathematica [C] time = 0.160776, size = 155, normalized size = 1.11

$$-\frac{e\sqrt{d^2-e^2x^2}\left(6e^3x^3\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+d\left(4d^2+3dex+8e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}+3e^3x^3\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right)\right)}{6d^4x^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(e*Sqrt[d^2 - e^2*x^2]*(d*(4*d^2 + 3*d*e*x + 8*e^2*x^2)*Sqrt[1 - (e^2*x^2)
/d^2] + 3*e^3*x^3*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]] + 6*e^3*x^3*Sqrt[1 - (e^
2*x^2)/d^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (e^2*x^2)/d^2]))/(6*d^4*x^3*
Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.059, size = 139, normalized size = 1.

$$-\frac{2e}{3dx^3}\sqrt{-x^2e^2+d^2}-\frac{4e^3}{3d^3x}\sqrt{-x^2e^2+d^2}-\frac{1}{4x^4}\sqrt{-x^2e^2+d^2}-\frac{7e^2}{8d^2x^2}\sqrt{-x^2e^2+d^2}-\frac{7e^4}{8d^2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -2/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-4/3*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x-1/4*(-
e^2*x^2+d^2)^(1/2)/x^4-7/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-7/8/d^2*e^4/(d^
2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83666, size = 184, normalized size = 1.31

$$\frac{21\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3+21\,de^2x^2+16\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 + 21*d*e^
2*x^2 + 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*x^4)
```

Sympy [C] time = 9.21469, size = 459, normalized size = 3.28

$$d^2\left(\begin{cases}-\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2}-\frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{fo}\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2}-\frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{ot}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**5/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*s
qrt(d**2/(e**2*x**2) - 1)) + 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) -
```

```
 3*e**4*acosh(d/(e*x))/(8*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I/(
4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x
**2) + 1)) - 3*I*e**3/(8*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asi
n(d/(e*x))/(8*d**5), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1
)/(3*d**2*x**2) - 2*e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2)/(Ab
s(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) -
2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**4), True)) + e**2*Piecewise((-e*
sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) - e**2*acosh(d/(e*x))/(2*d**3), Abs(d
**2)/(Abs(e**2)*Abs(x**2)) > 1), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1))
- I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**2*asin(d/(e*x))/(2*d**3
), True))
```

Giac [B] time = 1.15415, size = 412, normalized size = 2.94

$$\frac{x^4\left(\frac{16\left(de+\sqrt{-x^2e^2+d^2}e\right)e^8}{x} + \frac{48\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^6}{x^2} + \frac{144\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^4}{x^3} + 3\,e^{10}\right)e^2}{192\left(de+\sqrt{-x^2e^2+d^2}e\right)^4 d^3} - \frac{7\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{8\,d^3} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/x + 48*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^6/x^2 + 144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^4/x^3 + 3*e^
10)*e^2/((d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3) - 7/8*e^4*log(1/2*abs(-2*d*e
 - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/192*(144*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^9*e^26/x + 48*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^9*e^24/
x^2 + 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^9*e^22/x^3 + 3*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^4*d^9*e^20/x^4)*e^(-24)/d^12
```

3.43 $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=169

$$-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*x^5) - (e*Sqrt[d^2 - e^2*x^2])/(2*d*x^4) - (3*e^2*S
qrt[d^2 - e^2*x^2])/(5*d^2*x^3) - (3*e^3*Sqrt[d^2 - e^2*x^2])/(4*d^3*x^2) -
 (6*e^4*Sqrt[d^2 - e^2*x^2])/(5*d^4*x) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(4*d^4)
```

Rubi [A] time = 0.195176, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 835, 807, 266, 63, 208}

$$-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```


```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*x^5) - (e*Sqrt[d^2 - e^2*x^2])/(2*d*x^4) - (3*e^2*S
qrt[d^2 - e^2*x^2])/(5*d^2*x^3) - (3*e^3*Sqrt[d^2 - e^2*x^2])/(4*d^3*x^2) -
 (6*e^4*Sqrt[d^2 - e^2*x^2])/(5*d^4*x) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(4*d^4)
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{\int \frac{-10d^3e-9d^2e^2x}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}+\frac{\int \frac{36d^4e^2+30d^3e^3x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{\int \frac{-90d^5e^3-72d^4e^4x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{60d^6}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}+\frac{\int \frac{144d^6e^4+90d^5e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{120d^8}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\frac{\left(3e^5\right)\int \frac{}{x\sqrt{d^2}}}{4d^3}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\frac{\left(3e^5\right)\text{Subst}}{}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{\left(3e^3\right)\text{Subst}}{}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^5\tanh^{-1}(}{4d}
\end{aligned}$$

Mathematica [C] time = 0.0436116, size = 79, normalized size = 0.47

$$-\frac{\sqrt{d^2-e^2x^2}\left(10e^5x^5\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+3d^3e^2x^2+d^5+6de^4x^4\right)}{5d^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(d^5 + 3*d^3*e^2*x^2 + 6*d*e^4*x^4 + 10*e^5*x^5*Hyper
geometric2F1[1/2, 3, 3/2, 1 - (e^2*x^2)/d^2]))/(5*d^5*x^5)
```

Maple [A] time = 0.075, size = 164, normalized size = 1.

$$-\frac{3\,e^{2}}{5\,d^{2}x^{3}}\sqrt{-x^{2}e^{2}+d^{2}}-\frac{6\,e^{4}}{5\,d^{4}x}\sqrt{-x^{2}e^{2}+d^{2}}-\frac{1}{5\,x^{5}}\sqrt{-x^{2}e^{2}+d^{2}}-\frac{e}{2\,dx^{4}}\sqrt{-x^{2}e^{2}+d^{2}}-\frac{3\,e^{3}}{4\,d^{3}x^{2}}\sqrt{-x^{2}e^{2}+d^{2}}-\frac{3\,e^{5}}{4\,d^{3}}\ln\left(\frac{1}{x}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -3/5*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-6/5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x-1/
5*(-e^2*x^2+d^2)^(1/2)/x^5-1/2*e*(-e^2*x^2+d^2)^(1/2)/d/x^4-3/4*e^3*(-e^2*x
^2+d^2)^(1/2)/d^3/x^2-3/4/d^3*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86618, size = 207, normalized size = 1.22

$$\frac{15\,e^{5}x^{5}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)-\left(24\,e^{4}x^{4}+15\,de^{3}x^{3}+12\,d^{2}e^{2}x^{2}+10\,d^{3}ex+4\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{20\,d^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (24*e^4*x^4 + 15*d*e^
3*x^3 + 12*d^2*e^2*x^2 + 10*d^3*e*x + 4*d^4)*sqrt(-e^2*x^2 + d^2))/(d^4*x^5
)
```

Sympy [C] time = 8.22559, size = 520, normalized size = 3.08

$$d^{2}\left(\begin{cases}-\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{5d^{2}x^{4}}-\frac{4e^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{15d^{4}x^{2}}-\frac{8e^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{15d^{6}} & \text{for}\ \frac{|d^{2}|}{|e^{2}||x^{2}|}>1\\-\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{5d^{2}x^{4}}-\frac{4ie^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{15d^{4}x^{2}}-\frac{8ie^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{15d^{6}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{1}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e}{8d^{2}x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{3e^{3}}{8d^{4}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{3e^{4}\ldots}{\ldots}\\\frac{i}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie}{8d^{2}x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{3ie^{3}}{8d^{4}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\ldots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**6/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(5*d**2*x**4) - 4*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(15*d**4*x**2) - 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(15
*d**6), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(5*d**2*x**4) - 4*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(15*d**4*x**2) -
8*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(15*d**6), True)) + 2*d*e*Piecewise((-
1/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*sqrt(d**2/(e**2*x*
*2) - 1)) + 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) - 3*e**4*acosh(d/(
e*x))/(8*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I/(4*e*x**5*sqrt(-d*
*2/(e**2*x**2) + 1)) + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*
e**3/(8*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))/(8*d**
5), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) -
2*e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2)/(Abs(e**2)*Abs(x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**
2/(e**2*x**2) + 1)/(3*d**4), True))
```

Giac [B] time = 1.16473, size = 493, normalized size = 2.92

$$\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{10}}{x}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^8}{x^2}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^6}{x^3}+\frac{110\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^4}{x^4}+e^{12}\right)e^3}{160\left(de+\sqrt{-x^2e^2+d^2}e\right)^5d^4}-\frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2\cdots}\right.}{2\cdots}\right.}{4\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/160*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^10/x + 15*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e^8/x^2 + 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^6/x^3 + 110*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^4/x^4 + e^12)*e^3/((d*e + sqrt(-x^2*e^2 +
 d^2)*e)^5*d^4) - 3/4*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^
(-2)/abs(x))/d^4 - 1/160*(110*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^16*e^38/x +
40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^16*e^36/x^2 + 15*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^3*d^16*e^34/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^16*e^32/x
^4 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^16*e^30/x^5)*e^(-35)/d^20
```

3.44 $\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=143

$$\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}-\frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] (d^4*(d + e*x)^2)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (22*d^3*(d + e*x))/(15*e^
6*(d^2 - e^2*x^2)^(3/2)) + (2*d*(30*d + 23*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + Sqrt[d^2 - e^2*x^2]/e^6 - (2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^6
```

Rubi [A] time = 0.27113, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1814, 641, 217, 203}

$$\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}-\frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^2)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (22*d^3*(d + e*x))/(15*e^
6*(d^2 - e^2*x^2)^(3/2)) + (2*d*(30*d + 23*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + Sqrt[d^2 - e^2*x^2]/e^6 - (2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^6
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{16d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{30d^5}{e^5}+\frac{15d^4x}{e^4}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\right.}{e^5}$$

$$= \frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Mathematica [A] time = 0.233577, size = 111, normalized size = 0.78

$$\frac{-32d^2e^2x^2 - \frac{30(d-ex)^3(d+ex)\sin^{-1}\left(\frac{ex}{d}\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}} - 82d^3ex + 56d^4 + 76de^3x^3 - 15e^4x^4}{15e^6(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (56*d^4 - 82*d^3*e*x - 32*d^2*e^2*x^2 + 76*d*e^3*x^3 - 15*e^4*x^4 - (30*(d
- e*x)^3*(d + e*x)*ArcSin[(e*x)/d])/Sqrt[1 - (e^2*x^2)/d^2])/(15*e^6*(d - e
*x)^2*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.103, size = 193, normalized size = 1.4

$$-x^6\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} + 7\frac{d^2x^4}{e^2\left(-x^2e^2+d^2\right)^{5/2}} - \frac{28\,d^4x^2}{3\,e^4}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} + \frac{56\,d^6}{15\,e^6}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} + \frac{2\,dx^5}{5\,e}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -x^6/(-e^2*x^2+d^2)^(5/2)+7/e^2*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-28/3/e^4*d^4*x
^2/(-e^2*x^2+d^2)^(5/2)+56/15/e^6*d^6/(-e^2*x^2+d^2)^(5/2)+2/5*d/e*x^5/(-e^
2*x^2+d^2)^(5/2)-2/3*d/e^3*x^3/(-e^2*x^2+d^2)^(3/2)+2*d/e^5*x/(-e^2*x^2+d^2
)^(1/2)-2*d/e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [B] time = 1.55754, size = 390, normalized size = 2.73

$$\frac{2}{15}\,dex\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{x^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2\,dx\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)}{3\,e}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*d*e*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - x^6/(-e^2*x^2 + d^2)
^(5/2) - 2/3*d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d
^2)^(3/2)*e^4))/e + 7*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 28/3*d^4*x^2/(
(-e^2*x^2 + d^2)^(5/2)*e^4) + 56/15*d^6/((-e^2*x^2 + d^2)^(5/2)*e^6) + 8/15
*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^5) - 14/15*d*x/(sqrt(-e^2*x^2 + d^2)*e^5)
- 2*d*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^5)
```

Fricas [A] time = 2.01491, size = 397, normalized size = 2.78

$$\frac{56\,de^4x^4-112\,d^2e^3x^3+112\,d^4ex-56\,d^5+60\left(de^4x^4-2\,d^2e^3x^3+2\,d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(15\,e^4x^4-76\,de^3\right.}{15\left(e^{10}x^4-2\,de^9x^3+2\,d^3e^7x-d^4e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(56*d*e^4*x^4 - 112*d^2*e^3*x^3 + 112*d^4*e*x - 56*d^5 + 60*(d*e^4*x^4
 - 2*d^2*e^3*x^3 + 2*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + (15*e^4*x^4 - 76*d*e^3*x^3 + 32*d^2*e^2*x^2 + 82*d^3*e*x - 56*d^4)*sqr
t(-e^2*x^2 + d^2))/(e^10*x^4 - 2*d*e^9*x^3 + 2*d^3*e^7*x - d^4*e^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\,(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**5*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.16453, size = 143, normalized size = 1.

$$-2\,d\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}\left(d\right)-\frac{\left(56\,d^6e^{(-6)}+\left(30\,d^5e^{(-5)}-\left(140\,d^4e^{(-4)}+\left(70\,d^3e^{(-3)}-\left(105\,d^2e^{(-2)}+\left(46\,de^{(-1)}-15\,x\right)x\right.\right.\right.\right.\right.}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2*d*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/15*(56*d^6*e^(-6) + (30*d^5*e^(-5) - (
140*d^4*e^(-4) + (70*d^3*e^(-3) - (105*d^2*e^(-2) + (46*d*e^(-1) - 15*x)*x)
*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.45 $\int \frac{x^4(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=121

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] (d^3*(d + e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (17*d^2*(d + e*x))/(15*e^
5*(d^2 - e^2*x^2)^(3/2)) + (2*(15*d + 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2])
 - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rubi [A] time = 0.207675, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1814, 12, 217, 203}

$$\frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (17*d^2*(d + e*x))/(15*e^
5*(d^2 - e^2*x^2)^(3/2)) + (2*(15*d + 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2])
 - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}\,dx}{15d^4} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4} \\
&= \frac{d^3(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{17d^2(d+ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}
\end{aligned}$$

Mathematica [A] time = 0.214264, size = 96, normalized size = 0.79

$$\frac{-15d(d-ex)^2\sqrt{1-\frac{e^2x^2}{d^2}}\sin^{-1}\left(\frac{ex}{d}\right)-17d^2ex+16d^3-22de^2x^2+26e^3x^3}{15e^5(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (16*d^3 - 17*d^2*e*x - 22*d*e^2*x^2 + 26*e^3*x^3 - 15*d*(d - e*x)^2*Sqrt[1
- (e^2*x^2)/d^2]*ArcSin[(e*x)/d])/(15*e^5*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

Maple [B] time = 0.083, size = 236, normalized size = 2.

$$\frac{x^5}{5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}-\frac{x^3}{3e^2}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{6x}{5e^4}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{1}{e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+2\frac{dx^4}{e\left(-x^2e^2+d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*x^5/(-e^2*x^2+d^2)^(5/2)-1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)+6/5/e^4*x/(-e
^2*x^2+d^2)^(1/2)-1/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/
2))+2*d/e*x^4/(-e^2*x^2+d^2)^(5/2)-8/3*d^3/e^3*x^2/(-e^2*x^2+d^2)^(5/2)+16/
15*d^5/e^5/(-e^2*x^2+d^2)^(5/2)+1/2*d^2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/10*d
^4/e^4*x/(-e^2*x^2+d^2)^(5/2)+1/10*d^2/e^4*x/(-e^2*x^2+d^2)^(3/2)
```

Maxima [B] time = 1.54407, size = 420, normalized size = 3.47

$$\frac{1}{15}e^2x\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{1}{3}x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)+\frac{2\,dx^4}{\left(-e^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*x*(3*x^2/((-e^2*
x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 2*d*x^4/((-e^
2*x^2 + d^2)^(5/2)*e) + 1/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 8/3*d^3*
x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^4)
+ 16/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^5) + 11/30*d^2*x/((-e^2*x^2 + d^2)^(3
/2)*e^4) - 4/15*x/(sqrt(-e^2*x^2 + d^2)*e^4) - arcsin(e^2*x/sqrt(d^2*e^2))/
(sqrt(e^2)*e^4)
```

Fricas [A] time = 1.81285, size = 359, normalized size = 2.97

$$\frac{16\,e^4x^4-32\,de^3x^3+32\,d^3ex-16\,d^4+30\left(e^4x^4-2\,de^3x^3+2\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(26\,e^3x^3-22\,de^2x^2-17\right.}{15\left(e^9x^4-2\,de^8x^3+2\,d^3e^6x-d^4e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(16*e^4*x^4 - 32*d*e^3*x^3 + 32*d^3*e*x - 16*d^4 + 30*(e^4*x^4 - 2*d*e
^3*x^3 + 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (26*e
^3*x^3 - 22*d*e^2*x^2 - 17*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*x^4
 - 2*d*e^8*x^3 + 2*d^3*e^6*x - d^4*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\,(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

<u>Giac</u> [A] time = 1.16284, size = 128, normalized size = 1.06

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}\left(d\right)-\frac{\left(16\,d^{5}e^{(-5)}+\left(15\,d^{4}e^{(-4)}-\left(40\,d^{3}e^{(-3)}+\left(35\,d^{2}e^{(-2)}-2\left(15\,de^{(-1)}+13\,x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}-}}{15\left(x^{2}e^{2}-d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-5)*sgn(d) - 1/15*(16*d^5*e^(-5) + (15*d^4*e^(-4) - (40*d
^3*e^(-3) + (35*d^2*e^(-2) - 2*(15*d*e^(-1) + 13*x)*x)*x)*x)*x)*sqrt(-x^2*e
^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.46 $\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[Out] (d^2*(d + e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d + e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d + 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.172656, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1635, 637}

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d + e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d + 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{(d+ex)\left(\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}\\
&= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0637293, size = 63, normalized size = 0.65

$$\frac{-4d^2ex + 2d^3 + de^2x^2 + 2e^3x^3}{5de^4(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 2*e^3*x^3)/(5*d*e^4*(d - e*x)^2*Sqrt[d^2 -
 e^2*x^2])
```

Maple [A] time = 0.049, size = 65, normalized size = 0.7

$$\frac{(-ex+d)(ex+d)^3\left(2e^3x^3+de^2x^2-4d^2ex+2d^3\right)}{5de^4}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3+d*e^2*x^2-4*d^2*e*x+2*d^3)/d/e^4/(-e^2*x^
2+d^2)^(7/2)
```

Maxima [A] time = 1.01238, size = 209, normalized size = 2.15

$$\frac{x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{dx^3}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e}-\frac{d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{3d^3x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3}+\frac{2d^4}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{dx}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4/(-e^2*x^2 + d^2)^(5/2) + d*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - d^2*x^2/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 3/5*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2/5*d^
4/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 2/5
*x/(sqrt(-e^2*x^2 + d^2)*d*e^3)
```

Fricas [A] time = 1.86781, size = 230, normalized size = 2.37

$$\frac{2e^4x^4-4de^3x^3+4d^3ex-2d^4-\left(2e^3x^3+de^2x^2-4d^2ex+2d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(de^8x^4-2d^2e^7x^3+2d^4e^5x-d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 + d*e^2*x^2 -
 4*d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 - 2*d^2*e^7*x^3 + 2*d^
```

```
4*e^5*x - d^5*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.1506, size = 85, normalized size = 0.88

$$-\frac{\left(2\,d^4 e^{(-4)} + \left(x^2\left(\frac{2\,xe}{d}+5\right) - 5\,d^2 e^{(-2)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*(2*d^4*e^(-4) + (x^2*(2*x*e/d + 5) - 5*d^2*e^(-2))*x^2)*sqrt(-x^2*e^2
+ d^2)/(x^2*e^2 - d^2)^3
```

3.47 $\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=87

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] (d*(d + e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (7*(d + e*x))/(15*e^3*(d^2
- e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.120037, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1635, 778, 191}

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (7*(d + e*x))/(15*e^3*(d^2
- e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d(d+ex)^2}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{\left(\frac{2d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d(d+ex)^2}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{7(d+ex)}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15e^2}$$

$$= \frac{d(d+ex)^2}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{7(d+ex)}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0600976, size = 63, normalized size = 0.72

$$\frac{8d^2ex-4d^3-2de^2x^2+e^3x^3}{15d^2e^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (-4*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)/(15*d^2*e^3*(d - e*x)^2*Sqrt[d
^2 - e^2*x^2])
```

Maple [A] time = 0.05, size = 66, normalized size = 0.8

$$-\frac{(-ex+d)(ex+d)^3\left(-e^3x^3+2\,de^2x^2-8\,d^2ex+4\,d^3\right)}{15\,d^2e^3}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^3*(-e^3*x^3+2*d*e^2*x^2-8*d^2*e*x+4*d^3)/d^2/e^3/(-e
^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 1.00427, size = 177, normalized size = 2.03

$$\frac{x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{2\,dx^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{4\,d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} + \frac{x}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} + \frac{x}{15\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(-e^2*x^2 + d^2)^(5/2) + 2/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e) - 1/1
0*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4/15*d^3/((-e^2*x^2 + d^2)^(5/2)*e^3
) + 1/30*x/((-e^2*x^2 + d^2)^(3/2)*e^2) + 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2*
e^2)
```

Fricas [A] time = 1.78998, size = 235, normalized size = 2.7

$$-\frac{4\,e^4x^4-8\,de^3x^3+8\,d^3ex-4\,d^4+\left(e^3x^3-2\,de^2x^2+8\,d^2ex-4\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^7x^4-2\,d^3e^6x^3+2\,d^5e^4x-d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(4*e^4*x^4 - 8*d*e^3*x^3 + 8*d^3*e*x - 4*d^4 + (e^3*x^3 - 2*d*e^2*x^2
 + 8*d^2*e*x - 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 - 2*d^3*e^6*x^3 +
2*d^5*e^4*x - d^6*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.15725, size = 82, normalized size = 0.94

$$\frac{\left(4\,d^3e^{(-3)}-\left(x\left(\frac{x^2e^2}{d^2}+5\right)+10\,de^{(-1)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*d^3*e^(-3) - (x*(x^2*e^2/d^2 + 5) + 10*d*e^(-1))*x^2)*sqrt(-x^2*e^2
 + d^2)/(x^2*e^2 - d^2)^3
```

3.48 $\int \frac{x(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=89

$$\frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] (d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(15*d*e^2*(d^2 -
e^2*x^2)^(3/2)) - (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.033264, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {789, 639, 191}

$$\frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(15*d*e^2*(d^2 -
e^2*x^2)^(3/2)) - (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5e}$$

$$= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{15de}$$

$$= \frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0561763, size = 62, normalized size = 0.7

$$\frac{-2d^2ex+d^3+8de^2x^2-4e^3x^3}{15d^3e^2(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3 - 2*d^2*e*x + 8*d*e^2*x^2 - 4*e^3*x^3)/(15*d^3*e^2*(d - e*x)^2*Sqrt[d^
2 - e^2*x^2])
```

Maple [A] time = 0.05, size = 64, normalized size = 0.7

$$\frac{(-ex+d)(ex+d)^3\left(-4e^3x^3+8de^2x^2-2d^2ex+d^3\right)}{15d^3e^2}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^3*(-4*e^3*x^3+8*d*e^2*x^2-2*d^2*e*x+d^3)/d^3/e^2/(-e^
2*x^2+d^2)^(7/2)
```

Maxima [A] time = 1.01071, size = 147, normalized size = 1.65

$$\frac{x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{2dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{2x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de} - \frac{4x}{15\sqrt{-e^2x^2+d^2}d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2/(-e^2*x^2 + d^2)^(5/2) + 2/5*d*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*
d^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 2/15*x/((-e^2*x^2 + d^2)^(3/2)*d*e) - 4/
15*x/(sqrt(-e^2*x^2 + d^2)*d^3*e)
```

Fricas [A] time = 1.87344, size = 228, normalized size = 2.56

$$\frac{e^4x^4 - 2\,de^3x^3 + 2\,d^3ex - d^4 + \left(4\,e^3x^3 - 8\,de^2x^2 + 2\,d^2ex - d^3\right)\sqrt{-e^2x^2 + d^2}}{15\left(d^3e^6x^4 - 2\,d^4e^5x^3 + 2\,d^6e^3x - d^7e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x - d^4 + (4*e^3*x^3 - 8*d*e^2*x^2 +
2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 - 2*d^4*e^5*x^3 + 2*d^6
*e^3*x - d^7*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(d + ex\right)^2}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.17491, size = 86, normalized size = 0.97

$$\frac{\left(\left(2\,x\left(\frac{2\,x^2e^3}{d^3} - \frac{5\,e}{d}\right) - 5\right)x^2 - d^2e^{(-2)}\right)\sqrt{-x^2e^2 + d^2}}{15\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((2*x*(2*x^2*e^3/d^3 - 5*e/d) - 5)*x^2 - d^2*e^(-2))*sqrt(-x^2*e^2 + d
^2)/(x^2*e^2 - d^2)^3
```

3.49 $\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + x/(5*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0197757, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {653, 192, 191}

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + x/(5*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{3}{5}\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$
$$= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d^2}$$
$$= \frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0441357, size = 63, normalized size = 0.82

$$\frac{d^2ex+2d^3-4de^2x^2+2e^3x^3}{5d^4e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)/(5*d^4*e*(d - e*x)^2*Sqrt[d^2 -
 e^2*x^2])
```

Maple [A] time = 0.049, size = 65, normalized size = 0.8

$$\frac{(-ex+d)\,(ex+d)^3\left(2\,e^3x^3-4\,e^2x^2d+xd^2e+2\,d^3\right)}{5\,d^4e}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3-4*d*e^2*x^2+d^2*e*x+2*d^3)/d^4/e/(-e^2*x^
2+d^2)^(7/2)
```

Maxima [A] time = 0.987325, size = 105, normalized size = 1.36

$$\frac{2\,x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{2\,d}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2} + \frac{2\,x}{5\sqrt{-e^2x^2+d^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/(-e^2*x^2 + d^2)^(5/2) + 2/5*d/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*x/((-
e^2*x^2 + d^2)^(3/2)*d^2) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

Fricas [A] time = 1.93368, size = 230, normalized size = 2.99

$$\frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-\left(2\,e^3x^3-4\,de^2x^2+d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(d^4e^5x^4-2\,d^5e^4x^3+2\,d^7e^2x-d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 - 4*d*e^2*x^2
 + d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 - 2*d^5*e^4*x^3 + 2*
d^7*e^2*x - d^8*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.17718, size = 82, normalized size = 1.06

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(x^2\left(\frac{2\,x^2e^4}{d^4}-\frac{5\,e^2}{d^2}\right)+5\right)x+2\,de^{(-1)}\right)}{5\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*((x^2*(2*x^2*e^4/d^4 - 5*e^2/d^2) + 5)*x + 2*d*e^
(-1))/(x^2*e^2 - d^2)^3
```

3.50 $\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] (2*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d + 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d + 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rubi [A] time = 0.158799, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d + 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d + 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{-5d^2-8dex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{-15d^4e^2-16d^3e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d^4e^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [C] time = 0.0470325, size = 81, normalized size = 0.69

$$\frac{3d^5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)-40d^2e^3x^3+30d^4ex+3d^5+16e^5x^5}{15d^5\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (3*d^5 + 30*d^4*e*x - 40*d^2*e^3*x^3 + 16*e^5*x^5 + 3*d^5*Hypergeometric2F1
[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^5*(d^2 - e^2*x^2)^(5/2))
```

Maple [A] time = 0.058, size = 160, normalized size = 1.4

$$\frac{2}{5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{2ex}{5d}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{8ex}{15d^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{16ex}{15d^5}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{1}{3d^2}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{1}{d^4}\frac{1}{\sqrt{-x^2e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 2/5/(-e^2*x^2+d^2)^(5/2)+2/5/d*e*x/(-e^2*x^2+d^2)^(5/2)+8/15/d^3*e*x/(-e^2*
x^2+d^2)^(3/2)+16/15/d^5*e*x/(-e^2*x^2+d^2)^(1/2)+1/3/d^2/(-e^2*x^2+d^2)^(3
/2)+1/d^4/(-e^2*x^2+d^2)^(1/2)-1/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02699, size = 350, normalized size = 2.99

$$\frac{26e^4x^4-52de^3x^3+52d^3ex-26d^4+15\left(e^4x^4-2de^3x^3+2d^3ex-d^4\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(16e^3x^3-17de^2x^2-22d^2e\right.}{15\left(d^5e^4x^4-2d^6e^3x^3+2d^8ex-d^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*e^4*x^4 - 52*d*e^3*x^3 + 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 - 2*d*e
^3*x^3 + 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (16*e^3*x^3
- 17*d*e^2*x^2 - 22*d^2*e*x + 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4 -
2*d^6*e^3*x^3 + 2*d^8*e*x - d^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{x\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [A] time = 1.15945, size = 159, normalized size = 1.36

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{16\,xe^5}{d^5}+\frac{15\,e^4}{d^4}\right)-\frac{40\,e^3}{d^3}\right)x-\frac{35\,e^2}{d^2}\right)x+\frac{30\,e}{d}\right)x+26\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(16*x*e^5/d^5 + 15*e^4/d^4) - 40*e^3/d^3)*
x - 35*e^2/d^2)*x + 30*e/d)*x + 26)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d*e
- 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5
```

3.51 $\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] (2*e*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e*(10*d + 13*e*x))/(15*d^4
*(d^2 - e^2*x^2)^(3/2)) + (e*(30*d + 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
- Sqrt[d^2 - e^2*x^2]/(d^6*x) - (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rubi [A] time = 0.275118, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1805, 807, 266, 63, 208}

$$\frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e*(10*d + 13*e*x))/(15*d^4
*(d^2 - e^2*x^2)^(3/2)) + (e*(30*d + 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
- Sqrt[d^2 - e^2*x^2]/(d^6*x) - (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-8e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{(2e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}}{d^5}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2}}\right.}{d^5}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x}{e}}\right.}{}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-}}{d}\right.}{d^6}$$

Mathematica [C] time = 0.0542935, size = 90, normalized size = 0.62

$$\frac{6d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+105d^4e^2x^2-140d^2e^4x^4-15d^6+56e^6x^6}{15d^6x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-15*d^6 + 105*d^4*e^2*x^2 - 140*d^2*e^4*x^4 + 56*e^6*x^6 + 6*d^5*e*x*Hyper
geometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^6*x*(d^2 - e^2*x^2)^(
5/2))
```

Maple [A] time = 0.064, size = 193, normalized size = 1.3

$$\frac{7e^2x}{5d^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{28e^2x}{15d^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{56e^2x}{15d^6}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{2e}{5d}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{2e}{3d^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+2\frac{}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 7/5*e^2*x/d^2/(-e^2*x^2+d^2)^(5/2)+28/15*e^2/d^4*x/(-e^2*x^2+d^2)^(3/2)+56/
15*e^2/d^6*x/(-e^2*x^2+d^2)^(1/2)+2/5/d*e/(-e^2*x^2+d^2)^(5/2)+2/3/d^3*e/(-
e^2*x^2+d^2)^(3/2)+2/d^5*e/(-e^2*x^2+d^2)^(1/2)-2/d^5*e/(d^2)^(1/2)*ln((2*d
^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/x/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95366, size = 401, normalized size = 2.77

$$\frac{46e^5x^5-92de^4x^4+92d^3e^2x^2-46d^4ex+30\left(e^5x^5-2de^4x^4+2d^3e^2x^2-d^4ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(56e^4x^4-82de^3x^3\right.}{15\left(d^6e^4x^5-2d^7e^3x^4+2d^9ex^2-d^{10}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(46*e^5*x^5 - 92*d*e^4*x^4 + 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^5
 - 2*d*e^4*x^4 + 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x
) - (56*e^4*x^4 - 82*d*e^3*x^3 - 32*d^2*e^2*x^2 + 76*d^3*e*x - 15*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^6*e^4*x^5 - 2*d^7*e^3*x^4 + 2*d^9*e*x^2 - d^10*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [A] time = 1.16364, size = 254, normalized size = 1.75

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{41\,xe^6}{d^6}+\frac{30\,e^5}{d^5}\right)-\frac{95\,e^4}{d^4}\right)x-\frac{70\,e^3}{d^3}\right)x+\frac{60\,e^2}{d^2}\right)x+\frac{46\,e}{d}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{2\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{d^6}+\frac{xe\cdots}{2\left(de+\sqrt{-x^2\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(41*x*e^6/d^6 + 30*e^5/d^5) - 95*e^4/d^4)*
x - 70*e^3/d^3)*x + 60*e^2/d^2)*x + 46*e/d)/(x^2*e^2 - d^2)^3 - 2*e*log(1/2
*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6 + 1/2*x*e^3/((d*
e + sqrt(-x^2*e^2 + d^2)*e)*d^6) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1
)/(d^6*x)
```

3.52 $\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

```
[Out] (2*e^2*(d + e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d + 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d + 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rubi [A] time = 0.358016, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^2*(d + e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d + 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d + 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{8e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{36e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{\int \frac{60d^3e+135d^2e^2x}{x^2\sqrt{d^2-e^2x^2}}\,d}{30d^8}$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} +$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} +$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} -$$

$$= \frac{2e^2(d+ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} -$$

Mathematica [C] time = 0.0612163, size = 117, normalized size = 0.64

$$\frac{e\left(d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+d^5ex\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+60d^4e^2x^2-80d^2e^4x^4-10d^6+32e^6x^6\right)}{5d^7x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (e*(-10*d^6 + 60*d^4*e^2*x^2 - 80*d^2*e^4*x^4 + 32*e^6*x^6 + d^5*e*x*Hyperg
eometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2] + d^5*e*x*Hypergeometric2F1[-
5/2, 2, -3/2, 1 - (e^2*x^2)/d^2]))/(5*d^7*x*(d^2 - e^2*x^2)^(5/2))
```

Maple [A] time = 0.069, size = 224, normalized size = 1.2

$$\frac{9\,e^2}{10\,d^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{3\,e^2}{2\,d^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{9\,e^2}{2\,d^6}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{9\,e^2}{2\,d^6}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{1}{2\,x^2}(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 9/10*e^2/d^2/(-e^2*x^2+d^2)^(5/2)+3/2*e^2/d^4/(-e^2*x^2+d^2)^(3/2)+9/2*e^2/
d^6/(-e^2*x^2+d^2)^(1/2)-9/2*e^2/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x)-1/2/x^2/(-e^2*x^2+d^2)^(5/2)-2/d*e/x/(-e^2*x^2+d^2)^
(5/2)+12/5/d^3*e^3*x/(-e^2*x^2+d^2)^(5/2)+16/5/d^5*e^3*x/(-e^2*x^2+d^2)^(3/
2)+32/5/d^7*e^3*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0328, size = 440, normalized size = 2.42

$$\frac{54\,e^6x^6-108\,de^5x^5+108\,d^3e^3x^3-54\,d^4e^2x^2+45\left(e^6x^6-2\,de^5x^5+2\,d^3e^3x^3-d^4e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(64\,e^5x^5-83\right.}{10\left(d^7e^4x^6-2\,d^8e^3x^5+2\,d^{10}ex^3-d^{11}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*e^6*x^6 - 108*d*e^5*x^5 + 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(e
^6*x^6 - 2*d*e^5*x^5 + 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x^2
```

```
 + d^2))/x) - (64*e^5*x^5 - 83*d*e^4*x^4 - 58*d^2*e^3*x^3 + 94*d^3*e^2*x^2
- 10*d^4*e*x - 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 - 2*d^8*e^3*x^5 +
2*d^10*e*x^3 - d^11*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [A] time = 1.16959, size = 351, normalized size = 1.93

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(2\left(x\left(\frac{11\,xe^7}{d^7}+\frac{10\,e^6}{d^6}\right)-\frac{25\,e^5}{d^5}\right)x-\frac{45\,e^4}{d^4}\right)x+\frac{30\,e^3}{d^3}\right)x+\frac{27\,e^2}{d^2}\right)}{5\left(x^2e^2-d^2\right)^3}-\frac{9\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^7}+\frac{x^2\left(\frac{8\left(de+\sqrt{\ }\right.}{}\right.}{8\left(de+\sqrt{\ }\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*(((2*(x*(11*x*e^7/d^7 + 10*e^6/d^6) - 25*e^5/d^5)
*x - 45*e^4/d^4)*x + 30*e^3/d^3)*x + 27*e^2/d^2)/(x^2*e^2 - d^2)^3 - 9/2*e^
2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^7 + 1/8*x
^2*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2 + d
^2)*e)^2*d^7) - 1/8*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*e^8/x + (d*e + sq
rt(-x^2*e^2 + d^2)*e)^2*d^7*e^6/x^2)*e^(-8)/d^14
```

3.53 $\int \frac{(d+ex)^2}{x^4(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=209

$$\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}-\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}$$

```
[Out] (2*e^3*(d + e*x))/(5*d^4*(d^2 - e^2*x^2)^(5/2)) + (e^3*(20*d + 23*e*x))/(15
*d^6*(d^2 - e^2*x^2)^(3/2)) + (2*e^3*(45*d + 53*e*x))/(15*d^8*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^6*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d^7*x^
2) - (14*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^8*x) - (7*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^8
```

Rubi [A] time = 0.473886, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}-\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (2*e^3*(d + e*x))/(5*d^4*(d^2 - e^2*x^2)^(5/2)) + (e^3*(20*d + 23*e*x))/(15
*d^6*(d^2 - e^2*x^2)^(3/2)) + (2*e^3*(45*d + 53*e*x))/(15*d^8*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^6*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d^7*x^
2) - (14*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^8*x) - (7*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^8
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{10e^3x^3}{d}-\frac{8e^4x^4}{d^2}}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{60e^3x^3}{d}+\frac{46e^4x^4}{d^2}}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2-\frac{90e^3x^3}{d}}{x^4\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} + \frac{\int \frac{90d^3e+210d^2e^2x+270}{x^3\sqrt{d^2-e^2x^2}}}{45d^8}$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - \int$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - 14e$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - 14e$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - 14e$$

$$= \frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - 14e$$

Mathematica [C] time = 0.0599333, size = 105, normalized size = 0.5

$$\frac{6d^5e^3x^3\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)-55d^6e^2x^2+330d^4e^4x^4-440d^2e^6x^6-5d^8+176e^8x^8}{15d^8x^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-5*d^8 - 55*d^6*e^2*x^2 + 330*d^4*e^4*x^4 - 440*d^2*e^6*x^6 + 176*e^8*x^8
+ 6*d^5*e^3*x^3*Hypergeometric2F1[-5/2, 2, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^
8*x^3*(d^2 - e^2*x^2)^(5/2))
```

Maple [A] time = 0.069, size = 249, normalized size = 1.2

$$-\frac{1}{3x^3}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}-\frac{11e^2}{3d^2x}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{22e^4x}{5d^4}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{88e^4x}{15d^6}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{176e^4x}{15d^8}\frac{1}{\sqrt{-x^2e^2+d^2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/3/x^3/(-e^2*x^2+d^2)^(5/2)-11/3/d^2*e^2/x/(-e^2*x^2+d^2)^(5/2)+22/5/d^4*
e^4*x/(-e^2*x^2+d^2)^(5/2)+88/15/d^6*e^4*x/(-e^2*x^2+d^2)^(3/2)+176/15/d^8*
e^4*x/(-e^2*x^2+d^2)^(1/2)-1/d*e/x^2/(-e^2*x^2+d^2)^(5/2)+7/5/d^3*e^3/(-e^2
*x^2+d^2)^(5/2)+7/3/d^5*e^3/(-e^2*x^2+d^2)^(3/2)+7/d^7*e^3/(-e^2*x^2+d^2)^(
1/2)-7/d^7*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27404, size = 471, normalized size = 2.25

$$\frac{116\,e^7x^7-232\,de^6x^6+232\,d^3e^4x^4-116\,d^4e^3x^3+105\left(e^7x^7-2\,de^6x^6+2\,d^3e^4x^4-d^4e^3x^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(176\,e\right.}{15\left(d^8e^4x^7-2\,d^9e^3x^6+2\,d^{11}ex^4-d^{12}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(116*e^7*x^7 - 232*d*e^6*x^6 + 232*d^3*e^4*x^4 - 116*d^4*e^3*x^3 + 105
*(e^7*x^7 - 2*d*e^6*x^6 + 2*d^3*e^4*x^4 - d^4*e^3*x^3)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) - (176*e^6*x^6 - 247*d*e^5*x^5 - 122*d^2*e^4*x^4 + 246*d^3*e
^3*x^3 - 40*d^4*e^2*x^2 - 5*d^5*e*x - 5*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^4
*x^7 - 2*d^9*e^3*x^6 + 2*d^11*e*x^4 - d^12*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{x^4\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**4*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [A] time = 1.17848, size = 439, normalized size = 2.1

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(2\,x\left(\frac{53\,xe^8}{d^8}+\frac{45\,e^7}{d^7}\right)-\frac{235\,e^6}{d^6}\right)x-\frac{200\,e^5}{d^5}\right)x+\frac{135\,e^4}{d^4}\right)x+\frac{116\,e^3}{d^3}\right)}{15\left(x^2e^2-d^2\right)^3}+\frac{x^3\left(\frac{6\left(de+\sqrt{-x^2e^2+d^2}e\right)e^6}{x}+\frac{57\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^4}{x^2}\right.}{24\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((2*x*(53*x*e^8/d^8 + 45*e^7/d^7) - 235*e^6/d^
6)*x - 200*e^5/d^5)*x + 135*e^4/d^4)*x + 116*e^3/d^3)/(x^2*e^2 - d^2)^3 + 1
/24*x^3*(6*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^6/x + 57*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*e^4/x^2 + e^8)*e/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^8) - 7*e^3*
log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^8 - 1/24*(5
7*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^16*e^16/x + 6*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^2*d^16*e^14/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^16*e^12/x^3)*e^(-
15)/d^24
```

3.54 $\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=81

$$-\frac{1}{5}\sqrt{1-x^2}x^4-\frac{1}{2}\sqrt{1-x^2}x^3-\frac{3}{5}\sqrt{1-x^2}x^2-\frac{3}{20}(5x+8)\sqrt{1-x^2}+\frac{3}{4}\sin^{-1}(x)$$

```
[Out] (-3*x^2*Sqrt[1 - x^2])/5 - (x^3*Sqrt[1 - x^2])/2 - (x^4*Sqrt[1 - x^2])/5 -
(3*(8 + 5*x)*Sqrt[1 - x^2])/20 + (3*ArcSin[x])/4
```

Rubi [A] time = 0.0932834, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1809, 833, 780, 216}

$$-\frac{1}{5}\sqrt{1-x^2}x^4-\frac{1}{2}\sqrt{1-x^2}x^3-\frac{3}{5}\sqrt{1-x^2}x^2-\frac{3}{20}(5x+8)\sqrt{1-x^2}+\frac{3}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2*Sqrt[1 - x^2])/5 - (x^3*Sqrt[1 - x^2])/2 - (x^4*Sqrt[1 - x^2])/5 -
(3*(8 + 5*x)*Sqrt[1 - x^2])/20 + (3*ArcSin[x])/4
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{5}x^4\sqrt{1-x^2}-\frac{1}{5}\int \frac{(-9-10x)x^3}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}+\frac{1}{20}\int \frac{x^2(30+36x)}{\sqrt{1-x^2}}\,dx\\
&= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{1}{60}\int \frac{(-72-90x)x}{\sqrt{1-x^2}}\,dx\\
&= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3}{4}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3}{4}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0373061, size = 42, normalized size = 0.52

$$\frac{3}{4}\sin^{-1}(x)-\frac{1}{20}\sqrt{1-x^2}\left(4x^4+10x^3+12x^2+15x+24\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -(Sqrt[1 - x^2]*(24 + 15*x + 12*x^2 + 10*x^3 + 4*x^4))/20 + (3*ArcSin[x])/4
```

Maple [A] time = 0.051, size = 71, normalized size = 0.9

$$-\frac{x^4}{5}\sqrt{-x^2+1}-\frac{3\,x^2}{5}\sqrt{-x^2+1}-\frac{6}{5}\sqrt{-x^2+1}-\frac{x^3}{2}\sqrt{-x^2+1}-\frac{3\,x}{4}\sqrt{-x^2+1}+\frac{3\arcsin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/5*x^4*(-x^2+1)^(1/2)-3/5*x^2*(-x^2+1)^(1/2)-6/5*(-x^2+1)^(1/2)-1/2*x^3*(
-x^2+1)^(1/2)-3/4*x*(-x^2+1)^(1/2)+3/4*arcsin(x)
```

Maxima [A] time = 1.49215, size = 95, normalized size = 1.17

$$-\frac{1}{5}\sqrt{-x^2+1}x^4-\frac{1}{2}\sqrt{-x^2+1}x^3-\frac{3}{5}\sqrt{-x^2+1}x^2-\frac{3}{4}\sqrt{-x^2+1}x-\frac{6}{5}\sqrt{-x^2+1}+\frac{3}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)*x^4 - 1/2*sqrt(-x^2 + 1)*x^3 - 3/5*sqrt(-x^2 + 1)*x^2 -
 3/4*sqrt(-x^2 + 1)*x - 6/5*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

Fricas [A] time = 1.76712, size = 131, normalized size = 1.62

$$-\frac{1}{20}\left(4\,x^4+10\,x^3+12\,x^2+15\,x+24\right)\sqrt{-x^2+1}-\frac{3}{2}\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(4*x^4 + 10*x^3 + 12*x^2 + 15*x + 24)*sqrt(-x^2 + 1) - 3/2*arctan((sq
rt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 1.4386, size = 73, normalized size = 0.9

$$-\frac{x^4\sqrt{1-x^2}}{5}-\frac{x^3\sqrt{1-x^2}}{2}-\frac{3x^2\sqrt{1-x^2}}{5}-\frac{3x\sqrt{1-x^2}}{4}-\frac{6\sqrt{1-x^2}}{5}+\frac{3\,\mathrm{asin}\,(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**4*sqrt(1 - x**2)/5 - x**3*sqrt(1 - x**2)/2 - 3*x**2*sqrt(1 - x**2)/5 -
3*x*sqrt(1 - x**2)/4 - 6*sqrt(1 - x**2)/5 + 3*asin(x)/4
```

Giac [A] time = 1.1255, size = 46, normalized size = 0.57

$$-\frac{1}{20}\,((2\,((2\,x+5)x+6)x+15)x+24)\sqrt{-x^2+1}+\frac{3}{4}\,\arcsin\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/20*((2*((2*x + 5)*x + 6)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

3.55 $\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=63

$$-\frac{1}{4}\sqrt{1-x^2}x^3 - \frac{2}{3}\sqrt{1-x^2}x^2 - \frac{1}{24}(21x+32)\sqrt{1-x^2} + \frac{7}{8}\sin^{-1}(x)$$

```
[Out] (-2*x^2*Sqrt[1 - x^2])/3 - (x^3*Sqrt[1 - x^2])/4 - ((32 + 21*x)*Sqrt[1 - x^
2])/24 + (7*ArcSin[x])/8
```

Rubi [A] time = 0.080565, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1809, 833, 780, 216}

$$-\frac{1}{4}\sqrt{1-x^2}x^3 - \frac{2}{3}\sqrt{1-x^2}x^2 - \frac{1}{24}(21x+32)\sqrt{1-x^2} + \frac{7}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-2*x^2*Sqrt[1 - x^2])/3 - (x^3*Sqrt[1 - x^2])/4 - ((32 + 21*x)*Sqrt[1 - x^
2])/24 + (7*ArcSin[x])/8
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{4}\int \frac{(-7-8x)x^2}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}+\frac{1}{12}\int \frac{x(16+21x)}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{24}(32+21x)\sqrt{1-x^2}+\frac{7}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{24}(32+21x)\sqrt{1-x^2}+\frac{7}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0302614, size = 37, normalized size = 0.59

$$\frac{7}{8}\sin^{-1}(x)-\frac{1}{24}\sqrt{1-x^2}\left(6x^3+16x^2+21x+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -(Sqrt[1 - x^2]*(32 + 21*x + 16*x^2 + 6*x^3))/24 + (7*ArcSin[x])/8
```

Maple [A] time = 0.048, size = 57, normalized size = 0.9

$$-\frac{x^3}{4}\sqrt{-x^2+1}-\frac{7x}{8}\sqrt{-x^2+1}+\frac{7\arcsin(x)}{8}-\frac{2x^2}{3}\sqrt{-x^2+1}-\frac{4}{3}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/4*x^3*(-x^2+1)^(1/2)-7/8*x*(-x^2+1)^(1/2)+7/8*arcsin(x)-2/3*x^2*(-x^2+1)
^(1/2)-4/3*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.48661, size = 76, normalized size = 1.21

$$-\frac{1}{4}\sqrt{-x^2+1}x^3-\frac{2}{3}\sqrt{-x^2+1}x^2-\frac{7}{8}\sqrt{-x^2+1}x-\frac{4}{3}\sqrt{-x^2+1}+\frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 - 2/3*sqrt(-x^2 + 1)*x^2 - 7/8*sqrt(-x^2 + 1)*x - 4
/3*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

Fricas [A] time = 1.85394, size = 119, normalized size = 1.89

$$-\frac{1}{24}\left(6x^3+16x^2+21x+32\right)\sqrt{-x^2+1}-\frac{7}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*x^3 + 16*x^2 + 21*x + 32)*sqrt(-x^2 + 1) - 7/4*arctan((sqrt(-x^2 +
 1) - 1)/x)
```

Sympy [A] time = 0.762138, size = 60, normalized size = 0.95

$$-\frac{x^3\sqrt{1-x^2}}{4}-\frac{2x^2\sqrt{1-x^2}}{3}-\frac{7x\sqrt{1-x^2}}{8}-\frac{4\sqrt{1-x^2}}{3}+\frac{7\operatorname{asin}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**3*sqrt(1 - x**2)/4 - 2*x**2*sqrt(1 - x**2)/3 - 7*x*sqrt(1 - x**2)/8 - 4
*sqrt(1 - x**2)/3 + 7*asin(x)/8
```

Giac [A] time = 1.11217, size = 41, normalized size = 0.65

$$-\frac{1}{24}\left((2(3x+8)x+21)x+32\right)\sqrt{-x^2+1}+\frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x + 8)*x + 21)*x + 32)*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

3.56 $\int \frac{x(1+x)^2}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=41

$$-\frac{1}{3}\sqrt{1-x^2}x^2-\frac{1}{3}(3x+5)\sqrt{1-x^2}+\sin^{-1}(x)$$

```
[Out] -(x^2*Sqrt[1 - x^2])/3 - ((5 + 3*x)*Sqrt[1 - x^2])/3 + ArcSin[x]
```

Rubi [A] time = 0.0477031, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1809, 780, 216}

$$-\frac{1}{3}\sqrt{1-x^2}x^2-\frac{1}{3}(3x+5)\sqrt{1-x^2}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -(x^2*Sqrt[1 - x^2])/3 - ((5 + 3*x)*Sqrt[1 - x^2])/3 + ArcSin[x]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(1+x)^2}{\sqrt{1-x^2}} dx &= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}\int \frac{(-5-6x)x}{\sqrt{1-x^2}} dx \\
&= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}} dx \\
&= -\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0185539, size = 26, normalized size = 0.63

$$\sin^{-1}(x) - \frac{1}{3}\sqrt{1-x^2}\left(x^2+3x+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -(Sqrt[1 - x^2]*(5 + 3*x + x^2))/3 + ArcSin[x]
```

Maple [A] time = 0.047, size = 41, normalized size = 1.

$$-\frac{x^2}{3}\sqrt{-x^2+1} - \frac{5}{3}\sqrt{-x^2+1} - x\sqrt{-x^2+1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/3*x^2*(-x^2+1)^(1/2)-5/3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)+arcsin(x)
```

Maxima [A] time = 1.46933, size = 54, normalized size = 1.32

$$-\frac{1}{3}\sqrt{-x^2+1}x^2 - \sqrt{-x^2+1}x - \frac{5}{3}\sqrt{-x^2+1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - sqrt(-x^2 + 1)*x - 5/3*sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [A] time = 1.8519, size = 97, normalized size = 2.37

$$-\frac{1}{3}\left(x^2+3x+5\right)\sqrt{-x^2+1} - 2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^2 + 3*x + 5)*sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.426946, size = 37, normalized size = 0.9

$$-\frac{x^2\sqrt{1-x^2}}{3} - x\sqrt{1-x^2} - \frac{5\sqrt{1-x^2}}{3} + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**2*sqrt(1 - x**2)/3 - x*sqrt(1 - x**2) - 5*sqrt(1 - x**2)/3 + asin(x)
```

Giac [A] time = 1.14691, size = 28, normalized size = 0.68

$$-\frac{1}{3}\left((x+3)x+5\right)\sqrt{-x^2+1}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*((x + 3)*x + 5)*sqrt(-x^2 + 1) + arcsin(x)
```

3.57 $\int \frac{(1+x)^2}{\sqrt{1-x^2}}\, dx$

Optimal. Leaf size=40

$$-\frac{1}{2}\sqrt{1-x^2}(x+1)-\frac{3\sqrt{1-x^2}}{2}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] (-3*Sqrt[1 - x^2])/2 - ((1 + x)*Sqrt[1 - x^2])/2 + (3*ArcSin[x])/2
```

Rubi [A] time = 0.0122981, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {671, 641, 216}

$$-\frac{1}{2}\sqrt{1-x^2}(x+1)-\frac{3\sqrt{1-x^2}}{2}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-3*Sqrt[1 - x^2])/2 - ((1 + x)*Sqrt[1 - x^2])/2 + (3*ArcSin[x])/2
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^2}{\sqrt{1-x^2}}\, dx &= -\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\int \frac{1+x}{\sqrt{1-x^2}}\, dx\\ &= -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\, dx\\ &= -\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0150907, size = 25, normalized size = 0.62

$$\frac{1}{2}\left(3\sin^{-1}(x)-(x+4)\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-((4 + x)*Sqrt[1 - x^2]) + 3*ArcSin[x])/2
```

Maple [A] time = 0.057, size = 29, normalized size = 0.7

$$-\frac{x}{2}\sqrt{-x^2+1}+\frac{3\arcsin(x)}{2}-2\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*x*(-x^2+1)^(1/2)+3/2*arcsin(x)-2*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.50275, size = 38, normalized size = 0.95

$$-\frac{1}{2}\sqrt{-x^2+1}x-2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A] time = 2.02853, size = 86, normalized size = 2.15

$$-\frac{1}{2}\sqrt{-x^2+1}(x+4)-3\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) - 3*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.208012, size = 27, normalized size = 0.68

$$-\frac{x\sqrt{1-x^2}}{2}-2\sqrt{1-x^2}+\frac{3\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x*sqrt(1 - x**2)/2 - 2*sqrt(1 - x**2) + 3*asin(x)/2
```

Giac [A] time = 1.10593, size = 26, normalized size = 0.65

$$-\frac{1}{2}\sqrt{-x^2+1}(x+4)+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) + 3/2*arcsin(x)
```

3.58 $\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=32

$$-\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\sin^{-1}(x)$$

```
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0589199, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1809, 844, 216, 266, 63, 206}

$$-\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx &= -\sqrt{1-x^2}-\int \frac{-1-2x}{x\sqrt{1-x^2}}\,dx\\ &= -\sqrt{1-x^2}+2\int \frac{1}{\sqrt{1-x^2}}\,dx+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\ &= -\sqrt{1-x^2}+2\sin^{-1}(x)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx,x,x^2\right)\\ &= -\sqrt{1-x^2}+2\sin^{-1}(x)-\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\ &= -\sqrt{1-x^2}+2\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0105869, size = 32, normalized size = 1.

$$-\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] time = 0.048, size = 29, normalized size = 0.9

$$-\sqrt{-x^2+1}+2\arcsin(x)-\mathrm{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x/(-x^2+1)^(1/2),x)
```

```
[Out] -(-x^2+1)^(1/2)+2*arcsin(x)-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.48763, size = 55, normalized size = 1.72

$$-\sqrt{-x^2+1}+2\arcsin(x)-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 2.13574, size = 111, normalized size = 3.47

$$-\sqrt{-x^2+1} - 4\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1) - 4*arctan((sqrt(-x^2 + 1) - 1)/x) + log((sqrt(-x^2 + 1) -
1)/x)
```

Sympy [A] time = 4.91425, size = 31, normalized size = 0.97

$$-\sqrt{1-x^2} + \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} + 2\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), T
rue)) + 2*asin(x)
```

Giac [A] time = 1.14875, size = 46, normalized size = 1.44

$$-\sqrt{-x^2+1} + 2\arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

3.59 $\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\sqrt{1-x^2}}{x}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sin^{-1}(x)$$

```
[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0625413, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1807, 844, 216, 266, 63, 206}

$$-\frac{\sqrt{1-x^2}}{x}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{x} - \int \frac{-2-x}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{x} + 2\int \frac{1}{x\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) + \text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= -\frac{\sqrt{1-x^2}}{x} + \sin^{-1}(x) - 2\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0150139, size = 33, normalized size = 1.

$$-\frac{\sqrt{1-x^2}}{x} - 2\tanh^{-1}\left(\sqrt{1-x^2}\right) + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]

[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] time = 0.054, size = 30, normalized size = 0.9

$$\arcsin(x) - 2\,\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - \frac{1}{x}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^2/(-x^2+1)^(1/2),x)

[Out] arcsin(x)-2*arctanh(1/(-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x
```

Maxima [A] time = 1.4913, size = 57, normalized size = 1.73

$$-\frac{\sqrt{-x^2+1}}{x} + \arcsin(x) - 2\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/x + arcsin(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 2.16486, size = 124, normalized size = 3.76

$$-\frac{2\,x\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-2\,x\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*x*arctan((sqrt(-x^2 + 1) - 1)/x) - 2*x*log((sqrt(-x^2 + 1) - 1)/x) + sq
rt(-x^2 + 1))/x
```

Sympy [C] time = 3.96754, size = 51, normalized size = 1.55

$$\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for}\ \left|x^2\right|>1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases} + 2\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{\left|x^2\right|}>1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}\right) + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**2/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
+ 2*Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) + asin(x
)
```

Giac [A] time = 1.12314, size = 74, normalized size = 2.24

$$\frac{x}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\sqrt{-x^2+1}-1}{2\,x}+\arcsin(x)+2\,\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x + arcsin(x) + 2*log
(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

3.60 $\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -Sqrt[1 - x^2]/(2*x^2) - (2*Sqrt[1 - x^2])/x - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

Rubi [A] time = 0.0614837, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1807, 807, 266, 63, 206}

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2]/(2*x^2) - (2*Sqrt[1 - x^2])/x - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{1}{2}\int \frac{-4-3x}{x^2\sqrt{1-x^2}}\,dx\\ &= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}+\frac{3}{2}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\ &= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\ &= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0195676, size = 40, normalized size = 0.78

$$-\frac{\sqrt{1-x^2}(4x+1)}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

```
[Out] -((1 + 4*x)*Sqrt[1 - x^2])/(2*x^2) - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

Maple [A] time = 0.052, size = 42, normalized size = 0.8

$$-\frac{3}{2}\operatorname{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\frac{1}{2\,x^2}\sqrt{-x^2+1}-2\,\frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^3/(-x^2+1)^(1/2),x)
```

```
[Out] -3/2*arctanh(1/(-x^2+1)^(1/2))-1/2/x^2*(-x^2+1)^(1/2)-2*(-x^2+1)^(1/2)/x
```

Maxima [A] time = 1.48397, size = 73, normalized size = 1.43

$$-\frac{2\,\sqrt{-x^2+1}}{x}-\frac{\sqrt{-x^2+1}}{2\,x^2}-\frac{3}{2}\,\log\left(\frac{2\,\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/x - 1/2*sqrt(-x^2 + 1)/x^2 - 3/2*log(2*sqrt(-x^2 + 1)/abs
(x) + 2/abs(x))
```

Fricas [A] time = 1.98484, size = 97, normalized size = 1.9

$$\frac{3\,x^2\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}(4\,x+1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*x^2*log((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*(4*x + 1))/x^2
```

Sympy [C] time = 6.60171, size = 116, normalized size = 2.27

$$2\left(\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for}\ |x^2|>1\\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}-\frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for}\ \frac{1}{|x^2|}>1\\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i}{2x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise}\end{cases}+\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**3/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True)
) + Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2) > 1),
(I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**2)), Tr
ue)) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True))
```

Giac [B] time = 1.1394, size = 123, normalized size = 2.41

$$\frac{x^2\left(\frac{8\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{8\left(\sqrt{-x^2+1}-1\right)^2}-\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{8\,x^2}+\frac{3}{2}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*x^2*(8*(sqrt(-x^2 + 1) - 1)/x - 1)/(sqrt(-x^2 + 1) - 1)^2 - (sqrt(-x^2
+ 1) - 1)/x + 1/8*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3/2*log(-(sqrt(-x^2 + 1) - 1
)/abs(x))
```

3.61 $\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{5\sqrt{1-x^2}}{3x}-\frac{\sqrt{1-x^2}}{x^2}-\frac{\sqrt{1-x^2}}{3x^3}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -Sqrt[1 - x^2]/(3*x^3) - Sqrt[1 - x^2]/x^2 - (5*Sqrt[1 - x^2])/(3*x) - ArcT
anh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0737916, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{5\sqrt{1-x^2}}{3x}-\frac{\sqrt{1-x^2}}{x^2}-\frac{\sqrt{1-x^2}}{3x^3}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```


```
[Out] -Sqrt[1 - x^2]/(3*x^3) - Sqrt[1 - x^2]/x^2 - (5*Sqrt[1 - x^2])/(3*x) - ArcT
anh[Sqrt[1 - x^2]]
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{1}{3}\int \frac{-6-5x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{6}\int \frac{10+6x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0231655, size = 43, normalized size = 0.64

$$-\frac{\sqrt{1-x^2}\left(5x^2+3x+1\right)}{3x^3}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]*(1 + 3*x + 5*x^2))/(3*x^3) - ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] time = 0.051, size = 56, normalized size = 0.8

$$-\frac{1}{3\,x^3}\sqrt{-x^2+1}-\frac{5}{3\,x}\sqrt{-x^2+1}-\frac{1}{x^2}\sqrt{-x^2+1}-\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^4/(-x^2+1)^(1/2),x)
```

```
[Out] -1/3*(-x^2+1)^(1/2)/x^3-5/3*(-x^2+1)^(1/2)/x-1/x^2*(-x^2+1)^(1/2)-arctanh(1
/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.48349, size = 92, normalized size = 1.37

$$-\frac{5\sqrt{-x^2+1}}{3x}-\frac{\sqrt{-x^2+1}}{x^2}-\frac{\sqrt{-x^2+1}}{3x^3}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/3*sqrt(-x^2 + 1)/x - sqrt(-x^2 + 1)/x^2 - 1/3*sqrt(-x^2 + 1)/x^3 - log(2
*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 1.84516, size = 108, normalized size = 1.61

$$\frac{3x^3\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(5x^2+3x+1\right)\sqrt{-x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log((sqrt(-x^2 + 1) - 1)/x) - (5*x^2 + 3*x + 1)*sqrt(-x^2 + 1))/
x^3
```

Sympy [C] time = 7.73114, size = 128, normalized size = 1.91

$$\begin{cases}-\frac{\sqrt{1-x^2}}{x}-\frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for}\ x>-1\wedge x<1\end{cases}+\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for}\ \left|x^2\right|>1\\-\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}+2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}-\frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for}\ \frac{1}{|x^2|}>1\\\frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i}{2x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**4/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x,
True)) + 2*Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2)
 > 1), (I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**
2)), True))
```

Giac [B] time = 1.11811, size = 169, normalized size = 2.52

$$-\frac{x^3\left(\frac{6\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{21\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+1}-1\right)^3}-\frac{7\left(\sqrt{-x^2+1}-1\right)}{8x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{24x^3}+\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*x^3*(6*(sqrt(-x^2 + 1) - 1)/x - 21*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(s
qrt(-x^2 + 1) - 1)^3 - 7/8*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1
)^2/x^2 - 1/24*(sqrt(-x^2 + 1) - 1)^3/x^3 + log(-(sqrt(-x^2 + 1) - 1)/abs(x
))
```

3.62 $\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{4\sqrt{1-x^2}}{3x}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{4x^4}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -Sqrt[1 - x^2]/(4*x^4) - (2*Sqrt[1 - x^2])/(3*x^3) - (7*Sqrt[1 - x^2])/(8*x
^2) - (4*Sqrt[1 - x^2])/(3*x) - (7*ArcTanh[Sqrt[1 - x^2]])/8
```

Rubi [A] time = 0.0895911, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{4\sqrt{1-x^2}}{3x}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{4x^4}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2]/(4*x^4) - (2*Sqrt[1 - x^2])/(3*x^3) - (7*Sqrt[1 - x^2])/(8*x
^2) - (4*Sqrt[1 - x^2])/(3*x) - (7*ArcTanh[Sqrt[1 - x^2]])/8
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{1}{4}\int \frac{-8-7x}{x^4\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}+\frac{1}{12}\int \frac{21+16x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{1}{24}\int \frac{-32-21x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}+\frac{7}{8}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}+\frac{7}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.0400475, size = 73, normalized size = 0.82

$$-\sqrt{1-x^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^2\right)-\frac{\sqrt{1-x^2}\left(8x^2+3x+4\right)}{6x^3}-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]*(4 + 3*x + 8*x^2))/(6*x^3) - ArcTanh[Sqrt[1 - x^2]]/2 - Sqr
t[1 - x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x^2]
```

Maple [A] time = 0.053, size = 70, normalized size = 0.8

$$-\frac{2}{3\,x^3}\sqrt{-x^2+1}-\frac{4}{3\,x}\sqrt{-x^2+1}-\frac{1}{4\,x^4}\sqrt{-x^2+1}-\frac{7}{8\,x^2}\sqrt{-x^2+1}-\frac{7}{8}\operatorname{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^5/(-x^2+1)^(1/2),x)
```

```
[Out] -2/3*(-x^2+1)^(1/2)/x^3-4/3*(-x^2+1)^(1/2)/x-1/4/x^4*(-x^2+1)^(1/2)-7/8/x^2
*(-x^2+1)^(1/2)-7/8*arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.49207, size = 111, normalized size = 1.25

$$-\frac{4\sqrt{-x^2+1}}{3x}-\frac{7\sqrt{-x^2+1}}{8x^2}-\frac{2\sqrt{-x^2+1}}{3x^3}-\frac{\sqrt{-x^2+1}}{4x^4}-\frac{7}{8}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(-x^2 + 1)/x - 7/8*sqrt(-x^2 + 1)/x^2 - 2/3*sqrt(-x^2 + 1)/x^3 - 1
/4*sqrt(-x^2 + 1)/x^4 - 7/8*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 1.81457, size = 126, normalized size = 1.42

$$\frac{21x^4\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(32x^3+21x^2+16x+6\right)\sqrt{-x^2+1}}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*x^4*log((sqrt(-x^2 + 1) - 1)/x) - (32*x^3 + 21*x^2 + 16*x + 6)*sqr
t(-x^2 + 1))/x^4
```

Sympy [A] time = 12.4501, size = 223, normalized size = 2.51

$$2\left(\begin{cases}-\frac{\sqrt{1-x^2}}{x}-\frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for}\ x>-1\wedge x<1\end{cases}\right)+\begin{cases}-\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}-\frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for}\ \frac{1}{|x^2|}>1\\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i}{2x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise}\end{cases}+\begin{cases}-\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8}+\frac{3}{8x\sqrt{-1+\frac{1}{x^2}}} \cdots \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8}-\frac{3i}{8x\sqrt{1-\frac{1}{x^2}}}+ \cdots\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**5/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x
< 1))) + Piecewise((-acosh(1/x)/2 - sqrt(-1 + x**(-2))/(2*x), 1/Abs(x**2) >
 1), (I*asin(1/x)/2 - I/(2*x*sqrt(1 - 1/x**2)) + I/(2*x**3*sqrt(1 - 1/x**2)
), True)) + Piecewise((-3*acosh(1/x)/8 + 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*
x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1),
 (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)
) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

Giac [B] time = 1.1037, size = 220, normalized size = 2.47

$$\frac{x^4\left(\frac{16\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{48\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{144\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-3\right)}{192\left(\sqrt{-x^2+1}-1\right)^4}-\frac{3\left(\sqrt{-x^2+1}-1\right)}{4x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{12x^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(sqrt(-x^2 + 1) - 1)/x - 48*(sqrt(-x^2 + 1) - 1)^2/x^2 + 144*
(sqrt(-x^2 + 1) - 1)^3/x^3 - 3)/(sqrt(-x^2 + 1) - 1)^4 - 3/4*(sqrt(-x^2 + 1
) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1/12*(sqrt(-x^2 + 1) - 1)^3/x^3
 + 1/64*(sqrt(-x^2 + 1) - 1)^4/x^4 + 7/8*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

3.63 $\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{6\sqrt{1-x^2}}{5x}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{\sqrt{1-x^2}}{5x^5}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -Sqrt[1 - x^2]/(5*x^5) - Sqrt[1 - x^2]/(2*x^4) - (3*Sqrt[1 - x^2])/(5*x^3)
- (3*Sqrt[1 - x^2])/(4*x^2) - (6*Sqrt[1 - x^2])/(5*x) - (3*ArcTanh[Sqrt[1 -
 x^2]])/4
```

Rubi [A] time = 0.103015, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1807, 835, 807, 266, 63, 206}

$$-\frac{6\sqrt{1-x^2}}{5x}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{\sqrt{1-x^2}}{5x^5}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```


```
[Out] -Sqrt[1 - x^2]/(5*x^5) - Sqrt[1 - x^2]/(2*x^4) - (3*Sqrt[1 - x^2])/(5*x^3)
- (3*Sqrt[1 - x^2])/(4*x^2) - (6*Sqrt[1 - x^2])/(5*x) - (3*ArcTanh[Sqrt[1 -
 x^2]])/4
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{1}{5}\int \frac{-10-9x}{x^5\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}+\frac{1}{20}\int \frac{36+30x}{x^4\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{1}{60}\int \frac{-90-72x}{x^3\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}+\frac{1}{120}\int \frac{144+90x}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}+\frac{3}{4}\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.0177882, size = 50, normalized size = 0.47

$$-\frac{\sqrt{1-x^2}\left(10x^5\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^2\right)+6x^4+3x^2+1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]*(1 + 3*x^2 + 6*x^4 + 10*x^5*Hypergeometric2F1[1/2, 3, 3/2,
1 - x^2]))/(5*x^5)
```

Maple [A] time = 0.05, size = 84, normalized size = 0.8

$$-\frac{3}{5\,x^3}\sqrt{-x^2+1}-\frac{6}{5\,x}\sqrt{-x^2+1}-\frac{1}{5\,x^5}\sqrt{-x^2+1}-\frac{1}{2\,x^4}\sqrt{-x^2+1}-\frac{3}{4\,x^2}\sqrt{-x^2+1}-\frac{3}{4}\operatorname{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^6/(-x^2+1)^(1/2),x)
```

```
[Out] -3/5*(-x^2+1)^(1/2)/x^3-6/5*(-x^2+1)^(1/2)/x-1/5*(-x^2+1)^(1/2)/x^5-1/2/x^4
*(-x^2+1)^(1/2)-3/4/x^2*(-x^2+1)^(1/2)-3/4*arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.70761, size = 130, normalized size = 1.21

$$-\frac{6\sqrt{-x^2+1}}{5x}-\frac{3\sqrt{-x^2+1}}{4x^2}-\frac{3\sqrt{-x^2+1}}{5x^3}-\frac{\sqrt{-x^2+1}}{2x^4}-\frac{\sqrt{-x^2+1}}{5x^5}-\frac{3}{4}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -6/5*sqrt(-x^2 + 1)/x - 3/4*sqrt(-x^2 + 1)/x^2 - 3/5*sqrt(-x^2 + 1)/x^3 - 1
/2*sqrt(-x^2 + 1)/x^4 - 1/5*sqrt(-x^2 + 1)/x^5 - 3/4*log(2*sqrt(-x^2 + 1)/a
bs(x) + 2/abs(x))
```

Fricas [A] time = 1.85401, size = 138, normalized size = 1.29

$$\frac{15\,x^5\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(24\,x^4+15\,x^3+12\,x^2+10\,x+4\right)\sqrt{-x^2+1}}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*x^5*log((sqrt(-x^2 + 1) - 1)/x) - (24*x^4 + 15*x^3 + 12*x^2 + 10*x
 + 4)*sqrt(-x^2 + 1))/x^5
```

Sympy [C] time = 14.0711, size = 201, normalized size = 1.88

$$\begin{cases}-\frac{\sqrt{1-x^2}}{x}-\frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for}\ x>-1\wedge x<1\end{cases}+\begin{cases}-\frac{\sqrt{1-x^2}}{x}-\frac{2(1-x^2)^{\frac{3}{2}}}{3x^3}-\frac{(1-x^2)^{\frac{5}{2}}}{5x^5} & \text{for}\ x>-1\wedge x<1\end{cases}+2\left(\begin{cases}-\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8}+\frac{3}{8x\sqrt{-1\cdots}} \\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8}-\frac{3i}{8x\sqrt{1-\frac{1}{x^2}}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**6/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-sqrt(1 - x**2)/x - 2*(1 - x**2)**(3/2)/(3*x**3) - (1 - x
**2)**(5/2)/(5*x**5), (x > -1) & (x < 1))) + 2*Piecewise((-3*acosh(1/x)/8 +
 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqr
t(-1 + x**(-2))), 1/Abs(x**2) > 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/
```

```
x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

Giac [B] time = 1.13081, size = 269, normalized size = 2.51

$$-\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{15\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{40\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-\frac{110\left(\sqrt{-x^2+1}-1\right)^4}{x^4}-1\right)}{160\left(\sqrt{-x^2+1}-1\right)^5}-\frac{11\left(\sqrt{-x^2+1}-1\right)}{16\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-3($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/160*x^5*(5*(sqrt(-x^2 + 1) - 1)/x - 15*(sqrt(-x^2 + 1) - 1)^2/x^2 + 40*(
sqrt(-x^2 + 1) - 1)^3/x^3 - 110*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1)/(sqrt(-x^2
+ 1) - 1)^5 - 11/16*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2
 - 3/32*(sqrt(-x^2 + 1) - 1)^3/x^3 + 1/32*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1/16
0*(sqrt(-x^2 + 1) - 1)^5/x^5 + 3/4*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

3.64 $\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$

Optimal. Leaf size=134

$$-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -(e^2*(13*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d*(d^2 - e^2*x^2)^(3/2
))/(4*x^4) - (e*(d^2 - e^2*x^2)^(3/2))/x^3 - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]] + (13*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rubi [A] time = 0.215096, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -(e^2*(13*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d*(d^2 - e^2*x^2)^(3/2
))/(4*x^4) - (e*(d^2 - e^2*x^2)^(3/2))/x^3 - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]] + (13*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx &= -\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\int \frac{\sqrt{d^2-e^2x^2}\left(-12d^4e-13d^3e^2x-4d^2e^3x^2\right)}{x^4}\,dx}{4d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}+\frac{\int \frac{\left(39d^5e^2+12d^4e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{12d^4}\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{\int \frac{78d^7e^4+48d^6e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{48d^6}\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{1}{8}\left(13de^4\right)\int \frac{1}{x\sqrt{d^2-e^2}}\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-\frac{1}{16}\left(13de^4\right)\mathrm{Subst}\left(\int\right.\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\\
&= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{x^3}-e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.261596, size = 196, normalized size = 1.46

$$
-\frac{e\sqrt{d^2-e^2x^2}\left(2e^3x^3\left(d^2-e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{e^2x^2}{d^2}\right)+6d^5\sqrt{1-\frac{e^2x^2}{d^2}}+9d^4ex\sqrt{1-\frac{e^2x^2}{d^2}}+6d^2e^3x^3\sin^{-1}\left(\frac{ex}{d}\right.\right.}{6d^3x^3\sqrt{1-\frac{e^2x^2}{d^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -(e*Sqrt[d^2 - e^2*x^2]*(6*d^5*Sqrt[1 - (e^2*x^2)/d^2] + 9*d^4*e*x*Sqrt[1 -
 (e^2*x^2)/d^2] + 6*d^2*e^3*x^3*ArcSin[(e*x)/d] - 9*d^2*e^3*x^3*ArcTanh[Sqr
t[1 - (e^2*x^2)/d^2]] + 2*e^3*x^3*(d^2 - e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^2]*H
ypergeometric2F1[3/2, 3, 5/2, 1 - (e^2*x^2)/d^2]))/(6*d^3*x^3*Sqrt[1 - (e^2
*x^2)/d^2])
```

Maple [A] time = 0.064, size = 212, normalized size = 1.6

$$-\frac{e}{x^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{d}{4x^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{13e^2}{8dx^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{13e^4}{8d}\sqrt{-x^2e^2+d^2}+\frac{13de^4}{8}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x)
```

```
[Out] -e*(-e^2*x^2+d^2)^(3/2)/x^3-1/4*d*(-e^2*x^2+d^2)^(3/2)/x^4-13/8/d*e^2/x^2*(
-e^2*x^2+d^2)^(3/2)-13/8*e^4/d*(-e^2*x^2+d^2)^(1/2)+13/8*d*e^4/(d^2)^(1/2)*
ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-e^3/d^2/x*(-e^2*x^2+d^2)^(
3/2)-e^5/d^2*x*(-e^2*x^2+d^2)^(1/2)-e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76287, size = 230, normalized size = 1.72

$$\frac{16\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-13\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(11\,de^2x^2+8\,d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(16*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13*e^4*x^4*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - (11*d*e^2*x^2 + 8*d^2*e*x + 2*d^3)*sqrt(-e
^2*x^2 + d^2))/x^4
```

Sympy [C] time = 9.60595, size = 556, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(1/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/
(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e
*x))/(8*d**3), True)) + 3*d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(
3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Ab
s(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d*
*2/(e**2*x**2) + 1)/(3*d**2), True)) + 3*d*e**2*Piecewise((-d**2/(2*e*x**3*
sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*aco
sh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(
e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**3*Piecewise
((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x
**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [B] time = 1.17322, size = 398, normalized size = 2.97

$$-\arcsin\left(\frac{xe}{d}\right)e^4\mathrm{sgn}\left(d\right)+\frac{x^4\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)e^8}{x}+\frac{24\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^6}{x^2}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^4}{x^3}+e^{10}\right)e^2}{64\left(de+\sqrt{-x^2e^2+d^2}e\right)^4}-\frac{1}{64}\left(\frac{8\left(de+\sqrt{-x\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^4*sgn(d) + 1/64*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/
x + 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^6/x^2 + 8*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^3*e^4/x^3 + e^10)*e^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 - 1/64*(8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)*e^26/x + 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e
^24/x^2 + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^22/x^3 + (d*e + sqrt(-x^2*e^
2 + d^2)*e)^4*e^20/x^4)*e^(-24) + 13/8*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2
*e^2 + d^2)*e)*e^(-2)/abs(x))
```

3.65 $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=310

$$\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6}-\frac{124d^5x^2\left(d^2-\right.}{1287e}$$

```
[Out] (35*d^12*x*Sqrt[d^2 - e^2*x^2])/(2048*e^5) + (35*d^10*x*(d^2 - e^2*x^2)^(3/
2))/(3072*e^5) + (7*d^8*x*(d^2 - e^2*x^2)^(5/2))/(768*e^5) - (124*d^5*x^2*(
d^2 - e^2*x^2)^(7/2))/(1287*e^4) - (7*d^4*x^3*(d^2 - e^2*x^2)^(7/2))/(48*e^
3) - (31*d^3*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e^2) - (7*d^2*x^5*(d^2 - e^2*x
^2)^(7/2))/(24*e) - (3*d*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (e*x^7*(d^2 - e^2*
x^2)^(7/2))/14 - (d^6*(31744*d + 63063*e*x)*(d^2 - e^2*x^2)^(7/2))/(1153152
*e^6) + (35*d^14*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2048*e^6)
```

Rubi [A] time = 0.48728, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6}-\frac{124d^5x^2\left(d^2-\right.}{1287e}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```


```
[Out] (35*d^12*x*Sqrt[d^2 - e^2*x^2])/(2048*e^5) + (35*d^10*x*(d^2 - e^2*x^2)^(3/
2))/(3072*e^5) + (7*d^8*x*(d^2 - e^2*x^2)^(5/2))/(768*e^5) - (124*d^5*x^2*(
d^2 - e^2*x^2)^(7/2))/(1287*e^4) - (7*d^4*x^3*(d^2 - e^2*x^2)^(7/2))/(48*e^
3) - (31*d^3*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e^2) - (7*d^2*x^5*(d^2 - e^2*x
^2)^(7/2))/(24*e) - (3*d*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (e*x^7*(d^2 - e^2*
x^2)^(7/2))/14 - (d^6*(31744*d + 63063*e*x)*(d^2 - e^2*x^2)^(7/2))/(1153152
*e^6) + (35*d^14*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2048*e^6)
```


Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^5\left(d^2-e^2x^2\right)^{5/2}\left(-14d^3e^2-49d^2e^3x-42de^4x^2\right)dx}{14e^2}$$

$$= -\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^5\left(434d^3e^4+637d^2e^5x\right)\left(d^2-e^2x^2\right)^{5}}{182e^4}$$

$$= -\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^4\left(-3185d^4e^5\right.}{}$$

$$= -\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{14}ex^7\left(d^2-e^2x\right.$$

$$= -\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e^2x^2\right)^{7/2}}{24e}-\frac{3}{13}dx^6\left(d^2-e^2\right.$$

$$= -\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e\right.}{24e}$$

$$= -\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x^2\right)^{7/2}}{143e^2}-\frac{7d^2x^5\left(d^2-e\right.}{24e}$$

$$= \frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x^2\right)^{7/2}}{48e^3}-\frac{31d^3x^4\left(d^2-e^2x\right.}{143e^2}$$

$$= \frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right)^{7/2}}{1287e^4}-\frac{7d^4x^3\left(d^2-e^2x\right.}{48e^3}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right.}{1287e^4}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right.}{1287e^4}$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x\left(d^2-e^2x^2\right)^{3/2}}{3072e^5}+\frac{7d^8x\left(d^2-e^2x^2\right)^{5/2}}{768e^5}-\frac{124d^5x^2\left(d^2-e^2x^2\right.}{1287e^4}$$

Mathematica [A] time = 0.379081, size = 212, normalized size = 0.68

$$\sqrt{d^2-e^2x^2}\left(315315d^{13}\sin^{-1}\left(\frac{ex}{d}\right)-\sqrt{1-\frac{e^2x^2}{d^2}}\left(253952d^{11}e^2x^2+210210d^{10}e^3x^3+190464d^9e^4x^4+168168d^8e^5x^5-29163\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-(Sqrt[1 - (e^2*x^2)/d^2]*(507904*d^13 + 315315*d^12*
e*x + 253952*d^11*e^2*x^2 + 210210*d^10*e^3*x^3 + 190464*d^9*e^4*x^4 + 1681
68*d^8*e^5*x^5 - 2916352*d^7*e^6*x^6 - 7763184*d^6*e^7*x^7 - 2551808*d^5*e^
8*x^8 + 9499776*d^4*e^9*x^9 + 8773632*d^3*e^10*x^10 - 1427712*d^2*e^11*x^11
 - 4257792*d*e^12*x^12 - 1317888*e^13*x^13)) + 315315*d^13*ArcSin[(e*x)/d])
)/(18450432*e^6*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.154, size = 291, normalized size = 0.9

$$-\frac{ex^7}{14}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{7\,d^2x^5}{24\,e}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{7\,d^4x^3}{48\,e^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{7\,d^6x}{128\,e^5}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}+\frac{7\,d^8x}{768\,e^5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{35}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/14*e*x^7*(-e^2*x^2+d^2)^(7/2)-7/24*d^2*x^5*(-e^2*x^2+d^2)^(7/2)/e-7/48*d
^4*x^3*(-e^2*x^2+d^2)^(7/2)/e^3-7/128/e^5*d^6*x*(-e^2*x^2+d^2)^(7/2)+7/768*
d^8*x*(-e^2*x^2+d^2)^(5/2)/e^5+35/3072*d^10*x*(-e^2*x^2+d^2)^(3/2)/e^5+35/2
048*d^12*x*(-e^2*x^2+d^2)^(1/2)/e^5+35/2048/e^5*d^14/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3/13*d*x^6*(-e^2*x^2+d^2)^(7/2)-31/143*d^3
*x^4*(-e^2*x^2+d^2)^(7/2)/e^2-124/1287*d^5*x^2*(-e^2*x^2+d^2)^(7/2)/e^4-248
/9009*d^7/e^6*(-e^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 1.44528, size = 382, normalized size = 1.23

$$-\frac{1}{14}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^7-\frac{3}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^6-\frac{7\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^5}{24\,e}+\frac{35\,d^{14}\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2048\,\sqrt{e^2}e^5}+\frac{35\,\sqrt{-e^2x^2+d^2}d^{12}x}{2048\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/14*(-e^2*x^2 + d^2)^(7/2)*e*x^7 - 3/13*(-e^2*x^2 + d^2)^(7/2)*d*x^6 - 7/
24*(-e^2*x^2 + d^2)^(7/2)*d^2*x^5/e + 35/2048*d^14*arcsin(e^2*x/sqrt(d^2*e^
2))/(sqrt(e^2)*e^5) + 35/2048*sqrt(-e^2*x^2 + d^2)*d^12*x/e^5 - 31/143*(-e^
2*x^2 + d^2)^(7/2)*d^3*x^4/e^2 + 35/3072*(-e^2*x^2 + d^2)^(3/2)*d^10*x/e^5
- 7/48*(-e^2*x^2 + d^2)^(7/2)*d^4*x^3/e^3 + 7/768*(-e^2*x^2 + d^2)^(5/2)*d^
8*x/e^5 - 124/1287*(-e^2*x^2 + d^2)^(7/2)*d^5*x^2/e^4 - 7/128*(-e^2*x^2 + d
^2)^(7/2)*d^6*x/e^5 - 248/9009*(-e^2*x^2 + d^2)^(7/2)*d^7/e^6
```

Fricas [A] time = 1.88778, size = 529, normalized size = 1.71

$$-\frac{630630\,d^{14}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(1317888\,e^{13}x^{13}+4257792\,de^{12}x^{12}+1427712\,d^2e^{11}x^{11}-8773632\,d^3e^{10}x^{10}-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/18450432*(630630*d^14*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (13178
88*e^13*x^13 + 4257792*d*e^12*x^12 + 1427712*d^2*e^11*x^11 - 8773632*d^3*e^
10*x^10 - 9499776*d^4*e^9*x^9 + 2551808*d^5*e^8*x^8 + 7763184*d^6*e^7*x^7 +
 2916352*d^7*e^6*x^6 - 168168*d^8*e^5*x^5 - 190464*d^9*e^4*x^4 - 210210*d^1
0*e^3*x^3 - 253952*d^11*e^2*x^2 - 315315*d^12*e*x - 507904*d^13)*sqrt(-e^2*
x^2 + d^2))/e^6
```

Sympy [A] time = 139.604, size = 2280, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqr
t(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2)
 + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + 3
*d**6*e*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6
*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d
**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128
*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/
d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(
1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + d
**5*e**2*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**
2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(1
05*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e*
*2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) - 5*d**4*e**3*Piecewise((
-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x*
*2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x
**5/(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 +
 e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**1
1/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (7*d**10
*asin(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*
d**7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt
(1 - e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d
*x**9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d
**2)), True)) - 5*d**3*e**4*Piecewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3
465*e**10) - 64*d**8*x**2*sqrt(d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4
*sqrt(d**2 - e**2*x**2)/(1155*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(6
93*e**4) - d**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e
**2*x**2)/11, Ne(e, 0)), (x**10*sqrt(d**2)/10, True)) + d**2*e**5*Piecewise
((-21*I*d**12*acosh(e*x/d)/(1024*e**11) + 21*I*d**11*x/(1024*e**10*sqrt(-1
+ e**2*x**2/d**2)) - 7*I*d**9*x**3/(1024*e**8*sqrt(-1 + e**2*x**2/d**2)) -
7*I*d**7*x**5/(2560*e**6*sqrt(-1 + e**2*x**2/d**2)) - I*d**5*x**7/(640*e**4
*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**9/(960*e**2*sqrt(-1 + e**2*x**2/d**
2)) - 11*I*d*x**11/(120*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**13/(12*d*sqr
t(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (21*d**12*asin(e*x/
d)/(1024*e**11) - 21*d**11*x/(1024*e**10*sqrt(1 - e**2*x**2/d**2)) + 7*d**9
*x**3/(1024*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**5/(2560*e**6*sqrt(1
- e**2*x**2/d**2)) + d**5*x**7/(640*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x
**9/(960*e**2*sqrt(1 - e**2*x**2/d**2)) + 11*d*x**11/(120*sqrt(1 - e**2*x**
2/d**2)) - e**2*x**13/(12*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Pi
ecewise((-256*d**12*sqrt(d**2 - e**2*x**2)/(9009*e**12) - 128*d**10*x**2*sq
rt(d**2 - e**2*x**2)/(9009*e**10) - 32*d**8*x**4*sqrt(d**2 - e**2*x**2)/(30
03*e**8) - 80*d**6*x**6*sqrt(d**2 - e**2*x**2)/(9009*e**6) - 10*d**4*x**8*s
qrt(d**2 - e**2*x**2)/(1287*e**4) - d**2*x**10*sqrt(d**2 - e**2*x**2)/(143*
e**2) + x**12*sqrt(d**2 - e**2*x**2)/13, Ne(e, 0)), (x**12*sqrt(d**2)/12, T
rue)) + e**7*Piecewise((-33*I*d**14*acosh(e*x/d)/(2048*e**13) + 33*I*d**13*
x/(2048*e**12*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d**11*x**3/(2048*e**10*sqrt
(-1 + e**2*x**2/d**2)) - 11*I*d**9*x**5/(5120*e**8*sqrt(-1 + e**2*x**2/d**2
)) - 11*I*d**7*x**7/(8960*e**6*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d**5*x**9/
(13440*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**11/(1680*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 13*I*d*x**13/(168*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x*
*15/(14*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (33*d*
*14*asin(e*x/d)/(2048*e**13) - 33*d**13*x/(2048*e**12*sqrt(1 - e**2*x**2/d*
*2)) + 11*d**11*x**3/(2048*e**10*sqrt(1 - e**2*x**2/d**2)) + 11*d**9*x**5/(
5120*e**8*sqrt(1 - e**2*x**2/d**2)) + 11*d**7*x**7/(8960*e**6*sqrt(1 - e**2
*x**2/d**2)) + 11*d**5*x**9/(13440*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x*
*11/(1680*e**2*sqrt(1 - e**2*x**2/d**2)) + 13*d*x**13/(168*sqrt(1 - e**2*x*
```

```
*2/d**2)) - e**2*x**15/(14*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.13629, size = 230, normalized size = 0.74

$$\frac{35}{2048}\,d^{14}\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}\left(d\right)-\frac{1}{18450432}\left(507904\,d^{13}e^{(-6)}+\left(315315\,d^{12}e^{(-5)}+2\left(126976\,d^{11}e^{(-4)}+\left(105105\,d^{10}e^{(-4)}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/2048*d^14*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/18450432*(507904*d^13*e^(-6) +
 (315315*d^12*e^(-5) + 2*(126976*d^11*e^(-4) + (105105*d^10*e^(-3) + 4*(238
08*d^9*e^(-2) + (21021*d^8*e^(-1) - 2*(182272*d^7 + (485199*d^6*e + 8*(1993
6*d^5*e^2 - 3*(24739*d^4*e^3 + 2*(11424*d^3*e^4 - 11*(169*d^2*e^5 + 12*(13*
x*e^7 + 42*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.66 $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=281

$$\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5}-\frac{20d^4x^2\left(d^2-e^2x^2\right.}{143e^3}$$

```
[Out] (27*d^11*x*Sqrt[d^2 - e^2*x^2])/(1024*e^4) + (9*d^9*x*(d^2 - e^2*x^2)^(3/2)
)/(512*e^4) + (9*d^7*x*(d^2 - e^2*x^2)^(5/2))/(640*e^4) - (20*d^4*x^2*(d^2
- e^2*x^2)^(7/2))/(143*e^3) - (9*d^3*x^3*(d^2 - e^2*x^2)^(7/2))/(40*e^2) -
(45*d^2*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e) - (d*x^5*(d^2 - e^2*x^2)^(7/2))/
4 - (e*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (d^5*(12800*d + 27027*e*x)*(d^2 - e^
2*x^2)^(7/2))/(320320*e^5) + (27*d^13*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1
024*e^5)
```

Rubi [A] time = 0.405587, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5}-\frac{20d^4x^2\left(d^2-e^2x^2\right.}{143e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```


```
[Out] (27*d^11*x*Sqrt[d^2 - e^2*x^2])/(1024*e^4) + (9*d^9*x*(d^2 - e^2*x^2)^(3/2)
)/(512*e^4) + (9*d^7*x*(d^2 - e^2*x^2)^(5/2))/(640*e^4) - (20*d^4*x^2*(d^2
- e^2*x^2)^(7/2))/(143*e^3) - (9*d^3*x^3*(d^2 - e^2*x^2)^(7/2))/(40*e^2) -
(45*d^2*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e) - (d*x^5*(d^2 - e^2*x^2)^(7/2))/
4 - (e*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (d^5*(12800*d + 27027*e*x)*(d^2 - e^
2*x^2)^(7/2))/(320320*e^5) + (27*d^13*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1
024*e^5)
```


Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^4\left(d^2-e^2x^2\right)^{5/2}\left(-13d^3e^2-45d^2e^3x-39de^4x^2\right)dx}{13e^2}$$

$$= -\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^4\left(351d^3e^4+540d^2e^5x\right)\left(d^2-e^2x^2\right.}{156e^4}$$

$$= -\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^3\left(-2160d\right.}{}$$

$$= -\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{13}ex^6\left(d^2-e\right.$$

$$= -\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-\right.$$

$$= -\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}-\frac{1}{4}dx^5\left(d^2-\right.$$

$$= \frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}-\frac{45d^2x^4\left(d^2-e\right.}{143e}$$

$$= \frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x\right.}{40e^2}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right.}{143e^3}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right.}{143e^3}$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right.}{143e^3}$$

Mathematica [A] time = 0.360975, size = 200, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(-102400d^{10}e^2x^2-90090d^9e^3x^3-76800d^8e^4x^4+952952d^7e^5x^5+2498560d^6e^6x^6+816816d^5e^7x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-204800*d^12 - 135135*d^11*e
*x - 102400*d^10*e^2*x^2 - 90090*d^9*e^3*x^3 - 76800*d^8*e^4*x^4 + 952952*d
^7*e^5*x^5 + 2498560*d^6*e^6*x^6 + 816816*d^5*e^7*x^7 - 2938880*d^4*e^8*x^8
 - 2690688*d^3*e^9*x^9 + 430080*d^2*e^10*x^10 + 1281280*d*e^11*x^11 + 39424
0*e^12*x^12) + 135135*d^12*ArcSin[(e*x)/d]))/(5125120*e^5*Sqrt[1 - (e^2*x^2
)/d^2])
```

Maple [A] time = 0.112, size = 266, normalized size = 1.

$$-\frac{ex^6}{13}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{45\,d^2x^4}{143\,e}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{20\,d^4x^2}{143\,e^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{40\,d^6}{1001\,e^5}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{dx^5}{4}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{9}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/13*e*x^6*(-e^2*x^2+d^2)^(7/2)-45/143*d^2*x^4*(-e^2*x^2+d^2)^(7/2)/e-20/1
43*d^4*x^2*(-e^2*x^2+d^2)^(7/2)/e^3-40/1001/e^5*d^6*(-e^2*x^2+d^2)^(7/2)-1/
4*d*x^5*(-e^2*x^2+d^2)^(7/2)-9/40*d^3*x^3*(-e^2*x^2+d^2)^(7/2)/e^2-27/320*d
^5/e^4*x*(-e^2*x^2+d^2)^(7/2)+9/640*d^7*x*(-e^2*x^2+d^2)^(5/2)/e^4+9/512*d^
9*x*(-e^2*x^2+d^2)^(3/2)/e^4+27/1024*d^11*x*(-e^2*x^2+d^2)^(1/2)/e^4+27/102
4*d^13/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.52526, size = 348, normalized size = 1.24

$$-\frac{1}{13}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^6-\frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^5+\frac{27\,d^{13}\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{1024\,\sqrt{e^2}e^4}+\frac{27\,\sqrt{-e^2x^2+d^2}d^{11}x}{1024\,e^4}-\frac{45\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x^4}{143\,e}+\frac{9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/13*(-e^2*x^2 + d^2)^(7/2)*e*x^6 - 1/4*(-e^2*x^2 + d^2)^(7/2)*d*x^5 + 27/
1024*d^13*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^4) + 27/1024*sqrt(-e^2*x
^2 + d^2)*d^11*x/e^4 - 45/143*(-e^2*x^2 + d^2)^(7/2)*d^2*x^4/e + 9/512*(-e^
2*x^2 + d^2)^(3/2)*d^9*x/e^4 - 9/40*(-e^2*x^2 + d^2)^(7/2)*d^3*x^3/e^2 + 9/
640*(-e^2*x^2 + d^2)^(5/2)*d^7*x/e^4 - 20/143*(-e^2*x^2 + d^2)^(7/2)*d^4*x^
2/e^3 - 27/320*(-e^2*x^2 + d^2)^(7/2)*d^5*x/e^4 - 40/1001*(-e^2*x^2 + d^2)^
(7/2)*d^6/e^5
```

Fricas [A] time = 1.92356, size = 487, normalized size = 1.73

$$-\frac{270270\,d^{13}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(394240\,e^{12}x^{12}+1281280\,de^{11}x^{11}+430080\,d^2e^{10}x^{10}-2690688\,d^3e^9x^9-2938\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5125120*(270270*d^13*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (394240
*e^12*x^12 + 1281280*d*e^11*x^11 + 430080*d^2*e^10*x^10 - 2690688*d^3*e^9*x
^9 - 2938880*d^4*e^8*x^8 + 816816*d^5*e^7*x^7 + 2498560*d^6*e^6*x^6 + 95295
2*d^7*e^5*x^5 - 76800*d^8*e^4*x^4 - 90090*d^9*e^3*x^3 - 102400*d^10*e^2*x^2
 - 135135*d^11*e*x - 204800*d^12)*sqrt(-e^2*x^2 + d^2))/e^5
```

Sympy [C] time = 83.5954, size = 2035, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5
*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x*
*2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1
- e**2*x**2/d**2)), True)) + 3*d**6*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x
**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4
*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)
), (x**6*sqrt(d**2)/6, True)) + d**5*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)
/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x*
*3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(
8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin
(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*
x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**
2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqr
t(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((-16*d**8*sqrt(d**2
- e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2
*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x
**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)
/8, True)) - 5*d**3*e**4*Piecewise((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*
I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqr
t(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt(-1 + e**2*x**2/d**2
)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt
(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2)/Abs(d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9) - 7*d**9*x/(256
*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt(1 - e**2*x**2/
d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**7/(480*
e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*x**2/d**2)) - e
**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewise((-1
28*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*sqrt(d**2 - e**
2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(1155*e**6) - 8*d
```

```
**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d**2 - e**2*x**
2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x**10*sqrt(d**2
)/10, True)) + 3*d*e**6*Piecewise((-21*I*d**12*acosh(e*x/d)/(1024*e**11) +
21*I*d**11*x/(1024*e**10*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**9*x**3/(1024*e
**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**5/(2560*e**6*sqrt(-1 + e**2*x*
*2/d**2)) - I*d**5*x**7/(640*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**9/
(960*e**2*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d*x**11/(120*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**13/(12*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Ab
s(d**2) > 1), (21*d**12*asin(e*x/d)/(1024*e**11) - 21*d**11*x/(1024*e**10*s
qrt(1 - e**2*x**2/d**2)) + 7*d**9*x**3/(1024*e**8*sqrt(1 - e**2*x**2/d**2))
 + 7*d**7*x**5/(2560*e**6*sqrt(1 - e**2*x**2/d**2)) + d**5*x**7/(640*e**4*s
qrt(1 - e**2*x**2/d**2)) + d**3*x**9/(960*e**2*sqrt(1 - e**2*x**2/d**2)) +
11*d*x**11/(120*sqrt(1 - e**2*x**2/d**2)) - e**2*x**13/(12*d*sqrt(1 - e**2*
x**2/d**2)), True)) + e**7*Piecewise((-256*d**12*sqrt(d**2 - e**2*x**2)/(90
09*e**12) - 128*d**10*x**2*sqrt(d**2 - e**2*x**2)/(9009*e**10) - 32*d**8*x*
*4*sqrt(d**2 - e**2*x**2)/(3003*e**8) - 80*d**6*x**6*sqrt(d**2 - e**2*x**2)
/(9009*e**6) - 10*d**4*x**8*sqrt(d**2 - e**2*x**2)/(1287*e**4) - d**2*x**10
*sqrt(d**2 - e**2*x**2)/(143*e**2) + x**12*sqrt(d**2 - e**2*x**2)/13, Ne(e,
 0)), (x**12*sqrt(d**2)/12, True))
```

Giac [A] time = 1.13118, size = 216, normalized size = 0.77

$$\frac{27}{1024}\,d^{13}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}\,(d) - \frac{1}{5125120}\left(204800\,d^{12}e^{(-5)} + \left(135135\,d^{11}e^{(-4)} + 2\left(51200\,d^{10}e^{(-3)} + \left(45045\,d^{9}e^{(-2)} + 4\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 27/1024*d^13*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/5125120*(204800*d^12*e^(-5) +
(135135*d^11*e^(-4) + 2*(51200*d^10*e^(-3) + (45045*d^9*e^(-2) + 4*(9600*d^
8*e^(-1) - (119119*d^7 + 2*(156160*d^6*e + 7*(7293*d^5*e^2 - 8*(3280*d^4*e^
3 + (3003*d^3*e^4 - 10*(48*d^2*e^5 + 11*(4*x*e^7 + 13*d*e^6)*x)*x)*x)*x)*x)
*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.67 $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3}+\frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}-\frac{23d^3x^2\left(d^2\right.}{99}$$

```
[Out] (41*d^10*x*Sqrt[d^2 - e^2*x^2])/(1024*e^3) + (41*d^8*x*(d^2 - e^2*x^2)^(3/2
))/(1536*e^3) + (41*d^6*x*(d^2 - e^2*x^2)^(5/2))/(1920*e^3) - (23*d^3*x^2*(
d^2 - e^2*x^2)^(7/2))/(99*e^2) - (41*d^2*x^3*(d^2 - e^2*x^2)^(7/2))/(120*e)
 - (3*d*x^4*(d^2 - e^2*x^2)^(7/2))/11 - (e*x^5*(d^2 - e^2*x^2)^(7/2))/12 -
(d^4*(14720*d + 28413*e*x)*(d^2 - e^2*x^2)^(7/2))/(221760*e^4) + (41*d^12*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1024*e^4)
```

Rubi [A] time = 0.364037, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3}+\frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}-\frac{23d^3x^2\left(d^2\right.}{99}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```


```
[Out] (41*d^10*x*Sqrt[d^2 - e^2*x^2])/(1024*e^3) + (41*d^8*x*(d^2 - e^2*x^2)^(3/2
))/(1536*e^3) + (41*d^6*x*(d^2 - e^2*x^2)^(5/2))/(1920*e^3) - (23*d^3*x^2*(
d^2 - e^2*x^2)^(7/2))/(99*e^2) - (41*d^2*x^3*(d^2 - e^2*x^2)^(7/2))/(120*e)
 - (3*d*x^4*(d^2 - e^2*x^2)^(7/2))/11 - (e*x^5*(d^2 - e^2*x^2)^(7/2))/12 -
(d^4*(14720*d + 28413*e*x)*(d^2 - e^2*x^2)^(7/2))/(221760*e^4) + (41*d^12*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1024*e^4)
```


Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}dx = -\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^3\left(d^2-e^2x^2\right)^{5/2}\left(-12d^3e^2-41d^2e^3x-36de^4x^2\right)dx}{12e^2}$$

$$= -\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^3\left(276d^3e^4+451d^2e^5x\right)\left(d^2-e^2x^2\right)^{5}}{132e^4}$$

$$= -\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^2\left(-1353d^4e\right.}{}$$

$$= -\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2\right.$$

$$= -\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2\right.$$

$$= \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x\right.$$

$$= \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2} - \frac{41d^2x^3\left(d^2-e^2x\right.}{120e}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7}}{99e^2}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7}}{99e^2}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3} + \frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3} - \frac{23d^3x^2\left(d^2-e^2x^2\right)^{7}}{99e^2}$$

Mathematica [A] time = 0.31831, size = 189, normalized size = 0.75

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(-117760d^9e^2x^2-94710d^8e^3x^3+798720d^7e^4x^4+2053128d^6e^5x^5+665600d^5e^6x^6-2295216d^4e^7\right.\right.}{3548160}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-235520*d^11 - 142065*d^10*e
*x - 117760*d^9*e^2*x^2 - 94710*d^8*e^3*x^3 + 798720*d^7*e^4*x^4 + 2053128*
d^6*e^5*x^5 + 665600*d^5*e^6*x^6 - 2295216*d^4*e^7*x^7 - 2078720*d^3*e^8*x^
8 + 325248*d^2*e^9*x^9 + 967680*d*e^10*x^10 + 295680*e^11*x^11) + 142065*d^
11*ArcSin[(e*x)/d]))/(3548160*e^4*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.092, size = 241, normalized size = 1.

$$-\frac{ex^5}{12}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{41\,d^2x^3}{120\,e}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{41\,d^4x}{320\,e^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}+\frac{41\,d^6x}{1920\,e^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{41\,d^8x}{1536\,e^3}\left(-x^2e^2+d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/12*e*x^5*(-e^2*x^2+d^2)^(7/2)-41/120*d^2*x^3*(-e^2*x^2+d^2)^(7/2)/e-41/3
20/e^3*d^4*x*(-e^2*x^2+d^2)^(7/2)+41/1920*d^6*x*(-e^2*x^2+d^2)^(5/2)/e^3+41
/1536*d^8*x*(-e^2*x^2+d^2)^(3/2)/e^3+41/1024*d^10*x*(-e^2*x^2+d^2)^(1/2)/e^
3+41/1024/e^3*d^12/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3
/11*d*x^4*(-e^2*x^2+d^2)^(7/2)-23/99*d^3*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-46/69
3*d^5/e^4*(-e^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 1.48652, size = 315, normalized size = 1.25

$$-\frac{1}{12}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^5+\frac{41\,d^{12}\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{1024\,\sqrt{e^2}e^3}+\frac{41\,\sqrt{-e^2x^2+d^2}d^{10}x}{1024\,e^3}-\frac{3}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^4+\frac{41\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^8x}{1536\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-e^2*x^2 + d^2)^(7/2)*e*x^5 + 41/1024*d^12*arcsin(e^2*x/sqrt(d^2*e^2
))/(sqrt(e^2)*e^3) + 41/1024*sqrt(-e^2*x^2 + d^2)*d^10*x/e^3 - 3/11*(-e^2*x
^2 + d^2)^(7/2)*d*x^4 + 41/1536*(-e^2*x^2 + d^2)^(3/2)*d^8*x/e^3 - 41/120*(
-e^2*x^2 + d^2)^(7/2)*d^2*x^3/e + 41/1920*(-e^2*x^2 + d^2)^(5/2)*d^6*x/e^3
- 23/99*(-e^2*x^2 + d^2)^(7/2)*d^3*x^2/e^2 - 41/320*(-e^2*x^2 + d^2)^(7/2)*
d^4*x/e^3 - 46/693*(-e^2*x^2 + d^2)^(7/2)*d^5/e^4
```

Fricas [A] time = 2.02802, size = 455, normalized size = 1.81

$$-\frac{284130\,d^{12}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(295680\,e^{11}x^{11}+967680\,de^{10}x^{10}+325248\,d^2e^9x^9-2078720\,d^3e^8x^8-2295216\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3548160*(284130*d^12*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (295680
*e^11*x^11 + 967680*d*e^10*x^10 + 325248*d^2*e^9*x^9 - 2078720*d^3*e^8*x^8
- 2295216*d^4*e^7*x^7 + 665600*d^5*e^6*x^6 + 2053128*d^6*e^5*x^5 + 798720*d
^7*e^4*x^4 - 94710*d^8*e^3*x^3 - 117760*d^9*e^2*x^2 - 142065*d^10*e*x - 235
520*d^11)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] time = 66.9829, size = 1926, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + 3*d**6*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5)
 + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt
(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2
*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6
*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x
**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d*
*2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**5*e**2*Piecewi
se((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**
2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt
(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 5*d**4*e**3*P
iecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1
+ e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I
*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 +
e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x*
*2)/Abs(d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqr
t(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) +
d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*
x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**
4*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(
d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4
) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2
)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) + d**2*e**5*Piecewise((-7*I*d**1
0*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2))
 - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920
*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**
2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (7*d**10*asin(e*x
/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3
/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2
*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80
*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), Tr
ue)) + 3*d*e**6*Piecewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) -
 64*d**8*x**2*sqrt(d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 -
 e**2*x**2)/(1155*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) - d
**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/11
, Ne(e, 0)), (x**10*sqrt(d**2)/10, True)) + e**7*Piecewise((-21*I*d**12*aco
sh(e*x/d)/(1024*e**11) + 21*I*d**11*x/(1024*e**10*sqrt(-1 + e**2*x**2/d**2)
) - 7*I*d**9*x**3/(1024*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**5/(25
60*e**6*sqrt(-1 + e**2*x**2/d**2)) - I*d**5*x**7/(640*e**4*sqrt(-1 + e**2*x
**2/d**2)) - I*d**3*x**9/(960*e**2*sqrt(-1 + e**2*x**2/d**2)) - 11*I*d*x**1
1/(120*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**13/(12*d*sqrt(-1 + e**2*x**2/
```

```
d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (21*d**12*asin(e*x/d)/(1024*e**11) -
 21*d**11*x/(1024*e**10*sqrt(1 - e**2*x**2/d**2)) + 7*d**9*x**3/(1024*e**8*
sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**5/(2560*e**6*sqrt(1 - e**2*x**2/d**2)
) + d**5*x**7/(640*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**9/(960*e**2*sqr
t(1 - e**2*x**2/d**2)) + 11*d*x**11/(120*sqrt(1 - e**2*x**2/d**2)) - e**2*x
**13/(12*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.12354, size = 201, normalized size = 0.8

$$\frac{41}{1024}\,d^{12}\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}\left(d\right)-\frac{1}{3548160}\left(235520\,d^{11}e^{(-4)}+\left(142065\,d^{10}e^{(-3)}+2\left(58880\,d^{9}e^{(-2)}+\left(47355\,d^{8}e^{(-1)}-\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 41/1024*d^12*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/3548160*(235520*d^11*e^(-4) +
(142065*d^10*e^(-3) + 2*(58880*d^9*e^(-2) + (47355*d^8*e^(-1) - 4*(99840*d^
7 + (256641*d^6*e + 2*(41600*d^5*e^2 - 7*(20493*d^4*e^3 + 8*(2320*d^3*e^4 -
 3*(121*d^2*e^5 + 10*(11*x*e^7 + 36*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sq
rt(-x^2*e^2 + d^2)
```

3.68 $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=223

$$\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3}-\frac{37d^2x^2\left(d^2-e^2x\right.}{99e}$$

```
[Out] (19*d^9*x*Sqrt[d^2 - e^2*x^2])/(256*e^2) + (19*d^7*x*(d^2 - e^2*x^2)^(3/2))
/(384*e^2) + (19*d^5*x*(d^2 - e^2*x^2)^(5/2))/(480*e^2) - (37*d^2*x^2*(d^2
- e^2*x^2)^(7/2))/(99*e) - (3*d*x^3*(d^2 - e^2*x^2)^(7/2))/10 - (e*x^4*(d^2
 - e^2*x^2)^(7/2))/11 - (d^3*(5920*d + 13167*e*x)*(d^2 - e^2*x^2)^(7/2))/(5
5440*e^3) + (19*d^11*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^3)
```

Rubi [A] time = 0.305661, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1809, 833, 780, 195, 217, 203}

$$\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3}-\frac{37d^2x^2\left(d^2-e^2x\right.}{99e}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (19*d^9*x*Sqrt[d^2 - e^2*x^2])/(256*e^2) + (19*d^7*x*(d^2 - e^2*x^2)^(3/2))
/(384*e^2) + (19*d^5*x*(d^2 - e^2*x^2)^(5/2))/(480*e^2) - (37*d^2*x^2*(d^2
- e^2*x^2)^(7/2))/(99*e) - (3*d*x^3*(d^2 - e^2*x^2)^(7/2))/10 - (e*x^4*(d^2
 - e^2*x^2)^(7/2))/11 - (d^3*(5920*d + 13167*e*x)*(d^2 - e^2*x^2)^(7/2))/(5
5440*e^3) + (19*d^11*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^3)
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx=-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x^2\left(d^2-e^2x^2\right)^{5/2}\left(-11d^3e^2-37d^2e^3x-33de^4x^2\right)dx}{11e^2}$$

$$=-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}+\frac{\int x^2\left(209d^3e^4+370d^2e^5x\right)\left(d^2-e^2x\right.}{110e^4}$$

$$=-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{\int x\left(-740d^4\right.}{}$$

$$=-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{d^3(5920d+}{}$$

$$=\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{11}ex^4\left(d^2-e\right.$$

$$=\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-\right.$$

$$=\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2-e^2x\right.}{99e}$$

$$=\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2-e^2x\right.}{99e}$$

$$=\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2-e^2x\right.}{99e}$$

Mathematica [A] time = 0.283597, size = 178, normalized size = 0.8

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(-47360d^8e^2x^2+251790d^7e^3x^3+629760d^6e^4x^4+201432d^5e^5x^5-657920d^4e^6x^6-587664d^3e\right.\right.}{887040e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-94720*d^10 - 65835*d^9*e*x
- 47360*d^8*e^2*x^2 + 251790*d^7*e^3*x^3 + 629760*d^6*e^4*x^4 + 201432*d^5*
e^5*x^5 - 657920*d^4*e^6*x^6 - 587664*d^3*e^7*x^7 + 89600*d^2*e^8*x^8 + 266
112*d*e^9*x^9 + 80640*e^10*x^10) + 65835*d^10*ArcSin[(e*x)/d]))/(887040*e^3
*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.079, size = 216, normalized size = 1.

$$-\frac{ex^4}{11}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{37\,d^2x^2}{99\,e}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{74\,d^4}{693\,e^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{3\,dx^3}{10}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{19\,d^3x}{80\,e^2}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}+\frac{19}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/11*e*x^4*(-e^2*x^2+d^2)^(7/2)-37/99*d^2*x^2*(-e^2*x^2+d^2)^(7/2)/e-74/69
3/e^3*d^4*(-e^2*x^2+d^2)^(7/2)-3/10*d*x^3*(-e^2*x^2+d^2)^(7/2)-19/80*d^3/e^
2*x*(-e^2*x^2+d^2)^(7/2)+19/480*d^5*x*(-e^2*x^2+d^2)^(5/2)/e^2+19/384*d^7*x
*(-e^2*x^2+d^2)^(3/2)/e^2+19/256*d^9*x*(-e^2*x^2+d^2)^(1/2)/e^2+19/256*d^11
/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.47916, size = 281, normalized size = 1.26

$$\frac{19\,d^{11}\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{256\,\sqrt{e^2}e^2}+\frac{19\,\sqrt{-e^2x^2+d^2}d^9x}{256\,e^2}-\frac{1}{11}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}ex^4+\frac{19\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x}{384\,e^2}-\frac{3}{10}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx^3+\frac{19(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 19/256*d^11*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) + 19/256*sqrt(-e^2*
x^2 + d^2)*d^9*x/e^2 - 1/11*(-e^2*x^2 + d^2)^(7/2)*e*x^4 + 19/384*(-e^2*x^2
 + d^2)^(3/2)*d^7*x/e^2 - 3/10*(-e^2*x^2 + d^2)^(7/2)*d*x^3 + 19/480*(-e^2*
x^2 + d^2)^(5/2)*d^5*x/e^2 - 37/99*(-e^2*x^2 + d^2)^(7/2)*d^2*x^2/e - 19/80
*(-e^2*x^2 + d^2)^(7/2)*d^3*x/e^2 - 74/693*(-e^2*x^2 + d^2)^(7/2)*d^4/e^3
```

Fricas [A] time = 1.93752, size = 412, normalized size = 1.85

$$-\frac{131670\,d^{11}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(80640\,e^{10}x^{10}+266112\,de^9x^9+89600\,d^2e^8x^8-587664\,d^3e^7x^7-657920\,d^4e^6x^6+2\right.}{887040\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/887040*(131670*d^11*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (80640*e
^10*x^10 + 266112*d*e^9*x^9 + 89600*d^2*e^8*x^8 - 587664*d^3*e^7*x^7 - 6579
20*d^4*e^6*x^6 + 201432*d^5*e^5*x^5 + 629760*d^6*e^4*x^4 + 251790*d^7*e^3*x
```

```
^3 - 47360*d^8*e^2*x^2 - 65835*d^9*e*x - 94720*d^10)*sqrt(-e^2*x^2 + d^2))/
e^3
```

Sympy [C] time = 51.3383, size = 1688, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e
*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqr
t(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) +
 3*d**6*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*s
qrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)),
 (x**4*sqrt(d**2)/4, True)) + d**5*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16
*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**
2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) +
I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1),
 (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) +
d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x
**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3
*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d*
*2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x
**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 5*d**
3*e**4*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*
sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d*
*2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sq
rt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*
e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d
**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1
 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + d*
*2*e**5*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2
*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(10
5*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**
2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) + 3*d*e**6*Piecewise((-7*I
*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d
**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/
(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(1
0*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (7*d**10*asi
n(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7
*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 -
 e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**
9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)
), True)) + e**7*Piecewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10)
- 64*d**8*x**2*sqrt(d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2
- e**2*x**2)/(1155*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) -
d**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/1
1, Ne(e, 0)), (x**10*sqrt(d**2)/10, True))
```

Giac [A] time = 1.13116, size = 188, normalized size = 0.84

$$\frac{19}{256}\,d^{11}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\,(d)-\frac{1}{887040}\left(94720\,d^{10}e^{(-3)}+\left(65835\,d^{9}e^{(-2)}+2\left(23680\,d^{8}e^{(-1)}-\left(125895\,d^{7}+4\left(78720\,d\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 19/256*d^11*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/887040*(94720*d^10*e^(-3) + (65
835*d^9*e^(-2) + 2*(23680*d^8*e^(-1) - (125895*d^7 + 4*(78720*d^6*e + (2517
9*d^5*e^2 - 2*(41120*d^4*e^3 + 7*(5247*d^3*e^4 - 8*(100*d^2*e^5 + 9*(10*x*e
^7 + 33*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.69 $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal. Leaf size=230

$$\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-$$

```
[Out] (33*d^8*x*Sqrt[d^2 - e^2*x^2])/(256*e) + (11*d^6*x*(d^2 - e^2*x^2)^(3/2))/(
128*e) + (11*d^4*x*(d^2 - e^2*x^2)^(5/2))/(160*e) - (33*d^3*(d^2 - e^2*x^2)
^(7/2))/(560*e^2) - (11*d^2*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(240*e^2) - (d
*(d + e*x)^2*(d^2 - e^2*x^2)^(7/2))/(30*e^2) - ((d + e*x)^3*(d^2 - e^2*x^2)
^(7/2))/(10*e^2) + (33*d^10*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^2)
```

Rubi [A] time = 0.121367, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {795, 671, 641, 195, 217, 203}

$$\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (33*d^8*x*Sqrt[d^2 - e^2*x^2])/(256*e) + (11*d^6*x*(d^2 - e^2*x^2)^(3/2))/(
128*e) + (11*d^4*x*(d^2 - e^2*x^2)^(5/2))/(160*e) - (33*d^3*(d^2 - e^2*x^2)
^(7/2))/(560*e^2) - (11*d^2*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(240*e^2) - (d
*(d + e*x)^2*(d^2 - e^2*x^2)^(7/2))/(30*e^2) - ((d + e*x)^3*(d^2 - e^2*x^2)
^(7/2))/(10*e^2) + (33*d^10*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^2)
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(3d)\int(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx}{10e}$$

$$=-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{\left(11d^2\right)\int(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}{30e}$$

$$=-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{(33}{}$$

$$=-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex}{}$$

$$=\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d\right.}{30e}$$

$$=\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e\right.}{240e^2}$$

$$=\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-$$

$$=\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-$$

$$=\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-$$

Mathematica [A] time = 0.41101, size = 167, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(10240d^7e^2x^2+24570d^6e^3x^3+7680d^5e^4x^4-23352d^4e^5x^5-20480d^3e^6x^6+3024d^2e^7x^7-3465d^8\right.\right.}{26880e^2\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-6400*d^9 - 3465*d^8*e*x + 1
0240*d^7*e^2*x^2 + 24570*d^6*e^3*x^3 + 7680*d^5*e^4*x^4 - 23352*d^4*e^5*x^5
 - 20480*d^3*e^6*x^6 + 3024*d^2*e^7*x^7 + 8960*d*e^8*x^8 + 2688*e^9*x^9) +
3465*d^9*ArcSin[(e*x)/d]))/(26880*e^2*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.067, size = 191, normalized size = 0.8

$$-\frac{ex^{3}}{10}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{7}{2}}-\frac{33\,d^{2}x}{80\,e}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{7}{2}}+\frac{11\,d^{4}x}{160\,e}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}+\frac{11\,d^{6}x}{128\,e}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}+\frac{33\,d^{8}x}{256\,e}\sqrt{-x^{2}e^{2}+d^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/10*e*x^3*(-e^2*x^2+d^2)^(7/2)-33/80/e*d^2*x*(-e^2*x^2+d^2)^(7/2)+11/160*
d^4*x*(-e^2*x^2+d^2)^(5/2)/e+11/128*d^6*x*(-e^2*x^2+d^2)^(3/2)/e+33/256*d^8
*x*(-e^2*x^2+d^2)^(1/2)/e+33/256/e*d^10/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2))-1/3*d*x^2*(-e^2*x^2+d^2)^(7/2)-5/21*d^3*(-e^2*x^2+d^2)^
(7/2)/e^2
```

Maxima [A] time = 1.49361, size = 247, normalized size = 1.07

$$\frac{33\,d^{10}\arcsin\left(\frac{e^{2}x}{\sqrt{d^{2}e^{2}}}\right)}{256\,\sqrt{e^{2}}e}+\frac{33\,\sqrt{-e^{2}x^{2}+d^{2}}d^{8}x}{256\,e}+\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}d^{6}x}{128\,e}-\frac{1}{10}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}ex^{3}+\frac{11\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}x}{160\,e}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 33/256*d^10*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e) + 33/256*sqrt(-e^2*x^
2 + d^2)*d^8*x/e + 11/128*(-e^2*x^2 + d^2)^(3/2)*d^6*x/e - 1/10*(-e^2*x^2 +
 d^2)^(7/2)*e*x^3 + 11/160*(-e^2*x^2 + d^2)^(5/2)*d^4*x/e - 1/3*(-e^2*x^2 +
 d^2)^(7/2)*d*x^2 - 33/80*(-e^2*x^2 + d^2)^(7/2)*d^2*x/e - 5/21*(-e^2*x^2 +
 d^2)^(7/2)*d^3/e^2
```

Fricas [A] time = 1.86996, size = 360, normalized size = 1.57

$$-\frac{6930\,d^{10}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(2688\,e^{9}x^{9}+8960\,de^{8}x^{8}+3024\,d^{2}e^{7}x^{7}-20480\,d^{3}e^{6}x^{6}-23352\,d^{4}e^{5}x^{5}+7680\,d^{5}e\right.}{26880\,e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/26880*(6930*d^10*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2688*e^9*x
^9 + 8960*d*e^8*x^8 + 3024*d^2*e^7*x^7 - 20480*d^3*e^6*x^6 - 23352*d^4*e^5*
x^5 + 7680*d^5*e^4*x^4 + 24570*d^6*e^3*x^3 + 10240*d^7*e^2*x^2 - 3465*d^8*e
*x - 6400*d^9)*sqrt(-e^2*x^2 + d^2))/e^2
```

Sympy [A] time = 53.3296, size = 1561, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + 3*d**6*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*
d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs
(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2
/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e
**2*x**2/d**2)), True)) + d**5*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**
2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2
- e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 5*d**4*e**3*Piecewi
se((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/
d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*
sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) +
5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/
d**2)), True)) - 5*d**3*e**4*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105
*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**
2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*
sqrt(d**2)/6, True)) + d**2*e**5*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**
7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e
**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/
d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/d)/(
128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384
*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**
2*x**2/d**2)), True)) + 3*d*e**6*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)
/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*s
qrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**
2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) +
 e**7*Piecewise((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*
sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d*
*2)) - 7*I*d**5*x**5/(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(4
80*e**2*sqrt(-1 + e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**
2)) + I*e**2*x**11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (7*d**10*asin(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2
*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**
5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*
x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqr
t(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.14265, size = 173, normalized size = 0.75

$$\frac{33}{256}\, d^{10} \arcsin\left(\frac{xe}{d}\right) e^{(-2)} \mathrm{sgn}\left(d\right) - \frac{1}{26880}\left(6400\, d^{9} e^{(-2)} + \left(3465\, d^{8} e^{(-1)} - 2\left(5120\, d^{7} + \left(12285\, d^{6} e + 4\left(960\, d^{5} e^{2} - \left(2919\, d\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 33/256*d^10*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/26880*(6400*d^9*e^(-2) + (3465*
d^8*e^(-1) - 2*(5120*d^7 + (12285*d^6*e + 4*(960*d^5*e^2 - (2919*d^4*e^3 +
2*(1280*d^3*e^4 - 7*(27*d^2*e^5 + 8*(3*x*e^7 + 10*d*e^6)*x)*x)*x)*x)*x)*x)*
x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.70 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=188

$$\frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-$$

```
[Out] (55*d^7*x*Sqrt[d^2 - e^2*x^2])/128 + (55*d^5*x*(d^2 - e^2*x^2)^(3/2))/192 +
 (11*d^3*x*(d^2 - e^2*x^2)^(5/2))/48 - (11*d^2*(d^2 - e^2*x^2)^(7/2))/(56*e
) - (11*d*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(72*e) - ((d + e*x)^2*(d^2 - e^2
*x^2)^(7/2))/(9*e) + (55*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e)
```

Rubi [A] time = 0.082442, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {671, 641, 195, 217, 203}

$$\frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (55*d^7*x*Sqrt[d^2 - e^2*x^2])/128 + (55*d^5*x*(d^2 - e^2*x^2)^(3/2))/192 +
 (11*d^3*x*(d^2 - e^2*x^2)^(5/2))/48 - (11*d^2*(d^2 - e^2*x^2)^(7/2))/(56*e
) - (11*d*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(72*e) - ((d + e*x)^2*(d^2 - e^2
*x^2)^(7/2))/(9*e) + (55*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e)
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{1}{9}(11d)\int (d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx$$
$$= -\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{1}{8}\left(11d^2\right)\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}$$
$$= -\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{1}{8}\left(11d^3\right)\int$$
$$= \frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e\right.}{9e}$$
$$= \frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2\right.}{72e}$$
$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-$$
$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-$$
$$= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-$$

Mathematica [A] time = 0.350777, size = 156, normalized size = 0.83

$$\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(10240d^6e^2x^2+3066d^5e^3x^3-8448d^4e^4x^4-7224d^3e^5x^5+1024d^2e^6x^6+4599d^7ex-3712d^8+302\right.\right.}{8064e\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(-3712*d^8 + 4599*d^7*e*x + 1
0240*d^6*e^2*x^2 + 3066*d^5*e^3*x^3 - 8448*d^4*e^4*x^4 - 7224*d^3*e^5*x^5 +
 1024*d^2*e^6*x^6 + 3024*d*e^7*x^7 + 896*e^8*x^8) + 3465*d^8*ArcSin[(e*x)/d
]))/(8064*e*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.06, size = 154, normalized size = 0.8

$$-\frac{ex^2}{9}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{29\,d^2}{63\,e}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{3\,dx}{8}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}+\frac{11\,d^3x}{48}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{55\,d^5x}{192}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{55\,d^7}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/9*e*x^2*(-e^2*x^2+d^2)^(7/2)-29/63*d^2*(-e^2*x^2+d^2)^(7/2)/e-3/8*d*x*(-
e^2*x^2+d^2)^(7/2)+11/48*d^3*x*(-e^2*x^2+d^2)^(5/2)+55/192*d^5*x*(-e^2*x^2+
d^2)^(3/2)+55/128*d^7*x*(-e^2*x^2+d^2)^(1/2)+55/128*d^9/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A] time = 1.4854, size = 197, normalized size = 1.05

$$\frac{55\,d^9 \arcsin\left(\frac{e^2 x}{\sqrt{d^2e^2}}\right)}{128\,\sqrt{e^2}} + \frac{55}{128}\,\sqrt{-e^2x^2 + d^2}d^7x + \frac{55}{192}\left(-e^2x^2 + d^2\right)^{\frac{3}{2}}d^5x + \frac{11}{48}\left(-e^2x^2 + d^2\right)^{\frac{5}{2}}d^3x - \frac{1}{9}\left(-e^2x^2 + d^2\right)^{\frac{7}{2}}ex^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 55/128*d^9*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2) + 55/128*sqrt(-e^2*x^2 + d
^2)*d^7*x + 55/192*(-e^2*x^2 + d^2)^(3/2)*d^5*x + 11/48*(-e^2*x^2 + d^2)^(5
/2)*d^3*x - 1/9*(-e^2*x^2 + d^2)^(7/2)*e*x^2 - 3/8*(-e^2*x^2 + d^2)^(7/2)*d
*x - 29/63*(-e^2*x^2 + d^2)^(7/2)*d^2/e
```

Fricas [A] time = 2.01678, size = 324, normalized size = 1.72

$$-\frac{6930\,d^9 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(896\,e^8x^8 + 3024\,de^7x^7 + 1024\,d^2e^6x^6 - 7224\,d^3e^5x^5 - 8448\,d^4e^4x^4 + 3066\,d^5e^3x^3 -\right.}{8064\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(6930*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (896*e^8*x^8
+ 3024*d*e^7*x^7 + 1024*d^2*e^6*x^6 - 7224*d^3*e^5*x^5 - 8448*d^4*e^4*x^4 +
 3066*d^5*e^3*x^3 + 10240*d^6*e^2*x^2 + 4599*d^7*e*x - 3712*d^8)*sqrt(-e^2*
x^2 + d^2))/e
```

Sympy [C] time = 40.9155, size = 1290, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) +
 3*d**6*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)*
*(3/2)/(3*e**2), True)) + d**5*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3
) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e
**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**
2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt
```

```
(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((-2*d**4*sqrt(d**2 -
e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sq
rt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 5*d**3*e**4
*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e*
*2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x*
*5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d*
*2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(
16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d
**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e*
*2*x**2/d**2)), True)) + d**2*e**5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2
)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sq
rt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)),
(x**6*sqrt(d**2)/6, True)) + 3*d*e**6*Piecewise((-5*I*d**8*acosh(e*x/d)/(12
8*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(
384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*
x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*
sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x
/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3
/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x*
*2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1
- e**2*x**2/d**2)), True)) + e**7*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2
)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*
sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e*
*2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [A] time = 1.17023, size = 158, normalized size = 0.84

$$\frac{55}{128}\,d^9\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\,(d)-\frac{1}{8064}\left(3712\,d^8e^{(-1)}-\left(4599\,d^7+2\left(5120\,d^6e+\left(1533\,d^5e^2-4\left(1056\,d^4e^3+\left(903\,d^3e^4-\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 55/128*d^9*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/8064*(3712*d^8*e^(-1) - (4599*d^
7 + 2*(5120*d^6*e + (1533*d^5*e^2 - 4*(1056*d^4*e^3 + (903*d^3*e^4 - 2*(64*
d^2*e^5 + 7*(8*x*e^7 + 27*d*e^6)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.71 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=190

$$\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-\right.$$

```
[Out] (d^6*(128*d + 125*e*x)*Sqrt[d^2 - e^2*x^2])/128 + (d^4*(64*d + 125*e*x)*(d^
2 - e^2*x^2)^(3/2))/192 + (d^2*(48*d + 125*e*x)*(d^2 - e^2*x^2)^(5/2))/240
- (3*d*(d^2 - e^2*x^2)^(7/2))/7 - (e*x*(d^2 - e^2*x^2)^(7/2))/8 + (125*d^8*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/128 - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.30707, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1809, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (d^6*(128*d + 125*e*x)*Sqrt[d^2 - e^2*x^2])/128 + (d^4*(64*d + 125*e*x)*(d^
2 - e^2*x^2)^(3/2))/192 + (d^2*(48*d + 125*e*x)*(d^2 - e^2*x^2)^(5/2))/240
- (3*d*(d^2 - e^2*x^2)^(7/2))/7 - (e*x*(d^2 - e^2*x^2)^(7/2))/8 + (125*d^8*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/128 - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}dx = -\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-8d^3e^2-25d^2e^3x-24de^4x^2\right)}{x}dx}{8e^2}$$

$$= -\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} + \frac{\int \frac{\left(56d^3e^4+175d^2e^5x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}dx}{56e^4}$$

$$= \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2} - \frac{\int \frac{(-336d}{}}{}$$

$$= \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{3}{7}d\left(d^2-e^2\right.$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{240}d^2(48d+$$

Mathematica [A] time = 0.380262, size = 168, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(7424d^5e^2x^2-17710d^4e^3x^3-14592d^3e^4x^4+1960d^2e^5x^5+27195d^6ex+14848d^7+5760de^6x^6+1680e^7x^7\right.}{13440}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(14848*d^7 + 27195*d^6*e*x + 7424*d^5*e^2*x^2 - 17710*
d^4*e^3*x^3 - 14592*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 + 5760*d*e^6*x^6 + 1680*
e^7*x^7))/13440 + (125*d^7*Sqrt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(128*Sqrt[1
 - (e^2*x^2)/d^2]) - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Maple [A] time = 0.056, size = 231, normalized size = 1.2

$$-\frac{ex}{8}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + \frac{25\,d^2ex}{48}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} + \frac{125\,ed^4x}{192}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} + \frac{125\,ed^6x}{128}\sqrt{-x^2e^2+d^2} + \frac{125\,ed^8}{128}\arctan\left(x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x)
```

```
[Out] -1/8*e*x*(-e^2*x^2+d^2)^(7/2)+25/48*d^2*e*x*(-e^2*x^2+d^2)^(5/2)+125/192*e*
d^4*x*(-e^2*x^2+d^2)^(3/2)+125/128*e*d^6*x*(-e^2*x^2+d^2)^(1/2)+125/128*e*d
^8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3/7*d*(-e^2*x^2+d
```

```
^2)^(7/2)+1/5*d^3*(-e^2*x^2+d^2)^(5/2)+1/3*d^5*(-e^2*x^2+d^2)^(3/2)+d^7*(-e
^2*x^2+d^2)^(1/2)-d^9/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98322, size = 358, normalized size = 1.88

$$-\frac{125}{64}\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{13440}\left(1680\,e^7x^7+5760\,de^6x^6+1960\,d^2e^5x^5-14\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -125/64*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^8*log(-(d - sqrt(
-e^2*x^2 + d^2))/x) + 1/13440*(1680*e^7*x^7 + 5760*d*e^6*x^6 + 1960*d^2*e^5
*x^5 - 14592*d^3*e^4*x^4 - 17710*d^4*e^3*x^3 + 7424*d^5*e^2*x^2 + 27195*d^6
*e*x + 14848*d^7)*sqrt(-e^2*x^2 + d^2)
```

Sympy [C] time = 55.4003, size = 1273, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x,x)
```

```
[Out] d**7*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d
**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**
2/(e**2*x**2) + 1), True)) + 3*d**6*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e)
 - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x*
*2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqr
t(1 - e**2*x**2/d**2)/2, True)) + d**5*e**2*Piecewise((x**2*sqrt(d**2)/2, E
q(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) - 5*d**4*e**3*Pie
cewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**
2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt
(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8
*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e*
*2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*
e**4*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
```

```
4*sqrt(d**2)/4, True)) + d**2*e**5*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5
) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqr
t(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**
6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*
x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d
**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewi
se((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**
2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt
(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + e**7*Piecewis
e((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*
x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs
(d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e
**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x*
*5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d*
*2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A] time = 1.14547, size = 193, normalized size = 1.02

$$\frac{125}{128}\,d^8 \arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d) - d^8\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2\,|x|}\right) + \frac{1}{13440}\left(14848\,d^7 + \left(27195\,d^6e + 2\left(3712\,d^5e^2 -\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 125/128*d^8*arcsin(x*e/d)*sgn(d) - d^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/13440*(14848*d^7 + (27195*d^6*e + 2*(3712*d^5
*e^2 - (8855*d^4*e^3 + 4*(1824*d^3*e^4 - 5*(49*d^2*e^5 + 6*(7*x*e^7 + 24*d*
e^6)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.72 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=193

$$\frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2\right.$$

```
[Out] (3*d^5*e*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^3*e*(8*d - 5*e*x)*(d^2
 - e^2*x^2)^(3/2))/8 + (d*e*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/10 - (e*(d
^2 - e^2*x^2)^(7/2))/7 - (d*(d^2 - e^2*x^2)^(7/2))/x - (15*d^7*e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/16 - 3*d^7*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.305454, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 1809, 815, 844, 217, 203, 266, 63, 208}

$$\frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```


```
[Out] (3*d^5*e*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^3*e*(8*d - 5*e*x)*(d^2
 - e^2*x^2)^(3/2))/8 + (d*e*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/10 - (e*(d
^2 - e^2*x^2)^(7/2))/7 - (d*(d^2 - e^2*x^2)^(7/2))/x - (15*d^7*e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/16 - 3*d^7*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
```

```
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2}\,dx = -\frac{d(d^2-e^2x^2)^{7/2}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-3d^4e+3d^3e^2x-d^2e^3x^2)}{x}\,dx}{d^2}$$

$$= -\frac{1}{7}e(d^2-e^2x^2)^{7/2} - \frac{d(d^2-e^2x^2)^{7/2}}{x} + \frac{\int \frac{(21d^4e^3-21d^3e^4x)(d^2-e^2x^2)^{5/2}}{x}\,dx}{7d^2e^2}$$

$$= \frac{1}{10}de(6d-5ex)(d^2-e^2x^2)^{5/2} - \frac{1}{7}e(d^2-e^2x^2)^{7/2} - \frac{d(d^2-e^2x^2)^{7/2}}{x} - \frac{\int \frac{(-126d^6e^5+105d^5e^6x}{x}}{42d^2e^4}$$

$$= \frac{1}{8}d^3e(8d-5ex)(d^2-e^2x^2)^{3/2} + \frac{1}{10}de(6d-5ex)(d^2-e^2x^2)^{5/2} - \frac{1}{7}e(d^2-e^2x^2)^{7/2} - \frac{d(d^2}{}$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)(d^2-e^2x^2)^{3/2} + \frac{1}{10}de(6d-5ex)(d^2-e^2x^2$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)(d^2-e^2x^2)^{3/2} + \frac{1}{10}de(6d-5ex)(d^2-e^2x^2$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)(d^2-e^2x^2)^{3/2} + \frac{1}{10}de(6d-5ex)(d^2-e^2x^2$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)(d^2-e^2x^2)^{3/2} + \frac{1}{10}de(6d-5ex)(d^2-e^2x^2$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)(d^2-e^2x^2)^{3/2} + \frac{1}{10}de(6d-5ex)(d^2-e^2x^2$$

Mathematica [C] time = 0.576062, size = 221, normalized size = 1.15

$$-\frac{d^7\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{1}{560}e\sqrt{d^2-e^2x^2}\left(-992d^4e^2x^2-910d^3e^3x^3+96d^2e^4x^4+1155d^5ex+2496d^6+280\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(2496*d^6 + 1155*d^5*e*x - 992*d^4*e^2*x^2 - 910*d^3
*e^3*x^3 + 96*d^2*e^4*x^4 + 280*d*e^5*x^5 + 80*e^6*x^6))/560 + (15*d^6*e*Sq
rt[d^2 - e^2*x^2]*ArcSin[(e*x)/d])/(16*Sqrt[1 - (e^2*x^2)/d^2]) - 3*d^7*e*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d] - (d^7*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[
-5/2, -1/2, 1/2, (e^2*x^2)/d^2])/(x*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.061, size = 243, normalized size = 1.3

$$-\frac{e}{7}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{de^2x}{2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{5\,d^3e^2x}{8}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} - \frac{15\,d^5e^2x}{16}\sqrt{-x^2e^2+d^2} - \frac{15\,d^7e^2}{16}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x)
```

```
[Out] -1/7*e*(-e^2*x^2+d^2)^(7/2)-1/2*d*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/8*d^3*e^2*x*
(-e^2*x^2+d^2)^(3/2)-15/16*d^5*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/16*d^7*e^2/(e^
2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+3/5*d^2*e*(-e^2*x^2+d^2
)^(5/2)+d^4*e*(-e^2*x^2+d^2)^(3/2)+3*d^6*e*(-e^2*x^2+d^2)^(1/2)-3*d^8*e/(d^
2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-d*(-e^2*x^2+d^2)^
(7/2)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9589, size = 377, normalized size = 1.95

$$\frac{1050\,d^7 ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+1680\,d^7 ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+2496\,d^7 ex+\left(80\,e^7x^7+280\,de^6x^6+96\,d^2e^5x^5-77\right.}{560\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/560*(1050*d^7*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1680*d^7*e*
x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2496*d^7*e*x + (80*e^7*x^7 + 280*d*e
^6*x^6 + 96*d^2*e^5*x^5 - 770*d^3*e^4*x^4 - 992*d^4*e^3*x^3 + 525*d^5*e^2*x
^2 + 2496*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/x
```

Sympy [C] time = 22.3394, size = 1068, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**2,x)
```

```
[Out] d**7*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*s
qrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**
2)), True)) + 3*d**6*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d
*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(
x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x))
 + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + d**5*e**2*Piecewise((-I*d**2
*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*
d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x
/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - 5*d**4*e**3*Piecewise(
(x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), Tru
e)) - 5*d**3*e**4*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e*
```

```
*2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) +
I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1),
 (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*
d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**
2)), True)) + d**2*e**5*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4)
 - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)
/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + 3*d*e**6*Piecewise((-I*d**6*aco
sh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3
*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x
**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Ab
s(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x
**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sq
rt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
+ e**7*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*s
qrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**
2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

<u>Giac</u> [A] time = 1.17445, size = 269, normalized size = 1.39

$$-\frac{15}{16}d^7\arcsin\left(\frac{xe}{d}\right)\mathrm{esgn}\left(d\right)-3\,d^7e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)+\frac{d^7xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*d^7*arcsin(x*e/d)*e*sgn(d) - 3*d^7*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^7*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*
e) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*e^(-1)/x + 1/560*(2496*d^6*e +
(525*d^5*e^2 - 2*(496*d^4*e^3 + (385*d^3*e^4 - 4*(12*d^2*e^5 + 5*(2*x*e^7 +
 7*d*e^6)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.73 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=207

$$\frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{30}e^2(3d-8$$

```
[Out] (d^4*e^2*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^2*e^2*(4*d - 85*e*x)*(
d^2 - e^2*x^2)^(3/2))/24 + (e^2*(3*d - 85*e*x)*(d^2 - e^2*x^2)^(5/2))/30 -
(d*(d^2 - e^2*x^2)^(7/2))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(7/2))/x - (85*d^6
*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/16 - (d^6*e^2*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/2
```

Rubi [A] time = 0.312916, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{30}e^2(3d-8$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```


```
[Out] (d^4*e^2*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^2*e^2*(4*d - 85*e*x)*(
d^2 - e^2*x^2)^(3/2))/24 + (e^2*(3*d - 85*e*x)*(d^2 - e^2*x^2)^(5/2))/30 -
(d*(d^2 - e^2*x^2)^(7/2))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(7/2))/x - (85*d^6
*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/16 - (d^6*e^2*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/2
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-6d^4e-d^3e^2x-2d^2e^3x^2\right)}{x^2}\,dx}{2d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x} + \frac{\int \frac{\left(d^5e^2-34d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx}{2d^4}$$

$$= \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x} - \frac{\int \frac{\left(-6d^7e^4+170d^6\right.}{}}{12}$$

$$= \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(\right.$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(\right.$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(\right.$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(\right.$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(\right.$$

Mathematica [C] time = 0.686788, size = 259, normalized size = 1.25

$$-\frac{e\left(5040d^9\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)+ex\left(240\left(d^2-e^2x^2\right)^4\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)-7d\left(-1632d^5e^2x^2-295d^4e^3x^3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] -(e*(5040*d^9*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (e
^2*x^2)/d^2] + e*x*(-7*d*(1104*d^7 + 165*d^6*e*x - 1632*d^5*e^2*x^2 - 295*d
^4*e^3*x^3 + 672*d^3*e^4*x^4 + 170*d^2*e^5*x^5 - 144*d*e^6*x^6 - 40*e^7*x^7
 + 75*d^7*Sqrt[1 - (e^2*x^2)/d^2]*ArcSin[(e*x)/d] - 720*d^6*Sqrt[d^2 - e^2*
x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]) + 240*(d^2 - e^2*x^2)^4*Hypergeometric
2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2])))/(1680*d*x*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.063, size = 252, normalized size = 1.2

$$-\frac{17\,e^3x}{6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{85\,e^3d^2x}{24}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{85\,e^3d^4x}{16}\sqrt{-x^2e^2+d^2}-\frac{85\,e^3d^6}{16}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x)
```

```
[Out] -17/6*e^3*x*(-e^2*x^2+d^2)^(5/2)-85/24*e^3*d^2*x*(-e^2*x^2+d^2)^(3/2)-85/16
*e^3*d^4*x*(-e^2*x^2+d^2)^(1/2)-85/16*e^3*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-e^2*x^2+d^2)^(1/2))+1/10*d*e^2*(-e^2*x^2+d^2)^(5/2)+1/6*d^3*e^2*(-e^2
*x^2+d^2)^(3/2)+1/2*d^5*e^2*(-e^2*x^2+d^2)^(1/2)-1/2*d^7*e^2/(d^2)^(1/2)*ln
((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/2*d*(-e^2*x^2+d^2)^(7/2)/x
^2-3*e*(-e^2*x^2+d^2)^(7/2)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02622, size = 392, normalized size = 1.89

$$\frac{2550\, d^6 e^2 x^2 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 120\, d^6 e^2 x^2 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 544\, d^6 e^2 x^2 + \left(40\, e^7 x^7 + 144\, d e^6 x^6 + 50\, d^2 e^5 x^5 - 4 \text{[illegible]}\right.}{240\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/240*(2550*d^6*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 120*d^6
*e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 544*d^6*e^2*x^2 + (40*e^7*x^7
 + 144*d*e^6*x^6 + 50*d^2*e^5*x^5 - 448*d^3*e^4*x^4 - 645*d^4*e^3*x^3 + 544
*d^5*e^2*x^2 - 720*d^6*e*x - 120*d^7)*sqrt(-e^2*x^2 + d^2))/x^2
```

Sympy [C] time = 25.1735, size = 1073, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**3,x)
```

```
[Out] d**7*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs
(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x)
)/(2*d), True)) + 3*d**6*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I
*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Ab
s(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*
sqrt(1 - e**2*x**2/d**2)), True)) + d**5*e**2*Piecewise((d**2/(e*x*sqrt(d**
2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Ab
s(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) +
1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**
4*e**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
```

```
**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
 - 5*d**3*e**4*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x
**2)**(3/2)/(3*e**2), True)) + d**2*e**5*Piecewise((-I*d**4*acosh(e*x/d)/(8
*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-
1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**
2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1
- e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d
*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise((-2*d**4*sqrt(d**2
- e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*
sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + e**7*Piec
ewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x*
*2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(2
4*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e*
*4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2))
 + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x*
*2/d**2)), True))
```

Giac [A] time = 1.18552, size = 354, normalized size = 1.71

$$-\frac{85}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^2\mathrm{sgn}\left(d\right)-\frac{1}{2}\,d^6e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)-\frac{1}{8}\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}e\right)d^6e^8}{x}+\frac{\left(de+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -85/16*d^6*arcsin(x*e/d)*e^2*sgn(d) - 1/2*d^6*e^2*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(12*(d*e + sqrt(-x^2*e^2 + d^2)*
e)*d^6*e^8/x + (d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*e^6/x^2)*e^(-8) + 1/240
*(544*d^5*e^2 - (645*d^4*e^3 + 2*(224*d^3*e^4 - (25*d^2*e^5 + 4*(5*x*e^7 +
18*d*e^6)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2) + 1/8*(d^6*e^6 + 12*(d*e + sqrt(
-x^2*e^2 + d^2)*e)*d^6*e^4/x)*x^2/(d*e + sqrt(-x^2*e^2 + d^2)*e)^2
```

3.74 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=210

$$-\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}-\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{e^2(50d+39e$$

```
[Out] -(d^3*e^3*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/8 - (d*e^3*(26*d + 25*e*x)*(
d^2 - e^2*x^2)^(3/2))/12 - (e^2*(50*d + 39*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*
x) - (d*(d^2 - e^2*x^2)^(7/2))/(3*x^3) - (3*e*(d^2 - e^2*x^2)^(7/2))/(2*x^2
) - (25*d^5*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (13*d^5*e^3*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/2
```

Rubi [A] time = 0.313565, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}-\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{e^2(50d+39e$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] -(d^3*e^3*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/8 - (d*e^3*(26*d + 25*e*x)*(
d^2 - e^2*x^2)^(3/2))/12 - (e^2*(50*d + 39*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*
x) - (d*(d^2 - e^2*x^2)^(7/2))/(3*x^3) - (3*e*(d^2 - e^2*x^2)^(7/2))/(2*x^2
) - (25*d^5*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (13*d^5*e^3*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^4} dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-9d^4e-5d^3e^2x-3d^2e^3x^2\right)}{x^3} dx}{3d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} + \frac{\int \frac{\left(10d^5e^2-39d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^2} dx}{6d^4}$$

$$= -\frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2} - \frac{\int \frac{\left(78d^6e^3+100d^5e^4x\right)}{x}}{12d^4}$$

$$= -\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3} -$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2\right.}{30x}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2\right.}{30x}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2\right.}{30x}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2\right.}{30x}$$

$$= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)\left(d^2\right.}{30x}$$

Mathematica [C] time = 0.293106, size = 251, normalized size = 1.2

$$-\frac{d^7\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{3d^5e^2\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{3e^3\left(d^2-e^2x^2\right)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] (e^3*(Sqrt[d^2 - e^2*x^2]*(23*d^4 - 11*d^2*e^2*x^2 + 3*e^4*x^4) - 15*d^5*Ar
cTanh[Sqrt[d^2 - e^2*x^2]/d]))/15 - (d^7*Sqrt[d^2 - e^2*x^2]*Hypergeometric
2F1[-5/2, -3/2, -1/2, (e^2*x^2)/d^2])/(3*x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (3*
d^5*e^2*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (e^2*x^2)/d^
2])/(x*Sqrt[1 - (e^2*x^2)/d^2]) - (3*e^3*(d^2 - e^2*x^2)^(7/2)*Hypergeometr
ic2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2])/(7*d^2)
```

Maple [A] time = 0.066, size = 277, normalized size = 1.3

$$-\frac{13\,e^3}{10}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{13\,e^3d^2}{6}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} - \frac{13\,e^3d^4}{2}\sqrt{-x^2e^2+d^2} + \frac{13\,e^3d^6}{2}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x)
```

```
[Out] -13/10*e^3*(-e^2*x^2+d^2)^(5/2)-13/6*e^3*d^2*(-e^2*x^2+d^2)^(3/2)-13/2*e^3*
d^4*(-e^2*x^2+d^2)^(1/2)+13/2*e^3*d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)-1/3*d*(-e^2*x^2+d^2)^(7/2)/x^3-5/3/d*e^2/x*(-e^2*x^
2+d^2)^(7/2)-5/3/d*e^4*x*(-e^2*x^2+d^2)^(5/2)-25/12*d*e^4*x*(-e^2*x^2+d^2)^
(3/2)-25/8*d^3*e^4*x*(-e^2*x^2+d^2)^(1/2)-25/8*d^5*e^4/(e^2)^(1/2)*arctan((
e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3/2*e*(-e^2*x^2+d^2)^(7/2)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99381, size = 386, normalized size = 1.84

$$\frac{750\,d^5e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-780\,d^5e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-656\,d^5e^3x^3+\left(24\,e^7x^7+90\,de^6x^6+32\,d^2e^5x^5-3\right.}{120\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/120*(750*d^5*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 780*d^5*
e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 656*d^5*e^3*x^3 + (24*e^7*x^7
+ 90*d*e^6*x^6 + 32*d^2*e^5*x^5 - 345*d^3*e^4*x^4 - 656*d^4*e^3*x^3 - 80*d^
5*e^2*x^2 - 180*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^3
```

Sympy [C] time = 19.0665, size = 926, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**4,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d
**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2)
, True)) + 3*d**6*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1))
+ e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)
/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e*
*2*asin(d/(e*x))/(2*d), True)) + d**5*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2
*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), A
bs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x
/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((
d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e
**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d
```

```
**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) +
1), True)) - 5*d**3*e**4*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x*
*2/d**2)/2, True)) + d**2*e**5*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)),
(-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + 3*d*e**6*Piecewise((-I*d**4*
acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d
*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/
d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/
(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) -
 e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((-2*d**4
*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e*
*2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

Giac [A] time = 1.15548, size = 429, normalized size = 2.04

$$-\frac{25}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^3\mathrm{sgn}\left(d\right)+\frac{13}{2}\,d^5e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(d^5e^8+\frac{9\left(de+\sqrt{-x^2e^2+d^2}e\right)d^5e^6}{x}+\frac{9\left(de+\sqrt{-x^2e^2+d\right.}{x^2}\right.}{24\left(de+\sqrt{-x^2e^2+d^2}e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] -25/8*d^5*arcsin(x*e/d)*e^3*sgn(d) + 13/2*d^5*e^3*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/24*(d^5*e^8 + 9*(d*e + sqrt(-x^2*e
^2 + d^2)*e)*d^5*e^6/x + 9*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*e^4/x^2)*x^
3*e/(d*e + sqrt(-x^2*e^2 + d^2)*e)^3 - 1/24*(9*(d*e + sqrt(-x^2*e^2 + d^2)*
e)*d^5*e^16/x + 9*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*e^14/x^2 + (d*e + sq
rt(-x^2*e^2 + d^2)*e)^3*d^5*e^12/x^3)*e^(-15) - 1/120*(656*d^4*e^3 + (345*d
^3*e^4 - 2*(16*d^2*e^5 + 3*(4*x*e^7 + 15*d*e^6)*x)*x)*x)*sqrt(-x^2*e^2 + d^
2)
```

3.75 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x}-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3}-\frac{d\left(d^2-\right.}{4}$$

```
[Out] (-45*d^2*e^4*(d - e*x)*Sqrt[d^2 - e^2*x^2])/8 + (15*d*e^3*(2*d - e*x)*(d^2
- e^2*x^2)^(3/2))/(8*x) - (3*e^2*(3*d + 2*e*x)*(d^2 - e^2*x^2)^(5/2))/(8*x^
2) - (d*(d^2 - e^2*x^2)^(7/2))/(4*x^4) - (e*(d^2 - e^2*x^2)^(7/2))/x^3 + (4
5*d^4*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (45*d^4*e^4*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d])/8
```

Rubi [A] time = 0.319215, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x}-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3}-\frac{d\left(d^2-\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```


```
[Out] (-45*d^2*e^4*(d - e*x)*Sqrt[d^2 - e^2*x^2])/8 + (15*d*e^3*(2*d - e*x)*(d^2
- e^2*x^2)^(3/2))/(8*x) - (3*e^2*(3*d + 2*e*x)*(d^2 - e^2*x^2)^(5/2))/(8*x^
2) - (d*(d^2 - e^2*x^2)^(7/2))/(4*x^4) - (e*(d^2 - e^2*x^2)^(7/2))/x^3 + (4
5*d^4*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (45*d^4*e^4*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d])/8
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-12d^4e-9d^3e^2x-4d^2e^3x^2\right)}{x^4}dx}{4d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} + \frac{\int \frac{\left(27d^5e^2-36d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^3}dx}{12d^4}$$

$$= -\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3} - \frac{5\int \frac{\left(144d^6e^3+216d^5\right.}{x}}{192}$$

$$= \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{e\left(d^2\right.}{}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x\right.}{8x^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x\right.}{8x^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x\right.}{8x^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x\right.}{8x^2}$$

$$= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x\right.}{8x^2}$$

Mathematica [C] time = 0.10926, size = 195, normalized size = 0.93

$$\frac{e\sqrt{d^2-e^2x^2}\left(-\frac{7d^9\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{7d^7e^2\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x\sqrt{1-\frac{e^2x^2}{d^2}}} + 3\left(e^3x^2-d^2e\right)^3\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right) + \left(e^3x^2-d^2e\right)^3\,{}_2F_1\left(\right.\right.}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*((-7*d^9*Hypergeometric2F1[-5/2, -3/2, -1/2, (e^2*x^
2)/d^2])/(x^3*Sqrt[1 - (e^2*x^2)/d^2]) - (7*d^7*e^2*Hypergeometric2F1[-5/2,
 -1/2, 1/2, (e^2*x^2)/d^2])/(x*Sqrt[1 - (e^2*x^2)/d^2]) + 3*(-(d^2*e) + e^3
*x^2)^3*Hypergeometric2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2] + (-(d^2*e) + e^3
*x^2)^3*Hypergeometric2F1[3, 7/2, 9/2, 1 - (e^2*x^2)/d^2]))/(7*d^3)
```

Maple [A] time = 0.074, size = 302, normalized size = 1.4

$$-\frac{e}{x^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + 3\frac{e^3\left(-x^2e^2+d^2\right)^{7/2}}{d^2x} + 3\frac{e^5x\left(-x^2e^2+d^2\right)^{5/2}}{d^2} + \frac{15\,e^5x}{4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} + \frac{45\,d^2e^5x}{8}\sqrt{-x^2e^2+d^2} + \frac{4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x)
```

```
[Out] -e*(-e^2*x^2+d^2)^(7/2)/x^3+3/d^2*e^3/x*(-e^2*x^2+d^2)^(7/2)+3/d^2*e^5*x*(-
e^2*x^2+d^2)^(5/2)+15/4*e^5*x*(-e^2*x^2+d^2)^(3/2)+45/8*d^2*e^5*x*(-e^2*x^2
+d^2)^(1/2)+45/8*d^4*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1
/2))-1/4*d*(-e^2*x^2+d^2)^(7/2)/x^4-9/8/d*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-9/8/
d*e^4*(-e^2*x^2+d^2)^(5/2)-15/8*d*e^4*(-e^2*x^2+d^2)^(3/2)-45/8*d^3*e^4*(-e
^2*x^2+d^2)^(1/2)+45/8*d^5*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^
2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8945, size = 369, normalized size = 1.77

$$-\frac{90\,d^4e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+45\,d^4e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+48\,d^4e^4x^4-\left(2\,e^7x^7+8\,de^6x^6+3\,d^2e^5x^5-48\,d^3e^4x^4}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(90*d^4*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 45*d^4*e^4
*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 48*d^4*e^4*x^4 - (2*e^7*x^7 + 8*d
*e^6*x^6 + 3*d^2*e^5*x^5 - 48*d^3*e^4*x^4 + 48*d^4*e^3*x^3 - 3*d^5*e^2*x^2
- 8*d^6*e*x - 2*d^7)*sqrt(-e^2*x^2 + d^2))/x^4
```

Sympy [C] time = 20.5459, size = 1047, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**5,x)
```

```
[Out] d**7*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/
(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e
*x))/(8*d**3), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(
3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Ab
s(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d*
*2/(e**2*x**2) + 1)/(3*d**2), True)) + d**5*e**2*Piecewise((-d**2/(2*e*x**3
*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*ac
osh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/
```

```
(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) - 5*d**4*e**3*P
iecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(
d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1
- e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), T
rue)) - 5*d**3*e**4*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*ac
osh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x**
2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) +
I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + d**2*e**5*Piecewise((-I*d**2*ac
osh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)
/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + 3*d*e**6*Piecewise((x**2*
sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) +
e**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e
*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqr
t(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [B] time = 1.25771, size = 505, normalized size = 2.42

$$\frac{45}{8}\, d^4 \arcsin\left(\frac{xe}{d}\right) e^4 \mathrm{sgn}\left(d\right) + \frac{45}{8}\, d^4 e^4 \log\left(\frac{\left|-2\, de - 2\, \sqrt{-x^2 e^2 + d^2} e\right| e^{(-2)}}{2\, |x|}\right) + \frac{\left(d^4 e^{10} + \frac{8\left(de + \sqrt{-x^2e^2+d^2}e\right) d^4 e^8}{x} + \frac{8\left(de+\sqrt{-x^2e^2 \cdots}\right.}{x^2 \cdots}\right.}{64\left(de + \sqrt{-x \cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 45/8*d^4*arcsin(x*e/d)*e^4*sgn(d) + 45/8*d^4*e^4*log(1/2*abs(-2*d*e - 2*sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/64*(d^4*e^10 + 8*(d*e + sqrt(-x^2*e
^2 + d^2)*e)*d^4*e^8/x + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^6/x^2 - 1
84*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^4/x^3)*x^4*e^2/(d*e + sqrt(-x^2*e
^2 + d^2)*e)^4 + 1/64*(184*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^26/x - 8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^24/x^2 - 8*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d^4*e^22/x^3 - (d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^20/x^4)*e^(-24)
 - 1/8*(48*d^3*e^4 - (3*d^2*e^5 + 2*(x*e^7 + 4*d*e^6)*x)*x)*sqrt(-x^2*e^2 +
 d^2)
```

3.76 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=216

$$\frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x}+\frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2}-\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{d\left(d^2\right.}{}$$

```
[Out] (d^2*e^4*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x) + (d*e^3*(25*d - 52*e*x
)*(d^2 - e^2*x^2)^(3/2))/(24*x^2) - (e^2*(52*d + 25*e*x)*(d^2 - e^2*x^2)^(5
/2))/(60*x^3) - (d*(d^2 - e^2*x^2)^(7/2))/(5*x^5) - (3*e*(d^2 - e^2*x^2)^(7
/2))/(4*x^4) + (13*d^3*e^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (25*d^3*e
^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rubi [A] time = 0.313177, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 813, 844, 217, 203, 266, 63, 208}

$$\frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x}+\frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2}-\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{d\left(d^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```


```
[Out] (d^2*e^4*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x) + (d*e^3*(25*d - 52*e*x
)*(d^2 - e^2*x^2)^(3/2))/(24*x^2) - (e^2*(52*d + 25*e*x)*(d^2 - e^2*x^2)^(5
/2))/(60*x^3) - (d*(d^2 - e^2*x^2)^(7/2))/(5*x^5) - (3*e*(d^2 - e^2*x^2)^(7
/2))/(4*x^4) + (13*d^3*e^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (25*d^3*e
^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^6} dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-15d^4e-13d^3e^2x-5d^2e^3x^2\right)}{x^5} dx}{5d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} + \frac{\int \frac{\left(52d^5e^2-25d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^4} dx}{20d^4}$$

$$= -\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4} - \frac{\int \frac{\left(150d^6e^3+312d^5e^4x\right.}{x^3}}{72d^4}$$

$$= \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{3e\left(d\right.}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x\right.}{60x^3}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x\right.}{60x^3}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x\right.}{60x^3}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x\right.}{60x^3}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)\left(d^2-e^2x\right.}{60x^3}$$

Mathematica [C] time = 0.104517, size = 199, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{7d^{11}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{x^5\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{35d^9e^2\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3\sqrt{1-\frac{e^2x^2}{d^2}}} + 5e^5\left(e^2x^2-d^2\right)^3 {}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right) + 15e^5\left(e^2x^2-d^2\right)^3 {}_2\right.}{35d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((-7*d^11*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2
)/d^2])/(x^5*Sqrt[1 - (e^2*x^2)/d^2]) - (35*d^9*e^2*Hypergeometric2F1[-5/2,
 -3/2, -1/2, (e^2*x^2)/d^2])/(x^3*Sqrt[1 - (e^2*x^2)/d^2]) + 5*e^5*(-d^2 +
e^2*x^2)^3*Hypergeometric2F1[2, 7/2, 9/2, 1 - (e^2*x^2)/d^2] + 15*e^5*(-d^2
 + e^2*x^2)^3*Hypergeometric2F1[3, 7/2, 9/2, 1 - (e^2*x^2)/d^2]))/(35*d^4)
```

Maple [A] time = 0.086, size = 327, normalized size = 1.5

$$-\frac{13\,e^2}{15\,dx^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + \frac{52\,e^4}{15\,d^3x}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + \frac{52\,e^6x}{15\,d^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} + \frac{13\,e^6x}{3\,d}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} + \frac{13\,de^6x}{2}\sqrt{-x^2e^2+d^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x)
```

```
[Out] -13/15/d*e^2/x^3*(-e^2*x^2+d^2)^(7/2)+52/15/d^3*e^4/x*(-e^2*x^2+d^2)^(7/2)+
52/15/d^3*e^6*x*(-e^2*x^2+d^2)^(5/2)+13/3/d*e^6*x*(-e^2*x^2+d^2)^(3/2)+13/2
*d*e^6*x*(-e^2*x^2+d^2)^(1/2)+13/2*d^3*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2))-1/5*d*(-e^2*x^2+d^2)^(7/2)/x^5-3/4*e*(-e^2*x^2+d^2)^
(7/2)/x^4+5/8/d^2*e^3/x^2*(-e^2*x^2+d^2)^(7/2)+5/8/d^2*e^5*(-e^2*x^2+d^2)^(
5/2)+25/24*e^5*(-e^2*x^2+d^2)^(3/2)+25/8*d^2*e^5*(-e^2*x^2+d^2)^(1/2)-25/8*
d^4*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90585, size = 387, normalized size = 1.79

$$-\frac{1560\,d^3e^5x^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-375\,d^3e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-80\,d^3e^5x^5-\left(40\,e^7x^7+180\,de^6x^6+80\,d^2e^5x^5+\cdots\right.}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(1560*d^3*e^5*x^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 375*d^
3*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 80*d^3*e^5*x^5 - (40*e^7*x^7
 + 180*d*e^6*x^6 + 80*d^2*e^5*x^5 + 656*d^3*e^4*x^4 + 345*d^4*e^3*x^3 - 32*
d^5*e^2*x^2 - 90*d^6*e*x - 24*d^7)*sqrt(-e^2*x^2 + d^2))/x^5
```

Sympy [C] time = 17.522, size = 1197, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**6,x)
```

```
[Out] d**7*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-
15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*
d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x*
*5 + 15*d*e**2*x**7), True)) + 3*d**6*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**
2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**
2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(
```

```
Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) -
 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(
e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**5*e**2*Piecew
ise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) -
1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x
**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
5*d**4*e**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x
*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e*
*2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(
d/(e*x))/(2*d), True)) - 5*d**3*e**4*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/
d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**
2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) +
e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewise((d**2/(e*
x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2
) - 1), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**
2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True
)) + 3*d*e**6*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e*
*2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2,
 True)) + e**7*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x
**2)**(3/2)/(3*e**2), True))
```

Giac [B] time = 1.25596, size = 581, normalized size = 2.69

$$\frac{13}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^5\mathrm{sgn}\left(d\right)-\frac{25}{8}\,d^3e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(6\,d^3e^{12}+\frac{45\left(de+\sqrt{-x^2e^2+d^2}e\right)d^3e^{10}}{x}+\frac{50\left(de+\sqrt{-x^2\ldots}\right.}{x\ldots}\right.}{96\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 13/2*d^3*arcsin(x*e/d)*e^5*sgn(d) - 25/8*d^3*e^5*log(1/2*abs(-2*d*e - 2*sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/960*(6*d^3*e^12 + 45*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^3*e^10/x + 50*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*e^8/x
^2 - 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*e^6/x^3 - 2580*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^4*d^3*e^4/x^4)*x^5*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e)^5 +
1/960*(2580*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^38/x + 600*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^2*d^3*e^36/x^2 - 50*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*e^
34/x^3 - 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3*e^32/x^4 - 6*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^5*d^3*e^30/x^5)*e^(-35) + 1/6*(4*d^2*e^5 + (2*x*e^7 + 9*d
*e^6)*x)*sqrt(-x^2*e^2 + d^2)
```

3.77 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=214

$$-\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3}-\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{d\,(d}{}$$

```
[Out] -(d*e^5*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (d*e^3*(8*d + 85*e*x)*
(d^2 - e^2*x^2)^(3/2))/(48*x^3) - (e^2*(85*d + 12*e*x)*(d^2 - e^2*x^2)^(5/2
))/(120*x^4) - (d*(d^2 - e^2*x^2)^(7/2))/(6*x^6) - (3*e*(d^2 - e^2*x^2)^(7/
2))/(5*x^5) - (d^2*e^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (85*d^2*e^6*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/16
```

Rubi [A] time = 0.311736, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 813, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3}-\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{d\,(d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] -(d*e^5*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (d*e^3*(8*d + 85*e*x)*
(d^2 - e^2*x^2)^(3/2))/(48*x^3) - (e^2*(85*d + 12*e*x)*(d^2 - e^2*x^2)^(5/2
))/(120*x^4) - (d*(d^2 - e^2*x^2)^(7/2))/(6*x^6) - (3*e*(d^2 - e^2*x^2)^(7/
2))/(5*x^5) - (d^2*e^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (85*d^2*e^6*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/16
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
```

```
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-18d^4e-17d^3e^2x-6d^2e^3x^2\right)}{x^6}\,dx}{6d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} + \frac{\int \frac{\left(85d^5e^2-6d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx}{30d^4}$$

$$= -\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \frac{\int \frac{\left(48d^6e^3+340d^5\right.}{x}}{9}$$

$$= \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - 3e$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2\right.}{120x^4}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2\right.}{120x^4}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2\right.}{120x^4}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2\right.}{120x^4}$$

$$= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2\right.}{120x^4}$$

Mathematica [C] time = 0.242407, size = 286, normalized size = 1.34

$$-\frac{3d^6e\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{d^4e^3\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3x^3\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{3e^6\left(d^2-e^2x^2\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\right.}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] (-8*d^9 + 34*d^7*e^2*x^2 - 59*d^5*e^4*x^4 + 33*d^3*e^6*x^6 + 15*d^3*e^6*x^6
*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2)/d^2]])/(48*x^6*Sqrt[d^2
 - e^2*x^2]) - (3*d^6*e*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -
3/2, (e^2*x^2)/d^2])/(5*x^5*Sqrt[1 - (e^2*x^2)/d^2]) - (d^4*e^3*Sqrt[d^2 -
e^2*x^2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (e^2*x^2)/d^2])/(3*x^3*Sqrt[1
- (e^2*x^2)/d^2]) - (3*e^6*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[3, 7/2,
9/2, 1 - (e^2*x^2)/d^2])/(7*d^5)
```

Maple [A] time = 0.106, size = 352, normalized size = 1.6

$$\frac{e^3}{15\,d^2x^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{4\,e^5}{15\,d^4x}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{4\,e^7x}{15\,d^4}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{e^7x}{3\,d^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} - \frac{e^7d^2}{2}\arctan\left(x\sqrt{e^2}\frac{}{\sqrt{-x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x)
```

```
[Out] 1/15*e^3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/15*e^5/d^4/x*(-e^2*x^2+d^2)^(7/2)-4
/15*e^7/d^4*x*(-e^2*x^2+d^2)^(5/2)-1/3*e^7/d^2*x*(-e^2*x^2+d^2)^(3/2)-1/2*e
^7*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*e^7*x*(-e
^2*x^2+d^2)^(1/2)-3/5*e*(-e^2*x^2+d^2)^(7/2)/x^5+17/16/d^3*e^6*(-e^2*x^2+d^
2)^(5/2)+85/48/d*e^6*(-e^2*x^2+d^2)^(3/2)+85/16*d*e^6*(-e^2*x^2+d^2)^(1/2)-
1/6*d*(-e^2*x^2+d^2)^(7/2)/x^6-17/24/d*e^2/x^4*(-e^2*x^2+d^2)^(7/2)+17/16/d
^3*e^4/x^2*(-e^2*x^2+d^2)^(7/2)-85/16*d^3*e^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04116, size = 392, normalized size = 1.83

$$\frac{240\, d^2 e^6 x^6 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 1275\, d^2 e^6 x^6 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 720\, d^2 e^6 x^6 + \left(120\, e^7 x^7 + 720\, d e^6 x^6 - 544\, d^2 e^5 x^5 + \right.}{240\, x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(240*d^2*e^6*x^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 1275*d^2
*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 720*d^2*e^6*x^6 + (120*e^7*x^
7 + 720*d*e^6*x^6 - 544*d^2*e^5*x^5 + 645*d^3*e^4*x^4 + 448*d^4*e^3*x^3 - 5
0*d^5*e^2*x^2 - 144*d^6*e*x - 40*d^7)*sqrt(-e^2*x^2 + d^2))/x^6
```

Sympy [C] time = 23.5367, size = 1420, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**7,x)
```

```
[Out] d**7*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e
**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*
d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2
*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + 3*d**6*e*Piecewise(
(3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*
e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e*
*6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e
```

```
**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e*
*2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 +
15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*
e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*
e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*
x**7), True)) + d**5*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2)
- 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/
(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(
x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3
*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) - 5*d**4*e**3*Piecewise((-e*sqrt
(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2),
 Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3
*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - 5*d**3*e**4*
Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(
e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2
)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*
d), True)) + d**2*e**5*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*a
cosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d*
*2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt
(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise((d**2/(e*x*sqrt(d**2/(e*
*2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**
2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + e**7*Piece
wise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*
e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (
d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
```

Giac [B] time = 1.32402, size = 655, normalized size = 3.06

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^6\mathrm{sgn}\left(d\right)-\frac{85}{16}\,d^2e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(5\,d^2e^{14}+\frac{36\left(de+\sqrt{-x^2e^2+d^2}e\right)d^2e^{12}}{x}+\frac{45\left(de+\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^6*sgn(d) - 85/16*d^2*e^6*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/1920*(5*d^2*e^14 + 36*(d*e + sqrt(
-x^2*e^2 + d^2)*e)*d^2*e^12/x + 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^1
0/x^2 - 340*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*e^8/x^3 - 1215*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^4*d^2*e^6/x^4 + 1800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d
^2*e^4/x^5)*x^6*e^4/(d*e + sqrt(-x^2*e^2 + d^2)*e)^6 - 1/1920*(1800*(d*e +
sqrt(-x^2*e^2 + d^2)*e)*d^2*e^52/x - 1215*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*
d^2*e^50/x^2 - 340*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*e^48/x^3 + 45*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^4*d^2*e^46/x^4 + 36*(d*e + sqrt(-x^2*e^2 + d^2)*e
)^5*d^2*e^44/x^5 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2*e^42/x^6)*e^(-48)
 + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^7 + 6*d*e^6)
```

3.78 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=206

$$-\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}-\frac{d\left(d^2-e^2x\right.}{7x^7}$$

```
[Out] (-3*e^6*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (e^4*(16*d + 5*e*x)*(d
^2 - e^2*x^2)^(3/2))/(16*x^3) - (e^2*(24*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/
(40*x^5) - (d*(d^2 - e^2*x^2)^(7/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(7/2))/(2
*x^6) - 3*d*e^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (15*d*e^7*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/16
```

Rubi [A] time = 0.310915, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1807, 811, 813, 844, 217, 203, 266, 63, 208}

$$-\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}-\frac{d\left(d^2-e^2x\right.}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```


```
[Out] (-3*e^6*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (e^4*(16*d + 5*e*x)*(d
^2 - e^2*x^2)^(3/2))/(16*x^3) - (e^2*(24*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/
(40*x^5) - (d*(d^2 - e^2*x^2)^(7/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(7/2))/(2
*x^6) - 3*d*e^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (15*d*e^7*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/16
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^8}dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-21d^4e-21d^3e^2x-7d^2e^3x^2\right)}{x^7}dx}{7d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6} + \frac{\int \frac{\left(126d^5e^2+21d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^6}dx}{42d^4}$$

$$= -\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6} - \frac{\int \frac{\left(1008d^7e^4+210d^6e^5x\right)(}{x^4}}{336d^6}$$

$$= \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \frac{e\left(d^2-e^2\right.}{2x^6}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}$$

$$= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3} - \frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}$$

Mathematica [C] time = 0.168598, size = 247, normalized size = 1.2

$$-\frac{e^7\left(d^2-e^2x^2\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^6} - \frac{3d^5e^2\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7} + \frac{34d^6e^3x^2-59d^4e^5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(7/2))/(7*x^7) + (-8*d^8*e + 34*d^6*e^3*x^2 - 59*d^4*e^
5*x^4 + 33*d^2*e^7*x^6 + 15*d^2*e^7*x^6*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqr
t[1 - (e^2*x^2)/d^2]])/(16*x^6*Sqrt[d^2 - e^2*x^2]) - (3*d^5*e^2*Sqrt[d^2 -
 e^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2)/d^2])/(5*x^5*Sqrt[1
 - (e^2*x^2)/d^2]) - (e^7*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9
/2, 1 - (e^2*x^2)/d^2])/(7*d^6)
```

Maple [B] time = 0.131, size = 377, normalized size = 1.8

$$-\frac{3e^2}{5dx^5}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + \frac{2e^4}{5d^3x^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{8e^6}{5d^5x}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{8e^8x}{5d^5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - 2\frac{e^8x\left(-x^2e^2+d^2\right)^{3/2}}{d^3} - 3\frac{e^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x)
```

```
[Out] -3/5/d*e^2/x^5*(-e^2*x^2+d^2)^(7/2)+2/5/d^3*e^4/x^3*(-e^2*x^2+d^2)^(7/2)-8/
5/d^5*e^6/x*(-e^2*x^2+d^2)^(7/2)-8/5/d^5*e^8*x*(-e^2*x^2+d^2)^(5/2)-2/d^3*e
^8*x*(-e^2*x^2+d^2)^(3/2)-3/d*e^8*x*(-e^2*x^2+d^2)^(1/2)-3*d*e^8/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/8*e^3/d^2/x^4*(-e^2*x^2+d^2)
^(7/2)+3/16*e^5/d^4/x^2*(-e^2*x^2+d^2)^(7/2)+3/16*e^7/d^4*(-e^2*x^2+d^2)^(5
/2)+5/16*e^7/d^2*(-e^2*x^2+d^2)^(3/2)+15/16*e^7*(-e^2*x^2+d^2)^(1/2)-15/16*
e^7*d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/2*e*
(-e^2*x^2+d^2)^(7/2)/x^6-1/7*d*(-e^2*x^2+d^2)^(7/2)/x^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83584, size = 385, normalized size = 1.87

$$\frac{3360\, de^7x^7 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + 525\, de^7x^7 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + 560\, de^7x^7 + \left(560\, e^7x^7 - 2496\, de^6x^6 - 525\, d^2e^5x^5\right.}{560\, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/560*(3360*d*e^7*x^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 525*d*e^7
*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 560*d*e^7*x^7 + (560*e^7*x^7 - 24
96*d*e^6*x^6 - 525*d^2*e^5*x^5 + 992*d^3*e^4*x^4 + 770*d^4*e^3*x^3 - 96*d^5
*e^2*x^2 - 280*d^6*e*x - 80*d^7)*sqrt(-e^2*x^2 + d^2))/x^7
```

Sympy [C] time = 24.0499, size = 1537, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**8,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2)/(Abs(e**2)*
Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-
d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)
/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True))
 + 3*d**6*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24
*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2
) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))
/(16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d
```

```
**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**
3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2
/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**5*e**2*Pie
cewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) -
 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) +
 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7
) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**
5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 1
5*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*
d*e**2*x**7), True)) - 5*d**4*e**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e*
*2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*s
qrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e
**2)*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*
e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*
x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) - 5*d**3*e**4*Piecewise
((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/
(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2
) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + d**
2*e**5*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*A
bs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*
x))/(2*d), True)) + 3*d*e**6*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) +
 I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/
Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(
d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((d**2/(e*x*sqrt(d**2/(
e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d
**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1))
 + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Giac [B] time = 1.22846, size = 689, normalized size = 3.34

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{7}\mathrm{sgn}\left(d\right)-\frac{15}{16}\,de^{7}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(5\,de^{16}+\frac{35\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)de^{14}}{x}+\frac{49\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\right.}{x^{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^7*sgn(d) - 15/16*d*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/4480*(5*d*e^16 + 35*(d*e + sqrt(-x^2*e^2
 + d^2)*e)*d*e^14/x + 49*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^12/x^2 - 245*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*e^10/x^3 - 875*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^4*d*e^8/x^4 + 455*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d*e^6/x^5 + 9065*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d*e^4/x^6)*x^7*e^5/(d*e + sqrt(-x^2*e^2 + d
^2)*e)^7 - 1/4480*(9065*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^68/x + 455*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^2*d*e^66/x^2 - 875*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^3*d*e^64/x^3 - 245*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d*e^62/x^4 + 49*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)^5*d*e^60/x^5 + 35*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6
*d*e^58/x^6 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d*e^56/x^7)*e^(-63) + sqrt
(-x^2*e^2 + d^2)*e^7
```

3.79 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^9} dx$

Optimal. Leaf size=204

$$-\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-$$

```
[Out] -(e^6*(125*d + 128*e*x)*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (e^4*(125*d + 64*e
*x)*(d^2 - e^2*x^2)^(3/2))/(192*x^4) - (e^2*(125*d + 48*e*x)*(d^2 - e^2*x^2
)^(5/2))/(240*x^6) - (d*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (3*e*(d^2 - e^2*x^
2)^(7/2))/(7*x^7) - e^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (125*e^8*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])/128
```

Rubi [A] time = 0.303815, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1807, 811, 844, 217, 203, 266, 63, 208}

$$-\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```



```
[Out] -(e^6*(125*d + 128*e*x)*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (e^4*(125*d + 64*e
*x)*(d^2 - e^2*x^2)^(3/2))/(192*x^4) - (e^2*(125*d + 48*e*x)*(d^2 - e^2*x^2
)^(5/2))/(240*x^6) - (d*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (3*e*(d^2 - e^2*x^
2)^(7/2))/(7*x^7) - e^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (125*e^8*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])/128
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-24d^4e-25d^3e^2x-8d^2e^3x^2\right)}{x^8}\,dx}{8d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} + \frac{\int \frac{\left(175d^5e^2+56d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} - \int \frac{(1750d^7e^4+67}{}$$

$$= \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - 3$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)\left(d^2\right.}{240x^6}$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)\left(d^2\right.}{240x^6}$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)\left(d^2\right.}{240x^6}$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)\left(d^2\right.}{240x^6}$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)\left(d^2\right.}{240x^6}$$

Mathematica [C] time = 0.165984, size = 245, normalized size = 1.2

$$-\frac{e^8\left(d^2-e^2x^2\right)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)}{7d^7} - \frac{d^4e^3\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{5x^5\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{7x^7} + \frac{-59d^3e^6x^4+34}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] (-3*e*(d^2 - e^2*x^2)^(7/2))/(7*x^7) + (-8*d^7*e^2 + 34*d^5*e^4*x^2 - 59*d^
3*e^6*x^4 + 33*d*e^8*x^6 + 15*d*e^8*x^6*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqr
t[1 - (e^2*x^2)/d^2]])/(16*x^6*Sqrt[d^2 - e^2*x^2]) - (d^4*e^3*Sqrt[d^2 - e
^2*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (e^2*x^2)/d^2])/(5*x^5*Sqrt[1 -
 (e^2*x^2)/d^2]) - (e^8*(d^2 - e^2*x^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2
, 1 - (e^2*x^2)/d^2])/(7*d^7)
```

Maple [B] time = 0.176, size = 402, normalized size = 2.

$$-\frac{e^3}{5\,d^2x^5}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + \frac{2\,e^5}{15\,d^4x^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{8\,e^7}{15\,d^6x}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{8\,e^9x}{15\,d^6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{2\,e^9x}{3\,d^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x)
```

```
[Out] -1/5*e^3/d^2/x^5*(-e^2*x^2+d^2)^(7/2)+2/15*e^5/d^4/x^3*(-e^2*x^2+d^2)^(7/2)
-8/15*e^7/d^6/x*(-e^2*x^2+d^2)^(7/2)-8/15*e^9/d^6*x*(-e^2*x^2+d^2)^(5/2)-2/
3*e^9/d^4*x*(-e^2*x^2+d^2)^(3/2)-e^9/d^2*x*(-e^2*x^2+d^2)^(1/2)-e^9/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/8*d*(-e^2*x^2+d^2)^(7/2)/
x^8-25/48/d*e^2/x^6*(-e^2*x^2+d^2)^(7/2)+25/192/d^3*e^4/x^4*(-e^2*x^2+d^2)^
(7/2)-25/128/d^5*e^6/x^2*(-e^2*x^2+d^2)^(7/2)-25/128/d^5*e^8*(-e^2*x^2+d^2)
^(5/2)-125/384/d^3*e^8*(-e^2*x^2+d^2)^(3/2)-125/128/d*e^8*(-e^2*x^2+d^2)^(1
/2)+125/128*d*e^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))
/x)-3/7*e*(-e^2*x^2+d^2)^(7/2)/x^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27045, size = 382, normalized size = 1.87

$$\frac{26880\, e^8 x^8 \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - 13125\, e^8 x^8 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - \left(14848\, e^7 x^7 + 27195\, d e^6 x^6 + 7424\, d^2 e^5 x^5 - 17710\, d\right.}{13440\, x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/13440*(26880*e^8*x^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 13125*e^
8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (14848*e^7*x^7 + 27195*d*e^6*x^6
 + 7424*d^2*e^5*x^5 - 17710*d^3*e^4*x^4 - 14592*d^4*e^3*x^3 + 1960*d^5*e^2*
x^2 + 5760*d^6*e*x + 1680*d^7)*sqrt(-e^2*x^2 + d^2))/x^8
```

Sympy [C] time = 34.5318, size = 1742, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**9,x)
```

```
[Out] d**7*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2)/
(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1))
- 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(
-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) +
1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(
e*x))/(128*d**7), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1
```

```
)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d
**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(1
05*d**6), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2
) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e
**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**6), True)) + d**5*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(
d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(4
8*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x
**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2))
 > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)
) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(
16*d**5), True)) - 5*d**4*e**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2
)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2
)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/
(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)
/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*s
qrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt
(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 -
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - 5*d**3*e**4*Piecew
ise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e
**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d
/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**
3), True)) + d**2*e**5*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x
**2) + 1)/(3*d**2), True)) + 3*d*e**6*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/
(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x)
)/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
 + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**7*Piecewise((I*d/(x*s
qrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x
**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2))
 - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [B] time = 1.27234, size = 726, normalized size = 3.56

$$-\arcsin\left(\frac{xe}{d}\right)e^8\operatorname{sgn}(d) + \frac{x^8\left(\frac{720\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{16}}{x} + \frac{1120\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{14}}{x^2} - \frac{3696\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{12}}{x^3} - \frac{14280\left(de+\sqrt{-x^2e^2+d^2}\right.}{x^4}\right.}{215040\left(de+\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^8*sgn(d) + 1/215040*x^8*(720*(d*e + sqrt(-x^2*e^2 + d^2)*e
)*e^16/x + 1120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^14/x^2 - 3696*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^3*e^12/x^3 - 14280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^
10/x^4 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^8/x^5 + 77280*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^6*e^6/x^6 + 122640*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^4/x
^7 + 105*e^18)*e^6/(d*e + sqrt(-x^2*e^2 + d^2)*e)^8 - 1/215040*(122640*(d*e
 + sqrt(-x^2*e^2 + d^2)*e)*e^86/x + 77280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*
e^84/x^2 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^82/x^3 - 14280*(d*e + sqr
```

```
t(-x^2*e^2 + d^2)*e)^4*e^80/x^4 - 3696*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^7
8/x^5 + 1120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^76/x^6 + 720*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^7*e^74/x^7 + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*e^72/x^8
)*e^(-80) + 125/128*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/abs(x))
```

3.80 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=187

$$-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}$$

```
[Out] (-55*e^7*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (55*e^5*(d^2 - e^2*x^2)^(3/2))/(1
92*x^4) - (11*e^3*(d^2 - e^2*x^2)^(5/2))/(48*x^6) - (d*(d^2 - e^2*x^2)^(7/2
))/(9*x^9) - (3*e*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (29*e^2*(d^2 - e^2*x^2)^
(7/2))/(63*d*x^7) + (55*e^9*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d)
```

Rubi [A] time = 0.260319, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1807, 807, 266, 47, 63, 208}

$$-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6}-\frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (-55*e^7*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (55*e^5*(d^2 - e^2*x^2)^(3/2))/(1
92*x^4) - (11*e^3*(d^2 - e^2*x^2)^(5/2))/(48*x^6) - (d*(d^2 - e^2*x^2)^(7/2
))/(9*x^9) - (3*e*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (29*e^2*(d^2 - e^2*x^2)^
(7/2))/(63*d*x^7) + (55*e^9*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d)
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-27d^4e-29d^3e^2x-9d^2e^3x^2\right)}{x^9}\,dx}{9d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} + \frac{\int \frac{\left(232d^5e^2+99d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx}{72d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7} + \frac{1}{8}\left(11e^3\right)\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7}\,d$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7} + \frac{1}{16}\left(11e^3\right)\text{Subst}\left(\int \frac{\left(d^2-e\right.}{x}\right.$$

$$= -\frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{29e^2\left(d^2-e^2x^2\right)^{7/2}}{63dx^7} - \frac{1}{96}\left(55\right.$$

$$= \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{8x^8} - \frac{29e^2\left(d^2\right.}{6}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2\right.}{8}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2\right.}{8}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5\left(d^2-e^2x^2\right)^{3/2}}{192x^4} - \frac{11e^3\left(d^2-e^2x^2\right)^{5/2}}{48x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{9x^9} - \frac{3e\left(d^2\right.}{8}$$

Mathematica [C] time = 0.183328, size = 218, normalized size = 1.17

$$\frac{-16d^8e^2x^2-168d^7e^3x^3+1184d^6e^4x^4+714d^5e^5x^5-2336d^4e^6x^6-1239d^3e^7x^7+1744d^2e^8x^8+315de^9x^9\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-}}{1008dx^9\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (-112*d^10 - 16*d^8*e^2*x^2 - 168*d^7*e^3*x^3 + 1184*d^6*e^4*x^4 + 714*d^5*
e^5*x^5 - 2336*d^4*e^6*x^6 - 1239*d^3*e^7*x^7 + 1744*d^2*e^8*x^8 + 693*d*e^
9*x^9 - 464*e^10*x^10 + 315*d*e^9*x^9*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[
1 - (e^2*x^2)/d^2]])/(1008*d*x^9*Sqrt[d^2 - e^2*x^2]) - (3*e^9*(d^2 - e^2*x
^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2])/(7*d^8)
```

Maple [A] time = 0.253, size = 250, normalized size = 1.3

$$-\frac{3e}{8x^8}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{11e^3}{48d^2x^6}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}+\frac{11e^5}{192d^4x^4}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{11e^7}{128d^6x^2}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{11e^9}{128d^6}\left(-x^2e^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x)
```

```
[Out] -3/8*e*(-e^2*x^2+d^2)^(7/2)/x^8-11/48/d^2*e^3/x^6*(-e^2*x^2+d^2)^(7/2)+11/1
92/d^4*e^5/x^4*(-e^2*x^2+d^2)^(7/2)-11/128/d^6*e^7/x^2*(-e^2*x^2+d^2)^(7/2)
-11/128/d^6*e^9*(-e^2*x^2+d^2)^(5/2)-55/384/d^4*e^9*(-e^2*x^2+d^2)^(3/2)-55
/128/d^2*e^9*(-e^2*x^2+d^2)^(1/2)+55/128*e^9/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^
(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-29/63*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^7-1/9*d*(
-e^2*x^2+d^2)^(7/2)/x^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.41347, size = 328, normalized size = 1.75

$$-\frac{3465e^9x^9\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(3712e^8x^8-4599de^7x^7-10240d^2e^6x^6-3066d^3e^5x^5+8448d^4e^4x^4+7224d^5e^3x^3\right.}{8064dx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/8064*(3465*e^9*x^9*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (3712*e^8*x^8 -
4599*d*e^7*x^7 - 10240*d^2*e^6*x^6 - 3066*d^3*e^5*x^5 + 8448*d^4*e^4*x^4 +
7224*d^5*e^3*x^3 - 1024*d^6*e^2*x^2 - 3024*d^7*e*x - 896*d^8)*sqrt(-e^2*x^2
 + d^2))/(d*x^9)
```

Sympy [C] time = 34.8065, size = 1914, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**10,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d*
*2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(63*d**2*x**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I
*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e
**2*x**2) + 1)/(315*d**8), True)) + 3*d**6*e*Piecewise((-d**2/(8*e*x**9*sqr
t(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/
(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e*
*8*acosh(d/(e*x))/(128*d**7), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2
/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x
**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/
(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128*d**7), True)) + d**5*e**
2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*
x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**
2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2)/(Abs(e**2)*Abs
(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**
2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(1
05*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 5
*d**4*e**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24
*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2
) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))
/(16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d
**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**
3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2
/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 5*d**3*e**4*P
iecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7)
 - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7)
 + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x*
*7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7
), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x
**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 +
 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 1
5*d*e**2*x**7), True)) + d**2*e**5*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e*
*2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*s
qrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e
**2)*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*
e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*
x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + 3*d*e**6*Piecewise((-
e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*
d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) +
 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**7*P
iecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e
**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2)
) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d
), True))
```

Giac [B] time = 1.26285, size = 837, normalized size = 4.48

$$\frac{x^9\left(\frac{189\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{18}}{x}+\frac{324\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{16}}{x^2}-\frac{672\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{14}}{x^3}-\frac{3024\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^{12}}{x^4}-\frac{1512\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^{10}}{x^5}-\right.}{129024\left(de+\sqrt{-x^2e^2+d^2}e\right)^9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/129024*x^9*(189*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^18/x + 324*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^2*e^16/x^2 - 672*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^14/x^
3 - 3024*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^12/x^4 - 1512*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^5*e^10/x^5 + 9744*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^8/x^6 +
18144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^6/x^7 - 16632*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^8*e^4/x^8 + 28*e^20)*e^7/((d*e + sqrt(-x^2*e^2 + d^2)*e)^9*d) +
55/128*e^9*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d
+ 1/129024*(16632*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*e^106/x - 18144*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)^2*d^8*e^104/x^2 - 9744*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d^8*e^102/x^3 + 1512*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^8*e^100/x^4 +
 3024*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^8*e^98/x^5 + 672*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^6*d^8*e^96/x^6 - 324*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^8*e^9
4/x^7 - 189*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^8*e^92/x^8 - 28*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^9*d^8*e^90/x^9)*e^(-99)/d^9
```

3.81 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=225

$$-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4}-\frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6}-\frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7}-\frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}$$

```
[Out] (-33*e^8*Sqrt[d^2 - e^2*x^2])/(256*d*x^2) + (11*e^6*(d^2 - e^2*x^2)^(3/2))/
(128*d*x^4) - (11*e^4*(d^2 - e^2*x^2)^(5/2))/(160*d*x^6) - (d*(d^2 - e^2*x^
2)^(7/2))/(10*x^10) - (e*(d^2 - e^2*x^2)^(7/2))/(3*x^9) - (33*e^2*(d^2 - e^
2*x^2)^(7/2))/(80*d*x^8) - (5*e^3*(d^2 - e^2*x^2)^(7/2))/(21*d^2*x^7) + (33
*e^10*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(256*d^2)
```

Rubi [A] time = 0.298169, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 835, 807, 266, 47, 63, 208}

$$-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4}-\frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6}-\frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7}-\frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] (-33*e^8*Sqrt[d^2 - e^2*x^2])/(256*d*x^2) + (11*e^6*(d^2 - e^2*x^2)^(3/2))/
(128*d*x^4) - (11*e^4*(d^2 - e^2*x^2)^(5/2))/(160*d*x^6) - (d*(d^2 - e^2*x^
2)^(7/2))/(10*x^10) - (e*(d^2 - e^2*x^2)^(7/2))/(3*x^9) - (33*e^2*(d^2 - e^
2*x^2)^(7/2))/(80*d*x^8) - (5*e^3*(d^2 - e^2*x^2)^(7/2))/(21*d^2*x^7) + (33
*e^10*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(256*d^2)
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-30d^4e-33d^3e^2x-10d^2e^3x^2\right)}{x^{10}}dx}{10d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} + \frac{\int \frac{\left(297d^5e^2+150d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^9}dx}{90d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{\int \frac{\left(-1200d^6e^3-297d^5e^4x\right)\left(d^2-e^2x^2\right)^{5/}}{x^8}}{720d^6}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7} + \frac{\left(33e^4\right)\int}{}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2-e^2x^2\right)^{7/2}}{21d^2x^7} + \frac{\left(33e^4\right)S}{}$$

$$= -\frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2\right.}{21}$$

$$= \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-\right.}{80}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-\right.}{3}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-\right.}{3}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-\right.}{3}$$

Mathematica [C] time = 0.074063, size = 102, normalized size = 0.45

$$-\frac{e\left(d^2-e^2x^2\right)^{7/2}\left(9e^9x^9\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+3e^9x^9\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+5d^7e^2x^2+7d^9\right)}{21d^9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] -(e*(d^2 - e^2*x^2)^(7/2)*(7*d^9 + 5*d^7*e^2*x^2 + 9*e^9*x^9*Hypergeometric
2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2] + 3*e^9*x^9*Hypergeometric2F1[7/2, 6, 9
/2, 1 - (e^2*x^2)/d^2]))/(21*d^9*x^9)
```

Maple [A] time = 0.373, size = 278, normalized size = 1.2

$$-\frac{33\,e^2}{80\,dx^8}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{11\,e^4}{160\,d^3x^6}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + \frac{11\,e^6}{640\,d^5x^4}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{33\,e^8}{1280\,d^7x^2}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{33\,e^{10}}{1280\,d^7}\left(-x^2e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x)
```

```
[Out] -33/80*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^8-11/160/d^3*e^4/x^6*(-e^2*x^2+d^2)^(7/
2)+11/640/d^5*e^6/x^4*(-e^2*x^2+d^2)^(7/2)-33/1280/d^7*e^8/x^2*(-e^2*x^2+d^
2)^(7/2)-33/1280/d^7*e^10*(-e^2*x^2+d^2)^(5/2)-11/256/d^5*e^10*(-e^2*x^2+d^
2)^(3/2)-33/256/d^3*e^10*(-e^2*x^2+d^2)^(1/2)+33/256/d*e^10/(d^2)^(1/2)*ln(
(2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-5/21*e^3*(-e^2*x^2+d^2)^(7/2)
/d^2/x^7-1/3*e*(-e^2*x^2+d^2)^(7/2)/x^9-1/10*d*(-e^2*x^2+d^2)^(7/2)/x^10
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.09006, size = 367, normalized size = 1.63

$$-\frac{3465\,e^{10}x^{10}\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(6400\,e^9x^9+3465\,de^8x^8-10240\,d^2e^7x^7-24570\,d^3e^6x^6-7680\,d^4e^5x^5+23352\,d^5e\right.}{26880\,d^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/26880*(3465*e^10*x^10*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (6400*e^9*x^9
 + 3465*d*e^8*x^8 - 10240*d^2*e^7*x^7 - 24570*d^3*e^6*x^6 - 7680*d^4*e^5*x^
5 + 23352*d^5*e^4*x^4 + 20480*d^6*e^3*x^3 - 3024*d^7*e^2*x^2 - 8960*d^8*e*x
 - 2688*d^9)*sqrt(-e^2*x^2 + d^2))/(d^2*x^10)
```

Sympy [C] time = 54.1913, size = 2184, normalized size = 9.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**11,x)
```

```
[Out] d**7*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/(80*x**
9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*x**2) -
 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(768*d**
6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e**2*x**
2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2)/(Abs(e**2)*Abs(x**2
)) > 1), (I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*
sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2)
 + 1)) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(
768*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**
2/(e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + 3*d**6*e
*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x
**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4
```

```
) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(
e**2*x**2) - 1)/(315*d**8), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqr
t(-d**2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*
d**2*x**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**
7*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*
x**2) + 1)/(315*d**8), True)) + d**5*e**2*Piecewise((-d**2/(8*e*x**9*sqrt(d
**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(19
2*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(
e**2*x**2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*
acosh(d/(e*x))/(128*d**7), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(8
*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(38
4*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/
(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128*d**7), True)) - 5*d**4*e**3
*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x
**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2
) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2)/(Abs(e**2)*Abs(
x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2
/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(10
5*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 5*
d**3*e**4*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*
x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2)
 - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/
(16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d*
*2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3
/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/
(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**2*e**5*Piec
ewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) -
4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) +
2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7)
 - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x
**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15
*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), True)) + 3*d*e**6*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x
**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(
d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)
*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8
*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**7*Piecewise((-e*sqrt(d
**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), A
bs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x
**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

Giac [B] time = 1.28323, size = 922, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/430080*x^10*(280*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^20/x + 525*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^2*e^18/x^2 - 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^16/x
^3 - 3570*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^14/x^4 - 3360*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^5*e^12/x^5 + 5880*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^10/x^6
```

```
+ 16800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^8/x^7 + 10500*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^8*e^6/x^8 - 31920*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*e^4/x^9 + 4
2*e^22)*e^8/((d*e + sqrt(-x^2*e^2 + d^2)*e)^10*d^2) + 33/256*e^10*log(1/2*a
bs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 + 1/430080*(31920*
(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^18*e^128/x - 10500*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^2*d^18*e^126/x^2 - 16800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e^12
4/x^3 - 5880*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^18*e^122/x^4 + 3360*(d*e +
sqrt(-x^2*e^2 + d^2)*e)^5*d^18*e^120/x^5 + 3570*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^6*d^18*e^118/x^6 + 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^18*e^116/x^7
- 525*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^18*e^114/x^8 - 280*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^9*d^18*e^112/x^9 - 42*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*d^1
8*e^110/x^10)*e^(-120)/d^20
```

3.82 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=254

$$-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4}-\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6}-\frac{74e^4\left(d^2-e^2x^2\right)^{7/2}}{693d^3x^7}-\frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8}-\frac{37e^2\left(d^2-e^2x^2\right.}{99dx^9}$$

```
[Out] (-19*e^9*Sqrt[d^2 - e^2*x^2])/(256*d^2*x^2) + (19*e^7*(d^2 - e^2*x^2)^(3/2)
)/(384*d^2*x^4) - (19*e^5*(d^2 - e^2*x^2)^(5/2))/(480*d^2*x^6) - (d*(d^2 -
e^2*x^2)^(7/2))/(11*x^11) - (3*e*(d^2 - e^2*x^2)^(7/2))/(10*x^10) - (37*e^2
*(d^2 - e^2*x^2)^(7/2))/(99*d*x^9) - (19*e^3*(d^2 - e^2*x^2)^(7/2))/(80*d^2
*x^8) - (74*e^4*(d^2 - e^2*x^2)^(7/2))/(693*d^3*x^7) + (19*e^11*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d])/(256*d^3)
```

Rubi [A] time = 0.329047, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1807, 835, 807, 266, 47, 63, 208}

$$-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4}-\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6}-\frac{74e^4\left(d^2-e^2x^2\right)^{7/2}}{693d^3x^7}-\frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8}-\frac{37e^2\left(d^2-e^2x^2\right.}{99dx^9}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```


```
[Out] (-19*e^9*Sqrt[d^2 - e^2*x^2])/(256*d^2*x^2) + (19*e^7*(d^2 - e^2*x^2)^(3/2)
)/(384*d^2*x^4) - (19*e^5*(d^2 - e^2*x^2)^(5/2))/(480*d^2*x^6) - (d*(d^2 -
e^2*x^2)^(7/2))/(11*x^11) - (3*e*(d^2 - e^2*x^2)^(7/2))/(10*x^10) - (37*e^2
*(d^2 - e^2*x^2)^(7/2))/(99*d*x^9) - (19*e^3*(d^2 - e^2*x^2)^(7/2))/(80*d^2
*x^8) - (74*e^4*(d^2 - e^2*x^2)^(7/2))/(693*d^3*x^7) + (19*e^11*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d])/(256*d^3)
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{12}}dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-33d^4e-37d^3e^2x-11d^2e^3x^2\right)}{x^{11}}dx}{11d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} + \frac{\int \frac{\left(370d^5e^2+209d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}dx}{110d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{\int \frac{\left(-1881d^6e^3-740d^5e^4x\right)\left(d^2-e^2x^2\right)}{x^9}}{990d^6}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8} + \int \frac{(592}{}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8} - \frac{74e^4(}{6}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3\left(d^2-e^2x^2\right)^{7/2}}{80d^2x^8} - \frac{74e^4(}{6}$$

$$= -\frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2\left(d^2-e^2x^2\right)^{7/2}}{99dx^9} - \frac{19e^3(}{8}$$

$$= \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{37e^2(d^2}{9}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e(d^2}{1}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e(d^2}{1}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7\left(d^2-e^2x^2\right)^{3/2}}{384d^2x^4} - \frac{19e^5\left(d^2-e^2x^2\right)^{5/2}}{480d^2x^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{11x^{11}} - \frac{3e(d^2}{1}$$

Mathematica [C] time = 0.0676529, size = 112, normalized size = 0.44

$$-\frac{\left(d^2-e^2x^2\right)^{7/2}\left(99e^{11}x^{11}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+297e^{11}x^{11}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};1-\frac{e^2x^2}{d^2}\right)+259d^9e^2x^2+74d^7e^4x^4+63d^{11}\right)}{693d^{10}x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] -((d^2 - e^2*x^2)^(7/2)*(63*d^11 + 259*d^9*e^2*x^2 + 74*d^7*e^4*x^4 + 99*e^
11*x^11*Hypergeometric2F1[7/2, 5, 9/2, 1 - (e^2*x^2)/d^2] + 297*e^11*x^11*H
ypergeometric2F1[7/2, 6, 9/2, 1 - (e^2*x^2)/d^2]))/(693*d^10*x^11)
```

Maple [A] time = 0.569, size = 303, normalized size = 1.2

$$-\frac{d}{11\,x^{11}}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{37\,e^2}{99\,dx^9}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{74\,e^4}{693\,d^3x^7}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{19\,e^3}{80\,d^2x^8}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{19\,e^5}{480\,d^4x^6}\left(-x^2e^2+d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x)
```

```
[Out] -1/11*d*(-e^2*x^2+d^2)^(7/2)/x^11-37/99*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^9-74/6
93*e^4*(-e^2*x^2+d^2)^(7/2)/d^3/x^7-19/80*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^8-
19/480*e^5/d^4/x^6*(-e^2*x^2+d^2)^(7/2)+19/1920*e^7/d^6/x^4*(-e^2*x^2+d^2)^
(7/2)-19/1280*e^9/d^8/x^2*(-e^2*x^2+d^2)^(7/2)-19/1280*e^11/d^8*(-e^2*x^2+d
^2)^(5/2)-19/768*e^11/d^6*(-e^2*x^2+d^2)^(3/2)-19/256*e^11/d^4*(-e^2*x^2+d^
2)^(1/2)+19/256*e^11/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-3/10*e*(-e^2*x^2+d^2)^(7/2)/x^10
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.16834, size = 417, normalized size = 1.64

$$-\frac{65835\,e^{11}x^{11}\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(94720\,e^{10}x^{10}+65835\,de^9x^9+47360\,d^2e^8x^8-251790\,d^3e^7x^7-629760\,d^4e^6x^6-\right.}{887040\,d^3x^1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/887040*(65835*e^11*x^11*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (94720*e^10
*x^10 + 65835*d*e^9*x^9 + 47360*d^2*e^8*x^8 - 251790*d^3*e^7*x^7 - 629760*d
^4*e^6*x^6 - 201432*d^5*e^5*x^5 + 657920*d^6*e^4*x^4 + 587664*d^7*e^3*x^3 -
 89600*d^8*e^2*x^2 - 266112*d^9*e*x - 80640*d^10)*sqrt(-e^2*x^2 + d^2))/(d^
3*x^11)
```

Sympy [C] time = 54.7222, size = 2422, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**12,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(11*x**10) + e**3*sqrt(d**2/(
e**2*x**2) - 1)/(99*d**2*x**8) + 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(693*d**
4*x**6) + 16*e**7*sqrt(d**2/(e**2*x**2) - 1)/(1155*d**6*x**4) + 64*e**9*sqr
t(d**2/(e**2*x**2) - 1)/(3465*d**8*x**2) + 128*e**11*sqrt(d**2/(e**2*x**2)
- 1)/(3465*d**10), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(
e**2*x**2) + 1)/(11*x**10) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(99*d**2*x*
*8) + 8*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(693*d**4*x**6) + 16*I*e**7*sqrt
```

```
(-d**2/(e**2*x**2) + 1)/(1155*d**6*x**4) + 64*I*e**9*sqrt(-d**2/(e**2*x**2)
 + 1)/(3465*d**8*x**2) + 128*I*e**11*sqrt(-d**2/(e**2*x**2) + 1)/(3465*d**1
0), True)) + 3*d**6*e*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) -
1)) + 9*e/(80*x**9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d
**2/(e**2*x**2) - 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1))
+ 7*e**7/(768*d**6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sq
rt(d**2/(e**2*x**2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2)/(A
bs(e**2)*Abs(x**2)) > 1), (I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1))
- 9*I*e/(80*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(
-d**2/(e**2*x**2) + 1)) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) +
 1)) - 7*I*e**7/(768*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256
*d**8*x*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9),
True)) + d**5*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3
*sqrt(d**2/(e**2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) -
 1)/(105*d**4*x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 1
6*e**9*sqrt(d**2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2)/(Abs(e**2)*Abs(x**2
)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e*
*2*x**2) + 1)/(63*d**2*x**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d*
*4*x**4) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9
*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**8), True)) - 5*d**4*e**3*Piecewise((-d
**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x*
*2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d
**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x
**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2)/(Abs(e**2)*Abs(x**
2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*s
qrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(12
8*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128*d**7),
True)) - 5*d**3*e**4*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e*
*3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2)
 - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d
**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6)
 + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/
(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(10
5*d**6), True)) + d**2*e**5*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2
) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqr
t(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e*
*6*acosh(d/(e*x))/(16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/
(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x*
*2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*
d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)
) + 3*d*e**6*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 +
 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 +
 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 1
5*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/
d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**
2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-
15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15
*d**3*x**5 + 15*d*e**2*x**7), True)) + e**7*Piecewise((-d**2/(4*e*x**5*sqrt
(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8
*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**
2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1
)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True))
```

Giac [B] time = 1.2886, size = 1007, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/14192640*x^11*(4158*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^22/x + 8470*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^2*e^20/x^2 - 3465*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e
^18/x^3 - 40590*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^16/x^4 - 57750*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^5*e^14/x^5 + 6930*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^
12/x^6 + 138600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^10/x^7 + 244860*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^8*e^8/x^8 + 152460*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*
e^6/x^9 - 568260*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*e^4/x^10 + 630*e^24)*e^9
/((d*e + sqrt(-x^2*e^2 + d^2)*e)^11*d^3) + 19/256*e^11*log(1/2*abs(-2*d*e -
 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 + 1/14192640*(568260*(d*e + s
qrt(-x^2*e^2 + d^2)*e)*d^30*e^152/x - 152460*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^2*d^30*e^150/x^2 - 244860*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^30*e^148/x^3
- 138600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^30*e^146/x^4 - 6930*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^5*d^30*e^144/x^5 + 57750*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^6*d^30*e^142/x^6 + 40590*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^30*e^140/x^7 +
 3465*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^30*e^138/x^8 - 8470*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^9*d^30*e^136/x^9 - 4158*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*
d^30*e^134/x^10 - 630*(d*e + sqrt(-x^2*e^2 + d^2)*e)^11*d^30*e^132/x^11)*e^
(-143)/d^33
```

3.83 $\int \frac{x^5(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} dx$

Optimal. Leaf size=174

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}+\frac{x\sqrt{d^2-e^2x^2}}{2e^5}-\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] (d^4*(d + e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d + e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d + e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 + (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rubi [A] time = 0.404562, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1635, 1815, 641, 217, 203}

$$\frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}+\frac{x\sqrt{d^2-e^2x^2}}{2e^5}-\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d + e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d + e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 + (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{-\frac{195d^5}{e^3}-\frac{90d^4x}{e^2}}{\sqrt{d^2-e^2x^2}}\,dx}{30d^3e^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} - \ldots$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} - \ldots$$

$$= \frac{d^4(d+ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} - \ldots$$

Mathematica [A] time = 0.262394, size = 131, normalized size = 0.75

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(479d^2e^2x^2-717d^3ex+304d^4-45de^3x^3-15e^4x^4\right)-195d(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{30e^6(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(304*d^4 - 717*d^3*e*x + 479*d^2*e^2*x^
2 - 45*d*e^3*x^3 - 15*e^4*x^4) - 195*d*(d - e*x)^3*ArcSin[(e*x)/d]))/(30*e^
6*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [A] time = 0.137, size = 222, normalized size = 1.3

$$-\frac{ex^7}{2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{13\,d^2x^5}{10\,e}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}-\frac{13\,d^2x^3}{6\,e^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{13\,d^2x}{2\,e^5}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{13\,d^2}{2\,e^5}\arctan\left(x\sqrt{e^2}\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/2*e*x^7/(-e^2*x^2+d^2)^(5/2)+13/10/e*d^2*x^5/(-e^2*x^2+d^2)^(5/2)-13/6/e
^3*d^2*x^3/(-e^2*x^2+d^2)^(3/2)+13/2/e^5*d^2*x/(-e^2*x^2+d^2)^(1/2)-13/2/e^
5*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-3*d*x^6/(-e^2*
x^2+d^2)^(5/2)+19*d^3/e^2*x^4/(-e^2*x^2+d^2)^(5/2)-76/3*d^5/e^4*x^2/(-e^2*x
^2+d^2)^(5/2)+152/15*d^7/e^6/(-e^2*x^2+d^2)^(5/2)
```

Maxima [B] time = 1.50447, size = 429, normalized size = 2.47

$$-\frac{ex^7}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{13}{30}d^2ex\left(\frac{15\,x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20\,d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8\,d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{3\,dx^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{13\,d^2x\left(\frac{3\,x^2}{\left(-e^2x^2+d\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*e*x^7/(-e^2*x^2 + d^2)^(5/2) + 13/30*d^2*e*x*(15*x^4/((-e^2*x^2 + d^2)
^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 +
d^2)^(5/2)*e^6)) - 3*d*x^6/(-e^2*x^2 + d^2)^(5/2) - 13/6*d^2*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 19*d^3*x^
4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 76/3*d^5*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4)
+ 152/15*d^7/((-e^2*x^2 + d^2)^(5/2)*e^6) + 26/15*d^4*x/((-e^2*x^2 + d^2)^(
3/2)*e^5) - 91/30*d^2*x/(sqrt(-e^2*x^2 + d^2)*e^5) - 13/2*d^2*arcsin(e^2*x/
sqrt(d^2*e^2))/(sqrt(e^2)*e^5)
```

Fricas [A] time = 1.78923, size = 409, normalized size = 2.35

$$\frac{304\,d^2e^3x^3-912\,d^3e^2x^2+912\,d^4ex-304\,d^5+390\left(d^2e^3x^3-3\,d^3e^2x^2+3\,d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(15\,e^4x^4+\right.}{30\left(e^9x^3-3\,de^8x^2+3\,d^2e^7x-d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*e^3*x^3 - 912*d^3*e^2*x^2 + 912*d^4*e*x - 304*d^5 + 390*(d^2*
e^3*x^3 - 3*d^3*e^2*x^2 + 3*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) + (15*e^4*x^4 + 45*d*e^3*x^3 - 479*d^2*e^2*x^2 + 717*d^3*e*x - 30
4*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 - 3*d*e^8*x^2 + 3*d^2*e^7*x - d^3*e^6
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 (d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**5*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.20391, size = 159, normalized size = 0.91

$$-\frac{13}{2}\,d^2 \arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}\left(d\right) - \frac{\left(304\,d^7e^{(-6)} + \left(195\,d^6e^{(-5)} - \left(760\,d^5e^{(-4)} + \left(455\,d^4e^{(-3)} - \left(570\,d^3e^{(-2)} + \left(299\,d^2e^{(-1)}\right.\right.\right.\right.\right.\right.}{30\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -13/2*d^2*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/30*(304*d^7*e^(-6) + (195*d^6*e^(
-5) - (760*d^5*e^(-4) + (455*d^4*e^(-3) - (570*d^3*e^(-2) + (299*d^2*e^(-1)
 - 15*(x*e + 6*d)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.84 $\int \frac{x^4(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} dx$

Optimal. Leaf size=142

$$\frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] (d^3*(d + e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (6*d^2*(d + e*x)^2)/(5*e^
5*(d^2 - e^2*x^2)^(3/2)) + (24*d*(d + e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) + S
qrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rubi [A] time = 0.324808, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1635, 641, 217, 203}

$$\frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (6*d^2*(d + e*x)^2)/(5*e^
5*(d^2 - e^2*x^2)^(3/2)) + (24*d*(d + e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) + S
qrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{27d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}+\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,d\right.}{e^4}$$

$$= \frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] time = 0.211784, size = 119, normalized size = 0.84

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(-57d^2ex+24d^3+39de^2x^2-5e^3x^3\right)-15(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{5e^5(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(Sqrt[1 - (e^2*x^2)/d^2]*(24*d^3 - 57*d^2*e*x + 39*d*e^2*x^2 - 5
*e^3*x^3) - 15*(d - e*x)^3*ArcSin[(e*x)/d]))/(5*e^5*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [B] time = 0.114, size = 262, normalized size = 1.9

$$-ex^6\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} + 9\frac{d^2x^4}{e\left(-x^2e^2+d^2\right)^{5/2}} - 12\frac{d^4x^2}{e^3\left(-x^2e^2+d^2\right)^{5/2}} + \frac{24\,d^6}{5\,e^5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} + \frac{3\,dx^5}{5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -e*x^6/(-e^2*x^2+d^2)^(5/2)+9/e*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-12/e^3*d^4*x^2
/(-e^2*x^2+d^2)^(5/2)+24/5/e^5*d^6/(-e^2*x^2+d^2)^(5/2)+3/5*d*x^5/(-e^2*x^2
+d^2)^(5/2)-d/e^2*x^3/(-e^2*x^2+d^2)^(3/2)+16/5*d/e^4*x/(-e^2*x^2+d^2)^(1/2
```

```
)-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/2*d^3*x^
3/e^2/(-e^2*x^2+d^2)^(5/2)-3/10*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)+1/10*d^3/e^4
*x/(-e^2*x^2+d^2)^(3/2)
```

Maxima [B] time = 1.52637, size = 455, normalized size = 3.2

$$\frac{1}{5}de^2x\left(\frac{15x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{ex^6}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-dx\left(\frac{3x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*d*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - e*x^6/(-e^2*x^2 + d
^2)^(5/2) - d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^
2)^(3/2)*e^4)) + 9*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d^3*x^3/((-e^2*
x^2 + d^2)^(5/2)*e^2) - 12*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^5*
x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 24/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^5) + 9/
10*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 6/5*d*x/(sqrt(-e^2*x^2 + d^2)*e^4)
- 3*d*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^4)
```

Fricas [A] time = 1.66556, size = 367, normalized size = 2.58

$$\frac{24\,de^3x^3-72\,d^2e^2x^2+72\,d^3ex-24\,d^4+30\left(de^3x^3-3\,d^2e^2x^2+3\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(5\,e^3x^3-39\,de^2x^2\right.}{5\left(e^8x^3-3\,de^7x^2+3\,d^2e^6x-d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*d*e^3*x^3 - 72*d^2*e^2*x^2 + 72*d^3*e*x - 24*d^4 + 30*(d*e^3*x^3 -
3*d^2*e^2*x^2 + 3*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
+ (5*e^3*x^3 - 39*d*e^2*x^2 + 57*d^2*e*x - 24*d^3)*sqrt(-e^2*x^2 + d^2))/(e
^8*x^3 - 3*d*e^7*x^2 + 3*d^2*e^6*x - d^3*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(d+ex\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.16533, size = 144, normalized size = 1.01

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}\left(d\right)-\frac{\left(24\,d^{6}e^{(-5)}+\left(15\,d^{5}e^{(-4)}-\left(60\,d^{4}e^{(-3)}+\left(35\,d^{3}e^{(-2)}-\left(45\,d^{2}e^{(-1)}-\left(5\,xe-24\,d\right)x\right)x\right)x\right)x}{5\left(x^{2}e^{2}-d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/5*(24*d^6*e^(-5) + (15*d^5*e^(-4) - (6
0*d^4*e^(-3) + (35*d^3*e^(-2) - (45*d^2*e^(-1) - (5*x*e - 24*d)*x)*x)*x)*x)
*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.85 $\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=118

$$\frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] (d^2*(d + e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d + e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d + e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) - Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rubi [A] time = 0.215959, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1635, 778, 217, 203}

$$\frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d + e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d + e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) - Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\left(\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d+ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

$$= \frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] time = 0.16662, size = 112, normalized size = 0.95

$$\frac{(d+ex)\left(d\left(22d^2-51dex+32e^2x^2\right)\sqrt{1-\frac{e^2x^2}{d^2}}-15(d-ex)^3\sin^{-1}\left(\frac{ex}{d}\right)\right)}{15de^4(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(d*(22*d^2 - 51*d*e*x + 32*e^2*x^2)*Sqrt[1 - (e^2*x^2)/d^2] - 15
*(d - e*x)^3*ArcSin[(e*x)/d]))/(15*d*e^4*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sq
rt[1 - (e^2*x^2)/d^2])
```

Maple [B] time = 0.093, size = 234, normalized size = 2.

$$\frac{ex^5}{5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}} - \frac{x^3}{3e}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}} + \frac{8x}{5e^3}\frac{1}{\sqrt{-x^2e^2+d^2}} - \frac{1}{e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + 3\frac{dx^4}{\left(-x^2e^2+d^2\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5*e*x^5/(-e^2*x^2+d^2)^(5/2)-1/3/e*x^3/(-e^2*x^2+d^2)^(3/2)+8/5/e^3*x/(-e
^2*x^2+d^2)^(1/2)-1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/
2))+3*d*x^4/(-e^2*x^2+d^2)^(5/2)-11/3*d^3/e^2*x^2/(-e^2*x^2+d^2)^(5/2)+22/1
5*d^5/e^4/(-e^2*x^2+d^2)^(5/2)+3/2/e*d^2*x^3/(-e^2*x^2+d^2)^(5/2)-9/10/e^3*
d^4*x/(-e^2*x^2+d^2)^(5/2)+3/10/e^3*d^2*x/(-e^2*x^2+d^2)^(3/2)
```

Maxima [B] time = 1.56242, size = 417, normalized size = 3.53

$$\frac{1}{15}e^3x\left(\frac{15x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2}-\frac{20d^2x^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^4}+\frac{8d^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^6}\right)-\frac{1}{3}ex\left(\frac{3x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2}-\frac{2d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right)+\frac{3dx}{\left(-e^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*x*(3*x^2/((-e^
2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 3*d*x^4/(-e
^2*x^2 + d^2)^(5/2) + 3/2*d^2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - 11/3*d^3*x^2
/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*d^4*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2
2/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e^4) + 17/30*d^2*x/((-e^2*x^2 + d^2)^(3/2)
*e^3) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*e^3) - arcsin(e^2*x/sqrt(d^2*e^2))/(sq
rt(e^2)*e^3)
```

Fricas [A] time = 1.69453, size = 336, normalized size = 2.85

$$\frac{22e^3x^3-66de^2x^2+66d^2ex-22d^3+30\left(e^3x^3-3de^2x^2+3d^2ex-d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(32e^2x^2-51dex+22d\right.}{15\left(e^7x^3-3de^6x^2+3d^2e^5x-d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*e^3*x^3 - 66*d*e^2*x^2 + 66*d^2*e*x - 22*d^3 + 30*(e^3*x^3 - 3*d*e
^2*x^2 + 3*d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (32*e
^2*x^2 - 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 - 3*d*e^6*x^2 +
3*d^2*e^5*x - d^3*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.18498, size = 128, normalized size = 1.08

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{\left(22d^5e^{(-4)}+\left(15d^4e^{(-3)}-\left(55d^3e^{(-2)}+\left(35d^2e^{(-1)}-(32xe+45d)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-4)*sgn(d) - 1/15*(22*d^5*e^(-4) + (15*d^4*e^(-3) - (55*d
^3*e^(-2) + (35*d^2*e^(-1) - (32*x*e + 45*d)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^
2)/(x^2*e^2 - d^2)^3
```

3.86 $\int \frac{x^2(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=93

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

```
[Out] (d*(d + e*x)^3)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (8*(d + e*x)^2)/(15*e^3*(d^
2 - e^2*x^2)^(3/2)) + (7*(d + e*x))/(15*d*e^3*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.125844, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1635, 789, 637}

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^3)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (8*(d + e*x)^2)/(15*e^3*(d^
2 - e^2*x^2)^(3/2)) + (7*(d + e*x))/(15*d*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^2(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{3d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{7\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2}$$

$$= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0849899, size = 58, normalized size = 0.62

$$\frac{(d+ex)\left(2d^2-6dex+7e^2x^2\right)}{15de^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(2*d^2 - 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

Maple [A] time = 0.056, size = 55, normalized size = 0.6

$$\frac{(-ex+d)(ex+d)^4\left(7x^2e^2-6dex+2d^2\right)}{15de^3}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(7*e^2*x^2-6*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(7/2
)
```

Maxima [A] time = 1.0136, size = 208, normalized size = 2.24

$$\frac{ex^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{3dx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{d^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{7d^3x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} + \frac{2d^4}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} + \frac{7dx}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] e*x^4/(-e^2*x^2 + d^2)^(5/2) + 3/2*d*x^3/(-e^2*x^2 + d^2)^(5/2) - 1/3*d^2*x
^2/((-e^2*x^2 + d^2)^(5/2)*e) - 7/10*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 2
/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^3) + 7/30*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2
) + 7/15*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)
```

Fricas [A] time = 1.54293, size = 212, normalized size = 2.28

$$\frac{2\,e^3x^3-6\,de^2x^2+6\,d^2ex-2\,d^3-\left(7\,e^2x^2-6\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(de^6x^3-3\,d^2e^5x^2+3\,d^3e^4x-d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*e^3*x^3 - 6*d*e^2*x^2 + 6*d^2*e*x - 2*d^3 - (7*e^2*x^2 - 6*d*e*x +
2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 - 3*d^2*e^5*x^2 + 3*d^3*e^4*x - d^4
*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\left(d+ex\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.22847, size = 97, normalized size = 1.04

$$-\frac{\left(2\,d^4e^{(-3)}-\left(5\,d^2e^{(-1)}-\left(x\left(\frac{7\,xe^2}{d}+15\,e\right)+5\,d\right)x\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(2*d^4*e^(-3) - (5*d^2*e^(-1) - (x*(7*x*e^2/d + 15*e) + 5*d)*x)*x^2)*
sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

3.87 $\int \frac{x(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

```
[Out] (d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(5*e^2*(d^2 - e^2
*x^2)^(3/2)) - x/(5*d^2*e*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0355586, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {789, 653, 191}

$$\frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(5*e^2*(d^2 - e^2
*x^2)^(3/2)) - x/(5*d^2*e*Sqrt[d^2 - e^2*x^2])
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{3\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}$$

$$= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e}$$

$$= \frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.190062, size = 55, normalized size = 0.64

$$-\frac{(d+ex)\left(d^2-3dex+e^2x^2\right)}{5d^2e^2(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -((d + e*x)*(d^2 - 3*d*e*x + e^2*x^2))/(5*d^2*e^2*(d - e*x)^2*Sqrt[d^2 - e^
2*x^2])
```

Maple [A] time = 0.049, size = 52, normalized size = 0.6

$$-\frac{(-ex+d)\,(ex+d)^4\left(x^2e^2-3\,dex+d^2\right)}{5\,d^2e^2}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/5*(-e*x+d)*(e*x+d)^4*(e^2*x^2-3*d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(7/2)
```

Maxima [A] time = 0.99952, size = 173, normalized size = 2.01

$$\frac{ex^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{dx^2}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{3\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{d^3}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} - \frac{x}{10\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e} - \frac{x}{5\sqrt{-e^2x^2+d^2}d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^3/(-e^2*x^2 + d^2)^(5/2) + d*x^2/(-e^2*x^2 + d^2)^(5/2) + 3/10*d^2*
x/((-e^2*x^2 + d^2)^(5/2)*e) - 1/5*d^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/10*
x/((-e^2*x^2 + d^2)^(3/2)*e) - 1/5*x/(sqrt(-e^2*x^2 + d^2)*d^2*e)
```

Fricas [A] time = 1.65068, size = 204, normalized size = 2.37

$$-\frac{e^3x^3 - 3\,de^2x^2 + 3\,d^2ex - d^3 - \left(e^2x^2 - 3\,dex + d^2\right)\sqrt{-e^2x^2 + d^2}}{5\left(d^2e^5x^3 - 3\,d^3e^4x^2 + 3\,d^4e^3x - d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(e^3*x^3 - 3*d*e^2*x^2 + 3*d^2*e*x - d^3 - (e^2*x^2 - 3*d*e*x + d^2)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 - 3*d^3*e^4*x^2 + 3*d^4*e^3*x - d^5*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(d + ex)^3}{(-(-d + ex)\,(d + ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.23682, size = 81, normalized size = 0.94

$$\frac{\left(d^3e^{(-2)} + \left(x\left(\frac{x^2e^3}{d^2} - 5\,e\right) - 5\,d\right)x^2\right)\sqrt{-x^2e^2 + d^2}}{5\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/5*(d^3*e^(-2) + (x*(x^2*e^3/d^2 - 5*e) - 5*d)*x^2)*sqrt(-x^2*e^2 + d^2)/(
x^2*e^2 - d^2)^3
```

3.88 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rubi [A] time = 0.0483763, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {655, 659, 651}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Mathematica [A] time = 0.0673158, size = 58, normalized size = 0.56

$$\frac{(d+ex)\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

Maple [A] time = 0.048, size = 55, normalized size = 0.5

$$\frac{(-ex+d)(ex+d)^4\left(2x^2e^2-6dex+7d^2\right)}{15d^3e}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2
)
```

Maxima [A] time = 0.991146, size = 136, normalized size = 1.32

$$\frac{ex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{4\,dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{7\,d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d} + \frac{2\,x}{15\sqrt{-e^2x^2+d^2}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

Fricas [A] time = 1.56251, size = 215, normalized size = 2.09

$$\frac{7\,e^3x^3 - 21\,de^2x^2 + 21\,d^2ex - 7\,d^3 - \left(2\,e^2x^2 - 6\,dex + 7\,d^2\right)\sqrt{-e^2x^2 + d^2}}{15\left(d^3e^4x^3 - 3\,d^4e^3x^2 + 3\,d^5e^2x - d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.18938, size = 95, normalized size = 0.92

$$-\frac{\sqrt{-x^2e^2 + d^2}\left(7\,d^2e^{(-1)} + \left(\left(x\left(\frac{2\,x^2e^4}{d^3} - \frac{5\,e^2}{d}\right) + 5\,e\right)x + 15\,d\right)x\right)}{15\left(x^2e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(7*d^2*e^(-1) + ((x*(2*x^2*e^4/d^3 - 5*e^2/d) +
5*e)*x + 15*d)*x)/(x^2*e^2 - d^2)^3
```

3.89 $\int \frac{(d+ex)^3}{x(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=114

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] (4*(d + e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d + 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d + 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rubi [A] time = 0.158929, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(d + e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d + 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d + 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-11d^2ex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2-22d^4e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^3e^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}
\end{aligned}
$$

Mathematica [C] time = 0.0619555, size = 81, normalized size = 0.71

$$
\frac{3d^5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)-55d^2e^3x^3+45d^4ex+9d^5+22e^5x^5}{15d^4\left(d^2-e^2x^2\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (9*d^5 + 45*d^4*e*x - 55*d^2*e^3*x^3 + 22*e^5*x^5 + 3*d^5*Hypergeometric2F1
[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(15*d^4*(d^2 - e^2*x^2)^(5/2))
```

Maple [A] time = 0.066, size = 158, normalized size = 1.4

$$\frac{4\,ex}{5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{11\,ex}{15\,d^2}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{22\,ex}{15\,d^4}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{4\,d}{5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{1}{3\,d}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{1}{d^3}\frac{1}{\sqrt{\cdot}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*e*x/(-e^2*x^2+d^2)^(5/2)+11/15*e/d^2*x/(-e^2*x^2+d^2)^(3/2)+22/15*e/d^4
*x/(-e^2*x^2+d^2)^(1/2)+4/5*d/(-e^2*x^2+d^2)^(5/2)+1/3/d/(-e^2*x^2+d^2)^(3/
2)+1/d^3/(-e^2*x^2+d^2)^(1/2)-1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60921, size = 327, normalized size = 2.87

$$\frac{32\,e^3x^3-96\,de^2x^2+96\,d^2ex-32\,d^3+15\left(e^3x^3-3\,de^2x^2+3\,d^2ex-d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(22\,e^2x^2-51\,dex+32\,d^2\right)\cdots}{15\left(d^4e^3x^3-3\,d^5e^2x^2+3\,d^6ex-d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*e^3*x^3 - 96*d*e^2*x^2 + 96*d^2*e*x - 32*d^3 + 15*(e^3*x^3 - 3*d*e
^2*x^2 + 3*d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (22*e^2*x^2
- 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 - 3*d^5*e^2*x^2 + 3
*d^6*e*x - d^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{x\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [A] time = 1.2307, size = 158, normalized size = 1.39

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{22\,xe^5}{d^4}+\frac{15\,e^4}{d^3}\right)-\frac{55\,e^3}{d^2}\right)x-\frac{35\,e^2}{d}\right)x+45\,e\right)x+32\,d\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(22*x*e^5/d^4 + 15*e^4/d^3) - 55*e^3/d^2)*
x - 35*e^2/d)*x + 45*e)*x + 32*d)/(x^2*e^2 - d^2)^3 - log(1/2*abs(-2*d*e -
2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4
```

3.90 $\int \frac{(d+ex)^3}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] (4*e*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (e*(5*d + 7*e*x))/(5*d^3*(d^2
 - e^2*x^2)^(3/2)) + (e*(15*d + 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqrt
[d^2 - e^2*x^2]/(d^5*x) - (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rubi [A] time = 0.287314, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1805, 807, 266, 63, 208}

$$\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (4*e*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (e*(5*d + 7*e*x))/(5*d^3*(d^2
 - e^2*x^2)^(3/2)) + (e*(15*d + 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqrt
[d^2 - e^2*x^2]/(d^5*x) - (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```


Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{-5d^3-15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}\\
&= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{15d^3+45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}\\
&= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-15d^3-45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}\\
&= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{(3e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}\\
&= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{(3e)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\right.}{2d^4}\\
&= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x\right.}{d^4e}\\
&= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}
\end{aligned}$$

Mathematica [C] time = 0.0602409, size = 96, normalized size = 0.66

$$\frac{3d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+45d^4e^2x^2-60d^2e^4x^4+d^5ex-5d^6+24e^6x^6}{5d^5x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-5*d^6 + d^5*e*x + 45*d^4*e^2*x^2 - 60*d^2*e^4*x^4 + 24*e^6*x^6 + 3*d^5*e*
x*Hypergeometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2])/(5*d^5*x*(d^2 - e^2*
x^2)^(5/2))
```

Maple [A] time = 0.064, size = 190, normalized size = 1.3

$$\frac{4e}{5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{9e^2x}{5d}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{12e^2x}{5d^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{24e^2x}{5d^5}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{e}{d^2}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+3\frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*e/(-e^2*x^2+d^2)^(5/2)+9/5/d*e^2*x/(-e^2*x^2+d^2)^(5/2)+12/5/d^3*e^2*x/
(-e^2*x^2+d^2)^(3/2)+24/5/d^5*e^2*x/(-e^2*x^2+d^2)^(1/2)+e/d^2/(-e^2*x^2+d^
2)^(3/2)+3*e/d^4/(-e^2*x^2+d^2)^(1/2)-3*e/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-d/x/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69005, size = 374, normalized size = 2.58

$$\frac{24e^4x^4-72de^3x^3+72d^2e^2x^2-24d^3ex+15\left(e^4x^4-3de^3x^3+3d^2e^2x^2-d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(24e^3x^3-57de\right.}{5\left(d^5e^3x^4-3d^6e^2x^3+3d^7ex^2-d^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*e^4*x^4 - 72*d*e^3*x^3 + 72*d^2*e^2*x^2 - 24*d^3*e*x + 15*(e^4*x^4
- 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x)
 - (24*e^3*x^3 - 57*d*e^2*x^2 + 39*d^2*e*x - 5*d^3)*sqrt(-e^2*x^2 + d^2))/(
d^5*e^3*x^4 - 3*d^6*e^2*x^3 + 3*d^7*e*x^2 - d^8*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [A] time = 1.22195, size = 250, normalized size = 1.72

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{19\,xe^6}{d^5}+\frac{15\,e^5}{d^4}\right)-\frac{45\,e^4}{d^3}\right)x-\frac{35\,e^3}{d^2}\right)x+\frac{30\,e^2}{d}\right)x+24\,e\right)}{5\left(x^2e^2-d^2\right)^3}-\frac{3\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{d^5}+\frac{xe^3}{2\left(de+\sqrt{-x^2e^2\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*((((x*(19*x*e^6/d^5 + 15*e^5/d^4) - 45*e^4/d^3)*x
 - 35*e^3/d^2)*x + 30*e^2/d)*x + 24*e)/(x^2*e^2 - d^2)^3 - 3*e*log(1/2*abs(
-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5 + 1/2*x*e^3/((d*e + s
qrt(-x^2*e^2 + d^2)*e)*d^5) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/(d^
5*x)
```

3.91 $\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

```
[Out] (4*e^2*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d + 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d + 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) - (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rubi [A] time = 0.361906, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e^2*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d + 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d + 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) - (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-20de^2x^2-16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+75de^2x^2+62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3-45d^2ex-90de^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{90d^4e+195d^3e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{30d^8}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} + \frac{(13\ldots}{\ldots}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} + \frac{(13\ldots}{\ldots}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} - \frac{13\ldots}{\ldots}$$

$$= \frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} - \frac{13\ldots}{\ldots}$$

Mathematica [C] time = 0.0758142, size = 119, normalized size = 0.65

$$\frac{e\left(9d^5ex\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+3d^5ex\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};1-\frac{e^2x^2}{d^2}\right)+285d^4e^2x^2-380d^2e^4x^4-45d^6+152e^6x^6\right)}{15d^6x\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (e*(-45*d^6 + 285*d^4*e^2*x^2 - 380*d^2*e^4*x^4 + 152*e^6*x^6 + 9*d^5*e*x*H
ypergeometric2F1[-5/2, 1, -3/2, 1 - (e^2*x^2)/d^2] + 3*d^5*e*x*Hypergeometr
ic2F1[-5/2, 2, -3/2, 1 - (e^2*x^2)/d^2]))/(15*d^6*x*(d^2 - e^2*x^2)^(5/2))
```

Maple [A] time = 0.065, size = 222, normalized size = 1.2

$$\frac{19\,e^3x}{5\,d^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{76\,e^3x}{15\,d^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{152\,e^3x}{15\,d^6}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{13\,e^2}{10\,d}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}+\frac{13\,e^2}{6\,d^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 19/5*e^3*x/d^2/(-e^2*x^2+d^2)^(5/2)+76/15*e^3/d^4*x/(-e^2*x^2+d^2)^(3/2)+15
2/15*e^3/d^6*x/(-e^2*x^2+d^2)^(1/2)+13/10/d*e^2/(-e^2*x^2+d^2)^(5/2)+13/6/d
^3*e^2/(-e^2*x^2+d^2)^(3/2)+13/2/d^5*e^2/(-e^2*x^2+d^2)^(1/2)-13/2/d^5*e^2/
(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/2*d/x^2/(-e^
2*x^2+d^2)^(5/2)-3*e/x/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73885, size = 424, normalized size = 2.33

$$\frac{254\,e^5x^5-762\,de^4x^4+762\,d^2e^3x^3-254\,d^3e^2x^2+195\left(e^5x^5-3\,de^4x^4+3\,d^2e^3x^3-d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(304\right.}{30\left(d^6e^3x^5-3\,d^7e^2x^4+3\,d^8ex^3-d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*e^5*x^5 - 762*d*e^4*x^4 + 762*d^2*e^3*x^3 - 254*d^3*e^2*x^2 + 195
*(e^5*x^5 - 3*d*e^4*x^4 + 3*d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*
```

```
x^2 + d^2))/x) - (304*e^4*x^4 - 717*d*e^3*x^3 + 479*d^2*e^2*x^2 - 45*d^3*e*
x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 - 3*d^7*e^2*x^4 + 3*d^8*e*x^
3 - d^9*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [A] time = 1.22188, size = 350, normalized size = 1.92

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{107\,xe^7}{d^6}+\frac{90\,e^6}{d^5}\right)-\frac{245\,e^5}{d^4}\right)x-\frac{205\,e^4}{d^3}\right)x+\frac{150\,e^3}{d^2}\right)x+\frac{127\,e^2}{d}\right)}{15\left(x^2e^2-d^2\right)^3}-\frac{13\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^6}+\frac{x^2\left(\frac{12\left(de\cdots\right.}{\cdots}\right.}{8\left(de+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((((x*(107*x*e^7/d^6 + 90*e^6/d^5) - 245*e^5/d^4
)*x - 205*e^4/d^3)*x + 150*e^3/d^2)*x + 127*e^2/d)/(x^2*e^2 - d^2)^3 - 13/2
*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6 + 1/
8*x^2*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + e^6)/((d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*d^6) - 1/8*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6*e^8/x + (d*e
 + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*e^6/x^2)*e^(-8)/d^12
```

3.92 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=147

$$\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}+\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}$$

```
[Out] (4*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(4*e
^2) + (x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (d^3*(64*d - 45*e*x)*Sqrt[d^2 - e^2
*x^2])/(120*e^5) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^5)
```

Rubi [A] time = 0.141667, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 833, 780, 217, 203}

$$\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}+\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```


```
[Out] (4*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(4*e
^2) + (x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (d^3*(64*d - 45*e*x)*Sqrt[d^2 - e^2
*x^2])/(120*e^5) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^5)
```


Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^4(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x^3(4d^2e-5de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2}$$

$$= -\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{\int \frac{x^2(15d^3e^2-16d^2e^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4}$$

$$= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x(32d^4e^3-45d^3e^4x)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6}$$

$$= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{(3d^5)\int \frac{1}{\sqrt{d^2-e^2 …}}}{8e^4}$$

$$= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{(3d^5)\,\mathrm{Subst}\big(…}{…}$$

$$= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5} + \frac{3d^5\tan^{-1}\big(\frac{…}{\sqrt{d^2 …}}}{8e^5}$$

Mathematica [A] time = 0.150789, size = 91, normalized size = 0.62

$$\frac{\sqrt{d^2-e^2x^2}\left(32d^2e^2x^2-45d^3ex+64d^4-30de^3x^3+24e^4x^4\right)+45d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{120e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^4 - 45*d^3*e*x + 32*d^2*e^2*x^2 - 30*d*e^3*x^3 +
 24*e^4*x^4) + 45*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(120*e^5)
```

Maple [A] time = 0.064, size = 208, normalized size = 1.4

$$-\frac{x^2}{5e^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} - \frac{7d^2}{15e^5}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} + \frac{dx}{4e^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} - \frac{5d^3x}{8e^4}\sqrt{-x^2e^2+d^2} - \frac{5d^5}{8e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/5/e^3*x^2*(-e^2*x^2+d^2)^(3/2)-7/15*d^2/e^5*(-e^2*x^2+d^2)^(3/2)+1/4*d/e
^4*x*(-e^2*x^2+d^2)^(3/2)-5/8*d^3/e^4*x*(-e^2*x^2+d^2)^(1/2)-5/8*d^5/e^4/(e
```

```
^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d^4/e^5*(-(d/e+x)^2*e^
2+2*d*e*(d/e+x))^(1/2)+d^5/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2
*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57821, size = 207, normalized size = 1.41

$$-\frac{90\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,e^4x^4-30\,de^3x^3+32\,d^2e^2x^2-45\,d^3ex+64\,d^4\right)\sqrt{-e^2x^2+d^2}}{120\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(90*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (24*e^4*x^4 - 30
*d*e^3*x^3 + 32*d^2*e^2*x^2 - 45*d^3*e*x + 64*d^4)*sqrt(-e^2*x^2 + d^2))/e^
5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A] time = 1.19613, size = 104, normalized size = 0.71

$$\frac{3}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}\left(d\right)+\frac{1}{120}\left(64\,d^4e^{(-5)}-\left(45\,d^3e^{(-4)}-2\left(16\,d^2e^{(-3)}+3\left(4\,xe^{(-1)}-5\,de^{(-2)}\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/8*d^5*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/120*(64*d^4*e^(-5) - (45*d^3*e^(-4)
 - 2*(16*d^2*e^(-3) + 3*(4*x*e^(-1) - 5*d*e^(-2))*x)*x)*x)*sqrt(-x^2*e^2 +
d^2)
```

3.93 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=118

$$-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

```
[Out] -(d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e^2) + (x^3*Sqrt[d^2 - e^2*x^2])/(4*e) - (d
^2*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^4) - (3*d^4*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(8*e^4)
```

Rubi [A] time = 0.0989571, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 833, 780, 217, 203}

$$-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -(d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e^2) + (x^3*Sqrt[d^2 - e^2*x^2])/(4*e) - (d
^2*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^4) - (3*d^4*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(8*e^4)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= \int \frac{x^3(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{\int \frac{x^2\left(3d^2e-4de^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2} \\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} + \frac{\int \frac{x\left(8d^3e^2-9d^2e^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4} \\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{\left(3d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^3} \\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{\left(3d^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{}{\sqrt{d^2}}\right.}{8e^3} \\
&= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}
\end{aligned}$$

Mathematica [A] time = 0.115075, size = 80, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2x^2}\left(9d^2ex-16d^3-8de^2x^2+6e^3x^3\right)-9d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{24e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 + 9*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3) - 9*d^
4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^4)
```

Maple [A] time = 0.064, size = 185, normalized size = 1.6

$$-\frac{x}{4e^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{5d^2x}{8e^3}\sqrt{-x^2e^2+d^2}+\frac{5d^4}{8e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+\frac{d}{3e^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{d^3}{e^4}\sqrt{-\left(\frac{d}{e}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/4/e^3*x*(-e^2*x^2+d^2)^(3/2)+5/8*d^2/e^3*x*(-e^2*x^2+d^2)^(1/2)+5/8/e^3*
d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/3*d/e^4*(-e^2*
x^2+d^2)^(3/2)-d^3/e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-d^4/e^3/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54405, size = 177, normalized size = 1.5

$$\frac{18\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3-8\,de^2x^2+9\,d^2ex-16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(18*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 - 8*d*e
^2*x^2 + 9*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A] time = 1.15272, size = 89, normalized size = 0.75

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}\left(d\right)-\frac{1}{24}\left(16\,d^3e^{(-4)}-\left(9\,d^2e^{(-3)}+2\left(3\,xe^{(-1)}-4\,de^{(-2)}\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/8*d^4*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/24*(16*d^3*e^(-4) - (9*d^2*e^(-3)
+ 2*(3*x*e^(-1) - 4*d*e^(-2))*x)*x)*sqrt(-x^2*e^2 + d^2)
```

3.94 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=86

$$\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] (d*(2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (d^2 - e^2*x^2)^(3/2)/(3*e^3)
 + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rubi [A] time = 0.11044, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1639, 12, 785, 780, 217, 203}

$$\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (d^2 - e^2*x^2)^(3/2)/(3*e^3)
 + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{\int \frac{3de^3x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{d\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{e}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{\int \frac{x\left(d^2e-de^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}
\end{aligned}
$$

Mathematica [A] time = 0.0854072, size = 69, normalized size = 0.8

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^2-3dex+2e^2x^2\right)+3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 - 3*d*e*x + 2*e^2*x^2) + 3*d^3*ArcTan[(e*x)/Sqr
t[d^2 - e^2*x^2]])/(6*e^3)
```

Maple [B] time = 0.058, size = 160, normalized size = 1.9

$$-\frac{1}{3e^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{dx}{2e^2}\sqrt{-x^2e^2+d^2}-\frac{d^3}{2e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+\frac{d^2}{e^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}+\frac{d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/e^3-1/2*d*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/2/e^2*d^3/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d^2/e^3*(-(d/e+x)^2*
e^2+2*d*e*(d/e+x))^(1/2)+d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55548, size = 153, normalized size = 1.78

$$-\frac{6\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(2\,e^2x^2-3\,dex+4\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (2*e^2*x^2 - 3*d*e*
x + 4*d^2)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A] time = 1.1845, size = 73, normalized size = 0.85

$$\frac{1}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}\left(d\right)+\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\left(4\,d^2e^{(-3)}+\left(2\,xe^{(-1)}-3\,de^{(-2)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3
) + (2*x*e^(-1) - 3*d*e^(-2))*x)
```

3.95 $\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=62

$$-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

```
[Out] -((2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (d^2*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(2*e^2)
```

Rubi [A] time = 0.0407475, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {785, 780, 217, 203}

$$-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -((2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (d^2*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(2*e^2)
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\int \frac{x(d^2e-de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{de}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2 \int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$
$$= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Mathematica [A] time = 0.0665321, size = 57, normalized size = 0.92

$$\frac{(ex-2d)\sqrt{d^2-e^2x^2}-d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((-2*d + e*x)*Sqrt[d^2 - e^2*x^2] - d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(2*e^2)
```

Maple [B] time = 0.054, size = 140, normalized size = 2.3

$$\frac{x}{2e}\sqrt{-x^2e^2+d^2}+\frac{d^2}{2e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{d}{e^2}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}-\frac{d^2}{e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(\frac{d}{e}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*x*(-e^2*x^2+d^2)^(1/2)+1/2/e*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e
^2*x^2+d^2)^(1/2))-d/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-d^2/e/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57045, size = 127, normalized size = 2.05

$$\frac{2\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex-2\,d)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2)
*(e*x - 2*d))/e^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A] time = 1.23454, size = 58, normalized size = 0.94

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}\left(d\right)+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\left(xe^{(-1)}-2\,de^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^(-2)*sgn(d) + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^(-1) -
 2*d*e^(-2))
```

3.96 $\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] Sqrt[d^2 - e^2*x^2]/e + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rubi [A] time = 0.0161483, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {665, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/e + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{e}+d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{\sqrt{d^2-e^2x^2}}{e}+d\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= \frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}
\end{aligned}$$

Mathematica [A] time = 0.0245406, size = 43, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2x^2}+d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2] + d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Maple [A] time = 0.047, size = 77, normalized size = 1.7

$$\frac{1}{e}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}+d\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65665, size = 101, normalized size = 2.2

$$-\frac{2\,d\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -(2*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - sqrt(-e^2*x^2 + d^2))/e
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A] time = 1.21602, size = 42, normalized size = 0.91

$$d\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}\left(d\right)+\sqrt{-x^2e^2+d^2}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] d*arcsin(x*e/d)*e^(-1)*sgn(d) + sqrt(-x^2*e^2 + d^2)*e^(-1)
```

3.97 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)} dx$

Optimal. Leaf size=46

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.059482, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 844, 217, 203, 266, 63, 208}

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx &= \int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}\,dx \\
&= d\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right) \\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2} \\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.0426394, size = 46, normalized size = 1.

$$-\log\left(\sqrt{d^2-e^2x^2}+d\right) - \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + Log[x] - Log[d + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] time = 0.063, size = 137, normalized size = 3.

$$\frac{1}{d}\sqrt{-x^2e^2+d^2} - d\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{1}{d}\sqrt{-\left(\frac{d}{e}+x\right)^2 e^2+2\,de\left(\frac{d}{e}+x\right)} - e\arctan\left(x\sqrt{e^2}\frac{}{\sqrt{-(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x)
```

```
[Out] (-e^2*x^2+d^2)^(1/2)/d-d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)-1/d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-e/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55558, size = 111, normalized size = 2.41

$$2 \arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) + \log\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + log(-(d - sqrt(-e^2*x^2 + d^2
))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)), x)
```

Giac [A] time = 1.23569, size = 65, normalized size = 1.41

$$-\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d) - \log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*sgn(d) - log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/abs(x))
```

3.98 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)} dx$

Optimal. Leaf size=51

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d
```

Rubi [A] time = 0.0573296, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 807, 266, 63, 208}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx &= \int \frac{d-ex}{x^2\sqrt{d^2-e^2x^2}}\,dx\\ &= -\frac{\sqrt{d^2-e^2x^2}}{dx}-e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\ &= -\frac{\sqrt{d^2-e^2x^2}}{dx}-\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{d^2-e^2x^2}}{dx}+\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{dx}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0691569, size = 53, normalized size = 1.04

$$-\frac{\sqrt{d^2-e^2x^2}-ex\log\left(\sqrt{d^2-e^2x^2}+d\right)+ex\log(x)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2] + e*x*Log[x] - e*x*Log[d + Sqrt[d^2 - e^2*x^2]])/(d*
x))
```

Maple [B] time = 0.062, size = 222, normalized size = 4.4

$$-\frac{e}{d^2}\sqrt{-x^2e^2+d^2}+e\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{e}{d^2}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}+\frac{e^2}{d}\arctan\left(x\sqrt{e^2}\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] -e/d^2*(-e^2*x^2+d^2)^(1/2)+e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x)+e/d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+e^2/d/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/d^3/x*(-e^2*
x^2+d^2)^(3/2)-1/d^3*e^2*x*(-e^2*x^2+d^2)^(1/2)-1/d*e^2/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^2), x)
```

Fricas [A] time = 1.593, size = 97, normalized size = 1.9

$$-\frac{ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2 + d^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] -(e*x*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^2(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)), x)
```

Giac [B] time = 1.23455, size = 138, normalized size = 2.71

$$\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{d} + \frac{xe^3}{2\left(de + \sqrt{-x^2e^2 + d^2}e\right)d} - \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)e^{(-1)}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d + 1/2*x*e
^3/((d*e + sqrt(-x^2*e^2 + d^2)*e)*d) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
e^(-1)/(d*x)
```

3.99 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=82

$$\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(2*d*x^2) + (e*Sqrt[d^2 - e^2*x^2])/(d^2*x) - (e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^2)
```

Rubi [A] time = 0.0783041, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(2*d*x^2) + (e*Sqrt[d^2 - e^2*x^2])/(d^2*x) - (e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^2)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx &= \int \frac{d-ex}{x^3\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} - \frac{\int \frac{2d^2e-de^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} + \frac{e^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} + \frac{e^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2} + \frac{e\sqrt{d^2-e^2x^2}}{d^2x} - \frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}
\end{aligned}$$

Mathematica [A] time = 0.11493, size = 70, normalized size = 0.85

$$-\frac{(d-2ex)\sqrt{d^2-e^2x^2}+e^2x^2\log\left(\sqrt{d^2-e^2x^2}+d\right)-e^2x^2\log(x)}{2d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -((d - 2*e*x)*Sqrt[d^2 - e^2*x^2] - e^2*x^2*Log[x] + e^2*x^2*Log[d + Sqrt[d
^2 - e^2*x^2]])/(2*d^2*x^2)
```

Maple [B] time = 0.066, size = 254, normalized size = 3.1

$$\frac{e^2}{2\,d^3}\sqrt{-x^2e^2+d^2}-\frac{e^2}{2\,d}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{e^2}{d^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}-\frac{e^3}{d^2}\arctan\left(x\sqrt{e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] 1/2*e^2/d^3*(-e^2*x^2+d^2)^(1/2)-1/2*e^2/d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1
/2)*(-e^2*x^2+d^2)^(1/2))/x)-e^2/d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-e
^3/d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2
))-1/2/d^3/x^2*(-e^2*x^2+d^2)^(3/2)+e/d^4/x*(-e^2*x^2+d^2)^(3/2)+e^3/d^4*x*
(-e^2*x^2+d^2)^(1/2)+e^3/d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^3), x)
```

Fricas [A] time = 1.84987, size = 128, normalized size = 1.56

$$\frac{e^2x^2 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2 + d^2}(2\,ex - d)}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*x^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)*(2*e
*x - d))/(d^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^3(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.100 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=114

$$-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*d*x^3) + (e*Sqrt[d^2 - e^2*x^2])/(2*d^2*x^2) - (2*e
^2*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) + (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d^3)
```

Rubi [A] time = 0.110491, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(3*d*x^3) + (e*Sqrt[d^2 - e^2*x^2])/(2*d^2*x^2) - (2*e
^2*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) + (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d^3)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx &= \int \frac{d-ex}{x^4\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{3d^2e-2de^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} + \frac{\int \frac{4d^3e^2-3d^2e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{e^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{4d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}
\end{aligned}$$

Mathematica [A] time = 0.11598, size = 84, normalized size = 0.74

$$\frac{\left(-2d^2+3dex-4e^2x^2\right)\sqrt{d^2-e^2x^2}+3e^3x^3\log\left(\sqrt{d^2-e^2x^2}+d\right)-3e^3x^3\log(x)}{6d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] ((-2*d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2] - 3*e^3*x^3*Log[x] + 3*
e^3*x^3*Log[d + Sqrt[d^2 - e^2*x^2]])/(6*d^3*x^3)
```

Maple [B] time = 0.069, size = 280, normalized size = 2.5

$$-\frac{e^3}{2\,d^4}\sqrt{-x^2e^2+d^2}+\frac{e^3}{2\,d^2}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{1}{3\,d^3x^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{e^3}{d^4}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] -1/2*e^3/d^4*(-e^2*x^2+d^2)^(1/2)+1/2*e^3/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^3/x^3*(-e^2*x^2+d^2)^(3/2)+e^3/d^4*(-
(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+e^4/d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))+1/2*e/d^4/x^2*(-e^2*x^2+d^2)^(3/2)-e
^2/d^5/x*(-e^2*x^2+d^2)^(3/2)-e^4/d^5*x*(-e^2*x^2+d^2)^(1/2)-e^4/d^3/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^4), x)
```

Fricas [A] time = 1.92524, size = 157, normalized size = 1.38

$$-\frac{3\,e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(4\,e^2x^2-3\,dex+2\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(3*e^3*x^3*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (4*e^2*x^2 - 3*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^4(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.101 $\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=143

$$\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(4*d*x^4) + (e*Sqrt[d^2 - e^2*x^2])/(3*d^2*x^3) - (3*e
^2*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) + (2*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^4*x)
 - (3*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rubi [A] time = 0.135082, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 835, 807, 266, 63, 208}

$$\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(4*d*x^4) + (e*Sqrt[d^2 - e^2*x^2])/(3*d^2*x^3) - (3*e
^2*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) + (2*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^4*x)
 - (3*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx &= \int \frac{d-ex}{x^5\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}-\frac{\int \frac{4d^2e-3de^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}+\frac{\int \frac{9d^3e^2-8d^2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{\int \frac{16d^4e^3-9d^3e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}+\frac{\left(3e^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^3} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}+\frac{\left(3e^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x\ldots\right.}{16d^3} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{\left(3e^2\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d\ldots}\right.}{8d^3} \\
&= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}
\end{aligned}
$$

Mathematica [A] time = 0.136423, size = 95, normalized size = 0.66

$$
\frac{\sqrt{d^2-e^2x^2}\left(8d^2ex-6d^3-9de^2x^2+16e^3x^3\right)-9e^4x^4\log\left(\sqrt{d^2-e^2x^2}+d\right)+9e^4x^4\log(x)}{24d^4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 9*d*e^2*x^2 + 16*e^3*x^3) + 9*e^
4*x^4*Log[x] - 9*e^4*x^4*Log[d + Sqrt[d^2 - e^2*x^2]])/(24*d^4*x^4)
```

Maple [B] time = 0.077, size = 304, normalized size = 2.1

$$\frac{3e^4}{8d^5}\sqrt{-x^2e^2+d^2}-\frac{3e^4}{8d^3}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{e}{3d^4x^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{e^4}{d^5}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x)
```

```
[Out] 3/8*e^4/d^5*(-e^2*x^2+d^2)^(1/2)-3/8*e^4/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^
(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/3*e/d^4/x^3*(-e^2*x^2+d^2)^(3/2)-e^4/d^5*(
-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-e^5/d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*
x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/4/d^3/x^4*(-e^2*x^2+d^2)^(3/2)-5/
8/d^5*e^2/x^2*(-e^2*x^2+d^2)^(3/2)+e^3/d^6/x*(-e^2*x^2+d^2)^(3/2)+e^5/d^6*x
*(-e^2*x^2+d^2)^(1/2)+e^5/d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^
2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^5), x)
```

Fricas [A] time = 1.75541, size = 180, normalized size = 1.26

$$\frac{9e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16e^3x^3-9de^2x^2+8d^2ex-6d^3\right)\sqrt{-e^2x^2+d^2}}{24d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(9*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 - 9*d*e^2*
x^2 + 8*d^2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^5(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**5*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.102 $\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex} dx$

Optimal. Leaf size=113

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) + (d*(4*d - 3*e*x)*(d^2 - e^2*x^2)^(3/2
))/(12*e^3) - (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(8*e^3)
```

Rubi [A] time = 0.131154, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1639, 12, 785, 780, 195, 217, 203}

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```


```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) + (d*(4*d - 3*e*x)*(d^2 - e^2*x^2)^(3/2
))/(12*e^3) - (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(8*e^3)
```


Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\int \frac{5de^3x\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{5e^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{d\int \frac{x\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{e}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}-\frac{\int x\left(d^2e-de^2x\right)\sqrt{d^2-e^2x^2}\,dx}{e^2}\\
&= \frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{4e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{8e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{\cdots}{\sqrt{d^2\cdots}}\right)}{8e^2}\\
&= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}
\end{aligned}
$$

Mathematica [A] time = 0.132477, size = 112, normalized size = 0.99

$$
\frac{\sqrt{d^2-e^2x^2}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\left(8d^2e^2x^2-15d^3ex+16d^4+30de^3x^3-24e^4x^4\right)+15d^4\sin^{-1}\left(\frac{ex}{d}\right)\right)}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(16*d^4 - 15*d^3*e*x + 8*d^2*
e^2*x^2 + 30*d*e^3*x^3 - 24*e^4*x^4) + 15*d^4*ArcSin[(e*x)/d]))/(120*e^3*Sq
```

```
rt[1 - (e^2*x^2)/d^2])
```

Maple [B] time = 0.056, size = 222, normalized size = 2.

$$-\frac{1}{5\,e^{3}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}-\frac{dx}{4\,e^{2}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}-\frac{3\,d^{3}x}{8\,e^{2}}\sqrt{-x^{2}e^{2}+d^{2}}-\frac{3\,d^{5}}{8\,e^{2}}\arctan\left(x\sqrt{e^{2}}\frac{1}{\sqrt{-x^{2}e^{2}+d^{2}}}\right)\frac{1}{\sqrt{e^{2}}}+\frac{d^{2}}{3\,e^{3}}\left(-\left(\frac{d}{e}+x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x)
```

```
[Out] -1/5*(-e^2*x^2+d^2)^(5/2)/e^3-1/4*d*x*(-e^2*x^2+d^2)^(3/2)/e^2-3/8*d^3*x*(-
e^2*x^2+d^2)^(1/2)/e^2-3/8/e^2*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))+1/3*d^2/e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)+1/2*d^3/e^2
*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+1/2*d^5/e^2/(e^2)^(1/2)*arctan((e^2
)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95208, size = 205, normalized size = 1.81

$$-\frac{30\,d^{5}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)+\left(24\,e^{4}x^{4}-30\,de^{3}x^{3}-8\,d^{2}e^{2}x^{2}+15\,d^{3}ex-16\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{120\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*e^4*x^4 - 30
*d*e^3*x^3 - 8*d^2*e^2*x^2 + 15*d^3*e*x - 16*d^4)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [C] time = 6.75831, size = 280, normalized size = 2.48

$$d\left(\begin{cases}-\frac{id^{4}\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^{3}}+\frac{id^{3}x}{8e^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{3idx^{3}}{8\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\frac{ie^{2}x^{5}}{4d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\ \frac{|e^{2}x^{2}|}{|d^{2}|}>1\\ \frac{d^{4}\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^{3}}-\frac{d^{3}x}{8e^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{3dx^{3}}{8\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-\frac{e^{2}x^{5}}{4d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)-e\left(\left(\begin{cases}-\frac{2d^{4}\sqrt{d^{2}-e^{2}x^{2}}}{15e^{4}}-\frac{d^{2}x^{2}\sqrt{d^{2}-e^{2}x^{2}}}{15e^{2}}+\frac{x^{4}\sqrt{d^{2}-e^{2}x^{2}}}{5}\\ \frac{x^{4}\sqrt{d^{2}}}{4}\end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(3/2)/(e*x+d),x)
```

```
[Out] d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/
d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1
 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - e*
Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
 e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqr
t(d**2)/4, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.103 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex} dx$

Optimal. Leaf size=201

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}+\frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4}{}$$

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) + (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) + (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) + (d^3*(128*d - 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rubi [A] time = 0.160258, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 833, 780, 195, 217, 203}

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4}+\frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}+\frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3}-\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2}+\frac{x^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```


```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) + (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) + (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) + (d^3*(128*d - 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```


Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \int x^4(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$$
$$= \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{\int x^3\left(4d^2e-9de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{9e^2}$$
$$= -\frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} + \frac{\int x^2\left(27d^3e^2-32d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{72e^4}$$
$$= \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} - \frac{\int x\left(64d^4e^3-189d^3e^4x\right)\left(d^2 \cdots\right.}{504e^6}$$
$$= \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} + \frac{d^3(128d-315ex)\left(d^2-e^2x^2\right)\cdots}{5040e^5}$$
$$= \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} + \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{9e} + \frac{d^3(128d\cdots}{\cdots}$$
$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} + \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-\cdots\right.}{9\cdots}$$
$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} + \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-\cdots\right.}{9\cdots}$$
$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x\left(d^2-e^2x^2\right)^{3/2}}{64e^4} + \frac{4d^2x^2\left(d^2-e^2x^2\right)^{5/2}}{63e^3} - \frac{dx^3\left(d^2-e^2x^2\right)^{5/2}}{8e^2} + \frac{x^4\left(d^2-\cdots\right.}{9\cdots}$$

Mathematica [A] time = 0.191343, size = 135, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(512d^6e^2x^2-630d^5e^3x^3+384d^4e^4x^4+7560d^3e^5x^5-6400d^2e^6x^6-945d^7ex+1024d^8-5040de^7x^7+448\cdots\right.}{40320e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(1024*d^8 - 945*d^7*e*x + 512*d^6*e^2*x^2 - 630*d^5*e^
3*x^3 + 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 - 5040*d*e^7*
```

```
x^7 + 4480*e^8*x^8) + 945*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(40320*e^5
)
```

Maple [A] time = 0.066, size = 330, normalized size = 1.6

$$-\frac{x^2}{9e^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{11d^2}{63e^5}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}+\frac{dx}{8e^4}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{3d^3x}{16e^4}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{15d^5x}{64e^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{45d^7x}{128e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/9/e^3*x^2*(-e^2*x^2+d^2)^(7/2)-11/63*d^2/e^5*(-e^2*x^2+d^2)^(7/2)+1/8*d/
e^4*x*(-e^2*x^2+d^2)^(7/2)-3/16*d^3/e^4*x*(-e^2*x^2+d^2)^(5/2)-15/64*d^5*x*
(-e^2*x^2+d^2)^(3/2)/e^4-45/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4-45/128*d^9/e
^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/5*d^4/e^5*(-(d/
e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+1/4*d^5/e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^
(3/2)*x+3/8*d^7/e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+3/8*d^9/e^4/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79688, size = 323, normalized size = 1.61

$$-\frac{1890\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(4480\,e^8x^8-5040\,de^7x^7-6400\,d^2e^6x^6+7560\,d^3e^5x^5+384\,d^4e^4x^4-630\,d^5e^3x^3+512\right.}{40320\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (4480*e^8*x^
8 - 5040*d*e^7*x^7 - 6400*d^2*e^6*x^6 + 7560*d^3*e^5*x^5 + 384*d^4*e^4*x^4
- 630*d^5*e^3*x^3 + 512*d^6*e^2*x^2 - 945*d^7*e*x + 1024*d^8)*sqrt(-e^2*x^2
 + d^2))/e^5
```

Sympy [C] time = 22.2807, size = 833, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5
*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x*
*2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1
- e**2*x**2/d**2)), True)) - d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**
2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*s
qrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)),
 (x**6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128
*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(3
84*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x
**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/
d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/
(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**
2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 -
 e**2*x**2/d**2)), True)) + e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)
/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*s
qrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**
2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.104 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=172

$$-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}-\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}+\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{3d^8\tan}{}$$

```
[Out] (-3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) - (d^4*x*(d^2 - e^2*x^2)^(3/2))/(6
4*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) + (x^3*(d^2 - e^2*x^2)^(5/2)
)/(8*e) - (d^2*(32*d - 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) - (3*d^8*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rubi [A] time = 0.123488, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 833, 780, 195, 217, 203}

$$-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}-\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3}-\frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}-\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2}+\frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e}-\frac{3d^8\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) - (d^4*x*(d^2 - e^2*x^2)^(3/2))/(6
4*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) + (x^3*(d^2 - e^2*x^2)^(5/2)
)/(8*e) - (d^2*(32*d - 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) - (3*d^8*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \int x^3(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$$
$$= \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{\int x^2\left(3d^2e-8de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$
$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} + \frac{\int x\left(16d^3e^2-21d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{56e^4}$$
$$= -\frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{d^4\int\left(d^2-e^2x^2\right)^{3/2}}{16e^3}$$
$$= -\frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}$$
$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-3}{}$$
$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-3}{}$$
$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x\left(d^2-e^2x^2\right)^{3/2}}{64e^3} - \frac{dx^2\left(d^2-e^2x^2\right)^{5/2}}{7e^2} + \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{8e} - \frac{d^2(32d-3}{}$$

Mathematica [A] time = 0.139402, size = 124, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(-128d^5e^2x^2+70d^4e^3x^3+1024d^3e^4x^4-840d^2e^5x^5+105d^6ex-256d^7-640de^6x^6+560e^7x^7\right)-105d^8\tan}{4480e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^7 + 105*d^6*e*x - 128*d^5*e^2*x^2 + 70*d^4*e^3
*x^3 + 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 - 640*d*e^6*x^6 + 560*e^7*x^7) -
105*d^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4480*e^4)
```

Maple [B] time = 0.059, size = 305, normalized size = 1.8

$$-\frac{x}{8 e^{3}} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{7}{2}}+\frac{3 d^{2} x}{16 e^{3}} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{5}{2}}+\frac{15 d^{4} x}{64 e^{3}} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}}+\frac{45 d^{6} x}{128 e^{3}} \sqrt{-x^{2} e^{2}+d^{2}}+\frac{45 d^{8}}{128 e^{3}} \arctan\left(x \sqrt{e^{2}} \frac{1}{\sqrt{-x^{2} e^{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/8/e^3*x*(-e^2*x^2+d^2)^(7/2)+3/16*d^2/e^3*x*(-e^2*x^2+d^2)^(5/2)+15/64*d
^4*x*(-e^2*x^2+d^2)^(3/2)/e^3+45/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^3+45/128*
d^8/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/7*d/e^4*(-
e^2*x^2+d^2)^(7/2)-1/5*d^3/e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/4*d^4
/e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-3/8*d^6/e^3*(-(d/e+x)^2*e^2+2*d
*e*(d/e+x))^(1/2)*x-3/8*d^8/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^
2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65833, size = 288, normalized size = 1.67

$$\frac{210 d^{8} \arctan\left(-\frac{d-\sqrt{-e^{2} x^{2}+d^{2}}}{ex}\right)+\left(560 e^{7} x^{7}-640 d e^{6} x^{6}-840 d^{2} e^{5} x^{5}+1024 d^{3} e^{4} x^{4}+70 d^{4} e^{3} x^{3}-128 d^{5} e^{2} x^{2}+105 d^{6} e x-\right.}{4480 e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(210*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (560*e^7*x^7 -
640*d*e^6*x^6 - 840*d^2*e^5*x^5 + 1024*d^3*e^4*x^4 + 70*d^4*e^3*x^3 - 128*d
^5*e^2*x^2 + 105*d^6*e*x - 256*d^7)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] time = 20.617, size = 779, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) - d**2*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*a
sin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**
3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2
)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((-
8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**
2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2
 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + e**3*Piecewise((-5
*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/
d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(
192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d*
*2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2
) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x
**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(1
92*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2))
- e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.105 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=140

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) + (d*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^3) - (d^2 - e^2*x^2)^(7/
2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rubi [A] time = 0.150291, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1639, 12, 785, 780, 195, 217, 203}

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) + (d*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^3) - (d^2 - e^2*x^2)^(7/
2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
```

```
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{\int \frac{7de^3x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{7e^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{d\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{e}\\
&= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}-\frac{\int x\left(d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}\\
&= \frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^3\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{6e^2}\\
&= \frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^5\int \sqrt{d^2-e^2x^2}\,dx}{8e^2}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\int \frac{\ }{\sqrt{d^{\cdots}}}}{16\cdots}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\,\mathrm{Subs}\cdots}{\cdots}\\
&= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x\left(d^2-e^2x^2\right)^{3/2}}{24e^2}+\frac{d(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^3}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e^3}+\frac{d^7\tan^{-\cdots}}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.108896, size = 113, normalized size = 0.81

$$\frac{\sqrt{d^2-e^2x^2}\left(48d^4e^2x^2+490d^3e^3x^3-384d^2e^4x^4-105d^5ex+96d^6-280de^5x^5+240e^6x^6\right)+105d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1680e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(96*d^6 - 105*d^5*e*x + 48*d^4*e^2*x^2 + 490*d^3*e^3*x
^3 - 384*d^2*e^4*x^4 - 280*d*e^5*x^5 + 240*e^6*x^6) + 105*d^7*ArcTan[(e*x)/
```

```
Sqrt[d^2 - e^2*x^2]])/(1680*e^3)
```

Maple [B] time = 0.058, size = 282, normalized size = 2.

$$-\frac{1}{7e^{3}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{7}{2}}-\frac{dx}{6e^{2}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}-\frac{5d^{3}x}{24e^{2}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}-\frac{5d^{5}x}{16e^{2}}\sqrt{-x^{2}e^{2}+d^{2}}-\frac{5d^{7}}{16e^{2}}\arctan\left(x\sqrt{e^{2}}\frac{1}{\sqrt{-x^{2}e^{2}+d^{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/7*(-e^2*x^2+d^2)^(7/2)/e^3-1/6*d*x*(-e^2*x^2+d^2)^(5/2)/e^2-5/24*d^3*x*(
-e^2*x^2+d^2)^(3/2)/e^2-5/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2-5/16*d^7/e^2/(e
^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/5*d^2/e^3*(-(d/e+x)^
2*e^2+2*d*e*(d/e+x))^(5/2)+1/4*d^3/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)
*x+3/8*d^5/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+3/8*d^7/e^2/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58616, size = 262, normalized size = 1.87

$$-\frac{210\,d^{7}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(240\,e^{6}x^{6}-280\,de^{5}x^{5}-384\,d^{2}e^{4}x^{4}+490\,d^{3}e^{3}x^{3}+48\,d^{4}e^{2}x^{2}-105\,d^{5}ex+96\,d^{6}\right)\sqrt{-e^{2}x}}{1680\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (240*e^6*x^6 -
 280*d*e^5*x^5 - 384*d^2*e^4*x^4 + 490*d^3*e^3*x^3 + 48*d^4*e^2*x^2 - 105*d
^5*e*x + 96*d^6)*sqrt(-e^2*x^2 + d^2))/e^3
```

Sympy [C] time = 13.9106, size = 656, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e
*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqr
t(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) -
 d**2*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqr
t(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (
x**4*sqrt(d**2)/4, True)) - d*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5
) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqr
t(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**
6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*
x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d
**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**3*Piecewise((
-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x*
*2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**
2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.106 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=116

$$-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] -(d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) - (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e)
- ((6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) - (d^6*ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]])/(16*e^2)
```

Rubi [A] time = 0.0588302, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {785, 780, 195, 217, 203}

$$-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -(d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) - (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e)
- ((6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) - (d^6*ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]])/(16*e^2)
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx &= \frac{\int x\left(d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}\,dx}{de}\\ &= -\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}\\ &= -\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}\\ &= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{16e}\\ &= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\operatorname{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,x,\right.}{16e}\\ &= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x\left(d^2-e^2x^2\right)^{3/2}}{24e}-\frac{(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}\end{aligned}$$

Mathematica [A] time = 0.0939942, size = 102, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(96d^3e^2x^2-70d^2e^3x^3+15d^4ex-48d^5-48de^4x^4+40e^5x^5\right)-15d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{240e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 + 15*d^4*e*x + 96*d^3*e^2*x^2 - 70*d^2*e^3*x^
3 - 48*d*e^4*x^4 + 40*e^5*x^5) - 15*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(240*e^2)
```

Maple [B] time = 0.054, size = 260, normalized size = 2.2

$$\frac{x}{6e}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{5d^2x}{24e}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{5d^4x}{16e}\sqrt{-x^2e^2+d^2}+\frac{5d^6}{16e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{d}{5e^2}\left(-\left(\frac{d}{e}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/6/e*x*(-e^2*x^2+d^2)^(5/2)+5/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e+5/16*d^4*x*(
-e^2*x^2+d^2)^(1/2)/e+5/16/e*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))-1/5*d/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/4*d^2/e*(-(d/
e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-3/8*d^4/e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^
(1/2)*x-3/8*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d
```

```
/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64897, size = 228, normalized size = 1.97

$$\frac{30\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(40\,e^5x^5-48\,de^4x^4-70\,d^2e^3x^3+96\,d^3e^2x^2+15\,d^4ex-48\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(30*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 - 48*
d*e^4*x^4 - 70*d^2*e^3*x^3 + 96*d^3*e^2*x^2 + 15*d^4*e*x - 48*d^5)*sqrt(-e^
2*x^2 + d^2))/e^2
```

Sympy [A] time = 17.3989, size = 583, normalized size = 5.03

$$d^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\: e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\left(\begin{cases}- \\ x^4\end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) - d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d
**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**
2*x**2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(1
5*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**
2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + e**3*Piecewise((-I*d**6*
acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d
**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**
2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)
/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**
2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24
```

```
*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True
))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.107 $\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex} dx$

Optimal. Leaf size=100

$$\frac{3}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] (3*d^3*x*Sqrt[d^2 - e^2*x^2])/8 + (d*x*(d^2 - e^2*x^2)^(3/2))/4 + (d^2 - e^
2*x^2)^(5/2)/(5*e) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rubi [A] time = 0.0305591, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {665, 195, 217, 203}

$$\frac{3}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2}+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*d^3*x*Sqrt[d^2 - e^2*x^2])/8 + (d*x*(d^2 - e^2*x^2)^(3/2))/4 + (d^2 - e^
2*x^2)^(5/2)/(5*e) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + d\int \left(d^2-e^2x^2\right)^{3/2}\,dx$$
$$= \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{4}\left(3d^3\right)\int \sqrt{d^2-e^2x^2}\,dx$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{8}\left(3d^5\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{1}{8}\left(3d^5\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{}{\sqrt{d^2}}\right.$$
$$= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2} + \frac{\left(d^2-e^2x^2\right)^{5/2}}{5e} + \frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Mathematica [A] time = 0.0611791, size = 91, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}\left(-16d^2e^2x^2+25d^3ex+8d^4-10de^3x^3+8e^4x^4\right)+15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{40e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 + 25*d^3*e*x - 16*d^2*e^2*x^2 - 10*d*e^3*x^3 +
8*e^4*x^4) + 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(40*e)
```

Maple [A] time = 0.051, size = 147, normalized size = 1.5

$$\frac{1}{5e}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{5}{2}} + \frac{dx}{4}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}} + \frac{3d^3x}{8}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)} + \frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+1/4*d*(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(3/2)*x+3/8*d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+3/8*d^5/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [C] time = 1.49222, size = 147, normalized size = 1.47

$$-\frac{3i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{8e} + \frac{3}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x + \frac{3\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{4e} + \frac{1}{4}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx + \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^5*arcsin(e*x/d + 2)/e + 3/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
+ 3/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*
x + 1/5*(-e^2*x^2 + d^2)^(5/2)/e
```

Fricas [A] time = 1.60922, size = 200, normalized size = 2.

$$-\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(8\,e^4x^4-10\,de^3x^3-16\,d^2e^2x^2+25\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d^2}}{40\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (8*e^4*x^4 - 10*d
*e^3*x^3 - 16*d^2*e^2*x^2 + 25*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [C] time = 10.6007, size = 439, normalized size = 4.39

$$d^3\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\cdots\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) -
 d**2*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(
3/2)/(3*e**2), True)) - d*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I
*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x
**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Ab
s(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**
2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 -
e**2*x**2/d**2)), True)) + e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(
15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e*
*2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.108 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=113

$$\frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (d^2*(8*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d - 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 - (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.116023, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (d^2*(8*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d - 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 - (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx$$

$$= \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2+3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2}$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4-3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4}$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{8}\left(3d^4e\right)\int \frac{\ldots}{\sqrt{d^2\ldots}}$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^5\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \ldots$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^5\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4\tanh^{-1}\left(\ldots\right.$$

Mathematica [A] time = 0.0942631, size = 108, normalized size = 0.96

$$\frac{1}{24}\sqrt{d^2-e^2x^2}\left(-15d^2ex+32d^3-8de^2x^2+6e^3x^3\right) - d^4\log\left(\sqrt{d^2-e^2x^2}+d\right) - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + d^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 - 15*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3))/24 -
(3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + d^4*Log[x] - d^4*Log[d + Sqrt
[d^2 - e^2*x^2]]
```

Maple [B] time = 0.062, size = 245, normalized size = 2.2

$$\frac{1}{5d}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{d}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+d^3\sqrt{-x^2e^2+d^2}-d^5\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{1}{5d}\left(-\left(\frac{d}{e}+x\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x)
```

```
[Out] 1/5/d*(-e^2*x^2+d^2)^(5/2)+1/3*d*(-e^2*x^2+d^2)^(3/2)+d^3*(-e^2*x^2+d^2)^(1
/2)-d^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5/d*
(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/4*e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
3/2)*x-3/8*d^2*e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-3/8*d^4*e/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67327, size = 227, normalized size = 2.01

$$\frac{3}{4}d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{24}\left(6e^3x^3-8de^2x^2-15d^2ex+32d^3\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 3/4*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^4*log(-(d - sqrt(-e^2
*x^2 + d^2))/x) + 1/24*(6*e^3*x^3 - 8*d*e^2*x^2 - 15*d^2*e*x + 32*d^3)*sqrt
(-e^2*x^2 + d^2)
```

Sympy [C] time = 24.1036, size = 476, normalized size = 4.21

$$d^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\,\mathrm{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \frac{|d^2|}{|e^2||x^2|} > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\,\mathrm{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} -\frac{id^2\,\mathrm{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2x^2|}{|d^2|} \\ \frac{d^2\,\mathrm{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwis} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d),x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d
**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**
2/(e**2*x**2) + 1), True)) - d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) -
 I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2
/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(
1 - e**2*x**2/d**2)/2, True)) - d*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**
2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + e**3*Piecewise((-I*d
**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3
*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x
**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**
3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2
)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.109 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -(d*e*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d + e*x)*(d^2 - e^2*x^2)^(
3/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + d^3*e*ArcTanh
[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.117931, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {850, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] -(d*e*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d + e*x)*(d^2 - e^2*x^2)^(
3/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + d^3*e*ArcTanh
[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
```

```
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{\left(2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \frac{\int \frac{-4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \left(d^4e\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}\left(3d^3e^2\right)\int$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\left(d^4e\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right.$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{d^4\,\text{Subst}\left(\right.}{}$$

$$= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + d^3e\tanh^{-1}$$

Mathematica [A] time = 0.135068, size = 114, normalized size = 0.99

$$\sqrt{d^2-e^2x^2}\left(-\frac{4d^2e}{3}-\frac{d^3}{x}-\frac{1}{2}de^2x+\frac{e^3x^2}{3}\right)+d^3e\log\left(\sqrt{d^2-e^2x^2}+d\right)-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-4*d^2*e)/3 - d^3/x - (d*e^2*x)/2 + (e^3*x^2)/3) - (3
*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*Log[x] + d^3*e*Log[d +
Sqrt[d^2 - e^2*x^2]]
```

Maple [B] time = 0.064, size = 380, normalized size = 3.3

$$-\frac{e}{5d^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{e}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-ed^2\sqrt{-x^2e^2+d^2}+ed^4\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{e}{5d^2}\left(-\left(\frac{d}{e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x)
```

```
[Out] -1/5*e/d^2*(-e^2*x^2+d^2)^(5/2)-1/3*e*(-e^2*x^2+d^2)^(3/2)-e*d^2*(-e^2*x^2+
d^2)^(1/2)+e*d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x)+1/5*e/d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+1/4*e^2/d*(-(d/e+x)^2*e^2
+2*d*e*(d/e+x))^(3/2)*x+3/8*e^2*d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+3/
8*e^2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
1/2))-1/d^3/x*(-e^2*x^2+d^2)^(7/2)-1/d^3*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/4/d*e
^2*x*(-e^2*x^2+d^2)^(3/2)-15/8*d*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/8*d^3*e^2/(e
^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60561, size = 257, normalized size = 2.23

$$\frac{18\,d^3ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-6\,d^3ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-8\,d^3ex+\left(2\,e^3x^3-3\,de^2x^2-8\,d^2ex-6\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 6*d^3*e*x*log(-
(d - sqrt(-e^2*x^2 + d^2))/x) - 8*d^3*e*x + (2*e^3*x^3 - 3*d*e^2*x^2 - 8*d^
2*e*x - 6*d^3)*sqrt(-e^2*x^2 + d^2))/x
```

Sympy [C] time = 13.2584, size = 393, normalized size = 3.42

$$d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d),x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*s
qrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**
2)), True)) - d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*a
cosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x*
*2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) +
 I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((-I*d**2*acos
h(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqr
t(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(
2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + e**3*Piecewise((x**2*sqrt(d
**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.110 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=121

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d + e*x)*(d^2 - e^2*x^2)^(3
/2))/(2*x^2) + (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^2
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rubi [A] time = 0.112722, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 813, 844, 217, 203, 266, 63, 208}

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d + e*x)*(d^2 - e^2*x^2)^(3
/2))/(2*x^2) + (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^2
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx$$
$$= -\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{\left(4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx$$
$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{-8d^3e^2+8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}\left(3d^3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \frac{1}{2}\left(3d^2e^3\right)\int$$
$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{4}\left(3d^3e^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) +$$
$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}\left(3d^3\right)\mathrm{Subst}$$
$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.168974, size = 119, normalized size = 0.98

$$\frac{1}{2}\left(\frac{\sqrt{d^2-e^2x^2}\left(2d^2ex-d^3-2de^2x^2+e^3x^3\right)}{x^2} + 3d^2e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 3d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - 3d^2e^2\log(x)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^3 + 2*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/x^2 + 3*d
^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 3*d^2*e^2*Log[x] + 3*d^2*e^2*Log
[d + Sqrt[d^2 - e^2*x^2]])/2
```

Maple [B] time = 0.067, size = 411, normalized size = 3.4

$$-\frac{3e^2}{10d^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{e^2}{2d}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{3de^2}{2}\sqrt{-x^2e^2+d^2}+\frac{3e^2d^3}{2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{e^2}{5d^3}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x)
```

```
[Out] -3/10*e^2/d^3*(-e^2*x^2+d^2)^(5/2)-1/2*e^2/d*(-e^2*x^2+d^2)^(3/2)-3/2*d*e^2
*(-e^2*x^2+d^2)^(1/2)+3/2*e^2*d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)-1/5*e^2/d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/4*e^
3/d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-3/8*e^3*(-(d/e+x)^2*e^2+2*d*e*
(d/e+x))^(1/2)*x-3/8*e^3*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e
^2+2*d*e*(d/e+x))^(1/2))-1/2/d^3/x^2*(-e^2*x^2+d^2)^(7/2)+e/d^4/x*(-e^2*x^2
+d^2)^(7/2)+e^3/d^4*x*(-e^2*x^2+d^2)^(5/2)+5/4*e^3/d^2*x*(-e^2*x^2+d^2)^(3/
2)+15/8*e^3*x*(-e^2*x^2+d^2)^(1/2)+15/8*e^3*d^2/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64981, size = 270, normalized size = 2.23

$$-\frac{6d^2e^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3d^2e^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+2d^2e^2x^2-\left(e^3x^3-2de^2x^2+2d^2ex-d^3\right)\sqrt{-e^2x^2+d^2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*d^2*e^2*x
^2*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + 2*d^2*e^2*x^2 - (e^3*x^3 - 2*d*e^2*
x^2 + 2*d^2*e*x - d^3)*sqrt(-e^2*x^2 + d^2))/x^2
```

Sympy [C] time = 11.3905, size = 471, normalized size = 3.89

$$d^{3}\left(\begin{cases} - \frac{d^{2}}{2 e x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e}{2 x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \frac{\left|{d^{2}}\right|}{\left|{e^{2}}\right| \left|{x^{2}}\right|} > 1 \\- \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{2 x} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right) - d^{2} e \left(\begin{cases} \frac{i d}{x \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + i e \operatorname{acosh}{\left(\frac{e x}{d} \right)} - \frac{i e^{2} x}{d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \frac{\left|{e^{2} x \cdots}\right|}{\left|{d^{2}}\right|} \\- \frac{d}{x \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - e \operatorname{asin}{\left(\frac{e x}{d} \right)} + \frac{e^{2} x}{d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{other}\cdots \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs
(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x)
)/(2*d), True)) - d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e
*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(
d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sq
rt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e*
*2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**
2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + e**3*Piece
wise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*
e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (
d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.111 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=120

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (e^2*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d - 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rubi [A] time = 0.114397, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {850, 811, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (e^2*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d - 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
```

```
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)}\,dx = \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx$$
$$= -\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{\int \frac{\left(4d^3e^2-6d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{\int \frac{12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{1}{2}\left(3d^2e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + \left(de^4\right)\int \frac{}{\sqrt{d^2}}$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{1}{4}\left(3d^2e^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) +$$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}\left(3d^2e\right)\text{Subst}\Bigg($$
$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d}}{}\right.$$

Mathematica [A] time = 0.169105, size = 116, normalized size = 0.97

$$\left(\frac{d^2e}{2x^2} - \frac{d^3}{3x^3} + \frac{4de^2}{3x} + e^3\right)\sqrt{d^2-e^2x^2} - \frac{3}{2}de^3\log\left(\sqrt{d^2-e^2x^2}+d\right) + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}de^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (e^3 - d^3/(3*x^3) + (d^2*e)/(2*x^2) + (4*d*e^2)/(3*x))*Sqrt[d^2 - e^2*x^2]
 + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Log[x])/2 - (3*d*e^3*
Log[d + Sqrt[d^2 - e^2*x^2]])/2
```

Maple [B] time = 0.069, size = 439, normalized size = 3.7

$$\frac{3e^3}{10d^4}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} + \frac{e^3}{2d^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} + \frac{3e^3}{2}\sqrt{-x^2e^2+d^2} - \frac{3e^3d^2}{2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{1}{3d^3x^3}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x)
```

```
[Out] 3/10*e^3/d^4*(-e^2*x^2+d^2)^(5/2)+1/2*e^3/d^2*(-e^2*x^2+d^2)^(3/2)+3/2*e^3*
(-e^2*x^2+d^2)^(1/2)-3/2*e^3*d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)-1/3/d^3/x^3*(-e^2*x^2+d^2)^(7/2)+1/3/d^5*e^2/x*(-e^2*x^2
+d^2)^(7/2)+1/3/d^5*e^4*x*(-e^2*x^2+d^2)^(5/2)+5/12/d^3*e^4*x*(-e^2*x^2+d^2
)^(3/2)+5/8/d*e^4*x*(-e^2*x^2+d^2)^(1/2)+5/8*d*e^4/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/5*e^3/d^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
5/2)+1/4*e^4/d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+3/8*e^4/d*(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(1/2)*x+3/8*e^4*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
```

```
(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))+1/2*e/d^4/x^2*(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65882, size = 269, normalized size = 2.24

$$-\frac{12\,de^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-9\,de^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-6\,de^3x^3-\left(6\,e^3x^3+8\,de^2x^2+3\,d^2ex-2\,d^3\right)\sqrt{-e^2x^2+}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 9*d*e^3*x^3*
log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 6*d*e^3*x^3 - (6*e^3*x^3 + 8*d*e^2*x^2
 + 3*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/x^3
```

Sympy [C] time = 12.7885, size = 469, normalized size = 3.91

$$d^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-de$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d
**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2)
, True)) - d**2*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) +
e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(
Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2
*asin(d/(e*x))/(2*d), True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2
/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e*
*2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) +
 e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + e**3*Piecewise((d**2/(e*x*sq
rt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) -
1), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x*
*2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.112 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=119

$$\frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (e^2*(3*d - 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d - 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rubi [A] time = 0.114609, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {850, 811, 844, 217, 203, 266, 63, 208}

$$\frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}+e^4\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```


```
[Out] (e^2*(3*d - 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d - 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```


Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx\\
&= -\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - \frac{\int \frac{\left(6d^3e^2-8d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}\\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{\int \frac{12d^5e^4-32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}\\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{8}\left(3de^4\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^5\int \frac{1}{\sqrt{d^2-e^2
\end{aligned}$$

$$\begin{aligned}
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{16}\left(3de^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) -\\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{8}\left(3de^2\right)\text{Subst}\Bigg(\int\\
&= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4\tanh^{-1}\Bigg(\frac{\sqrt{d^2-}}{d}
\end{aligned}$$

Mathematica [A] time = 0.203241, size = 111, normalized size = 0.93

$$\frac{1}{24}\left(\frac{\sqrt{d^2-e^2x^2}\left(8d^2ex-6d^3+15de^2x^2-32e^3x^3\right)}{x^4} - 9e^4\log\left(\sqrt{d^2-e^2x^2}+d\right) - 24e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 9e^4\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x + 15*d*e^2*x^2 - 32*e^3*x^3))/x^4
 - 24*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + 9*e^4*Log[x] - 9*e^4*Log[d +
Sqrt[d^2 - e^2*x^2]])/24
```

Maple [B] time = 0.081, size = 463, normalized size = 3.9

$$\frac{e}{3\,d^4x^3}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{e^3}{3\,d^6x}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{e^5x}{3\,d^6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{5\,e^5x}{12\,d^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{3\,de^4}{8}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x)
```

```
[Out] 1/3*e/d^4/x^3*(-e^2*x^2+d^2)^(7/2)-1/3*e^3/d^6/x*(-e^2*x^2+d^2)^(7/2)-1/3*e
^5/d^6*x*(-e^2*x^2+d^2)^(5/2)-5/12*e^5/d^4*x*(-e^2*x^2+d^2)^(3/2)-3/8*d*e^4
/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/8/d^5*e^2/x
^2*(-e^2*x^2+d^2)^(7/2)-1/4*e^5/d^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-
3/8*e^5/d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-5/8*e^5/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/4/d^3/x^4*(-e^2*x^2+d^2)^(7/2)-1/5
*e^4/d^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-3/8*e^5/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))+3/40*e^4/d^5*(-e^2*x^2+d^
2)^(5/2)+1/8*e^4/d^3*(-e^2*x^2+d^2)^(3/2)+3/8*e^4/d*(-e^2*x^2+d^2)^(1/2)-5/
8*e^5/d^2*x*(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63645, size = 247, normalized size = 2.08

$$\frac{48\,e^4x^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+9\,e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(32\,e^3x^3-15\,de^2x^2-8\,d^2ex+6\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(48*e^4*x^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 9*e^4*x^4*log(
-(d - sqrt(-e^2*x^2 + d^2))/x) - (32*e^3*x^3 - 15*d*e^2*x^2 - 8*d^2*e*x + 6
*d^3)*sqrt(-e^2*x^2 + d^2))/x^4
```

Sympy [C] time = 15.7384, size = 552, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/
(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e
*x))/(8*d**3), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*
x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(
x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2
/(e**2*x**2) + 1)/(3*d**2), True)) - d*e**2*Piecewise((-d**2/(2*e*x**3*sqrt
(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d
/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2
*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*
d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 +
e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/
d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.113 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=108

$$-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] (-3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) + (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) -
(d^2 - e^2*x^2)^(5/2)/(5*d*x^5) + (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8
*d)
```

Rubi [A] time = 0.0893033, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 807, 266, 47, 63, 208}

$$-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (-3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) + (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) -
(d^2 - e^2*x^2)^(5/2)/(5*d*x^5) + (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8
*d)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right) \\
&= \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{1}{8}\left(3e^3\right)\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} - \frac{1}{16}\left(3e^5\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{1}{8}\left(3e^3\right)\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right) \\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5} + \frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}
$$

Mathematica [A] time = 0.159132, size = 106, normalized size = 0.98

$$
\frac{\sqrt{d^2-e^2x^2}\left(16d^2e^2x^2+10d^3ex-8d^4-25de^3x^3-8e^4x^4\right)+15e^5x^5\log\left(\sqrt{d^2-e^2x^2}+d\right)-15e^5x^5\log(x)}{40dx^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 + 10*d^3*e*x + 16*d^2*e^2*x^2 - 25*d*e^3*x^3 -
 8*e^4*x^4) - 15*e^5*x^5*Log[x] + 15*e^5*x^5*Log[d + Sqrt[d^2 - e^2*x^2]])/
(40*d*x^5)
```

Maple [B] time = 0.097, size = 493, normalized size = 4.6

$$\frac{e}{4\,d^{4}x^{4}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{7}{2}}+\frac{e^{3}}{8\,d^{6}x^{2}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{7}{2}}-\frac{1}{5\,d^{3}x^{5}}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{7}{2}}+\frac{e^{5}}{5\,d^{6}}\left(-\left(\frac{d}{e}+x\right)^{2}e^{2}+2\,de\left(\frac{d}{e}+x\right)\right)^{\frac{5}{2}}-\frac{e^{2}}{5\,d^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x)
```

```
[Out] 1/4*e/d^4/x^4*(-e^2*x^2+d^2)^(7/2)+1/8*e^3/d^6/x^2*(-e^2*x^2+d^2)^(7/2)-1/5
/d^3/x^5*(-e^2*x^2+d^2)^(7/2)+1/5*e^5/d^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5
/2)-1/5*e^2/d^5/x^3*(-e^2*x^2+d^2)^(7/2)-1/5*e^4/d^7/x*(-e^2*x^2+d^2)^(7/2)
-1/5*e^6/d^7*x*(-e^2*x^2+d^2)^(5/2)-1/4*e^6/d^5*x*(-e^2*x^2+d^2)^(3/2)-3/40
*e^5/d^6*(-e^2*x^2+d^2)^(5/2)+1/4*e^6/d^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3
/2)*x+3/8*e^6/d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+3/8*e^6/d/(e^2)^(1
/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-3/8*e^6/d^3*
x*(-e^2*x^2+d^2)^(1/2)-3/8*e^6/d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))-1/8*e^5/d^4*(-e^2*x^2+d^2)^(3/2)-3/8*e^5/d^2*(-e^2*x^2+d^2)^(1
/2)+3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63698, size = 204, normalized size = 1.89

$$-\frac{15\,e^{5}x^{5}\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(8\,e^{4}x^{4}+25\,de^{3}x^{3}-16\,d^{2}e^{2}x^{2}-10\,d^{3}ex+8\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{40\,dx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 + 25*d*e^
3*x^3 - 16*d^2*e^2*x^2 - 10*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(d*x^5)
```

Sympy [C] time = 14.1072, size = 785, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-
15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*
d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x*
*5 + 15*d*e**2*x**7), True)) - d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/
(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*
x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Ab
s(e**2)*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3
*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e*
*2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((
-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3
*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**3*
Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(
e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2
)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*
d), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.114 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=143

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}+\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) + (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rubi [A] time = 0.120188, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}+\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) + (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}-\frac{\int \frac{\left(6d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{6d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^2\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d} \\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^2\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)}{12d} \\
&= -\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^4\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right)}{16d} \\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}+\frac{e^6\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x\right.}{32d} \\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^4\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d}\right.}{16d} \\
&= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}
\end{aligned}
$$

Mathematica [A] time = 0.200641, size = 117, normalized size = 0.82

$$
\frac{\sqrt{d^2-e^2x^2}\left(70d^3e^2x^2-96d^2e^3x^3+48d^4ex-40d^5-15de^4x^4+48e^5x^5\right)-15e^6x^6\log\left(\sqrt{d^2-e^2x^2}+d\right)+15e^6x^6\log(x)}{240d^2x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 + 48*d^4*e*x + 70*d^3*e^2*x^2 - 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 + 48*e^5*x^5) + 15*e^6*x^6*Log[x] - 15*e^6*x^6*Log[d + Sqr
t[d^2 - e^2*x^2]])/(240*d^2*x^6)
```

Maple [B] time = 0.124, size = 521, normalized size = 3.6

$$-\frac{e^6}{16d}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{e}{5d^4x^5}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}+\frac{e^6}{80d^7}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{e^6}{48d^5}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x)
```

```
[Out] -1/16*e^6/d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/
5*e/d^4/x^5*(-e^2*x^2+d^2)^(7/2)+1/80*e^6/d^7*(-e^2*x^2+d^2)^(5/2)+1/48*e^6
/d^5*(-e^2*x^2+d^2)^(3/2)+1/16*e^6/d^3*(-e^2*x^2+d^2)^(1/2)-1/6/d^3/x^6*(-e
^2*x^2+d^2)^(7/2)-1/5*e^6/d^7*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+1/5*e^3/
d^6/x^3*(-e^2*x^2+d^2)^(7/2)+1/5*e^5/d^8/x*(-e^2*x^2+d^2)^(7/2)+1/5*e^7/d^8
*x*(-e^2*x^2+d^2)^(5/2)+1/4*e^7/d^6*x*(-e^2*x^2+d^2)^(3/2)+3/8*e^7/d^4*x*(-
e^2*x^2+d^2)^(1/2)+3/8*e^7/d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d
^2)^(1/2))-5/24*e^2/d^5/x^4*(-e^2*x^2+d^2)^(7/2)-3/16*e^4/d^7/x^2*(-e^2*x^2
+d^2)^(7/2)-1/4*e^7/d^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-3/8*e^7/d^4*
(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-3/8*e^7/d^2/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72457, size = 232, normalized size = 1.62

$$\frac{15\,e^6x^6\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(48\,e^5x^5-15\,de^4x^4-96\,d^2e^3x^3+70\,d^3e^2x^2+48\,d^4ex-40\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(15*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*x^5 - 15*d*e
^4*x^4 - 96*d^2*e^3*x^3 + 70*d^3*e^2*x^2 + 48*d^4*e*x - 40*d^5)*sqrt(-e^2*x
^2 + d^2))/(d^2*x^6)
```

Sympy [C] time = 20.1386, size = 930, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e
**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*
d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2
*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d**2*e*Piecewise((3
*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6
*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2
*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15
*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
*2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e*
*2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
*7), True)) - d*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1))
 + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2
*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)
) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) -
I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**3*Piecewise((-e*sqrt(d**2/(e**2*
x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2)/(
Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e
**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.115 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)}\,dx$

Optimal. Leaf size=172

$$-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}+\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] -(e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) + (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d
^2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) + (e*(d^2 - e^2*x^2)^(5/2))/(6*d^
2*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) + (e^7*ArcTanh[Sqrt[d^2
 - e^2*x^2]/d])/(16*d^3)
```

Rubi [A] time = 0.153946, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4}-\frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7}+\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] -(e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) + (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d
^2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) + (e*(d^2 - e^2*x^2)^(5/2))/(6*d^
2*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) + (e^7*ArcTanh[Sqrt[d^2
 - e^2*x^2]/d])/(16*d^3)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^8(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{x^8}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} - \frac{\int \frac{\left(7d^2e-2de^2x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx}{7d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} + \frac{\int \frac{\left(12d^3e^2-7d^2e^3x\right)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{42d^4}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^3\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^3\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d^2}\\
&= \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^5\,\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,\right.}{16d^2}\\
&= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} - \frac{e^7\,\mathrm{Su}}{}\\
&= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^5\,\mathrm{Su}}{}\\
&= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{24d^2x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{7dx^7} + \frac{e\left(d^2-e^2x^2\right)^{5/2}}{6d^2x^6} - \frac{2e^2\left(d^2-e^2x^2\right)^{5/2}}{35d^3x^5} + \frac{e^7\tan}{}
\end{aligned}
$$

Mathematica [A] time = 0.216027, size = 128, normalized size = 0.74

$$\frac{\sqrt{d^2-e^2x^2}\left(384d^4e^2x^2-490d^3e^3x^3-48d^2e^4x^4+280d^5ex-240d^6+105de^5x^5-96e^6x^6\right)+105e^7x^7\log\left(\sqrt{d^2-e^2x^2}+\right.}{1680d^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 + 280*d^5*e*x + 384*d^4*e^2*x^2 - 490*d^3*e^
3*x^3 - 48*d^2*e^4*x^4 + 105*d*e^5*x^5 - 96*e^6*x^6) - 105*e^7*x^7*Log[x] +
 105*e^7*x^7*Log[d + Sqrt[d^2 - e^2*x^2]])/(1680*d^3*x^7)
```

Maple [B] time = 0.149, size = 546, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x)
```

```
[Out] -1/5*e^4/d^7/x^3*(-e^2*x^2+d^2)^(7/2)-1/5*e^6/d^9/x*(-e^2*x^2+d^2)^(7/2)-1/
5*e^8/d^9*x*(-e^2*x^2+d^2)^(5/2)-1/4*e^8/d^7*x*(-e^2*x^2+d^2)^(3/2)-3/8*e^8
/d^5*x*(-e^2*x^2+d^2)^(1/2)-3/8*e^8/d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2))+1/6*e/d^4/x^6*(-e^2*x^2+d^2)^(7/2)+1/5*e^7/d^8*(-(d/e+x
)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/80*e^7/d^8*(-e^2*x^2+d^2)^(5/2)-1/48*e^7/d^6
*(-e^2*x^2+d^2)^(3/2)-1/16*e^7/d^4*(-e^2*x^2+d^2)^(1/2)+1/16*e^7/d^2/(d^2)^
(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/7/d^3/x^7*(-e^2*x^
2+d^2)^(7/2)-1/5*e^2/d^5/x^5*(-e^2*x^2+d^2)^(7/2)+5/24*e^3/d^6/x^4*(-e^2*x^
2+d^2)^(7/2)+3/16*e^5/d^8/x^2*(-e^2*x^2+d^2)^(7/2)+1/4*e^8/d^7*(-(d/e+x)^2*
e^2+2*d*e*(d/e+x))^(3/2)*x+3/8*e^8/d^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
*x+3/8*e^8/d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+
x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76293, size = 266, normalized size = 1.55

$$-\frac{105\,e^7x^7\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(96\,e^6x^6-105\,de^5x^5+48\,d^2e^4x^4+490\,d^3e^3x^3-384\,d^4e^2x^2-280\,d^5ex+240\,d^6\right)\sqrt{-e^2}}{1680\,d^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (96*e^6*x^6 - 105
*d*e^5*x^5 + 48*d^2*e^4*x^4 + 490*d^3*e^3*x^3 - 384*d^4*e^2*x^2 - 280*d^5*e
*x + 240*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*x^7)
```

Sympy [C] time = 26.6652, size = 1049, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2)/(Abs(e**2)*
Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-
d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)
/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True))
- d**2*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x
**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2)
- 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(
16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**
2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/
(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(
e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewis
e((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*
d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*
e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I
*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(
e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
+ 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 1
5*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**
3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), True)) + e**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e*
*2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.116 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx$

Optimal. Leaf size=201

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}+\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{3e^8\tan}{}$$

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) + (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) + (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rubi [A] time = 0.189864, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {850, 835, 807, 266, 47, 63, 208}

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}+\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}+\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{3e^8\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) + (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) + (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^9}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{\int \frac{(8d^2e-3de^2x)(d^2-e^2x^2)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} + \frac{\int \frac{(21d^3e^2-16d^2e^3x)(d^2-e^2x^2)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{\int \frac{(96d^4e^3-21d^3e^4x)(d^2-e^2x^2)^{3/2}}{x^6}\,dx}{336d^6}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} + \frac{e^4\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{16d^3}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} + \frac{e^4\,\mathrm{Subst}\left(\int \frac{(d^2-e^2}{x^3}\right.}{32d^3}$$

$$= -\frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} -$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3}{}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3}{}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3}{}$$

Mathematica [A] time = 0.236507, size = 139, normalized size = 0.69

$$\frac{\sqrt{d^2-e^2x^2}\left(840d^5e^2x^2-1024d^4e^3x^3-70d^3e^4x^4+128d^2e^5x^5+640d^6ex-560d^7-105de^6x^6+256e^7x^7\right)-105e^8x^8\log}{4480d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 640*d^6*e*x + 840*d^5*e^2*x^2 - 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 + 128*d^2*e^5*x^5 - 105*d*e^6*x^6 + 256*e^7*x^7) +
105*e^8*x^8*Log[x] - 105*e^8*x^8*Log[d + Sqrt[d^2 - e^2*x^2]])/(4480*d^4*x^
8)
```

Maple [B] time = 0.195, size = 571, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x)
```

```
[Out] -3/128*e^8/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
+1/7*e/d^4/x^7*(-e^2*x^2+d^2)^(7/2)+3/640*e^8/d^9*(-e^2*x^2+d^2)^(5/2)+1/12
8*e^8/d^7*(-e^2*x^2+d^2)^(3/2)+3/128*e^8/d^5*(-e^2*x^2+d^2)^(1/2)-1/5*e^8/d
^9*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/8/d^3/x^8*(-e^2*x^2+d^2)^(7/2)+1/
5*e^3/d^6/x^5*(-e^2*x^2+d^2)^(7/2)-13/64*e^4/d^7/x^4*(-e^2*x^2+d^2)^(7/2)-2
5/128*e^6/d^9/x^2*(-e^2*x^2+d^2)^(7/2)-1/4*e^9/d^8*(-(d/e+x)^2*e^2+2*d*e*(d
/e+x))^(3/2)*x-3/8*e^9/d^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-3/8*e^9/d
^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-3
/16/d^5*e^2/x^6*(-e^2*x^2+d^2)^(7/2)+1/5*e^5/d^8/x^3*(-e^2*x^2+d^2)^(7/2)+1
/5*e^7/d^10/x*(-e^2*x^2+d^2)^(7/2)+1/5*e^9/d^10*x*(-e^2*x^2+d^2)^(5/2)+1/4*
e^9/d^8*x*(-e^2*x^2+d^2)^(3/2)+3/8*e^9/d^6*x*(-e^2*x^2+d^2)^(1/2)+3/8*e^9/d
^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85352, size = 292, normalized size = 1.45

$$\frac{105\,e^8x^8\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(256\,e^7x^7-105\,de^6x^6+128\,d^2e^5x^5-70\,d^3e^4x^4-1024\,d^4e^3x^3+840\,d^5e^2x^2+640\,d^6ex-56\right.}{4480\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(105*e^8*x^8*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (256*e^7*x^7 - 105
*d*e^6*x^6 + 128*d^2*e^5*x^5 - 70*d^3*e^4*x^4 - 1024*d^4*e^3*x^3 + 840*d^5*
e^2*x^2 + 640*d^6*e*x - 560*d^7)*sqrt(-e^2*x^2 + d^2))/(d^4*x^8)
```

Sympy [C] time = 30.168, size = 1171, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2)/
(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1))
- 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(
-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) +
```

```
1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(
e*x))/(128*d**7), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/
(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**
2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105
*d**6), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**
5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**6), True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/
(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**
2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2)
- 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1)
, (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**
2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I
*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d*
*5), True)) + e**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x*
*5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x
**5 + 15*d*e**2*x**7), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2
*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x*
*2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d*
*2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2
)/(-15*d**3*x**5 + 15*d*e**2*x**7), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.117 $\int \frac{x\sqrt{1-x^2}}{1+x}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rubi [A] time = 0.0170649, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {785, 780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(1 + x),x]

[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{1-x^2}}{1+x}\,dx &= \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0336552, size = 26, normalized size = 0.96

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(1 + x),x]
```

```
[Out] (-1 + x/2)*Sqrt[1 - x^2] - ArcSin[x]/2
```

Maple [A] time = 0.046, size = 34, normalized size = 1.3

$$\frac{x}{2}\sqrt{-x^2+1}-\frac{\arcsin(x)}{2}-\sqrt{-(1+x)^2+2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(1+x),x)
```

```
[Out] 1/2*x*(-x^2+1)^(1/2)-1/2*arcsin(x)-(-(1+x)^2+2+2*x)^(1/2)
```

Maxima [A] time = 1.48329, size = 38, normalized size = 1.41

$$\frac{1}{2}\sqrt{-x^2+1}x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Fricas [A] time = 1.61252, size = 82, normalized size = 3.04

$$\frac{1}{2}\sqrt{-x^2+1}(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 2.86234, size = 29, normalized size = 1.07

$$\begin{cases} \frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2} & \text{for}\ x>-1\wedge x<1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(1+x),x)
```

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2, (x > -1) & (x <
 1)))
```

Giac [A] time = 1.28953, size = 26, normalized size = 0.96

$$\frac{1}{2}\sqrt{-x^2+1}(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

3.118 $\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)} \, dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{1-a^2x^2}(1-ax)}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a\sin^{-1}(ax)$$

```
[Out] -(((1 - a*x)*Sqrt[1 - a^2*x^2])/x) - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rubi [A] time = 0.0721489, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {850, 813, 844, 216, 266, 63, 208}

$$-\frac{\sqrt{1-a^2x^2}(1-ax)}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] -(((1 - a*x)*Sqrt[1 - a^2*x^2])/x) - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^2(1-ax)}\,dx &= \int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}-\frac{1}{2}\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}+a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx-a^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}-a\sin^{-1}(ax)+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right) \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}-a\sin^{-1}(ax)-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x}-a\sin^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0358729, size = 49, normalized size = 0.96

$$\frac{\sqrt{1-a^2x^2}(ax-1)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-a\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] ((-1 + a*x)*Sqrt[1 - a^2*x^2])/x - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2*x
^2]]
```

Maple [B] time = 0.067, size = 238, normalized size = 4.7

$$\frac{a}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}}+a\sqrt{-a^2x^2+1}-a\mathrm{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{a}{3}\left(-\left(x-a^{-1}\right)^2a^2-2a\left(x-a^{-1}\right)\right)^{\frac{3}{2}}+\frac{a^2x}{2}\sqrt{-\left(x-a^{-1}\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x)
```

```
[Out] 1/3*a*(-a^2*x^2+1)^(3/2)+a*(-a^2*x^2+1)^(1/2)-a*arctanh(1/(-a^2*x^2+1)^(1/2
))-1/3*a*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(3/2)+1/2*a^2*(-(x-1/a)^2*a^2-2*a*(x-
1/a))^(1/2)*x+1/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x-1/a)^2*a^2-2*a*
(x-1/a))^(1/2))-1/x*(-a^2*x^2+1)^(5/2)-a^2*x*(-a^2*x^2+1)^(3/2)-3/2*a^2*x*(
-a^2*x^2+1)^(1/2)-3/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/
2))
```

Maxima [A] time = 1.45781, size = 111, normalized size = 2.18

$$-\frac{a^{2}\arcsin\left(\frac{a^{2}x}{\sqrt{a^{2}}}\right)}{\sqrt{a^{2}}}-a\log\left(\frac{2\sqrt{-a^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^{2}x^{2}+1}a-\frac{\sqrt{-a^{2}x^{2}+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -a^2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - a*log(2*sqrt(-a^2*x^2 + 1)/abs(x)
+ 2/abs(x)) + sqrt(-a^2*x^2 + 1)*a - sqrt(-a^2*x^2 + 1)/x
```

Fricas [A] time = 1.62952, size = 169, normalized size = 3.31

$$\frac{2\,ax\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+ax\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)+ax+\sqrt{-a^{2}x^{2}+1}(ax-1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*x*log((sqrt(-a^2*x^2 + 1)
 - 1)/x) + a*x + sqrt(-a^2*x^2 + 1)*(a*x - 1))/x
```

Sympy [C] time = 6.29764, size = 170, normalized size = 3.33

$$a\left(\begin{cases}i\sqrt{a^{2}x^{2}-1}-\log(ax)+\frac{\log(a^{2}x^{2})}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^{2}x^{2}\right|>1\\ \sqrt{-a^{2}x^{2}+1}+\frac{\log(a^{2}x^{2})}{2}-\log\left(\sqrt{-a^{2}x^{2}+1}+1\right) & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ia^{2}x}{\sqrt{a^{2}x^{2}-1}}+ia\,\mathrm{acosh}(ax)+\frac{i}{x\sqrt{a^{2}x^{2}-1}} & \text{for } |a\\ \frac{a^{2}x}{\sqrt{-a^{2}x^{2}+1}}-a\,\mathrm{asin}(ax)-\frac{1}{x\sqrt{-a^{2}x^{2}+1}} & \text{other}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/x**2/(-a*x+1),x)
```

```
[Out] a*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1
/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - lo
g(sqrt(-a**2*x**2 + 1) + 1), True)) + Piecewise((-I*a**2*x/sqrt(a**2*x**2 -
 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**
```

```
2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1)), True))
```

Giac [B] time = 1.38024, size = 169, normalized size = 3.31

$$\frac{a^4x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{a^2\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{|a|}-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}+\sqrt{-a^2x^2+1}a-\frac{\sqrt{-a^2x^2+1}|a|+a}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*arcsin(a*x)*sgn(a)
/abs(a) - a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))
/abs(a) + sqrt(-a^2*x^2 + 1)*a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs
(a))
```

3.119 $\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=118

$$-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}+\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

```
[Out] (x^3*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) - (4*x^2*Sqrt[d^2 - e^2*x^2])/(3*
e^3) - (d*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^5) - (3*d^3*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

Rubi [A] time = 0.0961943, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {850, 819, 833, 780, 217, 203}

$$-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}+\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^3*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) - (4*x^2*Sqrt[d^2 - e^2*x^2])/(3*
e^3) - (d*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^5) - (3*d^3*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x^2\left(3d^3-4d^2ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} + \frac{\int \frac{x\left(8d^4e-9d^3e^2x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^2e^4} \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{\left(3d^3\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^4} \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{\left(3d^3\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^4} \\
&= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}
\end{aligned}$$

Mathematica [A] time = 0.095506, size = 91, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(-7d^2ex-16d^3+de^2x^2-2e^3x^3\right)-9d^3(d+ex)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 7*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3) - 9*d^3*
(d + e*x)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(6*e^5*(d + e*x))
```

Maple [A] time = 0.063, size = 147, normalized size = 1.3

$$-\frac{x^2}{3e^3}\sqrt{-x^2e^2+d^2} - \frac{5d^2}{3e^5}\sqrt{-x^2e^2+d^2} + \frac{dx}{2e^4}\sqrt{-x^2e^2+d^2} - \frac{3d^3}{2e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{d^3}{e^6}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3-5/3/e^5*d^2*(-e^2*x^2+d^2)^(1/2)+1/2*d/e^
4*x*(-e^2*x^2+d^2)^(1/2)-3/2*d^3/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2
*x^2+d^2)^(1/2))-d^3/e^6/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69335, size = 236, normalized size = 2.

$$-\frac{16\,d^3ex+16\,d^4-18\left(d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,e^3x^3-de^2x^2+7\,d^2ex+16\,d^3\right)\sqrt{-e^2x^2+d^2}}{6\left(e^6x+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^3*e*x + 16*d^4 - 18*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) + (2*e^3*x^3 - d*e^2*x^2 + 7*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2
+ d^2))/(e^6*x + d*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.120 $\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=91

$$\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

```
[Out] (x^2*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) + ((4*d - 3*e*x)*Sqrt[d^2 - e^2*x
^2])/(2*e^4) + (3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rubi [A] time = 0.0635089, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^2*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) + ((4*d - 3*e*x)*Sqrt[d^2 - e^2*x
^2])/(2*e^4) + (3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{\left(3d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^3} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^3} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}
\end{aligned}$$

Mathematica [A] time = 0.0673267, size = 80, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^2+dex-e^2x^2\right)+3d^2(d+ex)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^2 + d*e*x - e^2*x^2) + 3*d^2*(d + e*x)*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4*(d + e*x))
```

Maple [A] time = 0.058, size = 120, normalized size = 1.3

$$-\frac{x}{2e^3}\sqrt{-x^2e^2+d^2}+\frac{3d^2}{2e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+\frac{d}{e^4}\sqrt{-x^2e^2+d^2}+\frac{d^2}{e^5}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2/e^3*x*(-e^2*x^2+d^2)^(1/2)+3/2*d^2/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*
x/(-e^2*x^2+d^2)^(1/2))+d/e^4*(-e^2*x^2+d^2)^(1/2)+d^2/e^5/(d/e+x)*(-(d/e+x
)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56663, size = 205, normalized size = 2.25

$$\frac{4\,d^2ex + 4\,d^3 - 6\left(d^2ex + d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(e^2x^2 - dex - 4\,d^2\right)\sqrt{-e^2x^2 + d^2}}{2\left(e^5x + de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*d^2*e*x + 4*d^3 - 6*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (e^2*x^2 - d*e*x - 4*d^2)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-\left(-d + ex\right)\left(d + ex\right)}\left(d + ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.121 $\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e^3) - (d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rubi [A] time = 0.0973203, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1639, 12, 793, 217, 203}

$$-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e^3) - (d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{\int \frac{de^3x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.0796284, size = 59, normalized size = 0.77

$$-\frac{\frac{\sqrt{d^2-e^2x^2}(2d+ex)}{d+ex}+d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((((2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(d + e*x) + d*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/e^3)
```

Maple [A] time = 0.053, size = 97, normalized size = 1.3

$$-\frac{1}{e^3}\sqrt{-x^2e^2+d^2}-\frac{d}{e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{d}{e^4}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^3-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2))-d/e^4/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59906, size = 177, normalized size = 2.3

$$-\frac{2\,dex+2\,d^2-2\left(dex+d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex+2\,d)}{e^4x+de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*d*e*x + 2*d^2 - 2*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + sqrt(-e^2*x^2 + d^2)*(e*x + 2*d))/(e^4*x + d*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.122 $\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^2
```

Rubi [A] time = 0.0221245, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {793, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^2
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}\\ &= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}\\ &= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\end{aligned}$$

Mathematica [A] time = 0.0318285, size = 49, normalized size = 0.94

$$\frac{\frac{\sqrt{d^2-e^2x^2}}{d+ex}+\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]/(d + e*x) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Maple [A] time = 0.053, size = 74, normalized size = 1.4

$$\frac{1}{e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+\frac{1}{e^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 1/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/e^3/(d/e+x)*(-
(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63051, size = 143, normalized size = 2.75

$$\frac{ex-2\,(ex+d)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d+\sqrt{-e^2x^2+d^2}}{e^3x+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x - 2*(e*x + d)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d + sqrt(-e^
2*x^2 + d^2))/(e^3*x + d*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.123 $\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rubi [A] time = 0.0107353, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {651}

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Mathematica [A] time = 0.0064883, size = 32, normalized size = 1.03

$$-\frac{\sqrt{d^2-e^2x^2}}{d^2e+de^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d^2*e + d*e^2*x))
```

Maple [A] time = 0.047, size = 29, normalized size = 0.9

$$-\frac{-ex+d}{de}\frac{1}{\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e*x+d)/d/e/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55864, size = 72, normalized size = 2.32

$$-\frac{ex + d + \sqrt{-e^2x^2 + d^2}}{de^2x + d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(e*x + d + sqrt(-e^2*x^2 + d^2))/(d*e^2*x + d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-d + ex)(d + ex)}(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.124 $\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] Sqrt[d^2 - e^2*x^2]/(d^2*(d + e*x)) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^2
```

Rubi [A] time = 0.0443948, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {857, 12, 266, 63, 208}

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(d^2*(d + e*x)) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^2
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\int \frac{de^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d} \\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d} \\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{de^2} \\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.0394156, size = 52, normalized size = 0.96

$$\frac{\frac{\sqrt{d^2-e^2x^2}}{d+ex} - \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]/(d + e*x) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Maple [A] time = 0.056, size = 88, normalized size = 1.6

$$-\frac{1}{d}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} + \frac{1}{ed^2}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/e/d^2/(
d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x), x)
```

Fricas [A] time = 1.64702, size = 131, normalized size = 2.43

$$\frac{ex + (ex + d)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + d + \sqrt{-e^2x^2 + d^2}}{d^2ex + d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + (e*x + d)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + d + sqrt(-e^2*x^2 + d
^2))/(d^2*e*x + d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(-d + ex)(d + ex)}(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.125 $\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(d^3*x) + Sqrt[d^2 - e^2*x^2]/(d^2*x*(d + e*x)) +
(e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^3
```

Rubi [A] time = 0.063554, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {857, 807, 266, 63, 208}

$$-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(d^3*x) + Sqrt[d^2 - e^2*x^2]/(d^2*x*(d + e*x)) +
(e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^3
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{\int \frac{-2de^2+e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} - \frac{e\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^2e}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.0556433, size = 62, normalized size = 0.77

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{(d+2ex)\sqrt{d^2-e^2x^2}}{x(d+ex)}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-(((d + 2*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x))) + e*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/d^3
```

Maple [A] time = 0.059, size = 108, normalized size = 1.3

$$\frac{e}{d^2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{1}{d^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1} - \frac{1}{d^3x}\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] e/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/d^3/(d
/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2}(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^2), x)
```

Fricas [A] time = 1.56651, size = 176, normalized size = 2.17

$$-\frac{e^2x^2 + dex + \left(e^2x^2 + dex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \sqrt{-e^2x^2 + d^2}(2\,ex + d)}{d^3ex^2 + d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(e^2*x^2 + d*e*x + (e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) +
sqrt(-e^2*x^2 + d^2)*(2*e*x + d))/(d^3*e*x^2 + d^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{-(-d + ex)(d + ex)}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.126 $\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=113

$$\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

```
[Out] (-3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^4*x) +
Sqrt[d^2 - e^2*x^2]/(d^2*x^2*(d + e*x)) - (3*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2
]/d])/(2*d^4)
```

Rubi [A] time = 0.0907612, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 835, 807, 266, 63, 208}

$$\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^4*x) +
Sqrt[d^2 - e^2*x^2]/(d^2*x^2*(d + e*x)) - (3*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2
]/d])/(2*d^4)
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{\int \frac{-3de^2+2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\int \frac{-4d^2e^3+3de^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4e^2} \\
&= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\left(3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^3} \\
&= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\left(3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{4d^3} \\
&= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^3} \\
&= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}
\end{aligned}
$$

Mathematica [A] time = 0.338108, size = 127, normalized size = 1.12

$$
-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4} - \frac{de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}\tanh^{-1}\left(\sqrt{1-\frac{e^2x^2}{d^2}}\right) - 2d^2ex + d^3 - 3de^2x^2 + 4e^3x^3}{2d^4x^2\sqrt{d^2-e^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4) - (d^3 - 2*d^2*e*x - 3*d*e^2*x^
2 + 4*e^3*x^3 + d*e^2*x^2*Sqrt[1 - (e^2*x^2)/d^2]*ArcTanh[Sqrt[1 - (e^2*x^2
)/d^2]])/(2*d^4*x^2*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.068, size = 133, normalized size = 1.2

$$-\frac{3e^{2}}{2d^{3}}\ln\left(\frac{1}{x}\left(2d^{2}+2\sqrt{d^{2}}\sqrt{-x^{2}e^{2}+d^{2}}\right)\right)\frac{1}{\sqrt{d^{2}}}+\frac{e}{d^{4}}\sqrt{-\left(\frac{d}{e}+x\right)^{2}e^{2}+2de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1}-\frac{1}{2d^{3}x^{2}}\sqrt{-x^{2}e^{2}+d^{2}}+\frac{e}{d^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -3/2*e^2/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+e
/d^4/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/
d^3/x^2+e*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^{2}x^{2}+d^{2}}(ex+d)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^3), x)
```

Fricas [A] time = 1.59989, size = 220, normalized size = 1.95

$$\frac{2e^{3}x^{3}+2de^{2}x^{2}+3\left(e^{3}x^{3}+de^{2}x^{2}\right)\log\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)+\left(4e^{2}x^{2}+dex-d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{2\left(d^{4}ex^{3}+d^{5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^3*x^3 + 2*d*e^2*x^2 + 3*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (4*e^2*x^2 + d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^3
 + d^5*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.127 $\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}-\frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6}-\frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] (x^4*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x^2*(4*d - 5*e*x))/(3*e^4*
Sqrt[d^2 - e^2*x^2]) - ((16*d - 15*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^6) - (5*d
^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rubi [A] time = 0.105259, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}-\frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6}-\frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^4*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x^2*(4*d - 5*e*x))/(3*e^4*
Sqrt[d^2 - e^2*x^2]) - ((16*d - 15*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^6) - (5*d
^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^5(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x^3\left(4d^3-5d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2} \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4} \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{\left(5d^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^5} \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\right.}{2e^5} \\
&= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}
\end{aligned}
$$

Mathematica [A] time = 0.185987, size = 106, normalized size = 0.83

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\left(-23d^2e^2x^2+d^3ex+16d^4-3de^3x^3+3e^4x^4\right)}{(ex-d)(d+ex)^2}-15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(16*d^4 + d^3*e*x - 23*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*
e^4*x^4))/((-d + e*x)*(d + e*x)^2) - 15*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(6*e^6)
```

Maple [A] time = 0.066, size = 208, normalized size = 1.6

$$
-\frac{x^3}{2e^3}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{7d^2x}{2e^5}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{5d^2}{2e^5}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+\frac{dx^2}{e^4}\frac{1}{\sqrt{-x^2e^2+d^2}}-3\frac{d^3}{e^6\sqrt{-x^2e^2+}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/2/e^3*x^3/(-e^2*x^2+d^2)^(1/2)+7/2/e^5*d^2*x/(-e^2*x^2+d^2)^(1/2)-5/2/e^
5*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d/e^4*x^2/(-e^
2*x^2+d^2)^(1/2)-3*d^3/e^6/(-e^2*x^2+d^2)^(1/2)+1/3*d^4/e^7/(d/e+x)/(-(d/e+
x)^2*e^2+2*d*e*(d/e+x))^(1/2)-2/3*d^2/e^5/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1
/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66397, size = 381, normalized size = 2.98

$$-\frac{16\,d^2e^3x^3+16\,d^3e^2x^2-16\,d^4ex-16\,d^5-30\left(d^2e^3x^3+d^3e^2x^2-d^4ex-d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(3\,e^4x^4-3\,de^3x^3-\right.}{6\left(e^9x^3+de^8x^2-d^2e^7x-d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^2*e^3*x^3 + 16*d^3*e^2*x^2 - 16*d^4*e*x - 16*d^5 - 30*(d^2*e^3*x
^3 + d^3*e^2*x^2 - d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
 - (3*e^4*x^4 - 3*d*e^3*x^3 - 23*d^2*e^2*x^2 + d^3*e*x + 16*d^4)*sqrt(-e^2*
x^2 + d^2))/(e^9*x^3 + d*e^8*x^2 - d^2*e^7*x - d^3*e^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, 1]
```

3.128 $\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] (x^3*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x*(3*d - 4*e*x))/(3*e^4*Sq
rt[d^2 - e^2*x^2]) + (8*Sqrt[d^2 - e^2*x^2])/(3*e^5) + (d*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/e^5
```

Rubi [A] time = 0.0938843, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 641, 217, 203}

$$\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^3*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x*(3*d - 4*e*x))/(3*e^4*Sq
rt[d^2 - e^2*x^2]) + (8*Sqrt[d^2 - e^2*x^2])/(3*e^5) + (d*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/e^5
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx\\
&= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{x^2\left(3d^3-4d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}\\
&= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{\int \frac{3d^5-8d^4ex}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4}\\
&= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^4}\\
&= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}
\end{aligned}$$

Mathematica [A] time = 0.150518, size = 93, normalized size = 0.82

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(5d^2ex+8d^3-7de^2x^2-3e^3x^3\right)}{(d-ex)(d+ex)^2}+3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^3 + 5*d^2*e*x - 7*d*e^2*x^2 - 3*e^3*x^3))/((d -
e*x)*(d + e*x)^2) + 3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^5)
```

Maple [A] time = 0.062, size = 179, normalized size = 1.6

$$-\frac{x^2}{e^3}\frac{1}{\sqrt{-x^2e^2+d^2}}+3\frac{d^2}{e^5\sqrt{-x^2e^2+d^2}}-2\frac{dx}{e^4\sqrt{-x^2e^2+d^2}}+\frac{d}{e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{d^3}{3e^6}\left(\frac{d}{e}+x\right)^{-1}\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/e^3*x^2/(-e^2*x^2+d^2)^(1/2)+3*d^2/e^5/(-e^2*x^2+d^2)^(1/2)-2*d/e^4*x/(-
e^2*x^2+d^2)^(1/2)+d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1
/2))-1/3*d^3/e^6/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+2/3*d/e^4/(-(
d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71407, size = 346, normalized size = 3.06

$$\frac{8\,de^3x^3 + 8\,d^2e^2x^2 - 8\,d^3ex - 8\,d^4 - 6\left(de^3x^3 + d^2e^2x^2 - d^3ex - d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(3\,e^3x^3 + 7\,de^2x^2 - 5\,d^2ex - \right.}{3\left(e^8x^3 + de^7x^2 - d^2e^6x - d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*d*e^3*x^3 + 8*d^2*e^2*x^2 - 8*d^3*e*x - 8*d^4 - 6*(d*e^3*x^3 + d^2*e
^2*x^2 - d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^3*
x^3 + 7*d*e^2*x^2 - 5*d^2*e*x - 8*d^3)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + d*e
^7*x^2 - d^2*e^6*x - d^3*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, 1]
```

3.129 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] (x^2*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (2*d - 3*e*x)/(3*e^4*Sqrt[d
^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rubi [A] time = 0.0702345, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 217, 203}

$$\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^2*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (2*d - 3*e*x)/(3*e^4*Sqrt[d
^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx \\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2} \\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3} \\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3} \\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}
\end{aligned}$$

Mathematica [A] time = 0.136403, size = 80, normalized size = 0.9

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-2d^2+dex+4e^2x^2\right)}{(d-ex)(d+ex)^2} - 3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-2*d^2 + d*e*x + 4*e^2*x^2))/((d - e*x)*(d + e*x)^2)
 - 3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^4)
```

Maple [A] time = 0.056, size = 153, normalized size = 1.7

$$2\frac{x}{e^3\sqrt{-x^2e^2+d^2}} - \frac{1}{e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{d}{e^4}\frac{1}{\sqrt{-x^2e^2+d^2}} + \frac{d^2}{3e^5}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 2/e^3*x/(-e^2*x^2+d^2)^(1/2)-1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))-d/e^4/(-e^2*x^2+d^2)^(1/2)+1/3*d^2/e^5/(d/e+x)/(-(d/e+x)^2*e
^2+2*d*e*(d/e+x))^(1/2)-2/3/e^3/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59925, size = 312, normalized size = 3.51

$$-\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3-6\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(4\,e^2x^2+dex-2\,d^2\right)\sqrt{-e^2x^2}}{3\left(e^7x^3+de^6x^2-d^2e^5x-d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 - 6*(e^3*x^3 + d*e^2*x^2
- d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*e^2*x^2 + d
*e*x - 2*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + d*e^6*x^2 - d^2*e^5*x - d^3*
e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.130 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/(3*e^3*Sqrt[d^2 - e^2*x^2]) - x^2/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0346268, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {855, 12, 261}

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] 2/(3*e^3*Sqrt[d^2 - e^2*x^2]) - x^2/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{\int \frac{2dx}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3de}\\ &= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{x}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}\\ &= \frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0568615, size = 60, normalized size = 1.

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^2+2dex-e^2x^2\right)}{3de^3(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 + 2*d*e*x - e^2*x^2))/(3*d*e^3*(d - e*x)*(d + e
*x)^2)
```

Maple [A] time = 0.048, size = 48, normalized size = 0.8

$$\frac{(-ex+d)\left(-x^2e^2+2\,dex+2\,d^2\right)}{3\,de^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/3*(-e*x+d)*(-e^2*x^2+2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6291, size = 203, normalized size = 3.38

$$\frac{2\,e^3x^3+2\,de^2x^2-2\,d^2ex-2\,d^3+\left(e^2x^2-2\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(de^6x^3+d^2e^5x^2-d^3e^4x-d^4e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^3*x^3 + 2*d*e^2*x^2 - 2*d^2*e*x - 2*d^3 + (e^2*x^2 - 2*d*e*x - 2*d
^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + d^2*e^5*x^2 - d^3*e^4*x - d^4*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.25927, size = 1, normalized size = 0.02

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

3.131 $\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] x/(3*d^2*e*Sqrt[d^2 - e^2*x^2]) + 1/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0203455, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {793, 191}

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] x/(3*d^2*e*Sqrt[d^2 - e^2*x^2]) + 1/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}\\ &= \frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0543328, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(d^2+dex+e^2x^2\right)}{3d^2e^2(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^2 + d*e*x + e^2*x^2))/(3*d^2*e^2*(d - e*x)*(d + e*x
)^2)
```

Maple [A] time = 0.048, size = 44, normalized size = 0.8

$$\frac{(-ex+d)\left(x^2e^2+dex+d^2\right)}{3\,d^2e^2}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/3*(-e*x+d)*(e^2*x^2+d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56244, size = 189, normalized size = 3.26

$$\frac{e^3x^3+de^2x^2-d^2ex-d^3-\left(e^2x^2+dex+d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(d^2e^5x^3+d^3e^4x^2-d^4e^3x-d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 - (e^2*x^2 + d*e*x + d^2)*sqrt(-e^
2*x^2 + d^2))/(d^2*e^5*x^3 + d^3*e^4*x^2 - d^4*e^3*x - d^5*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(-(-d+ex)(d+ex)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.132 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0146012, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 191}

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d}\\ &= \frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0379513, size = 58, normalized size = 1.

$$-\frac{\left(d^2-2dex-2e^2x^2\right)\sqrt{d^2-e^2x^2}}{3d^3e(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -((d^2 - 2*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*d^3*e*(d - e*x)*(d +
e*x)^2)
```

Maple [A] time = 0.048, size = 46, normalized size = 0.8

$$-\frac{(-ex+d)\left(-2x^2e^2-2dex+d^2\right)}{3d^3e}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/3*(-e*x+d)*(-2*e^2*x^2-2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.5293, size = 193, normalized size = 3.33

$$-\frac{e^3x^3+de^2x^2-d^2ex-d^3+\left(2e^2x^2+2dex-d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(d^3e^4x^3+d^4e^3x^2-d^5e^2x-d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 + (2*e^2*x^2 + 2*d*e*x - d^2)*sqr
t(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + d^4*e^3*x^2 - d^5*e^2*x - d^6*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-(-d+ex)(d+ex)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.133 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] (3*d - 2*e*x)/(3*d^4*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*(d + e*x)*Sqrt[d^2 - e
^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^4
```

Rubi [A] time = 0.0754648, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 12, 266, 63, 208}

$$\frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (3*d - 2*e*x)/(3*d^4*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*(d + e*x)*Sqrt[d^2 - e
^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^4
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] && !IGtQ[n
, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-3de^2+2e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}\\ &= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{3d^3e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}\\ &= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}\\ &= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3}\\ &= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^3e^2}\\ &= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}\end{aligned}$$

Mathematica [A] time = 0.108529, size = 83, normalized size = 0.94

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(4d^2+dex-2e^2x^2\right)}{(d-ex)(d+ex)^2} - 3\log\left(\sqrt{d^2-e^2x^2}+d\right) + 3\log(x)}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (((4*d^2 + d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/((d - e*x)*(d + e*x)^2)
+ 3*Log[x] - 3*Log[d + Sqrt[d^2 - e^2*x^2]])/(3*d^4)
```

Maple [A] time = 0.057, size = 142, normalized size = 1.6

$$\frac{1}{d^3}\frac{1}{\sqrt{-x^2e^2+d^2}} - \frac{1}{d^3}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} + \frac{1}{3ed^2}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}} - \frac{2ex}{3d^4\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/d^3/(-e^2*x^2+d^2)^(1/2)-1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)+1/3/d^2/e/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-2
/3/d^4*e/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x), x)
```

Fricas [A] time = 1.60024, size = 301, normalized size = 3.42

$$\frac{4\,e^3x^3+4\,de^2x^2-4\,d^2ex-4\,d^3+3\left(e^3x^3+de^2x^2-d^2ex-d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(2\,e^2x^2-dex-4\,d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(d^4e^3x^3+d^5e^2x^2-d^6ex-d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*e^3*x^3 + 4*d*e^2*x^2 - 4*d^2*e*x - 4*d^3 + 3*(e^3*x^3 + d*e^2*x^2 -
 d^2*e*x - d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (2*e^2*x^2 - d*e*x - 4
*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + d^5*e^2*x^2 - d^6*e*x - d^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.134 $\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{3d^5x}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] (4*d - 3*e*x)/(3*d^4*x*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x*(d + e*x)*Sqrt[d^2
 - e^2*x^2]) - (8*Sqrt[d^2 - e^2*x^2])/(3*d^5*x) + (e*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/d^5
```

Rubi [A] time = 0.102529, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 807, 266, 63, 208}

$$\frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{3d^5x}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```


```
[Out] (4*d - 3*e*x)/(3*d^4*x*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x*(d + e*x)*Sqrt[d^2
 - e^2*x^2]) - (8*Sqrt[d^2 - e^2*x^2])/(3*d^5*x) + (e*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/d^5
```


Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-4de^2+3e^3x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-8d^3e^4+3d^2e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x,\right.}{2d^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2}\right.}{d^4e}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [A] time = 0.125512, size = 101, normalized size = 0.84

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(7d^2ex+3d^3-5de^2x^2-8e^3x^3\right)}{x(ex-d)(d+ex)^2}+3e\log\left(\sqrt{d^2-e^2x^2}+d\right)-3e\log(x)}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^3 + 7*d^2*e*x - 5*d*e^2*x^2 - 8*e^3*x^3))/(x*(-d
 + e*x)*(d + e*x)^2) - 3*e*Log[x] + 3*e*Log[d + Sqrt[d^2 - e^2*x^2]])/(3*d^
```

5)

Maple [A] time = 0.062, size = 188, normalized size = 1.6

$$-\frac{e}{d^4}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{e}{d^4}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{1}{3\,d^3}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}}+\frac{2\,e^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -e/d^4/(-e^2*x^2+d^2)^(1/2)+e/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)-1/3/d^3/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+2/
3*e^2/d^5/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-1/d^3/x/(-e^2*x^2+d^2)^(1/
2)+2/d^5*e^2*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x)
```

Fricas [A] time = 1.60876, size = 354, normalized size = 2.95

$$-\frac{4\,e^4x^4+4\,de^3x^3-4\,d^2e^2x^2-4\,d^3ex+3\left(e^4x^4+de^3x^3-d^2e^2x^2-d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8\,e^3x^3+5\,de^2x^2-7\,d^2\right.}{3\left(d^5e^3x^4+d^6e^2x^3-d^7ex^2-d^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*e^4*x^4 + 4*d*e^3*x^3 - 4*d^2*e^2*x^2 - 4*d^3*e*x + 3*(e^4*x^4 + d*
e^3*x^3 - d^2*e^2*x^2 - d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^
3*x^3 + 5*d*e^2*x^2 - 7*d^2*e*x - 3*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^4
 + d^6*e^2*x^3 - d^7*e*x^2 - d^8*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.22125, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

3.135 $\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

```
[Out] (5*d - 4*e*x)/(3*d^4*x^2*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x^2*(d + e*x)*Sqrt
[d^2 - e^2*x^2]) - (5*Sqrt[d^2 - e^2*x^2])/(2*d^5*x^2) + (8*e*Sqrt[d^2 - e^
2*x^2])/(3*d^6*x) - (5*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rubi [A] time = 0.129039, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```


```
[Out] (5*d - 4*e*x)/(3*d^4*x^2*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x^2*(d + e*x)*Sqrt
[d^2 - e^2*x^2]) - (5*Sqrt[d^2 - e^2*x^2])/(2*d^5*x^2) + (8*e*Sqrt[d^2 - e^
2*x^2])/(3*d^6*x) - (5*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```


Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-5de^2+4e^3x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{-16d^4e^5+15d^3e^6x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^8e^4}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} + \frac{\left(5e^2\right)\int}{}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} + \frac{\left(5e^2\right)\mathrm{S}}{}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{5\,\mathrm{Subs}}{}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{5e^2\tan}{}$$

Mathematica [A] time = 0.115383, size = 115, normalized size = 0.76

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-23d^2e^2x^2-3d^3ex+3d^4+de^3x^3+16e^4x^4\right)}{x^2(ex-d)(d+ex)^2}-15e^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+15e^2\log(x)}{6d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x - 23*d^2*e^2*x^2 + d*e^3*x^3 + 16*
e^4*x^4))/(x^2*(-d + e*x)*(d + e*x)^2) + 15*e^2*Log[x] - 15*e^2*Log[d + Sqr
t[d^2 - e^2*x^2]])/(6*d^6)
```

Maple [A] time = 0.063, size = 216, normalized size = 1.4

$$\frac{5e^2}{2d^5}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{5e^2}{2d^5}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{e}{3d^4}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}}-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 5/2/d^5*e^2/(-e^2*x^2+d^2)^(1/2)-5/2/d^5*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^
(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/3*e/d^4/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(1/2)-2/3*e^3/d^6/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-1/2/d^3/x^2/(
-e^2*x^2+d^2)^(1/2)+e/d^4/x/(-e^2*x^2+d^2)^(1/2)-2*e^3/d^6*x/(-e^2*x^2+d^2)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^3), x)
```

Fricas [A] time = 1.70155, size = 394, normalized size = 2.59

$$\frac{14e^5x^5+14de^4x^4-14d^2e^3x^3-14d^3e^2x^2+15\left(e^5x^5+de^4x^4-d^2e^3x^3-d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16e^4x^4+de^3x\right.}{6\left(d^6e^3x^5+d^7e^2x^4-d^8ex^3-d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(14*e^5*x^5 + 14*d*e^4*x^4 - 14*d^2*e^3*x^3 - 14*d^3*e^2*x^2 + 15*(e^5*
x^5 + d*e^4*x^4 - d^2*e^3*x^3 - d^3*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (16*e^4*x^4 + d*e^3*x^3 - 23*d^2*e^2*x^2 - 3*d^3*e*x + 3*d^4)*sqrt(-
e^2*x^2 + d^2))/(d^6*e^3*x^5 + d^7*e^2*x^4 - d^8*e*x^3 - d^9*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.136 $\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

```
[Out] (x^6*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d - 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d - 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d - 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) + (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rubi [A] time = 0.160423, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 780, 217, 203}

$$\frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```


```
[Out] (x^6*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d - 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d - 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d - 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) + (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```


Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^5\left(6d^3-7d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^3\left(24d^5-35d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x\left(48d^7-105d^6ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} +$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} +$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} +$$

Mathematica [A] time = 0.261823, size = 128, normalized size = 0.79

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-249d^4e^2x^2-4d^3e^3x^3+176d^2e^4x^4-9d^5ex+96d^6+15de^5x^5-15e^6x^6\right)}{(d-ex)^2(d+ex)^3}+105d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{30e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(96*d^6 - 9*d^5*e*x - 249*d^4*e^2*x^2 - 4*d^3*e^3*x^3
 + 176*d^2*e^4*x^4 + 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^2*(d + e*x)^3)
+ 105*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(30*e^8)
```

Maple [B] time = 0.087, size = 318, normalized size = 2.

$$-\frac{x^5}{2e^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{7d^2x^3}{6e^5}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}-\frac{19d^2x}{6e^7}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{7d^2}{2e^7}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+\frac{dx^4}{e^4}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/2/e^3*x^5/(-e^2*x^2+d^2)^(3/2)+7/6/e^5*d^2*x^3/(-e^2*x^2+d^2)^(3/2)-19/6
/e^7*d^2*x/(-e^2*x^2+d^2)^(1/2)+7/2/e^7*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*
x/(-e^2*x^2+d^2)^(1/2))+d/e^4*x^4/(-e^2*x^2+d^2)^(3/2)-5*d^3/e^6*x^2/(-e^2*
x^2+d^2)^(3/2)+3*d^5/e^8/(-e^2*x^2+d^2)^(3/2)+2/3*d^4/e^7*x/(-e^2*x^2+d^2)^
(3/2)+1/5*d^6/e^9/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-4/15*d^4/e^7
/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-8/15*d^2/e^7/(-(d/e+x)^2*e^2+2*d*e*
(d/e+x))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00057, size = 575, normalized size = 3.55

$$\frac{96d^2e^5x^5+96d^3e^4x^4-192d^4e^3x^3-192d^5e^2x^2+96d^6ex+96d^7-210\left(d^2e^5x^5+d^3e^4x^4-2d^4e^3x^3-2d^5e^2x^2+d^6e}{30\left(e^{13}x^5+de^{12}x^4-2d^2e^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*e^5*x^5 + 96*d^3*e^4*x^4 - 192*d^4*e^3*x^3 - 192*d^5*e^2*x^2 +
 96*d^6*e*x + 96*d^7 - 210*(d^2*e^5*x^5 + d^3*e^4*x^4 - 2*d^4*e^3*x^3 - 2*d
^5*e^2*x^2 + d^6*e*x + d^7)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15
*e^6*x^6 - 15*d*e^5*x^5 - 176*d^2*e^4*x^4 + 4*d^3*e^3*x^3 + 249*d^4*e^2*x^2
 + 9*d^5*e*x - 96*d^6)*sqrt(-e^2*x^2 + d^2))/(e^13*x^5 + d*e^12*x^4 - 2*d^2
*e^11*x^3 - 2*d^3*e^10*x^2 + d^4*e^9*x + d^5*e^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**7/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.137 $\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=148

$$\frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

```
[Out] (x^5*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d - 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d - 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) - (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rubi [A] time = 0.137417, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 641, 217, 203}

$$\frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^5*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d - 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d - 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) - (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^4\left(5d^3-6d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^2\left(15d^5-24d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^7-48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,d}{e^6}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\,\text{Subst}\left(\int \frac{}{1-}\right.}{}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-}}\right.}{e^7}$$

Mathematica [A] time = 0.206691, size = 115, normalized size = 0.78

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(-87d^3e^2x^2-52d^2e^3x^3+33d^4ex+48d^5+38de^4x^4+15e^5x^5\right)}{(d-ex)^2(d+ex)^3}+15d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(48*d^5 + 33*d^4*e*x - 87*d^3*e^2*x^2 - 52*d^2*e^3*x
^3 + 38*d*e^4*x^4 + 15*e^5*x^5))/((d - e*x)^2*(d + e*x)^3) + 15*d*ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^7)
```

Maple [B] time = 0.073, size = 288, normalized size = 2.

$$-\frac{x^4}{e^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+5\frac{d^2x^2}{e^5\left(-x^2e^2+d^2\right)^{3/2}}-3\frac{d^4}{e^7\left(-x^2e^2+d^2\right)^{3/2}}-\frac{dx^3}{3e^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{2dx}{3e^6}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{d}{e^6}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/e^3*x^4/(-e^2*x^2+d^2)^(3/2)+5/e^5*d^2*x^2/(-e^2*x^2+d^2)^(3/2)-3*d^4/e^
7/(-e^2*x^2+d^2)^(3/2)-1/3*d/e^4*x^3/(-e^2*x^2+d^2)^(3/2)+2/3*d/e^6*x/(-e^2
*x^2+d^2)^(1/2)-d/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)-2/3*d^3/e^6*x/(-e^2*x^2+d^2)^(3/2)-1/5*d^5/e^8/(d/e+x)/(-(d/e+x)^2*e^2+2*
d*e*(d/e+x))^(3/2)+4/15*d^3/e^6/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+8/15
*d/e^6/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80288, size = 543, normalized size = 3.67

$$-\frac{48\,de^5x^5+48\,d^2e^4x^4-96\,d^3e^3x^3-96\,d^4e^2x^2+48\,d^5ex+48\,d^6-30\,(de^5x^5+d^2e^4x^4-2\,d^3e^3x^3-2\,d^4e^2x^2+d^5ex+}{15\,(e^{12}x^5+de^{11}x^4-2\,d^2e^{10}x^3-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(48*d*e^5*x^5 + 48*d^2*e^4*x^4 - 96*d^3*e^3*x^3 - 96*d^4*e^2*x^2 + 48
*d^5*e*x + 48*d^6 - 30*(d*e^5*x^5 + d^2*e^4*x^4 - 2*d^3*e^3*x^3 - 2*d^4*e^2
*x^2 + d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*e^5*x
^5 + 38*d*e^4*x^4 - 52*d^2*e^3*x^3 - 87*d^3*e^2*x^2 + 33*d^4*e*x + 48*d^5)*
sqrt(-e^2*x^2 + d^2))/(e^12*x^5 + d*e^11*x^4 - 2*d^2*e^10*x^3 - 2*d^3*e^9*x
^2 + d^4*e^8*x + d^5*e^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**6/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, 1]
```

3.138 $\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] (x^4*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d - 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d - 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) + Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rubi [A] time = 0.101513, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 217, 203}

$$\frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^4*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d - 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d - 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) + Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= \int \frac{x^5(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^3\left(4d^3-5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2} \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4} \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5} \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^5} \\
&= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}
\end{aligned}
$$

Mathematica [A] time = 0.150955, size = 103, normalized size = 0.84

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\left(-27d^2e^2x^2-7d^3ex+8d^4+8de^3x^3+23e^4x^4\right)}{(d-ex)^2(d+ex)^3}+15\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(8*d^4 - 7*d^3*e*x - 27*d^2*e^2*x^2 + 8*d*e^3*x^3 + 2
3*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 15*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(15*e^6)
```

Maple [B] time = 0.066, size = 259, normalized size = 2.1

$$
\frac{x^3}{3e^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}} - \frac{2x}{3e^5}\frac{1}{\sqrt{-x^2e^2+d^2}} + \frac{1}{e^5}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{dx^2}{e^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}} + \frac{d^3}{3e^6}\left(-x^2e^2+d^2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3/e^3*x^3/(-e^2*x^2+d^2)^(3/2)-2/3/e^5*x/(-e^2*x^2+d^2)^(1/2)+1/e^5/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-d/e^4*x^2/(-e^2*x^2+d^2)^
(3/2)+1/3*d^3/e^6/(-e^2*x^2+d^2)^(3/2)+2/3*d^2/e^5*x/(-e^2*x^2+d^2)^(3/2)+1
/5*d^4/e^7/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-4/15*d^2/e^5/(-(d/e
+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-8/15/e^5/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1
/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71697, size = 497, normalized size = 4.07

$$\frac{8\,e^5x^5+8\,de^4x^4-16\,d^2e^3x^3-16\,d^3e^2x^2+8\,d^4ex+8\,d^5-30\left(e^5x^5+de^4x^4-2\,d^2e^3x^3-2\,d^3e^2x^2+d^4ex+d^5\right)\arctan}{15\left(e^{11}x^5+de^{10}x^4-2\,d^2e^9x^3-2\,d^3e^8x^2+d^4e^7x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*x
 + 8*d^5 - 30*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*
x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (23*e^4*x^4 + 8*d*e^3*
x^3 - 27*d^2*e^2*x^2 - 7*d^3*e*x + 8*d^4)*sqrt(-e^2*x^2 + d^2))/(e^11*x^5 +
 d*e^10*x^4 - 2*d^2*e^9*x^3 - 2*d^3*e^8*x^2 + d^4*e^7*x + d^5*e^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.139 $\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}} dx$

Optimal. Leaf size=85

$$-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] -(x^4*(d - e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*d^2)/(15*e^5*(d^2 - e^2
*x^2)^(3/2)) - 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0742674, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {850, 805, 266, 43}

$$-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -(x^4*(d - e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*d^2)/(15*e^5*(d^2 - e^2
*x^2)^(3/2)) - 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$
$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{x^3}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}$$
$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\left(d^2-e^2x\right)^{5/2}}\,dx, x, x^2\right)}{5e}$$
$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \left(\frac{d^2}{e^2\left(d^2-e^2x\right)^{5/2}} - \frac{1}{e^2\left(d^2-e^2x\right)^{3/2}}\right)dx, x, x^2\right)}{5e}$$
$$= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}} + \frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0948815, size = 82, normalized size = 0.96

$$-\frac{\sqrt{d^2-e^2x^2}\left(-12d^2e^2x^2+8d^3ex+8d^4-12de^3x^3+3e^4x^4\right)}{15de^5(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(8*d^4 + 8*d^3*e*x - 12*d^2*e^2*x^2 - 12*d*e^3*x^3 +
3*e^4*x^4))/(15*d*e^5*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] time = 0.055, size = 70, normalized size = 0.8

$$-\frac{(-ex+d)\left(3\,x^4e^4-12\,x^3de^3-12\,d^2x^2e^2+8\,d^3xe+8\,d^4\right)}{15\,de^5}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(3*e^4*x^4-12*d*e^3*x^3-12*d^2*e^2*x^2+8*d^3*e*x+8*d^4)/d/e^
5/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.61291, size = 344, normalized size = 4.05

$$-\frac{8\,e^5x^5+8\,de^4x^4-16\,d^2e^3x^3-16\,d^3e^2x^2+8\,d^4ex+8\,d^5+\left(3\,e^4x^4-12\,de^3x^3-12\,d^2e^2x^2+8\,d^3ex+8\,d^4\right)\sqrt{-e^2x^2+d}}{15\left(de^{10}x^5+d^2e^9x^4-2\,d^3e^8x^3-2\,d^4e^7x^2+d^5e^6x+d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*e^5*x^5 + 8*d*e^4*x^4 - 16*d^2*e^3*x^3 - 16*d^3*e^2*x^2 + 8*d^4*e*
x + 8*d^5 + (3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2 + 8*d^3*e*x + 8*d^4)
*sqrt(-e^2*x^2 + d^2))/(d*e^10*x^5 + d^2*e^9*x^4 - 2*d^3*e^8*x^3 - 2*d^4*e^
7*x^2 + d^5*e^6*x + d^6*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.140 $\int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (x^2*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d - 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) - x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0694935, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {850, 819, 778, 191}

$$\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^2*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d - 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) - x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3}$$

$$= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0784474, size = 82, normalized size = 0.9

$$\frac{\sqrt{d^2-e^2x^2}\left(3d^2e^2x^2-2d^3ex-2d^4+3de^3x^3+3e^4x^4\right)}{15d^2e^4(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 - 2*d^3*e*x + 3*d^2*e^2*x^2 + 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] time = 0.048, size = 70, normalized size = 0.8

$$-\frac{(-ex+d)\left(-3\,x^4e^4-3\,x^3de^3-3\,x^2d^2e^2+2\,d^3xe+2\,d^4\right)}{15\,d^2e^4}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(-3*e^4*x^4-3*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^2/e
^4/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.65836, size = 340, normalized size = 3.74

$$-\frac{2e^5x^5+2de^4x^4-4d^2e^3x^3-4d^3e^2x^2+2d^4ex+2d^5-\left(3e^4x^4+3de^3x^3+3d^2e^2x^2-2d^3ex-2d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^9x^5+d^3e^8x^4-2d^4e^7x^3-2d^5e^6x^2+d^6e^5x+d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x
+ 2*d^5 - (3*e^4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2*d^3*e*x - 2*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^2*e^9*x^5 + d^3*e^8*x^4 - 2*d^4*e^7*x^3 - 2*d^5*e^6*x
^2 + d^6*e^5*x + d^7*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},\mathrm{undef},1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.141 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] -x^2/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (2*(d + e*x))/(15*d*e^3*(d^2
 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0533926, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {855, 778, 191}

$$-\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -x^2/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (2*(d + e*x))/(15*d*e^3*(d^2
 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x(2d+2ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de}$$

$$= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de^2}$$

$$= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0689794, size = 82, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\left(-3d^2e^2x^2+2d^3ex+2d^4+2de^3x^3+2e^4x^4\right)}{15d^3e^3(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^4 + 2*d^3*e*x - 3*d^2*e^2*x^2 + 2*d*e^3*x^3 + 2*e
^4*x^4))/(15*d^3*e^3*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] time = 0.048, size = 70, normalized size = 0.7

$$\frac{(-ex+d)\left(2e^4x^4+2x^3de^3-3x^2d^2e^2+2xd^3e+2d^4\right)}{15d^3e^3}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/15*(-e*x+d)*(2*e^4*x^4+2*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^3/e^3
/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74563, size = 339, normalized size = 3.57

$$\frac{2e^5x^5+2de^4x^4-4d^2e^3x^3-4d^3e^2x^2+2d^4ex+2d^5+\left(2e^4x^4+2de^3x^3-3d^2e^2x^2+2d^3ex+2d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^8x^5+d^4e^7x^4-2d^5e^6x^3-2d^6e^5x^2+d^7e^4x+d^8e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*e^5*x^5 + 2*d*e^4*x^4 - 4*d^2*e^3*x^3 - 4*d^3*e^2*x^2 + 2*d^4*e*x +
 2*d^5 + (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2*d^3*e*x + 2*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^3*e^8*x^5 + d^4*e^7*x^4 - 2*d^5*e^6*x^3 - 2*d^6*e^5*x^
2 + d^7*e^4*x + d^8*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.142 $\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}+\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)) + 1/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/
2)) + (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0287794, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {793, 192, 191}

$$\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}+\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)) + 1/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/
2)) + (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}$$
$$= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e}$$
$$= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0583992, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(3d^2e^2x^2+3d^3ex+3d^4-2de^3x^3-2e^4x^4\right)}{15d^4e^2(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 3*d^3*e*x + 3*d^2*e^2*x^2 - 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] time = 0.05, size = 70, normalized size = 0.8

$$\frac{(-ex+d)\left(-2\,e^4x^4-2\,e^3x^3d+3\,x^2d^2e^2+3\,xd^3e+3\,d^4\right)}{15\,d^4e^2}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/15*(-e*x+d)*(-2*e^4*x^4-2*d*e^3*x^3+3*d^2*e^2*x^2+3*d^3*e*x+3*d^4)/d^4/e^
2/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7433, size = 339, normalized size = 3.99

$$\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5-\left(2\,e^4x^4+2\,de^3x^3-3\,d^2e^2x^2-3\,d^3ex-3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^4e^7x^5+d^5e^6x^4-2\,d^6e^5x^3-2\,d^7e^4x^2+d^8e^3x+d^9e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x +
 3*d^5 - (2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 - 3*d^3*e*x - 3*d^4)*sqrt
(-e^2*x^2 + d^2))/(d^4*e^7*x^5 + d^5*e^6*x^4 - 2*d^6*e^5*x^3 - 2*d^7*e^4*x^
2 + d^8*e^3*x + d^9*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.143 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
3/2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0218122, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
3/2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3}$$

$$= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0426635, size = 82, normalized size = 1.

$$-\frac{\sqrt{d^2-e^2x^2}\left(-12d^2e^2x^2-12d^3ex+3d^4+8de^3x^3+8e^4x^4\right)}{15d^5e(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(3*d^4 - 12*d^3*e*x - 12*d^2*e^2*x^2 + 8*d*e^3*x^3 +
8*e^4*x^4))/(15*d^5*e*(d - e*x)^2*(d + e*x)^3)
```

Maple [A] time = 0.048, size = 70, normalized size = 0.9

$$-\frac{(-ex+d)\left(8\,e^4x^4+8\,e^3x^3d-12\,e^2x^2d^2-12\,xd^3e+3\,d^4\right)}{15\,d^5e}\left(-x^2e^2+d^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/15*(-e*x+d)*(8*e^4*x^4+8*d*e^3*x^3-12*d^2*e^2*x^2-12*d^3*e*x+3*d^4)/d^5/
e/(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72673, size = 342, normalized size = 4.17

$$-\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5+\left(8\,e^4x^4+8\,de^3x^3-12\,d^2e^2x^2-12\,d^3ex+3\,d^4\right)\sqrt{-e^2x^2+d^2}}{15\left(d^5e^6x^5+d^6e^5x^4-2\,d^7e^4x^3-2\,d^8e^3x^2+d^9e^2x+d^{10}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x
+ 3*d^5 + (8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 - 12*d^3*e*x + 3*d^4)*s
qrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + d^6*e^5*x^4 - 2*d^7*e^4*x^3 - 2*d^8*e^3
*x^2 + d^9*e^2*x + d^10*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.144 $\int \frac{1}{x(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] (5*d - 4*e*x)/(15*d^4*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*(d + e*x)*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^6
```

Rubi [A] time = 0.10675, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 12, 266, 63, 208}

$$\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}+\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```


```
[Out] (5*d - 4*e*x)/(15*d^4*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*(d + e*x)*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^6
```


Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-5de^2+4e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^5e^6}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^6}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-\cdots}}\cdots\right)}{2d^5\cdots}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\cdots\right)}{\cdots}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x\cdots}}{d}\right)}{d^6}$$

Mathematica [A] time = 0.095976, size = 106, normalized size = 0.89

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-27d^2e^2x^2+8d^3ex+23d^4-7de^3x^3+8e^4x^4\right)}{(d-ex)^2(d+ex)^3} - 15\log\left(\sqrt{d^2-e^2x^2}+d\right) + 15\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(23*d^4 + 8*d^3*e*x - 27*d^2*e^2*x^2 - 7*d*e^3*x^3 +
8*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 15*Log[x] - 15*Log[d + Sqrt[d^2 - e
^2*x^2]])/(15*d^6)
```

Maple [A] time = 0.059, size = 196, normalized size = 1.7

$$\frac{1}{3d^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}+\frac{1}{d^5}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{1}{d^5}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{1}{5ed^2}\left(\frac{d}{e}+x\right)^{-1}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 1/3/d^3/(-e^2*x^2+d^2)^(3/2)+1/d^5/(-e^2*x^2+d^2)^(1/2)-1/d^5/(d^2)^(1/2)*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5/d^2/e/(d/e+x)/(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(3/2)-4/15/d^4*e/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)
*x-8/15/d^6*e/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x), x)
```

Fricas [B] time = 1.70179, size = 493, normalized size = 4.14

$$\frac{23\,e^5x^5+23\,de^4x^4-46\,d^2e^3x^3-46\,d^3e^2x^2+23\,d^4ex+23\,d^5+15\left(e^5x^5+de^4x^4-2\,d^2e^3x^3-2\,d^3e^2x^2+d^4ex+d^5\right)\log\left(-\right.}{15\left(d^6e^5x^5+d^7e^4x^4-2\,d^8e^3x^3-2\,d^9e^2x^2+d^{10}ex+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*e^5*x^5 + 23*d*e^4*x^4 - 46*d^2*e^3*x^3 - 46*d^3*e^2*x^2 + 23*d^4*
e*x + 23*d^5 + 15*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^
4*e*x + d^5)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^4*x^4 - 7*d*e^3*x^3
- 27*d^2*e^2*x^2 + 8*d^3*e*x + 23*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^5*x^5 +
 d^7*e^4*x^4 - 2*d^8*e^3*x^3 - 2*d^9*e^2*x^2 + d^10*e*x + d^11)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.145 $\int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

```
[Out] (6*d - 5*e*x)/(15*d^4*x*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x*(d + e*x)*(d^2
- e^2*x^2)^(3/2)) + (8*d - 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[
d^2 - e^2*x^2])/(5*d^7*x) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rubi [A] time = 0.135612, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {857, 823, 807, 266, 63, 208}

$$\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (6*d - 5*e*x)/(15*d^4*x*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x*(d + e*x)*(d^2
- e^2*x^2)^(3/2)) + (8*d - 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[
d^2 - e^2*x^2])/(5*d^7*x) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-6de^2+5e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-24d^3e^4+15d^2e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-48d^5e^6+15}{x^2\sqrt{d^2-e^2}}}{15d^{10}e}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x}}{5d^7x}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x}}{5d^7x}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x}}{5d^7x}$$

$$= \frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x}}{5d^7x}$$

Mathematica [A] time = 0.133733, size = 122, normalized size = 0.79

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(-52d^3e^2x^2-87d^2e^3x^3+38d^4ex+15d^5+33de^4x^4+48e^5x^5\right)}{x(d-ex)^2(d+ex)^3} - 15e\log\left(\sqrt{d^2-e^2x^2}+d\right) + 15e\log(x)}{15d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(15*d^5 + 38*d^4*e*x - 52*d^3*e^2*x^2 - 87*d^2*e^3*x
^3 + 33*d*e^4*x^4 + 48*e^5*x^5))/(x*(d - e*x)^2*(d + e*x)^3) + 15*e*Log[x]
- 15*e*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^7)
```

Maple [A] time = 0.059, size = 268, normalized size = 1.7

$$-\frac{e}{3\,d^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}-\frac{e}{d^6}\frac{1}{\sqrt{-x^2e^2+d^2}}+\frac{e}{d^6}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{1}{5\,d^3}\left(\frac{d}{e}+x\right)^{-1}\left(-\left(\frac{d}{e}+x\right)^2e^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -1/3*e/d^4/(-e^2*x^2+d^2)^(3/2)-e/d^6/(-e^2*x^2+d^2)^(1/2)+e/d^6/(d^2)^(1/2
)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5/d^3/(d/e+x)/(-(d/e+x
)^2*e^2+2*d*e*(d/e+x))^(3/2)+4/15*e^2/d^5/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3
/2)*x+8/15*e^2/d^7/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-1/d^3/x/(-e^2*x^2
+d^2)^(3/2)+4/3/d^5*e^2*x/(-e^2*x^2+d^2)^(3/2)+8/3/d^7*e^2*x/(-e^2*x^2+d^2)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^2), x)
```

Fricas [A] time = 1.94252, size = 549, normalized size = 3.56

$$-\frac{23\,e^6x^6+23\,de^5x^5-46\,d^2e^4x^4-46\,d^3e^3x^3+23\,d^4e^2x^2+23\,d^5ex+15\left(e^6x^6+de^5x^5-2\,d^2e^4x^4-2\,d^3e^3x^3+d^4e^2x^2+d^5\right.}{15\left(d^7e^5x^6+d^8e^4x^5-2\,d^9e^3x^4-2\,d^{10}e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(23*e^6*x^6 + 23*d*e^5*x^5 - 46*d^2*e^4*x^4 - 46*d^3*e^3*x^3 + 23*d^4
*e^2*x^2 + 23*d^5*e*x + 15*(e^6*x^6 + d*e^5*x^5 - 2*d^2*e^4*x^4 - 2*d^3*e^3
*x^3 + d^4*e^2*x^2 + d^5*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (48*e^5*
x^5 + 33*d*e^4*x^4 - 87*d^2*e^3*x^3 - 52*d^3*e^2*x^2 + 38*d^4*e*x + 15*d^5)
*sqrt(-e^2*x^2 + d^2))/(d^7*e^5*x^6 + d^8*e^4*x^5 - 2*d^9*e^3*x^4 - 2*d^10*
```

```
e^2*x^3 + d^11*e*x^2 + d^12*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.146 $\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{7e^2\tanh^{-1}}{2d}$$

```
[Out] (7*d - 6*e*x)/(15*d^4*x^2*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^2*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (35*d - 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) -
(7*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) + (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x)
- (7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rubi [A] time = 0.168233, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{7e^2\tanh^{-1}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```


```
[Out] (7*d - 6*e*x)/(15*d^4*x^2*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^2*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (35*d - 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) -
(7*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) + (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x)
- (7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```


Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
```

```
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-7de^2+6e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-35d^3e^4+24d^2e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-105d^5e^6+}{x^3\sqrt{d^2-}}}{15d^{1}}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x}}{2d^7x^2}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x}}{2d^7x^2}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x}}{2d^7x^2}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x}}{2d^7x^2}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x}}{2d^7x^2}$$

Mathematica [A] time = 0.143633, size = 137, normalized size = 0.74

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(176d^4e^2x^2-4d^3e^3x^3-249d^2e^4x^4+15d^5ex-15d^6-9de^5x^5+96e^6x^6\right)}{x^2(d-ex)^2(d+ex)^3} - 105e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 105e^2\log(x)}{30d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^6 + 15*d^5*e*x + 176*d^4*e^2*x^2 - 4*d^3*e^3*x
^3 - 249*d^2*e^4*x^4 - 9*d*e^5*x^5 + 96*e^6*x^6))/(x^2*(d - e*x)^2*(d + e*x
)^3) + 105*e^2*Log[x] - 105*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(30*d^8)
```

Maple [A] time = 0.065, size = 298, normalized size = 1.6

$$\frac{7e^2}{6d^5}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}} + \frac{7e^2}{2d^7}\frac{1}{\sqrt{-x^2e^2+d^2}} - \frac{7e^2}{2d^7}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} + \frac{e}{5d^4}\left(\frac{d}{e}+x\right)^{-1}\left(-\left(\frac{d}{e}+x\right)^2e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] 7/6*e^2/d^5/(-e^2*x^2+d^2)^(3/2)+7/2*e^2/d^7/(-e^2*x^2+d^2)^(1/2)-7/2*e^2/d
^7/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/5*e/d^4/(
d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-4/15*e^3/d^6/(-(d/e+x)^2*e^2+2*
d*e*(d/e+x))^(3/2)*x-8/15*e^3/d^8/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-1/
2/d^3/x^2/(-e^2*x^2+d^2)^(3/2)+e/d^4/x/(-e^2*x^2+d^2)^(3/2)-4/3*e^3/d^6*x/(
-e^2*x^2+d^2)^(3/2)-8/3*e^3/d^8*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^3), x)
```

Fricas [A] time = 2.17023, size = 595, normalized size = 3.2

$$\frac{116\,e^7x^7+116\,de^6x^6-232\,d^2e^5x^5-232\,d^3e^4x^4+116\,d^4e^3x^3+116\,d^5e^2x^2+105\left(e^7x^7+de^6x^6-2\,d^2e^5x^5-2\,d^3e^4x^4+\right.}{30\left(d^8e^5x^7+d^9e^4x^6-2\,d^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*e^7*x^7 + 116*d*e^6*x^6 - 232*d^2*e^5*x^5 - 232*d^3*e^4*x^4 + 116
*d^4*e^3*x^3 + 116*d^5*e^2*x^2 + 105*(e^7*x^7 + d*e^6*x^6 - 2*d^2*e^5*x^5 -
 2*d^3*e^4*x^4 + d^4*e^3*x^3 + d^5*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))
/x) + (96*e^6*x^6 - 9*d*e^5*x^5 - 249*d^2*e^4*x^4 - 4*d^3*e^3*x^3 + 176*d^4
*e^2*x^2 + 15*d^5*e*x - 15*d^6)*sqrt(-e^2*x^2 + d^2))/(d^8*e^5*x^7 + d^9*e^
4*x^6 - 2*d^10*e^3*x^5 - 2*d^11*e^2*x^4 + d^12*e*x^3 + d^13*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.147 $\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}+\frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2\right.}$$

```
[Out] (8*d - 7*e*x)/(15*d^4*x^3*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^3*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (48*d - 35*e*x)/(15*d^6*x^3*Sqrt[d^2 - e^2*x^2]) -
(64*Sqrt[d^2 - e^2*x^2])/(15*d^7*x^3) + (7*e*Sqrt[d^2 - e^2*x^2])/(2*d^8*x^
2) - (128*e^2*Sqrt[d^2 - e^2*x^2])/(15*d^9*x) + (7*e^3*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/(2*d^9)
```

Rubi [A] time = 0.209699, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {857, 823, 835, 807, 266, 63, 208}

$$-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}+\frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (8*d - 7*e*x)/(15*d^4*x^3*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^3*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (48*d - 35*e*x)/(15*d^6*x^3*Sqrt[d^2 - e^2*x^2]) -
(64*Sqrt[d^2 - e^2*x^2])/(15*d^7*x^3) + (7*e*Sqrt[d^2 - e^2*x^2])/(2*d^8*x^
2) - (128*e^2*Sqrt[d^2 - e^2*x^2])/(15*d^9*x) + (7*e^3*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/(2*d^9)
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-8de^2+7e^3x}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-48d^3e^4+35d^2e^5x}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-192d^5}{x^4}}{1}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-}}{15d^7}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-}}{15d^7}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-}}{15d^7}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-}}{15d^7}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-}}{15d^7}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-}}{15d^7}$$

Mathematica [A] time = 0.182137, size = 148, normalized size = 0.69

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(75d^5e^2x^2+236d^4e^3x^3-244d^3e^4x^4-489d^2e^5x^5-5d^6ex+10d^7+151de^6x^6+256e^7x^7\right)}{x^3(d-ex)^2(d+ex)^3} - 105e^3\log\left(\sqrt{d^2-e^2x^2}+d\right) + 105e^3\log(x)}{30d^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(10*d^7 - 5*d^6*e*x + 75*d^5*e^2*x^2 + 236*d^4*e^3*x
^3 - 244*d^3*e^4*x^4 - 489*d^2*e^5*x^5 + 151*d*e^6*x^6 + 256*e^7*x^7))/(x^3
*(d - e*x)^2*(d + e*x)^3) + 105*e^3*Log[x] - 105*e^3*Log[d + Sqrt[d^2 - e^2
*x^2]])/(30*d^9)
```

Maple [A] time = 0.069, size = 326, normalized size = 1.5

$$-\frac{7e^3}{6d^6}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}} - \frac{7e^3}{2d^8}\frac{1}{\sqrt{-x^2e^2+d^2}} + \frac{7e^3}{2d^8}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{1}{3d^3x^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}} - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] -7/6*e^3/d^6/(-e^2*x^2+d^2)^(3/2)-7/2*e^3/d^8/(-e^2*x^2+d^2)^(1/2)+7/2*e^3/
d^8/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^3/x^
3/(-e^2*x^2+d^2)^(3/2)-3/d^5*e^2/x/(-e^2*x^2+d^2)^(3/2)+4/d^7*e^4*x/(-e^2*x
^2+d^2)^(3/2)+8/d^9*e^4*x/(-e^2*x^2+d^2)^(1/2)-1/5*e^2/d^5/(d/e+x)/(-(d/e+x
)^2*e^2+2*d*e*(d/e+x))^(3/2)+4/15*e^4/d^7/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3
/2)*x+8/15*e^4/d^9/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+1/2*e/d^4/x^2/(-e
^2*x^2+d^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4), x)
```

Fricas [A] time = 2.4647, size = 626, normalized size = 2.91

$$-\frac{116\,e^8x^8+116\,de^7x^7-232\,d^2e^6x^6-232\,d^3e^5x^5+116\,d^4e^4x^4+116\,d^5e^3x^3+105\left(e^8x^8+de^7x^7-2\,d^2e^6x^6-2\,d^3e^5x^5+d\right.}{30\left(d^9e^5x^8+d^{10}e^4x^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(116*e^8*x^8 + 116*d*e^7*x^7 - 232*d^2*e^6*x^6 - 232*d^3*e^5*x^5 + 11
6*d^4*e^4*x^4 + 116*d^5*e^3*x^3 + 105*(e^8*x^8 + d*e^7*x^7 - 2*d^2*e^6*x^6
- 2*d^3*e^5*x^5 + d^4*e^4*x^4 + d^5*e^3*x^3)*log(-(d - sqrt(-e^2*x^2 + d^2)
)/x) + (256*e^7*x^7 + 151*d*e^6*x^6 - 489*d^2*e^5*x^5 - 244*d^3*e^4*x^4 + 2
36*d^4*e^3*x^3 + 75*d^5*e^2*x^2 - 5*d^6*e*x + 10*d^7)*sqrt(-e^2*x^2 + d^2))
/(d^9*e^5*x^8 + d^10*e^4*x^7 - 2*d^11*e^3*x^6 - 2*d^12*e^2*x^5 + d^13*e*x^4
 + d^14*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.148 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}-\frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (x^2*(d - e*x))/(7*e^2*(d^2 - e^2*x^2)^(7/2)) - (2*d - 3*e*x)/(35*e^4*(d^2
- e^2*x^2)^(5/2)) - x/(35*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) - (2*x)/(35*d^4*e^
3*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0773119, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {850, 819, 778, 192, 191}

$$\frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}-\frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x^2*(d - e*x))/(7*e^2*(d^2 - e^2*x^2)^(7/2)) - (2*d - 3*e*x)/(35*e^4*(d^2
- e^2*x^2)^(5/2)) - x/(35*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) - (2*x)/(35*d^4*e^
3*Sqrt[d^2 - e^2*x^2])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx \\
&= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7d^2e^2} \\
&= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{3\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35e^3} \\
&= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{35d^2e^3} \\
&= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.137547, size = 104, normalized size = 0.88

$$-\frac{\sqrt{d^2-e^2x^2}\left(-5d^4e^2x^2-5d^3e^3x^3-5d^2e^4x^4+2d^5ex+2d^6+2de^5x^5+2e^6x^6\right)}{35d^4e^4(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(2*d^6 + 2*d^5*e*x - 5*d^4*e^2*x^2 - 5*d^3*e^3*x^3 -
5*d^2*e^4*x^4 + 2*d*e^5*x^5 + 2*e^6*x^6))/(35*d^4*e^4*(d - e*x)^3*(d + e*x)
^4)
```

Maple [A] time = 0.049, size = 92, normalized size = 0.8

$$-\frac{(-ex+d)\left(2e^6x^6+2e^5x^5d-5x^4d^2e^4-5x^3d^3e^3-5x^2d^4e^2+2d^5xe+2d^6\right)}{35d^4e^4}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/35*(-e*x+d)*(2*e^6*x^6+2*d*e^5*x^5-5*d^2*e^4*x^4-5*d^3*e^3*x^3-5*d^4*e^2
*x^2+2*d^5*e*x+2*d^6)/d^4/e^4/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13383, size = 475, normalized size = 4.03

$$-\frac{2\,e^{7}x^{7}+2\,de^{6}x^{6}-6\,d^{2}e^{5}x^{5}-6\,d^{3}e^{4}x^{4}+6\,d^{4}e^{3}x^{3}+6\,d^{5}e^{2}x^{2}-2\,d^{6}ex-2\,d^{7}-\left(2\,e^{6}x^{6}+2\,de^{5}x^{5}-5\,d^{2}e^{4}x^{4}-5\,d^{3}e^{3}x^{3}-5}{35\left(d^{4}e^{11}x^{7}+d^{5}e^{10}x^{6}-3\,d^{6}e^{9}x^{5}-3\,d^{7}e^{8}x^{4}+3\,d^{8}e^{7}x^{3}+3\,d^{9}e^{6}x^{2}-d^{10}e^{5}x-d^{11}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/35*(2*e^7*x^7 + 2*d*e^6*x^6 - 6*d^2*e^5*x^5 - 6*d^3*e^4*x^4 + 6*d^4*e^3*
x^3 + 6*d^5*e^2*x^2 - 2*d^6*e*x - 2*d^7 - (2*e^6*x^6 + 2*d*e^5*x^5 - 5*d^2*
e^4*x^4 - 5*d^3*e^3*x^3 - 5*d^4*e^2*x^2 + 2*d^5*e*x + 2*d^6)*sqrt(-e^2*x^2
+ d^2))/(d^4*e^11*x^7 + d^5*e^10*x^6 - 3*d^6*e^9*x^5 - 3*d^7*e^8*x^4 + 3*d^
8*e^7*x^3 + 3*d^9*e^6*x^2 - d^10*e^5*x - d^11*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.149 $\int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] -x^2/(7*d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (2*(d + 2*e*x))/(35*d*e^3*(d
^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(1
05*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0575934, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {855, 778, 192, 191}

$$-\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] -x^2/(7*d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (2*(d + 2*e*x))/(35*d*e^3*(d
^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(1
05*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{\int \frac{x(2d+4ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7de}$$
$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35de^2}$$
$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)}}{105d^3e}$$
$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8x}{105d^5e^2\sqrt{d^2}}$$

Mathematica [A] time = 0.0842236, size = 104, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(-15d^4e^2x^2+20d^3e^3x^3+20d^2e^4x^4+6d^5ex+6d^6-8de^5x^5-8e^6x^6\right)}{105d^5e^3(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(6*d^6 + 6*d^5*e*x - 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 +
 20*d^2*e^4*x^4 - 8*d*e^5*x^5 - 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^3*(d + e
*x)^4)
```

Maple [A] time = 0.049, size = 92, normalized size = 0.8

$$\frac{(-ex+d)\left(-8\,e^6x^6-8\,e^5x^5d+20\,e^4x^4d^2+20\,x^3d^3e^3-15\,x^2d^4e^2+6\,xd^5e+6\,d^6\right)}{105\,d^5e^3}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/105*(-e*x+d)*(-8*e^6*x^6-8*d*e^5*x^5+20*d^2*e^4*x^4+20*d^3*e^3*x^3-15*d^4
*e^2*x^2+6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.06759, size = 485, normalized size = 3.94

$$\frac{6\,e^{7}x^{7} + 6\,de^{6}x^{6} - 18\,d^{2}e^{5}x^{5} - 18\,d^{3}e^{4}x^{4} + 18\,d^{4}e^{3}x^{3} + 18\,d^{5}e^{2}x^{2} - 6\,d^{6}ex - 6\,d^{7} + \left(8\,e^{6}x^{6} + 8\,de^{5}x^{5} - 20\,d^{2}e^{4}x^{4} - 20\,d}{105\,\left(d^{5}e^{10}x^{7} + d^{6}e^{9}x^{6} - 3\,d^{7}e^{8}x^{5} - 3\,d^{8}e^{7}x^{4} + 3\,d^{9}e^{6}x^{3} + 3\,d^{10}e^{5}x^{2} - d^{11}e^{4}x - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*e^7*x^7 + 6*d*e^6*x^6 - 18*d^2*e^5*x^5 - 18*d^3*e^4*x^4 + 18*d^4*e
^3*x^3 + 18*d^5*e^2*x^2 - 6*d^6*e*x - 6*d^7 + (8*e^6*x^6 + 8*d*e^5*x^5 - 20
*d^2*e^4*x^4 - 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 - 6*d^5*e*x - 6*d^6)*sqrt(-e
^2*x^2 + d^2))/(d^5*e^10*x^7 + d^6*e^9*x^6 - 3*d^7*e^8*x^5 - 3*d^8*e^7*x^4
+ 3*d^9*e^6*x^3 + 3*d^10*e^5*x^2 - d^11*e^4*x - d^12*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{7}{2}}\left(d + ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.150 $\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=66

$$\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}+\frac{3\sin^{-1}(ax)}{2a^4}$$

```
[Out] (x^2*(1 - a*x))/(a^2*Sqrt[1 - a^2*x^2]) + ((4 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
2*a^4) + (3*ArcSin[a*x])/(2*a^4)
```

Rubi [A] time = 0.0538498, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {850, 819, 780, 216}

$$\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}+\frac{3\sin^{-1}(ax)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (x^2*(1 - a*x))/(a^2*Sqrt[1 - a^2*x^2]) + ((4 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
2*a^4) + (3*ArcSin[a*x])/(2*a^4)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx &= \int \frac{x^3(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx\\
&= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\int \frac{x(2-3ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3}\\
&= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\sin^{-1}(ax)}{2a^4}
\end{aligned}
$$

Mathematica [A] time = 0.0573529, size = 54, normalized size = 0.82

$$
\frac{\sqrt{1-a^2x^2}\left(-a^2x^2+ax+4\right)+3(ax+1)\sin^{-1}(ax)}{2a^4(ax+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(4 + a*x - a^2*x^2) + 3*(1 + a*x)*ArcSin[a*x])/(2*a^4*(1
 + a*x))
```

Maple [A] time = 0.056, size = 100, normalized size = 1.5

$$
-\frac{x}{2a^3}\sqrt{-a^2x^2+1}+\frac{3}{2a^3}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{a^4}\sqrt{-a^2x^2+1}+\frac{1}{a^5\left(x+a^{-1}\right)}\sqrt{-\left(x+a^{-1}\right)^2a^2+2a\left(x+\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/a^3*x*(-a^2*x^2+1)^(1/2)+3/2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^
2*x^2+1)^(1/2))+1/a^4*(-a^2*x^2+1)^(1/2)+1/a^5/(x+1/a)*(-(x+1/a)^2*a^2+2*a*
(x+1/a))^(1/2)
```

Maxima [A] time = 1.46735, size = 92, normalized size = 1.39

$$
\frac{\sqrt{-a^2x^2+1}}{a^5x+a^4}-\frac{\sqrt{-a^2x^2+1}x}{2a^3}+\frac{3\arcsin(ax)}{2a^4}+\frac{\sqrt{-a^2x^2+1}}{a^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^5*x + a^4) - 1/2*sqrt(-a^2*x^2 + 1)*x/a^3 + 3/2*arcsi
n(a*x)/a^4 + sqrt(-a^2*x^2 + 1)/a^4
```

Fricas [A] time = 1.67148, size = 169, normalized size = 2.56

$$\frac{4\,ax-6\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(a^2x^2-ax-4\right)\sqrt{-a^2x^2+1}+4}{2\left(a^5x+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x - 6*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a^2*x^2
- a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)/(a^5*x + a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [A] time = 1.2575, size = 105, normalized size = 1.59

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a^3}-\frac{2}{a^4}\right)+\frac{3\arcsin(ax)\,\mathrm{sgn}(a)}{2\,a^3|a|}-\frac{2}{a^3\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x/a^3 - 2/a^4) + 3/2*arcsin(a*x)*sgn(a)/(a^3*abs(a
)) - 2/(a^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.151 $\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=55

$$-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - Sqrt[1 - a^2*x^2]/(a^3*(1 + a*x)) - ArcSin[a*x]/
a^3
```

Rubi [A] time = 0.0863924, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1639, 12, 793, 216}

$$-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - Sqrt[1 - a^2*x^2]/(a^3*(1 + a*x)) - ArcSin[a*x]/
a^3
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\int \frac{a^3x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a^4}\\ &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a}\\ &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\ &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{\sin^{-1}(ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0551428, size = 37, normalized size = 0.67

$$-\frac{\frac{\sqrt{1-a^2x^2}(ax+2)}{ax+1}+\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]

[Out] -((((2 + a*x)*Sqrt[1 - a^2*x^2])/(1 + a*x) + ArcSin[a*x])/a^3)
```

Maple [A] time = 0.048, size = 84, normalized size = 1.5

$$-\frac{1}{a^3}\sqrt{-a^2x^2+1}-\frac{1}{a^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}-\frac{1}{a^4\left(x+a^{-1}\right)}\sqrt{-\left(x+a^{-1}\right)^2a^2+2\,a\left(x+a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x)

[Out] -(-a^2*x^2+1)^(1/2)/a^3-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-1/a^4/(x+1/a)*(-(x+1/a)^2*a^2+2*a*(x+1/a))^(1/2)
```

Maxima [A] time = 1.47628, size = 70, normalized size = 1.27

$$-\frac{\sqrt{-a^2x^2+1}}{a^4x+a^3}-\frac{\arcsin(ax)}{a^3}-\frac{\sqrt{-a^2x^2+1}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -sqrt(-a^2*x^2 + 1)/(a^4*x + a^3) - arcsin(a*x)/a^3 - sqrt(-a^2*x^2 + 1)/a^
3
```

Fricas [A] time = 1.62199, size = 151, normalized size = 2.75

$$-\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}(ax+2)+2}{a^4x+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-(ax-1)(ax+1)}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [A] time = 1.2485, size = 95, normalized size = 1.73

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2|a|}-\frac{\sqrt{-a^2x^2+1}}{a^3}+\frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.152 $\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=34

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}+\frac{\sin^{-1}(ax)}{a^2}$$

```
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + ArcSin[a*x]/a^2
```

Rubi [A] time = 0.0174339, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {793, 216}

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}+\frac{\sin^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + ArcSin[a*x]/a^2
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a}\\ &= \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\sin^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0247696, size = 31, normalized size = 0.91

$$\frac{\frac{\sqrt{1-a^2x^2}}{ax+1}+\sin^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]/(1 + a*x) + ArcSin[a*x])/a^2
```

Maple [A] time = 0.053, size = 65, normalized size = 1.9

$$\frac{1}{a}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+\frac{1}{a^3\left(x+a^{-1}\right)}\sqrt{-\left(x+a^{-1}\right)^2a^2+2a\left(x+a^{-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^3/(x+1/a)*(-(x
+1/a)^2*a^2+2*a*(x+1/a))^(1/2)
```

Maxima [A] time = 1.45976, size = 45, normalized size = 1.32

$$\frac{\sqrt{-a^2x^2+1}}{a^3x+a^2}+\frac{\arcsin(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + arcsin(a*x)/a^2
```

Fricas [A] time = 1.63347, size = 134, normalized size = 3.94

$$\frac{ax-2(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}+1}{a^3x+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 +
 1) + 1)/(a^3*x + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-(ax-1)(ax+1)}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [A] time = 1.25238, size = 70, normalized size = 2.06

$$\frac{\arcsin(ax)\operatorname{sgn}(a)}{a|a|} - \frac{2}{a\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a*abs(a)) - 2/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*
x) + 1)*abs(a))
```

3.153 $\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

Rubi [A] time = 0.0095005, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {651}

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

Mathematica [A] time = 0.0061963, size = 25, normalized size = 0.96

$$-\frac{\sqrt{1-a^2x^2}}{a^2x+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a + a^2*x))
```

Maple [A] time = 0.044, size = 22, normalized size = 0.9

$$\frac{ax-1}{a}\frac{1}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] (a*x-1)/a/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.47253, size = 31, normalized size = 1.19

$$-\frac{\sqrt{-a^2x^2+1}}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*x^2 + 1)/(a^2*x + a)
```

Fricas [A] time = 1.54348, size = 61, normalized size = 2.35

$$-\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*x + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [A] time = 1.25429, size = 46, normalized size = 1.77

$$\frac{2}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

3.154 $\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0397202, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {857, 12, 266, 63, 208}

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]

[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{1-ax}+\frac{\int \frac{a^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= \frac{\sqrt{1-a^2x^2}}{1-ax}+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{1-ax}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{1-a^2x^2}}{1-ax}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= \frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0301259, size = 41, normalized size = 1.

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Maple [A] time = 0.053, size = 58, normalized size = 1.4

$$-\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{1}{a}\sqrt{-\left(x-a^{-1}\right)^2a^2-2\,a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -arctanh(1/(-a^2*x^2+1)^(1/2))-1/a/(x-1/a)*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-a^2x^2+1}(ax-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x)
```

Fricas [A] time = 1.58115, size = 116, normalized size = 2.83

$$\frac{ax + (ax - 1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2 + 1} - 1}{ax - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1) - 1)/
(a*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^2\sqrt{-a^2x^2 + 1} - x\sqrt{-a^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x)
```

Giac [A] time = 1.29234, size = 100, normalized size = 2.44

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \frac{2a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 2
*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.155 $\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=64

$$-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] (-2*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x^2]/(x*(1 - a*x)) - a*ArcTanh[Sqrt
[1 - a^2*x^2]]
```

Rubi [A] time = 0.0534527, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {857, 807, 266, 63, 208}

$$-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x^2]/(x*(1 - a*x)) - a*ArcTanh[Sqrt
[1 - a^2*x^2]]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - \frac{\int \frac{-2a^2-a^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2} \\
&= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0445593, size = 50, normalized size = 0.78

$$\frac{(1-2ax)\sqrt{1-a^2x^2}}{x(ax-1)} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 - 2*a*x)*Sqrt[1 - a^2*x^2])/(x*(-1 + a*x)) - a*ArcTanh[Sqrt[1 - a^2*x^2
]]
```

Maple [A] time = 0.058, size = 73, normalized size = 1.1

$$-a\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \sqrt{-\left(x-a^{-1}\right)^2a^2 - 2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1} - \frac{1}{x}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -a*arctanh(1/(-a^2*x^2+1)^(1/2))-1/(x-1/a)*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(1/
2)-(-a^2*x^2+1)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-a^2x^2+1}(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2), x)
```

Fricas [A] time = 1.55587, size = 151, normalized size = 2.36

$$\frac{a^2x^2 - ax + \left(a^2x^2 - ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}(2\,ax - 1)}{ax^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + (a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^
2*x^2 + 1)*(2*a*x - 1))/(a*x^2 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^3\sqrt{-a^2x^2+1} - x^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x)
```

Giac [B] time = 1.31984, size = 203, normalized size = 3.17

$$-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|} - \frac{\left(a^2 - \frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}|a| + a\right)\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a| + a}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) -
 1/2*(a^2 - 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^2 + 1)
*abs(a) + a)*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 1/2*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a))
```

3.156 $\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x
^2]/(x^2*(1 - a*x)) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rubi [A] time = 0.0792337, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {857, 835, 807, 266, 63, 208}

$$-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x
^2]/(x^2*(1 - a*x)) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{\int \frac{-3a^2-2a^3x}{x^3\sqrt{1-a^2x^2}}\,dx}{a^2} \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{\int \frac{4a^3+3a^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{2a^2} \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{2}\left(3a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0588669, size = 63, normalized size = 0.7

$$\frac{1}{2}\left(\frac{\left(-4a^2x^2+ax+1\right)\sqrt{1-a^2x^2}}{x^2(ax-1)} - 3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (((1 + a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/(x^2*(-1 + a*x)) - 3*a^2*ArcTanh
[Sqrt[1 - a^2*x^2]])/2
```

Maple [A] time = 0.057, size = 94, normalized size = 1.

$$-\frac{3a^2}{2}\text{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - a\sqrt{-\left(x-a^{-1}\right)^2a^2-2a\left(x-a^{-1}\right)}\left(x-a^{-1}\right)^{-1} - \frac{1}{2x^2}\sqrt{-a^2x^2+1} - \frac{a}{x}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -3/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-a/(x-1/a)*(-(x-1/a)^2*a^2-2*a*(x-1/a
))^(1/2)-1/2*(-a^2*x^2+1)^(1/2)/x^2-a*(-a^2*x^2+1)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-a^2x^2+1}(ax-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^3), x)
```

Fricas [A] time = 1.63976, size = 192, normalized size = 2.13

$$\frac{2\,a^3x^3-2\,a^2x^2+3\left(a^3x^3-a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(4\,a^2x^2-ax-1\right)\sqrt{-a^2x^2+1}}{2\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*x^3 - 2*a^2*x^2 + 3*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (4*a^2*x^2 - a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x)
```

Giac [B] time = 1.26761, size = 288, normalized size = 3.2

$$-\frac{\left(a^3+\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a}{x}-\frac{20\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}-\frac{3\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(a^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 20*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 3/2*a^3*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(4*(sqrt(-a^2*x^2 +
1)*abs(a) + a)*a*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2
))/a^2
```

3.157 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=229

$$-\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{d^5(256d-315ex)\left(d^2-e^2x^2\right)^{3/2}}{2016e^6}-\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right.}{21e^2}$$

```
[Out] (-5*d^7*x*Sqrt[d^2 - e^2*x^2])/(64*e^5) - (4*d^4*x^2*(d^2 - e^2*x^2)^(3/2))
/(21*e^4) + (5*d^3*x^3*(d^2 - e^2*x^2)^(3/2))/(24*e^3) - (5*d^2*x^4*(d^2 -
e^2*x^2)^(3/2))/(21*e^2) + (d*x^5*(d^2 - e^2*x^2)^(3/2))/(4*e) - (x^6*(d^2
- e^2*x^2)^(3/2))/9 - (d^5*(256*d - 315*e*x)*(d^2 - e^2*x^2)^(3/2))/(2016*e
^6) - (5*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(64*e^6)
```

Rubi [A] time = 0.314594, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$-\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{d^5(256d-315ex)\left(d^2-e^2x^2\right)^{3/2}}{2016e^6}-\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4}+\frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3}-\frac{5d^2x^4\left(d^2-e^2x^2\right.}{21e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (-5*d^7*x*Sqrt[d^2 - e^2*x^2])/(64*e^5) - (4*d^4*x^2*(d^2 - e^2*x^2)^(3/2))
/(21*e^4) + (5*d^3*x^3*(d^2 - e^2*x^2)^(3/2))/(24*e^3) - (5*d^2*x^4*(d^2 -
e^2*x^2)^(3/2))/(21*e^2) + (d*x^5*(d^2 - e^2*x^2)^(3/2))/(4*e) - (x^6*(d^2
- e^2*x^2)^(3/2))/9 - (d^5*(256*d - 315*e*x)*(d^2 - e^2*x^2)^(3/2))/(2016*e
^6) - (5*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(64*e^6)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5 \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^5(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$
$$= -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^5\left(-15d^2e^2+18de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{9e^2}$$
$$= \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^4\left(-90d^3e^3+120d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{72e^4}$$
$$= -\frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^3\left(-480d^4e^4+630d^3e^5\right.}{504e^6}$$
$$= \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^2}{}$$
$$= -\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e} - \frac{1}{9}$$
$$= -\frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx^5\left(d^2-e^2x^2\right)^{3/2}}{4e} - \frac{1}{9}$$
$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx}{}$$
$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx}{}$$
$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2\left(d^2-e^2x^2\right)^{3/2}}{21e^4} + \frac{5d^3x^3\left(d^2-e^2x^2\right)^{3/2}}{24e^3} - \frac{5d^2x^4\left(d^2-e^2x^2\right)^{3/2}}{21e^2} + \frac{dx}{}$$

Mathematica [A] time = 0.230254, size = 135, normalized size = 0.59

$$\frac{\sqrt{d^2-e^2x^2}\left(-256d^6e^2x^2+210d^5e^3x^3-192d^4e^4x^4+168d^3e^5x^5+512d^2e^6x^6+315d^7ex-512d^8-1008de^7x^7+448e^8x\right.}{4032e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-512*d^8 + 315*d^7*e*x - 256*d^6*e^2*x^2 + 210*d^5*e^
3*x^3 - 192*d^4*e^4*x^4 + 168*d^3*e^5*x^5 + 512*d^2*e^6*x^6 - 1008*d*e^7*x^
7 + 448*e^8*x^8) - 315*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4032*e^6)
```

Maple [A] time = 0.076, size = 375, normalized size = 1.6

$$-\frac{x^2}{9e^4}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{29d^2}{63e^6}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + \frac{dx}{4e^5}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} - \frac{17d^3x}{24e^5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}} - \frac{85d^5x}{96e^5}\left(-x^2e^2+d^2\right)^{\frac{3}{2}} - \frac{85}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -1/9/e^4*x^2*(-e^2*x^2+d^2)^(7/2)-29/63*d^2/e^6*(-e^2*x^2+d^2)^(7/2)+1/4*d/
e^5*x*(-e^2*x^2+d^2)^(7/2)-17/24*d^3/e^5*x*(-e^2*x^2+d^2)^(5/2)-85/96*d^5/e
```

```
^5*x*(-e^2*x^2+d^2)^(3/2)-85/64*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^5-85/64*d^9/e^
5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+2/3/e^6*d^4*(-(d/e
+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+5/6/e^5*d^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
3/2)*x+5/4/e^5*d^7*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+5/4/e^5*d^9/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/3*d^4/e
^8/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6114, size = 313, normalized size = 1.37

$$\frac{630\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(448\,e^8x^8-1008\,de^7x^7+512\,d^2e^6x^6+168\,d^3e^5x^5-192\,d^4e^4x^4+210\,d^5e^3x^3-256\,d^6e^2x^2\cdots\right.}{4032\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(630*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (448*e^8*x^8 -
1008*d*e^7*x^7 + 512*d^2*e^6*x^6 + 168*d^3*e^5*x^5 - 192*d^4*e^4*x^4 + 210*
d^5*e^3*x^3 - 256*d^6*e^2*x^2 + 315*d^7*e*x - 512*d^8)*sqrt(-e^2*x^2 + d^2)
)/e^6
```

Sympy [A] time = 21.4452, size = 573, normalized size = 2.5

$$d^2\left(\begin{cases}-\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for } e\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7}+\frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}}-\cdots\\ \frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7}-\frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5d^5x^3}{384e^4\sqrt{1-\cdots}}\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqr
t(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2)
 + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 2
*d*e*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sq
rt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2
)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt
(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e
**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e*
```

```
*6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2
*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d
**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4)
 - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)
/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.158 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=200

$$\frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^4(1024d-1365ex)\left(d^2-e^2x^2\right)^{3/2}}{6720e^5}+\frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3}-\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2}+\frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e}$$

```
[Out] (13*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (8*d^3*x^2*(d^2 - e^2*x^2)^(3/2)
)/(35*e^3) - (13*d^2*x^3*(d^2 - e^2*x^2)^(3/2))/(48*e^2) + (2*d*x^4*(d^2 -
e^2*x^2)^(3/2))/(7*e) - (x^5*(d^2 - e^2*x^2)^(3/2))/8 + (d^4*(1024*d - 1365
*e*x)*(d^2 - e^2*x^2)^(3/2))/(6720*e^5) + (13*d^8*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(128*e^5)
```

Rubi [A] time = 0.269624, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$\frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^4(1024d-1365ex)\left(d^2-e^2x^2\right)^{3/2}}{6720e^5}+\frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3}-\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2}+\frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```


```
[Out] (13*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (8*d^3*x^2*(d^2 - e^2*x^2)^(3/2)
)/(35*e^3) - (13*d^2*x^3*(d^2 - e^2*x^2)^(3/2))/(48*e^2) + (2*d*x^4*(d^2 -
e^2*x^2)^(3/2))/(7*e) - (x^5*(d^2 - e^2*x^2)^(3/2))/8 + (d^4*(1024*d - 1365
*e*x)*(d^2 - e^2*x^2)^(3/2))/(6720*e^5) + (13*d^8*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(128*e^5)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^4(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^4\left(-13d^2e^2+16de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{8e^2}$$

$$= \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^3\left(-64d^3e^3+91d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{56e^4}$$

$$= -\frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^2\left(-273d^4e^4+384d^3\right.}{336e^6}$$

$$= \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \int$$

$$= \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{d^4}{}$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x$$

$$= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^3} - \frac{13d^2x^3\left(d^2-e^2x^2\right)^{3/2}}{48e^2} + \frac{2dx^4\left(d^2-e^2x^2\right)^{3/2}}{7e} - \frac{1}{8}x$$

Mathematica [A] time = 0.171357, size = 124, normalized size = 0.62

$$\frac{\sqrt{d^2-e^2x^2}\left(1024d^5e^2x^2-910d^4e^3x^3+768d^3e^4x^4+1960d^2e^5x^5-1365d^6ex+2048d^7-3840de^6x^6+1680e^7x^7\right)+1365d^{8}\cdots}{13440e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2048*d^7 - 1365*d^6*e*x + 1024*d^5*e^2*x^2 - 910*d^4*
e^3*x^3 + 768*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 - 3840*d*e^6*x^6 + 1680*e^7*x^
7) + 1365*d^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(13440*e^5)
```

Maple [B] time = 0.068, size = 350, normalized size = 1.8

$$-\frac{x}{8\,e^4}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}+\frac{25\,d^2x}{48\,e^4}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{125\,d^4x}{192\,e^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{125\,d^6x}{128\,e^4}\sqrt{-x^2e^2+d^2}+\frac{125\,d^8}{128\,e^4}\arctan\left(x\sqrt{e^2}\frac{\cdots}{\sqrt{-\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -1/8/e^4*x*(-e^2*x^2+d^2)^(7/2)+25/48*d^2/e^4*x*(-e^2*x^2+d^2)^(5/2)+125/19
2/e^4*d^4*x*(-e^2*x^2+d^2)^(3/2)+125/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^4+125
/128/e^4*d^8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+2/7*d/e
^5*(-e^2*x^2+d^2)^(7/2)-7/15/e^5*d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-7
/12/e^4*d^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-7/8/e^4*d^6*(-(d/e+x)^2*
e^2+2*d*e*(d/e+x))^(1/2)*x-7/8/e^4*d^8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(
d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))+1/3*d^3/e^7/(d/e+x)^2*(-(d/e+x)^2*e^2+2*
d*e*(d/e+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60569, size = 300, normalized size = 1.5

$$-\frac{2730\,d^8\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(1680\,e^7x^7-3840\,de^6x^6+1960\,d^2e^5x^5+768\,d^3e^4x^4-910\,d^4e^3x^3+1024\,d^5e^2x^2-136\cdots\right.}{13440\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/13440*(2730*d^8*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (1680*e^7*x^
7 - 3840*d*e^6*x^6 + 1960*d^2*e^5*x^5 + 768*d^3*e^4*x^4 - 910*d^4*e^3*x^3 +
 1024*d^5*e^2*x^2 - 1365*d^6*e*x + 2048*d^7)*sqrt(-e^2*x^2 + d^2))/e^5
```

Sympy [C] time = 27.2561, size = 694, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5
*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x*
*2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1
- e**2*x**2/d**2)), True)) - 2*d*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2
)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sq
rt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)),
(x**6*sqrt(d**2)/6, True)) + e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e*
*7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*
e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2
/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt
(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (5*d**8*asin(e*x/d)/
(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(38
4*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d
**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e*
*2*x**2/d**2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.159 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal. Leaf size=171

$$-\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{d^3(88d-105ex)(d^2-e^2x^2)^{3/2}}{420e^4}-\frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2}+\frac{dx^3(d^2-e^2x^2)^{3/2}}{3e}-\frac{1}{7}x^4(d^2-e^2x^2)^{3/2}-\frac{d^7 \cdots}{}$$

```
[Out] -(d^5*x*Sqrt[d^2 - e^2*x^2])/(8*e^3) - (11*d^2*x^2*(d^2 - e^2*x^2)^(3/2))/(
35*e^2) + (d*x^3*(d^2 - e^2*x^2)^(3/2))/(3*e) - (x^4*(d^2 - e^2*x^2)^(3/2))
/7 - (d^3*(88*d - 105*e*x)*(d^2 - e^2*x^2)^(3/2))/(420*e^4) - (d^7*ArcTan[(
e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^4)
```

Rubi [A] time = 0.215467, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$-\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{d^3(88d-105ex)(d^2-e^2x^2)^{3/2}}{420e^4}-\frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2}+\frac{dx^3(d^2-e^2x^2)^{3/2}}{3e}-\frac{1}{7}x^4(d^2-e^2x^2)^{3/2}-\frac{d^7 \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```


```
[Out] -(d^5*x*Sqrt[d^2 - e^2*x^2])/(8*e^3) - (11*d^2*x^2*(d^2 - e^2*x^2)^(3/2))/(
35*e^2) + (d*x^3*(d^2 - e^2*x^2)^(3/2))/(3*e) - (x^4*(d^2 - e^2*x^2)^(3/2))
/7 - (d^3*(88*d - 105*e*x)*(d^2 - e^2*x^2)^(3/2))/(420*e^4) - (d^7*ArcTan[(
e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^4)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int x^3(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$
$$= -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^3\left(-11d^2e^2+14de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{7e^2}$$
$$= \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^2\left(-42d^3e^3+66d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{42e^4}$$
$$= -\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x\left(-132d^4e^4+210d^3e^5\ldots\right.}{210e^6}$$
$$= -\frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{d^3(88d-105ex)\left(d^2-e^2x\ldots\right.}{420e^4}$$
$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{d^3(88d\ldots}{}$$
$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{d^3(88d\ldots}{}$$
$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2\left(d^2-e^2x^2\right)^{3/2}}{35e^2} + \frac{dx^3\left(d^2-e^2x^2\right)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{d^3(88d\ldots}{}$$

Mathematica [A] time = 0.13492, size = 113, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(-88d^4e^2x^2+70d^3e^3x^3+144d^2e^4x^4+105d^5ex-176d^6-280de^5x^5+120e^6x^6\right)-105d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{840e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-176*d^6 + 105*d^5*e*x - 88*d^4*e^2*x^2 + 70*d^3*e^3*
x^3 + 144*d^2*e^4*x^4 - 280*d*e^5*x^5 + 120*e^6*x^6) - 105*d^7*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(840*e^4)
```

Maple [B] time = 0.065, size = 327, normalized size = 1.9

$$-\frac{1}{7e^4}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{dx}{3e^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{5d^3x}{12e^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{5d^5x}{8e^3}\sqrt{-x^2e^2+d^2}-\frac{5d^7}{8e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -1/7/e^4*(-e^2*x^2+d^2)^(7/2)-1/3*d/e^3*x*(-e^2*x^2+d^2)^(5/2)-5/12/e^3*d^3
*x*(-e^2*x^2+d^2)^(3/2)-5/8*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^3-5/8/e^3*d^7/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+4/15/e^4*d^2*(-(d/e+x)^2
*e^2+2*d*e*(d/e+x))^(5/2)+1/3/e^3*d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*
x+1/2/e^3*d^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+1/2/e^3*d^7/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/3*d^2/e^6/(d
/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52085, size = 259, normalized size = 1.51

$$\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(120\,e^6x^6-280\,de^5x^5+144\,d^2e^4x^4+70\,d^3e^3x^3-88\,d^4e^2x^2+105\,d^5ex-176\,d^6\right)\sqrt{-e^2x^2}}{840\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (120*e^6*x^6 - 2
80*d*e^5*x^5 + 144*d^2*e^4*x^4 + 70*d^3*e^3*x^3 - 88*d^4*e^2*x^2 + 105*d^5*
e*x - 176*d^6)*sqrt(-e^2*x^2 + d^2))/e^4
```

Sympy [A] time = 19.365, size = 452, normalized size = 2.64

$$d^2\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5}{24\sqrt{}} \\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) - 2*d*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x*
*7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*as
in(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3
/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)
) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piecewise((-8*d
**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/
(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 -
e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.160 $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=142

$$\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{3d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] (3*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (2*d*x^2*(d^2 - e^2*x^2)^(3/2))/(5
*e) - (x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^2*(32*d - 45*e*x)*(d^2 - e^2*x^2)^
(3/2))/(120*e^3) + (3*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rubi [A] time = 0.176956, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1809, 833, 780, 195, 217, 203}

$$\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{3d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (3*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (2*d*x^2*(d^2 - e^2*x^2)^(3/2))/(5
*e) - (x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^2*(32*d - 45*e*x)*(d^2 - e^2*x^2)^
(3/2))/(120*e^3) + (3*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2 \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= \int x^2(d-ex)^2\sqrt{d^2-e^2x^2}\,dx \\
&= -\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}-\frac{\int x^2\left(-9d^2e^2+12de^3x\right)\sqrt{d^2-e^2x^2}\,dx}{6e^2} \\
&= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{\int x\left(-24d^3e^3+45d^2e^4x\right)\sqrt{d^2-e^2x^2}\,dx}{30e^4} \\
&= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{\left(3d^4\right)\int\sqrt{d^2-e^2}}{8e^2} \\
&= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3} \\
&= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3} \\
&= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}
\end{aligned}
$$

Mathematica [A] time = 0.120862, size = 102, normalized size = 0.72

$$
\frac{\sqrt{d^2-e^2x^2}\left(32d^3e^2x^2+50d^2e^3x^3-45d^4ex+64d^5-96de^4x^4+40e^5x^5\right)+45d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{240e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(64*d^5 - 45*d^4*e*x + 32*d^3*e^2*x^2 + 50*d^2*e^3*x^3
 - 96*d*e^4*x^4 + 40*e^5*x^5) + 45*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(
240*e^3)
```

Maple [B] time = 0.061, size = 303, normalized size = 2.1

$$\frac{x}{6e^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{5d^2x}{24e^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{5d^4x}{16e^2}\sqrt{-x^2e^2+d^2}+\frac{5d^6}{16e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{d}{15e^3}\left(-\left(\frac{d}{e}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 1/6/e^2*x*(-e^2*x^2+d^2)^(5/2)+5/24/e^2*d^2*x*(-e^2*x^2+d^2)^(3/2)+5/16*d^4
*x*(-e^2*x^2+d^2)^(1/2)/e^2+5/16/e^2*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(
-e^2*x^2+d^2)^(1/2))-1/15*d/e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/12*d
^2/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-1/8*d^4/e^2*(-(d/e+x)^2*e^2+2
*d*e*(d/e+x))^(1/2)*x-1/8*d^6/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x
)^2*e^2+2*d*e*(d/e+x))^(1/2))+1/3*d/e^5/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/
e+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58316, size = 230, normalized size = 1.62

$$-\frac{90\,d^6\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(40\,e^5x^5-96\,de^4x^4+50\,d^2e^3x^3+32\,d^3e^2x^2-45\,d^4ex+64\,d^5\right)\sqrt{-e^2x^2+d^2}}{240\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(90*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (40*e^5*x^5 - 96
*d*e^4*x^4 + 50*d^2*e^3*x^3 + 32*d^3*e^2*x^2 - 45*d^4*e*x + 64*d^5)*sqrt(-e
^2*x^2 + d^2))/e^3
```

Sympy [C] time = 22.1104, size = 544, normalized size = 3.83

$$d^2\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d}}{} \\ \frac{x^4\sqrt{d^2}}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**4*asin(e
*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqr
t(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) -
 2*d*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt
(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x
**4*sqrt(d**2)/4, True)) + e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**6*a
sin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**
3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2
)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.161 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}$$

```
[Out] -(d^3*x*Sqrt[d^2 - e^2*x^2])/(4*e) - (d*x*(d^2 - e^2*x^2)^(3/2))/(6*e) - (2
*(d^2 - e^2*x^2)^(5/2))/(15*e^2) - (d^2 - e^2*x^2)^(7/2)/(3*e^2*(d + e*x)^2
) - (d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^2)
```

Rubi [A] time = 0.0566443, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {793, 665, 195, 217, 203}

$$-\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -(d^3*x*Sqrt[d^2 - e^2*x^2])/(4*e) - (d*x*(d^2 - e^2*x^2)^(3/2))/(6*e) - (2
*(d^2 - e^2*x^2)^(5/2))/(15*e^2) - (d^2 - e^2*x^2)^(7/2)/(3*e^2*(d + e*x)^2
) - (d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^2)
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{2\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx}{3e}\\
&= -\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{(2d)\int \left(d^2-e^2x^2\right)^{3/2}\,dx}{3e}\\
&= -\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{2e}\\
&= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{4e}\\
&= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\right.}{4e}\\
&= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx\left(d^2-e^2x^2\right)^{3/2}}{6e}-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{15e^2}-\frac{\left(d^2-e^2x^2\right)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}\end{aligned}$$

Mathematica [A] time = 0.0808211, size = 91, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(16d^2e^2x^2+15d^3ex-28d^4-30de^3x^3+12e^4x^4\right)-15d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{60e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-28*d^4 + 15*d^3*e*x + 16*d^2*e^2*x^2 - 30*d*e^3*x^3
+ 12*e^4*x^4) - 15*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(60*e^2)
```

Maple [A] time = 0.059, size = 198, normalized size = 1.5

$$-\frac{2}{15\,e^2}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{\frac{5}{2}}-\frac{dx}{6\,e}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}-\frac{d^3x}{4\,e}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] -2/15/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/6/e*d*(-(d/e+x)^2*e^2+2*d*
e*(d/e+x))^(3/2)*x-1/4/e*d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-1/4/e*d
```

```
^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1
/3/e^4/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)
```

Maxima [C] time = 1.4719, size = 225, normalized size = 1.65

$$\frac{i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{4\,e^2}-\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x}{4\,e}-\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{4\left(e^3x+de^2\right)}-\frac{\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{2\,e^2}+\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}dx}{4\,e}-\frac{5\left(-e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*d^5*arcsin(e*x/d + 2)/e^2 - 1/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
/e - 1/4*(-e^2*x^2 + d^2)^(5/2)*d/(e^3*x + d*e^2) - 1/2*sqrt(e^2*x^2 + 4*d*
e*x + 3*d^2)*d^4/e^2 + 1/4*(-e^2*x^2 + d^2)^(3/2)*d*x/e - 5/12*(-e^2*x^2 +
d^2)^(3/2)*d^2/e^2 + 1/5*(-e^2*x^2 + d^2)^(5/2)/e^2
```

Fricas [A] time = 1.60258, size = 204, normalized size = 1.5

$$\frac{30\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(12\,e^4x^4-30\,de^3x^3+16\,d^2e^2x^2+15\,d^3ex-28\,d^4\right)\sqrt{-e^2x^2+d^2}}{60\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(30*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (12*e^4*x^4 - 30*d
*e^3*x^3 + 16*d^2*e^2*x^2 + 15*d^3*e*x - 28*d^4)*sqrt(-e^2*x^2 + d^2))/e^2
```

Sympy [A] time = 11.1704, size = 323, normalized size = 2.38

$$d^2\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{2d}{} \\ \frac{x^4\sqrt{}}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) - 2*d*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**
3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/
d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d*
*2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2
*x**2/d**2)), True)) + e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e
**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x
```

```
**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.162 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=108

$$\frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] (5*d^2*x*Sqrt[d^2 - e^2*x^2])/8 + (5*d*(d^2 - e^2*x^2)^(3/2))/(12*e) + ((d
- e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (5*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(8*e)
```

Rubi [A] time = 0.0418491, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {655, 671, 641, 195, 217, 203}

$$\frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (5*d^2*x*Sqrt[d^2 - e^2*x^2])/8 + (5*d*(d^2 - e^2*x^2)^(3/2))/(12*e) + ((d
- e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (5*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(8*e)
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= \int (d-ex)^2\sqrt{d^2-e^2x^2}\,dx\\
&= \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{4}(5d)\int (d-ex)\sqrt{d^2-e^2x^2}\,dx\\
&= \frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{4}\left(5d^2\right)\int \sqrt{d^2-e^2x^2}\,dx\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{8}\left(5d^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{8}\left(5d^4\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,\right.\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}
\end{aligned}$$

Mathematica [A] time = 0.0578734, size = 80, normalized size = 0.74

$$\frac{\sqrt{d^2-e^2x^2}\left(9d^2ex+16d^3-16de^2x^2+6e^3x^3\right)+15d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{24e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^3 + 9*d^2*e*x - 16*d*e^2*x^2 + 6*e^3*x^3) + 15*d
^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e)
```

Maple [B] time = 0.054, size = 194, normalized size = 1.8

$$\frac{1}{3e^3d}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{7}{2}}\left(\frac{d}{e}+x\right)^{-2}+\frac{1}{3de}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{5}{2}}+\frac{5x}{12}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] 1/3/e^3/d/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+1/3/e/d*(-(d/e+x)^
2*e^2+2*d*e*(d/e+x))^(5/2)+5/12*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+5/8*
d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+5/8*d^4/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [C] time = 1.7967, size = 161, normalized size = 1.49

$$-\frac{5i\,d^4\arcsin\left(\frac{ex}{d}+2\right)}{8\,e}+\frac{5}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}d^2x+\frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}d^3}{4\,e}+\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{4\left(e^2x+de\right)}+\frac{5\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d}{12\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -5/8*I*d^4*arcsin(e*x/d + 2)/e + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^2*x
+ 5/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3/e + 1/4*(-e^2*x^2 + d^2)^(5/2)/(e
^2*x + d*e) + 5/12*(-e^2*x^2 + d^2)^(3/2)*d/e
```

Fricas [A] time = 1.55863, size = 177, normalized size = 1.64

$$-\frac{30\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,e^3x^3-16\,de^2x^2+9\,d^2ex+16\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(30*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (6*e^3*x^3 - 16*d
*e^2*x^2 + 9*d^2*e*x + 16*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [C] time = 8.56547, size = 354, normalized size = 3.28

$$d^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\ldots\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\ldots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**
2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) -
 2*d*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3
/2)/(3*e**2), True)) + e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d
```

```
**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**
2*x**2/d**2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.163 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2} dx$

Optimal. Leaf size=96

$$d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] d*(d - e*x)*Sqrt[d^2 - e^2*x^2] - (d^2 - e^2*x^2)^(3/2)/3 - d^3*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.161523, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1809, 815, 844, 217, 203, 266, 63, 208}

$$d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```


```
[Out] d*(d - e*x)*Sqrt[d^2 - e^2*x^2] - (d^2 - e^2*x^2)^(3/2)/3 - d^3*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= -\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-3d^2e^2+6de^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{3e^2}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{6d^4e^4-6d^3e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{6e^4}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + d^4\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \left(d^3e\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^4\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \left(d^3e\right)\mathrm{Subst}\Big(\int$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^4\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d}\right.}{e^2}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] time = 0.109484, size = 96, normalized size = 1.

$$\sqrt{d^2-e^2x^2}\left(\frac{2d^2}{3}-dex+\frac{e^2x^2}{3}\right)-d^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+d^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+d^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((2*d^2)/3 - d*e*x + (e^2*x^2)/3) - d^3*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]] + d^3*Log[x] - d^3*Log[d + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] time = 0.067, size = 290, normalized size = 3.

$$\frac{1}{5d^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{1}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+d^2\sqrt{-x^2e^2+d^2}-d^4\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{8}{15d^2}\left(-\left(\frac{d}{e}+x\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x)
```

```
[Out] 1/5/d^2*(-e^2*x^2+d^2)^(5/2)+1/3*(-e^2*x^2+d^2)^(3/2)+d^2*(-e^2*x^2+d^2)^(1
/2)-d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-8/15/d
^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-2/3/d*e*(-(d/e+x)^2*e^2+2*d*e*(d/e+
x))^(3/2)*x-d*e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-d^3*e/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/3/e^2/d^2/(d/e+x
)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70291, size = 196, normalized size = 2.04

$$2d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+d^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{1}{3}\left(e^2x^2-3dex+2d^2\right)\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 2*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + d^3*log(-(d - sqrt(-e^2*x
^2 + d^2))/x) + 1/3*(e^2*x^2 - 3*d*e*x + 2*d^2)*sqrt(-e^2*x^2 + d^2)
```

Sympy [C] time = 12.045, size = 272, normalized size = 2.83

$$d^2 \left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \frac{|d^2|}{|e^2||x^2|} > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases} \right) - 2de \left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \frac{|e^2}{|} \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{other} \end{cases} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*d
**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**
2/(e**2*x**2) + 1), True)) - 2*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) -
I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/
d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1
 - e**2*x**2/d**2)/2, True)) + e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2,
0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.164 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{(d^2-e^2x^2)^{3/2}}{x}-\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -(e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/2 - (d^2 - e^2*x^2)^(3/2)/x - (d^2*e*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + 2*d^2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d
]
```

Rubi [A] time = 0.16069, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{(d^2-e^2x^2)^{3/2}}{x}-\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```


```
[Out] -(e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/2 - (d^2 - e^2*x^2)^(3/2)/x - (d^2*e*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + 2*d^2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d
]
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^2}\,dx$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{\int \frac{(2d^3e+d^2e^2x)\sqrt{d^2-e^2x^2}}{x}\,dx}{d^2}$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} + \frac{\int \frac{-4d^5e^3-d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2e^2}$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - (2d^3e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}(d^2e^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - (d^3e)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - \frac{1}{2}(d^2e^2)\,\text{Subs}$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,\right.}{e}$$
$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] time = 0.142649, size = 100, normalized size = 0.95

$$\left(-\frac{d^2}{x}-2de+\frac{e^2x}{2}\right)\sqrt{d^2-e^2x^2}+2d^2e\log\left(\sqrt{d^2-e^2x^2}+d\right)-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2d^2e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] (-2*d*e - d^2/x + (e^2*x)/2)*Sqrt[d^2 - e^2*x^2] - (d^2*e*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/2 - 2*d^2*e*Log[x] + 2*d^2*e*Log[d + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] time = 0.073, size = 425, normalized size = 4.1

$$-\frac{2e}{5d^3}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{2e}{3d}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-2de\sqrt{-x^2e^2+d^2}+2\frac{d^3e}{\sqrt{d^2}}\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}}{x}\right)+\frac{11e}{15d^3}\left(-\left(\frac{d}{e}+x\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x)
```

```
[Out] -2/5/d^3*e*(-e^2*x^2+d^2)^(5/2)-2/3/d*e*(-e^2*x^2+d^2)^(3/2)-2*d*e*(-e^2*x^
2+d^2)^(1/2)+2*d^3*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/
2))/x)+11/15/d^3*e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+11/12/d^2*e^2*(-(d/
e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+11/8*e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
1/2)*x+11/8*d^2*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*
(d/e+x))^(1/2))+1/3/d^3/e/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-1/
d^4/x*(-e^2*x^2+d^2)^(7/2)-1/d^4*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/4/d^2*e^2*x*(
-e^2*x^2+d^2)^(3/2)-15/8*e^2*x*(-e^2*x^2+d^2)^(1/2)-15/8*d^2*e^2/(e^2)^(1/2
```

```
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59977, size = 231, normalized size = 2.2

$$\frac{2\,d^2ex\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-4\,d^2ex\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-4\,d^2ex+\left(e^2x^2-4\,dex-2\,d^2\right)\sqrt{-e^2x^2+d^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*e*x*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 4*d^2*e*x*log(-(
d - sqrt(-e^2*x^2 + d^2))/x) - 4*d^2*e*x + (e^2*x^2 - 4*d*e*x - 2*d^2)*sqrt
(-e^2*x^2 + d^2))/x
```

Sympy [C] time = 9.24709, size = 354, normalized size = 3.37

$$d^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \frac{|d^2|}{|e^2||x} \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherw}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (-d/(x*s
qrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**
2)), True)) - 2*d*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*ac
osh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/(Abs(e**2)*Abs(x**
2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) +
I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + e**2*Piecewise((-I*d**2*acosh(e
*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d**2) > 1), (d**2*asin(e*x/d)/(2*e
) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.165 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^2}\,dx$

Optimal. Leaf size=110

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - (d^2 - e^2*x^2)^(3/2)/(2*x^2) +
 2*d*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (d*e^2*ArcTanh[Sqrt[d^2 - e^2*
x^2]/d])/2
```

Rubi [A] time = 0.162917, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 813, 844, 217, 203, 266, 63, 208}

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - (d^2 - e^2*x^2)^(3/2)/(2*x^2) +
 2*d*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (d*e^2*ArcTanh[Sqrt[d^2 - e^2*
x^2]/d])/2
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^3}\,dx$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{2x^2} - \frac{\int \frac{(4d^3e-d^2e^2x)\sqrt{d^2-e^2x^2}}{x^2}dx}{2d^2}$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{\int \frac{2d^4e^2+8d^3e^3x}{x\sqrt{d^2-e^2x^2}}dx}{4d^2}$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{1}{2}(d^2e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}dx + (2de^3)\int \frac{1}{\sqrt{d^2-e^2x^2}}d$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{1}{4}(d^2e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}dx,x,x^2\right) + (2de^3)\,\text{Su}$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^2\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}dx\right.$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right.$$

Mathematica [A] time = 0.152905, size = 102, normalized size = 0.93

$$\left(-\frac{d^2}{2x^2}+\frac{2de}{x}+e^2\right)\sqrt{d^2-e^2x^2}-\frac{1}{2}de^2\log\left(\sqrt{d^2-e^2x^2}+d\right)+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}de^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (e^2 - d^2/(2*x^2) + (2*d*e)/x)*Sqrt[d^2 - e^2*x^2] + 2*d*e^2*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]] + (d*e^2*Log[x])/2 - (d*e^2*Log[d + Sqrt[d^2 - e^2*x^2
]])/2
```

Maple [B] time = 0.07, size = 456, normalized size = 4.2

$$\frac{e^2}{10\,d^4}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}+\frac{e^2}{6\,d^2}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+\frac{e^2}{2}\sqrt{-x^2e^2+d^2}-\frac{d^2e^2}{2}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{14\,e^2}{15\,d^4}\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x)
```

```
[Out] 1/10/d^4*e^2*(-e^2*x^2+d^2)^(5/2)+1/6/d^2*e^2*(-e^2*x^2+d^2)^(3/2)+1/2*e^2*
(-e^2*x^2+d^2)^(1/2)-1/2*d^2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)-14/15/d^4*e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-7/6/d
^3*e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-7/4/d*e^3*(-(d/e+x)^2*e^2+2*d
*e*(d/e+x))^(1/2)*x-7/4*d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*
e^2+2*d*e*(d/e+x))^(1/2))-1/3/d^4/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^
(7/2)-1/2/d^4/x^2*(-e^2*x^2+d^2)^(7/2)+2/d^5*e/x*(-e^2*x^2+d^2)^(7/2)+2/d^5
*e^3*x*(-e^2*x^2+d^2)^(5/2)+5/2/d^3*e^3*x*(-e^2*x^2+d^2)^(3/2)+15/4/d*e^3*x
*(-e^2*x^2+d^2)^(1/2)+15/4*d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
```

```
+d^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62689, size = 240, normalized size = 2.18

$$-\frac{8\,de^2x^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-de^2x^2\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-2\,de^2x^2-\left(2\,e^2x^2+4\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*d*e^2*x^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - d*e^2*x^2*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - 2*d*e^2*x^2 - (2*e^2*x^2 + 4*d*e*x - d^2)
*sqrt(-e^2*x^2 + d^2))/x^2
```

Sympy [C] time = 8.92536, size = 355, normalized size = 3.23

$$d^2\left(\begin{cases}-\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \frac{|d^2|}{|e^2||x^2|}>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \frac{|e^2x^2|}{|d^2|}>\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwis}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d
**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs
(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x)
)/(2*d), True)) - 2*d*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*
acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2)/Abs(d
**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqr
t(1 - e**2*x**2/d**2)), True)) + e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*
x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2)/
(Abs(e**2)*Abs(x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*
d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.166 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^2}\,dx$

Optimal. Leaf size=102

$$\frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (d^2 - e^2*x^2)^(3/2)/(3*x^3) - e^3
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.162587, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1807, 811, 844, 217, 203, 266, 63, 208}

$$\frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3}+e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] (e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (d^2 - e^2*x^2)^(3/2)/(3*x^3) - e^3
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^4}\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} - \frac{\int \frac{\left(6d^3e-3d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{3d^2}$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} + \frac{\int \frac{12d^5e^3-12d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{12d^4}$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} + \left(de^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^4\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} + \frac{1}{2}\left(de^3\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e^4\operatorname{Subst}\left(\int\right.$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} - e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - (de)\operatorname{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \right.$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{3x^3} - e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A] time = 0.187797, size = 96, normalized size = 0.94

$$-\frac{\sqrt{d^2-e^2x^2}\left(d^2-3dex+2e^2x^2\right)}{3x^3}-e^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+e^3\left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+e^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(d^2 - 3*d*e*x + 2*e^2*x^2))/(3*x^3) - e^3*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + e^3*Log[x] - e^3*Log[d + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] time = 0.076, size = 479, normalized size = 4.7

$$-\frac{5e^2}{3d^6x}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{5e^4x}{3d^6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{25e^4x}{12d^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{25e^4x}{8d^2}\sqrt{-x^2e^2+d^2}+\frac{17e^4x}{12d^4}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x)
```

```
[Out] -5/3/d^6*e^2/x*(-e^2*x^2+d^2)^(7/2)-5/3/d^6*e^4*x*(-e^2*x^2+d^2)^(5/2)-25/1
2/d^4*e^4*x*(-e^2*x^2+d^2)^(3/2)-25/8/d^2*e^4*x*(-e^2*x^2+d^2)^(1/2)+17/12/
d^4*e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+17/8/d^2*e^4*(-(d/e+x)^2*e^2
+2*d*e*(d/e+x))^(1/2)*x-d*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x)+1/d^5*e/x^2*(-e^2*x^2+d^2)^(7/2)+1/3/d^5*e/(d/e+x)^2*(-(d/e
+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+17/15/d^5*e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))
^(5/2)+17/8*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(1/2))-1/3/d^4/x^3*(-e^2*x^2+d^2)^(7/2)-25/8*e^4/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/3/d^3*e^3*(-e^2*x^2+d^2)^(3/2)+1/d*e^3*
(-e^2*x^2+d^2)^(1/2)+1/5/d^5*e^3*(-e^2*x^2+d^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60153, size = 217, normalized size = 2.13

$$\frac{6e^3x^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+3e^3x^3\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-\left(2e^2x^2-3dex+d^2\right)\sqrt{-e^2x^2+d^2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*e^3*x^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*e^3*x^3*log(-(
d - sqrt(-e^2*x^2 + d^2))/x) - (2*e^2*x^2 - 3*d*e*x + d^2)*sqrt(-e^2*x^2 +
d^2))/x^3
```

Sympy [C] time = 8.45791, size = 347, normalized size = 3.4

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \frac{|d^2|}{|e^2||x^2|}>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right)-2de\left(\begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \frac{|d^2|}{|e^2||x^2|}>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)+e^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d
**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2)
, True)) - 2*d*e*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e
/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2)/(A
bs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*
asin(d/(e*x))/(2*d), True)) + e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d*
*2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*
x**2)/Abs(d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e*
*2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.167 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^2}\,dx$

Optimal. Leaf size=108

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] (-5*e^2*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d^2 - e^2*x^2)^(3/2)/(4*x^4) + (2*e
*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3) + (5*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
(8*d)
```

Rubi [A] time = 0.146877, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1807, 807, 266, 47, 63, 208}

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```


```
[Out] (-5*e^2*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d^2 - e^2*x^2)^(3/2)/(4*x^4) + (2*e
*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3) + (5*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
(8*d)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^5}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\int \frac{\left(8d^3e-5d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{4d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{1}{4}\left(5e^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{1}{8}\left(5e^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right) \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}-\frac{1}{16}\left(5e^4\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right) \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{1}{8}\left(5e^2\right)\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right) \\
&= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{4x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{3dx^3}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}
$$

Mathematica [A] time = 0.188358, size = 95, normalized size = 0.88

$$
-\frac{\sqrt{d^2-e^2x^2}\left(-16d^2ex+6d^3+9de^2x^2+16e^3x^3\right)-15e^4x^4\log\left(\sqrt{d^2-e^2x^2}+d\right)+15e^4x^4\log(x)}{24dx^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(6*d^3 - 16*d^2*e*x + 9*d*e^2*x^2 + 16*e^3*x^3) + 15*
e^4*x^4*Log[x] - 15*e^4*x^4*Log[d + Sqrt[d^2 - e^2*x^2]])/(24*d*x^4)
```

Maple [B] time = 0.115, size = 513, normalized size = 4.8

$$-\frac{9e^2}{8d^6x^2}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-\frac{5e^5x}{3d^5}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}-\frac{5e^5x}{2d^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}-\frac{5e^5}{2d}\arctan\left(x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x)
```

```
[Out] -9/8/d^6*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/3/d^5*e^5*(-(d/e+x)^2*e^2+2*d*e*(d/
e+x))^(3/2)*x-5/2/d^3*e^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-5/2/d*e^5/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/4/
d^4/x^4*(-e^2*x^2+d^2)^(7/2)-4/3/d^6*e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/
2)-1/8/d^6*e^4*(-e^2*x^2+d^2)^(5/2)-5/24/d^4*e^4*(-e^2*x^2+d^2)^(3/2)-5/8/d
^2*e^4*(-e^2*x^2+d^2)^(1/2)+5/8*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e
^2*x^2+d^2)^(1/2))/x)+2/3/d^5*e/x^3*(-e^2*x^2+d^2)^(7/2)+4/3/d^7*e^3/x*(-e^
2*x^2+d^2)^(7/2)+4/3/d^7*e^5*x*(-e^2*x^2+d^2)^(5/2)+5/3/d^5*e^5*x*(-e^2*x^2
+d^2)^(3/2)+5/2/d^3*e^5*x*(-e^2*x^2+d^2)^(1/2)+5/2/d*e^5/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/3/d^6*e^2/(d/e+x)^2*(-(d/e+x)^2*e^2+
2*d*e*(d/e+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60306, size = 181, normalized size = 1.68

$$-\frac{15e^4x^4\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16e^3x^3+9de^2x^2-16d^2ex+6d^3\right)\sqrt{-e^2x^2+d^2}}{24dx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(15*e^4*x^4*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3 + 9*d*e^
2*x^2 - 16*d^2*e*x + 6*d^3)*sqrt(-e^2*x^2 + d^2))/(d*x^4)
```

Sympy [C] time = 11.8741, size = 432, normalized size = 4.

$$d^2\left(\begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \frac{|d^2|}{|e^2||x^2|}>1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for} \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{oth}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/
(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e
*x))/(8*d**3), True)) - 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x
**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2)/(Abs(e**2)*Abs(x
**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/
(e**2*x**2) + 1)/(3*d**2), True)) + e**2*Piecewise((-d**2/(2*e*x**3*sqrt(d*
*2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e
*x))/(2*d), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x*
*2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.168 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^2}\,dx$

Optimal. Leaf size=140

$$\frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}-\frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5}-\frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

```
[Out] (e^3*Sqrt[d^2 - e^2*x^2])/(4*d*x^2) - (d^2 - e^2*x^2)^(3/2)/(5*x^5) + (e*(d
^2 - e^2*x^2)^(3/2))/(2*d*x^4) - (7*e^2*(d^2 - e^2*x^2)^(3/2))/(15*d^2*x^3)
 - (e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(4*d^2)
```

Rubi [A] time = 0.176687, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}-\frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5}-\frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (e^3*Sqrt[d^2 - e^2*x^2])/(4*d*x^2) - (d^2 - e^2*x^2)^(3/2)/(5*x^5) + (e*(d
^2 - e^2*x^2)^(3/2))/(2*d*x^4) - (7*e^2*(d^2 - e^2*x^2)^(3/2))/(15*d^2*x^3)
 - (e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(4*d^2)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^6}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} - \frac{\int \frac{\left(10d^3e-7d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^5}\,dx}{5d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} + \frac{\int \frac{\left(28d^4e^2-10d^3e^3x\right)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{20d^4}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} - \frac{e^3\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx}{2d}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} - \frac{e^3\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)}{4d}$$
$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} + \frac{e^5\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x,\right.}{8d}$$
$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} - \frac{e^3\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{}\right.}{4d}$$
$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{5x^5} + \frac{e\left(d^2-e^2x^2\right)^{3/2}}{2dx^4} - \frac{7e^2\left(d^2-e^2x^2\right)^{3/2}}{15d^2x^3} - \frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

Mathematica [A] time = 0.190551, size = 106, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(-16d^2e^2x^2+30d^3ex-12d^4-15de^3x^3+28e^4x^4\right)-15e^5x^5\log\left(\sqrt{d^2-e^2x^2}+d\right)+15e^5x^5\log(x)}{60d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-12*d^4 + 30*d^3*e*x - 16*d^2*e^2*x^2 - 15*d*e^3*x^3
+ 28*e^4*x^4) + 15*e^5*x^5*Log[x] - 15*e^5*x^5*Log[d + Sqrt[d^2 - e^2*x^2]]
)/(60*d^2*x^5)
```

Maple [B] time = 0.1, size = 541, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x)
```

```
[Out] 1/2/d^5*e/x^4*(-e^2*x^2+d^2)^(7/2)+5/4/d^7*e^3/x^2*(-e^2*x^2+d^2)^(7/2)-1/4
/d*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+23/15/d
^7*e^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-1/5/d^4/x^5*(-e^2*x^2+d^2)^(7/2
)+1/20/d^7*e^5*(-e^2*x^2+d^2)^(5/2)+1/12/d^5*e^5*(-e^2*x^2+d^2)^(3/2)+1/4/d
^3*e^5*(-e^2*x^2+d^2)^(1/2)-13/15/d^6*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-23/15/d^
8*e^4/x*(-e^2*x^2+d^2)^(7/2)-23/15/d^8*e^6*x*(-e^2*x^2+d^2)^(5/2)-23/12/d^6
*e^6*x*(-e^2*x^2+d^2)^(3/2)-23/8/d^4*e^6*x*(-e^2*x^2+d^2)^(1/2)-23/8/d^2*e^
```

```
6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+23/12/d^6*e^6*(-(d
/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+23/8/d^4*e^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+
x))^(1/2)*x+23/8/d^2*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2
*d*e*(d/e+x))^(1/2))+1/3/d^7*e^3/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65197, size = 208, normalized size = 1.49

$$\frac{15\,e^5x^5\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(28\,e^4x^4-15\,de^3x^3-16\,d^2e^2x^2+30\,d^3ex-12\,d^4\right)\sqrt{-e^2x^2+d^2}}{60\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*e^5*x^5*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (28*e^4*x^4 - 15*d*e^
3*x^3 - 16*d^2*e^2*x^2 + 30*d^3*e*x - 12*d^4)*sqrt(-e^2*x^2 + d^2))/(d^2*x^
5)
```

Sympy [C] time = 13.0823, size = 668, normalized size = 4.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-
15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*
d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x*
*5 + 15*d*e**2*x**7), True)) - 2*d*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(
e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x
*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs
(e**2)*Abs(x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*
I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**
2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**2*Piecewise((-e*
```

```
sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d*
*2), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1
)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.169 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3}-\frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}+\frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] (-3*e^4*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (d^2 - e^2*x^2)^(3/2)/(6*x^6) +
 (2*e*(d^2 - e^2*x^2)^(3/2))/(5*d*x^5) - (3*e^2*(d^2 - e^2*x^2)^(3/2))/(8*d
^2*x^4) + (4*e^3*(d^2 - e^2*x^2)^(3/2))/(15*d^3*x^3) + (3*e^6*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/(16*d^3)
```

Rubi [A] time = 0.207751, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$-\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3}-\frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}+\frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] (-3*e^4*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (d^2 - e^2*x^2)^(3/2)/(6*x^6) +
 (2*e*(d^2 - e^2*x^2)^(3/2))/(5*d*x^5) - (3*e^2*(d^2 - e^2*x^2)^(3/2))/(8*d
^2*x^4) + (4*e^3*(d^2 - e^2*x^2)^(3/2))/(15*d^3*x^3) + (3*e^6*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/(16*d^3)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^7}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} - \frac{\int \frac{\left(12d^3e-9d^2e^2x\right)\sqrt{d^2-e^2x^2}}{x^6}dx}{6d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} + \frac{\int \frac{\left(45d^4e^2-24d^3e^3x\right)\sqrt{d^2-e^2x^2}}{x^5}dx}{30d^4}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} - \frac{\int \frac{\left(96d^5e^3-45d^4e^4x\right)\sqrt{d^2-e^2x^2}}{x^4}dx}{120d^6}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} + \frac{\left(3e^4\right)\int \frac{\sqrt{d^2-e^2x^2}}{x^3}}{8d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} + \frac{\left(3e^4\right)\text{Subst}\left(\int\right.}{16}$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} -$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} +$$
$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6} + \frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5} - \frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4} + \frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3} +$$

Mathematica [A] time = 0.264799, size = 117, normalized size = 0.69

$$-\frac{\sqrt{d^2-e^2x^2}\left(50d^3e^2x^2+32d^2e^3x^3-96d^4ex+40d^5-45de^4x^4+64e^5x^5\right)-45e^6x^6\log\left(\sqrt{d^2-e^2x^2}+d\right)+45e^6x^6\log(}{240d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(40*d^5 - 96*d^4*e*x + 50*d^3*e^2*x^2 + 32*d^2*e^3*x^
3 - 45*d*e^4*x^4 + 64*e^5*x^5) + 45*e^6*x^6*Log[x] - 45*e^6*x^6*Log[d + Sqr
t[d^2 - e^2*x^2]])/(240*d^3*x^6)
```

Maple [B] time = 0.12, size = 566, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x)
```

```
[Out] 16/15/d^7*e^3/x^3*(-e^2*x^2+d^2)^(7/2)+26/15/d^9*e^5/x*(-e^2*x^2+d^2)^(7/2)
+26/15/d^9*e^7*x*(-e^2*x^2+d^2)^(5/2)+13/6/d^7*e^7*x*(-e^2*x^2+d^2)^(3/2)+1
3/4/d^5*e^7*x*(-e^2*x^2+d^2)^(1/2)+13/4/d^3*e^7/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-e^2*x^2+d^2)^(1/2))-1/6/d^4/x^6*(-e^2*x^2+d^2)^(7/2)-1/3/d^8*e^4/(d
/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-3/80/d^8*e^6*(-e^2*x^2+d^2)^(5
```

```
/2)-1/16/d^6*e^6*(-e^2*x^2+d^2)^(3/2)-3/16/d^4*e^6*(-e^2*x^2+d^2)^(1/2)-26/
15/d^8*e^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+3/16/d^2*e^6/(d^2)^(1/2)*ln
((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+2/5/d^5*e/x^5*(-e^2*x^2+d^2)
^(7/2)-17/24/d^6*e^2/x^4*(-e^2*x^2+d^2)^(7/2)-23/16/d^8*e^4/x^2*(-e^2*x^2+d
^2)^(7/2)-13/6/d^7*e^7*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-13/4/d^5*e^7*
(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-13/4/d^3*e^7/(e^2)^(1/2)*arctan((e^2
)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70395, size = 234, normalized size = 1.38

$$-\frac{45\, e^{6} x^{6} \log\left(-\frac{d-\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)+\left(64\, e^{5} x^{5}-45\, d e^{4} x^{4}+32\, d^{2} e^{3} x^{3}+50\, d^{3} e^{2} x^{2}-96\, d^{4} e x+40\, d^{5}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{240\, d^{3} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(45*e^6*x^6*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (64*e^5*x^5 - 45*d*
e^4*x^4 + 32*d^2*e^3*x^3 + 50*d^3*e^2*x^2 - 96*d^4*e*x + 40*d^5)*sqrt(-e^2*
x^2 + d^2))/(d^3*x^6)
```

Sympy [C] time = 18.4442, size = 816, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e
**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*
d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2
*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 2*d*e*Piecewise((3*
I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*
x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4
*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*
x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*
```

```
e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**
2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**
7), True)) + e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) +
3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x*
*2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2)/(Abs(e**2)*Abs(x**2)) >
 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d
**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
**4*asin(d/(e*x))/(8*d**3), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.170 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2} dx$

Optimal. Leaf size=198

$$\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{22e^4\left(d^2-e^2x^2\right)^{3/2}}{105d^4x^3}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}-\frac{e^7\tanh$$

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) - (d^2 - e^2*x^2)^(3/2)/(7*x^7) + (e*
(d^2 - e^2*x^2)^(3/2))/(3*d*x^6) - (11*e^2*(d^2 - e^2*x^2)^(3/2))/(35*d^2*x
^5) + (e^3*(d^2 - e^2*x^2)^(3/2))/(4*d^3*x^4) - (22*e^4*(d^2 - e^2*x^2)^(3/
2))/(105*d^4*x^3) - (e^7*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rubi [A] time = 0.235818, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {852, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{22e^4\left(d^2-e^2x^2\right)^{3/2}}{105d^4x^3}+\frac{e^3\left(d^2-e^2x^2\right)^{3/2}}{4d^3x^4}-\frac{11e^2\left(d^2-e^2x^2\right)^{3/2}}{35d^2x^5}+\frac{e\left(d^2-e^2x^2\right)^{3/2}}{3dx^6}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7x^7}-\frac{e^7\tanh$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) - (d^2 - e^2*x^2)^(3/2)/(7*x^7) + (e*
(d^2 - e^2*x^2)^(3/2))/(3*d*x^6) - (11*e^2*(d^2 - e^2*x^2)^(3/2))/(35*d^2*x
^5) + (e^3*(d^2 - e^2*x^2)^(3/2))/(4*d^3*x^4) - (22*e^4*(d^2 - e^2*x^2)^(3/
2))/(105*d^4*x^3) - (e^7*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^8}\,dx$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} - \frac{\int \frac{(14d^3e-11d^2e^2x)\sqrt{d^2-e^2x^2}}{x^7}dx}{7d^2}$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} + \frac{\int \frac{(66d^4e^2-42d^3e^3x)\sqrt{d^2-e^2x^2}}{x^6}dx}{42d^4}$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} - \frac{\int \frac{(210d^5e^3-132d^4e^4x)\sqrt{d^2-e^2x^2}}{x^5}dx}{210d^6}$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} + \frac{\int \frac{(528d^6e^4-210d^5e^5x)\sqrt{d^2}}{x^4}}{840d^8}$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3} -$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3} -$$
$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(}{1}$$
$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(}{1}$$
$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(}{1}$$

Mathematica [A] time = 0.261762, size = 128, normalized size = 0.65

$$\frac{\sqrt{d^2-e^2x^2}\left(-144d^4e^2x^2-70d^3e^3x^3+88d^2e^4x^4+280d^5ex-120d^6-105de^5x^5+176e^6x^6\right)-105e^7x^7\log\left(\sqrt{d^2-e^2x^2}+d\right.}{840d^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^6 + 280*d^5*e*x - 144*d^4*e^2*x^2 - 70*d^3*e^3
*x^3 + 88*d^2*e^4*x^4 - 105*d*e^5*x^5 + 176*e^6*x^6) + 105*e^7*x^7*Log[x] -
 105*e^7*x^7*Log[d + Sqrt[d^2 - e^2*x^2]])/(840*d^4*x^7)
```

Maple [B] time = 0.154, size = 591, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x)
```

```
[Out] -1/7/d^4/x^7*(-e^2*x^2+d^2)^(7/2)-19/15/d^8*e^4/x^3*(-e^2*x^2+d^2)^(7/2)-29
/15/d^10*e^6/x*(-e^2*x^2+d^2)^(7/2)-29/15/d^10*e^8*x*(-e^2*x^2+d^2)^(5/2)+1
```

```
/3/d^5*e/x^6*(-e^2*x^2+d^2)^(7/2)-1/8/d^3*e^7/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+29/15/d^9*e^7*(-(d/e+x)^2*e^2+2*d*e*(d/e+x)
)^(5/2)+1/40/d^9*e^7*(-e^2*x^2+d^2)^(5/2)+1/24/d^7*e^7*(-e^2*x^2+d^2)^(3/2)
+1/8/d^5*e^7*(-e^2*x^2+d^2)^(1/2)-29/12/d^8*e^8*x*(-e^2*x^2+d^2)^(3/2)-29/8
/d^6*e^8*x*(-e^2*x^2+d^2)^(1/2)-29/8/d^4*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))-3/5/d^6*e^2/x^5*(-e^2*x^2+d^2)^(7/2)+11/12/d^7*e^3
/x^4*(-e^2*x^2+d^2)^(7/2)+13/8/d^9*e^5/x^2*(-e^2*x^2+d^2)^(7/2)+29/12/d^8*e
^8*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+29/8/d^6*e^8*(-(d/e+x)^2*e^2+2*d*
e*(d/e+x))^(1/2)*x+29/8/d^4*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^
2*e^2+2*d*e*(d/e+x))^(1/2))+1/3/d^9*e^5/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/
e+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70271, size = 263, normalized size = 1.33

$$\frac{105\, e^7 x^7 \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \left(176\, e^6x^6 - 105\, de^5x^5 + 88\, d^2e^4x^4 - 70\, d^3e^3x^3 - 144\, d^4e^2x^2 + 280\, d^5ex - 120\, d^6\right)\sqrt{-e^2x^2}}{840\, d^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*e^7*x^7*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (176*e^6*x^6 - 105*
d*e^5*x^5 + 88*d^2*e^4*x^4 - 70*d^3*e^3*x^3 - 144*d^4*e^2*x^2 + 280*d^5*e*x
 - 120*d^6)*sqrt(-e^2*x^2 + d^2))/(d^4*x^7)
```

Sympy [C] time = 18.2054, size = 843, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2)/(Abs(e**2)*
Abs(x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-
d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)
/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True))
 - 2*d*e*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x*
*5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) -
```

```
 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(1
6*d**5), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + e**2*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2)/Abs(d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 1
5*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e
**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x
**7), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.171 $\int \frac{x^4}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] -(d^3*(d - e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (17*d^2*(d - e*x))/(15*e
^5*(d^2 - e^2*x^2)^(3/2)) - (2*(15*d - 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2]
) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rubi [A] time = 0.237506, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1814, 12, 217, 203}

$$-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -(d^3*(d - e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (17*d^2*(d - e*x))/(15*e
^5*(d^2 - e^2*x^2)^(3/2)) - (2*(15*d - 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2]
) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)\left(\frac{2d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-}}\right.}{e^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A] time = 0.172455, size = 106, normalized size = 0.86

$$\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{10e^5(d+ex)^3}+\frac{31d}{60e^5(d+ex)^2}-\frac{1}{8e^5(ex-d)}-\frac{193}{120e^5(d+ex)}\right)-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(-1/(8*e^5*(-d + e*x)) - d^2/(10*e^5*(d + e*x)^3) + (31
*d)/(60*e^5*(d + e*x)^2) - 193/(120*e^5*(d + e*x))) - ArcTan[(e*x)/Sqrt[d^2
```

```
- e^2*x^2]]/e^5
```

Maple [A] time = 0.069, size = 198, normalized size = 1.6

$$4\frac{x}{e^4\sqrt{-x^2e^2+d^2}}-\frac{1}{e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-2\frac{d}{e^5\sqrt{-x^2e^2+d^2}}+\frac{17\,d^2}{15\,e^6}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 4/e^4*x/(-e^2*x^2+d^2)^(1/2)-1/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))-2*d/e^5/(-e^2*x^2+d^2)^(1/2)+17/15/e^6*d^2/(d/e+x)/(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(1/2)-34/15/e^4/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*
x-1/5*d^3/e^7/(d/e+x)^2/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59017, size = 360, normalized size = 2.93

$$-\frac{16\,e^4x^4+32\,de^3x^3-32\,d^3ex-16\,d^4-30\left(e^4x^4+2\,de^3x^3-2\,d^3ex-d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(26\,e^3x^3+22\,de^2x^2\right.}{15\left(e^9x^4+2\,de^8x^3-2\,d^3e^6x-d^4e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(16*e^4*x^4 + 32*d*e^3*x^3 - 32*d^3*e*x - 16*d^4 - 30*(e^4*x^4 + 2*d*
e^3*x^3 - 2*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (26*
e^3*x^3 + 22*d*e^2*x^2 - 17*d^2*e*x - 16*d^3)*sqrt(-e^2*x^2 + d^2))/(e^9*x^
4 + 2*d*e^8*x^3 - 2*d^3*e^6*x - d^4*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.172 $\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[Out] (d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d - e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d - 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.204107, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {852, 1635, 637}

$$\frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d - e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d - 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^3(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)\left(-\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{-\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0867032, size = 70, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^2ex+2d^3+de^2x^2-2e^3x^3\right)}{5de^4(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + 4*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(5*d*e^4*
(d - e*x)*(d + e*x)^3)
```

Maple [A] time = 0.047, size = 65, normalized size = 0.7

$$\frac{(-ex+d)\left(-2\,e^3x^3+de^2x^2+4\,d^2ex+2\,d^3\right)}{(5\,ex+5\,d)\,de^4}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/5*(-e*x+d)*(-2*e^3*x^3+d*e^2*x^2+4*d^2*e*x+2*d^3)/(e*x+d)/d/e^4/(-e^2*x^2
+d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58563, size = 230, normalized size = 2.32

$$\frac{2\,e^4x^4+4\,de^3x^3-4\,d^3ex-2\,d^4+\left(2\,e^3x^3-de^2x^2-4\,d^2ex-2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(de^8x^4+2\,d^2e^7x^3-2\,d^4e^5x-d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 - d*e^2*x^2 -
 4*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d*e^8*x^4 + 2*d^2*e^7*x^3 - 2*d^
4*e^5*x - d^5*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.173 $\int \frac{x^2}{(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$

Optimal. Leaf size=89

$$-\frac{d}{5e^3(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{7}{15e^3(d+ex)\sqrt{d^2-e^2x^2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2]) - d/(5*e^3*(d + e*x)^2*Sqrt[d^2 - e^2*x^
2]) + 7/(15*e^3*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.141929, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {852, 1635, 778, 191}

$$-\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -(d*(d - e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) + (7*(d - e*x))/(15*e^3*(d^2
 - e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^2(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx\\
&= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{2d^2}{e^2}-\frac{5dx}{e}\right)(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\
&= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2}\\
&= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0710843, size = 70, normalized size = 0.79

$$
\frac{\sqrt{d^2-e^2x^2}\left(8d^2ex+4d^3+2de^2x^2+e^3x^3\right)}{15d^2e^3(d-ex)(d+ex)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^3 + 8*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*e
^3*(d - e*x)*(d + e*x)^3)
```

Maple [A] time = 0.05, size = 65, normalized size = 0.7

$$
\frac{(-ex+d)\left(e^3x^3+2\,de^2x^2+8\,xd^2e+4\,d^3\right)}{(15\,ex+15\,d)\,d^2e^3}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e^3*x^3+2*d*e^2*x^2+8*d^2*e*x+4*d^3)/(e*x+d)/d^2/e^3/(-e^2*x
^2+d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68399, size = 234, normalized size = 2.63

$$\frac{4\,e^4x^4+8\,de^3x^3-8\,d^3ex-4\,d^4-\left(e^3x^3+2\,de^2x^2+8\,d^2ex+4\,d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^7x^4+2\,d^3e^6x^3-2\,d^5e^4x-d^6e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*e^4*x^4 + 8*d*e^3*x^3 - 8*d^3*e*x - 4*d^4 - (e^3*x^3 + 2*d*e^2*x^2
+ 8*d^2*e*x + 4*d^3)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*x^4 + 2*d^3*e^6*x^3 - 2
*d^5*e^4*x - d^6*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.174 $\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}$$

```
[Out] (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2]) + 1/(5*e^2*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2]) - 2/(15*d*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0364116, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {793, 659, 191}

$$\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2]) + 1/(5*e^2*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2]) - 2/(15*d*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{2\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e}$$

$$= \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}$$

$$= \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}} + \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0627545, size = 69, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^2ex+d^3+8de^2x^2+4e^3x^3\right)}{15d^3e^2(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 8*d*e^2*x^2 + 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)*(d + e*x)^3)
```

Maple [A] time = 0.049, size = 64, normalized size = 0.7

$$\frac{(-ex+d)\left(4e^3x^3+8de^2x^2+2d^2ex+d^3\right)}{(15ex+15d)d^3e^2}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/15*(-e*x+d)*(4*e^3*x^3+8*d*e^2*x^2+2*d^2*e*x+d^3)/(e*x+d)/d^3/e^2/(-e^2*x
^2+d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59638, size = 228, normalized size = 2.51

$$\frac{e^4x^4+2de^3x^3-2d^3ex-d^4-\left(4e^3x^3+8de^2x^2+2d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^6x^4+2d^4e^5x^3-2d^6e^3x-d^7e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(e^4*x^4 + 2*d*e^3*x^3 - 2*d^3*e*x - d^4 - (4*e^3*x^3 + 8*d*e^2*x^2 +
2*d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^4 + 2*d^4*e^5*x^3 - 2*d^6
*e^3*x - d^7*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.175 $\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

```
[Out] (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2]) - 1/(5*d*e*(d + e*x)^2*Sqrt[d^2 - e^2*x^2
]) - 1/(5*d^2*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0311912, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 191}

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2]) - 1/(5*d*e*(d + e*x)^2*Sqrt[d^2 - e^2*x^2
]) - 1/(5*d^2*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{3\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d}\\
&= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d^2}\\
&= \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0397725, size = 70, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(d^2ex-2d^3+4de^2x^2+2e^3x^3\right)}{5d^4e(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 + d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e
*(d - e*x)*(d + e*x)^3)
```

Maple [A] time = 0.05, size = 66, normalized size = 0.7

$$-\frac{(-ex+d)\left(-2\,e^3x^3-4\,e^2x^2d-xd^2e+2\,d^3\right)}{(5\,ex+5\,d)\,d^4e}\left(-x^2e^2+d^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -1/5*(-e*x+d)*(-2*e^3*x^3-4*d*e^2*x^2-d^2*e*x+2*d^3)/(e*x+d)/d^4/e/(-e^2*x^
2+d^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54651, size = 231, normalized size = 2.54

$$-\frac{2\,e^4x^4+4\,de^3x^3-4\,d^3ex-2\,d^4+\left(2\,e^3x^3+4\,de^2x^2+d^2ex-2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\left(d^4e^5x^4+2\,d^5e^4x^3-2\,d^7e^2x-d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 + 4*d*e^2*x^
2 + d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 + 2*d^5*e^4*x^3 - 2
*d^7*e^2*x - d^8*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.176 $\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] (2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d - 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rubi [A] time = 0.177683, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d - 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\int \frac{(d-ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$=\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{-5d^2+8dex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$=\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{-15d^4e^2+16d^3e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2}$$

$$=\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$=\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$=\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}+\frac{\text{Subst}\Big(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2}{2d^4}$$

$$=\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\text{Subst}\Big(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2}}{d^4e^2}$$

$$=\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Mathematica [A] time = 0.0979339, size = 95, normalized size = 0.81

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(22d^2ex+26d^3-17de^2x^2-16e^3x^3\right)}{(d-ex)(d+ex)^3}-15\log\left(\sqrt{d^2-e^2x^2}+d\right)+15\log(x)}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(26*d^3 + 22*d^2*e*x - 17*d*e^2*x^2 - 16*e^3*x^3))/((
d - e*x)*(d + e*x)^3) + 15*Log[x] - 15*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^
5)
```

Maple [A] time = 0.062, size = 187, normalized size = 1.6

$$\frac{1}{d^4}\frac{1}{\sqrt{-x^2e^2+d^2}}-\frac{1}{d^4}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{8}{15d^3e}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}}-\frac{16}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 1/d^4/(-e^2*x^2+d^2)^(1/2)-1/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)+8/15/d^3/e/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-
16/15/d^5*e/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+1/5/d^2/e^2/(d/e+x)^2/(-
(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x), x)
```

Fricas [A] time = 1.6897, size = 350, normalized size = 2.97

$$\frac{26e^4x^4+52de^3x^3-52d^3ex-26d^4+15\left(e^4x^4+2de^3x^3-2d^3ex-d^4\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(16e^3x^3+17de^2x^2-22\right.}{15\left(d^5e^4x^4+2d^6e^3x^3-2d^8ex-d^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*e^4*x^4 + 52*d*e^3*x^3 - 52*d^3*e*x - 26*d^4 + 15*(e^4*x^4 + 2*d*e
^3*x^3 - 2*d^3*e*x - d^4)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (16*e^3*x^3
+ 17*d*e^2*x^2 - 22*d^2*e*x - 26*d^3)*sqrt(-e^2*x^2 + d^2))/(d^5*e^4*x^4 +
2*d^6*e^3*x^3 - 2*d^8*e*x - d^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.177 $\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] (-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (e*(10*d - 13*e*x))/(15*d^
4*(d^2 - e^2*x^2)^(3/2)) - (e*(30*d - 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
 - Sqrt[d^2 - e^2*x^2]/(d^6*x) + (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rubi [A] time = 0.300216, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (e*(10*d - 13*e*x))/(15*d^
4*(d^2 - e^2*x^2)^(3/2)) - (e*(30*d - 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
 - Sqrt[d^2 - e^2*x^2]/(d^6*x) + (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+10dex-8e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2-30dex+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2+30dex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{(2e)\int \frac{}{x\sqrt{}}}{d}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{e\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{2\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{2e\tanh^{-}}{}$$

Mathematica [A] time = 0.118418, size = 112, normalized size = 0.77

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(32d^2e^2x^2+76d^3ex+15d^4-82de^3x^3-56e^4x^4\right)}{x(ex-d)(d+ex)^3}+30e\log\left(\sqrt{d^2-e^2x^2}+d\right)-30e\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^4 + 76*d^3*e*x + 32*d^2*e^2*x^2 - 82*d*e^3*x^3
- 56*e^4*x^4))/(x*(-d + e*x)*(d + e*x)^3) - 30*e*Log[x] + 30*e*Log[d + Sqrt
[d^2 - e^2*x^2]])/(15*d^6)
```

Maple [A] time = 0.067, size = 234, normalized size = 1.6

$$-2\,\frac{e}{d^5\sqrt{-x^2e^2+d^2}}+2\,\frac{e}{d^5\sqrt{d^2}}\ln\left(\frac{2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}}{x}\right)-\frac{13}{15\,d^4}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}}+\frac{26\,e}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] -2/d^5*e/(-e^2*x^2+d^2)^(1/2)+2/d^5*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)-13/15/d^4/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1
/2)+26/15/d^6*e^2/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-1/5/d^3/e/(d/e+x)^
2/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-1/d^4/x/(-e^2*x^2+d^2)^(1/2)+2/d^6*e
^2*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^2), x)
```

Fricas [A] time = 1.69005, size = 402, normalized size = 2.75

$$-\frac{46\,e^5x^5+92\,de^4x^4-92\,d^3e^2x^2-46\,d^4ex+30\left(e^5x^5+2\,de^4x^4-2\,d^3e^2x^2-d^4ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(56\,e^4x^4+82\,\right.}{15\left(d^6e^4x^5+2\,d^7e^3x^4-2\,d^9ex^2-d^{10}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(46*e^5*x^5 + 92*d*e^4*x^4 - 92*d^3*e^2*x^2 - 46*d^4*e*x + 30*(e^5*x^
5 + 2*d*e^4*x^4 - 2*d^3*e^2*x^2 - d^4*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/
x) + (56*e^4*x^4 + 82*d*e^3*x^3 - 32*d^2*e^2*x^2 - 76*d^3*e*x - 15*d^4)*sqr
t(-e^2*x^2 + d^2))/(d^6*e^4*x^5 + 2*d^7*e^3*x^4 - 2*d^9*e*x^2 - d^10*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}} \left(d+ex\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.178 $\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=183

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

```
[Out] (2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d - 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d - 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rubi [A] time = 0.374615, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d - 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d - 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+10dex-10e^2x^2+\frac{8e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2-30dex+45e^2x^2-\frac{36e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2+30dex-60e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{\int \frac{-60d^3e-}{x^2\sqrt{d}}}{3}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e\sqrt{d^2-}}{d^7x}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e\sqrt{d^2-}}{d^7x}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e\sqrt{d^2-}}{d^7x}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{2e\sqrt{d^2-}}{d^7x}$$

Mathematica [A] time = 0.148093, size = 127, normalized size = 0.69

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-94d^3e^2x^2-58d^2e^3x^3-10d^4ex+5d^5+83de^4x^4+64e^5x^5\right)}{x^2(ex-d)(d+ex)^3} - 45e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 45e^2\log(x)}{10d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 10*d^4*e*x - 94*d^3*e^2*x^2 - 58*d^2*e^3*x^3
 + 83*d*e^4*x^4 + 64*e^5*x^5))/(x^2*(-d + e*x)*(d + e*x)^3) + 45*e^2*Log[x]
 - 45*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(10*d^7)
```

Maple [A] time = 0.068, size = 259, normalized size = 1.4

$$\frac{9\,e^2}{2\,d^6}\frac{1}{\sqrt{-x^2e^2+d^2}} - \frac{9\,e^2}{2\,d^6}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} + \frac{6\,e}{5\,d^5}\left(\frac{d}{e}+x\right)^{-1}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}} - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

```
[Out] 9/2*e^2/d^6/(-e^2*x^2+d^2)^(1/2)-9/2*e^2/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^
(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+6/5/d^5*e/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(1/2)-12/5/d^7*e^3/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+1/5/d^4/(d/e
+x)^2/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-1/2/d^4/x^2/(-e^2*x^2+d^2)^(1/2)
+2/d^5*e/x/(-e^2*x^2+d^2)^(1/2)-4/d^7*e^3*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(ex+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^3), x)
```

Fricas [A] time = 1.75807, size = 440, normalized size = 2.4

$$\frac{54\,e^6x^6+108\,de^5x^5-108\,d^3e^3x^3-54\,d^4e^2x^2+45\left(e^6x^6+2\,de^5x^5-2\,d^3e^3x^3-d^4e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(64\,e^5x^5+8\right.}{10\left(d^7e^4x^6+2\,d^8e^3x^5-2\,d^{10}ex^3-d^{11}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*e^6*x^6 + 108*d*e^5*x^5 - 108*d^3*e^3*x^3 - 54*d^4*e^2*x^2 + 45*(e
^6*x^6 + 2*d*e^5*x^5 - 2*d^3*e^3*x^3 - d^4*e^2*x^2)*log(-(d - sqrt(-e^2*x^2
 + d^2))/x) + (64*e^5*x^5 + 83*d*e^4*x^4 - 58*d^2*e^3*x^3 - 94*d^3*e^2*x^2
- 10*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^7*e^4*x^6 + 2*d^8*e^3*x^5 -
2*d^10*e*x^3 - d^11*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{3}{2}}(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.179 $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=177

$$\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}-\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] (d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d - e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d - e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 - (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) + (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rubi [A] time = 0.439337, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}-\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d - e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d - e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 - (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) + (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^5(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)\left(-\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{\frac{195d^5}{e^3}-\frac{90d^4x}{e^2}}{\sqrt{d^2-e^2x^2}}}{30d^3e^2}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x}}{2e^5}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x}}{2e^5}$$

$$= \frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x}}{2e^5}$$

Mathematica [A] time = 0.198671, size = 98, normalized size = 0.55

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(479d^2e^2x^2+717d^3ex+304d^4+45de^3x^3-15e^4x^4\right)}{(d+ex)^3}+195d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{30e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(304*d^4 + 717*d^3*e*x + 479*d^2*e^2*x^2 + 45*d*e^3*x
^3 - 15*e^4*x^4))/(d + e*x)^3 + 195*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(30*e^6)
```

Maple [A] time = 0.069, size = 212, normalized size = 1.2

$$-\frac{x}{2e^5}\sqrt{-x^2e^2+d^2}+\frac{13d^2}{2e^5}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+3\frac{d\sqrt{-x^2e^2+d^2}}{e^6}+\frac{127d^2}{15e^7}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*x*(-e^2*x^2+d^2)^(1/2)/e^5+13/2/e^5*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))+3*d*(-e^2*x^2+d^2)^(1/2)/e^6+127/15/e^7*d^2/(d/e+x
)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-23/15/e^8*d^3/(d/e+x)^2*(-(d/e+x)^2*
e^2+2*d*e*(d/e+x))^(1/2)+1/5*d^4/e^9/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x
))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75441, size = 409, normalized size = 2.31

$$\frac{304\,d^2e^3x^3+912\,d^3e^2x^2+912\,d^4ex+304\,d^5-390\left(d^2e^3x^3+3\,d^3e^2x^2+3\,d^4ex+d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(15\,e^4x^4-\right.}{30\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*e^3*x^3 + 912*d^3*e^2*x^2 + 912*d^4*e*x + 304*d^5 - 390*(d^2*
e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (15*e^4*x^4 - 45*d*e^3*x^3 - 479*d^2*e^2*x^2 - 717*d^3*e*x - 30
4*d^4)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.180 $\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] -(d^3*(d - e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (6*d^2*(d - e*x)^2)/(5*e
^5*(d^2 - e^2*x^2)^(3/2)) - (24*d*(d - e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) -
Sqrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rubi [A] time = 0.365028, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1635, 641, 217, 203}

$$-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(d^3*(d - e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (6*d^2*(d - e*x)^2)/(5*e
^5*(d^2 - e^2*x^2)^(3/2)) - (24*d*(d - e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) -
Sqrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(\frac{3d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)\left(\frac{27d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}-\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\int \frac{1}{\sqrt{d^2-e^2x^2}}}{e^4}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan^{-1}\left(\frac{e}{\sqrt{d^2-}}\right.}{e^5}$$

Mathematica [A] time = 0.159434, size = 85, normalized size = 0.58

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(57d^2ex+24d^3+39de^2x^2+5e^3x^3\right)}{(d+ex)^3}+15d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{5e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(24*d^3 + 57*d^2*e*x + 39*d*e^2*x^2 + 5*e^3*x^3))/(d
 + e*x)^3 + 15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(5*e^5)
```

Maple [A] time = 0.061, size = 187, normalized size = 1.3

$$-\frac{1}{e^5}\sqrt{-x^2e^2+d^2} - 3\frac{d}{e^4\sqrt{e^2}}\arctan\left(\frac{\sqrt{e^2}x}{\sqrt{-x^2e^2+d^2}}\right) - \frac{24\,d}{5\,e^6}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1} + \frac{6\,d^2}{5\,e^7}\sqrt{-\left(\frac{d}{e}+x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^5-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))-24/5/e^6*d/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+6/5/e
^7*d^2/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-1/5*d^3/e^8/(d/e+x)^3
*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70304, size = 369, normalized size = 2.53

$$-\frac{24\,de^3x^3 + 72\,d^2e^2x^2 + 72\,d^3ex + 24\,d^4 - 30\left(de^3x^3 + 3\,d^2e^2x^2 + 3\,d^3ex + d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(5\,e^3x^3 + 39\,de^2x\right.}{5\left(e^8x^3 + 3\,de^7x^2 + 3\,d^2e^6x + d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*d*e^3*x^3 + 72*d^2*e^2*x^2 + 72*d^3*e*x + 24*d^4 - 30*(d*e^3*x^3 +
 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
 + (5*e^3*x^3 + 39*d*e^2*x^2 + 57*d^2*e*x + 24*d^3)*sqrt(-e^2*x^2 + d^2))/(
e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.181 $\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] (d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d - e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d - e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) + Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rubi [A] time = 0.261021, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1635, 778, 217, 203}

$$\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d - e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d - e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) + Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\left(-\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d-ex)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e}}\right.}{e^3}$$

$$= \frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Mathematica [A] time = 0.116699, size = 73, normalized size = 0.61

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(22d^2+51dex+32e^2x^2\right)}{(d+ex)^3}+15\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(22*d^2 + 51*d*e*x + 32*e^2*x^2))/(d + e*x)^3 + 15*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^4)
```

Maple [A] time = 0.065, size = 163, normalized size = 1.4

$$\frac{1}{e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + \frac{32}{15\,e^5}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1} - \frac{13\,d}{15\,e^6}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+32/15/e^5/(d/e
+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-13/15/e^6*d/(d/e+x)^2*(-(d/e+x)^2*
e^2+2*d*e*(d/e+x))^(1/2)+1/5*d^2/e^7/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x
))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57298, size = 336, normalized size = 2.8

$$\frac{22 e^3x^3 + 66 de^2x^2 + 66 d^2ex + 22 d^3 - 30 (e^3x^3 + 3 de^2x^2 + 3 d^2ex + d^3) \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (32 e^2x^2 + 51 dex + 22}{15 (e^7x^3 + 3 de^6x^2 + 3 d^2e^5x + d^3e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*e^3*x^3 + 66*d*e^2*x^2 + 66*d^2*e*x + 22*d^3 - 30*(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (32*e
^2*x^2 + 51*d*e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/(e^7*x^3 + 3*d*e^6*x^2 +
3*d^2*e^5*x + d^3*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.182 $\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

```
[Out] -(d*Sqrt[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3) + (8*Sqrt[d^2 - e^2*x^2])/(15*
e^3*(d + e*x)^2) - (7*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x))
```

Rubi [A] time = 0.128102, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1639, 793, 659, 651}

$$-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(d*Sqrt[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3) + (8*Sqrt[d^2 - e^2*x^2])/(15*
e^3*(d + e*x)^2) - (7*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x))
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{\int \frac{2d^2e^2+de^3x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{(7d)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e^2}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}+\frac{7\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15e^2}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0670098, size = 52, normalized size = 0.55

$$-\frac{\sqrt{d^2-e^2x^2}\left(2d^2+6dex+7e^2x^2\right)}{15de^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(2*d^2 + 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d + e*x)^3)
```

Maple [A] time = 0.05, size = 55, normalized size = 0.6

$$-\frac{(-ex+d)\left(7x^2e^2+6dex+2d^2\right)}{15e^3d(ex+d)^2}\frac{1}{\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(7*e^2*x^2+6*d*e*x+2*d^2)/(e*x+d)^2/d/e^3/(-e^2*x^2+d^2)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51837, size = 213, normalized size = 2.24

$$-\frac{2\,e^{3}x^{3}+6\,de^{2}x^{2}+6\,d^{2}ex+2\,d^{3}+\left(7\,e^{2}x^{2}+6\,dex+2\,d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\left(de^{6}x^{3}+3\,d^{2}e^{5}x^{2}+3\,d^{3}e^{4}x+d^{4}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*e^3*x^3 + 6*d*e^2*x^2 + 6*d^2*e*x + 2*d^3 + (7*e^2*x^2 + 6*d*e*x +
 2*d^2)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^
4*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.183 $\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*e^2*(d + e*x)^3) - Sqrt[d^2 - e^2*x^2]/(5*d*e^2*(d +
 e*x)^2) - Sqrt[d^2 - e^2*x^2]/(5*d^2*e^2*(d + e*x))
```

Rubi [A] time = 0.0446837, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*e^2*(d + e*x)^3) - Sqrt[d^2 - e^2*x^2]/(5*d*e^2*(d +
 e*x)^2) - Sqrt[d^2 - e^2*x^2]/(5*d^2*e^2*(d + e*x))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3} + \frac{3\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx}{5e}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3} - \frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2} + \frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{5de}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3} - \frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2} - \frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}$$

Mathematica [A] time = 0.0546501, size = 49, normalized size = 0.51

$$-\frac{\sqrt{d^2-e^2x^2}\left(d^2+3dex+e^2x^2\right)}{5d^2e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + e^2*x^2))/(5*d^2*e^2*(d + e*x)^3)
```

Maple [A] time = 0.048, size = 52, normalized size = 0.5

$$-\frac{(-ex+d)\left(x^2e^2+3\,dex+d^2\right)}{5\,e^2d^2\,(ex+d)^2}\frac{1}{\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*(-e*x+d)*(e^2*x^2+3*d*e*x+d^2)/(e*x+d)^2/d^2/e^2/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62409, size = 204, normalized size = 2.1

$$-\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3+\left(e^2x^2+3\,dex+d^2\right)\sqrt{-e^2x^2+d^2}}{5\left(d^2e^5x^3+3\,d^3e^4x^2+3\,d^4e^3x+d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 + (e^2*x^2 + 3*d*e*x + d^2)*s
qrt(-e^2*x^2 + d^2))/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.184 $\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*d*e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*
e*(d + e*x)^2) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))
```

Rubi [A] time = 0.0367133, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2]/(5*d*e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*
e*(d + e*x)^2) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}+\frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}+\frac{2\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}\end{aligned}$$

Mathematica [A] time = 0.0309776, size = 52, normalized size = 0.52

$$-\frac{\sqrt{d^2-e^2x^2}\left(7d^2+6dex+2e^2x^2\right)}{15d^3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]*(7*d^2 + 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d + e*x)^3)
```

Maple [A] time = 0.049, size = 55, normalized size = 0.6

$$-\frac{(-ex+d)\left(2x^2e^2+6dex+7d^2\right)}{15ed^3(ex+d)^2}\frac{1}{\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(2*e^2*x^2+6*d*e*x+7*d^2)/(e*x+d)^2/d^3/e/(-e^2*x^2+d^2)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5662, size = 216, normalized size = 2.16

$$-\frac{7e^3x^3+21de^2x^2+21d^2ex+7d^3+\left(2e^2x^2+6dex+7d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^3e^4x^3+3d^4e^3x^2+3d^5e^2x+d^6e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*e^3*x^3 + 21*d*e^2*x^2 + 21*d^2*e*x + 7*d^3 + (2*e^2*x^2 + 6*d*e*x
 + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + 3*d^4*e^3*x^2 + 3*d^5*e^2*x
+ d^6*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.185 $\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=115

$$\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}+\frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] (4*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d - 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d - 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rubi [A] time = 0.178962, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}+\frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d - 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d - 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= \int \frac{(d-ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+11d^2ex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2+22d^4e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2} \\
&= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4} \\
&= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3} \\
&= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^3} \\
&= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2}\right.}{d^3e^2} \\
&= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}
\end{aligned}
$$

Mathematica [A] time = 0.127278, size = 76, normalized size = 0.66

$$\frac{\frac{\sqrt{d^2-e^2x^2}(32d^2+51dex+22e^2x^2)}{(d+ex)^3}-15\log\left(\sqrt{d^2-e^2x^2}+d\right)+15\log(x)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(32*d^2 + 51*d*e*x + 22*e^2*x^2))/(d + e*x)^3 + 15*Lo
g[x] - 15*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^4)
```

Maple [A] time = 0.063, size = 179, normalized size = 1.6

$$-\frac{1}{d^3}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{22}{15ed^4}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1}+\frac{7}{15e^2d^3}\sqrt{-\left(\frac{d}{e}+x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+22/15/e
/d^4/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+7/15/e^2/d^3/(d/e+x)^2*(-
(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+1/5/e^3/d^2/(d/e+x)^3*(-(d/e+x)^2*e^2+2*
d*e*(d/e+x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x), x)
```

Fricas [A] time = 1.618, size = 327, normalized size = 2.84

$$\frac{32e^3x^3+96de^2x^2+96d^2ex+32d^3+15\left(e^3x^3+3de^2x^2+3d^2ex+d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(22e^2x^2+51dex+32d\right.}{15\left(d^4e^3x^3+3d^5e^2x^2+3d^6ex+d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*e^3*x^3 + 96*d*e^2*x^2 + 96*d^2*e*x + 32*d^3 + 15*(e^3*x^3 + 3*d*e
^2*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (22*e^2*x^2
+ 51*d*e*x + 32*d^2)*sqrt(-e^2*x^2 + d^2))/(d^4*e^3*x^3 + 3*d^5*e^2*x^2 + 3
```

```
*d^6*e*x + d^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.186 $\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] (-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d - 7*e*x))/(5*d^3*(d^
2 - e^2*x^2)^(3/2)) - (e*(15*d - 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqr
t[d^2 - e^2*x^2]/(d^5*x) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rubi [A] time = 0.304573, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d - 7*e*x))/(5*d^3*(d^
2 - e^2*x^2)^(3/2)) - (e*(15*d - 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqr
t[d^2 - e^2*x^2]/(d^5*x) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{(3e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}}{d^4}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{(3e)\,\mathrm{Subst}\left(\int \frac{}{x}\right.}{2}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-}\right.}{}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-}}{}\right.}{d^5}$$

Mathematica [A] time = 0.19203, size = 92, normalized size = 0.63

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(39d^2ex+5d^3+57de^2x^2+24e^3x^3\right)}{x(d+ex)^3} - 15e\log\left(\sqrt{d^2-e^2x^2}+d\right) + 15e\log(x)}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(5*d^3 + 39*d^2*e*x + 57*d*e^2*x^2 + 24*e^3*x^3))/(x
*(d + e*x)^3) + 15*e*Log[x] - 15*e*Log[d + Sqrt[d^2 - e^2*x^2]])/(5*d^5)
```

Maple [A] time = 0.069, size = 199, normalized size = 1.4

$$3\frac{e}{d^4\sqrt{d^2}}\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}}{x}\right)-\frac{19}{5d^5}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1}-\frac{4}{5d^4e}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] 3*e/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-19/5/d
^5/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-4/5/d^4/e/(d/e+x)^2*(-(d/e+
x)^2*e^2+2*d*e*(d/e+x))^(1/2)-1/5/e^2/d^3/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(
d/e+x))^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^2), x)
```

Fricas [A] time = 1.55484, size = 375, normalized size = 2.57

$$-\frac{24e^4x^4+72de^3x^3+72d^2e^2x^2+24d^3ex+15\left(e^4x^4+3de^3x^3+3d^2e^2x^2+d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(24e^3x^3+57\right.}{5\left(d^5e^3x^4+3d^6e^2x^3+3d^7ex^2+d^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*e^4*x^4 + 72*d*e^3*x^3 + 72*d^2*e^2*x^2 + 24*d^3*e*x + 15*(e^4*x^4
 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x
) + (24*e^3*x^3 + 57*d*e^2*x^2 + 39*d^2*e*x + 5*d^3)*sqrt(-e^2*x^2 + d^2))/
(d^5*e^3*x^4 + 3*d^6*e^2*x^3 + 3*d^7*e*x^2 + d^8*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{-(-d+ex)(d+ex)}(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A] time = 1.16536, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

3.187 $\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=183

$$\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

```
[Out] (4*e^2*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d - 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d - 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) + (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rubi [A] time = 0.380112, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```


```
[Out] (4*e^2*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d - 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d - 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) + (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-20de^2x^2+16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+75de^2x^2-62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45d^2ex-90de^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{-90d^4e+1}{x^2\sqrt{d^2-}}}{30}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{d^2-e}}{d^6x}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{d^2-e}}{d^6x}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{d^2-e}}{d^6x}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{3e\sqrt{d^2-e}}{d^6x}$$

Mathematica [A] time = 0.191481, size = 107, normalized size = 0.58

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(479d^2e^2x^2+45d^3ex-15d^4+717de^3x^3+304e^4x^4\right)}{x^2(d+ex)^3} - 195e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 195e^2\log(x)}{30d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 45*d^3*e*x + 479*d^2*e^2*x^2 + 717*d*e^3*x
^3 + 304*e^4*x^4))/(x^2*(d + e*x)^3) + 195*e^2*Log[x] - 195*e^2*Log[d + Sqr
t[d^2 - e^2*x^2]])/(30*d^6)
```

Maple [A] time = 0.066, size = 222, normalized size = 1.2

$$-\frac{13\,e^2}{2\,d^5}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} + \frac{107\,e}{15\,d^6}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}\left(\frac{d}{e}+x\right)^{-1} + \frac{17}{15\,d^5}\sqrt{-\left(\frac{d}{e}+x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -13/2/d^5*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+
107/15*e/d^6/(d/e+x)*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+17/15/d^5/(d/e+x)
^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2+1/
5/d^4/e/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+3*e*(-e^2*x^2+d^2)^(
1/2)/d^6/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x^3), x)
```

Fricas [A] time = 1.82406, size = 424, normalized size = 2.32

$$\frac{254\,e^5x^5+762\,de^4x^4+762\,d^2e^3x^3+254\,d^3e^2x^2+195\left(e^5x^5+3\,de^4x^4+3\,d^2e^3x^3+d^3e^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(304\,e^4x\right.}{30\left(d^6e^3x^5+3\,d^7e^2x^4+3\,d^8ex^3+d^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*e^5*x^5 + 762*d*e^4*x^4 + 762*d^2*e^3*x^3 + 254*d^3*e^2*x^2 + 195
*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (304*e^4*x^4 + 717*d*e^3*x^3 + 479*d^2*e^2*x^2 + 45*d^3*e*
x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^5 + 3*d^7*e^2*x^4 + 3*d^8*e*x^
3 + d^9*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A] time = 1.16422, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] +Infinity
```

3.188 $\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=204

$$\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}+\frac{18d^3\tan^{-1}}{}$$

```
[Out] (d^4*(d - e*x)^4)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (8*d^3*(d - e*x)^3)/(5*e^
6*(d^2 - e^2*x^2)^(3/2)) + (10*d^2*(d - e*x)^2)/(e^6*Sqrt[d^2 - e^2*x^2]) +
 (59*d^2*Sqrt[d^2 - e^2*x^2])/(3*e^6) - (2*d*x*Sqrt[d^2 - e^2*x^2])/e^5 + (
x^2*Sqrt[d^2 - e^2*x^2])/(3*e^4) + (18*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^6
```

Rubi [A] time = 0.593206, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}+\frac{18d^3\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (8*d^3*(d - e*x)^3)/(5*e^
6*(d^2 - e^2*x^2)^(3/2)) + (10*d^2*(d - e*x)^2)/(e^6*Sqrt[d^2 - e^2*x^2]) +
 (59*d^2*Sqrt[d^2 - e^2*x^2])/(3*e^6) - (2*d*x*Sqrt[d^2 - e^2*x^2])/e^5 + (
x^2*Sqrt[d^2 - e^2*x^2])/(3*e^4) + (18*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^6
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
```

```
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^5(d-ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(-\frac{60d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)\left(-\frac{240d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3x^2}{e^3}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{\int \frac{\frac{720d^6}{e^3}-\frac{885d^5x}{e^2}+\frac{180d^4x^2}{e}}{\sqrt{d^2-e^2x^2}}}{45d^3e^2}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} -$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} +$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} +$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} +$$

Mathematica [A] time = 0.2061, size = 109, normalized size = 0.53

$$\frac{\frac{\sqrt{d^2-e^2x^2}(674d^3e^2x^2+70d^2e^3x^3+1002d^4ex+424d^5-15de^4x^4+5e^5x^5)}{(d+ex)^3}+270d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(424*d^5 + 1002*d^4*e*x + 674*d^3*e^2*x^2 + 70*d^2*e^
3*x^3 - 15*d*e^4*x^4 + 5*e^5*x^5))/(d + e*x)^3 + 270*d^3*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(15*e^6)
```

Maple [A] time = 0.069, size = 297, normalized size = 1.5

$$-\frac{1}{3e^6}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-2\frac{dx\sqrt{-x^2e^2+d^2}}{e^5}-2\frac{d^3}{e^5\sqrt{e^2}}\arctan\left(\frac{\sqrt{e^2}x}{\sqrt{-x^2e^2+d^2}}\right)+\frac{d^4}{5e^{10}}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/3/e^6*(-e^2*x^2+d^2)^(3/2)-2*d*x*(-e^2*x^2+d^2)^(1/2)/e^5-2/e^5*d^3/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/5*d^4/e^10/(d/e+x)^4*(
-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-8/5/e^9*d^3/(d/e+x)^3*(-(d/e+x)^2*e^2+2
*d*e*(d/e+x))^(3/2)+20/e^6*d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+20/e^5*
d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))+
10/e^8*d^2/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89485, size = 435, normalized size = 2.13

$$\frac{424\,d^3e^3x^3+1272\,d^4e^2x^2+1272\,d^5ex+424\,d^6-540\left(d^3e^3x^3+3\,d^4e^2x^2+3\,d^5ex+d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(5\,e^5x^5\right.}{15\left(e^9x^3+3\,de^8x^2+3\,d^2e^7x+d^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(424*d^3*e^3*x^3 + 1272*d^4*e^2*x^2 + 1272*d^5*e*x + 424*d^6 - 540*(d^
3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^2 + d
^2))/(e*x)) + (5*e^5*x^5 - 15*d*e^4*x^4 + 70*d^2*e^3*x^3 + 674*d^3*e^2*x^2
+ 1002*d^4*e*x + 424*d^5)*sqrt(-e^2*x^2 + d^2))/(e^9*x^3 + 3*d*e^8*x^2 + 3*
d^2*e^7*x + d^3*e^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.189 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=160

$$-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{19d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

```
[Out] -(d^3*(d - e*x)^4)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (19*d^2*(d - e*x)^3)/(15
*e^5*(d^2 - e^2*x^2)^(3/2)) - (6*d*(d - e*x)^2)/(e^5*Sqrt[d^2 - e^2*x^2]) -
 ((20*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (19*d^2*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(2*e^5)
```

Rubi [A] time = 0.415125, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1635, 780, 217, 203}

$$-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{19d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -(d^3*(d - e*x)^4)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) + (19*d^2*(d - e*x)^3)/(15
*e^5*(d^2 - e^2*x^2)^(3/2)) - (6*d*(d - e*x)^2)/(e^5*Sqrt[d^2 - e^2*x^2]) -
 ((20*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (19*d^2*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(2*e^5)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^4(d-ex)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(\frac{45d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{135d^4}{e^4}-\frac{15d^3x}{e^3}\right)(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{\left(19d^2\right)\int \frac{}{\sqrt{}}}{2e}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{\left(19d^2\right)\text{Sub}}{}$$

$$= -\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5} - \frac{19d^2\tan^{-1}}{2e}$$

Mathematica [A] time = 0.203246, size = 98, normalized size = 0.61

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(713d^2e^2x^2+1059d^3ex+448d^4+75de^3x^3-15e^4x^4\right)}{(d+ex)^3}+285d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{30e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(448*d^4 + 1059*d^3*e*x + 713*d^2*e^2*x^2 + 75*d*e^3
*x^3 - 15*e^4*x^4))/(d + e*x)^3 + 285*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]
)/(30*e^5)
```

Maple [A] time = 0.1, size = 273, normalized size = 1.7

$$\frac{x}{2e^4}\sqrt{-x^2e^2+d^2}+\frac{d^2}{2e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-x^2e^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-\frac{d^3}{5e^9}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+x\right)^{-4}+\frac{19d^2}{15e^8}\left(-\left(\frac{d}{e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] 1/2/e^4*x*(-e^2*x^2+d^2)^(1/2)+1/2/e^4*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2))-1/5*d^3/e^9/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))
^(3/2)+19/15/e^8*d^2/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-10/e^5*
d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-10/e^4*d^2/(e^2)^(1/2)*arctan((e^2)^
(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-6/e^7*d/(d/e+x)^2*(-(d/e+x)^2
*e^2+2*d*e*(d/e+x))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75078, size = 414, normalized size = 2.59

$$-\frac{448d^2e^3x^3+1344d^3e^2x^2+1344d^4ex+448d^5-570\left(d^2e^3x^3+3d^3e^2x^2+3d^4ex+d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(15e^4x\right.}{30\left(e^8x^3+3de^7x^2+3d^2e^6x+d^3e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/30*(448*d^2*e^3*x^3 + 1344*d^3*e^2*x^2 + 1344*d^4*e*x + 448*d^5 - 570*(d
^2*e^3*x^3 + 3*d^3*e^2*x^2 + 3*d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) - (15*e^4*x^4 - 75*d*e^3*x^3 - 713*d^2*e^2*x^2 - 1059*d^3*e*x
- 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3
*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.190 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$

Optimal. Leaf size=148

$$\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}+\frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] (8*d*Sqrt[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (d^2*(d^2 - e^2*x^2)^(3/2))/(5*
e^4*(d + e*x)^4) - (14*d*(d^2 - e^2*x^2)^(3/2))/(15*e^4*(d + e*x)^3) - (d^2
 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2) + (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^4
```

Rubi [A] time = 0.246061, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1639, 1637, 659, 651, 663, 217, 203}

$$\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}+\frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (8*d*Sqrt[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (d^2*(d^2 - e^2*x^2)^(3/2))/(5*
e^4*(d + e*x)^4) - (14*d*(d^2 - e^2*x^2)^(3/2))/(15*e^4*(d + e*x)^3) - (d^2
 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2) + (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^4
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{\int \frac{\sqrt{d^2-e^2x^2}\left(2d^3e^2+5d^2e^3x+4de^4x^2\right)}{(d+ex)^4}\,dx}{e^5}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{\int \left(\frac{d^3e^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}-\frac{3d^2e^2\sqrt{d^2-e^2x^2}}{(d+ex)^3}+\frac{4de^2\sqrt{d^2-e^2x^2}}{(d+ex)^2}\right)dx}{e^5}\\
&= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}-\frac{(4d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^3}+\frac{\left(3d^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^3}-\frac{d^3\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^3}\\
&= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{(4d)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}-\frac{d^2}{}\\
&= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{(4d)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\right.}{e^3}\\
&= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2\left(d^2-e^2x^2\right)^{3/2}}{5e^4(d+ex)^4}-\frac{14d\left(d^2-e^2x^2\right)^{3/2}}{15e^4(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}
\end{aligned}$$

Mathematica [A] time = 0.144645, size = 85, normalized size = 0.57

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(222d^2ex+94d^3+149de^2x^2+15e^3x^3\right)}{(d+ex)^3}+60d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(94*d^3 + 222*d^2*e*x + 149*d*e^2*x^2 + 15*e^3*x^3))/
(d + e*x)^3 + 60*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^4)
```

Maple [A] time = 0.063, size = 212, normalized size = 1.4

$$\frac{d^2}{5e^8}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+x\right)^{-4}-\frac{14d}{15e^7}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+x\right)^{-3}+4\frac{1}{e^4}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] 1/5*d^2/e^8/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-14/15*d/e^7/(d/e
+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)+4/e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(1/2)+4/e^3*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(
d/e+x))^(1/2))+3/e^6/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78005, size = 377, normalized size = 2.55

$$\frac{94\,de^3x^3+282\,d^2e^2x^2+282\,d^3ex+94\,d^4-120\left(de^3x^3+3\,d^2e^2x^2+3\,d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(15\,e^3x^3+149\right.}{15\left(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(94*d*e^3*x^3 + 282*d^2*e^2*x^2 + 282*d^3*e*x + 94*d^4 - 120*(d*e^3*x^
3 + 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*
x)) + (15*e^3*x^3 + 149*d*e^2*x^2 + 222*d^2*e*x + 94*d^3)*sqrt(-e^2*x^2 + d
^2))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.191 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=115

$$\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^3
*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]]/e^3
```

Rubi [A] time = 0.146687, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1637, 659, 651, 663, 217, 203}

$$\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^3
*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]]/e^3
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
```

```
2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx &= \int \left(\frac{d^2\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4}-\frac{2d\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3}+\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2}\right)dx\\
&= \frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2}-\frac{(2d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2}+\frac{d^2\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}+\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e^3(d+ex)^3}-\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}+\frac{d\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}+\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^4}+\frac{3\left(d^2-e^2x^2\right)^{3/2}}{5e^3(d+ex)^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.129219, size = 73, normalized size = 0.63

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(8d^2+19dex+13e^2x^2\right)}{(d+ex)^3}+5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(8*d^2 + 19*d*e*x + 13*e^2*x^2))/(d + e*x)^3 + 5*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(5*e^3)
```

Maple [B] time = 0.066, size = 214, normalized size = 1.9

$$-\frac{d}{5e^7}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+x\right)^{-4}+\frac{3}{5e^6}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+x\right)^{-3}-\frac{1}{e^5d}\left(-\left(\frac{d}{e}+x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/5*d/e^7/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)+3/5/e^6/(d/e+x)^3
*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-1/e^5/d/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d
*e*(d/e+x))^(3/2)-1/e^3/d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)-1/e^2/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77875, size = 332, normalized size = 2.89

$$-\frac{8\,e^3x^3+24\,de^2x^2+24\,d^2ex+8\,d^3-10\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(13\,e^2x^2+19\,dex+8\,d^2\right.}{5\left(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/5*(8*e^3*x^3 + 24*d*e^2*x^2 + 24*d^2*e*x + 8*d^3 - 10*(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (13*e^2
*x^2 + 19*d*e*x + 8*d^2)*sqrt(-e^2*x^2 + d^2))/(e^6*x^3 + 3*d*e^5*x^2 + 3*d
^2*e^4*x + d^3*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(-d+ex)\,(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.192 $\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$

Optimal. Leaf size=64

$$\frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2-e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

```
[Out] (d^2 - e^2*x^2)^(3/2)/(5*e^2*(d + e*x)^4) - (4*(d^2 - e^2*x^2)^(3/2))/(15*d
*e^2*(d + e*x)^3)
```

Rubi [A] time = 0.0274207, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {793, 651}

$$\frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2-e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(3/2)/(5*e^2*(d + e*x)^4) - (4*(d^2 - e^2*x^2)^(3/2))/(15*d
*e^2*(d + e*x)^3)
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned} \int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\, dx &= \frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4} + \frac{4\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\, dx}{5e} \\ &= \frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2-e^2x^2)^{3/2}}{15de^2(d+ex)^3} \end{aligned}$$

Mathematica [A] time = 0.0509858, size = 50, normalized size = 0.78

$$-\frac{(d^2+3dex-4e^2x^2)\sqrt{d^2-e^2x^2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -((d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d*e^2*(d + e*x)^3)
```

Maple [A] time = 0.046, size = 42, normalized size = 0.7

$$-\frac{(4ex+d)(-ex+d)}{15(ex+d)^3de^2}\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/15*(-e*x+d)*(4*e*x+d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d/e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62987, size = 205, normalized size = 3.2

$$-\frac{e^3x^3+3de^2x^2+3d^2ex+d^3-\left(4e^2x^2-3dex-d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(de^5x^3+3d^2e^4x^2+3d^3e^3x+d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3 - (4*e^2*x^2 - 3*d*e*x - d^2
)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.193 $\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$

Optimal. Leaf size=67

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}$$

```
[Out] -(d^2 - e^2*x^2)^(3/2)/(5*d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d^2*
e*(d + e*x)^3)
```

Rubi [A] time = 0.0236743, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {659, 651}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] -(d^2 - e^2*x^2)^(3/2)/(5*d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d^2*
e*(d + e*x)^3)
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}+\frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3} dx}{5d}\\ &= -\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}\end{aligned}$$

Mathematica [A] time = 0.0297135, size = 51, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(-4d^2+3dex+e^2x^2\right)}{15d^2e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^2 + 3*d*e*x + e^2*x^2))/(15*d^2*e*(d + e*x)^3)
```

Maple [A] time = 0.045, size = 43, normalized size = 0.6

$$-\frac{(ex+4\,d)\,(-ex+d)}{15\,(ex+d)^3\,d^2e}\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d^2/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69081, size = 213, normalized size = 3.18

$$-\frac{4\,e^3x^3+12\,de^2x^2+12\,d^2ex+4\,d^3-\left(e^2x^2+3\,dex-4\,d^2\right)\sqrt{-e^2x^2+d^2}}{15\left(d^2e^4x^3+3\,d^3e^3x^2+3\,d^4e^2x+d^5e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(4*e^3*x^3 + 12*d*e^2*x^2 + 12*d^2*e*x + 4*d^3 - (e^2*x^2 + 3*d*e*x -
 4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e^2*x +
d^5*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.194 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx$

Optimal. Leaf size=110

$$-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

```
[Out] (8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*x)/(5*d*(d^2 - e^2*x^2)^(3
/2)) + (5*d - 8*e*x)/(5*d^3*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x
^2]/d]/d^3
```

Rubi [A] time = 0.219763, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*x)/(5*d*(d^2 - e^2*x^2)^(3
/2)) + (5*d - 8*e*x)/(5*d^3*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x
^2]/d]/d^3
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+12d^3ex+5d^2e^2x^2}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-24d^3ex}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{15d^6e^2}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^8e^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2d^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{d^2e^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Mathematica [A] time = 0.148624, size = 76, normalized size = 0.69

$$\frac{\frac{\sqrt{d^2-e^2x^2}(13d^2+19dex+8e^2x^2)}{(d+ex)^3}-5\log\left(\sqrt{d^2-e^2x^2}+d\right)+5\log(x)}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(13*d^2 + 19*d*e*x + 8*e^2*x^2))/(d + e*x)^3 + 5*Log[
x] - 5*Log[d + Sqrt[d^2 - e^2*x^2]])/(5*d^3)
```

Maple [B] time = 0.067, size = 196, normalized size = 1.8

$$\frac{1}{d^4}\sqrt{-x^2e^2+d^2}-\frac{1}{d^2}\ln\left(\frac{1}{x}\left(2\,d^2+2\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{1}{5\,e^4d^2}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}\left(\frac{d}{e}+x\right)^{-4}+\frac{}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x)
```

```
[Out] 1/d^4*(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x)+1/5/e^4/d^2/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/
2)+2/5/e^3/d^3/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)+1/e^2/d^4/(d/
e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x), x)
```

Fricas [A] time = 1.75794, size = 323, normalized size = 2.94

$$\frac{13\,e^3x^3+39\,de^2x^2+39\,d^2ex+13\,d^3+5\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(8\,e^2x^2+19\,dex+13\,d^2\right)}{5\left(d^3e^3x^3+3\,d^4e^2x^2+3\,d^5ex+d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(13*e^3*x^3 + 39*d*e^2*x^2 + 39*d^2*e*x + 13*d^3 + 5*(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^2*x^2 + 1
```

```
9*d*e*x + 13*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^3*x^3 + 3*d^4*e^2*x^2 + 3*d^
5*e*x + d^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.195 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx$

Optimal. Leaf size=143

$$-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] (-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*(5*d - 8*e*x))/(15*d^2*(d
^2 - e^2*x^2)^(3/2)) - (e*(60*d - 79*e*x))/(15*d^4*Sqrt[d^2 - e^2*x^2]) - S
qrt[d^2 - e^2*x^2]/(d^4*x) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4
```

Rubi [A] time = 0.305805, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*(5*d - 8*e*x))/(15*d^2*(d
^2 - e^2*x^2)^(3/2)) - (e*(60*d - 79*e*x))/(15*d^4*Sqrt[d^2 - e^2*x^2]) - S
qrt[d^2 - e^2*x^2]/(d^4*x) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-27d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+64d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(4e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(2e)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx\right.}{d^3}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4\operatorname{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{\right.}{d^3e}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [A] time = 0.226226, size = 92, normalized size = 0.64

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(149d^2ex+15d^3+222de^2x^2+94e^3x^3\right)}{x(d+ex)^3} - 60e\log\left(\sqrt{d^2-e^2x^2}+d\right) + 60e\log(x)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(15*d^3 + 149*d^2*e*x + 222*d*e^2*x^2 + 94*e^3*x^3))
/(x*(d + e*x)^3) + 60*e*Log[x] - 60*e*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^4
)
```

Maple [B] time = 0.068, size = 361, normalized size = 2.5

$$-4\,\frac{e\sqrt{-x^2e^2+d^2}}{d^5}+4\,\frac{e}{d^3\sqrt{d^2}}\ln\left(\frac{2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}}{x}\right)+\frac{e}{d^5}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)}+\frac{e^2}{d^4}\arctan\left(x\sqrt{e^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x)
```

```
[Out] -4/d^5*e*(-e^2*x^2+d^2)^(1/2)+4/d^3*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x)+1/d^5*e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+1/d^4*
e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-
1/5/e^3/d^3/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-11/15/e^2/d^4/(d
/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-3/d^5/e/(d/e+x)^2*(-(d/e+x)^2*
e^2+2*d*e*(d/e+x))^(3/2)-1/d^6/x*(-e^2*x^2+d^2)^(3/2)-1/d^6*e^2*x*(-e^2*x^2
+d^2)^(1/2)-1/d^4*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^2), x)
```

Fricas [A] time = 1.71911, size = 386, normalized size = 2.7

$$-\frac{104\,e^4x^4+312\,de^3x^3+312\,d^2e^2x^2+104\,d^3ex+60\left(e^4x^4+3\,de^3x^3+3\,d^2e^2x^2+d^3ex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(94\,e^3x^3\right.}{15\left(d^4e^3x^4+3\,d^5e^2x^3+3\,d^6ex^2+d^7x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(104*e^4*x^4 + 312*d*e^3*x^3 + 312*d^2*e^2*x^2 + 104*d^3*e*x + 60*(e^
4*x^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + d^3*e*x)*log(-(d - sqrt(-e^2*x^2 + d^
2))/x) + (94*e^3*x^3 + 222*d*e^2*x^2 + 149*d^2*e*x + 15*d^3)*sqrt(-e^2*x^2
```

```
+ d^2))/(d^4*e^3*x^4 + 3*d^5*e^2*x^3 + 3*d^6*e*x^2 + d^7*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^2(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)**4), x)
```

Giac [A] time = 1.1692, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

3.196 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=183

$$\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}-\frac{19e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}$$

```
[Out] (8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (4*e^2*(10*d - 13*e*x))/(15
*d^3*(d^2 - e^2*x^2)^(3/2)) + (e^2*(135*d - 164*e*x))/(15*d^5*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^4*x^2) + (4*e*Sqrt[d^2 - e^2*x^2])/(d^5*
x) - (19*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^5)
```

Rubi [A] time = 0.391562, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}-\frac{19e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```


```
[Out] (8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (4*e^2*(10*d - 13*e*x))/(15
*d^3*(d^2 - e^2*x^2)^(3/2)) + (e^2*(135*d - 164*e*x))/(15*d^5*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^4*x^2) + (4*e*Sqrt[d^2 - e^2*x^2])/(d^5*
x) - (19*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^5)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+32de^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-104de^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}dx}{15d^4}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}dx}{15d^6}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{\int \frac{-120d^5e+285d^4e^2x}{x^2\sqrt{d^2-e^2x^2}}dx}{30d^8}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(19e}{}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(19e}{}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} - \frac{19\,\mathrm{S}}{}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} - \frac{19e^2}{}$$

Mathematica [A] time = 0.242391, size = 107, normalized size = 0.58

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(713d^2e^2x^2+75d^3ex-15d^4+1059de^3x^3+448e^4x^4\right)}{x^2(d+ex)^3} - 285e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 285e^2\log(x)}{30d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 75*d^3*e*x + 713*d^2*e^2*x^2 + 1059*d*e^3*
x^3 + 448*e^4*x^4))/(x^2*(d + e*x)^3) + 285*e^2*Log[x] - 285*e^2*Log[d + Sq
rt[d^2 - e^2*x^2]])/(30*d^5)
```

Maple [B] time = 0.073, size = 389, normalized size = 2.1

$$\frac{19\,e^2}{2\,d^6}\sqrt{-x^2e^2+d^2} - \frac{19\,e^2}{2\,d^4}\ln\left(\frac{1}{x}\left(2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - 4\,\frac{e^2}{d^6}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)} - 4\,\frac{e^3}{d^5\sqrt{e^2}}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x)
```

```
[Out] 19/2/d^6*e^2*(-e^2*x^2+d^2)^(1/2)-19/2/d^4*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2
)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-4/d^6*e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
1/2)-4/d^5*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+
x))^(1/2))+1/5/e^2/d^4/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)+16/15
/e/d^5/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)+6/d^6/(d/e+x)^2*(-(d/
e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)-1/2/d^6/x^2*(-e^2*x^2+d^2)^(3/2)+4/d^7*e/x*
(-e^2*x^2+d^2)^(3/2)+4/d^7*e^3*x*(-e^2*x^2+d^2)^(1/2)+4/d^5*e^3/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)^4 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^3), x)
```

Fricas [A] time = 1.65852, size = 428, normalized size = 2.34

$$\frac{398\, e^5x^5 + 1194\, de^4x^4 + 1194\, d^2e^3x^3 + 398\, d^3e^2x^2 + 285 \left(e^5x^5 + 3\, de^4x^4 + 3\, d^2e^3x^3 + d^3e^2x^2\right) \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + \left(448\, e\right.}{30 \left(d^5e^3x^5 + 3\, d^6e^2x^4 + 3\, d^7ex^3 + d^8x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/30*(398*e^5*x^5 + 1194*d*e^4*x^4 + 1194*d^2*e^3*x^3 + 398*d^3*e^2*x^2 + 2
85*(e^5*x^5 + 3*d*e^4*x^4 + 3*d^2*e^3*x^3 + d^3*e^2*x^2)*log(-(d - sqrt(-e^
2*x^2 + d^2))/x) + (448*e^4*x^4 + 1059*d*e^3*x^3 + 713*d^2*e^2*x^2 + 75*d^3
*e*x - 15*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^3*x^5 + 3*d^6*e^2*x^4 + 3*d^7*e
*x^3 + d^8*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^3 (d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)**4), x)
```

Giac [A] time = 1.19897, size = 1, normalized size = 0.01

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

3.197 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=210

$$-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{18e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] (-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (4*e^3*(5*d - 6*e*x))/(5
*d^4*(d^2 - e^2*x^2)^(3/2)) - (e^3*(80*d - 93*e*x))/(5*d^6*Sqrt[d^2 - e^2*x
^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^4*x^3) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^5*x^2
) - (29*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^6*x) + (18*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^6
```

Rubi [A] time = 0.494123, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{18e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```


```
[Out] (-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (4*e^3*(5*d - 6*e*x))/(5
*d^4*(d^2 - e^2*x^2)^(3/2)) - (e^3*(80*d - 93*e*x))/(5*d^6*Sqrt[d^2 - e^2*x
^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^4*x^3) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^5*x^2
) - (29*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^6*x) + (18*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^6
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx=\int \frac{(d-ex)^4}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+40de^3x^3-32e^4x^4}{x^4\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-180de^3x^3+144e^4x^4}{x^4\left(d^2-e^2x^2\right)^{3/2}}dx}{15d^4}$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x^2+240de^3x^3}{x^4\sqrt{d^2-e^2x^2}}dx}{15d^6}$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{\int \frac{-180d^5e+435d^4e^2x-720d^3e^3x^2}{x^3\sqrt{d^2-e^2x^2}}}{45d^8}$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\int \frac{-870d^6e}{x^2\sqrt{}}}{9}$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{29e^2\sqrt{d^2}}{3d^6}$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{29e^2\sqrt{d^2}}{3d^6}$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{29e^2\sqrt{d^2}}{3d^6}$$

$$=-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{29e^2\sqrt{d^2}}{3d^6}$$

Mathematica [A] time = 0.297746, size = 118, normalized size = 0.56

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(70d^3e^2x^2+674d^2e^3x^3-15d^4ex+5d^5+1002de^4x^4+424e^5x^5\right)}{x^3(d+ex)^3}-270e^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+270e^3\log(x)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 15*d^4*e*x + 70*d^3*e^2*x^2 + 674*d^2*e^3*x
^3 + 1002*d*e^4*x^4 + 424*e^5*x^5))/(x^3*(d + e*x)^3) + 270*e^3*Log[x] - 27
0*e^3*Log[d + Sqrt[d^2 - e^2*x^2]])/(15*d^6)
```

Maple [B] time = 0.076, size = 412, normalized size = 2.

$$-18\,\frac{e^3\sqrt{-x^2e^2+d^2}}{d^7}+18\,\frac{e^3}{d^5\sqrt{d^2}}\ln\left(\frac{2\,d^2+2\,\sqrt{d^2}\sqrt{-x^2e^2+d^2}}{x}\right)-\frac{1}{3\,d^6x^3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}+10\,\frac{e^3}{d^7}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x)
```

```
[Out] -18/d^7*e^3*(-e^2*x^2+d^2)^(1/2)+18/d^5*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(
1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/3/d^6/x^3*(-e^2*x^2+d^2)^(3/2)+10/d^7*e^3*(
-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)+10/d^6*e^4/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/5/d^5/e/(d/e+x)^4*(-(d/e+x)^2*
e^2+2*d*e*(d/e+x))^(3/2)-7/5/d^6/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
3/2)-10/d^7*e/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)+2/d^7*e/x^2*(-
e^2*x^2+d^2)^(3/2)-10/d^8*e^2/x*(-e^2*x^2+d^2)^(3/2)-10/d^8*e^4*x*(-e^2*x^2
+d^2)^(1/2)-10/d^6*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)^4 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x^4), x)
```

Fricas [A] time = 1.92596, size = 448, normalized size = 2.13

$$-\frac{324\, e^6x^6 + 972\, de^5x^5 + 972\, d^2e^4x^4 + 324\, d^3e^3x^3 + 270\, \left(e^6x^6 + 3\, de^5x^5 + 3\, d^2e^4x^4 + d^3e^3x^3\right) \log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) + (42}{15\, \left(d^6e^3x^6 + 3\, d^7e^2x^5 + 3\, d^8ex^4 + d^9x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(324*e^6*x^6 + 972*d*e^5*x^5 + 972*d^2*e^4*x^4 + 324*d^3*e^3*x^3 + 27
0*(e^6*x^6 + 3*d*e^5*x^5 + 3*d^2*e^4*x^4 + d^3*e^3*x^3)*log(-(d - sqrt(-e^2
*x^2 + d^2))/x) + (424*e^5*x^5 + 1002*d*e^4*x^4 + 674*d^2*e^3*x^3 + 70*d^3*
e^2*x^2 - 15*d^4*e*x + 5*d^5)*sqrt(-e^2*x^2 + d^2))/(d^6*e^3*x^6 + 3*d^7*e^
2*x^5 + 3*d^8*e*x^4 + d^9*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^4 (d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)**4), x)
```

Giac [A] time = 1.24081, size = 1, normalized size = 0.

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] +Infinity
```

3.198 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=252

$$\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}+\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}$$

```
[Out] (d^4*(d - e*x)^4)/(e^6*Sqrt[d^2 - e^2*x^2]) + (515*d^6*Sqrt[d^2 - e^2*x^2])
/(21*e^6) - (49*d^5*x*Sqrt[d^2 - e^2*x^2])/(4*e^5) + (121*d^4*x^2*Sqrt[d^2
- e^2*x^2])/(21*e^4) - (17*d^3*x^3*Sqrt[d^2 - e^2*x^2])/(6*e^3) + (11*d^2*x
^4*Sqrt[d^2 - e^2*x^2])/(7*e^2) - (2*d*x^5*Sqrt[d^2 - e^2*x^2])/(3*e) + (x^
6*Sqrt[d^2 - e^2*x^2])/7 + (65*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^
6)
```

Rubi [A] time = 0.662574, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}+\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(e^6*Sqrt[d^2 - e^2*x^2]) + (515*d^6*Sqrt[d^2 - e^2*x^2])
/(21*e^6) - (49*d^5*x*Sqrt[d^2 - e^2*x^2])/(4*e^5) + (121*d^4*x^2*Sqrt[d^2
- e^2*x^2])/(21*e^4) - (17*d^3*x^3*Sqrt[d^2 - e^2*x^2])/(6*e^3) + (11*d^2*x
^4*Sqrt[d^2 - e^2*x^2])/(7*e^2) - (2*d*x^5*Sqrt[d^2 - e^2*x^2])/(3*e) + (x^
6*Sqrt[d^2 - e^2*x^2])/7 + (65*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^
6)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
```

```
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5 \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^5(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{d^4x}{e^4}-\frac{d^3x^2}{e^3}+\frac{d^2x^3}{e^2}-\frac{dx^4}{e}\right)}{\sqrt{d^2-e^2x^2}}dx}{d}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{28d^8}{e^3}-\frac{91d^7x}{e^2}+\frac{112d^6x^2}{e}-77d^5x^3+56d^4ex^4-55d^3e^2x^5+28d^2e^3x^6}{\sqrt{d^2-e^2x^2}}dx}{7de^2}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} - \frac{\int \frac{-\frac{168d^8}{e}+546d^7x-672d^6ex^2+462d^5e^2x^3-476d^4e^3x}{\sqrt{d^2-e^2x^2}}}{42de^4}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{\int 840d^8e-2730d^7e^2x+}{}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-}}{3e}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-}}{7e^2}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-}}{6e^3}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-}}{6e^3}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-}}{6e^3}$$

Mathematica [A] time = 0.263833, size = 131, normalized size = 0.52

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-293d^5e^2x^2+162d^4e^3x^3-106d^3e^4x^4+76d^2e^5x^5+779d^6ex+2144d^7-44de^6x^6+12e^7x^7\right)}{d+ex}+1365d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{84e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(2144*d^7 + 779*d^6*e*x - 293*d^5*e^2*x^2 + 162*d^4*e
^3*x^3 - 106*d^3*e^4*x^4 + 76*d^2*e^5*x^5 - 44*d*e^6*x^6 + 12*e^7*x^7))/(d
+ e*x) + 1365*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(84*e^6)
```

Maple [A] time = 0.073, size = 416, normalized size = 1.7

$$-\frac{1}{7e^6}\left(-x^2e^2+d^2\right)^{\frac{7}{2}} + \frac{28d^2}{3e^6}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{5}{2}} + \frac{35d^3x}{3e^5}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}} + \frac{35d^5x}{2e^5}\sqrt{-(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -1/7/e^6*(-e^2*x^2+d^2)^(7/2)+28/3/e^6*d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
5/2)+35/3/e^5*d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+35/2/e^5*d^5*(-(d/
e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+35/2/e^5*d^7/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-2/3/e^5*d*x*(-e^2*x^2+d^2)^(5/2)
-5/6/e^5*d^3*x*(-e^2*x^2+d^2)^(3/2)-5/4*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^5-5/4/
e^5*d^7/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d^4/e^10/(d/
e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+8*d^3/e^9/(d/e+x)^3*(-(d/e+x)^2
*e^2+2*d*e*(d/e+x))^(7/2)+22/3*d^2/e^8/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e
+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66641, size = 351, normalized size = 1.39

$$\frac{2144\,d^7ex + 2144\,d^8 - 2730\left(d^7ex + d^8\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \left(12\,e^7x^7 - 44\,de^6x^6 + 76\,d^2e^5x^5 - 106\,d^3e^4x^4 + 162\,d^4e^3}{84\left(e^7x + de^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/84*(2144*d^7*e*x + 2144*d^8 - 2730*(d^7*e*x + d^8)*arctan(-(d - sqrt(-e^2
*x^2 + d^2))/(e*x)) + (12*e^7*x^7 - 44*d*e^6*x^6 + 76*d^2*e^5*x^5 - 106*d^3
*e^4*x^4 + 162*d^4*e^3*x^3 - 293*d^5*e^2*x^2 + 779*d^6*e*x + 2144*d^7)*sqrt
(-e^2*x^2 + d^2))/(e^7*x + d*e^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.199 $\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=224

$$-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4}-\frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{4dx^4\sqrt{d^2-e^2x^2}}{5e}+$$

```
[Out] -((d^3*(d - e*x)^4)/(e^5*Sqrt[d^2 - e^2*x^2])) - (337*d^5*Sqrt[d^2 - e^2*x^
2])/(15*e^5) + (175*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^4) - (71*d^3*x^2*Sqrt[
d^2 - e^2*x^2])/(15*e^3) + (47*d^2*x^3*Sqrt[d^2 - e^2*x^2])/(24*e^2) - (4*d
*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (x^5*Sqrt[d^2 - e^2*x^2])/6 - (239*d^6*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rubi [A] time = 0.53395, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4}-\frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{4dx^4\sqrt{d^2-e^2x^2}}{5e}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d^3*(d - e*x)^4)/(e^5*Sqrt[d^2 - e^2*x^2])) - (337*d^5*Sqrt[d^2 - e^2*x^
2])/(15*e^5) + (175*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^4) - (71*d^3*x^2*Sqrt[
d^2 - e^2*x^2])/(15*e^3) + (47*d^2*x^3*Sqrt[d^2 - e^2*x^2])/(24*e^2) - (4*d
*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (x^5*Sqrt[d^2 - e^2*x^2])/6 - (239*d^6*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
```

```
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^4(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{d^3x}{e^3}+\frac{d^2x^2}{e^2}-\frac{dx^3}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-\frac{24d^7}{e^2}+\frac{78d^6x}{e}-96d^5x^2+66d^4ex^3-47d^3e^2x^4+24d^2e^3x^5}{\sqrt{d^2-e^2x^2}}\,dx}{6de^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{120d^7-390d^6ex+480d^5e^2x^2-426d^4e^3x^3+235d^3}{\sqrt{d^2-e^2x^2}}}{30de^4}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int -480d^7e^2+1560d^6}{}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-}}{5e}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2}}{24e^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2}}{24e^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2}}{24e^2}$$

Mathematica [A] time = 0.189041, size = 125, normalized size = 0.56

$$\frac{\sqrt{d^2-e^2x^2}\left(769d^4e^2x^2-426d^3e^3x^3+278d^2e^4x^4-2047d^5ex-5632d^6-152de^5x^5+40e^6x^6\right)-3585d^6(d+ex)\tan^{-1}}{240e^5(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5632*d^6 - 2047*d^5*e*x + 769*d^4*e^2*x^2 - 426*d^3*
e^3*x^3 + 278*d^2*e^4*x^4 - 152*d*e^5*x^5 + 40*e^6*x^6) - 3585*d^6*(d + e*x
)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(240*e^5*(d + e*x))
```

Maple [B] time = 0.071, size = 393, normalized size = 1.8

$$-\frac{d^3}{e^9}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{\frac{7}{2}}\left(\frac{d}{e}+x\right)^{-4}-7\,\frac{d^2}{e^8}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{7/2}\left(\frac{d}{e}+x\right)^{-3}-\frac{22\,d}{3\,e^7}\left(-\left(\frac{d}{e}+x\right)^2e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -d^3/e^9/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-7*d^2/e^8/(d/e+x)^3
*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-22/3*d/e^7/(d/e+x)^2*(-(d/e+x)^2*e^2+
2*d*e*(d/e+x))^(7/2)+5/16*d^4*x*(-e^2*x^2+d^2)^(1/2)/e^4+5/16/e^4*d^6/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+5/24/e^4*d^2*x*(-e^2*x^2+
d^2)^(3/2)-122/15/e^5*d*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+1/6/e^4*x*(-e^
2*x^2+d^2)^(5/2)-61/6/e^4*d^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-61/4/e
^4*d^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-61/4/e^4*d^6/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67713, size = 333, normalized size = 1.49

$$-\frac{5632\,d^6ex+5632\,d^7-7170\left(d^6ex+d^7\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(40\,e^6x^6-152\,de^5x^5+278\,d^2e^4x^4-426\,d^3e^3x^3+769\right.}{240\left(e^6x+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/240*(5632*d^6*e*x + 5632*d^7 - 7170*(d^6*e*x + d^7)*arctan(-(d - sqrt(-e
^2*x^2 + d^2))/(e*x)) - (40*e^6*x^6 - 152*d*e^5*x^5 + 278*d^2*e^4*x^4 - 426
*d^3*e^3*x^3 + 769*d^4*e^2*x^2 - 2047*d^5*e*x - 5632*d^6)*sqrt(-e^2*x^2 + d
```

```
^2))/(e^6*x + d*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (-(-d+ex)(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.200 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=192

$$\frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4}-\frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3}+\frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}+\frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}}-\frac{dx^3\sqrt{d^2-e^2x^2}}{e}+\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{27d^5\,\mathrm{ta}}{}$$

```
[Out] (d^2*(d - e*x)^4)/(e^4*Sqrt[d^2 - e^2*x^2]) + (101*d^4*Sqrt[d^2 - e^2*x^2])
/(5*e^4) - (19*d^3*x*Sqrt[d^2 - e^2*x^2])/(2*e^3) + (18*d^2*x^2*Sqrt[d^2 -
e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/e + (x^4*Sqrt[d^2 - e^2*x^2
])/5 + (27*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rubi [A] time = 0.435268, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1635, 1815, 641, 217, 203}

$$\frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4}-\frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3}+\frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}+\frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}}-\frac{dx^3\sqrt{d^2-e^2x^2}}{e}+\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{27d^5\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^2*(d - e*x)^4)/(e^4*Sqrt[d^2 - e^2*x^2]) + (101*d^4*Sqrt[d^2 - e^2*x^2])
/(5*e^4) - (19*d^3*x*Sqrt[d^2 - e^2*x^2])/(2*e^3) + (18*d^2*x^2*Sqrt[d^2 -
e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/e + (x^4*Sqrt[d^2 - e^2*x^2
])/5 + (27*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^3(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^3}{e^3}+\frac{d^2x}{e^2}-\frac{dx^2}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{20d^6}{e}-65d^5x+80d^4ex^2-54d^3e^2x^3+20d^2e^3x^4}{\sqrt{d^2-e^2x^2}}\,dx}{5de^2}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-80d^6e+260d^5e^2x-380d^4e^3x^2+216d^3e^4x^3}{\sqrt{d^2-e^2x^2}}\,dx}{20de^4}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{240d^6e^3-1212d^5e^4x+}{\sqrt{d^2-e^2x^2}}}{60de^6}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} -$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e}$$

Mathematica [A] time = 0.157789, size = 109, normalized size = 0.57

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-29d^3e^2x^2+16d^2e^3x^3+77d^4ex+212d^5-8de^4x^4+2e^5x^5\right)}{d+ex}+135d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{10e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(212*d^5 + 77*d^4*e*x - 29*d^3*e^2*x^2 + 16*d^2*e^3*x
^3 - 8*d*e^4*x^4 + 2*e^5*x^5))/(d + e*x) + 135*d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(10*e^4)
```

Maple [A] time = 0.066, size = 285, normalized size = 1.5

$$\frac{36}{5e^4}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{5}{2}}+9\frac{dx}{e^3}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{3/2}+\frac{27d^3x}{2e^3}\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] 36/5/e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+9/e^3*d*(-(d/e+x)^2*e^2+2*d*e
*(d/e+x))^(3/2)*x+27/2/e^3*d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+27/2/
e^3*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/
2))+d^2/e^8/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+6*d/e^7/(d/e+x)^
3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+7/e^6/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*
e*(d/e+x))^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68878, size = 292, normalized size = 1.52

$$\frac{212\,d^5ex+212\,d^6-270\left(d^5ex+d^6\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,e^5x^5-8\,de^4x^4+16\,d^2e^3x^3-29\,d^3e^2x^2+77\,d^4ex+212\,d\right.}{10\left(e^5x+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/10*(212*d^5*e*x + 212*d^6 - 270*(d^5*e*x + d^6)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (2*e^5*x^5 - 8*d*e^4*x^4 + 16*d^2*e^3*x^3 - 29*d^3*e^2*x
^2 + 77*d^4*e*x + 212*d^5)*sqrt(-e^2*x^2 + d^2))/(e^5*x + d*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.201 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=182

$$-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^4\tan$$

```
[Out] -((d*(d - e*x)^4)/(e^3*Sqrt[d^2 - e^2*x^2])) - (95*d^3*Sqrt[d^2 - e^2*x^2])
/(8*e^3) - (95*d^2*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) - (19*d*(d - e*x
)^2*Sqrt[d^2 - e^2*x^2])/(12*e^3) - ((d - e*x)^3*Sqrt[d^2 - e^2*x^2])/(4*e^
3) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rubi [A] time = 0.219433, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1635, 795, 671, 641, 217, 203}

$$-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^4\tan$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```


```
[Out] -((d*(d - e*x)^4)/(e^3*Sqrt[d^2 - e^2*x^2])) - (95*d^3*Sqrt[d^2 - e^2*x^2])
/(8*e^3) - (95*d^2*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) - (19*d*(d - e*x
)^2*Sqrt[d^2 - e^2*x^2])/(12*e^3) - ((d - e*x)^3*Sqrt[d^2 - e^2*x^2])/(4*e^
3) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^2(d-ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{4d^2}{e^2}-\frac{dx}{e}\right)(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(19d)\int \frac{(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{\left(95d^2\right)\int \frac{(d-ex)^2}{\sqrt{d^2-e^2x^2}}\,dx}{12e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - (d$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - (d$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - (d$$

Mathematica [A] time = 0.135274, size = 103, normalized size = 0.57

$$\sqrt{d^2-e^2x^2}\left(-\frac{8d^4}{e^3(d+ex)}+\frac{31d^2x}{8e^2}-\frac{32d^3}{3e^3}-\frac{4dx^2}{3e}+\frac{x^3}{4}\right)-\frac{95d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-32*d^3)/(3*e^3) + (31*d^2*x)/(8*e^2) - (4*d*x^2)/(3*
e) + x^3/4 - (8*d^4)/(e^3*(d + e*x))) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(8*e^3)
```

Maple [A] time = 0.065, size = 288, normalized size = 1.6

$$-\frac{d}{e^7}\left(-\left(\frac{d}{e}+x\right)^2 e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{\frac{7}{2}}\left(\frac{d}{e}+x\right)^{-4}-5\,\frac{1}{e^6}\left(-\left(\frac{d}{e}+x\right)^2 e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{7/2}\left(\frac{d}{e}+x\right)^{-3}-\frac{19}{3\,de^5}\left(-\left(\frac{d}{e}+x\right)^2 e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -d/e^7/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-5/e^6/(d/e+x)^3*(-(d/
e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-19/3/d/e^5/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*
(d/e+x))^(7/2)-19/3/d/e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-95/12/e^2*(-
(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-95/8*d^2/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/
e+x))^(1/2)*x-95/8*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2
+2*d*e*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59704, size = 273, normalized size = 1.5

$$-\frac{448\,d^4ex+448\,d^5-570\left(d^4ex+d^5\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,e^4x^4-26\,de^3x^3+61\,d^2e^2x^2-163\,d^3ex-448\,d^4\right)\sqrt{-e^2x}}{24\left(e^4x+de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(448*d^4*e*x + 448*d^5 - 570*(d^4*e*x + d^5)*arctan(-(d - sqrt(-e^2*x
^2 + d^2))/(e*x)) - (6*e^4*x^4 - 26*d*e^3*x^3 + 61*d^2*e^2*x^2 - 163*d^3*e*
x - 448*d^4)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (-(-d+ex)(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.202 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=130

$$\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] (10*d*x*Sqrt[d^2 - e^2*x^2])/e + (20*(d^2 - e^2*x^2)^(3/2))/(3*e^2) + (8*(d
^2 - e^2*x^2)^(5/2))/(e^2*(d + e*x)^2) + (d^2 - e^2*x^2)^(7/2)/(e^2*(d + e*
x)^4) + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rubi [A] time = 0.0641239, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {793, 663, 665, 195, 217, 203}

$$\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (10*d*x*Sqrt[d^2 - e^2*x^2])/e + (20*(d^2 - e^2*x^2)^(3/2))/(3*e^2) + (8*(d
^2 - e^2*x^2)^(5/2))/(e^2*(d + e*x)^2) + (d^2 - e^2*x^2)^(7/2)/(e^2*(d + e*
x)^4) + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx &= \frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{4\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx}{e}\\
&= \frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{20\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{d+ex}\,dx}{e}\\
&= \frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{(20d)\int \sqrt{d^2-e^2x^2}\,dx}{e}\\
&= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{\left(10d^3\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,d
\end{aligned}$$

$$\begin{aligned}
&= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{\left(10d^3\right)\mathrm{Subst}\left(\int \frac{}{1}\right.}{}\\
&= \frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20\left(d^2-e^2x^2\right)^{3/2}}{3e^2}+\frac{8\left(d^2-e^2x^2\right)^{5/2}}{e^2(d+ex)^2}+\frac{\left(d^2-e^2x^2\right)^{7/2}}{e^2(d+ex)^4}+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2}}\right.}{e^2}
\end{aligned}$$

Mathematica [A] time = 0.112286, size = 83, normalized size = 0.64

$$\frac{1}{3}\sqrt{d^2-e^2x^2}\left(\frac{24d^3}{e^2(d+ex)}+\frac{23d^2}{e^2}-\frac{6dx}{e}+x^2\right)+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((23*d^2)/e^2 - (6*d*x)/e + x^2 + (24*d^3)/(e^2*(d + e
*x))))/3 + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Maple [B] time = 0.06, size = 290, normalized size = 2.2

$$\frac{1}{e^6}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{\frac{7}{2}}\left(\frac{d}{e}+x\right)^{-4}+4\,\frac{1}{de^5}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{7/2}\left(\frac{d}{e}+x\right)^{-3}+\frac{16}{3\,e^4d^2}\left(-\left(\frac{d}{e}+x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] 1/e^6/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+4/d/e^5/(d/e+x)^3*(-(d
/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+16/3/d^2/e^4/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d
*e*(d/e+x))^(7/2)+16/3/d^2/e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+20/3/d/
e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+10*d/e*(-(d/e+x)^2*e^2+2*d*e*(d/e+
x))^(1/2)*x+10*d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e
*(d/e+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67388, size = 236, normalized size = 1.82

$$\frac{47\,d^3ex+47\,d^4-60\left(d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(e^3x^3-5\,de^2x^2+17\,d^2ex+47\,d^3\right)\sqrt{-e^2x^2+d^2}}{3\left(e^3x+de^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(47*d^3*e*x + 47*d^4 - 60*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) + (e^3*x^3 - 5*d*e^2*x^2 + 17*d^2*e*x + 47*d^3)*sqrt(-e^2*x^2
+ d^2))/(e^3*x + d*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.203 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=113

$$-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3}-\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)}-\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] (-15*d*Sqrt[d^2 - e^2*x^2])/(2*e) - (5*(d^2 - e^2*x^2)^(3/2))/(2*e*(d + e*x
)) - (2*(d^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^3) - (15*d^2*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(2*e)
```

Rubi [A] time = 0.0480503, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {663, 665, 217, 203}

$$-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{e(d+ex)^3}-\frac{5\left(d^2-e^2x^2\right)^{3/2}}{2e(d+ex)}-\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (-15*d*Sqrt[d^2 - e^2*x^2])/(2*e) - (5*(d^2 - e^2*x^2)^(3/2))/(2*e*(d + e*x
)) - (2*(d^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^3) - (15*d^2*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(2*e)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = -\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - 5\int \frac{(d^2-e^2x^2)^{3/2}}{(d+ex)^2}\,dx$$
$$= -\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d)\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{1}{2}(15d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{}{\sqrt{d^2}}\right.$$
$$= -\frac{15d\sqrt{d^2-e^2x^2}}{2e} - \frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)} - \frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3} - \frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Mathematica [A] time = 0.0763588, size = 75, normalized size = 0.66

$$\sqrt{d^2-e^2x^2}\left(-\frac{8d^2}{e(d+ex)} - \frac{4d}{e} + \frac{x}{2}\right) - \frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*((-4*d)/e + x/2 - (8*d^2)/(e*(d + e*x))) - (15*d^2*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Maple [B] time = 0.056, size = 284, normalized size = 2.5

$$-\frac{1}{e^5d}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{7}{2}}\left(\frac{d}{e}+x\right)^{-4} - 3\frac{1}{e^4d^2}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{7/2}\left(\frac{d}{e}+x\right)^{-3} - 4\frac{1}{e^3d^3}\left(-\left(\frac{d}{e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)
```

```
[Out] -1/e^5/d/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-3/e^4/d^2/(d/e+x)^3
*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-4/e^3/d^3/(d/e+x)^2*(-(d/e+x)^2*e^2+2
*d*e*(d/e+x))^(7/2)-4/e/d^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-5/d^2*(-(d
/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-15/2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2
)*x-15/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x)
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64021, size = 212, normalized size = 1.88

$$-\frac{24\,d^2ex + 24\,d^3 - 30\left(d^2ex + d^3\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(e^2x^2 - 7\,dex - 24\,d^2\right)\sqrt{-e^2x^2 + d^2}}{2\left(e^2x + de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^2*e*x + 24*d^3 - 30*(d^2*e*x + d^3)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) - (e^2*x^2 - 7*d*e*x - 24*d^2)*sqrt(-e^2*x^2 + d^2))/(e^2*x +
 d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{5}{2}}}{\left(d + ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.204 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^4}\,dx$

Optimal. Leaf size=89

$$\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (8*d*(d - e*x))/Sqrt[d^2 - e^2*x^2] + Sqrt[d^2 - e^2*x^2] + 4*d*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.211394, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1805, 1809, 844, 217, 203, 266, 63, 208}

$$\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/Sqrt[d^2 - e^2*x^2] + Sqrt[d^2 - e^2*x^2] + 4*d*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-d^4-4d^3ex+d^2e^2x^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+\frac{\int \frac{d^4e^2+4d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+(4de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+\frac{1}{2}d^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)+(4de)\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x
\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2}\\
&= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] time = 0.169612, size = 79, normalized size = 0.89

$$\sqrt{d^2-e^2x^2}\left(\frac{8d}{d+ex}+1\right)-d\log\left(\sqrt{d^2-e^2x^2}+d\right)+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(1 + (8*d)/(d + e*x)) + 4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]] + d*Log[x] - d*Log[d + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] time = 0.075, size = 378, normalized size = 4.3

$$\frac{32}{15d^4}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{\frac{5}{2}}+4\frac{de}{\sqrt{e^2}}\arctan\left(\sqrt{e^2}x\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)}}\right)+\sqrt{-x^2e^2+d^2}+\frac{1}{5d^4}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x)
```

```
[Out] 32/15/d^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+4*d*e/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))+(-e^2*x^2+d^2)^(1/2)+1/5/d
^4*(-e^2*x^2+d^2)^(5/2)+1/3/d^2*(-e^2*x^2+d^2)^(3/2)+4/d*e*(-(d/e+x)^2*e^2+
2*d*e*(d/e+x))^(1/2)*x+1/e^4/d^2/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
7/2)+2/e^3/d^3/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+7/3/e^2/d^4/(
d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(
d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+8/3/d^3*e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x)
)^(3/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x), x)
```

Fricas [A] time = 1.65044, size = 236, normalized size = 2.65

$$\frac{9dex+9d^2-8\left(dex+d^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(dex+d^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}(ex+9d)}{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] (9*d*e*x + 9*d^2 - 8*(d*e*x + d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (d*e*x + d^2)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)
*(e*x + 9*d))/(e*x + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d+ex)(d+ex))^{\frac{5}{2}}}{x(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.205 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4} dx$

Optimal. Leaf size=94

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] (-8*e*(d - e*x))/Sqrt[d^2 - e^2*x^2] - Sqrt[d^2 - e^2*x^2]/x - e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + 4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rubi [A] time = 0.216331, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {852, 1805, 1807, 844, 217, 203, 266, 63, 208}

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/Sqrt[d^2 - e^2*x^2] - Sqrt[d^2 - e^2*x^2]/x - e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + 4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-d^4+4d^3ex+d^2e^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}+\frac{\int \frac{-4d^5e-d^4e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-(4de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx-e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-(2de)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)-e^2\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\right.\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{(4d)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e}\\
&= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A] time = 0.213147, size = 84, normalized size = 0.89

$$\sqrt{d^2-e^2x^2}\left(-\frac{8e}{d+ex}-\frac{1}{x}\right)+4e\log\left(\sqrt{d^2-e^2x^2}+d\right)-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-4e\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]*(-x^(-1) - (8*e)/(d + e*x)) - e*ArcTan[(e*x)/Sqrt[d^2 -
 e^2*x^2]] - 4*e*Log[x] + 4*e*Log[d + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] time = 0.072, size = 515, normalized size = 5.5

$$-\frac{e^2x}{d^6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}-\frac{5\,e^2x}{4\,d^4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}-\frac{15\,e^2x}{8\,d^2}\sqrt{-x^2e^2+d^2}+\frac{7\,e^2x}{12\,d^4}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}+\frac{7\,e^2x}{8\,d^2}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x)
```

```
[Out] -1/d^6*e^2*x*(-e^2*x^2+d^2)^(5/2)-5/4/d^4*e^2*x*(-e^2*x^2+d^2)^(3/2)-15/8/d
^2*e^2*x*(-e^2*x^2+d^2)^(1/2)+7/12/d^4*e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(
3/2)*x+7/8/d^2*e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-1/e^3/d^3/(d/e+x)
^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-1/e^2/d^4/(d/e+x)^3*(-(d/e+x)^2*e^2
+2*d*e*(d/e+x))^(7/2)-1/3/e/d^5/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7
/2)+4*d*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/d^
6/x*(-e^2*x^2+d^2)^(7/2)-15/8*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))-4/5/d^5*e*(-e^2*x^2+d^2)^(5/2)-4/3/d^3*e*(-e^2*x^2+d^2)^(3/2)
-4/d*e*(-e^2*x^2+d^2)^(1/2)+7/15/d^5*e*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)
+7/8*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2), x)
```

Fricas [A] time = 1.59943, size = 262, normalized size = 2.79

$$-\frac{8\,e^2x^2+8\,dex-2\left(e^2x^2+dex\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+4\left(e^2x^2+dex\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\sqrt{-e^2x^2+d^2}(9\,ex+d)}{ex^2+dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -(8*e^2*x^2 + 8*d*e*x - 2*(e^2*x^2 + d*e*x)*arctan(-(d - sqrt(-e^2*x^2 + d^
2))/(e*x)) + 4*(e^2*x^2 + d*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(
-e^2*x^2 + d^2)*(9*e*x + d))/(e*x^2 + d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d+ex)(d+ex))^{\frac{5}{2}}}{x^2(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**2*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.206 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=110

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

```
[Out] (8*e^2*(d - e*x))/(d*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*x^2) + (
4*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (15*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d)
```

Rubi [A] time = 0.215146, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(d*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*x^2) + (
4*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (15*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^3(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx \\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2} \\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{\int \frac{-8d^5e+15d^4e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4} \\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{2}\left(15e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{4}\left(15e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) \\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right) \\
&= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.247161, size = 85, normalized size = 0.77

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\left(-d^2+7dex+24e^2x^2\right)}{x^2(d+ex)} - 15e^2\log\left(\sqrt{d^2-e^2x^2}+d\right) + 15e^2\log(x)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^2 + 7*d*e*x + 24*e^2*x^2))/(x^2*(d + e*x)) + 15*e
^2*Log[x] - 15*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/(2*d)
```

Maple [B] time = 0.084, size = 504, normalized size = 4.6

$$-\frac{1}{2d^6x^2}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}-2\frac{1}{d^6}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{7/2}\left(\frac{d}{e}+x\right)^{-2}-4\frac{e^2}{d^6}\left(-\left(\frac{d}{e}+x\right)^2e^2+2de\left(\frac{d}{e}+x\right)\right)^{5/2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x)
```

```
[Out] -1/2/d^6/x^2*(-e^2*x^2+d^2)^(7/2)-2/d^6/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/
e+x))^(7/2)-4/d^6*e^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+3/2/d^6*e^2*(-e^
2*x^2+d^2)^(5/2)+5/2/d^4*e^2*(-e^2*x^2+d^2)^(3/2)+15/2/d^2*e^2*(-e^2*x^2+d^
2)^(1/2)-15/2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))
/x)-5/d^5*e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x-15/2/d^3*e^3*(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(1/2)*x-15/2/d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(
-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))+1/e^2/d^4/(d/e+x)^4*(-(d/e+x)^2*e^2+2*
d*e*(d/e+x))^(7/2)+5/d^5*e^3*x*(-e^2*x^2+d^2)^(3/2)+15/2/d^3*e^3*x*(-e^2*x^
2+d^2)^(1/2)+15/2/d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/
2))+4/d^7*e/x*(-e^2*x^2+d^2)^(7/2)+4/d^7*e^3*x*(-e^2*x^2+d^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^3), x)
```

Fricas [A] time = 1.62552, size = 225, normalized size = 2.05

$$\frac{16\,e^3x^3+16\,de^2x^2+15\left(e^3x^3+de^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(24\,e^2x^2+7\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{2\left(dex^3+d^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(16*e^3*x^3 + 16*d*e^2*x^2 + 15*(e^3*x^3 + d*e^2*x^2)*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + (24*e^2*x^2 + 7*d*e*x - d^2)*sqrt(-e^2*x^2 + d^2))/(d*e
```

```
*x^3 + d^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^3\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**3*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.207 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=137

$$-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] (-8*e^3*(d - e*x))/(d^2*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*x^3)
+ (2*e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (23*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^2*x
) + (10*e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rubi [A] time = 0.297347, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(d^2*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*x^3)
+ (2*e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (23*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^2*x
) + (10*e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^4(d+ex)^4}\,dx &= \int \frac{(d-ex)^4}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx\\
&= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{\int \frac{-12d^5e+23d^4e^2x-24d^3e^3x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^4}\\
&= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\int \frac{-46d^6e^2+60d^5e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^6}\\
&= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{\left(10e^3\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}\\
&= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{\left(5e^3\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,\right.}{d}\\
&= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{(10e)\operatorname{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d\right.}}{d}\\
&= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.274926, size = 94, normalized size = 0.69

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(-5d^2ex+d^3+17de^2x^2+47e^3x^3\right)}{x^3(d+ex)}-30e^3\log\left(\sqrt{d^2-e^2x^2}+d\right)+30e^3\log(x)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(d^3 - 5*d^2*e*x + 17*d*e^2*x^2 + 47*e^3*x^3))/(x^3*
(d + e*x)) + 30*e^3*Log[x] - 30*e^3*Log[d + Sqrt[d^2 - e^2*x^2]])/(3*d^2)
```

Maple [B] time = 0.087, size = 575, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x)
```

```
[Out] 65/6/d^6*e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+65/4/d^4*e^4*(-(d/e+x)^
2*e^2+2*d*e*(d/e+x))^(1/2)*x+65/4/d^2*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))-1/3/d^6/x^3*(-e^2*x^2+d^2)^(7/2)+10/d
*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+26/3/d^7*
e^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+1/d^6/(d/e+x)^3*(-(d/e+x)^2*e^2+2*
d*e*(d/e+x))^(7/2)-2/d^7*e^3*(-e^2*x^2+d^2)^(5/2)-10/3/d^5*e^3*(-e^2*x^2+d^
2)^(3/2)-10/d^3*e^3*(-e^2*x^2+d^2)^(1/2)-26/3/d^8*e^2/x*(-e^2*x^2+d^2)^(7/2
)-26/3/d^8*e^4*x*(-e^2*x^2+d^2)^(5/2)-65/6/d^6*e^4*x*(-e^2*x^2+d^2)^(3/2)-6
5/4/d^4*e^4*x*(-e^2*x^2+d^2)^(1/2)-65/4/d^2*e^4/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-e^2*x^2+d^2)^(1/2))+2/d^7*e/x^2*(-e^2*x^2+d^2)^(7/2)-1/d^5/e/(d/e+x
)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+14/3/d^7*e/(d/e+x)^2*(-(d/e+x)^2*e
^2+2*d*e*(d/e+x))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^4), x)
```

Fricas [A] time = 1.57278, size = 252, normalized size = 1.84

$$-\frac{24\,e^4x^4+24\,de^3x^3+30\left(e^4x^4+de^3x^3\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(47\,e^3x^3+17\,de^2x^2-5\,d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{3\left(d^2ex^4+d^3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*e^4*x^4 + 24*d*e^3*x^3 + 30*(e^4*x^4 + d*e^3*x^3)*log(-(d - sqrt(-
e^2*x^2 + d^2))/x) + (47*e^3*x^3 + 17*d*e^2*x^2 - 5*d^2*e*x + d^3)*sqrt(-e^
2*x^2 + d^2))/(d^2*e*x^4 + d^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^4\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**4*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.208 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx$

Optimal. Leaf size=170

$$\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

```
[Out] (8*e^4*(d - e*x))/(d^3*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(4*x^4) +
 (4*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (31*e^2*Sqrt[d^2 - e^2*x^2])/(8*d^2*
x^2) + (32*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) - (95*e^4*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/(8*d^3)
```

Rubi [A] time = 0.394445, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] (8*e^4*(d - e*x))/(d^3*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(4*x^4) +
 (4*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (31*e^2*Sqrt[d^2 - e^2*x^2])/(8*d^2*
x^2) + (32*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) - (95*e^4*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/(8*d^3)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^5(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^5\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{\int \frac{-16d^5e+31d^4e^2x-32d^3e^3x^2+32d^2e^4x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^4}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{-93d^6e^2+128d^5e^3x-96d^4e^4x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^6}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{\int \frac{-256d^7e^3+285d^6e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^8}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{\left(95e^4\right)\int$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{\left(95e^4\right)\text{Su}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{\left(95e^2\right)\text{Su}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{95e^4\tanh$$

Mathematica [A] time = 0.305262, size = 107, normalized size = 0.63

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-61d^2e^2x^2+26d^3ex-6d^4+163de^3x^3+448e^4x^4\right)}{x^4(d+ex)} - 285e^4\log\left(\sqrt{d^2-e^2x^2}+d\right) + 285e^4\log(x)}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^4 + 26*d^3*e*x - 61*d^2*e^2*x^2 + 163*d*e^3*x^3
 + 448*e^4*x^4))/(x^4*(d + e*x)) + 285*e^4*Log[x] - 285*e^4*Log[d + Sqrt[d^
2 - e^2*x^2]])/(24*d^3)
```

Maple [B] time = 0.093, size = 600, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x)
```

```
[Out] 4/3/d^7*e/x^3*(-e^2*x^2+d^2)^(7/2)+44/3/d^9*e^3/x*(-e^2*x^2+d^2)^(7/2)+44/3
/d^9*e^5*x*(-e^2*x^2+d^2)^(5/2)+55/3/d^7*e^5*x*(-e^2*x^2+d^2)^(3/2)+55/2/d^
```

```
5*e^5*x*(-e^2*x^2+d^2)^(1/2)+55/2/d^3*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2))-37/8/d^8*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-1/4/d^6/x^4*(-e
^2*x^2+d^2)^(7/2)+1/d^6/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-44/3
/d^8*e^4*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+19/8/d^8*e^4*(-e^2*x^2+d^2)^(
5/2)+95/24/d^6*e^4*(-e^2*x^2+d^2)^(3/2)+95/8/d^4*e^4*(-e^2*x^2+d^2)^(1/2)-9
5/8/d^2*e^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-2/
d^7*e/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-23/3/d^8*e^2/(d/e+x)^2
*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)-55/3/d^7*e^5*(-(d/e+x)^2*e^2+2*d*e*(d
/e+x))^(3/2)*x-55/2/d^5*e^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x-55/2/d^3
*e^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^5), x)
```

Fricas [A] time = 1.64302, size = 286, normalized size = 1.68

$$\frac{192\,e^5x^5+192\,de^4x^4+285\left(e^5x^5+de^4x^4\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(448\,e^4x^4+163\,de^3x^3-61\,d^2e^2x^2+26\,d^3ex-6\,d^4\right)\sqrt{-e^2}}{24\left(d^3ex^5+d^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(192*e^5*x^5 + 192*d*e^4*x^4 + 285*(e^5*x^5 + d*e^4*x^4)*log(-(d - sqr
t(-e^2*x^2 + d^2))/x) + (448*e^4*x^4 + 163*d*e^3*x^3 - 61*d^2*e^2*x^2 + 26*
d^3*e*x - 6*d^4)*sqrt(-e^2*x^2 + d^2))/(d^3*e*x^5 + d^4*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^5\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**5*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.209 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx$

Optimal. Leaf size=196

$$-\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}+\frac{e\sqrt{d^2-e^2x^2}}{dx^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}+\frac{27e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

```
[Out] (-8*e^5*(d - e*x))/(d^4*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(5*x^5)
+ (e*Sqrt[d^2 - e^2*x^2])/(d*x^4) - (13*e^2*Sqrt[d^2 - e^2*x^2])/(5*d^2*x^3
) + (11*e^3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) - (66*e^4*Sqrt[d^2 - e^2*x^2])
/(5*d^4*x) + (27*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^4)
```

Rubi [A] time = 0.516622, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 1807, 807, 266, 63, 208}

$$-\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}+\frac{e\sqrt{d^2-e^2x^2}}{dx^4}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}+\frac{27e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```


```
[Out] (-8*e^5*(d - e*x))/(d^4*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(5*x^5)
+ (e*Sqrt[d^2 - e^2*x^2])/(d*x^4) - (13*e^2*Sqrt[d^2 - e^2*x^2])/(5*d^2*x^3
) + (11*e^3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) - (66*e^4*Sqrt[d^2 - e^2*x^2])
/(5*d^4*x) + (27*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^4)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^6\left(d^2-e^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4+\frac{8e^5x^5}{d}}{x^6\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{\int \frac{-20d^5e+39d^4e^2x-40d^3e^3x^2+40d^2e^4x^3-40de^5x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^4}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{\int \frac{-156d^6e^2+220d^5e^3x-160d^4e^4x^2+160d^3e^5x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^6}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{\int \frac{-660d^7e^3+792d^6e^4x-480d^5e^5x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{60d^8}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{\int \frac{-1584d^8e^4+16}{x^2\sqrt{d^2-e^2}}}{120d^1}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2}}{5d^4x}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2}}{5d^4x}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2}}{5d^4x}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{66e^4\sqrt{d^2-e^2}}{5d^4x}$$

Mathematica [A] time = 0.371896, size = 118, normalized size = 0.6

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(16d^3e^2x^2-29d^2e^3x^3-8d^4ex+2d^5+77de^4x^4+212e^5x^5\right)}{x^5(d+ex)} - 135e^5\log\left(\sqrt{d^2-e^2x^2}+d\right) + 135e^5\log(x)}{10d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2]*(2*d^5 - 8*d^4*e*x + 16*d^3*e^2*x^2 - 29*d^2*e^3*x^3
 + 77*d*e^4*x^4 + 212*e^5*x^5))/(x^5*(d + e*x)) + 135*e^5*Log[x] - 135*e^5*
Log[d + Sqrt[d^2 - e^2*x^2]])/(10*d^4)
```

Maple [B] time = 0.112, size = 628, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x)
```

```
[Out] -1/5/d^6/x^5*(-e^2*x^2+d^2)^(7/2)+111/5/d^9*e^5*(-(d/e+x)^2*e^2+2*d*e*(d/e+
x))^(5/2)-27/10/d^9*e^5*(-e^2*x^2+d^2)^(5/2)-9/2/d^7*e^5*(-e^2*x^2+d^2)^(3/
2)-27/2/d^5*e^5*(-e^2*x^2+d^2)^(1/2)-1/d^7*e/(d/e+x)^4*(-(d/e+x)^2*e^2+2*d*
e*(d/e+x))^(7/2)+1/d^7*e/x^4*(-e^2*x^2+d^2)^(7/2)+17/2/d^9*e^3/x^2*(-e^2*x^
2+d^2)^(7/2)+3/d^8*e^2/(d/e+x)^3*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+11/d^
9*e^3/(d/e+x)^2*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(7/2)+111/4/d^8*e^6*(-(d/e+x
)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+333/8/d^6*e^6*(-(d/e+x)^2*e^2+2*d*e*(d/e+x))
^(1/2)*x+333/8/d^4*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(d/e+x)^2*e^2+2*d
*e*(d/e+x))^(1/2))+27/2/d^3*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x)-16/5/d^8*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-111/5/d^10*e^4/x*(-
e^2*x^2+d^2)^(7/2)-111/5/d^10*e^6*x*(-e^2*x^2+d^2)^(5/2)-111/4/d^8*e^6*x*(-
e^2*x^2+d^2)^(3/2)-333/8/d^6*e^6*x*(-e^2*x^2+d^2)^(1/2)-333/8/d^4*e^6/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^4x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^6), x)
```

Fricas [A] time = 1.62542, size = 305, normalized size = 1.56

$$-\frac{80\,e^6x^6+80\,de^5x^5+135\left(e^6x^6+de^5x^5\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)+\left(212\,e^5x^5+77\,de^4x^4-29\,d^2e^3x^3+16\,d^3e^2x^2-8\,d^4ex\cdots\right.}{10\left(d^4ex^6+d^5x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/10*(80*e^6*x^6 + 80*d*e^5*x^5 + 135*(e^6*x^6 + d*e^5*x^5)*log(-(d - sqrt
(-e^2*x^2 + d^2))/x) + (212*e^5*x^5 + 77*d*e^4*x^4 - 29*d^2*e^3*x^3 + 16*d^
3*e^2*x^2 - 8*d^4*e*x + 2*d^5)*sqrt(-e^2*x^2 + d^2))/(d^4*e*x^6 + d^5*x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^6\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**6*(d + e*x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.210 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx$

Optimal. Leaf size=95

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}-\frac{\sin^{-1}(ax)}{a^3}$$

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*(1 - a*x
)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*(1 - a*x)^3) - ArcSin[a*x]/a^3
```

Rubi [A] time = 0.132497, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1637, 659, 651, 663, 216}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}-\frac{\sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```


```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*(1 - a*x
)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*(1 - a*x)^3) - ArcSin[a*x]/a^3
```


Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx &= \int \left(\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^2(-1+ax)^3}+\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^2}\right)dx\\
&= \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^2}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^2}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^2}\\
&= \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}-\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^3(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^4}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3(1-ax)^3}-\frac{\sin^{-1}(ax)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.126385, size = 50, normalized size = 0.53

$$\frac{\frac{\left(-13a^2x^2+19ax-8\right)\sqrt{1-a^2x^2}}{(ax-1)^3}-5\sin^{-1}(ax)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] (((-8 + 19*a*x - 13*a^2*x^2)*Sqrt[1 - a^2*x^2])/(-1 + a*x)^3 - 5*ArcSin[a*x
])/(5*a^3)
```

Maple [B] time = 0.064, size = 200, normalized size = 2.1

$$\frac{1}{a^5}\left(-\left(x-a^{-1}\right)^2a^2-2\,a\left(x-a^{-1}\right)\right)^{\frac{3}{2}}\left(x-a^{-1}\right)^{-2}+\frac{1}{a^3}\sqrt{-\left(x-a^{-1}\right)^2a^2-2\,a\left(x-a^{-1}\right)}-\frac{1}{a^2}\arctan\left(x\sqrt{a^2}\frac{}{\sqrt{-\left(x-a^{-1}\right)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x)
```

```
[Out] 1/a^5/(x-1/a)^2*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(3/2)+1/a^3*(-(x-1/a)^2*a^2-2*
a*(x-1/a))^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x-1/a)^2*a^2-2*a
*(x-1/a))^(1/2))+3/5/a^6/(x-1/a)^3*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(3/2)+1/5/a
^7/(x-1/a)^4*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}x^2}{(ax-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4, x)
```

Fricas [A] time = 1.60036, size = 278, normalized size = 2.93

$$\frac{8\,a^3x^3 - 24\,a^2x^2 + 24\,ax + 10\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \left(13\,a^2x^2 - 19\,ax + 8\right)\sqrt{-a^2x^2 + 1} - 8}{5\left(a^6x^3 - 3\,a^5x^2 + 3\,a^4x - a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(8*a^3*x^3 - 24*a^2*x^2 + 24*a*x + 10*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (13*a^2*x^2 - 19*a*x + 8)*sqrt(-a
^2*x^2 + 1) - 8)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}}{(ax-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**4,x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x - 1)**4, x)
```

Giac [A] time = 1.11031, size = 217, normalized size = 2.28

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2|a|} - \frac{2\left(\frac{35\left(\sqrt{-a^2x^2+1}|a|+a\right)}{a^2x} - \frac{55\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{a^4x^2} + \frac{25\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{a^6x^3} - \frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^8x^4} - 8\right)}{5\,a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - 2/5*(35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 55*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 25*(sqrt(-a^2*x^
2 + 1)*abs(a) + a)^3/(a^6*x^3) - 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x
^4) - 8)/(a^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

3.211 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx$

Optimal. Leaf size=88

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}$$

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*a^3*
(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*(1 - a*x)^3)
```

Rubi [A] time = 0.123396, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1639, 793, 659, 651}

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*a^3*
(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*(1 - a*x)^3)
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}+\frac{\int \frac{\left(4a^2-3a^3x\right)\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx}{a^4}\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx}{7a^2}\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{35a^2}\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.0769959, size = 50, normalized size = 0.57

$$\frac{\sqrt{1-a^2x^2}\left(23a^3x^3+13a^2x^2-8ax+2\right)}{105a^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 - 8*a*x + 13*a^2*x^2 + 23*a^3*x^3))/(105*a^3*(-1 + a*
x)^4)
```

Maple [A] time = 0.047, size = 44, normalized size = 0.5

$$\frac{\left(23\,a^2x^2-10\,ax+2\right)(ax+1)}{105\,(ax-1)^4\,a^3}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x)
```

```
[Out] 1/105*(-a^2*x^2+1)^(1/2)*(23*a^2*x^2-10*a*x+2)*(a*x+1)/(a*x-1)^4/a^3
```

Maxima [B] time = 1.12352, size = 207, normalized size = 2.35

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3\right)}+\frac{29\sqrt{-a^2x^2+1}}{35\left(a^6x^3-3\,a^5x^2+3\,a^4x-a^3\right)}+\frac{82\sqrt{-a^2x^2+1}}{105\left(a^5x^2-2\,a^4x+a^3\right)}+\frac{23\sqrt{-a^2x^2+1}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4*a^4*x + a^3) +
29/35*sqrt(-a^2*x^2 + 1)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3) + 82/105*sqr
t(-a^2*x^2 + 1)/(a^5*x^2 - 2*a^4*x + a^3) + 23/105*sqrt(-a^2*x^2 + 1)/(a^4*
x - a^3)
```

Fricas [A] time = 1.60369, size = 223, normalized size = 2.53

$$\frac{2\,a^4x^4-8\,a^3x^3+12\,a^2x^2-8\,ax+\left(23\,a^3x^3+13\,a^2x^2-8\,ax+2\right)\sqrt{-a^2x^2+1}+2}{105\left(a^7x^4-4\,a^6x^3+6\,a^5x^2-4\,a^4x+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/105*(2*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 8*a*x + (23*a^3*x^3 + 13*a^2*x^
2 - 8*a*x + 2)*sqrt(-a^2*x^2 + 1) + 2)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4
*a^4*x + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2\sqrt{-a^2x^2+1}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**5,x)
```

```
[Out] -Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 5*a**4*x**4 + 10*a**3*x**3
 - 10*a**2*x**2 + 5*a*x - 1), x)
```

Giac [C] time = 1.10011, size = 258, normalized size = 2.93

$$\frac{1}{420}\left(-\frac{92i\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}\left(a\right)}{a^4}-\frac{140\left(-\frac{2}{ax-1}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}\left(a\right)-\left(15\left(\frac{2}{ax-1}+1\right)^3\sqrt{-\frac{2}{ax-1}-1}-42\left(\frac{2}{ax-1}+1\right)^2\sqrt{-\frac{2}{ax-}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="giac")
```

```
[Out] 1/420*(-92*I*sgn(1/(a*x - 1))*sgn(a)/a^4 - (140*(-2/(a*x - 1) - 1)^(3/2)*sg
n(1/(a*x - 1))*sgn(a) - (15*(2/(a*x - 1) + 1)^3*sqrt(-2/(a*x - 1) - 1) - 42
*(2/(a*x - 1) + 1)^2*sqrt(-2/(a*x - 1) - 1) - 35*(-2/(a*x - 1) - 1)^(3/2))*
sgn(1/(a*x - 1))*sgn(a) - 28*(3*(2/(a*x - 1) + 1)^2*sqrt(-2/(a*x - 1) - 1)
+ 5*(-2/(a*x - 1) - 1)^(3/2))*sgn(1/(a*x - 1))*sgn(a))/a^4)*abs(a)
```

3.212 $\int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=209

$$\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{1001de^4(d+ex)\left(d^2-e^2\right.}$$

```
[Out] (-24*x)/(5005*d^3*e^3*(d^2 - e^2*x^2)^(5/2)) + d^2/(13*e^4*(d + e*x)^4*(d^2
 - e^2*x^2)^(5/2)) - (30*d)/(143*e^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + 2
1/(143*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + 4/(1001*d*e^4*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) - (32*x)/(5005*d^5*e^3*(d^2 - e^2*x^2)^(3/2)) - (64*x)/
(5005*d^7*e^3*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.314612, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1639, 793, 659, 192, 191}

$$\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{1001de^4(d+ex)\left(d^2-e^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (-24*x)/(5005*d^3*e^3*(d^2 - e^2*x^2)^(5/2)) + d^2/(13*e^4*(d + e*x)^4*(d^2
 - e^2*x^2)^(5/2)) - (30*d)/(143*e^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + 2
1/(143*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + 4/(1001*d*e^4*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) - (32*x)/(5005*d^5*e^3*(d^2 - e^2*x^2)^(3/2)) - (64*x)/
(5005*d^7*e^3*Sqrt[d^2 - e^2*x^2])
```


Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
```

```
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{2d^3e^2-3d^2e^3x-12de^4x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7e^5}$$

$$= -\frac{3d}{14e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{-20d^3e^6+36d^2e^7x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{56e^9}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{3d}{14e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)}$$

$$= \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$$= -\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

Mathematica [A] time = 0.198111, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(315d^7e^2x^2-540d^6e^3x^3+160d^5e^4x^4+776d^4e^5x^5+384d^3e^6x^6-224d^2e^7x^7+360d^8ex+90d^9-256de^8x^8-\right.}{5005d^7e^4(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(90*d^9 + 360*d^8*e*x + 315*d^7*e^2*x^2 - 540*d^6*e^3*
x^3 + 160*d^5*e^4*x^4 + 776*d^4*e^5*x^5 + 384*d^3*e^6*x^6 - 224*d^2*e^7*x^7
 - 256*d*e^8*x^8 - 64*e^9*x^9))/(5005*d^7*e^4*(d - e*x)^3*(d + e*x)^7)
```

Maple [A] time = 0.055, size = 132, normalized size = 0.6

$$\frac{(-ex+d)\left(-64\,e^9x^9-256\,e^8x^8d-224\,e^7x^7d^2+384\,e^6x^6d^3+776\,e^5x^5d^4+160\,x^4d^5e^4-540\,x^3d^6e^3+315\,x^2d^7e^2+36\right.}{5005\,e^4d^7\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/5005*(-e*x+d)*(-64*e^9*x^9-256*d*e^8*x^8-224*d^2*e^7*x^7+384*d^3*e^6*x^6+
776*d^4*e^5*x^5+160*d^5*e^4*x^4-540*d^6*e^3*x^3+315*d^7*e^2*x^2+360*d^8*e*x
+90*d^9)/(e*x+d)^3/d^7/e^4/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.57104, size = 703, normalized size = 3.36

$$\frac{90\,e^{10}x^{10}+360\,de^9x^9+270\,d^2e^8x^8-720\,d^3e^7x^7-1260\,d^4e^6x^6+1260\,d^6e^4x^4+720\,d^7e^3x^3-270\,d^8e^2x^2-360\,d^9ex-}{5005\left(d^7e^{14}x^{10}+4\,d^8e^{13}x^9+3\,d^9e^{12}x^8-8\,d^{10}e^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5005*(90*e^10*x^10 + 360*d*e^9*x^9 + 270*d^2*e^8*x^8 - 720*d^3*e^7*x^7 -
1260*d^4*e^6*x^6 + 1260*d^6*e^4*x^4 + 720*d^7*e^3*x^3 - 270*d^8*e^2*x^2 - 3
60*d^9*e*x - 90*d^10 + (64*e^9*x^9 + 256*d*e^8*x^8 + 224*d^2*e^7*x^7 - 384*
d^3*e^6*x^6 - 776*d^4*e^5*x^5 - 160*d^5*e^4*x^4 + 540*d^6*e^3*x^3 - 315*d^7
*e^2*x^2 - 360*d^8*e*x - 90*d^9)*sqrt(-e^2*x^2 + d^2))/(d^7*e^14*x^10 + 4*d
^8*e^13*x^9 + 3*d^9*e^12*x^8 - 8*d^10*e^11*x^7 - 14*d^11*e^10*x^6 + 14*d^13
*e^8*x^4 + 8*d^14*e^7*x^3 - 3*d^15*e^6*x^2 - 4*d^16*e^5*x - d^17*e^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.213 $\int \frac{x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287d^2e^3(d+ex)\left(d\right.}$$

```
[Out] (14*x)/(2145*d^4*e^2*(d^2 - e^2*x^2)^(5/2)) - d/(13*e^3*(d + e*x)^4*(d^2 -
e^2*x^2)^(5/2)) + 17/(143*e^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*
d*e^3*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*d^2*e^3*(d + e*x)*(d^2 -
 e^2*x^2)^(5/2)) + (56*x)/(6435*d^6*e^2*(d^2 - e^2*x^2)^(3/2)) + (112*x)/(6
435*d^8*e^2*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.207456, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1639, 793, 659, 192, 191}

$$-\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287d^2e^3(d+ex)\left(d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (14*x)/(2145*d^4*e^2*(d^2 - e^2*x^2)^(5/2)) - d/(13*e^3*(d + e*x)^4*(d^2 -
e^2*x^2)^(5/2)) + 17/(143*e^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*
d*e^3*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*d^2*e^3*(d + e*x)*(d^2 -
 e^2*x^2)^(5/2)) + (56*x)/(6435*d^6*e^2*(d^2 - e^2*x^2)^(3/2)) + (112*x)/(6
435*d^8*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
```

```
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{3d^2e^2-5de^3x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx}{8e^4}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{8e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{(7d)\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{104e^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx}{143e^2}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287de^3(d+ex)^2\left(d^2-e^2\right.}$$

$$= -\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{1287de^3(d+ex)^2\left(d^2-e^2\right.}$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} -$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} -$$

$$= \frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} -$$

Mathematica [A] time = 0.111982, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(700d^7e^2x^2+945d^6e^3x^3-280d^5e^4x^4-1358d^4e^5x^5-672d^3e^6x^6+392d^2e^7x^7+800d^8ex+200d^9+448de^8x^8\right.}{6435d^8e^3(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(200*d^9 + 800*d^8*e*x + 700*d^7*e^2*x^2 + 945*d^6*e^3
*x^3 - 280*d^5*e^4*x^4 - 1358*d^4*e^5*x^5 - 672*d^3*e^6*x^6 + 392*d^2*e^7*x
^7 + 448*d*e^8*x^8 + 112*e^9*x^9))/(6435*d^8*e^3*(d - e*x)^3*(d + e*x)^7)
```

Maple [A] time = 0.056, size = 132, normalized size = 0.6

$$\frac{(-ex+d)\left(112\,e^{9}x^{9}+448\,e^{8}x^{8}d+392\,e^{7}x^{7}d^{2}-672\,e^{6}x^{6}d^{3}-1358\,e^{5}x^{5}d^{4}-280\,e^{4}x^{4}d^{5}+945\,x^{3}d^{6}e^{3}+700\,x^{2}d^{7}e^{2}+80}{6435\,e^{3}d^{8}\,(ex+d)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/6435*(-e*x+d)*(112*e^9*x^9+448*d*e^8*x^8+392*d^2*e^7*x^7-672*d^3*e^6*x^6-
1358*d^4*e^5*x^5-280*d^5*e^4*x^4+945*d^6*e^3*x^3+700*d^7*e^2*x^2+800*d^8*e*
x+200*d^9)/(e*x+d)^3/d^8/e^3/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.99928, size = 713, normalized size = 3.41

$$\frac{200\,e^{10}x^{10}+800\,de^{9}x^{9}+600\,d^{2}e^{8}x^{8}-1600\,d^{3}e^{7}x^{7}-2800\,d^{4}e^{6}x^{6}+2800\,d^{6}e^{4}x^{4}+1600\,d^{7}e^{3}x^{3}-600\,d^{8}e^{2}x^{2}-800\,d^{9}e}{6435\left(d^{8}e^{13}x^{10}+4\,d^{9}e^{12}x^{9}+3\,d^{10}e^{11}x^{8}-8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/6435*(200*e^10*x^10 + 800*d*e^9*x^9 + 600*d^2*e^8*x^8 - 1600*d^3*e^7*x^7
- 2800*d^4*e^6*x^6 + 2800*d^6*e^4*x^4 + 1600*d^7*e^3*x^3 - 600*d^8*e^2*x^2
- 800*d^9*e*x - 200*d^10 - (112*e^9*x^9 + 448*d*e^8*x^8 + 392*d^2*e^7*x^7 -
 672*d^3*e^6*x^6 - 1358*d^4*e^5*x^5 - 280*d^5*e^4*x^4 + 945*d^6*e^3*x^3 + 7
00*d^7*e^2*x^2 + 800*d^8*e*x + 200*d^9)*sqrt(-e^2*x^2 + d^2))/(d^8*e^13*x^1
0 + 4*d^9*e^12*x^9 + 3*d^10*e^11*x^8 - 8*d^11*e^10*x^7 - 14*d^12*e^9*x^6 +
14*d^14*e^7*x^4 + 8*d^15*e^6*x^3 - 3*d^16*e^5*x^2 - 4*d^17*e^4*x - d^18*e^3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.214 $\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=211

$$\frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}+\frac{256x}{6435d^7e\left(d^2-e^2x^2\right)^{3/2}}+\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^3e^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{}{1287d^2e^2(}$$

```
[Out] (64*x)/(2145*d^5*e*(d^2 - e^2*x^2)^(5/2)) + 1/(13*e^2*(d + e*x)^4*(d^2 - e^
2*x^2)^(5/2)) - 4/(143*d*e^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*
d^2*e^2*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*d^3*e^2*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) + (256*x)/(6435*d^7*e*(d^2 - e^2*x^2)^(3/2)) + (512*x)/
(6435*d^9*e*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.104949, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {793, 659, 192, 191}

$$\frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}+\frac{256x}{6435d^7e\left(d^2-e^2x^2\right)^{3/2}}+\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^3e^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{}{1287d^2e^2(}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (64*x)/(2145*d^5*e*(d^2 - e^2*x^2)^(5/2)) + 1/(13*e^2*(d + e*x)^4*(d^2 - e^
2*x^2)^(5/2)) - 4/(143*d*e^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*
d^2*e^2*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*d^3*e^2*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) + (256*x)/(6435*d^7*e*(d^2 - e^2*x^2)^(3/2)) + (512*x)/
(6435*d^9*e*Sqrt[d^2 - e^2*x^2])
```


Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{13e}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{32\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx}{143de}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e\right.}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e\right.}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} -$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} -$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} -$$

Mathematica [A] time = 0.0939344, size = 137, normalized size = 0.65

$$\frac{\sqrt{d^2-e^2x^2}\left(3200d^7e^2x^2+4320d^6e^3x^3-1280d^5e^4x^4-6208d^4e^5x^5-3072d^3e^6x^6+1792d^2e^7x^7-20d^8ex-5d^9+2048de^8\right.}{6435d^9e^2(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^9 - 20*d^8*e*x + 3200*d^7*e^2*x^2 + 4320*d^6*e^3
*x^3 - 1280*d^5*e^4*x^4 - 6208*d^4*e^5*x^5 - 3072*d^3*e^6*x^6 + 1792*d^2*e^
7*x^7 + 2048*d*e^8*x^8 + 512*e^9*x^9))/(6435*d^9*e^2*(d - e*x)^3*(d + e*x)^
7)
```

Maple [A] time = 0.055, size = 132, normalized size = 0.6

$$-\frac{(-ex+d)\left(-512\,e^9x^9-2048\,e^8x^8d-1792\,e^7x^7d^2+3072\,e^6x^6d^3+6208\,e^5x^5d^4+1280\,e^4x^4d^5-4320\,e^3x^3d^6-3200\,x^2d^7\right.}{6435\,e^2d^9\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/6435*(-e*x+d)*(-512*e^9*x^9-2048*d*e^8*x^8-1792*d^2*e^7*x^7+3072*d^3*e^6
*x^6+6208*d^4*e^5*x^5+1280*d^5*e^4*x^4-4320*d^6*e^3*x^3-3200*d^7*e^2*x^2+20
*d^8*e*x+5*d^9)/(e*x+d)^3/d^9/e^2/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.40865, size = 697, normalized size = 3.3

$$-\frac{5 e^{10} x^{10} + 20 d e^9 x^9 + 15 d^2 e^8 x^8 - 40 d^3 e^7 x^7 - 70 d^4 e^6 x^6 + 70 d^6 e^4 x^4 + 40 d^7 e^3 x^3 - 15 d^8 e^2 x^2 - 20 d^9 e x - 5 d^{10} + (512}{6435 \left(d^9 e^{12} x^{10} + 4 d^{10} e^{11} x^9 + 3 d^{11} e^{10} x^8 - 8 d^{12} e^9 x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/6435*(5*e^10*x^10 + 20*d*e^9*x^9 + 15*d^2*e^8*x^8 - 40*d^3*e^7*x^7 - 70*
d^4*e^6*x^6 + 70*d^6*e^4*x^4 + 40*d^7*e^3*x^3 - 15*d^8*e^2*x^2 - 20*d^9*e*x
 - 5*d^10 + (512*e^9*x^9 + 2048*d*e^8*x^8 + 1792*d^2*e^7*x^7 - 3072*d^3*e^6
*x^6 - 6208*d^4*e^5*x^5 - 1280*d^5*e^4*x^4 + 4320*d^6*e^3*x^3 + 3200*d^7*e^
2*x^2 - 20*d^8*e*x - 5*d^9)*sqrt(-e^2*x^2 + d^2))/(d^9*e^12*x^10 + 4*d^10*e
^11*x^9 + 3*d^11*e^10*x^8 - 8*d^12*e^9*x^7 - 14*d^13*e^8*x^6 + 14*d^15*e^6*
x^4 + 8*d^16*e^5*x^3 - 3*d^17*e^4*x^2 - 4*d^18*e^3*x - d^19*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{7}{2}} \left(d + ex\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.215 $\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=205

$$\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d\right.}$$

```
[Out] (48*x)/(715*d^6*(d^2 - e^2*x^2)^(5/2)) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x
^2)^(5/2)) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^3*e
*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^(3/2)) + (128*x)/(715*d^10*Sqrt[
d^2 - e^2*x^2])
```

Rubi [A] time = 0.0906988, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {659, 192, 191}

$$\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (48*x)/(715*d^6*(d^2 - e^2*x^2)^(5/2)) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x
^2)^(5/2)) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^3*e
*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^(3/2)) + (128*x)/(715*d^10*Sqrt[
d^2 - e^2*x^2])
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}} dx = -\frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{9\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}} dx}{13d}$$

$$= -\frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{72\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}} dx}{143d^2}$$

$$= -\frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^3e(d+ex)^2 \left(d^2-e^2\right.}$$

$$= -\frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{8}{143d^3e(d+ex)^2 \left(d^2-e^2\right.}$$

$$= \frac{48x}{715d^6 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{}{14}$$

$$= \frac{48x}{715d^6 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{}{14}$$

$$= \frac{48x}{715d^6 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{}{14}$$

Mathematica [A] time = 0.0729396, size = 137, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2}\left(800d^7e^2x^2+1080d^6e^3x^3-320d^5e^4x^4-1552d^4e^5x^5-768d^3e^6x^6+448d^2e^7x^7-5d^8ex-180d^9+512de^8x^8+\right.}{715d^{10}e(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-180*d^9 - 5*d^8*e*x + 800*d^7*e^2*x^2 + 1080*d^6*e^3
*x^3 - 320*d^5*e^4*x^4 - 1552*d^4*e^5*x^5 - 768*d^3*e^6*x^6 + 448*d^2*e^7*x
^7 + 512*d*e^8*x^8 + 128*e^9*x^9))/(715*d^10*e*(d - e*x)^3*(d + e*x)^7)
```

Maple [A] time = 0.052, size = 132, normalized size = 0.6

$$-\frac{(-ex+d)\left(-128\,e^9x^9-512\,e^8x^8d-448\,e^7x^7d^2+768\,e^6x^6d^3+1552\,e^5x^5d^4+320\,e^4x^4d^5-1080\,e^3x^3d^6-800\,e^2x^2d^7+5\right.}{715\,ed^{10}\,(ex+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/715*(-e*x+d)*(-128*e^9*x^9-512*d*e^8*x^8-448*d^2*e^7*x^7+768*d^3*e^6*x^6
+1552*d^4*e^5*x^5+320*d^5*e^4*x^4-1080*d^6*e^3*x^3-800*d^7*e^2*x^2+5*d^8*e*
x+180*d^9)/(e*x+d)^3/d^10/e/(-e^2*x^2+d^2)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.90228, size = 709, normalized size = 3.46

$$-\frac{180 e^{10} x^{10} + 720 d e^{9} x^{9} + 540 d^{2} e^{8} x^{8} - 1440 d^{3} e^{7} x^{7} - 2520 d^{4} e^{6} x^{6} + 2520 d^{6} e^{4} x^{4} + 1440 d^{7} e^{3} x^{3} - 540 d^{8} e^{2} x^{2} - 720 d^{9}}{715 \left(d^{10} e^{11} x^{10} + 4 d^{11} e^{10} x^{9} + 3 d^{12} e^{9} x^{8} - 8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/715*(180*e^10*x^10 + 720*d*e^9*x^9 + 540*d^2*e^8*x^8 - 1440*d^3*e^7*x^7
- 2520*d^4*e^6*x^6 + 2520*d^6*e^4*x^4 + 1440*d^7*e^3*x^3 - 540*d^8*e^2*x^2
- 720*d^9*e*x - 180*d^10 + (128*e^9*x^9 + 512*d*e^8*x^8 + 448*d^2*e^7*x^7 -
 768*d^3*e^6*x^6 - 1552*d^4*e^5*x^5 - 320*d^5*e^4*x^4 + 1080*d^6*e^3*x^3 +
800*d^7*e^2*x^2 - 5*d^8*e*x - 180*d^9)*sqrt(-e^2*x^2 + d^2))/(d^10*e^11*x^1
0 + 4*d^11*e^10*x^9 + 3*d^12*e^9*x^8 - 8*d^13*e^8*x^7 - 14*d^14*e^7*x^6 + 1
4*d^16*e^5*x^4 + 8*d^17*e^4*x^3 - 3*d^18*e^3*x^2 - 4*d^19*e^2*x - d^20*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{7}{2}}\left(d + ex\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.216 $\int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}}+\frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}}+\frac{273d-512ex}{819d^9\left(d^2-e^2x^2\right)^{3/2}}+\frac{273d-640ex}{1365d^7\left(d^2-e^2x^2\right)^{5/2}}+\frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}}+\frac{1}{117d^3}$$

```
[Out] (8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*x)/(13*d*(d^2 - e^2*x^2)
^(11/2)) + (13*d - 40*e*x)/(117*d^3*(d^2 - e^2*x^2)^(9/2)) + (117*d - 320*e
*x)/(819*d^5*(d^2 - e^2*x^2)^(7/2)) + (273*d - 640*e*x)/(1365*d^7*(d^2 - e^
2*x^2)^(5/2)) + (273*d - 512*e*x)/(819*d^9*(d^2 - e^2*x^2)^(3/2)) + (819*d
- 1024*e*x)/(819*d^11*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
/d^11
```

Rubi [A] time = 0.384538, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {852, 1805, 823, 12, 266, 63, 208}

$$-\frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}}+\frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}}+\frac{273d-512ex}{819d^9\left(d^2-e^2x^2\right)^{3/2}}+\frac{273d-640ex}{1365d^7\left(d^2-e^2x^2\right)^{5/2}}+\frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}}+\frac{1}{117d^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*x)/(13*d*(d^2 - e^2*x^2)
^(11/2)) + (13*d - 40*e*x)/(117*d^3*(d^2 - e^2*x^2)^(9/2)) + (117*d - 320*e
*x)/(819*d^5*(d^2 - e^2*x^2)^(7/2)) + (273*d - 640*e*x)/(1365*d^7*(d^2 - e^
2*x^2)^(5/2)) + (273*d - 512*e*x)/(819*d^9*(d^2 - e^2*x^2)^(3/2)) + (819*d
- 1024*e*x)/(819*d^11*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
/d^11
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{15/2}}\,dx$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+44d^3ex+13d^2e^2x^2}{x\left(d^2-e^2x^2\right)^{13/2}}\,dx}{13d^2}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-440d^3ex}{x\left(d^2-e^2x^2\right)^{11/2}}\,dx}{143d^4}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{\int \frac{1287d^6e^2-3520d^5e^3x}{x\left(d^2-e^2x^2\right)^{9/2}}\,dx}{1287d^8e^2}$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} +$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} +$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} +$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} +$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} +$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} +$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} +$$

$$= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117d-320ex}{819d^5\left(d^2-e^2x^2\right)^{7/2}} +$$

Mathematica [A] time = 0.201152, size = 161, normalized size = 0.69

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-4466d^7e^2x^2-56304d^6e^3x^3-34156d^5e^4x^4+40240d^4e^5x^5+45735d^3e^6x^6-1540d^2e^7x^7+22976d^8ex+9839d^9-16385de^8x^8-5120e^9x^9\right)}{(d-ex)^3(d+ex)^7} - 4095\log\left(\sqrt{d}\right.}{4095d^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(9839*d^9 + 22976*d^8*e*x - 4466*d^7*e^2*x^2 - 56304*
d^6*e^3*x^3 - 34156*d^5*e^4*x^4 + 40240*d^4*e^5*x^5 + 45735*d^3*e^6*x^6 - 1
```

```
540*d^2*e^7*x^7 - 16385*d*e^8*x^8 - 5120*e^9*x^9))/((d - e*x)^3*(d + e*x)^7
) + 4095*Log[x] - 4095*Log[d + Sqrt[d^2 - e^2*x^2]])/(4095*d^11)
```

Maple [A] time = 0.075, size = 385, normalized size = 1.7

$$\frac{320}{819\,d^5 e}\left(\frac{d}{e}+x\right)^{-1}\left(-\left(\frac{d}{e}+x\right)^2 e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{-\frac{5}{2}}-\frac{128\,ex}{273\,d^7}\left(-\left(\frac{d}{e}+x\right)^2 e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{-\frac{5}{2}}-\frac{512\,ex}{819\,d^9}\left(-\left(\frac{d}{e}+x\right)^2 e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 320/819/d^5/e/(d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-128/273/d^7*e/(-
(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)*x-512/819/d^9*e/(-(d/e+x)^2*e^2+2*d*e*(d
/e+x))^(3/2)*x-1024/819/d^11*e/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(1/2)*x+2/13/
e^3/d^3/(d/e+x)^3/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+29/117/e^2/d^4/(d/e+
x)^2/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+1/13/e^4/d^2/(d/e+x)^4/(-(d/e+x)^
2*e^2+2*d*e*(d/e+x))^(5/2)-1/d^10/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)+1/5/d^6/(-e^2*x^2+d^2)^(5/2)+1/3/d^8/(-e^2*x^2+d^2)^(3/
2)+1/d^10/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^4 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x), x)
```

Fricas [B] time = 5.72544, size = 995, normalized size = 4.25

$$\frac{9839\,e^{10}x^{10}+39356\,de^9x^9+29517\,d^2e^8x^8-78712\,d^3e^7x^7-137746\,d^4e^6x^6+137746\,d^6e^4x^4+78712\,d^7e^3x^3-29517\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4095*(9839*e^10*x^10 + 39356*d*e^9*x^9 + 29517*d^2*e^8*x^8 - 78712*d^3*e^
7*x^7 - 137746*d^4*e^6*x^6 + 137746*d^6*e^4*x^4 + 78712*d^7*e^3*x^3 - 29517
*d^8*e^2*x^2 - 39356*d^9*e*x - 9839*d^10 + 4095*(e^10*x^10 + 4*d*e^9*x^9 +
3*d^2*e^8*x^8 - 8*d^3*e^7*x^7 - 14*d^4*e^6*x^6 + 14*d^6*e^4*x^4 + 8*d^7*e^3
*x^3 - 3*d^8*e^2*x^2 - 4*d^9*e*x - d^10)*log(-(d - sqrt(-e^2*x^2 + d^2))/x)
 + (5120*e^9*x^9 + 16385*d*e^8*x^8 + 1540*d^2*e^7*x^7 - 45735*d^3*e^6*x^6 -
 40240*d^4*e^5*x^5 + 34156*d^5*e^4*x^4 + 56304*d^6*e^3*x^3 + 4466*d^7*e^2*x
```

```
^2 - 22976*d^8*e*x - 9839*d^9)*sqrt(-e^2*x^2 + d^2))/(d^11*e^10*x^10 + 4*d^
12*e^9*x^9 + 3*d^13*e^8*x^8 - 8*d^14*e^7*x^7 - 14*d^15*e^6*x^6 + 14*d^17*e^
4*x^4 + 8*d^18*e^3*x^3 - 3*d^19*e^2*x^2 - 4*d^20*e*x - d^21)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.217 $\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}-\frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}}-\cdots$$

```
[Out] (-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*(13*d - 24*e*x))/(143*d
^2*(d^2 - e^2*x^2)^(11/2)) - (e*(572*d - 1103*e*x))/(1287*d^4*(d^2 - e^2*x^
2)^(9/2)) - (e*(5148*d - 10111*e*x))/(9009*d^6*(d^2 - e^2*x^2)^(7/2)) - (e*
(12012*d - 23225*e*x))/(15015*d^8*(d^2 - e^2*x^2)^(5/2)) - (e*(12012*d - 21
583*e*x))/(9009*d^10*(d^2 - e^2*x^2)^(3/2)) - (e*(36036*d - 52175*e*x))/(90
09*d^12*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(d^12*x) + (4*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/d^12
```

Rubi [A] time = 0.679868, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {852, 1805, 807, 266, 63, 208}

$$-\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}-\frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*(13*d - 24*e*x))/(143*d
^2*(d^2 - e^2*x^2)^(11/2)) - (e*(572*d - 1103*e*x))/(1287*d^4*(d^2 - e^2*x^
2)^(9/2)) - (e*(5148*d - 10111*e*x))/(9009*d^6*(d^2 - e^2*x^2)^(7/2)) - (e*
(12012*d - 23225*e*x))/(15015*d^8*(d^2 - e^2*x^2)^(5/2)) - (e*(12012*d - 21
583*e*x))/(9009*d^10*(d^2 - e^2*x^2)^(3/2)) - (e*(36036*d - 52175*e*x))/(90
09*d^12*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(d^12*x) + (4*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/d^12
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{15/2}}\,dx$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+52d^3ex-83d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{13/2}}\,dx}{13d^2}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-572d^3ex+960d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{11/2}}\,dx}{143d^4}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{-1287d^4+514}{x^2(d^2}}{12}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \frac{e(5148d-1}{9009d^6\left(d^2-\right.}$$

Mathematica [A] time = 0.230546, size = 183, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2x^2}\left(1014094d^8e^2x^2 - 700504d^7e^3x^3 - 3157776d^6e^4x^4 - 1301264d^5e^5x^5 + 2748320d^4e^6x^6 + 2496180d^3e^7x^7 -\right.}{45045d^{12}x(ex-d)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(45045*d^10 + 546316*d^9*e*x + 1014094*d^8*e^2*x^2 - 7
00504*d^7*e^3*x^3 - 3157776*d^6*e^4*x^4 - 1301264*d^5*e^5*x^5 + 2748320*d^4
```

```
*e^6*x^6 + 2496180*d^3*e^7*x^7 - 350000*d^2*e^8*x^8 - 1043500*d*e^9*x^9 - 3
05920*e^10*x^10))/(45045*d^12*x*(-d + e*x)^3*(d + e*x)^7) - (4*e*Log[x])/d^
12 + (4*e*Log[d + Sqrt[d^2 - e^2*x^2]])/d^12
```

Maple [B] time = 0.071, size = 484, normalized size = 1.8

$$\frac{20222\,e^2x}{15015\,d^8}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{-\frac{5}{2}}+\frac{80888\,e^2x}{45045\,d^{10}}\left(-\left(\frac{d}{e}+x\right)^2e^2+2\,de\left(\frac{d}{e}+x\right)\right)^{-\frac{3}{2}}+\frac{161776\,e^2x}{45045\,d^{12}}\frac{1}{\sqrt{-\left(\frac{d}{e}+x\right)^2e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 20222/15015/d^8*e^2/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)*x+80888/45045/d^10
*e^2/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(3/2)*x+161776/45045/d^12*e^2/(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(1/2)*x-1/d^6/x/(-e^2*x^2+d^2)^(5/2)-10111/9009/d^6/(
d/e+x)/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)+4/d^11*e/(d^2)^(1/2)*ln((2*d^2+
2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-4/3/d^9*e/(-e^2*x^2+d^2)^(3/2)-4/d^1
1*e/(-e^2*x^2+d^2)^(1/2)-4/5/d^7*e/(-e^2*x^2+d^2)^(5/2)-1/13/d^3/e^3/(d/e+x
)^4/(-(d/e+x)^2*e^2+2*d*e*(d/e+x))^(5/2)-35/143/d^4/e^2/(d/e+x)^3/(-(d/e+x)
^2*e^2+2*d*e*(d/e+x))^(5/2)-709/1287/d^5/e/(d/e+x)^2/(-(d/e+x)^2*e^2+2*d*e*
(d/e+x))^(5/2)+6/5/d^8*e^2*x/(-e^2*x^2+d^2)^(5/2)+8/5/d^10*e^2*x/(-e^2*x^2+
d^2)^(3/2)+16/5/d^12*e^2*x/(-e^2*x^2+d^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2), x)
```

Fricas [A] time = 8.26641, size = 1116, normalized size = 4.12

$$-\frac{366136\,e^{11}x^{11}+1464544\,de^{10}x^{10}+1098408\,d^2e^9x^9-2929088\,d^3e^8x^8-5125904\,d^4e^7x^7+5125904\,d^6e^5x^5+2929088\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/45045*(366136*e^11*x^11 + 1464544*d*e^10*x^10 + 1098408*d^2*e^9*x^9 - 29
29088*d^3*e^8*x^8 - 5125904*d^4*e^7*x^7 + 5125904*d^6*e^5*x^5 + 2929088*d^7
*e^4*x^4 - 1098408*d^8*e^3*x^3 - 1464544*d^9*e^2*x^2 - 366136*d^10*e*x + 18
0180*(e^11*x^11 + 4*d*e^10*x^10 + 3*d^2*e^9*x^9 - 8*d^3*e^8*x^8 - 14*d^4*e^
```

```
7*x^7 + 14*d^6*e^5*x^5 + 8*d^7*e^4*x^4 - 3*d^8*e^3*x^3 - 4*d^9*e^2*x^2 - d^
10*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (305920*e^10*x^10 + 1043500*d*
e^9*x^9 + 350000*d^2*e^8*x^8 - 2496180*d^3*e^7*x^7 - 2748320*d^4*e^6*x^6 +
1301264*d^5*e^5*x^5 + 3157776*d^6*e^4*x^4 + 700504*d^7*e^3*x^3 - 1014094*d^
8*e^2*x^2 - 546316*d^9*e*x - 45045*d^10)*sqrt(-e^2*x^2 + d^2))/(d^12*e^10*x
^11 + 4*d^13*e^9*x^10 + 3*d^14*e^8*x^9 - 8*d^15*e^7*x^8 - 14*d^16*e^6*x^7 +
 14*d^18*e^4*x^5 + 8*d^19*e^3*x^4 - 3*d^20*e^2*x^3 - 4*d^21*e*x^2 - d^22*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.218 $\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx$

Optimal. Leaf size=102

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}-\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -((a*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]) - (c^2*(1 - a^2*x^2)^(3/2))/(x*(
c - a*c*x)^(3/2)) + a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c -
a*c*x]]
```

Rubi [A] time = 0.111614, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {879, 865, 875, 208}

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}-\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] -((a*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]) - (c^2*(1 - a^2*x^2)^(3/2))/(x*(
c - a*c*x)^(3/2)) + a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c -
a*c*x]]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx &= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}(ac)\int \frac{\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}\,dx \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}a\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} - \left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right) \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)
\end{aligned}$$

Mathematica [A] time = 0.101095, size = 93, normalized size = 0.91

$$\frac{\sqrt{1-a^2x^2}\left(a\sqrt{c}x\tanh^{-1}\left(\sqrt{c}\sqrt{\frac{ax+1}{c}}\right)-c(2ax+1)\sqrt{\frac{ax+1}{c}}\right)}{x\sqrt{\frac{ax+1}{c}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-(c*Sqrt[(1 + a*x)/c]*(1 + 2*a*x)) + a*Sqrt[c]*x*ArcTan
h[Sqrt[c]*Sqrt[(1 + a*x)/c]]))/(x*Sqrt[(1 + a*x)/c]*Sqrt[c - a*c*x])
```

Maple [A] time = 0.198, size = 95, normalized size = 0.9

$$\frac{1}{(ax-1)x}\left(-\text{Artanh}\left(\sqrt{c(ax+1)}\frac{1}{\sqrt{c}}\right)xac+2xa\sqrt{c(ax+1)}\sqrt{c}+\sqrt{c(ax+1)}\sqrt{c}\right)\sqrt{-c(ax-1)}\sqrt{-a^2x^2+1}\frac{1}{\sqrt{c(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] (-arctanh((c*(a*x+1))^(1/2)/c^(1/2))*x*a*c+2*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+
(c*(a*x+1))^(1/2)*c^(1/2))*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c
*(a*x+1))^(1/2)/x/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/x^2, x)
```

Fricas [A] time = 1.59443, size = 473, normalized size = 4.64

$$\left[\frac{\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}(2ax+1)}{2\left(ax^2-x\right)}, \frac{\left(a^2x^2-ax\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\cdots}{\cdots}\right)\cdots}{\cdots}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x), ((a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt
(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\,(ax-1)}\sqrt{-(ax-1)\,(ax+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/x**2, x)
```

Giac [A] time = 1.14903, size = 149, normalized size = 1.46

$$-\frac{\left(\left(\frac{c\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{acx+c}+\frac{\sqrt{acx+c}}{ax}\right)a^2c^2-\frac{a^2c^3\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+3\sqrt{2}a^2\sqrt{-c}c^{\frac{5}{2}}}{\sqrt{-c}}\right)|c|}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -((c*arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(a*c*x + c) + sqrt(a
*c*x + c)/(a*x))*a^2*c^2 - (a^2*c^3*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) + 3*sq
rt(2)*a^2*sqrt(-c)*c^(5/2))/sqrt(-c))*abs(c)/(a*c^3)
```

3.219 $\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=39

$$-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rubi [A] time = 0.0399762, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {875, 208}

$$-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx &= \left(2a^2c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\ &= -2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A] time = 0.033967, size = 67, normalized size = 1.72

$$-\frac{2\sqrt{c}\sqrt{\frac{ax}{c}+\frac{1}{c}}\sqrt{c-acx}\tanh^{-1}\left(\sqrt{c}\sqrt{\frac{ax}{c}+\frac{1}{c}}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[c^(-1) + (a*x)/c]*Sqrt[c - a*c*x]*ArcTanh[Sqrt[c]*Sqrt[c^(
-1) + (a*x)/c]])/Sqrt[1 - a^2*x^2]
```

Maple [A] time = 0.141, size = 58, normalized size = 1.5

$$2\,\frac{\sqrt{-c\,(ax-1)}\sqrt{-a^2x^2+1}\sqrt{c}}{(ax-1)\,\sqrt{c\,(ax+1)}}\mathrm{Artanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)*c^(1/2)*a
rctanh((c*(a*x+1))^(1/2)/c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-acx+c}}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [A] time = 1.59948, size = 252, normalized size = 6.46

$$\left[\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2\,c}{ax^2-x}\right), -2\,\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{-c}}{a^2cx^2-c}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sq
rt(c) - 2*c)/(a*x^2 - x)), -2*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)*sqrt(-c)/(a^2*c*x^2 - c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/x/(-a**2*x**2+1)**(1/2),x)
```

[Out] Integral(sqrt(-c*(a*x - 1))/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)

Giac [A] time = 1.1336, size = 69, normalized size = 1.77

$$-\frac{2\,c^2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] -2*c^2*(arctan(sqrt(2)*sqrt(c)/sqrt(-c))/sqrt(-c) - arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c))/abs(c)

3.220 $\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rubi [A] time = 0.0100174, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{1-ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx\\ &= \sqrt{x}\sqrt{1-ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0162579, size = 35, normalized size = 1.

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Maple [B] time = 0.159, size = 62, normalized size = 1.8

$$\sqrt{x}\sqrt{-ax+1}+\frac{1}{2}\sqrt{(-ax+1)\,x}\arctan\left(\sqrt{a}\left(x-\frac{1}{2\,a}\right)\frac{1}{\sqrt{-ax^2+x}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-ax+1}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x+1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-a*x+1)^(1/2)+1/2*((-a*x+1)*x)^(1/2)/(-a*x+1)^(1/2)/x^(1/2)/a^(1/2
)*arctan(a^(1/2)*(x-1/2/a)/(-a*x^2+x)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57807, size = 248, normalized size = 7.09

$$\left[\frac{2\sqrt{-ax+1}a\sqrt{x}-\sqrt{-a}\log\left(-2\,ax+2\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{2\,a},\ \frac{\sqrt{-ax+1}a\sqrt{x}-\sqrt{a}\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-a*x + 1)*a*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt(-a*x + 1)*s
qrt(-a)*sqrt(x) + 1))/a, (sqrt(-a*x + 1)*a*sqrt(x) - sqrt(a)*arctan(sqrt(-a
*x + 1)/(sqrt(a)*sqrt(x))))/a]
```

Sympy [A] time = 1.95977, size = 83, normalized size = 2.37

$$\begin{cases} \frac{iax^{\frac{3}{2}}}{\sqrt{ax-1}} - \frac{i\sqrt{x}}{\sqrt{ax-1}} - \frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{for } |ax| > 1 \\ \sqrt{x}\sqrt{-ax+1} + \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*a*x**(3/2)/sqrt(a*x - 1) - I*sqrt(x)/sqrt(a*x - 1) - I*acosh(s
qrt(a)*sqrt(x))/sqrt(a), Abs(a*x) > 1), (sqrt(x)*sqrt(-a*x + 1) + asin(sqrt
(a)*sqrt(x))/sqrt(a), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.221 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rubi [A] time = 0.0288091, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {848, 50, 54, 216}

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx &= \int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx \\
&= \sqrt{x}\sqrt{1-ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx \\
&= \sqrt{x}\sqrt{1-ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.0101975, size = 35, normalized size = 1.

$$\sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Maple [B] time = 0.156, size = 76, normalized size = 2.2

$$\frac{1}{2}\sqrt{-a^2x^2+1}\sqrt{x}\left(2\sqrt{a}\sqrt{-x(ax-1)} + \arctan\left(\frac{2ax-1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-x(ax-1)}}\right)\right)\frac{1}{\sqrt{ax+1}}\frac{1}{\sqrt{-x(ax-1)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)/(a*x+1)^(1/2)*(2*a^(1/2)*(-x*(a*x-1))^(1/2)+
arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/(-x*(a*x-1))^(1/2)/a^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\sqrt{ax+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x)
```

Fricas [B] time = 1.80723, size = 486, normalized size = 13.89

$$\left[\frac{4\sqrt{-a^2x^2+1}\sqrt{ax+1}a\sqrt{x} - (ax+1)\sqrt{-a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}(2ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7ax+1}{ax+1}\right)}{4\left(a^2x+a\right)}, \frac{2\sqrt{-a^2x^2+1}\sqrt{ax+1}a\sqrt{x} - (a}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(-a)*log
(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*sqrt(-a)*sqrt
(x) - 7*a*x + 1)/(a*x + 1)))/(a^2*x + a), 1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(a*
x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(a*x +
 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^2*x + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(a*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(a*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.222 $\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx$

Optimal. Leaf size=34

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rubi [A] time = 0.0087351, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {50, 54, 215}

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{1+ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx\\ &= \sqrt{x}\sqrt{1+ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.013254, size = 34, normalized size = 1.

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Maple [B] time = 0.167, size = 57, normalized size = 1.7

$$\sqrt{x}\sqrt{ax+1}+\frac{1}{2}\sqrt{(ax+1)\,x}\ln\left(\left(\frac{1}{2}+ax\right)\frac{1}{\sqrt{a}}+\sqrt{ax^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{ax+1}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a*x+1)^(1/2)+1/2*((a*x+1)*x)^(1/2)/(a*x+1)^(1/2)/x^(1/2)*ln((1/2+a
*x)/a^(1/2)+(a*x^2+x)^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59336, size = 244, normalized size = 7.18

$$\left[\frac{2\sqrt{ax+1}a\sqrt{x}+\sqrt{a}\log\left(2\,ax+2\sqrt{ax+1}\sqrt{a}\sqrt{x}+1\right)}{2\,a},\ \frac{\sqrt{ax+1}a\sqrt{x}-\sqrt{-a}\arctan\left(\frac{\sqrt{ax+1}\sqrt{-a}}{a\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a*x + 1)*a*sqrt(x) + sqrt(a)*log(2*a*x + 2*sqrt(a*x + 1)*sqrt(
a)*sqrt(x) + 1))/a, (sqrt(a*x + 1)*a*sqrt(x) - sqrt(-a)*arctan(sqrt(a*x + 1
)*sqrt(-a)/(a*sqrt(x))))/a]
```

Sympy [A] time = 1.92732, size = 29, normalized size = 0.85

$$\sqrt{x}\sqrt{ax+1}+\frac{\operatorname{asinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(a*x + 1) + asinh(sqrt(a)*sqrt(x))/sqrt(a)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.223 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=34

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rubi [A] time = 0.028346, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {848, 50, 54, 215}

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx &= \int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx\\ &= \sqrt{x}\sqrt{1+ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx\\ &= \sqrt{x}\sqrt{1+ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0128162, size = 34, normalized size = 1.

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Maple [B] time = 0.139, size = 86, normalized size = 2.5

$$-\frac{1}{2\,ax-2}\sqrt{-a^2x^2+1}\sqrt{x}\sqrt{-ax+1}\left(2\sqrt{(ax+1)\,x}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{(ax+1)\,x}\sqrt{a}+2\,ax+1\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{(ax+1)\,x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)*(-a*x+1)^(1/2)*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)
^(1/2)/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\sqrt{-ax+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)), x)
```

Fricas [B] time = 1.79546, size = 494, normalized size = 14.53

$$\left[-\frac{4\sqrt{-a^2x^2+1}\sqrt{-ax+1}a\sqrt{x}-(ax-1)\sqrt{a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}(2ax+1)\sqrt{-ax+1}\sqrt{a}\sqrt{x}-7ax-1}{ax-1}\right)}{4\left(a^2x-a\right)},-\frac{2\sqrt{-a^2x^2+1}\sqrt{-ax+1}a\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(a)*lo
g(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x + 1)*sqrt(-a*x + 1)*sqrt(a)*sqr
t(x) - 7*a*x - 1)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(-a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(-
a*x + 1)*sqrt(-a)*sqrt(x)/(2*a^2*x^2 - a*x - 1)))/(a^2*x - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(-a*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.224 $\int \sqrt{x}\sqrt{1-ax}\,dx$

Optimal. Leaf size=63

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

```
[Out] -(Sqrt[x]*Sqrt[1 - a*x])/(4*a) + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rubi [A] time = 0.0150052, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 - a*x])/(4*a) + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\sqrt{1-ax}\,dx &= \frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx\\ &= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}\\ &= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx,x,\sqrt{x}\right)}{4a}\\ &= -\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.021092, size = 49, normalized size = 0.78

$$\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}(2ax-1)+\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*a
^(3/2))
```

Maple [A] time = 0.135, size = 79, normalized size = 1.3

$$\frac{1}{2}x^{\frac{3}{2}}\sqrt{-ax+1}-\frac{1}{4\,a}\sqrt{x}\sqrt{-ax+1}+\frac{1}{8}\sqrt{(-ax+1)\,x}\arctan\left(\sqrt{a}\left(x-\frac{1}{2\,a}\right)\frac{1}{\sqrt{-ax^2+x}}\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{-ax+1}}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-a*x+1)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*(-a*x+1)^(1/2)/a+1/8/a^(3/2)*((-a*x+
1)*x)^(1/2)/(-a*x+1)^(1/2)/x^(1/2)*arctan(a^(1/2)*(x-1/2/a)/(-a*x^2+x)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57949, size = 292, normalized size = 4.63

$$\left[\frac{2\left(2\,a^2x-a\right)\sqrt{-ax+1}\sqrt{x}-\sqrt{-a}\log\left(-2\,ax+2\,\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{8\,a^2},\frac{\left(2\,a^2x-a\right)\sqrt{-ax+1}\sqrt{x}-\sqrt{a}\arctan\left(\frac{\sqrt{-ax+}}{\sqrt{a}\sqrt{x}}\right.}{4\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(2*a^2*x - a)*sqrt(-a*x + 1)*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt
(-a*x + 1)*sqrt(-a)*sqrt(x) + 1))/a^2, 1/4*((2*a^2*x - a)*sqrt(-a*x + 1)*sq
rt(x) - sqrt(a)*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x))))/a^2]
```

Sympy [A] time = 3.54705, size = 148, normalized size = 2.35

$$\begin{cases} \frac{iax^{\frac{5}{2}}}{2\sqrt{ax-1}} - \frac{3ix^{\frac{3}{2}}}{4\sqrt{ax-1}} + \frac{i\sqrt{x}}{4a\sqrt{ax-1}} - \frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{for } |ax| > 1 \\ -\frac{ax^{\frac{5}{2}}}{2\sqrt{-ax+1}} + \frac{3x^{\frac{3}{2}}}{4\sqrt{-ax+1}} - \frac{\sqrt{x}}{4a\sqrt{-ax+1}} + \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((I*a*x**(5/2)/(2*sqrt(a*x - 1)) - 3*I*x**(3/2)/(4*sqrt(a*x - 1))
+ I*sqrt(x)/(4*a*sqrt(a*x - 1)) - I*acosh(sqrt(a)*sqrt(x))/(4*a**(3/2)), Ab
s(a*x) > 1), (-a*x**(5/2)/(2*sqrt(-a*x + 1)) + 3*x**(3/2)/(4*sqrt(-a*x + 1)
) - sqrt(x)/(4*a*sqrt(-a*x + 1)) + asin(sqrt(a)*sqrt(x))/(4*a**(3/2)), True
))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.225 $\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx$

Optimal. Leaf size=63

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

```
[Out] -(Sqrt[x]*Sqrt[1 - a*x])/(4*a) + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rubi [A] time = 0.0329066, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {848, 50, 54, 216}

$$\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 - a*x])/(4*a) + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx = \int \sqrt{x}\sqrt{1-ax}\,dx$$
$$= \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)}{4a}$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Mathematica [A] time = 0.0077062, size = 49, normalized size = 0.78

$$\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}(2ax-1) + \sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*a
^(3/2))
```

Maple [B] time = 0.135, size = 92, normalized size = 1.5

$$\frac{1}{8}\sqrt{x}\sqrt{-a^2x^2+1}\left(4\,xa^{3/2}\sqrt{-x\,(ax-1)} - 2\,\sqrt{a}\sqrt{-x\,(ax-1)} + \arctan\left(\frac{2\,ax-1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{-x\,(ax-1)}}\right)\right)a^{-\frac{3}{2}}\frac{1}{\sqrt{ax+1}}\frac{1}{\sqrt{-x\,(ax-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x)
```

```
[Out] 1/8*x^(1/2)*(-a^2*x^2+1)^(1/2)/a^(3/2)*(4*x*a^(3/2)*(-x*(a*x-1))^(1/2)-2*a^
(1/2)*(-x*(a*x-1))^(1/2)+arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/
(a*x+1)^(1/2)/(-x*(a*x-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{x}}{\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(x)/sqrt(a*x + 1), x)
```

Fricas [B] time = 1.75568, size = 525, normalized size = 8.33

$$\left[\frac{4\sqrt{-a^2x^2+1}\left(2a^2x-a\right)\sqrt{ax+1}\sqrt{x}-(ax+1)\sqrt{-a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}(2ax-1)\sqrt{ax+1}\sqrt{-a}\sqrt{x}-7ax+1}{ax+1}\right)}{16\left(a^3x+a^2\right)}, \frac{2\sqrt{-a^2x^2+1}\left(2a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-a^2*x^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)
*sqrt(-a)*log(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*
sqrt(-a)*sqrt(x) - 7*a*x + 1)/(a*x + 1)))/(a^3*x + a^2), 1/8*(2*sqrt(-a^2*x
^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sq
rt(-a^2*x^2 + 1)*sqrt(a*x + 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^3
*x + a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}\sqrt{-(ax-1)(ax+1)}}{\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-a**2*x**2+1)**(1/2)/(a*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-(a*x - 1)*(a*x + 1))/sqrt(a*x + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.226 $\int (gx)^m (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=250

$$\frac{d^6e(4m+29)\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+9)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}+\frac{d^7(4m+11)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1}{g(m+1)(m+8)\sqrt{1}}$$

```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m)) - (e*(g*x)^(2 + m)*(
d^2 - e^2*x^2)^(7/2))/(g^2*(9 + m)) + (d^7*(11 + 4*m)*(g*x)^(1 + m)*Sqrt[d^
2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/
(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^6*e*(29 + 4*m)*(g*x)^(2 +
m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x
^2)/d^2])/(g^2*(2 + m)*(9 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rubi [A] time = 0.385916, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1809, 808, 365, 364}

$$\frac{d^6e(4m+29)\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+9)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}+\frac{d^7(4m+11)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1}{g(m+1)(m+8)\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m)) - (e*(g*x)^(2 + m)*(
d^2 - e^2*x^2)^(7/2))/(g^2*(9 + m)) + (d^7*(11 + 4*m)*(g*x)^(1 + m)*Sqrt[d^
2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/
(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^6*e*(29 + 4*m)*(g*x)^(2 +
m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x
^2)/d^2])/(g^2*(2 + m)*(9 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} - \frac{\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}\left(-d^3e^2(9+m)-d^2e^3(29+4m\right.}{e^2(9+m)}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{\int (gx)^m\left(d^3e^4(9+m)(11\right.}{}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{\left(d^3(11+4m)\right)\int (gx)^m\left(d^2\right.}{8+m}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{\left(d^7(11+4m)\sqrt{d^2-e^2x^2}\right)}{(8+m)\sqrt{}}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)} + \frac{d^7(11+4m)(gx)^{1+m}\sqrt{d^2-}}{g(1+m)(8}$$

Mathematica [A] time = 0.214215, size = 199, normalized size = 0.8

$$\frac{d^4x\sqrt{d^2-e^2x^2}(gx)^m\left(\frac{d^3\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{m+1}+ex\left(\frac{3d^2\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{m+2}+ex\left(\frac{3d\,{}_2F_1\left(-\frac{5}{2},\frac{m+3}{2};\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}+\frac{ex\,{}_2F_1\left(-\frac{5}{2},\frac{m+4}{2};\frac{m+6}{2};\right.}{m+4}\right.\right.\right.}{\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*((d^3*Hypergeometric2F1[-5/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[-5/2, (
2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[
-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F
1[-5/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/Sqrt[1 - (e^2*x^2
)/d^2]
```

Maple [F] time = 0.572, size = 0, normalized size = 0.

$$\int \left(gx\right)^m\left(ex+d\right)^3\left(-x^2e^2+d^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^3\,(gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^7x^7+3\,de^6x^6+d^2e^5x^5-5\,d^3e^4x^4-5\,d^4e^3x^3+d^5e^2x^2+3\,d^6ex+d^7\right)\sqrt{-e^2x^2+d^2}\,(gx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^7*x^7 + 3*d*e^6*x^6 + d^2*e^5*x^5 - 5*d^3*e^4*x^4 - 5*d^4*e^3*x
^3 + d^5*e^2*x^2 + 3*d^6*e*x + d^7)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

Sympy [C] time = 134.441, size = 513, normalized size = 2.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**8*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**7*e*g**m*x**2*x
**m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2
*I*pi)/d**2)/(2*gamma(m/2 + 2)) + d**6*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)
*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(
2*gamma(m/2 + 5/2)) - 5*d**5*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2
, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3)
) - 5*d**4*e**4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m
/2 + 7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d**3*e
**5*g**m*x**6*x**m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x
**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 4)) + 3*d**2*e**6*g**m*x**7*x**m
*gamma(m/2 + 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/(2*gamma(m/2 + 9/2)) + d*e**7*g**m*x**8*x**m*gamma(m/2 + 4)
*hyper((-1/2, m/2 + 4), (m/2 + 5,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*ga
mma(m/2 + 5))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}} (ex + d)^3 \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)
```

3.227 $\int (gx)^m (d+ex)^2 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal. Leaf size=206

$$\frac{2d^5e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^6(2m+9)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{\left(d^2-e^2x^2\right)^{7/}}{g(m+}$$

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m))) + (d^6*(9 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (2*d^5*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rubi [A] time = 0.209129, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1809, 808, 365, 364}

$$\frac{2d^5e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^6(2m+9)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{\left(d^2-e^2x^2\right)^{7/}}{g(m+}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m))) + (d^6*(9 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (2*d^5*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} - \frac{\int (gx)^m\left(-d^2e^2(9+2m)-2de^3(8+m)x\right)\left(d^2-e^2x^2\right)^{5/2}}{e^2(8+m)}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} + \frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}\,dx}{g} + \frac{\left(d^2(9+2m)\right)\int (g}{8}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} + \frac{\left(2d^5e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{\left(d^6(9-\right.}{}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)} + \frac{d^6(9+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right.}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A] time = 0.123601, size = 174, normalized size = 0.84

$$\frac{d^4x\sqrt{d^2-e^2x^2}(gx)^m\left(d^2\left(m^2+5m+6\right){}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\left(2d(m+3){}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)+e(\right.\right.}{(m+1)(m+2)(m+3)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-
5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hyperg
eometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hyperg
eometric2F1[-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*
(3 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F] time = 0.527, size = 0, normalized size = 0.

$$\int (gx)^m(ex+d)^2\left(-x^2e^2+d^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)^2(gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^{6}x^{6}+2\,de^{5}x^{5}-d^{2}e^{4}x^{4}-4\,d^{3}e^{3}x^{3}-d^{4}e^{2}x^{2}+2\,d^{5}ex+d^{6}\right)\sqrt{-e^{2}x^{2}+d^{2}}\left(gx\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^6*x^6 + 2*d*e^5*x^5 - d^2*e^4*x^4 - 4*d^3*e^3*x^3 - d^4*e^2*x^2
 + 2*d^5*e*x + d^6)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

Sympy [C] time = 101.046, size = 442, normalized size = 2.15

$$\frac{d^{7}g^{m}xx^{m}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{d^{6}eg^{m}x^{2}x^{m}\Gamma\left(\frac{m}{2}+1\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{\Gamma\left(\frac{m}{2}+2\right)}-\frac{d^{5}e^{2}g^{m}x^{3}x^{m}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_{2}F_{1}\left(\right.}{2\Gamma\left(\frac{m}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**6*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/gamma(m/2 + 2) - d**5*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*hyper
((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamm
a(m/2 + 5/2)) - 2*d**4*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2
+ 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 3) - d**3*e
**4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2,),
e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d**2*e**5*g**m*x**
6*x**m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/gamma(m/2 + 4) + d*e**6*g**m*x**7*x**m*gamma(m/2 + 7/2)*hyp
er((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*ga
mma(m/2 + 9/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}(ex+d)^{2}\left(gx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)
```

3.228 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=162

$$\frac{d^4e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^5\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] (d^5*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^4*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rubi [A] time = 0.0835558, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {808, 365, 364}

$$\frac{d^4e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^5\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^5*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^4*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}dx = d\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}dx + \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}dx}{g}$$
$$= \frac{\left(d^5\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{\left(d^4e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}$$
$$= \frac{d^5(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^4e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{2+m}{2}\right.}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A] time = 0.0575928, size = 121, normalized size = 0.75

$$\frac{d^4x\sqrt{d^2-e^2x^2}(gx)^m\left(d(m+2)\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)\right)}{(m+1)(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d*(2 + m)*Hypergeometric2F1[-5/2, (1 +
m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[-5/2, (2 +
m)/2, (4 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F] time = 0.364, size = 0, normalized size = 0.

$$\int \left(gx\right)^m(ex+d)\left(-x^2e^2+d^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^5x^5+de^4x^4-2\,d^2e^3x^3-2\,d^3e^2x^2+d^4ex+d^5\right)\sqrt{-e^2x^2+d^2}\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*x + d
^5)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

Sympy [C] time = 62.4386, size = 374, normalized size = 2.31

$$\frac{d^6 g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d^5 e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} - \frac{d^4 e^2 g^m x^3 x^m \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2}{\Gamma\left(\frac{m}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**6*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**5*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/(2*gamma(m/2 + 2)) - d**4*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*h
yper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gam
ma(m/2 + 5/2) - d**3*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 +
2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 3) + d**2*e**
4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d*e**5*g**m*x**6*x**
m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/(2*gamma(m/2 + 4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}} (ex + d) \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*(g*x)^m, x)
```

3.229 $\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=80

$$\frac{d^4\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] (d^4*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rubi [A] time = 0.0248633, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {365, 364}

$$\frac{d^4\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx &= \frac{\left(d^4\sqrt{d^2-e^2x^2}\right)\int (gx)^m \left(1-\frac{e^2x^2}{d^2}\right)^{5/2} \, dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}\\
&= \frac{d^4(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0192638, size = 78, normalized size = 0.98

$$\frac{d^4x\sqrt{d^2-e^2x^2}(gx)^m\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+1}{2}+1;\frac{e^2x^2}{d^2}\right)}{(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, 1 + (
1 + m)/2, (e^2*x^2)/d^2])/((1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int \left(gx\right)^{m} \left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}} \left(gx\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^{4}x^{4}-2\,d^{2}e^{2}x^{2}+d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}\left(gx\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((e^4*x^4 - 2*d^2*e^2*x^2 + d^4)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

Sympy [C] time = 47.7421, size = 61, normalized size = 0.76

$$\frac{d^{5}g^{m}xx^{m}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**5*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-5/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2 + d^2\right)^{\frac{5}{2}} \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)
```

3.230 $\int \frac{(gx)^m(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=163

$$\frac{d^3\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] (d^3*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (d^2*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rubi [A] time = 0.1405, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {892, 82, 126, 365, 364}

$$\frac{d^3\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^3*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (d^2*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int (gx)^m(d-ex)^{5/2}(d+ex)^{3/2}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\ &= \frac{\left(d\sqrt{d^2-e^2x^2}\right)\int (gx)^m(d-ex)^{3/2}(d+ex)^{3/2}\,dx}{\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}(d-ex)^{3/2}(d+ex)^{3}}{g\sqrt{d-ex}\sqrt{d+ex}}\\ &= d\int (gx)^m\left(d^2-e^2x^2\right)^{3/2}dx - \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}dx}{g}\\ &= \frac{\left(d^3\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{3/2}dx}{\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{\left(d^2e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{3/2}dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}}\\ &= \frac{d^3(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{d^2e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2}{}\right.}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}\end{aligned}$$

Mathematica [A] time = 0.0591541, size = 122, normalized size = 0.75

$$\frac{d^2x\sqrt{d^2-e^2x^2}(gx)^m\left(d(m+2)\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\,{}_2F_1\left(-\frac{3}{2},\frac{m}{2}+1;\frac{m}{2}+2;\frac{e^2x^2}{d^2}\right)\right)}{(m+1)(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^2*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(-(e*(1 + m)*x*Hypergeometric2F1[-3/2, 1
 + m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[-3/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F] time = 0.574, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{ex+d}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e^3x^3 - de^2x^2 - d^2ex + d^3)\sqrt{-e^2x^2 + d^2}\,(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 - d*e^2*x^2 - d^2*e*x + d^3)*sqrt(-e^2*x^2 + d^2)*(g*x)^m
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)
```

3.231 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{2de\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^2(2m+5)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+4)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{\left(d^2-e^2x^2\right)^{3}}{g(m}$$

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(3/2))/(g*(4 + m))) + (d^2*(5 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(4 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (2*d*e*(g*x)^
(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rubi [A] time = 0.215488, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {852, 1809, 808, 365, 364}

$$-\frac{2de\sqrt{d^2-e^2x^2}(gx)^{m+2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^2(2m+5)\sqrt{d^2-e^2x^2}(gx)^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+4)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{\left(d^2-e^2x^2\right)^{3}}{g(m}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```


```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(3/2))/(g*(4 + m))) + (d^2*(5 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(4 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (2*d*e*(g*x)^
(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int (gx)^m(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{\int (gx)^m\left(-d^2e^2(5+2m)+2de^3(4+m)x\right)\sqrt{d^2-e^2x^2}\,dx}{e^2(4+m)}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{(2de)\int (gx)^{1+m}\sqrt{d^2-e^2x^2}\,dx}{g} + \frac{\left(d^2(5+2m)\right)\int (gx)^m\sqrt{d^2-e^2}}{4+m}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{\left(2de\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{\left(d^2(5+2m)\sqrt{d^2-e}\right.}{(4+n}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} + \frac{d^2(5+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(4+m)\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2de(}{}$$

Mathematica [A] time = 0.121718, size = 173, normalized size = 0.85

$$\frac{x\sqrt{d^2-e^2x^2}(gx)^m\left(d^2\left(m^2+5m+6\right)\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\left(2d(m+3)\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)-e(m\right.\right.}{(m+1)(m+2)(m+3)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-1/2,
 (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeome
tric2F1[-1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hypergeome
tric2F1[-1/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 +
 m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F] time = 0.606, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(ex+d)^2}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2-2\,dex+d^2\right)\sqrt{-e^2x^2+d^2}\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2, x)
```

3.232 $\int \frac{(gx)^m(d^2-e^2x^2)^{5/2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=250

$$-\frac{d^2e(4m+11)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+3)\sqrt{d^2-e^2x^2}}+\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}+\frac{d^3(4m+5)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\right.}{g(m+1)(m+2)\sqrt{d^2-e}}$$

```
[Out] (-3*d*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2])/(g*(2 + m)) + (e*(g*x)^(2 + m)*Sqr
t[d^2 - e^2*x^2])/(g^2*(3 + m)) + (d^3*(5 + 4*m)*(g*x)^(1 + m)*Sqrt[1 - (e^
2*x^2)/d^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/(g
*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2]) - (d^2*e*(11 + 4*m)*(g*x)^(2 + m)*Sqr
t[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)
/d^2])/(g^2*(2 + m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.380661, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {852, 1809, 808, 365, 364}

$$-\frac{d^2e(4m+11)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+3)\sqrt{d^2-e^2x^2}}+\frac{e\sqrt{d^2-e^2x^2}(gx)^{m+2}}{g^2(m+3)}+\frac{d^3(4m+5)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\right.}{g(m+1)(m+2)\sqrt{d^2-e}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```



```
[Out] (-3*d*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2])/(g*(2 + m)) + (e*(g*x)^(2 + m)*Sqr
t[d^2 - e^2*x^2])/(g^2*(3 + m)) + (d^3*(5 + 4*m)*(g*x)^(1 + m)*Sqrt[1 - (e^
2*x^2)/d^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/(g
*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2]) - (d^2*e*(11 + 4*m)*(g*x)^(2 + m)*Sqr
t[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)
/d^2])/(g^2*(2 + m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
```

```
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = \int \frac{(gx)^m(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} - \frac{\int \frac{(gx)^m\left(-d^3e^2(3+m)+d^2e^3(11+4m)x-3de^4(3+m)x^2\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2(3+m)}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\int \frac{(gx)^m\left(d^3e^4(3+m)(5+4m)-d^2e^5(2+m)(11+4m)x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^4(2+m)(3+m)}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\left(d^3(5+4m)\right)\int \frac{(gx)^m}{\sqrt{d^2-e^2x^2}}\,dx}{2+m} - \frac{\left(d^2e(11+4m\right.}{g(3}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\left(d^3(5+4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\sqrt{1-\frac{e^2x^2}{d^2}}}\,dx}{(2+m)\sqrt{d^2-e^2x^2}} - \frac{\left(d^2\right.}{}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{d^3(5+4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\right.}{g(1+m)(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.203725, size = 245, normalized size = 0.98

$$\frac{x\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(d^3\left(m^3+9m^2+26m+24\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\left(3d^2\left(m^2+7m+12\right)\,{}_2F_1\left(\frac{1}{2}\right.\right.\right.}{(m+1)(m+2)(m+3)(m+4)(d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2]*(d^3*(24 + 26*m + 9*
m^2 + m^3)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*
(1 + m)*x*(3*d^2*(12 + 7*m + m^2)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2] + e*(2 + m)*x*(-3*d*(4 + m)*Hypergeometric2F1[1/2, (3 +
m)/2, (5 + m)/2, (e^2*x^2)/d^2] + e*(3 + m)*x*Hypergeometric2F1[1/2, (4 + m
)/2, (6 + m)/2, (e^2*x^2)/d^2]))))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(d - e*
x)*(d + e*x))
```

Maple [F] time = 0.548, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(ex+d)^3} \left(-x^2e^2+d^2\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}} (gx)^m}{(ex+d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^2x^2-2\,dex+d^2\right)\sqrt{-e^2x^2+d^2}\,(gx)^m}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e*x + d),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\left(gx\right)^m}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)
```

3.233 $\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=213

$$\frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(m+1)\sqrt{d^2-e^2x^2}}+\frac{4(d+ex)(g}{5g\left(d^2-e^2x\right.}$$

```
[Out] (4*(g*x)^(1 + m)*(d + e*x))/(5*g*(d^2 - e^2*x^2)^(5/2)) + ((1 - 4*m)*(g*x)^
(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2
, (e^2*x^2)/d^2])/(5*d^3*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(7 - 4*m)*(g*x
)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.21292, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1806, 808, 365, 364}

$$\frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(m+1)\sqrt{d^2-e^2x^2}}+\frac{4(d+ex)(g}{5g\left(d^2-e^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```


```
[Out] (4*(g*x)^(1 + m)*(d + e*x))/(5*g*(d^2 - e^2*x^2)^(5/2)) + ((1 - 4*m)*(g*x)^
(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2
, (e^2*x^2)/d^2])/(5*d^3*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(7 - 4*m)*(g*x
)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```


Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] && !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
```

```
&&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(gx)^m\left(-d^3(1-4m)-d^2e(7-4m)x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{5}(d(1-4m))\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{5/2}}\,dx + \frac{(e(7-4m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5g}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{\left((1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\left(e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^4g\sqrt{d^2-e^2x^2}}$$

$$= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{(1-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(7-4m)(gx)^{2+m}\sqrt{1-\frac{e}{}}}{5d^4g^2(2+}$$

Mathematica [A] time = 0.203517, size = 199, normalized size = 0.93

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(\frac{d^3\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{m+1}+ex\left(\frac{3d^2\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{m+2}+ex\left(\frac{3d\,{}_2F_1\left(\frac{7}{2},\frac{m+3}{2};\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}+\frac{ex\,{}_2F_1\left(\frac{7}{2},\frac{m+4}{2};\frac{m+6}{2};\frac{e^2x^2}{d^2}\right)}{m+4}\right)\right)\right)}{d^6\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[7/2, (1 + m)/2,
(3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[7/2, (2
+ m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[7/
2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[7
/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*Sqrt[d^2 - e^2*x
^2])
```

Maple [F] time = 0.522, size = 0, normalized size = 0.

$$\int \left(gx\right)^{m}\left(ex+d\right)^{3}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 (gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^5x^5-3\,de^4x^4+2\,d^2e^3x^3+2\,d^3e^2x^2-3\,d^4ex+d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^5*x^5 - 3*d*e^4*x^4 + 2*d^2*e^3*x^
3 + 2*d^3*e^2*x^2 - 3*d^4*e*x + d^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((g*x)**m*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 (gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.234 $\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=216

$$\frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^5g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(m+1)\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)(gx)^{m}}{5dg\left(d^2-e^2x^2\right)}$$

```
[Out] (2*(g*x)^(1 + m)*(d + e*x))/(5*d*g*(d^2 - e^2*x^2)^(5/2)) + ((3 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (2*e*(3 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(5*d^5*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.208358, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1806, 808, 365, 364}

$$\frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^5g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(m+1)\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)(gx)^{m}}{5dg\left(d^2-e^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(g*x)^(1 + m)*(d + e*x))/(5*d*g*(d^2 - e^2*x^2)^(5/2)) + ((3 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (2*e*(3 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(5*d^5*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(gx)^m\left(-d^2(3-2m)-2de(3-m)x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2e(3-m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5dg} - \frac{1}{5}(-3+2m)\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{\left(2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^5g\sqrt{d^2-e^2x^2}} - \frac{\left((-3+2m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{5}}}{5d^4\sqrt{d^2-e^2x^2}}$$

$$= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{(3-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(1+m)\sqrt{d^2-e^2x^2}} + \frac{2e(3-m)(gx)^{2+m}\sqrt{}}{5d^5g^2(}$$

Mathematica [A] time = 0.121814, size = 174, normalized size = 0.81

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)+e(m+2)\right.\right.}{d^6(m+1)(m+2)(m+3)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[7
/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hyperge
ometric2F1[7/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeo
metric2F1[7/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^6*(1 + m)*(2 + m)
*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F] time = 0.507, size = 0, normalized size = 0.

$$\int (gx)^m (ex+d)^2 \left(-x^2e^2+d^2\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2 (gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^6x^6-2\,de^5x^5-d^2e^4x^4+4\,d^3e^3x^3-d^4e^2x^2-2\,d^5ex+d^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^6*x^6 - 2*d*e^5*x^5 - d^2*e^4*x^4
+ 4*d^3*e^3*x^3 - d^4*e^2*x^2 - 2*d^5*e*x + d^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((g*x)**m*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2 (gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.235 $\int \frac{(gx)^m(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=124

$$\frac{e(gx)^{m+2}\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^2g^2(m+2)\left(d^2-e^2x^2\right)^{5/2}}+\frac{(gx)^{m+1}\,{}_2F_1\left(1,\frac{m-4}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{dg(m+1)\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] ((g*x)^(1 + m)*Hypergeometric2F1[1, (-4 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/
(d*g*(1 + m)*(d^2 - e^2*x^2)^(5/2)) + (e*(g*x)^(2 + m)*Hypergeometric2F1[1,
 (-3 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(d^2*g^2*(2 + m)*(d^2 - e^2*x^2)^(5
/2))
```

Rubi [A] time = 0.0819333, antiderivative size = 162, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {808, 365, 364}

$$\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^6g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^5g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^5*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(g*x)^(2
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^6*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = d\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx + \frac{e\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{g}$$

$$= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^5\sqrt{d^2-e^2x^2}} + \frac{\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^6g\sqrt{d^2-e^2x^2}}$$

$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^5g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0566058, size = 121, normalized size = 0.98

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(d(m+2)\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)\right)}{d^6(m+1)(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d*(2 + m)*Hypergeometric2F1[7/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[7/2, (2 + m)
/2, (4 + m)/2, (e^2*x^2)/d^2]))/(d^6*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int \left(gx\right)^{m}(ex+d)\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(gx\right)^{m}}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\left(gx\right)^m}{e^7x^7-de^6x^6-3\,d^2e^5x^5+3\,d^3e^4x^4+3\,d^4e^3x^3-3\,d^5e^2x^2-d^6ex+d^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^7*x^7 - d*e^6*x^6 - 3*d^2*e^5*x^5
+ 3*d^3*e^4*x^4 + 3*d^4*e^3*x^3 - 3*d^5*e^2*x^2 - d^6*e*x + d^7), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(gx\right)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.236 $\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2-e^2x^2}}$$

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^6*g*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0252806, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {365, 364}

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^6*g*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^6\sqrt{d^2-e^2x^2}}\\ &= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(1+m)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0191127, size = 78, normalized size = 0.98

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^m\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+1}{2}+1;\frac{e^2x^2}{d^2}\right)}{d^6(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, 1 + (1
 + m)/2, (e^2*x^2)/d^2])/(d^6*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int (gx)^m \left(-x^2e^2 + d^2\right)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{\left(-e^2x^2 + d^2\right)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-e^2x^2 + d^2}\,(gx)^m}{e^8x^8 - 4\,d^2e^6x^6 + 6\,d^4e^4x^4 - 4\,d^6e^2x^2 + d^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^8*x^8 - 4*d^2*e^6*x^6 + 6*d^4*e^4*
x^4 - 4*d^6*e^2*x^2 + d^8), x)
```

Sympy [C] time = 63.9377, size = 60, normalized size = 0.75

$$\frac{g^m x x^m \Gamma\left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\begin{matrix} \frac{7}{2}, \frac{m}{2} + \frac{1}{2} \\ \frac{m}{2} + \frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] g**m*x*x**m*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**2*x**
2*exp_polar(2*I*pi)/d**2)/(2*d**7*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

3.237 $\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(m+1)\sqrt{d^2-e^2x^2}}-\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^8g^2(m+2)\sqrt{d^2-e^2x^2}}$$

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^7*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(g*x)^(2
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^8*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.145606, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {892, 82, 126, 365, 364}

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(m+1)\sqrt{d^2-e^2x^2}}-\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^8g^2(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^7*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(g*x)^(2
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^8*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```


Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\left(\sqrt{d-ex}\sqrt{d+ex}\right)\int \frac{(gx)^m}{(d-ex)^{7/2}(d+ex)^{9/2}}\,dx}{\sqrt{d^2-e^2x^2}}$$

$$= \frac{\left(d\sqrt{d-ex}\sqrt{d+ex}\right)\int \frac{(gx)^m}{(d-ex)^{9/2}(d+ex)^{9/2}}\,dx}{\sqrt{d^2-e^2x^2}} - \frac{\left(e\sqrt{d-ex}\sqrt{d+ex}\right)\int \frac{(gx)^{1+m}}{(d-ex)^{9/2}(d+ex)^{9/2}}\,dx}{g\sqrt{d^2-e^2x^2}}$$

$$= d\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{9/2}}\,dx - \frac{e\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{g}$$

$$= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{d^7\sqrt{d^2-e^2x^2}} - \frac{\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{d^8g\sqrt{d^2-e^2x^2}}$$

$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{9}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^8g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.0600845, size = 122, normalized size = 0.75

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(d(m+2)\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\,{}_2F_1\left(\frac{9}{2},\frac{m}{2}+1;\frac{m}{2}+2;\frac{e^2x^2}{d^2}\right)\right)}{d^8(m+1)(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(-(e*(1 + m)*x*Hypergeometric2F1[9/2, 1
+ m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[9/2, (1 + m)/
2, (3 + m)/2, (e^2*x^2)/d^2]))/(d^8*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F] time = 0.608, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{ex+d}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^9x^9+de^8x^8-4\,d^2e^7x^7-4\,d^3e^6x^6+6\,d^4e^5x^5+6\,d^5e^4x^4-4\,d^6e^3x^3-4\,d^7e^2x^2+d^8ex+d^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^9*x^9 + d*e^8*x^8 - 4*d^2*e^7*x^7
- 4*d^3*e^6*x^6 + 6*d^4*e^5*x^5 + 6*d^5*e^4*x^4 - 4*d^6*e^3*x^3 - 4*d^7*e^2
*x^2 + d^8*e*x + d^9), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

3.238 $\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{9d^9g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(m+1)\sqrt{d^2-e^2x^2}}+\frac{2(d-ex)(gx)}{9dg\left(d^2-e^2x\right.}$$

```
[Out] (2*(g*x)^(1 + m)*(d - e*x))/(9*d*g*(d^2 - e^2*x^2)^(9/2)) + ((7 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(9*d^8*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (2*e*(7 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(9*d^9*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.231118, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {852, 1806, 808, 365, 364}

$$-\frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{9d^9g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(m+1)\sqrt{d^2-e^2x^2}}+\frac{2(d-ex)(gx)}{9dg\left(d^2-e^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(g*x)^(1 + m)*(d - e*x))/(9*d*g*(d^2 - e^2*x^2)^(9/2)) + ((7 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(9*d^8*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (2*e*(7 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(9*d^9*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
```

```
] && !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(gx)^m(d-ex)^2}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$$

$$= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{(gx)^m\left(-d^2(7-2m)+2de(7-m)x\right)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9d^2}$$

$$= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{(2e(7-m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9dg} - \frac{1}{9}(-7+2m)\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{9/2}}\,dx$$

$$= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{\left(2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{9d^9g\sqrt{d^2-e^2x^2}} - \frac{\left((-7+2m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int}{9d^8\sqrt{d^2-e^2x}}$$

$$= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} + \frac{(7-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(1+m)\sqrt{d^2-e^2x^2}} - \frac{2e(7-m)(g}{}$$

Mathematica [A] time = 0.13423, size = 176, normalized size = 0.81

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{11}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{11}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)-e(m+2\right.\right.}{d^{10}(m+1)(m+2)(m+3)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[1
1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hyperg
eometric2F1[11/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hyperg
eometric2F1[11/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^10*(1 + m)*(2
+ m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F] time = 0.59, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(ex+d)^2}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^{10}x^{10}+2\,de^9x^9-3\,d^2e^8x^8-8\,d^3e^7x^7+2\,d^4e^6x^6+12\,d^5e^5x^5+2\,d^6e^4x^4-8\,d^7e^3x^3-3\,d^8e^2x^2+2\,d^9ex+d^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^10*x^10 + 2*d*e^9*x^9 - 3*d^2*e^8*
x^8 - 8*d^3*e^7*x^7 + 2*d^4*e^6*x^6 + 12*d^5*e^5*x^5 + 2*d^6*e^4*x^4 - 8*d^
7*e^3*x^3 - 3*d^8*e^2*x^2 + 2*d^9*e*x + d^10), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.239 $\int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{11}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{11}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{11d^9g(m+1)\sqrt{d^2-e^2x^2}}+\frac{4(d-ex)(}{11g\left(d^2-e\right.}$$

```
[Out] (4*(g*x)^(1 + m)*(d - e*x))/(11*g*(d^2 - e^2*x^2)^(11/2)) + ((7 - 4*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (1 + m)/2, (3 + m
)/2, (e^2*x^2)/d^2])/(11*d^9*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(25 - 4*m)
*(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (2 + m)/2, (
4 + m)/2, (e^2*x^2)/d^2])/(11*d^10*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.229159, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {852, 1806, 808, 365, 364}

$$-\frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2}\,{}_2F_1\left(\frac{11}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1}\,{}_2F_1\left(\frac{11}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{11d^9g(m+1)\sqrt{d^2-e^2x^2}}+\frac{4(d-ex)(}{11g\left(d^2-e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(g*x)^(1 + m)*(d - e*x))/(11*g*(d^2 - e^2*x^2)^(11/2)) + ((7 - 4*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (1 + m)/2, (3 + m
)/2, (e^2*x^2)/d^2])/(11*d^9*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(25 - 4*m)
*(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (2 + m)/2, (
4 + m)/2, (e^2*x^2)/d^2])/(11*d^10*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(gx)^m(d-ex)^3}{\left(d^2-e^2x^2\right)^{13/2}}\,dx$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} - \frac{\int \frac{(gx)^m\left(-d^3(7-4m)+d^2e(25-4m)x\right)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx}{11d^2}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{1}{11}(d(7-4m))\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{11/2}}\,dx - \frac{(e(25-4m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{11/2}}}{11g}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{\left((7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{11/2}}\,dx}{11d^9\sqrt{d^2-e^2x^2}} - \frac{\left(e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)}{11d^{10}g\sqrt{d^2-}}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{(7-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{11}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{11d^9g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(25-4m}{}$$

Mathematica [A] time = 0.224017, size = 200, normalized size = 0.93

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m\left(\frac{d^3\,{}_2F_1\left(\frac{13}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{m+1}+ex\left(ex\left(\frac{3d\,{}_2F_1\left(\frac{13}{2},\frac{m+3}{2};\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}-\frac{ex\,{}_2F_1\left(\frac{13}{2},\frac{m+4}{2};\frac{m+6}{2};\frac{e^2x^2}{d^2}\right)}{m+4}\right)-\frac{3d^2\,{}_2F_1\left(\frac{13}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{m+2}\right.\right.}{d^{12}\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[13/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[13/2,
(2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1
[13/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric2
F1[13/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^12*Sqrt[d^2 -
 e^2*x^2])
```

Maple [F] time = 0.645, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(ex+d)^3} \left(-x^2e^2+d^2\right)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-e^2x^2+d^2}\,(gx)^m}{e^{11}x^{11}+3\,de^{10}x^{10}-d^2e^9x^9-11\,d^3e^8x^8-6\,d^4e^7x^7+14\,d^5e^6x^6+14\,d^6e^5x^5-6\,d^7e^4x^4-11\,d^8e^3x^3-d^9e^2x^2+3\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e^11*x^11 + 3*d*e^10*x^10 - d^2*e^9*
x^9 - 11*d^3*e^8*x^8 - 6*d^4*e^7*x^7 + 14*d^5*e^6*x^6 + 14*d^6*e^5*x^5 - 6*
d^7*e^4*x^4 - 11*d^8*e^3*x^3 - d^9*e^2*x^2 + 3*d^10*e*x + d^11), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

3.240 $\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=148

$$\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^5\left(d^2-e^2x^2\right)^{p+1}}{2e^6(p+1)}+\frac{d^3\left(d^2-e^2x^2\right)^{p+2}}{e^6(p+2)}-\frac{d\left(d^2-e^2x^2\right)^{p+3}}{2e^6(p+3)}$$

```
[Out] -(d^5*(d^2 - e^2*x^2)^(1 + p))/(2*e^6*(1 + p)) + (d^3*(d^2 - e^2*x^2)^(2 +
p))/(e^6*(2 + p)) - (d*(d^2 - e^2*x^2)^(3 + p))/(2*e^6*(3 + p)) + (e*x^7*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^
2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0940171, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 266, 43, 365, 364}

$$\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^5\left(d^2-e^2x^2\right)^{p+1}}{2e^6(p+1)}+\frac{d^3\left(d^2-e^2x^2\right)^{p+2}}{e^6(p+2)}-\frac{d\left(d^2-e^2x^2\right)^{p+3}}{2e^6(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^5*(d^2 - e^2*x^2)^(1 + p))/(2*e^6*(1 + p)) + (d^3*(d^2 - e^2*x^2)^(2 +
p))/(e^6*(2 + p)) - (d*(d^2 - e^2*x^2)^(3 + p))/(2*e^6*(3 + p)) + (e*x^7*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^
2*x^2)/d^2)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^5\left(d^2-e^2x^2\right)^p\,dx + e\int x^6\left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\mathrm{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^6\left(1-\frac{e^2x^2}{d^2}\right.$$
$$= \frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}d\,\mathrm{Subst}\left(\int\left(\frac{d^4\left(d^2-e^2x\right)^p}{e^4}\right.\right.$$
$$= -\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{2e^6(1+p)} + \frac{d^3\left(d^2-e^2x^2\right)^{2+p}}{e^6(2+p)} - \frac{d\left(d^2-e^2x^2\right)^{3+p}}{2e^6(3+p)} + \frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\right.$$

Mathematica [A] time = 0.0985123, size = 132, normalized size = 0.89

$$\frac{\left(d^2-e^2x^2\right)^p\left(2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{7d\left(d^2-e^2x^2\right)\left(2d^2e^2(p+1)x^2+2d^4+e^4\left(p^2+3p+2\right)x^4\right)}{(p+1)(p+2)(p+3)}\right)}{14e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-7*d*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 +
e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (2*e^7*x^7*Hypergeome
tric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

Maple [F] time = 0.472, size = 0, normalized size = 0.

$$\int x^5(ex+d)\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e\int x^6e^{(p\log(ex+d)+p\log(-ex+d))}\,dx + \frac{\left(\left(p^2+3\,p+2\right)e^6x^6-\left(p^2+p\right)d^2e^4x^4-2\,d^4e^2px^2-2\,d^6\right)\left(-e^2x^2+d^2\right)^pd}{2\left(p^3+6\,p^2+11\,p+6\right)e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] e*integrate(x^6*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*((p^2 + 3*p
+ 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*d^6)*(-e^2*x^2 +
 d^2)^p*d/((p^3 + 6*p^2 + 11*p + 6)*e^6)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^6 + dx^5\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^6 + d*x^5)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 8.67009, size = 967, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4
*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x*
*2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e*
*8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e
**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e
**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*
e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2
) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d**2*e**6
 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**
2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*x**4
/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e**6) -
d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(p, -1))
, (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p +
12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*
p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*
e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d**2 -
e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e**6*p**
2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e
**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22
*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e*x**7*hyper((7/2, -p), (
9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)\left(-e^2x^2 + d^2\right)^p x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^5, x)
```

3.241 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=147

$$\frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{2e^5(p+1)}+\frac{d^2\left(d^2-e^2x^2\right)^{p+2}}{e^5(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^5(p+3)}$$

```
[Out] -(d^4*(d^2 - e^2*x^2)^(1 + p))/(2*e^5*(1 + p)) + (d^2*(d^2 - e^2*x^2)^(2 +
p))/(e^5*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^5*(3 + p)) + (d*x^5*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^
2)/d^2)^p)
```

Rubi [A] time = 0.0909236, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 365, 364, 266, 43}

$$\frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{2e^5(p+1)}+\frac{d^2\left(d^2-e^2x^2\right)^{p+2}}{e^5(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^5(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^4*(d^2 - e^2*x^2)^(1 + p))/(2*e^5*(1 + p)) + (d^2*(d^2 - e^2*x^2)^(2 +
p))/(e^5*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^5*(3 + p)) + (d*x^5*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^
2)/d^2)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^4\left(d^2-e^2x^2\right)^p\,dx + e\int x^5\left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}e\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right.$$
$$= \frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{d^4\left(d^2-e^2x\right)^p}{e^4}\right.\right.$$
$$= -\frac{d^4\left(d^2-e^2x^2\right)^{1+p}}{2e^5(1+p)} + \frac{d^2\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)} - \frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^5(3+p)} + \frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x}{d^2}\right.$$

Mathematica [A] time = 0.091418, size = 129, normalized size = 0.88

$$\frac{1}{10}\left(d^2-e^2x^2\right)^p\left(2dx^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{5\left(d^2-e^2x^2\right)\left(2d^2e^2(p+1)x^2+2d^4+e^4\left(p^2+3p+2\right)x^4\right)}{e^5(p+1)(p+2)(p+3)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 + e^
4*(2 + 3*p + p^2)*x^4))/(e^5*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeome
tric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/10
```

Maple [F] time = 0.458, size = 0, normalized size = 0.

$$\int x^4(ex+d)\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^5+dx^4\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^5 + d*x^4)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 7.92963, size = 967, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
5 + e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**
4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2
 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**
8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**
2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e**6 - 8*d**2*e**8*x**2
+ 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*
x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d**2*
e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8
*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*
x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e**6
) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(p,
-1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*
p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e
**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)**p
/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d**
2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e**6
*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p +
12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1
2*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.242 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=120

$$\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}$$

```
[Out] -(d^3*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(1 + p)) + (d*(d^2 - e^2*x^2)^(2 + p)
)/(2*e^4*(2 + p)) + (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2
, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0714619, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 266, 43, 365, 364}

$$\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^3*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(1 + p)) + (d*(d^2 - e^2*x^2)^(2 + p)
)/(2*e^4*(2 + p)) + (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2
, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^3\left(d^2-e^2x^2\right)^p\,dx + e\int x^4\left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{p}$$
$$= \frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}d\,\text{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right)^p}{e^2}-\right.\right.$$
$$= -\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)} + \frac{d\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)} + \frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2}\right.$$

Mathematica [A] time = 0.0847818, size = 106, normalized size = 0.88

$$\frac{\left(d^2-e^2x^2\right)^p\left(2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{5d\left(d^2-e^2x^2\right)\left(d^2+e^2(p+1)x^2\right)}{(p+1)(p+2)}\right)}{10e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/((1 + p)
*(2 + p)) + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p))/(10*e^4)
```

Maple [F] time = 0.442, size = 0, normalized size = 0.

$$\int x^3\left(ex+d\right)\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e\int x^4e^{\left(p\log(ex+d)+p\log(-ex+d)\right)}\,dx + \frac{\left(e^4\left(p+1\right)x^4-d^2e^2px^2-d^4\right)\left(-e^2x^2+d^2\right)^pd}{2\left(p^2+3p+2\right)e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] e*integrate(x^4*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*(e^4*(p + 1)
*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d/((p^2 + 3*p + 2)*e^4)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^4 + dx^3\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 5.72122, size = 382, normalized size = 3.18

$$d\left(\begin{cases} \frac{x^4\left(d^2\right)^p}{4} & \text{for } e = 0 \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p = -2 \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p = -1 \\ -\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise} \end{cases}\right) + \frac{d^{2p}ex^5{}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2/(-
2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6
*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (
-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2),
Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4)
- d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4)
+ e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**
4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) + d*
*(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)\left(-e^2x^2 + d^2\right)^p x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^3, x)
```

3.243 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=119

$$\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^3(p+2)}$$

```
[Out] -(d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(1 + p)) + (d^2 - e^2*x^2)^(2 + p)/(2
*e^3*(2 + p)) + (d*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e
^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0671999, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 365, 364, 266, 43}

$$\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^3(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(1 + p)) + (d^2 - e^2*x^2)^(2 + p)/(2
*e^3*(2 + p)) + (d*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e
^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^2\left(d^2-e^2x^2\right)^p\,dx + e\int x^3\left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}e\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$
$$= \frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right)^p}{e^2} - \frac{(d^2-e^2x)^{1+p}}{e^2}\right)dx, x, x^2\right)$$
$$= -\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)} + \frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)} + \frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A] time = 0.0806659, size = 103, normalized size = 0.87

$$\frac{1}{6}\left(d^2-e^2x^2\right)^p\left(2dx^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{3\left(d^2-e^2x^2\right)\left(d^2+e^2(p+1)x^2\right)}{e^3(p+1)(p+2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/(e^3*(1 +
p)*(2 + p)) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p))/6
```

Maple [F] time = 0.454, size = 0, normalized size = 0.

$$\int x^2(ex+d)\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^3+dx^2\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 5.01855, size = 382, normalized size = 3.21

$$\frac{dd^{2p}x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}+e\begin{cases}\frac{x^4\left(d^2\right)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4\left(d^2-e^2x^2\right)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
3 + e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**
2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*
e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)
), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**
2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e*
*4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e*
*4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
 e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.244 $\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=89

$$\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0336116, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {764, 261, 365, 364}

$$\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x\left(d^2-e^2x^2\right)^p\,dx + e\int x^2\left(d^2-e^2x^2\right)^p\,dx$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)} + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)} + \frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A] time = 0.0472064, size = 89, normalized size = 1.

$$\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.436, size = 0, normalized size = 0.

$$\int x(ex+d)\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^2+dx\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [A] time = 3.84194, size = 85, normalized size = 0.96

$$d\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{d^{2p}ex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2
)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), Tr
ue)) + d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)
/d**2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x, x)
```

3.245 $\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=83

$$dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeomet
ric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

Rubi [A] time = 0.0230472, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {641, 246, 245}

$$dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeomet
ric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+d\int \left(d^2-e^2x^2\right)^p\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+\left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0515144, size = 83, normalized size = 1.

$$dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeomet
ric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int (ex+d)\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex+d)\left(-e^2x^2+d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [A] time = 3.6246, size = 82, normalized size = 0.99

$$dd^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+e\left(\begin{cases}\frac{x^2\left(d^2\right)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e
*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)
**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

3.246 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x} dx$

Optimal. Leaf size=104

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

```
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rubi [A] time = 0.0550911, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 266, 65, 246, 245}

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}dx &= d\int \frac{\left(d^2-e^2x^2\right)^p}{x}dx+e\int \left(d^2-e^2x^2\right)^p dx\\ &= \frac{1}{2}d\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}dx,x,x^2\right)+\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p dx\\ &= ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1\right.}{2d(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0397086, size = 104, normalized size = 1.

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Maple [F] time = 0.45, size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(-x^2e^2+d^2\right)^p}{x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

Sympy [C] time = 7.74988, size = 78, normalized size = 0.75

$$-\frac{de^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+d^{2p}ex{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(
e**2*x**2))/(2*gamma(1 - p)) + d**(2*p)*e*x*hyper((1/2, -p), (3/2,), e**2*x
**2*exp_polar(2*I*pi)/d**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

3.247 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

```
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

Rubi [A] time = 0.0573612, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 365, 364, 266, 65}

$$-\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx = d\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}\,dx + e\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx$$
$$= \frac{1}{2}e\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}\,dx$$
$$= -\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1-\right.}{2d^2(1+p)}$$

Mathematica [A] time = 0.0546103, size = 108, normalized size = 1.

$$-\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

Maple [F] time = 0.461, size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(-x^2e^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

Sympy [C] time = 4.99063, size = 82, normalized size = 0.76

$$-\frac{dd^{2p}{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{ee^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**2,x)
```

```
[Out] -d*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x -
 e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(
e**2*x**2))/(2*gamma(1 - p))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.248 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3} dx$

Optimal. Leaf size=110

$$-\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

```
[Out] -((e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e^2*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2,
 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*(1 + p))
```

Rubi [A] time = 0.0587357, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {764, 266, 65, 365, 364}

$$-\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -((e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e^2*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2,
 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*(1 + p))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^3}\,dx = d\int \frac{\left(d^2-e^2x^2\right)^p}{x^3}\,dx + e\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x^2}\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^p}{x^2}\,dx$$
$$= -\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(2,1+p;2+p,\ldots\right)}{2d^3(1+p)}$$

Mathematica [A] time = 0.0561906, size = 106, normalized size = 0.96

$$\frac{1}{2}e\left(d^2-e^2x^2\right)^p\left(\frac{e\left(e^2x^2-d^2\right){}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{d^3(p+1)} - \frac{2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*((-2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/
(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*Hypergeometric2F1[2, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2])/(d^3*(1 + p))))/2
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(-x^2e^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

Sympy [C] time = 5.13574, size = 85, normalized size = 0.77

$$-\frac{de^{2p}x^{2p}e^{i\pi p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2x^2\Gamma\left(2-p\right)}-\frac{d^{2p}e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,),
d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - d**(2*p)*e*hyper((-1/2, -p), (1/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.249 $\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=178

$$\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^6\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{5d^4\left(d^2-e^2x^2\right)^{p+2}}{2e^6(p+2)}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+3}}{e^6(p+3)}+$$

```
[Out] -((d^6*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p))) + (5*d^4*(d^2 - e^2*x^2)^(2
+ p))/(2*e^6*(2 + p)) - (2*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (d^
2 - e^2*x^2)^(4 + p)/(2*e^6*(4 + p)) + (2*d*e*x^7*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.145467, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1652, 446, 77, 12, 365, 364}

$$\frac{2}{7}dex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^6\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{5d^4\left(d^2-e^2x^2\right)^{p+2}}{2e^6(p+2)}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+3}}{e^6(p+3)}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```


```
[Out] -((d^6*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p))) + (5*d^4*(d^2 - e^2*x^2)^(2
+ p))/(2*e^6*(2 + p)) - (2*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (d^
2 - e^2*x^2)^(4 + p)/(2*e^6*(4 + p)) + (2*d*e*x^7*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^2*x^2)/d^2)^p)
```


Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= \int 2dex^6\left(d^2-e^2x^2\right)^p\,dx+\int x^5\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx\\ &= \frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^6\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^6\left(d^2-e^2x\right)^p}{e^4}-\frac{5d^4\left(d^2-e^2x\right)^{1+p}}{e^4}+\frac{4d^2\left(d^2-e^2x\right)^{2+p}}{e^4}-\frac{\left(d^2-e^2x\right)^{3+p}}{e^4}\right)\right.\\ &= -\frac{d^6\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}+\frac{5d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{2d^2\left(d^2-e^2x^2\right)^{3+p}}{e^6(3+p)}+\frac{\left(d^2-e^2x^2\right)^{4+p}}{2e^6(4+p)}+\frac{2}{7}dex^7\end{aligned}$$

Mathematica [A] time = 0.140939, size = 159, normalized size = 0.89

$$\frac{\left(d^2-e^2x^2\right)^p\left(4de^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)+\frac{7\left(d^2-e^2x^2\right)^4}{p+4}-\frac{28d^2\left(d^2-e^2x^2\right)^3}{p+3}+\frac{35d^4\left(d^2-e^2x^2\right)^2}{p+2}-\frac{14d^6\left(d^2-e^2x^2\right)}{p+1}\right)}{14e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-14*d^6*(d^2 - e^2*x^2))/(1 + p) + (35*d^4*(d^2 - e^2*
x^2)^2)/(2 + p) - (28*d^2*(d^2 - e^2*x^2)^3)/(3 + p) + (7*(d^2 - e^2*x^2)^4
)/(4 + p) + (4*d*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1
 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

Maple [F] time = 0.67, size = 0, normalized size = 0.

$$\int x^5(ex+d)^2\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(p^2+3p+2\right)e^6x^6-\left(p^2+p\right)d^2e^4x^4-2d^4e^2px^2-2d^6\right)\left(-e^2x^2+d^2\right)^pd^2}{2\left(p^3+6p^2+11p+6\right)e^6}+\int\left(e^2x^7+2dex^6\right)e^{\left(p\log(ex+d)+p\log(-ex+d)\right)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*
d^6)*(-e^2*x^2 + d^2)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*e^6) + integrate((e^2
*x^7 + 2*d*e*x^6)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^7+2dex^6+d^2x^5\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^7 + 2*d*e*x^6 + d^2*x^5)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 15.0089, size = 2916, normalized size = 16.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2
+ 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**
2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**
2*e**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 +
 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x
**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d**2*e
**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*
x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*x
**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e**6)
 - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(p, -
1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
 + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/
(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d**2
```

```
 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e**6*
p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 1
2*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 +
 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12
*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 2*d*d**(2*p)*e*x**7*hyper((7/2,
 -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + e**2*Piecewise((x**8*(d
**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10
*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e + x)/(-12*d**6
*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 2*d**6/(
-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) +
18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d*
*2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e + x)/(-12*d**6*e
**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e*
*4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x
**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(d/e + x)/(-12*d**6*e**8 + 36*d
**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 9*d**2*e**4*x**4/(-1
2*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*
e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12
*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*
e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 9*e**6*x**6/(-12*d**6*e*
*8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6), Eq(p, -4)),
(-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6
*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 3*d**
6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2*log(
-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x
**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6*d**2*
e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) -
6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x*
*4) + 6*d**2*e**4*x**4/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 2
*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4), Eq(p, -3)), (-
6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6*log(d/e + x)/(-
4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d**4
*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d**4*e**2*x**2*l
og(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e**4*x**4/(-4*d**2*e**8
+ 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*x**2), Eq(p, -2)), (-d*
*6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d**4*x**2/(2*e**6)
 - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8*(d**2 - e**2*x*
*2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) -
 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*
e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4*(d**2 - e**2*x**2
)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3
*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e*
*8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2 - e**2*x**2)**p
/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**
2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**
8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 - e**2*x**2)**p/(
2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + e**8*p*
*3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 +
100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4
 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 11*e**8*p*x**8*(d*
*2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p
+ 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3
+ 70*e**8*p**2 + 100*e**8*p + 48*e**8), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(-e^2x^2+d^2\right)^p x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^5, x)
```

3.250 $\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=185

$$\frac{2d^2(p+6)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{x^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{d^5\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{2d^3\left(d^2-e^2x^2\right)^{p+2}}{e^5(p+2)}-$$

```
[Out] -((d^5*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p))) - (x^5*(d^2 - e^2*x^2)^(1 +
p))/(7 + 2*p) + (2*d^3*(d^2 - e^2*x^2)^(2 + p))/(e^5*(2 + p)) - (d*(d^2 - e
^2*x^2)^(3 + p))/(e^5*(3 + p)) + (2*d^2*(6 + p)*x^5*(d^2 - e^2*x^2)^p*Hyper
geometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)
^p)
```

Rubi [A] time = 0.173406, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1652, 459, 365, 364, 12, 266, 43}

$$\frac{2d^2(p+6)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{x^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{d^5\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{2d^3\left(d^2-e^2x^2\right)^{p+2}}{e^5(p+2)}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^5*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p))) - (x^5*(d^2 - e^2*x^2)^(1 +
p))/(7 + 2*p) + (2*d^3*(d^2 - e^2*x^2)^(2 + p))/(e^5*(2 + p)) - (d*(d^2 - e
^2*x^2)^(3 + p))/(e^5*(3 + p)) + (2*d^2*(6 + p)*x^5*(d^2 - e^2*x^2)^p*Hyper
geometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)
^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \int 2dex^5\left(d^2-e^2x^2\right)^p\,dx+\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+(2de)\int x^5\left(d^2-e^2x^2\right)^p\,dx+\frac{\left(2d^2(6+p)\right)\int x^4\left(d^2-e^2x^2\right)^p\,dx}{7+2p}$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+(de)\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\frac{\left(2d^2(6+p)\left(d^2-e^2x\right.\right.}{}$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{2d^2(6+p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(7+2p)}+(de$$

$$= -\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{2d^3\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)}-\frac{d\left(d^2-e^2x^2\right)^{3+p}}{e^5(3+p)}+\frac{2a}{}$$

Mathematica [A] time = 0.138504, size = 186, normalized size = 1.01

$$\frac{1}{35}\left(d^2-e^2x^2\right)^p\left(7d^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+5e^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{35d\left(d^2-e^2x^2\right)}{e^5(p+3)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*d^5*(d^2 - e^2*x^2))/(e^5*(1 + p)) + (70*d^3*(d^2
- e^2*x^2)^2)/(e^5*(2 + p)) - (35*d*(d^2 - e^2*x^2)^3)/(e^5*(3 + p)) + (7*d
^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^
p + (5*e^2*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^
2)/d^2)^p))/35
```

Maple [F] time = 0.782, size = 0, normalized size = 0.

$$\int x^{4} (ex+d)^{2} \left(-x^{2} e^{2}+d^{2}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^{2}\left(-e^{2} x^{2}+d^{2}\right)^{p} x^{4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(e^{2} x^{6}+2 \, d e x^{5}+d^{2} x^{4}\right)\left(-e^{2} x^{2}+d^{2}\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^6 + 2*d*e*x^5 + d^2*x^4)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 9.58863, size = 1010, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/5 + 2*d*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)
/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e**6 - 8*d**2*e*
*8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6
- 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e**6 - 8*d**2*e**
8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 +
```

```
2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-
2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 +
 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2)
+ e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/
(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2),
 Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 2
2*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3
 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x
**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x*
*4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
 + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e**2*x**7*hype
r((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(-e^2x^2+d^2\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.251 $\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=149

$$\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}+\frac{3d^2\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^4(p+3)}$$

```
[Out] -((d^4*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p))) + (3*d^2*(d^2 - e^2*x^2)^(2
+ p))/(2*e^4*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^4*(3 + p)) + (2*d*e*x^
5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 -
 (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.129887, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1652, 446, 77, 12, 365, 364}

$$\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^4\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}+\frac{3d^2\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}-\frac{\left(d^2-e^2x^2\right)^{p+3}}{2e^4(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^4*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p))) + (3*d^2*(d^2 - e^2*x^2)^(2
+ p))/(2*e^4*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^4*(3 + p)) + (2*d*e*x^
5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 -
 (e^2*x^2)/d^2)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= \int 2dex^4\left(d^2-e^2x^2\right)^p\,dx+\int x^3\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^4\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{2d^4\left(d^2-e^2x\right)^p}{e^2}-\frac{3d^2\left(d^2-e^2x\right)^{1+p}}{e^2}+\frac{\left(d^2-e^2x\right)^{2+p}}{e^2}\right)dx,x,x^2\right)+\left(2de\right.\\ &= -\frac{d^4\left(d^2-e^2x^2\right)^{1+p}}{e^4(1+p)}+\frac{3d^2\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}-\frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^4(3+p)}+\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\right.\end{aligned}$$

Mathematica [A] time = 0.135975, size = 138, normalized size = 0.93

$$\frac{\left(d^2-e^2x^2\right)^p\left(4de^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{5\left(d^2-e^2x^2\right)\left(d^2e^2\left(p^2+6p+5\right)x^2+d^4(p+5)+e^4\left(p^2+3p+2\right)x^4\right)}{(p+1)(p+2)(p+3)}\right)}{10e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(d^4*(5 + p) + d^2*e^2*(5 + 6*p + p
^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (4*d*e^5*x^
5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(
10*e^4)
```

Maple [F] time = 0.688, size = 0, normalized size = 0.

$$\int x^3(ex+d)^2\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^4(p+1)x^4 - d^2e^2px^2 - d^4\right)\left(-e^2x^2 + d^2\right)^p d^2}{2\left(p^2 + 3p + 2\right)e^4} + \int \left(e^2x^5 + 2\,dex^4\right)e^{\left(p\log(ex+d) + p\log(-ex+d)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^2/((p^2 +
3*p + 2)*e^4) + integrate((e^2*x^5 + 2*d*e*x^4)*e^(p*log(e*x + d) + p*log(-
e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^5 + 2\,dex^4 + d^2x^3\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^5 + 2*d*e*x^4 + d^2*x^3)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 9.37601, size = 1323, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2
/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2))
, (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2
), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) +
 2*d*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d
**2)/5 + e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x
)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e**6 - 8*d**2*e
**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e**6 - 8*d**2*e*
```

```
*8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 +
 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(
-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6
+ 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2)
 + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)
/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2)
, Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 +
22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**
3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*
x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x
**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6
) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e
**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*
p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(-e^2x^2+d^2\right)^p x^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^3, x)
```

3.252 $\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=155

$$\frac{2d^2(p+4)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{x^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{e^3(p+1)}+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{e^3(p+2)}$$

```
[Out] -((d^3*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p))) - (x^3*(d^2 - e^2*x^2)^(1 + 
p))/(5 + 2*p) + (d*(d^2 - e^2*x^2)^(2 + p))/(e^3*(2 + p)) + (2*d^2*(4 + p)*
x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5
 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.138683, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1652, 459, 365, 364, 12, 266, 43}

$$\frac{2d^2(p+4)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{x^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{e^3(p+1)}+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{e^3(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^3*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p))) - (x^3*(d^2 - e^2*x^2)^(1 + 
p))/(5 + 2*p) + (d*(d^2 - e^2*x^2)^(2 + p))/(e^3*(2 + p)) + (2*d^2*(4 + p)*
x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5
 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, 
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = \int 2dex^3\left(d^2-e^2x^2\right)^p\,dx + \int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx$$

$$= -\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p} + (2de)\int x^3\left(d^2-e^2x^2\right)^p\,dx + \frac{\left(2d^2(4+p)\right)\int x^2\left(d^2-e^2x^2\right)^p\,dx}{5+2p}$$

$$= -\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p} + (de)\,\mathrm{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \frac{\left(2d^2(4+p)\left(d^2-e^2x^2\right.\right.}{}$$

$$= -\frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p} + \frac{2d^2(4+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(5+2p)} + (de$$

$$= -\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)} - \frac{x^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p} + \frac{d\left(d^2-e^2x^2\right)^{2+p}}{e^3(2+p)} + \frac{2d^2(4+p)x^3\left(d^2-e^2x^2\right.}{}$$

Mathematica [A] time = 0.116613, size = 168, normalized size = 1.08

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(5d^2e^3\left(p^2+3p+2\right)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+3e^5\left(p^2+3p+2\right)x^5\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-15d\left(d\right.\right.}{15e^3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-15*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(d^2 + e^2*
(1 + p)*x^2) + 5*d^2*e^3*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2] + 3*e^5*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2
, (e^2*x^2)/d^2]))/(15*e^3*(1 + p)*(2 + p)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.653, size = 0, normalized size = 0.

$$\int x^{2} \left(ex+d\right)^{2} \left(-x^{2} e^{2}+d^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^{2}\left(-e^{2} x^{2}+d^{2}\right)^{p} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^{2} x^{4}+2\, d e x^{3}+d^{2} x^{2}\right)\left(-e^{2} x^{2}+d^{2}\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 6.48471, size = 425, normalized size = 2.74

$$\frac{d^{2} d^{2p} x^{3} {}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{e^{2} x^{2} e^{2i\pi}}{d^{2}}\right)}{3} + 2de\begin{cases} \frac{x^{4}\left(d^{2}\right)^{p}}{4} & \text{for}\: e = 0 \\ -\frac{d^{2}\log\left(-\frac{d}{e}+x\right)}{-2d^{2}e^{4}+2e^{6}x^{2}} - \frac{d^{2}\log\left(\frac{d}{e}+x\right)}{-2d^{2}e^{4}+2e^{6}x^{2}} - \frac{d^{2}}{-2d^{2}e^{4}+2e^{6}x^{2}} + \frac{e^{2}x^{2}\log\left(-\frac{d}{e}+x\right)}{-2d^{2}e^{4}+2e^{6}x^{2}} + \frac{e^{2}x^{2}\log\left(\frac{d}{e}+x\right)}{-2d^{2}e^{4}+2e^{6}x^{2}} & \text{for}\: p = -2 \\ -\frac{d^{2}\log\left(-\frac{d}{e}+x\right)}{2e^{4}} - \frac{d^{2}\log\left(\frac{d}{e}+x\right)}{2e^{4}} - \frac{x^{2}}{2e^{2}} & \text{for}\: p = -1 \\ -\frac{d^{4}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} - \frac{d^{2}e^{2}px^{2}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} + \frac{e^{4}px^{4}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} + \frac{e^{4}x^{4}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/3 + 2*d*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(
-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2
) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e*
*4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq
```

```
(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2
/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*
e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4),
True)) + d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*
I*pi)/d**2)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.253 $\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^2(p+2)}$$

```
[Out] -((d^2*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p))) + (d^2 - e^2*x^2)^(2 + p)/(2
*e^2*(2 + p)) + (2*d*e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0913054, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1652, 444, 43, 12, 365, 364}

$$\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^2(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^2*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p))) + (d^2 - e^2*x^2)^(2 + p)/(2
*e^2*(2 + p)) + (2*d*e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x(d+ex)^2\left(d^2-e^2x^2\right)^p dx &= \int 2dex^2\left(d^2-e^2x^2\right)^p dx+\int x\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^2\left(d^2-e^2x^2\right)^p dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(2d^2\left(d^2-e^2x\right)^p-\left(d^2-e^2x\right)^{1+p}\right)dx,x,x^2\right)+\left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right.\right.\\ &= -\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{e^2(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^2(2+p)}+\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2}\right.\end{aligned}$$

Mathematica [A] time = 0.0813278, size = 110, normalized size = 0.93

$$\frac{\left(d^2-e^2x^2\right)^p\left(4de^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{3\left(d^2-e^2x^2\right)\left(d^2(p+3)+e^2(p+1)x^2\right)}{(p+1)(p+2)}\right)}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2*(3 + p) + e^2*(1 + p)*x^2))/((
1 + p)*(2 + p)) + (4*d*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^
2])/(1 - (e^2*x^2)/d^2)^p))/(6*e^2)
```

Maple [F] time = 0.666, size = 0, normalized size = 0.

$$\int x\left(ex+d\right)^2\left(-x^2e^2+d^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^3 + 2\,dex^2 + d^2x\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^3 + 2*d*e*x^2 + d^2*x)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [A] time = 5.59443, size = 440, normalized size = 3.73

$$d^2\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2 = 0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1\\ \log\left(d^2 - e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right) + \frac{2dd^{2p}ex^3{}_2F_1\left(\begin{matrix}\frac{3}{2}, -p\\ \frac{5}{2}\end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3} + e^2\left(\begin{cases}\frac{x^4(d^2)^p}{4}\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \cdots\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \cdots\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} \cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x
**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
 True)) + 2*d*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(
2*I*pi)/d**2)/3 + e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-
d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(
-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6
*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e*
*4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e
**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2\left(-e^2x^2 + d^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x, x)
```

3.254 $\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=71

$$-\frac{d2^{p+2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\, {}_2F_1\left(-p-2,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[Out] -((2^(2 + p)*d*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[-2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rubi [A] time = 0.0306384, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{d2^{p+2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\, {}_2F_1\left(-p-2,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((2^(2 + p)*d*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[-2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx &= \left(d(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{2+p}\, dx\\ &= -\frac{2^{2+p}d\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\, {}_2F_1\left(-2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0607756, size = 134, normalized size = 1.89

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(3d^2e(p+1)x\, {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+e^3(p+1)x^3\, {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-3d\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p\right)}{3e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-3*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p + 3*d^2*e*(1
 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + e^3*(1 + p)*x^3*Hy
pergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2]))/(3*e*(1 + p)*(1 - (e^2*x^2)/
d^2)^p)
```

Maple [F] time = 0.642, size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(-x^2e^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(-e^2x^2+d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [A] time = 3.8257, size = 124, normalized size = 1.75

$$d^2d^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+2de\left(\begin{cases}\frac{x^2\left(d^2\right)^p}{2} & \text{for}\ e^2=0\\ -\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{d^{2p}e^2x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
+ 2*d*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**
2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**
2), True)) + d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p, x)
```

3.255 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x}\,dx$

Optimal. Leaf size=128

$$2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2(p+1)}$$

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*(1 + p)) + (2*d*e*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
 p))
```

Rubi [A] time = 0.0951983, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1652, 446, 80, 65, 12, 246, 245}

$$2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*(1 + p)) + (2*d*e*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
 p))
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
```

```
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x}\,dx = \int 2de\left(d^2-e^2x^2\right)^p\,dx + \int \frac{\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)}{x}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)}{x}\,dx, x, x^2\right) + (2de)\int \left(d^2-e^2x^2\right)^p\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} + \frac{1}{2}d^2\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + \left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)$$

$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} + 2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{\left(d^2-e^2x^2\right)^{1+p}\ {}_2}{}$$

Mathematica [A] time = 0.0706256, size = 103, normalized size = 0.8

$$\frac{1}{2}\left(d^2-e^2x^2\right)^p\left(4dex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)\left({}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)+1\right)}{p+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
)/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)*(1 + Hypergeometric2F1[1, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2]))/(1 + p)))/2
```

Maple [F] time = 0.592, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\left(-x^2e^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(-e^2x^2+d^2\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x, x)
```

Sympy [A] time = 7.83054, size = 136, normalized size = 1.06

$$-\frac{d^2e^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+2dd^{2p}ex{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+e^2\left(\begin{cases}\frac{x^2\left(d^2\right)^p}{2} & \text{for } e^2 \\ -\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{other}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**
2/(e**2*x**2))/(2*gamma(1 - p)) + 2*d*d**(2*p)*e*x*hyper((1/2, -p), (3/2,),
 e**2*x**2*exp_polar(2*I*pi)/d**2) + e**2*Piecewise((x**2*(d**2)**p/2, Eq(e
**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (lo
g(d**2 - e**2*x**2), True))/(2*e**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

3.256 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx$

Optimal. Leaf size=128

$$-2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{x}$$

```
[Out] -((d^2 - e^2*x^2)^(1 + p)/x) - (2*e^2*p*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e*(d^2 - e^2*x^2)
^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(d*(1 + p))
```

Rubi [A] time = 0.115745, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1807, 764, 266, 65, 246, 245}

$$-2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d^2 - e^2*x^2)^(1 + p)/x) - (2*e^2*p*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e*(d^2 - e^2*x^2)
^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(d*(1 + p))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\int \frac{\left(-2d^3e+2d^2e^2px\right)\left(d^2-e^2x^2\right)^p}{x}\,dx}{d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x} + (2de)\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx - \left(2e^2p\right)\int \left(d^2-e^2x^2\right)^p\,dx$$

$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x} + (de)\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) - \left(2e^2p\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right.$$

$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{x} - 2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{e\left(d^2-e^2x^2\right)^{1+p}}{}$$

Mathematica [A] time = 0.0771194, size = 153, normalized size = 1.2

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(ex\left(de(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)\right)-d^3(p\right.}{d(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-(d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2
)/d^2]) + e*x*(d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
 - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 - (e^2*x^2)/d^2])))/(d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.587, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\left(-x^2e^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2+2\,dex+d^2)(-e^2x^2+d^2)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x^2, x)
```

Sympy [C] time = 6.27673, size = 116, normalized size = 0.91

$$-\frac{d^2d^{2p}{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{dee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{\Gamma(1-p)}+d^{2p}e^2x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**2,x)
```

```
[Out] -d**2*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
x - d*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d
**2/(e**2*x**2))/gamma(1 - p) + d**(2*p)*e**2*x*hyper((1/2, -p), (3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.257 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3} dx$

Optimal. Leaf size=139

$$-\frac{2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(1-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2x^2}$$

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*x^2) - (2*d*e*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(1 - p)*
(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^
2])/(2*d^2*(1 + p))
```

Rubi [A] time = 0.124664, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1807, 764, 365, 364, 266, 65}

$$-\frac{2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(1-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -(d^2 - e^2*x^2)^(1 + p)/(2*x^2) - (2*d*e*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(1 - p)*
(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^
2])/(2*d^2*(1 + p))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x^3}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{\int \frac{\left(-4d^3e-2d^2e^2(1-p)x\right)\left(d^2-e^2x^2\right)^p}{x^2}\,dx}{2d^2}\\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2}+(2de)\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}\,dx+\left(e^2(1-p)\right)\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2}+\frac{1}{2}\left(e^2(1-p)\right)\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x,x^2\right)+\left(2de\left(d^2-e^2x^2\right)^p\left(1\right.\right.\\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(1-p)\left(d^2-e\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.087817, size = 131, normalized size = 0.94

$$\frac{e\left(d^2-e^2x^2\right)^p\left(\frac{e\left(e^2x^2-d^2\right)\left({}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)+{}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)\right)}{p+1}-\frac{4d^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*((-4*d^3*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*(Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e
^2*x^2)/d^2]))/(1 + p)))/(2*d^2)
```

Maple [F] time = 0.632, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\left(-x^2e^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2+2\,dex+d^2)(-e^2x^2+d^2)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p/x^3, x)
```

Sympy [C] time = 7.07993, size = 139, normalized size = 1.

$$-\frac{d^2e^{2p}x^{2p}e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2x^2\Gamma(2-p)}-\frac{2dd^{2p}e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{e^2e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,
), d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - 2*d*d**(2*p)*e*hyper((-1/2, -p
), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - e**2*e**(2*p)*x**(2*p)*exp
(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*gamma(1 -
 p))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.258 $\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=222

$$\frac{2d^2e(3p+17)x^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7(2p+9)}-\frac{ex^7\left(d^2-e^2x^2\right)^{p+1}}{2p+9}-\frac{2d^7\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{11d^5\left(d^2-e^2x^2\right.}{2e^6(p+2)}$$

```
[Out] (-2*d^7*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) - (e*x^7*(d^2 - e^2*x^2)^(1
+ p))/(9 + 2*p) + (11*d^5*(d^2 - e^2*x^2)^(2 + p))/(2*e^6*(2 + p)) - (5*d^3
*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (3*d*(d^2 - e^2*x^2)^(4 + p))/(2*
e^6*(4 + p)) + (2*d^2*e*(17 + 3*p)*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[
7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(9 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.191047, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1652, 446, 77, 459, 365, 364}

$$\frac{2d^2e(3p+17)x^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7(2p+9)}-\frac{ex^7\left(d^2-e^2x^2\right)^{p+1}}{2p+9}-\frac{2d^7\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{11d^5\left(d^2-e^2x^2\right.}{2e^6(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```


```
[Out] (-2*d^7*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) - (e*x^7*(d^2 - e^2*x^2)^(1
+ p))/(9 + 2*p) + (11*d^5*(d^2 - e^2*x^2)^(2 + p))/(2*e^6*(2 + p)) - (5*d^3
*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (3*d*(d^2 - e^2*x^2)^(4 + p))/(2*
e^6*(4 + p)) + (2*d^2*e*(17 + 3*p)*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[
7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(9 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```


Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^5\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^6\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=-\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{1}{2}\mathrm{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\frac{\left(2d^2e(17\right.}{}$$

$$=-\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{1}{2}\mathrm{Subst}\left(\int\left(\frac{4d^7\left(d^2-e^2x\right)^p}{e^4}-\frac{11d^5\left(d^2-e^2x\right)^{1+p}}{e^4}+\frac{10d^3\left(d^2\right.}{}\right.\right.$$

$$=-\frac{2d^7\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}-\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{11d^5\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{5d^3\left(d^2-e^2x^2\right)^{3+p}}{e^6(3+p)}$$

Mathematica [A] time = 0.268009, size = 205, normalized size = 0.92

$$\frac{\left(d^2-e^2x^2\right)^p\left(54d^2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)+14e^9x^9\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{9}{2},-p;\frac{11}{2};\frac{e^2x^2}{d^2}\right)+\frac{189d\left(d^2-e^2x^2\right)^4}{p+4}-\frac{630(}{}\right.}{126e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-252*d^7*(d^2 - e^2*x^2))/(1 + p) + (693*d^5*(d^2 - e^
2*x^2)^2)/(2 + p) + (189*d*(d^2 - e^2*x^2)^4)/(4 + p) - (630*(d^3 - d*e^2*x
^2)^3)/(3 + p) + (54*d^2*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/
d^2])/(1 - (e^2*x^2)/d^2)^p + (14*e^9*x^9*Hypergeometric2F1[9/2, -p, 11/2,
(e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(126*e^6)
```

Maple [F] time = 0.587, size = 0, normalized size = 0.

$$\int x^5\left(ex+d\right)^3\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(p^2+3p+2\right)e^6x^6-\left(p^2+p\right)d^2e^4x^4-2d^4e^2px^2-2d^6\right)\left(-e^2x^2+d^2\right)^p d^3}{2\left(p^3+6p^2+11p+6\right)e^6}+\int\left(e^3x^8+3de^2x^7+3d^2ex^6\right)e^{\left(p\log(ex+d)+p\log(\cdot\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*e^6*x^6 - (p^2 + p)*d^2*e^4*x^4 - 2*d^4*e^2*p*x^2 - 2*
d^6)*(-e^2*x^2 + d^2)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*e^6) + integrate((e^3
*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6)*e^(p*log(e*x + d) + p*log(-e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^3x^8+3de^2x^7+3d^2ex^6+d^3x^5\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(-e^2*x^2 + d^2)^p
, x)
```

Sympy [B] time = 19.1311, size = 2958, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2
+ 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**
2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**
2*e**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 +
 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x
**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d**2*e
**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*
x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*x
```

```
**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e**6)
 - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(p, -
1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
 + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/
(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d**2
 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e**6*
p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 1
2*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 +
 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12
*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d**2*d**(2*p)*e*x**7*hyper((7
/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + 3*d*e**2*Piecewise((
x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**
4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e + x)/(-
12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 2
*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x
**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2
 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e + x)/(-12
*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*
d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*
e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(d/e + x)/(-12*d**6*e**8
 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 9*d**2*e**4*x
**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**
6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**
2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e + x)/(-12*d**6*e**8 + 3
6*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 9*e**6*x**6/(-12*
d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6), Eq(p,
 -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x*
*4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4)
 - 3*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x*
*2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4
*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) -
6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x
**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e
**12*x**4) + 6*d**2*e**4*x**4/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x*
*4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4), Eq(p, -
3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6*log(d/e
 + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**10*x**2) +
 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d**4*e**2
*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e**4*x**4/(-4*d**
2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*x**2), Eq(p, -2)
), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d**4*x**2/(
2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8*(d**2 -
e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*
e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**
3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4*(d**2 - e*
*2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e*
*8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3
 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2 - e**2*x
**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8)
 - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 +
 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 - e**2*x**
2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) +
e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*
p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**2 - e**2*x**2)**p/(2*e*
*8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 11*e**8*p*x
**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*
e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**
8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True)) + d**(2*p)*e**3*x**9*
```

```
hyper((9/2, -p), (11/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/9
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^5, x)
```

3.259 $\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=218

$$\frac{2d^3(p+11)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{2d^6\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{9d^4\left(d^2-e^2x\right.}{2e^5(p+}$$

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) - (3*d*x^5*(d^2 - e^2*x^2)^(
1 + p))/(7 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(2 + p))/(2*e^5*(2 + p)) - (3*d^
2*(d^2 - e^2*x^2)^(3 + p))/(e^5*(3 + p)) + (d^2 - e^2*x^2)^(4 + p)/(2*e^5*(
4 + p)) + (2*d^3*(11 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p,
7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.177168, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1652, 459, 365, 364, 446, 77}

$$\frac{2d^3(p+11)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{2d^6\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{9d^4\left(d^2-e^2x\right.}{2e^5(p+}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) - (3*d*x^5*(d^2 - e^2*x^2)^(
1 + p))/(7 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(2 + p))/(2*e^5*(2 + p)) - (3*d^
2*(d^2 - e^2*x^2)^(3 + p))/(e^5*(3 + p)) + (d^2 - e^2*x^2)^(4 + p)/(2*e^5*(
4 + p)) + (2*d^3*(11 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p,
7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx &= \int x^4\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^5\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)dx\\ &= -\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\operatorname{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(3d^2e+e^3x\right)dx,x,x^2\right)+\frac{\left(2d^3(11+p\right.}{}\\ &= -\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{4d^6\left(d^2-e^2x\right)^p}{e^3}-\frac{9d^4\left(d^2-e^2x\right)^{1+p}}{e^3}+\frac{6d^2\left(d^2-e\right.}{e^3}\right.\right.\\ &= -\frac{2d^6\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{9d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^5(2+p)}-\frac{3d^2\left(d^2-e^2x^2\right)^{3+p}}{e^5(3+p)}+\end{aligned}$$

Mathematica [A] time = 0.268524, size = 219, normalized size = 1.

$$\frac{1}{70}\left(d^2-e^2x^2\right)^p\left(14d^3x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+30de^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{35\left(d^2-e^2x^2\right)(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*(d^2 - e^2*x^2)*(6*d^6*(5 + p) + 6*d^4*e^2*(5 + 6*
p + p^2)*x^2 + 3*d^2*e^4*(10 + 17*p + 8*p^2 + p^3)*x^4 + e^6*(6 + 11*p + 6*
p^2 + p^3)*x^6))/(e^5*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyperg
eometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (30*d*e^2
*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p)
)/70
```

Maple [F] time = 0.586, size = 0, normalized size = 0.

$$\int x^4\left(ex+d\right)^3\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^7+3\,de^2x^6+3\,d^2ex^5+d^3x^4\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(-e^2*x^2 + d^2)^p
, x)
```

Sympy [B] time = 17.9917, size = 2958, normalized size = 13.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/5 + 3*d**2*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e
**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e
**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e**6 - 8*d**2*
e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6
 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4
/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**
6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**
2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e +
x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**
```

```
2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**
2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p
*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e*
*6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22
*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*
e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**
6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d*d**(2*p)*e**2*x*
*7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + e**3*Piec
ewise((x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 +
 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e
+ x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x*
*6) - 2*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*
e**14*x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**1
0*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e +
x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6
) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 3
6*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(d/e + x)/(-12*d*
*6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 9*d**2
*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e*
*14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 -
 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e + x)/(-12*d**6*e
**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 9*e**6*x**
6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6)
, Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e
**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12
*x**4) - 3*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*
e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) +
12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x
**4) - 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*
e**12*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**
2 + 4*e**12*x**4) + 6*d**2*e**4*x**4/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e
**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4),
Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6*l
og(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**10*
x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d*
*4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e**4*x**4/
(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*x**2), Eq
(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d**4
*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8*(
d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*
p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e
**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4*(d*
*2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p
+ 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**
8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2 -
 e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48
*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8
*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 - e
**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e
**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 7
0*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**8*(d**2 - e**2*x**2)**
p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 11*e
**8*p*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2
 + 100*e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 +
 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^4, x)
```

3.260 $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=193

$$\frac{2d^2e(3p+13)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{2d^5\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}+\frac{7d^3\left(d^2-e^2x^2\right)}{2e^4(p+2)}$$

```
[Out] (-2*d^5*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) - (e*x^5*(d^2 - e^2*x^2)^(1
+ p))/(7 + 2*p) + (7*d^3*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) - (3*d*(d
^2 - e^2*x^2)^(3 + p))/(2*e^4*(3 + p)) + (2*d^2*e*(13 + 3*p)*x^5*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (
e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.182401, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1652, 446, 77, 459, 365, 364}

$$\frac{2d^2e(3p+13)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{ex^5\left(d^2-e^2x^2\right)^{p+1}}{2p+7}-\frac{2d^5\left(d^2-e^2x^2\right)^{p+1}}{e^4(p+1)}+\frac{7d^3\left(d^2-e^2x^2\right)}{2e^4(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^5*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) - (e*x^5*(d^2 - e^2*x^2)^(1
+ p))/(7 + 2*p) + (7*d^3*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) - (3*d*(d
^2 - e^2*x^2)^(3 + p))/(2*e^4*(3 + p)) + (2*d^2*e*(13 + 3*p)*x^5*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (
e^2*x^2)/d^2)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^3\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^4\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\operatorname{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\frac{\left(2d^2e(13\right.}{}$$

$$=-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{4d^5\left(d^2-e^2x\right)^p}{e^2}-\frac{7d^3\left(d^2-e^2x\right)^{1+p}}{e^2}+\frac{3d\left(d^2-\right.}{e^2}\right.\right.$$

$$=-\frac{2d^5\left(d^2-e^2x^2\right)^{1+p}}{e^4(1+p)}-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{7d^3\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}-\frac{3d\left(d^2-e^2x^2\right)^{3+p}}{2e^4(3+p)}+$$

Mathematica [A] time = 0.195975, size = 187, normalized size = 0.97

$$\frac{\left(d^2-e^2x^2\right)^p\left(42d^2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+10e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{35d\left(d^2-e^2x^2\right)\left(d^2e^2\left(p^2+10\right.\right.}{(p+}\right.}{70e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*d*(d^2 - e^2*x^2)*(d^4*(9 + p) + d^2*e^2*(9 + 10*p
 + p^2)*x^2 + 3*e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (42*d
^2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d
^2)^p + (10*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e
^2*x^2)/d^2)^p))/(70*e^4)
```

Maple [F] time = 0.588, size = 0, normalized size = 0.

$$\int x^3\left(ex+d\right)^3\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^4(p+1)x^4 - d^2e^2px^2 - d^4\right)\left(-e^2x^2 + d^2\right)^p d^3}{2\left(p^2 + 3p + 2\right)e^4} + \int \left(e^3x^6 + 3\,de^2x^5 + 3\,d^2ex^4\right)e^{\left(p\log(ex+d) + p\log(-ex+d)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^3/((p^2 +
3*p + 2)*e^4) + integrate((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4)*e^(p*log(e*
x + d) + p*log(-e*x + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^6 + 3\,de^2x^5 + 3\,d^2ex^4 + d^3x^3\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)*(-e^2*x^2 + d^2)^p
, x)
```

Sympy [B] time = 12.729, size = 1365, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2
/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2))
, (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2
), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) +
 3*d**2*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi
)/d**2)/5 + 3*d*e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-
d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e +
```

```
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e**6 - 8
*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d*
*4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2
*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**
2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**
8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d
/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(
4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*
p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e
**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2
- e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e
**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p +
12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(
2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e**3*x
**7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^3, x)
```

3.261 $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=189

$$\frac{2d^3(p+7)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{3dx^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}-\frac{2d^4\left(d^2-e^2x^2\right)^{p+1}}{e^3(p+1)}+\frac{5d^2\left(d^2-e^2x^2\right)^{p+}}{2e^3(p+2)}$$

```
[Out] (-2*d^4*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) - (3*d*x^3*(d^2 - e^2*x^2)^(
1 + p))/(5 + 2*p) + (5*d^2*(d^2 - e^2*x^2)^(2 + p))/(2*e^3*(2 + p)) - (d^2
- e^2*x^2)^(3 + p)/(2*e^3*(3 + p)) + (2*d^3*(7 + p)*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5 + 2*p)*(1 - (e^2*x^2)/
d^2)^p)
```

Rubi [A] time = 0.168645, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1652, 459, 365, 364, 446, 77}

$$\frac{2d^3(p+7)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{3dx^3\left(d^2-e^2x^2\right)^{p+1}}{2p+5}-\frac{2d^4\left(d^2-e^2x^2\right)^{p+1}}{e^3(p+1)}+\frac{5d^2\left(d^2-e^2x^2\right)^{p+}}{2e^3(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```


```
[Out] (-2*d^4*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) - (3*d*x^3*(d^2 - e^2*x^2)^(
1 + p))/(5 + 2*p) + (5*d^2*(d^2 - e^2*x^2)^(2 + p))/(2*e^3*(2 + p)) - (d^2
- e^2*x^2)^(3 + p)/(2*e^3*(3 + p)) + (2*d^3*(7 + p)*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5 + 2*p)*(1 - (e^2*x^2)/
d^2)^p)
```


Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = \int x^2\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^3\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$= -\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(3d^2e+e^3x\right)dx,x,x^2\right)+\frac{\left(2d^3(7+\right.}{}$$

$$= -\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^4\left(d^2-e^2x\right)^p}{e}-\frac{5d^2\left(d^2-e^2x\right)^{1+p}}{e}+\frac{\left(d^2-e^2\right.}{e}\right.\right.$$

$$= -\frac{2d^4\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)}-\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{5d^2\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)}-\frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^3(3+p)}+$$

Mathematica [A] time = 0.206004, size = 187, normalized size = 0.99

$$\frac{1}{30}\left(d^2-e^2x^2\right)^p\left(10d^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+18de^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{15\left(d^2-e^2x\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-15*(d^2 - e^2*x^2)*(d^4*(11 + 3*p) + d^2*e^2*(11 + 14
*p + 3*p^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/(e^3*(1 + p)*(2 + p)*(3 + p)) +
 (10*d^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)
/d^2)^p + (18*d*e^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1
- (e^2*x^2)/d^2)^p))/30
```

Maple [F] time = 0.583, size = 0, normalized size = 0.

$$\int x^2(ex+d)^3\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 \left(-e^2x^2 + d^2\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^5 + 3\,de^2x^4 + 3\,d^2ex^3 + d^3x^2\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)*(-e^2*x^2 + d^2)^p
, x)
```

Sympy [B] time = 10.4822, size = 1365, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/3 + 3*d**2*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x
)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x
**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2),
 Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x
**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4
), True)) + 3*d*d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/5 + e**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**
4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*lo
g(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - d**4/(4*d**4*e
**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*
```

```
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x
)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e +
x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e +
x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 2*e**4*x**4/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(
-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x*
*2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/
(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6
+ 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4
*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) -
 x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1
2*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)*
*p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4
*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) -
 d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**
6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**
2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^2, x)
```

3.262 $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=116

$$-\frac{3d^3 2^{p+3} \left(d^2-e^2x^2\right)^{p+1} \left(\frac{ex}{d}+1\right)^{-p-1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e^2(p+1)(2p+5)}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)}$$

```
[Out] -(((d + e*x)^3*(d^2 - e^2*x^2)^(1 + p))/(e^2*(5 + 2*p))) - (3*2^(3 + p)*d^3
*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[-3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])/(e^2*(1 + p)*(5 + 2*p))
```

Rubi [A] time = 0.0663772, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {795, 678, 69}

$$-\frac{3d^3 2^{p+3} \left(d^2-e^2x^2\right)^{p+1} \left(\frac{ex}{d}+1\right)^{-p-1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e^2(p+1)(2p+5)}-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(((d + e*x)^3*(d^2 - e^2*x^2)^(1 + p))/(e^2*(5 + 2*p))) - (3*2^(3 + p)*d^3
*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[-3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])/(e^2*(1 + p)*(5 + 2*p))
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int x(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)} + \frac{(3d)\int(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx}{e(5+2p)}$$
$$= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)} + \frac{\left(3d^3(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int(d-ex)^p\,(}{e(5+2p)}$$
$$= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)} - \frac{3\,2^{3+p}d^3\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(-3-p,1+p;2\right.}{e^2(1+p)(5+2p)}$$

Mathematica [A] time = 0.290134, size = 159, normalized size = 1.37

$$\frac{\left(d^2-e^2x^2\right)^p\left(10d^2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)-\frac{5d\left(d^2-e^2x^2\right)\left(d^2(p+5)+3e^2(\right.}{(p+1)(p+2)}}{10e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2*(5 + p) + 3*e^2*(1 + p)*x^2)
)/((1 + p)*(2 + p)) + (10*d^2*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*
x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/
2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(10*e^2)
```

Maple [F] time = 0.593, size = 0, normalized size = 0.

$$\int x\left(ex+d\right)^3\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^3x^4+3\,de^2x^3+3\,d^2ex^2+d^3x\right)\left(-e^2x^2+d^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x)*(-e^2*x^2 + d^2)^p,
x)
```

Sympy [A] time = 7.5621, size = 479, normalized size = 4.13

$$d^3\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+d^2d^{2p}ex^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+3de^2\left(\begin{cases}\frac{x^4(d^2)^p}{4}\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-}{2e^4p^2+6e^4p}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x
**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
 True)) + d**2*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar
(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*lo
g(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4
+ 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x
)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2
*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 +
6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*
p + 4*e**4), True)) + d**(2*p)*e**3*x**5*hyper((5/2, -p), (7/2,), e**2*x**2
*exp_polar(2*I*pi)/d**2)/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x, x)
```

3.263 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=73

$$-\frac{d^2 2^{p+3} \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

```
[Out] -((2^(3 + p)*d^2*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeomet
ric2F1[-3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rubi [A] time = 0.0256134, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{d^2 2^{p+3} \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((2^(3 + p)*d^2*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeomet
ric2F1[-3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned} \int (d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx &= \left(d^2(d-ex)^{-1-p} \left(1+\frac{ex}{d}\right)^{-1-p} \left(d^2-e^2x^2\right)^{1+p}\right) \int (d-ex)^p \left(1+\frac{ex}{d}\right)^{3+p} \, dx \\ &= -\frac{2^{3+p} d^2 \left(1+\frac{ex}{d}\right)^{-1-p} \left(d^2-e^2x^2\right)^{1+p} {}_2F_1\left(-3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)} \end{aligned}$$

Mathematica [B] time = 0.176499, size = 155, normalized size = 2.12

$$\frac{1}{2}\left(d^2-e^2x^2\right)^p \left(2d^3x\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+2de^2x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+\frac{\left(e^2x^2-d^2\right)\left(d^2(3\right.}{e(p+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(((-d^2 + e^2*x^2)*(d^2*(7 + 3*p) + e^2*(1 + p)*x^2))/(e
*(1 + p)*(2 + p)) + (2*d^3*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
)/(1 - (e^2*x^2)/d^2)^p + (2*d*e^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2
*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/2
```

Maple [F] time = 0.618, size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(-x^2e^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-e^2x^2+d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p, x)
```

Sympy [B] time = 7.05073, size = 476, normalized size = 6.52

$$d^3d^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+3d^2e\left(\begin{cases}\frac{x^2\left(d^2\right)^p}{2} & \text{for}\ e^2=0\\ -\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+dd^{2p}e^2x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
+ 3*d**2*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 -
e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*
e**2), True)) + d*d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp
_polar(2*I*pi)/d**2) + e**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*
log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e +
 x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/
(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*
p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*
p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p, x)
```

3.264 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x}\,dx$

Optimal. Leaf size=171

$$\frac{2d^2e(3p+5)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{d\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{ex\left(d^2-e^2x\right.}{2p+3}$$

```
[Out] (-3*d*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (e*x*(d^2 - e^2*x^2)^(1 + p))/
(3 + 2*p) + (2*d^2*e*(5 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -
p, 3/2, (e^2*x^2)/d^2])/((3 + 2*p)*(1 - (e^2*x^2)/d^2)^p) - (d*(d^2 - e^2*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
p))
```

Rubi [A] time = 0.128589, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1652, 446, 80, 65, 388, 246, 245}

$$\frac{2d^2e(3p+5)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{d\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{ex\left(d^2-e^2x\right.}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (-3*d*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (e*x*(d^2 - e^2*x^2)^(1 + p))/
(3 + 2*p) + (2*d^2*e*(5 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -
p, 3/2, (e^2*x^2)/d^2])/((3 + 2*p)*(1 - (e^2*x^2)/d^2)^p) - (d*(d^2 - e^2*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
p))
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x}\,dx &= \int \frac{\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)}{x}\,dx+\int \left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\ &= -\frac{ex\left(d^2-e^2x^2\right)^{1+p}}{3+2p}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)}{x}\,dx,x,x^2\right)+\frac{\left(2d^2e(5+3p)\right)}{3}\\ &= -\frac{3d\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)}-\frac{ex\left(d^2-e^2x^2\right)^{1+p}}{3+2p}+\frac{1}{2}d^3\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x,x^2\right)+\frac{\left(2d^2e\right.}{}\\ &= -\frac{3d\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)}-\frac{ex\left(d^2-e^2x^2\right)^{1+p}}{3+2p}+\frac{2d^2e(5+3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\right.}{3+2p}\end{aligned}$$

Mathematica [A] time = 0.152175, size = 169, normalized size = 0.99

$$\frac{1}{6}\left(d^2-e^2x^2\right)^p\left(18d^2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{3d\left(d^2-e^2x^2\right)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-9*d*(d^2 - e^2*x^2))/(1 + p) + (18*d^2*e*x*Hypergeome
tric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*d*(d^2 - e
^2*x^2)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(1 + p) + (2
*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2
```

```
)^p))/6
```

Maple [F] time = 0.59, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(-x^2e^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-e^2x^2+d^2\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x, x)
```

Sympy [A] time = 11.8515, size = 178, normalized size = 1.04

$$-\frac{d^3e^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+3d^2d^{2p}ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+3de^2\left(\begin{cases}\frac{x^2\left(d^2\right)^p}{2} & \text{for } e^2 \\ -\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+1}}{p+1} & \text{for } p\neq-1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{other}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d**3*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**
2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d**2*d**(2*p)*e*x*hyper((1/2, -p), (3/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**2*(d**2)**p/
2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(d**2 - e**2*x**2), True))/(2*e**2), True)) + d**(2*p)*e**3*x**3*hy
per((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x, x)
```

3.265 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^2} dx$

Optimal. Leaf size=159

$$2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{3e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{e\left(d^2-e^2x\right.}{2(p+}$$

```
[Out] -(e*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (d*(d^2 - e^2*x^2)^(1 + p))/x +
(2*d*e^2*(1 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x
^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometr
ic2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 + p))
```

Rubi [A] time = 0.184813, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1807, 1652, 446, 80, 65, 12, 246, 245}

$$2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{3e\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{e\left(d^2-e^2x\right.}{2(p+}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```


```
[Out] -(e*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (d*(d^2 - e^2*x^2)^(1 + p))/x +
(2*d*e^2*(1 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x
^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometr
ic2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 + p))
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-3d^4e-2d^3e^2(1-p)x-d^2e^3x^2\right)}{x}\,dx}{d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\int -2d^3e^2(1-p)\left(d^2-e^2x^2\right)^p\,dx}{d^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-3d^4e-d^2e^3x^2\right)}{x}\,dx}{d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(-3d^4e-d^2e^3x\right)}{x}\,dx, x, x^2\right)}{2d^2} + \left(2de^2(1-p)\right)\int \left(d^2-e^2 \right.$$

$$= -\frac{e\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} - \frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} + \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + \left(2d\right.$$

$$= -\frac{e\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} - \frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} + 2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, \right.$$

Mathematica [A] time = 0.0970604, size = 158, normalized size = 0.99

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(ex\left(6de(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(3\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)+\right.\right.\right.}{2(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-2*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^
2)/d^2] + e*x*(6*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^
2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(1 + 3*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 - (e^2*x^2)/d^2]))))/(2*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.61, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 \left(-x^2e^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-e^2x^2+d^2\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x^2,
x)
```

Sympy [A] time = 7.36551, size = 177, normalized size = 1.11

$$-\frac{d^3d^{2p}{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{3d^2ee^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+3dd^{2p}e^2x{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+e^3\left(\left\{\begin{matrix}\frac{x^2\left(d^2\right)^p}{2}\\ -\frac{\begin{cases}\frac{\left(d^2-e^2x^2\right)^{p+}}{p+1}\\ \log\left(d^2-\right.\end{cases}}{}\end{matrix}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**2,x)
```

```
[Out] -d**3*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
x - 3*d**2*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p
,), d**2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d*d**(2*p)*e**2*x*hyper((1/2, -p
), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e**3*Piecewise((x**2*(d**2)*
*p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p,
 -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^2, x)
```

3.266 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^3}\,dx$

Optimal. Leaf size=166

$$-2e^3(3p+1)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}-\frac{3e}{}$$

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(1 + p))/x - (2
*e^3*(1 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2
)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e^2*(3 - p)*(d^2 - e^2*x^2)^(1 + p)*Hyperg
eometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rubi [A] time = 0.215386, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1807, 764, 266, 65, 246, 245}

$$-2e^3(3p+1)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}-\frac{3e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```


```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(1 + p))/x - (2
*e^3*(1 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2
)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e^2*(3 - p)*(d^2 - e^2*x^2)^(1 + p)*Hyperg
eometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
```

```
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^3}\,dx &= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-6d^4e-2d^3e^2(3-p)x-2d^2e^3x^2\right)}{x^2}\,dx}{2d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\frac{\int \frac{\left(2d^5e^2(3-p)-4d^4e^3(1+3p)x\right)\left(d^2-e^2x^2\right)^p}{x}\,dx}{2d^4}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\left(de^2(3-p)\right)\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx-\left(2e^3(1+3p)\right)\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\frac{1}{2}\left(de^2(3-p)\right)\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x,x^2\right)\\
&= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}-2e^3(1+3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},\right.\end{aligned}$$

Mathematica [A] time = 0.114169, size = 182, normalized size = 1.1

$$\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(ex\left(2de(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(3\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)\right.\right.\right.}{2d(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*(-6*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*
x^2)/d^2] + e*x*(2*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/
d^2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(3*Hypergeometric2F1[1, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e^2*x^
2)/d^2]))))/(2*d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.614, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(-x^2e^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-e^2x^2+d^2\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p/x^3,
x)
```

Sympy [C] time = 8.37004, size = 177, normalized size = 1.07

$$-\frac{d^3e^{2p}x^{2p}e^{i\pi p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2x^2\Gamma\left(2-p\right)}-\frac{3d^2d^{2p}e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}-\frac{3de^2e^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma\left(1-p\right)}+d^{2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d**3*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,
), d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - 3*d**2*d**(2*p)*e*hyper((-1/2,
 -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - 3*d*e**2*e**(2*p)*x**(2
*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*ga
mma(1 - p)) + d**(2*p)*e**3*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(
2*I*pi)/d**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p/x^3, x)
```

3.267 $\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=148

$$\frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d}+\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^5p}-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^5(p+2)}$$

```
[Out] (d^4*(d^2 - e^2*x^2)^p)/(2*e^5*p) - (d^2*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 +
 p)) + (d^2 - e^2*x^2)^(2 + p)/(2*e^5*(2 + p)) + (x^5*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^2)/d^2])/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.108949, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {850, 764, 365, 364, 266, 43}

$$\frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d}+\frac{d^4\left(d^2-e^2x^2\right)^p}{2e^5p}-\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{e^5(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+2}}{2e^5(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (d^4*(d^2 - e^2*x^2)^p)/(2*e^5*p) - (d^2*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 +
 p)) + (d^2 - e^2*x^2)^(2 + p)/(2*e^5*(2 + p)) + (x^5*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^2)/d^2])/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^p}{d+ex}\,dx = \int x^4(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx$$
$$= d\int x^4\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^5\left(d^2-e^2x^2\right)^{-1+p}\,dx$$
$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-1+p}\,dx,x,x^2\right)\right) + \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}}{d}$$
$$= \frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d} - \frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{d^4\left(d^2-e^2x\right)^{-1+p}}{e^4} - \frac{2d^2\left(d\right.}{}\right.\right.$$
$$= \frac{d^4\left(d^2-e^2x^2\right)^p}{2e^5p} - \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)} + \frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^5(2+p)} + \frac{x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1\right.}{5d}$$

Mathematica [C] time = 0.113559, size = 66, normalized size = 0.45

$$\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(5;-p,1-p;6;\frac{ex}{d},-\frac{ex}{d}\right)}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 1 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.656, size = 0, normalized size = 0.

$$\int \frac{x^4\left(-x^2e^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p x^4}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2 + d^2\right)^p x^4}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

Sympy [C] time = 22.9988, size = 4446, normalized size = 30.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((-6*0**p*d**4*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p)*gamma(-p
 - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamm
a(p + 3)) + 6*0**p*d**4*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p)*gamm
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*
gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*
gamma(p + 3)) + 12*0**p*d**4*d**(2*p)*p*acoth(d/(e*x))*gamma(-p)*gamma(-p -
 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(
p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(
p + 3)) - 6*0**p*d**4*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p)*gamma(-p - 1
/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p
+ 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p
+ 3)) + 6*0**p*d**4*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(-p)*gamma(-p -
 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(
p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(
p + 3)) + 12*0**p*d**4*d**(2*p)*acoth(d/(e*x))*gamma(-p)*gamma(-p - 1/2)*ga
mma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3))
- 12*0**p*d**3*d**(2*p)*e*p*x*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(
p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*
e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 12*0**p*d**3*d*
*(2*p)*e*x*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamm
```

```
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**2*d**(2*p)*e**2*p*x**2*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gam
ma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*
gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**2*d**(2*p)*e**2*x**2*gamma(-p)*gamma
(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*g
amma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3)) - 4*0**p*d*d**(2*p)*e**3*p*x**3*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gam
ma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) -
4*0**p*d*d**(2*p)*e**3*x**3*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p
+ 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e*
*5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 3*0**p*d**(2*p)*e
**4*p*x**4*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamm
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 3*0**p*d**(2*p)*e**4*x**4*gamma(-p
)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p -
 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p
 + 1)*gamma(p + 3)) + 12*d**4*e**(2*p)*x**(2*p)*(d**2/(e**2*x**2) - 1)**p*g
amma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-
p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3)) + 12*d**2*e**2*e**(2*p)*p*x**2*x**(2*p)*(d**2/(e**2*
x**2) - 1)**p*gamma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*ga
mma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma
(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*d*e**3*e**(2*p)*p**2*x**3*x**(2*p
)*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 3/2)*gamma(p + 3)*hyper((1 - p,
 -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p -
1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p
+ 1)*gamma(p + 3)) + 6*d*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p
)*gamma(p)*gamma(-p - 3/2)*gamma(p + 3)*hyper((1 - p, -p - 3/2), (-p - 1/2,
), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamm
a(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6
*e**4*e**(2*p)*p**2*x**4*x**(2*p)*(d**2/(e**2*x**2) - 1)**p*gamma(-p)*gamma
(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma
(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma
(p + 3)) + 6*e**4*e**(2*p)*p*x**4*x**(2*p)*(d**2/(e**2*x**2) - 1)**p*gamma(
-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1
/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p +
 1)*gamma(p + 3)), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-6*0**p*d**4*d**(
2*p)*p*log(d**2/(e**2*x**2))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p
 + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e
**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**4*d**(
2*p)*p*log(-d**2/(e**2*x**2) + 1)*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*ga
mma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) +
 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 12*0**p*d**
4*d**(2*p)*p*atanh(d/(e*x))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p
+ 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e*
*5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 6*0**p*d**4*d**(2
*p)*log(d**2/(e**2*x**2))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p +
3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5
*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**4*d**(2*p
)*log(-d**2/(e**2*x**2) + 1)*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p
 + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e
**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 12*0**p*d**4*d**
(2*p)*atanh(d/(e*x))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(1
2*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamm
a(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 12*0**p*d**3*d**(2*p)*e*
p*x*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p
)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p -
1/2)*gamma(p + 1)*gamma(p + 3)) - 12*0**p*d**3*d**(2*p)*e*x*gamma(-p)*gamma
```

```
(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*g
amma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3)) + 6*0**p*d**2*d**(2*p)*e**2*p*x**2*gamma(-p)*gamma(-p - 1/2)*g
amma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*
gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3))
 + 6*0**p*d**2*d**(2*p)*e**2*x**2*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*ga
mma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) +
 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 4*0**p*d*d*
*(2*p)*e**3*p*x**3*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*
e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(
-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 4*0**p*d*d**(2*p)*e**3*x**
3*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*
gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/
2)*gamma(p + 1)*gamma(p + 3)) + 3*0**p*d**(2*p)*e**4*p*x**4*gamma(-p)*gamma
(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*g
amma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3)) + 3*0**p*d**(2*p)*e**4*x**4*gamma(-p)*gamma(-p - 1/2)*gamma(p
+ 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p
 + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 12*d
**4*e**(2*p)*x**(2*p)*(-d**2/(e**2*x**2) + 1)**p*exp(I*pi*p)*gamma(-p)*gamm
a(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamm
a(p + 3)) + 12*d**2*e**2*e**(2*p)*p*x**2*x**(2*p)*(-d**2/(e**2*x**2) + 1)**
p*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*ga
mma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma
(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*d*e**3*e**(2*p)*p**2*x**3*x**(2*p
)*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 3/2)*gamma(p + 3)*hyper((1 - p,
 -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p -
1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p
+ 1)*gamma(p + 3)) + 6*d*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p
)*gamma(p)*gamma(-p - 3/2)*gamma(p + 3)*hyper((1 - p, -p - 3/2), (-p - 1/2,
), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamm
a(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6
*e**4*e**(2*p)*p**2*x**4*x**(2*p)*(-d**2/(e**2*x**2) + 1)**p*exp(I*pi*p)*ga
mma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p
 - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma
(p + 1)*gamma(p + 3)) + 6*e**4*e**(2*p)*p*x**4*x**(2*p)*(-d**2/(e**2*x**2)
+ 1)**p*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(12*e**
5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)
*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

3.268 $\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=121

$$-\frac{ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}-\frac{d^3\left(d^2-e^2x^2\right)^p}{2e^4p}+\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}$$

```
[Out] -(d^3*(d^2 - e^2*x^2)^p)/(2*e^4*p) + (d*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(1
+ p)) - (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^
2)/d^2])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0930484, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {850, 764, 266, 43, 365, 364}

$$-\frac{ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}-\frac{d^3\left(d^2-e^2x^2\right)^p}{2e^4p}+\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -(d^3*(d^2 - e^2*x^2)^p)/(2*e^4*p) + (d*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(1
+ p)) - (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^
2)/d^2])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 (d^2-e^2x^2)^p}{d+ex}\,dx &= \int x^3(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= d\int x^3\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^4\left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= \frac{1}{2}d\,\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-1+p}\,dx, x, x^2\right) - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d^2}\\ &= -\frac{ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{1}{2}d\,\text{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right)^{-1+p}}{e^2} - \frac{\left(d^2-e\right.}{e^2}\right.\right.\\ &= -\frac{d^3\left(d^2-e^2x^2\right)^p}{2e^4p} + \frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)} - \frac{ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}\end{aligned}$$

Mathematica [B] time = 0.317096, size = 245, normalized size = 2.02

$$\frac{\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(6d^2e(p+1)x\left(\frac{ex}{d}+1\right)^p\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+2e^3(p+1)x^3\left(\frac{ex}{d}+1\right)^p\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right.\right.}{6e^4(p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(6*d^2*e*(1 + p)*x*(1 + (e*x)/d)^p*Hypergeometric2F1[1/2
, -p, 3/2, (e^2*x^2)/d^2] + 2*e^3*(1 + p)*x^3*(1 + (e*x)/d)^p*Hypergeometri
c2F1[3/2, -p, 5/2, (e^2*x^2)/d^2] + 3*d*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e
^2*x^2)/d^2)^p) + d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + d*(d - e*x)*(2 - (2*e
^2*x^2)/d^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(
6*e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.648, size = 0, normalized size = 0.

$$\int \frac{x^3\left(-x^2e^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^3}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

Sympy [C] time = 15.9521, size = 5100, normalized size = 42.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gam
ma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*g
amma(p + 1)) - 3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p -
1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
- 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*p*acoth(d/(e*x))*gamma(-p - 1/2
)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1
/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p - 1
/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p -
 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(
-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamm
a(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*acoth(d/(e*x))*gamma(-p -
1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
- 1/2)*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p - 1/2)*gamma(p +
 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p
 + 1)) + 6*0**p*d**2*d**(2*p)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*ga
mma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*
d*d**(2*p)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*
e**2*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p +
```

```
1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*p*x**3*gam
ma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*g
amma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*x**3*gamma(-p - 1/2)*ga
mma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*
gamma(p + 1)) - 3*d**3*d**(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(
p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
 - 1/2)*gamma(p + 1)) - 3*d*d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*e
xp(I*pi*p)*gamma(p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1)
+ 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)
*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,),
 d**2/(e**2*x**2))/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
 - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p
)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(
6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)
), (Abs(e**2*x**2)/Abs(d**2) > 1) & (Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1)),
 (3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)
/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p +
1)) - 3*0**p*d**3*d**(2*p)*p*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*gam
ma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*g
amma(p + 1)) - 6*0**p*d**3*d**(2*p)*p*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(
p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamm
a(p + 1)) + 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamm
a(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*ga
mma(p + 1)) - 3*0**p*d**3*d**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/
2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p -
1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*atanh(d/(e*x))*gamma(-p - 1/2)*ga
mma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*
gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p - 1/2)*gamma(p + 1)/(6*
e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1))
+ 6*0**p*d**2*d**(2*p)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p
- 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2
*p)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamm
a(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*e**2*x*
*2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*
e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*p*x**3*gamma(-p -
 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
 - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*x**3*gamma(-p - 1/2)*gamma(p +
 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p
 + 1)) - 3*d**3*d**(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamm
a(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)
*gamma(p + 1)) - 3*d*d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*exp(I*pi
*p)*gamma(p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**
4*gamma(-p - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)*exp(I*
pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(
e**2*x**2))/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)
*gamma(p + 1)) - 3*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma
(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*e**4*
p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)), Abs(
e**2*x**2)/Abs(d**2) > 1), (3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2))*ga
mma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*
gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2)
 - 1)*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) +
 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*p*acoth(d/(e*x
))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*
e**4*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x*
*2))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) +
6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*
x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p +
 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*acoth(d/(
```

```
e*x))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) +
 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p
 - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(
-p - 1/2)*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*x*gamma(-p - 1/2)*gamma(p
+ 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(
p + 1)) - 3*0**p*d*d**(2*p)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**
4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3
*0**p*d*d**(2*p)*e**2*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p
- 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p
)*e**3*p*x**3*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(
p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*x**3*g
amma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4
*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**3*d**(2*p)*(1 - e**2*x**2/d**2)**p*ga
mma(p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamm
a(-p - 1/2)*gamma(p + 1)) - 3*d*d**(2*p)*e**2*p*x**2*(1 - e**2*x**2/d**2)**
p*gamma(p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*
gamma(-p - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)*exp(I*pi
*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e*
*2*x**2))/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*g
amma(p + 1)) - 3*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*e**4*p*
gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)), Abs(d*
*2)/(Abs(e**2)*Abs(x**2)) > 1), (3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2
))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*
e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*p*log(-d**2/(e**2
*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p
+ 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*p*atanh(
d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1
) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p)*log(d**2/(e
**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p +
 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*log(-d**2
/(e**2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*ga
mma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*at
anh(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p
 + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*p*x*g
amma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4
*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*x*gamma(-p - 1/2)*g
amma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)
*gamma(p + 1)) - 3*0**p*d*d**(2*p)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)
/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p +
1)) - 3*0**p*d*d**(2*p)*e**2*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*ga
mma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*
d**(2*p)*e**3*p*x**3*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)
*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3
*x**3*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) +
 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**3*d**(2*p)*(1 - e**2*x**2/d**2
)**p*gamma(p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e*
*4*gamma(-p - 1/2)*gamma(p + 1)) - 3*d*d**(2*p)*e**2*p*x**2*(1 - e**2*x**2/
d**2)**p*gamma(p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) +
6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)*e
xp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d
**2/(e**2*x**2))/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p -
 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*
gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*
e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)),
 True))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)
```

3.269 $\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=119

$$\frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d}+\frac{d^2\left(d^2-e^2x^2\right)^p}{2e^3p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}$$

```
[Out] (d^2*(d^2 - e^2*x^2)^p)/(2*e^3*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^3*(1 + p))
 + (x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2]
)/(3*d*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0854197, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {850, 764, 365, 364, 266, 43}

$$\frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d}+\frac{d^2\left(d^2-e^2x^2\right)^p}{2e^3p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (d^2*(d^2 - e^2*x^2)^p)/(2*e^3*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^3*(1 + p))
 + (x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2]
)/(3*d*(1 - (e^2*x^2)/d^2)^p)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2 \left(d^2-e^2x^2\right)^p}{d+ex}\,dx = \int x^2(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx$$

$$= d\int x^2\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^3\left(d^2-e^2x^2\right)^{-1+p}\,dx$$

$$= -\left(\frac{1}{2}e\,\mathrm{Subst}\left(\int x\left(d^2-e^2x\right)^{-1+p}\,dx, x, x^2\right)\right) + \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d}$$

$$= \frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d} - \frac{1}{2}e\,\mathrm{Subst}\left(\int\left(\frac{d^2\left(d^2-e^2x\right)^{-1+p}}{e^2} - \frac{\left(d^2-e^2x\right)}{e^2}\right.\right.$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^p}{2e^3p} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)} + \frac{x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d}$$

Mathematica [A] time = 0.260615, size = 198, normalized size = 1.66

$$-\frac{\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2de(p+1)x\left(\frac{ex}{d}+1\right)^p\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+d(d-ex)\left(2-\frac{2e^2x^2}{d^2}\right)^p\,{}_2F_1\left(1-p,p+1;p\right.\right.}{2e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -((d^2 - e^2*x^2)^p*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e^2*x^2)/d^2)^p) + d^
2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + 2*d*e*(1 + p)*x*(1 + (e*x)/d)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + d*(d - e*x)*(2 - (2*e^2*x^2)/d^2)^p
*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(2*e^3*(1 + p)*(
1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.67, size = 0, normalized size = 0.

$$\int \frac{x^2\left(-x^2e^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

Sympy [C] time = 10.6224, size = 4134, normalized size = 34.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma
(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma
(p + 1)) + 0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*ga
mma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*ga
mma(p + 1)) + 2*0**p*d**2*d**(2*p)*p*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p
+ 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p
+ 1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1
)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1
)) + 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p +
1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p +
1)) + 2*0**p*d**2*d**(2*p)*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*
0**p*d*d**(2*p)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*
gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*x*
gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*
gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamma(1/2 - p)*gam
ma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gam
ma(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*
gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d**2*d*
```

```
*(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(2*e**3
*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*
e**(2*p)*p**2*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p,
 -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p
+ 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(
I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/
(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*
gamma(p + 1)) + d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*g
amma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma
(1/2 - p)*gamma(p + 1)), (Abs(e**2*x**2)/Abs(d**2) > 1) & (Abs(d**2)/(Abs(e
**2)*Abs(x**2)) > 1)), (-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(1
/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1
/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamm
a(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamm
a(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*p*acoth(d/(e*x))*gamma(1/2
- p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2
- p)*gamma(p + 1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2 - p
)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p
)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(1/2 -
p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 -
p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*acoth(d/(e*x))*gamma(1/2 - p)*gamma
(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma
(p + 1)) - 2*0**p*d*d**(2*p)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*ga
mma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*
d**(2*p)*e*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p +
 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamma
(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma
(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(p +
1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p +
1)) + d**2*d**(2*p)*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(1/2 - p)/(2*e**3
*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*
e**(2*p)*p**2*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p,
 -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p
+ 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(
I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/
(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*
gamma(p + 1)) + d**(2*p)*e**2*p*x**2*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma
(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gam
ma(p + 1)), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-0**p*d**2*d**(2*p)*p*lo
g(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*ga
mma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*p*log
(-d**2/(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p
)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)
*p*atanh(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamm
a(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p)*log(d**
2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p
 + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*log(-d**2/
(e**2*x**2) + 1)*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma
(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*atanh(
d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1)
+ 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*p*x*gamma(1/2 -
 p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 -
 p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*x*gamma(1/2 - p)*gamma(p + 1)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0*
*p*d**(2*p)*e**2*p*x**2*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p
)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*x
**2*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e
**3*gamma(1/2 - p)*gamma(p + 1)) + d**2*d**(2*p)*(-1 + e**2*x**2/d**2)**p*e
xp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) +
```

```
2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p**2*x*x**(2*p)*exp(I*pi
*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**
2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamm
a(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*
hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2
- p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d**(2*p)*e**2*p*x
**2*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(2*e**3*p*
gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)), Abs(e**2
*x**2)/Abs(d**2) > 1), (-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(1
/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1
/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*p*log(-d**2/(e**2*x**2) + 1)*gam
ma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gam
ma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*p*atanh(d/(e*x))*gamma(1/2
 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2
 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2 -
p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 -
p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(1/2
- p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2
- p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*atanh(d/(e*x))*gamma(1/2 - p)*gam
ma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gam
ma(p + 1)) - 2*0**p*d*d**(2*p)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*
gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*
d*d**(2*p)*e*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p
 + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gam
ma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gam
ma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(p
+ 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p
+ 1)) + d**2*d**(2*p)*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(1/2 - p)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*
e*e**(2*p)*p**2*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 -
p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(
p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*ex
p(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**
2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p
)*gamma(p + 1)) + d**(2*p)*e**2*p*x**2*(1 - e**2*x**2/d**2)**p*gamma(p)*gam
ma(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*g
amma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d), x)
```

3.270 $\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=90

$$-\frac{ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^p}{2e^2p}$$

```
[Out] -(d*(d^2 - e^2*x^2)^p)/(2*e^2*p) - (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0581513, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {785, 764, 261, 365, 364}

$$-\frac{ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}-\frac{d\left(d^2-e^2x^2\right)^p}{2e^2p}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^p)/(2*e^2*p) - (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d^2-e^2x^2\right)^p}{d+ex}\,dx &= \frac{\int x\left(d^2e-de^2x\right)\left(d^2-e^2x^2\right)^{-1+p}\,dx}{de}\\
&= d\int x\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^2\left(d^2-e^2x^2\right)^{-1+p}\,dx\\
&= -\frac{d\left(d^2-e^2x^2\right)^p}{2e^2p} - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^p}{2e^2p} - \frac{ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}
\end{aligned}$$

Mathematica [A] time = 0.112274, size = 147, normalized size = 1.63

$$\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2e(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p{}_2F_1\left(1-p,p+\right.\right.}{e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (2^(-1 + p)*(d^2 - e^2*x^2)^p*(2*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hy
pergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^2*(1 + p)*(1 + (
e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.662, size = 0, normalized size = 0.

$$\int \frac{x\left(-x^2e^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^px}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

Sympy [C] time = 6.05309, size = 430, normalized size = 4.78

$$\begin{cases} \frac{0^p d d^{2p}\log\left(\frac{d^2}{e^2x^2}\right)}{2e^2} - \frac{0^p d d^{2p}\log\left(\frac{d^2}{e^2x^2}-1\right)}{2e^2} - \frac{0^p d d^{2p}\operatorname{acoth}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p}x}{e} - \frac{e^{2p}pxx^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p,-p-\frac{1}{2}\\ \frac{1}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} - \frac{d^{2p}x^2\Gamma(p)\Gamma(1-p){}_3F}{2d\Gamma(} \\ \frac{0^p d d^{2p}\log\left(\frac{d^2}{e^2x^2}\right)}{2e^2} - \frac{0^p d d^{2p}\log\left(-\frac{d^2}{e^2x^2}+1\right)}{2e^2} - \frac{0^p d d^{2p}\operatorname{atanh}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p}x}{e} - \frac{e^{2p}pxx^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p,-p-\frac{1}{2}\\ \frac{1}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} - \frac{d^{2p}x^2\Gamma(p)\Gamma(1-p)}{2dI} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((0**p*d*d**(2*p)*log(d**2/(e**2*x**2))/(2*e**2) - 0**p*d*d**(2*p)
*log(d**2/(e**2*x**2) - 1)/(2*e**2) - 0**p*d*d**(2*p)*acoth(d/(e*x))/e**2 +
 0**p*d**(2*p)*x/e - e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p -
1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e*gamma(1/2
- p)*gamma(p + 1)) - d**(2*p)*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p
), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d*gamma(-p)*gamma(p + 1)),
Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (0**p*d*d**(2*p)*log(d**2/(e**2*x**2)
)/(2*e**2) - 0**p*d*d**(2*p)*log(-d**2/(e**2*x**2) + 1)/(2*e**2) - 0**p*d*d
**(2*p)*atanh(d/(e*x))/e**2 + 0**p*d**(2*p)*x/e - e**(2*p)*p*x*x**(2*p)*exp
(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2
/(e**2*x**2))/(2*e*gamma(1/2 - p)*gamma(p + 1)) - d**(2*p)*x**2*gamma(p)*ga
mma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(
2*d*gamma(-p)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d), x)
```

3.271 $\int \frac{\left(d^2-e^2x^2\right)^p}{d+ex}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

```
[Out] -((2^(-1 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)))
```

Rubi [A] time = 0.0328738, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x),x]
```

```
[Out] -((2^(-1 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{d+ex}\,dx &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-1+p}\,dx}{d^2}\\ &= -\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0391966, size = 75, normalized size = 1.03

$$-\frac{2^{p-1}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x),x]
```

```
[Out] -((2^(-1 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.702, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

Sympy [C] time = 6.79971, size = 323, normalized size = 4.42

$$\begin{cases} \frac{0^p\log\left(-1+\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p\operatorname{acoth}\left(\frac{ex}{d}\right)}{e} + \frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\ \frac{3}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2,1,1-p\\ 2,2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^2\Gamma(-p)\Gamma(p+1)} & \text{for}\ \frac{|e^2x^2|}{|d^2|} > \\ \frac{0^p\log\left(1-\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p\operatorname{atanh}\left(\frac{ex}{d}\right)}{e} + \frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\ \frac{3}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2,1,1-p\\ 2,2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^2\Gamma(-p)\Gamma(p+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((0**p*log(-1 + e**2*x**2/d**2)/(2*e) + 0**p*acoth(e*x/d)/e + d*e*
*(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), d**2/(e**2*x**2))/(2*e**2*x*gamma(3/2 - p)*gamma(p + 1)) + d
**(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2
*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), Abs(e**2*x**2)/Ab
s(d**2) > 1), (0**p*log(1 - e**2*x**2/d**2)/(2*e) + 0**p*atanh(e*x/d)/e + d
*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2
- p), (3/2 - p,), d**2/(e**2*x**2))/(2*e**2*x*gamma(3/2 - p)*gamma(p + 1))
+ d**(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x
**2*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d), x)
```

3.272 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx$

Optimal. Leaf size=104

$$-\frac{ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{\left(d^2-e^2x^2\right)^p{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

```
[Out] -((e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 1 - p, 3/2, (e^2*x^2)/d^2])
/(d^2*(1 - (e^2*x^2)/d^2)^p)) - ((d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p,
1 + p, 1 - (e^2*x^2)/d^2])/(2*d*p)
```

Rubi [A] time = 0.0679128, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {850, 764, 266, 65, 246, 245}

$$-\frac{ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{\left(d^2-e^2x^2\right)^p{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] -((e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 1 - p, 3/2, (e^2*x^2)/d^2])
/(d^2*(1 - (e^2*x^2)/d^2)^p)) - ((d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p,
1 + p, 1 - (e^2*x^2)/d^2])/(2*d*p)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{x(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx\\ &= d\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx - e\int \left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= \frac{1}{2}d\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x}\,dx, x, x^2\right) - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}dx}{d^2}\\ &= -\frac{ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2} - \frac{\left(d^2-e^2x^2\right)^p{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2dp}\end{aligned}$$

Mathematica [A] time = 0.118403, size = 151, normalized size = 1.45

$$\frac{2^{p-1}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(p(d-ex)\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)+d(p+1)\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\Big(\right.}{d^2p(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] (2^(-1 + p)*(d^2 - e^2*x^2)^p*(p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeome
tric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*(1 + p)*(1/2 + (e*x)/(2*d
))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^2*p*(1 + p)*(1 -
d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

Maple [F] time = 0.671, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^2 + d*x), x)
```

Sympy [C] time = 7.5121, size = 359, normalized size = 3.45

$$\begin{cases} -\frac{0^p d^{2p}\log\left(\frac{d^2}{e^2x^2}-1\right)}{2d}-\frac{0^p d^{2p}\operatorname{acoth}\left(\frac{d}{ex}\right)}{d}+\frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x^2\Gamma(2-p)\Gamma(p+1)}-\frac{e^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\\frac{3}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2ex\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} & \text{fo} \\ -\frac{0^p d^{2p}\log\left(-\frac{d^2}{e^2x^2}+1\right)}{2d}-\frac{0^p d^{2p}\operatorname{atanh}\left(\frac{d}{ex}\right)}{d}+\frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x^2\Gamma(2-p)\Gamma(p+1)}-\frac{e^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\\frac{3}{2}-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2ex\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**(2*p)*log(d**2/(e**2*x**2) - 1)/(2*d) - 0**p*d**(2*p)*a
coth(d/(e*x))/d + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*h
yper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*e**2*x**2*gamma(2 - p)*
gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyp
er((1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*e*x*gamma(3/2 - p)*ga
mma(p + 1)), Abs(d**2)/(Abs(e**2)*Abs(x**2)) > 1), (-0**p*d**(2*p)*log(-d**
2/(e**2*x**2) + 1)/(2*d) - 0**p*d**(2*p)*atanh(d/(e*x))/d + d*e**(2*p)*p*x*
*(2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d*
*2/(e**2*x**2))/(2*e**2*x**2*gamma(2 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,),
d**2/(e**2*x**2))/(2*e*x*gamma(3/2 - p)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{(ex + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x), x)
```

3.273 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx$

Optimal. Leaf size=106

$$\frac{e\left(d^2-e^2x^2\right)^p {}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}-\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx}$$

```
[Out] -((((d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
*x*(1 - (e^2*x^2)/d^2)^p)) + (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p, 1
 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rubi [A] time = 0.0742381, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {850, 764, 365, 364, 266, 65}

$$\frac{e\left(d^2-e^2x^2\right)^p {}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}-\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] -((((d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
*x*(1 - (e^2*x^2)/d^2)^p)) + (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p, 1
 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{-1+p}}{x^2}\,dx \\
&= d\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x^2}\,dx - e\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x}\,dx \\
&= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x}\,dx, x, x^2\right)\right) + \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}}{x^2}\,dx}{d} \\
&= -\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx} + \frac{e\left(d^2-e^2x^2\right)^p{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}
\end{aligned}$$

Mathematica [A] time = 0.190567, size = 167, normalized size = 1.58

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{de\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}-\frac{2d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{e2^p(ex-d)\left(\frac{ex}{d}+1\right)^{-p}{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{p+1}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-2*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[1 - p, 1
+ p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (d*e*Hypergeometr
ic2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(2*d^3)
```

Maple [F] time = 0.671, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x^2\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^3 + d*x^2), x)
```

Sympy [C] time = 8.34782, size = 452, normalized size = 4.26

$$\begin{cases} -\frac{0^p d^{2p}}{dx} - \frac{0^p d^{2p} e \log\left(\frac{e^2x^2}{d^2}\right)}{2d^2} + \frac{0^p d^{2p} e \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2d^2} + \frac{0^p d^{2p} e \operatorname{acoth}\left(\frac{ex}{d}\right)}{d^2} + \frac{d e^{2p} p x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{3}{2}-p\right) {}_2F_1\left(\begin{matrix} 1-p, \frac{3}{2}-p \\ \frac{5}{2}-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2e^2x^3\Gamma\left(\frac{5}{2}-p\right)\Gamma(p+1)} - \frac{e^{2p} p x^{2p} e^{i\pi p}\Gamma(p)\Gamma(1- \cdots}{2ex^2\Gamma(\cdots} \\ -\frac{0^p d^{2p}}{dx} - \frac{0^p d^{2p} e \log\left(\frac{e^2x^2}{d^2}\right)}{2d^2} + \frac{0^p d^{2p} e \log\left(1-\frac{e^2x^2}{d^2}\right)}{2d^2} + \frac{0^p d^{2p} e \operatorname{atanh}\left(\frac{ex}{d}\right)}{d^2} + \frac{d e^{2p} p x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{3}{2}-p\right) {}_2F_1\left(\begin{matrix} 1-p, \frac{3}{2}-p \\ \frac{5}{2}-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2e^2x^3\Gamma\left(\frac{5}{2}-p\right)\Gamma(p+1)} - \frac{e^{2p} p x^{2p} e^{i\pi p}\Gamma(p)\Gamma(1-p \cdots}{2ex^2\Gamma(2 \cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**(2*p)/(d*x) - 0**p*d**(2*p)*e*log(e**2*x**2/d**2)/(2*d*
*2) + 0**p*d**(2*p)*e*log(-1 + e**2*x**2/d**2)/(2*d**2) + 0**p*d**(2*p)*e*a
coth(e*x/d)/d**2 + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p
)*hyper((1 - p, 3/2 - p), (5/2 - p,), d**2/(e**2*x**2))/(2*e**2*x**3*gamma(
5/2 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1 -
 p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*e*x**2*gamma(2 - p
)*gamma(p + 1)), Abs(e**2*x**2)/Abs(d**2) > 1), (-0**p*d**(2*p)/(d*x) - 0**
p*d**(2*p)*e*log(e**2*x**2/d**2)/(2*d**2) + 0**p*d**(2*p)*e*log(1 - e**2*x*
*2/d**2)/(2*d**2) + 0**p*d**(2*p)*e*atanh(e*x/d)/d**2 + d*e**(2*p)*p*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,),
d**2/(e**2*x**2))/(2*e**2*x**3*gamma(5/2 - p)*gamma(p + 1)) - e**(2*p)*p*x*
*(2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d*
*2/(e**2*x**2))/(2*e*x**2*gamma(2 - p)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

3.274 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx$

Optimal. Leaf size=108

$$\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x}-\frac{e^2\left(d^2-e^2x^2\right)^p{}_2F_1\left(2,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

```
[Out] (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
^2*x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2, p
, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*p)
```

Rubi [A] time = 0.0798012, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {850, 764, 266, 65, 365, 364}

$$\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x}-\frac{e^2\left(d^2-e^2x^2\right)^p{}_2F_1\left(2,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
^2*x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2, p
, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*p)
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^p}{x^3(d+ex)}\,dx &= \int \frac{(d-ex)\left(d^2-e^2x^2\right)^{-1+p}}{x^3}\,dx\\
&= d\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x^3}\,dx - e\int \frac{\left(d^2-e^2x^2\right)^{-1+p}}{x^2}\,dx\\
&= \frac{1}{2}d\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x^2}\,dx, x, x^2\right) - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}}{x^2}\,dx}{d^2}\\
&= \frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{e^2\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(2,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}
\end{aligned}$$

Mathematica [B] time = 0.615091, size = 219, normalized size = 2.03

$$\frac{\left(d^2-e^2x^2\right)^p\left(\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{d^3\,{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2}+e^2\left(\frac{(d-ex)\left(2-\frac{2d^2}{e^2x^2}\right)^p\left(\frac{ex}{d}+1\right)^{-p}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{p+1}+\frac{d\,{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)\right)+\cdots}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((2*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + ((d^3*Hypergeometric2F1[1 - p, -p, 2 - p, d^
2/(e^2*x^2)])/((-1 + p)*x^2) + e^2*(((2 - (2*d^2)/(e^2*x^2))^p*(d - e*x)*Hy
pergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (d*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/p))/(1 - d^2/(e
^2*x^2))^p))/(2*d^4)
```

Maple [F] time = 0.678, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x^3\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e*x^4 + d*x^3), x)
```

Sympy [C] time = 9.28783, size = 500, normalized size = 4.63

$$\begin{cases} -\frac{0^p d^{2p}}{2dx^2} + \frac{0^p d^{2p} e}{d^2 x} + \frac{0^p d^{2p} e^2 \log\left(\frac{e^2x^2}{d^2}\right)}{2d^3} - \frac{0^p d^{2p} e^2 \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2d^3} - \frac{0^p d^{2p} e^2 \operatorname{acoth}\left(\frac{ex}{d}\right)}{d^3} + \frac{d e^{2p} p x^{2p} e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p, 2-p\\ 3-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x^4\Gamma(3-p)\Gamma(p+1)} - \cdots \\ -\frac{0^p d^{2p}}{2dx^2} + \frac{0^p d^{2p} e}{d^2 x} + \frac{0^p d^{2p} e^2 \log\left(\frac{e^2x^2}{d^2}\right)}{2d^3} - \frac{0^p d^{2p} e^2 \log\left(1-\frac{e^2x^2}{d^2}\right)}{2d^3} - \frac{0^p d^{2p} e^2 \operatorname{atanh}\left(\frac{ex}{d}\right)}{d^3} + \frac{d e^{2p} p x^{2p} e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p, 2-p\\ 3-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2e^2x^4\Gamma(3-p)\Gamma(p+1)} - \cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**(2*p)/(2*d*x**2) + 0**p*d**(2*p)*e/(d**2*x) + 0**p*d**(
2*p)*e**2*log(e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*log(-1 + e**2*x
**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*acoth(e*x/d)/d**3 + d*e**(2*p)*p*x*
*(2*p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), d*
*2/(e**2*x**2))/(2*e**2*x**4*gamma(3 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,),
d**2/(e**2*x**2))/(2*e*x**3*gamma(5/2 - p)*gamma(p + 1)), Abs(e**2*x**2)/Ab
s(d**2) > 1), (-0**p*d**(2*p)/(2*d*x**2) + 0**p*d**(2*p)*e/(d**2*x) + 0**p*
d**(2*p)*e**2*log(e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*log(1 - e**
2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*atanh(e*x/d)/d**3 + d*e**(2*p)*p
*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,),
 d**2/(e**2*x**2))/(2*e**2*x**4*gamma(3 - p)*gamma(p + 1)) - e**(2*p)*p*x**
```

```
(2*p)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,
), d**2/(e**2*x**2))/(2*e*x**3*gamma(5/2 - p)*gamma(p + 1)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

3.275 $\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=179

$$-\frac{2ex^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(\frac{7}{2},2-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7d^3}+\frac{d^6\left(d^2-e^2x^2\right)^{p-1}}{e^6(1-p)}+\frac{5d^4\left(d^2-e^2x^2\right)^p}{2e^6p}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{(d^2 \cdots}{2e \cdots}$$

```
[Out] (d^6*(d^2 - e^2*x^2)^(-1 + p))/(e^6*(1 - p)) + (5*d^4*(d^2 - e^2*x^2)^p)/(2
*e^6*p) - (2*d^2*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (d^2 - e^2*x^2)^(
2 + p)/(2*e^6*(2 + p)) - (2*e*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2,
2 - p, 9/2, (e^2*x^2)/d^2])/(7*d^3*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.188684, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1652, 446, 77, 12, 365, 364}

$$-\frac{2ex^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(\frac{7}{2},2-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7d^3}+\frac{d^6\left(d^2-e^2x^2\right)^{p-1}}{e^6(1-p)}+\frac{5d^4\left(d^2-e^2x^2\right)^p}{2e^6p}-\frac{2d^2\left(d^2-e^2x^2\right)^{p+1}}{e^6(p+1)}+\frac{(d^2 \cdots}{2e \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```



```
[Out] (d^6*(d^2 - e^2*x^2)^(-1 + p))/(e^6*(1 - p)) + (5*d^4*(d^2 - e^2*x^2)^p)/(2
*e^6*p) - (2*d^2*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (d^2 - e^2*x^2)^(
2 + p)/(2*e^6*(2 + p)) - (2*e*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2,
2 - p, 9/2, (e^2*x^2)/d^2])/(7*d^3*(1 - (e^2*x^2)/d^2)^p)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx &= \int x^5(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \int -2dex^6\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^5\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)dx\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)dx,x,x^2\right)-(2de)\int x^6\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{2d^6\left(d^2-e^2x\right)^{-2+p}}{e^4}-\frac{5d^4\left(d^2-e^2x\right)^{-1+p}}{e^4}+\frac{4d^2\left(d^2-e^2x\right)^{p}}{e^4}-\frac{\left(d^2-e^2x\right)^{1+p}}{e^4}\right)dx\right.\\
&= \frac{d^6\left(d^2-e^2x^2\right)^{-1+p}}{e^6(1-p)}+\frac{5d^4\left(d^2-e^2x^2\right)^{p}}{2e^6p}-\frac{2d^2\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{2ex^7\left(d^2-e^2x\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.168477, size = 66, normalized size = 0.37

$$
\frac{x^6(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(6;-p,2-p;7;\frac{ex}{d},-\frac{ex}{d}\right)}{6d^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x^6*(d - e*x)^p*(d + e*x)^p*AppellF1[6, -p, 2 - p, 7, (e*x)/d, -((e*x)/d)]
)/(6*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.705, size = 0, normalized size = 0.

$$\int \frac{x^5 \left(-x^2 e^2 + d^2\right)^p}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2 x^2 + d^2\right)^p x^5}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^5/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2 x^2 + d^2\right)^p x^5}{e^2 x^2 + 2\, dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^5/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{\left(d + ex\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2 x^2 + d^2\right)^p x^5}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^5/(e*x + d)^2, x)
```

3.276 $\int \frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=184

$$\frac{2(p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)}-\frac{x^5\left(d^2-e^2x^2\right)^{p-1}}{2p+3}-\frac{d^5\left(d^2-e^2x^2\right)^{p-1}}{e^5(1-p)}-\frac{2d^3\left(d^2-e^2x^2\right)^p}{e^5p}+\frac{d}{}$$

```
[Out] -((d^5*(d^2 - e^2*x^2)^(-1 + p))/(e^5*(1 - p))) - (x^5*(d^2 - e^2*x^2)^(-1
+ p))/(3 + 2*p) - (2*d^3*(d^2 - e^2*x^2)^p)/(e^5*p) + (d*(d^2 - e^2*x^2)^(1
 + p))/(e^5*(1 + p)) + (2*(4 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5
/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^2*(3 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.205062, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {852, 1652, 459, 365, 364, 12, 266, 43}

$$\frac{2(p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)}-\frac{x^5\left(d^2-e^2x^2\right)^{p-1}}{2p+3}-\frac{d^5\left(d^2-e^2x^2\right)^{p-1}}{e^5(1-p)}-\frac{2d^3\left(d^2-e^2x^2\right)^p}{e^5p}+\frac{d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^5*(d^2 - e^2*x^2)^(-1 + p))/(e^5*(1 - p))) - (x^5*(d^2 - e^2*x^2)^(-1
+ p))/(3 + 2*p) - (2*d^3*(d^2 - e^2*x^2)^p)/(e^5*p) + (d*(d^2 - e^2*x^2)^(1
 + p))/(e^5*(1 + p)) + (2*(4 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5
/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^2*(3 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4 \left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx = \int x^4(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$

$$= \int -2dex^5\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int x^4\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - (2de)\int x^5\left(d^2-e^2x^2\right)^{-2+p}\,dx + \frac{\left(2d^2(4+p)\right)\int x^4\left(d^2-e^2x^2\right)^{-2+p}\,dx}{3+2p}$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - (de)\,\mathrm{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-2+p}\,dx, x, x^2\right) + \frac{\left(2(4+p)\left(d^2-e^2x^2\right)^p\left(1-\right.\right.}{d^2}$$

$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} + \frac{2(4+p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(3+2p)} - (de)\,\mathrm{Subst}$$

$$= -\frac{d^5\left(d^2-e^2x^2\right)^{-1+p}}{e^5(1-p)} - \frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - \frac{2d^3\left(d^2-e^2x^2\right)^p}{e^5p} + \frac{d\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)} + \frac{2(4+p)x^5}{}$$

Mathematica [C] time = 0.125365, size = 66, normalized size = 0.36

$$\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(5;-p,2-p;6;\frac{ex}{d},-\frac{ex}{d}\right)}{5d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 2 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.694, size = 0, normalized size = 0.

$$\int \frac{x^4 \left(-x^2e^2 + d^2\right)^p}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p x^4}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2 + d^2\right)^p x^4}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{\left(d + ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^2, x)
```

3.277 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2ex^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3}+\frac{d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4(1-p)}+\frac{3d^2\left(d^2-e^2x^2\right)^p}{2e^4p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}$$

```
[Out] (d^4*(d^2 - e^2*x^2)^(-1 + p))/(e^4*(1 - p)) + (3*d^2*(d^2 - e^2*x^2)^p)/(2
*e^4*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^4*(1 + p)) - (2*e*x^5*(d^2 - e^2*x^2
)^p*Hypergeometric2F1[5/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^3*(1 - (e^2*x^2
)/d^2)^p)
```

Rubi [A] time = 0.164881, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1652, 446, 77, 12, 365, 364}

$$-\frac{2ex^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3}+\frac{d^4\left(d^2-e^2x^2\right)^{p-1}}{e^4(1-p)}+\frac{3d^2\left(d^2-e^2x^2\right)^p}{2e^4p}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^4*(d^2 - e^2*x^2)^(-1 + p))/(e^4*(1 - p)) + (3*d^2*(d^2 - e^2*x^2)^p)/(2
*e^4*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^4*(1 + p)) - (2*e*x^5*(d^2 - e^2*x^2
)^p*Hypergeometric2F1[5/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^3*(1 - (e^2*x^2
)/d^2)^p)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx &= \int x^3(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \int -2dex^4\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^3\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)dx\\
&= \frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)dx,x,x^2\right)-(2de)\int x^4\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^4\left(d^2-e^2x\right)^{-2+p}}{e^2}-\frac{3d^2\left(d^2-e^2x\right)^{-1+p}}{e^2}+\frac{\left(d^2-e^2x\right)^p}{e^2}\right)dx,x,x^2\right)-\frac{\left(2e\left(d^2-e^{\cdots}\right.\right.}{\cdots}\\
&= \frac{d^4\left(d^2-e^2x^2\right)^{-1+p}}{e^4(1-p)}+\frac{3d^2\left(d^2-e^2x^2\right)^p}{2e^4p}-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)}-\frac{2ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\cdots\right.}{5d^3}\end{aligned}$$

Mathematica [B] time = 0.313731, size = 332, normalized size = 2.21

$$2^{p-2}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-8de(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-6d(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p{}_2F_1\left(1-\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*d^2*(1/2 + (e*x)/(2*d))^p - 2*d^2*(1/2 + (
e*x)/(2*d))^p*(1 - (e^2*x^2)/d^2)^p + 2*e^2*x^2*(1/2 + (e*x)/(2*d))^p*(1 -
(e^2*x^2)/d^2)^p - 8*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[
1/2, -p, 3/2, (e^2*x^2)/d^2] - 6*d*(d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d^2*(1 - (e^2*x^2)/d^2)^p
*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - d*e*x*(1 - (e^2*
```

```
x^2)/d^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^4*
(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.705, size = 0, normalized size = 0.

$$\int \frac{x^3 \left(-x^2 e^2 + d^2\right)^p}{\left(ex + d\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2 x^2 + d^2\right)^p x^3}{\left(ex + d\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2 x^2 + d^2\right)^p x^3}{e^2 x^2 + 2\, dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{\left(d + ex\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)
```

3.278 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=156

$$\frac{2(p+2)x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)}-\frac{x^3\left(d^2-e^2x^2\right)^{p-1}}{2p+1}-\frac{d^3\left(d^2-e^2x^2\right)^{p-1}}{e^3(1-p)}-\frac{d\left(d^2-e^2x^2\right)^p}{e^3p}$$

```
[Out] -((d^3*(d^2 - e^2*x^2)^(-1 + p))/(e^3*(1 - p))) - (x^3*(d^2 - e^2*x^2)^(-1
+ p))/(1 + 2*p) - (d*(d^2 - e^2*x^2)^p)/(e^3*p) + (2*(2 + p)*x^3*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[3/2, 2 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 + 2*p)
*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.185173, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {852, 1652, 459, 365, 364, 12, 266, 43}

$$\frac{2(p+2)x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)}-\frac{x^3\left(d^2-e^2x^2\right)^{p-1}}{2p+1}-\frac{d^3\left(d^2-e^2x^2\right)^{p-1}}{e^3(1-p)}-\frac{d\left(d^2-e^2x^2\right)^p}{e^3p}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^3*(d^2 - e^2*x^2)^(-1 + p))/(e^3*(1 - p))) - (x^3*(d^2 - e^2*x^2)^(-1
+ p))/(1 + 2*p) - (d*(d^2 - e^2*x^2)^p)/(e^3*p) + (2*(2 + p)*x^3*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[3/2, 2 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 + 2*p)
*(1 - (e^2*x^2)/d^2)^p)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx &= \int x^2(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx \\
&= \int -2dex^3\left(d^2-e^2x^2\right)^{-2+p}\,dx+\int x^2\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)dx \\
&= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p}-(2de)\int x^3\left(d^2-e^2x^2\right)^{-2+p}\,dx+\frac{\left(2d^2(2+p)\right)\int x^2\left(d^2-e^2x^2\right)^{-2+p}\,dx}{1+2p} \\
&= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p}-(de)\,\mathrm{Subst}\left(\int x\left(d^2-e^2x\right)^{-2+p}\,dx,x,x^2\right)+\frac{\left(2(2+p)\left(d^2-e^2x^2\right)^p\left(1-\right.\right.}{d^2(} \\
&= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p}+\frac{2(2+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(1+2p)}-(de)\,\mathrm{Subst} \\
&= -\frac{d^3\left(d^2-e^2x^2\right)^{-1+p}}{e^3(1-p)}-\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p}-\frac{d\left(d^2-e^2x^2\right)^p}{e^3p}+\frac{2(2+p)x^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right.}{3d^2(1+2}
\end{aligned}
$$

Mathematica [A] time = 0.175926, size = 177, normalized size = 1.13

$$
\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(4e(p+1)x\left(\frac{ex}{2d}+\frac{1}{2}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(4\,{}_2F_1\left(1-p,p\right.\right.\right.}{e^3(p+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(4*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(4
*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometric2F
1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(e^3*(1 + p)*(1 + (e*x)/d)^p*(1
- (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.662, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(-x^2 e^2 + d^2\right)^p}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2 x^2 + d^2\right)^p x^2}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2 x^2 + d^2\right)^p x^2}{e^2 x^2 + 2\, dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p x^2}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)
```

3.279 $\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=115

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^2(1-p)(d+ex)^2}-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^2\left(1-p^2\right)}$$

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(1 - p)*(d + e*x)^2) - (2^(-1 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^2*e^2*(1 - p^2))
```

Rubi [A] time = 0.0539433, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {793, 678, 69}

$$\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^2(1-p)(d+ex)^2}-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^2\left(1-p^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(1 - p)*(d + e*x)^2) - (2^(-1 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^2*e^2*(1 - p^2))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) && !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx = \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1-p)(d+ex)^2} + \frac{\int \frac{\left(d^2-e^2x^2\right)^p}{d+ex}\,dx}{e(1-p)}$$
$$= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1-p)(d+ex)^2} + \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-1+p}dx}{d^2e(1-p)}$$
$$= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1-p)(d+ex)^2} - \frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e^2\left(1-p^2\right)}$$

Mathematica [A] time = 0.096461, size = 102, normalized size = 0.89

$$\frac{2^{p-2}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left({}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)-2\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)\right)}{de^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[1 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F] time = 0.683, size = 0, normalized size = 0.

$$\int \frac{x\left(-x^2e^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x}{e^2x^2+2\,dex+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}x}{\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```

3.280 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

```
[Out] -((2^(-2 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)))
```

Rubi [A] time = 0.0347262, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-2+p}\,dx}{d^3}\\ &= -\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0447244, size = 75, normalized size = 1.03

$$-\frac{2^{p-2}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^{p}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.66, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^2x^2+2\,dex+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

3.281 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2} dx$

Optimal. Leaf size=128

$$-\frac{2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{\left(d^2-e^2x^2\right)^p {}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}+\frac{\left(d^2-e^2x^2\right)^{p-1}}{1-p}$$

```
[Out] (d^2 - e^2*x^2)^(-1 + p)/(1 - p) - (2*e*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^3*(1 - (e^2*x^2)/d^2)^p) - ((d^2 - e
^2*x^2)^p*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rubi [A] time = 0.124908, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {852, 1652, 446, 79, 65, 12, 246, 245}

$$-\frac{2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{\left(d^2-e^2x^2\right)^p {}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}+\frac{\left(d^2-e^2x^2\right)^{p-1}}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] (d^2 - e^2*x^2)^(-1 + p)/(1 - p) - (2*e*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^3*(1 - (e^2*x^2)/d^2)^p) - ((d^2 - e
^2*x^2)^p*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{x(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx\\
&= \int -2de\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int \frac{\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)}{x}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)}{x}\,dx, x, x^2\right) - (2de)\int \left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{\left(d^2-e^2x^2\right)^{-1+p}}{1-p} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x}\,dx, x, x^2\right) - \frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\right.}{d^3}\\
&= \frac{\left(d^2-e^2x^2\right)^{-1+p}}{1-p} - \frac{2ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{\left(d^2-e^2x^2\right)^p{}_2F_1\left(1,p;1\right.}{2d^2p}\end{aligned}$$

Mathematica [A] time = 0.167914, size = 201, normalized size = 1.57

$$\frac{2^{p-2}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(2p(d-ex)\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)+p(d-ex)\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F\right.}{d^3p(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + p*(1 - d^2/(e^2*x^2))^p*(
d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d*(1 +
 p)*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))
/(d^3*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

Maple [F] time = 0.645, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2} e^{2}+d^{2}\right)^{p}}{x \left(e x +d \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{p}}{{\left(e x + d\right)}^{2} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{p}}{e^{2} x^{3} + 2 \, d e x^{2} + d^{2} x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{x \left(d + e x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{(ex + d)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x), x)
```

3.282 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2} dx$

Optimal. Leaf size=137

$$-\frac{e\left(d^2-e^2x^2\right)^{p-1} {}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}+\frac{2e^2(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{x}$$

```
[Out] -((d^2 - e^2*x^2)^(-1 + p)/x) + (2*e^2*(2 - p)*x*(d^2 - e^2*x^2)^p*Hypergeo
metric2F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^4*(1 - (e^2*x^2)/d^2)^p) - (e
*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2
])/(d*(1 - p))
```

Rubi [A] time = 0.161907, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 266, 65, 246, 245}

$$-\frac{e\left(d^2-e^2x^2\right)^{p-1} {}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}+\frac{2e^2(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] -((d^2 - e^2*x^2)^(-1 + p)/x) + (2*e^2*(2 - p)*x*(d^2 - e^2*x^2)^p*Hypergeo
metric2F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^4*(1 - (e^2*x^2)/d^2)^p) - (e
*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2
])/(d*(1 - p))
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)^2}\,dx = \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - \frac{\int \frac{\left(2d^3e-2d^2e^2(2-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx}{d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - (2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx + \left(2e^2(2-p)\right)\int \left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} - (de)\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx, x, x^2\right) + \frac{\left(2e^2(2-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)\right.}{d^4}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{x} + \frac{2e^2(2-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4} - \frac{e\left(d^2-e^2x^2\right)^{-1}}{}$$

Mathematica [C] time = 0.112843, size = 82, normalized size = 0.6

$$\frac{\left(1-\frac{d^2}{e^2x^2}\right)^{-p}(d-ex)^p(d+ex)^pF_1\left(3-2p;-p,2-p;4-2p;\frac{d}{ex},-\frac{d}{ex}\right)}{e^2(2p-3)x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] ((d - e*x)^p*(d + e*x)^p*AppellF1[3 - 2*p, -p, 2 - p, 4 - 2*p, d/(e*x), -(d
/(e*x))])/(e^2*(-3 + 2*p)*(1 - d^2/(e^2*x^2))^p*x^3)
```

Maple [F] time = 0.656, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2} e^{2}+d^{2}\right)^{p}}{x^{2} \left(e x +d \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{2} x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{e^{2} x^{4}+2\, d e x^{3}+d^{2} x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x^{2}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{2} x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^2), x)
```

3.283 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx$

Optimal. Leaf size=143

$$\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{2x^2}$$

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(2*x^2) + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(d^3*x*(1 - (e^2*x^2)/d^2)^p) + (e^2*(3
 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2
)/d^2])/(2*d^2*(1 - p))
```

Rubi [A] time = 0.16156, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 365, 364, 266, 65}

$$\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(2*x^2) + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(d^3*x*(1 - (e^2*x^2)/d^2)^p) + (e^2*(3
 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2
)/d^2])/(2*d^2*(1 - p))
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
```

```
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^p}{x^3(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2}-\frac{\int \frac{\left(4d^3e-2d^2e^2(3-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx}{2d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2}-(2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx+\left(e^2(3-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2}+\frac{1}{2}\left(e^2(3-p)\right)\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx,x,x^2\right)-\frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right.\right.}{d^3} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2}+\frac{2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x}+\frac{e^2(3-p)\left(d^2-e^2x^2\right)^{-1}}{2c}
\end{aligned}$$

Mathematica [A] time = 0.538843, size = 283, normalized size = 1.98

$$\frac{\left(d^2-e^2x^2\right)^p\left(\frac{2d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2}+\frac{6de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}+\frac{8d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{3e^22^{p+1}(d-e}{\cdots}\right.}{4d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + (2*d^3*Hypergeometric2F1[1 - p, -p, 2 - p, d
^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(1 + p)*e^2*(d -
 e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
```

```
+ (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d*e^2*Hypergeometric2F1[-p,
 -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(4*d^5)
```

Maple [F] time = 0.668, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x^3\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^2x^5+2\,dex^4+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^5 + 2*d*e*x^4 + d^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x^3\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{\left(ex + d\right)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^3), x)
```

3.284 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{e^3\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}-\frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{3x^3}$$

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(3*x^3) - (2*e^2*(4 - p)*(d^2 - e^2*x^2)^p*Hyperg
eometric2F1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(3*d^4*x*(1 - (e^2*x^2)/d^2)^
p) - (e^3*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^2
*x^2)/d^2])/(d^3*(1 - p))
```

Rubi [A] time = 0.171583, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 266, 65, 365, 364}

$$-\frac{e^3\left(d^2-e^2x^2\right)^{p-1}{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}-\frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(3*x^3) - (2*e^2*(4 - p)*(d^2 - e^2*x^2)^p*Hyperg
eometric2F1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(3*d^4*x*(1 - (e^2*x^2)/d^2)^
p) - (e^3*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^2
*x^2)/d^2])/(d^3*(1 - p))
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^2}\,dx = \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^4}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{3x^3} - \frac{\int \frac{\left(6d^3e-2d^2e^2(4-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx}{3d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{3x^3} - (2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx + \frac{1}{3}\left(2e^2(4-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{3x^3} - (de)\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x^2}\,dx, x, x^2\right) + \frac{\left(2e^2(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)\right.}{3d^4}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{3x^3} - \frac{2e^2(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x} - \frac{e^3\left(d^2-e^2x^2\right)^{-}}{}$$

Mathematica [B] time = 0.450979, size = 334, normalized size = 2.3

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{12d^3e\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} - \frac{24de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} - \frac{4d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3} - \frac{36d^2e^2}{}\right.}{12d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-4*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^2
])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (36*d^2*e^2*Hypergeometric2F1[-1/2, -p, 1/
2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (12*d^3*e*Hypergeometric2F1[
1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3
```

```
*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/
(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1
[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (24*d*e
^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^
p)))/(12*d^6)
```

Maple [F] time = 0.683, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2} e^{2}+d^{2}\right)^{p}}{x^{4} \left(e x +d \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d \right)^{2} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{e^{2} x^{6}+2\, d e x^{5}+d^{2} x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^6 + 2*d*e*x^5 + d^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+e x\right)\left(d+e x\right)\right)^{p}}{x^{4} \left(d+e x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^4), x)
```

3.285 $\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^2}\,dx$

Optimal. Leaf size=145

$$\frac{e^4(5-p)\left(d^2-e^2x^2\right)^{p-1}\,{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{4x^4}$$

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(4*x^4) + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-3/2, 2 - p, -1/2, (e^2*x^2)/d^2])/(3*d^3*x^3*(1 - (e^2*x^2)/d^2)^p) + (e
^4*(5 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^
2*x^2)/d^2])/(4*d^4*(1 - p))
```

Rubi [A] time = 0.171681, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 365, 364, 266, 65}

$$\frac{e^4(5-p)\left(d^2-e^2x^2\right)^{p-1}\,{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3}-\frac{\left(d^2-e^2x^2\right)^{p-1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

```
[Out] -(d^2 - e^2*x^2)^(-1 + p)/(4*x^4) + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-3/2, 2 - p, -1/2, (e^2*x^2)/d^2])/(3*d^3*x^3*(1 - (e^2*x^2)/d^2)^p) + (e
^4*(5 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^
2*x^2)/d^2])/(4*d^4*(1 - p))
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
```

```
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^5}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4} - \frac{\int \frac{\left(8d^3e-2d^2e^2(5-p)x\right)\left(d^2-e^2x^2\right)^{-2+p}}{x^4}\,dx}{4d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4} - (2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^4}\,dx + \frac{1}{2}\left(e^2(5-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4} + \frac{1}{4}\left(e^2(5-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x^2}\,dx, x, x^2\right) - \frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right.\right.}{d^3} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{4x^4} + \frac{2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3} + \frac{e^4(5-p)\left(d^2-e^2x^2\right)}{}
\end{aligned}$$

Mathematica [B] time = 0.551028, size = 389, normalized size = 2.68

$$\left(d^2-e^2x^2\right)^p\left(\frac{18d^3e^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} + \frac{6d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right)}{(p-2)x^4} + \frac{30de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{8d^4e(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (48*d^2*e^3*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (18*d^3*e^2*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
```

```
(15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d^5*Hypergeometric2F1[2 - p, -p,
3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (3*2^p*e^4*(d
 - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(
1 + (e*x)/d)^p) + (30*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]
)/(p*(1 - d^2/(e^2*x^2))^p)))/(12*d^7)
```

Maple [F] time = 0.695, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2} e^{2}+d^{2}\right)^{p}}{x^{5} \left(e x +d \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)

[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{p}}{{\left(e x + d\right)}^{2} x^{5}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="maxima")

[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(-e^{2} x^{2} + d^{2}\right)}^{p}}{e^{2} x^{7} + 2 \, d e x^{6} + d^{2} x^{5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="fricas")

[Out] integral((-e^2*x^2 + d^2)^p/(e^2*x^7 + 2*d*e*x^6 + d^2*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{x^{5} \left(d + e x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)
```

3.286 $\int \frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=220

$$\frac{2(p+8)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3(2p+1)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}-\frac{2d^6\left(d^2-e^2x^2\right)^{p-2}}{e^5(2-p)}+\frac{9d^4\left(d^2-e^2x^2\right)^{p-}}{2e^5(1-p)}$$

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (3*d*x^5*(d^2 - e^2*x^2)^
(-2 + p))/(1 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(-1 + p))/(2*e^5*(1 - p)) + (3
*d^2*(d^2 - e^2*x^2)^p)/(e^5*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^5*(1 + p)) +
 (2*(8 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^3*(1 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.229109, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1652, 459, 365, 364, 446, 77}

$$\frac{2(p+8)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3(2p+1)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}-\frac{2d^6\left(d^2-e^2x^2\right)^{p-2}}{e^5(2-p)}+\frac{9d^4\left(d^2-e^2x^2\right)^{p-}}{2e^5(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (3*d*x^5*(d^2 - e^2*x^2)^
(-2 + p))/(1 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(-1 + p))/(2*e^5*(1 - p)) + (3
*d^2*(d^2 - e^2*x^2)^p)/(e^5*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^5*(1 + p)) +
 (2*(8 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^3*(1 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx &= \int x^4(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx\\
&= \int x^4\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)dx+\int x^5\left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)dx\\
&= -\frac{3dx^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\operatorname{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-3+p}\left(-3d^2e-e^3x\right)dx,x,x^2\right)+\frac{\left(2d^3(8+p\right.}{}\\
&= -\frac{3dx^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{4d^6\left(d^2-e^2x\right)^{-3+p}}{e^3}+\frac{9d^4\left(d^2-e^2x\right)^{-2+p}}{e^3}-\frac{6d^2\left(d^2-\right.}{e^{}}\right.\right.\\
&= -\frac{2d^6\left(d^2-e^2x^2\right)^{-2+p}}{e^5(2-p)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{9d^4\left(d^2-e^2x^2\right)^{-1+p}}{2e^5(1-p)}+\frac{3d^2\left(d^2-e^2x^2\right)^p}{e^5p}-\frac{\left(d^2\right.}{2}
\end{aligned}$$

Mathematica [C] time = 0.146326, size = 66, normalized size = 0.3

$$\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(5;-p,3-p;6;\frac{ex}{d},-\frac{ex}{d}\right)}{5d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 3 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d^3*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.698, size = 0, normalized size = 0.

$$\int \frac{x^4 \left(-x^2 e^2 + d^2\right)^p}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2 x^2 + d^2\right)^p x^4}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2 x^2 + d^2\right)^p x^4}{e^3 x^3 + 3\, de^2 x^2 + 3\, d^2 ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(-(-d + ex)(d + ex)\right)^p}{(d + ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^3, x)
```

3.287 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=194

$$-\frac{2e(3p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)}+\frac{ex^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}+\frac{2d^5\left(d^2-e^2x^2\right)^{p-2}}{e^4(2-p)}-\frac{7d^3\left(d^2-e^2x^2\right)}{2e^4(1-p)}$$

```
[Out] (2*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^4*(2 - p)) + (e*x^5*(d^2 - e^2*x^2)^(-2
 + p))/(1 + 2*p) - (7*d^3*(d^2 - e^2*x^2)^(-1 + p))/(2*e^4*(1 - p)) - (3*d*
(d^2 - e^2*x^2)^p)/(2*e^4*p) - (2*e*(4 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```

Rubi [A] time = 0.205627, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1652, 446, 77, 459, 365, 364}

$$-\frac{2e(3p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)}+\frac{ex^5\left(d^2-e^2x^2\right)^{p-2}}{2p+1}+\frac{2d^5\left(d^2-e^2x^2\right)^{p-2}}{e^4(2-p)}-\frac{7d^3\left(d^2-e^2x^2\right)}{2e^4(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```


```
[Out] (2*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^4*(2 - p)) + (e*x^5*(d^2 - e^2*x^2)^(-2
 + p))/(1 + 2*p) - (7*d^3*(d^2 - e^2*x^2)^(-1 + p))/(2*e^4*(1 - p)) - (3*d*
(d^2 - e^2*x^2)^p)/(2*e^4*p) - (2*e*(4 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx &= \int x^3(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx\\ &= \int x^3\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)dx+\int x^4\left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)dx\\ &= \frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\mathrm{Subst}\left(\int x\left(d^2-e^2x\right)^{-3+p}\left(d^3+3de^2x\right)dx,x,x^2\right)-\frac{\left(2d^2e(4+3p)\right)\int}{}\\ &= \frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}+\frac{1}{2}\mathrm{Subst}\left(\int\left(\frac{4d^5\left(d^2-e^2x\right)^{-3+p}}{e^2}-\frac{7d^3\left(d^2-e^2x\right)^{-2+p}}{e^2}+\frac{3d\left(d^2-e^2x\right)^{-}}{e^2}\right.\right.\\ &= \frac{2d^5\left(d^2-e^2x^2\right)^{-2+p}}{e^4(2-p)}+\frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p}-\frac{7d^3\left(d^2-e^2x^2\right)^{-1+p}}{2e^4(1-p)}-\frac{3d\left(d^2-e^2x^2\right)^p}{2e^4p}-\frac{2e(4+3}{}\end{aligned}$$

Mathematica [C] time = 0.121968, size = 66, normalized size = 0.34

$$\frac{x^4(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(4;-p,3-p;5;\frac{ex}{d},-\frac{ex}{d}\right)}{4d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x^4*(d - e*x)^p*(d + e*x)^p*AppellF1[4, -p, 3 - p, 5, (e*x)/d, -((e*x)/d)]
)/(4*d^3*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.671, size = 0, normalized size = 0.

$$\int \frac{x^3 \left(-x^2e^2 + d^2\right)^p}{\left(ex + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p x^3}{\left(ex + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2 + d^2\right)^p x^3}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{\left(d + ex\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^3, x)
```

3.288 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=157

$$\frac{2^{p-3}(p+4)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^3p(d+ex)^2}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^3(2-p)(d+ex)^3}$$

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(2 - p)*(d + e*x)^3) - (d^2 - e^2*x^2)^
(1 + p)/(2*e^3*p*(d + e*x)^2) + (2^(-3 + p)*(4 + p)*(1 + (e*x)/d)^(-1 - p)*
(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2
*d)])/(d^2*e^3*(2 - p)*p*(1 + p))
```

Rubi [A] time = 0.183063, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1639, 793, 678, 69}

$$\frac{2^{p-3}(p+4)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^3p(d+ex)^2}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^3(2-p)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(2 - p)*(d + e*x)^3) - (d^2 - e^2*x^2)^
(1 + p)/(2*e^3*p*(d + e*x)^2) + (2^(-3 + p)*(4 + p)*(1 + (e*x)/d)^(-1 - p)*
(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2
*d)])/(d^2*e^3*(2 - p)*p*(1 + p))
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
```

```
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{x^2\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3p(d+ex)^2}-\frac{\int \frac{\left(2d^2e^2+2de^3(1+p)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx}{2e^4p}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3p(d+ex)^2}-\frac{(d(4+p))\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx}{2e^2(2-p)p}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3p(d+ex)^2}-\frac{\left((4+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int(d}{2d^2e^2(2-p)p}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3p(d+ex)^2}+\frac{2^{-3+p}(4+p)\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(2-p,1\right.}{d^2e^3(2-p)p(1+p)}$$

Mathematica [A] time = 0.133383, size = 130, normalized size = 0.83

$$-\frac{2^{p-3}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(4\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)-4\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)+{}_2F_1\left(3-p,p\right.\right.}{de^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[1 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d*e
^3*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.668, size = 0, normalized size = 0.

$$\int \frac{x^2\left(-x^2e^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^2}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(-(-d+ex)(d+ex)\right)^p}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^2}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)
```

3.289 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=118

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(2-p)(d+ex)^3}-\frac{3\,2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(p+1)}$$

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(2 - p)*(d + e*x)^3) - (3*2^(-3 + p)*(1 + (e
*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(d^3*e^2*(2 - p)*(1 + p))
```

Rubi [A] time = 0.0533851, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {793, 678, 69}

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(2-p)(d+ex)^3}-\frac{3\,2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(2 - p)*(d + e*x)^3) - (3*2^(-3 + p)*(1 + (e
*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(d^3*e^2*(2 - p)*(1 + p))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx &= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(2-p)(d+ex)^3}+\frac{3\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx}{2e(2-p)}\\ &= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(2-p)(d+ex)^3}+\frac{\left(3(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-2+p}\,dx}{2d^3e(2-p)}\\ &= \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^2(2-p)(d+ex)^3}-\frac{3\,2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0876527, size = 102, normalized size = 0.86

$$\frac{2^{p-3}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left({}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)-2\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)\right)}{d^2e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d^2*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F] time = 0.711, size = 0, normalized size = 0.

$$\int \frac{x\left(-x^2e^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}x}{\left(ex+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

3.290 $\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

```
[Out] -((2^(-3 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)))
```

Rubi [A] time = 0.0289771, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-3+p}\,dx}{d^4}\\ &= -\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0448501, size = 75, normalized size = 1.03

$$-\frac{2^{p-3}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.665, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2} e^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{e^{3} x^{3}+3\, d e^{2} x^{2}+3\, d^{2} e x+d^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

3.291 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$

Optimal. Leaf size=175

$$\frac{(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}-\frac{ex(d^2-e^2x^2)^{p}}{3-2p}$$

```
[Out] (2*d*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (e*x*(d^2 - e^2*x^2)^(-2 + p))/(3
- 2*p) - (2*e*(4 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3
/2, (e^2*x^2)/d^2])/(d^4*(3 - 2*p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2
)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d*(1 - p)
)
```

Rubi [A] time = 0.155502, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {852, 1652, 446, 79, 65, 388, 246, 245}

$$\frac{(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}-\frac{ex(d^2-e^2x^2)^{p}}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (2*d*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (e*x*(d^2 - e^2*x^2)^(-2 + p))/(3
- 2*p) - (2*e*(4 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3
/2, (e^2*x^2)/d^2])/(d^4*(3 - 2*p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2
)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d*(1 - p)
)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{x(d+ex)^3}\,dx &= \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx\\ &= \int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)}{x}\,dx + \int \left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)dx\\ &= -\frac{ex\left(d^2-e^2x^2\right)^{-2+p}}{3-2p} + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}\left(d^3+3de^2x\right)}{x}\,dx, x, x^2\right) - \frac{\left(2d^2e(4-3p)\right)\int\left(d^2-\right.}{3-2p}\\ &= \frac{2d\left(d^2-e^2x^2\right)^{-2+p}}{2-p} - \frac{ex\left(d^2-e^2x^2\right)^{-2+p}}{3-2p} + \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx, x, x^2\right) - \frac{\left(2e(4-3p)\left(d^2\right.\right.}{}\\ &= \frac{2d\left(d^2-e^2x^2\right)^{-2+p}}{2-p} - \frac{ex\left(d^2-e^2x^2\right)^{-2+p}}{3-2p} - \frac{2e(4-3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right.}{d^4(3-2p)}\end{aligned}$$

Mathematica [C] time = 0.0971274, size = 82, normalized size = 0.47

$$\frac{\left(1-\frac{d^2}{e^2x^2}\right)^{-p}(d-ex)^p(d+ex)^pF_1\left(3-2p;-p,3-p;4-2p;\frac{d}{ex},-\frac{d}{ex}\right)}{e^3(2p-3)x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] ((d - e*x)^p*(d + e*x)^p*AppellF1[3 - 2*p, -p, 3 - p, 4 - 2*p, d/(e*x), -(d
/(e*x))])/(e^3*(-3 + 2*p)*(1 - d^2/(e^2*x^2))^p*x^3)
```

Maple [F] time = 0.65, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^3x^4+3\,de^2x^3+3\,d^2ex^2+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)
```

3.292 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{3e\left(d^2-e^2x^2\right)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}+\frac{2e^2(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{2e\left(d^2-e^2\right.}{2-\ldots}$$

```
[Out] (-2*e*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (d*(d^2 - e^2*x^2)^(-2 + p))/x +
(2*e^2*(4 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2*
x^2)/d^2])/(d^5*(1 - (e^2*x^2)/d^2)^p) - (3*e*(d^2 - e^2*x^2)^(-1 + p)*Hype
rgeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 - p))
```

Rubi [A] time = 0.225759, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {852, 1807, 1652, 446, 79, 65, 12, 246, 245}

$$-\frac{3e\left(d^2-e^2x^2\right)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}+\frac{2e^2(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{2e\left(d^2-e^2\right.}{2-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] (-2*e*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (d*(d^2 - e^2*x^2)^(-2 + p))/x +
(2*e^2*(4 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2*
x^2)/d^2])/(d^5*(1 - (e^2*x^2)/d^2)^p) - (3*e*(d^2 - e^2*x^2)^(-1 + p)*Hype
rgeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 - p))
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{x} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(3d^4e-2d^3e^2(4-p)x+d^2e^3x^2\right)}{x}\,dx}{d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{x} - \frac{\int -2d^3e^2(4-p)\left(d^2-e^2x^2\right)^{-3+p}\,dx}{d^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(3d^4e+d^2e^3x^2\right)}{x}\,dx}{d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{x} - \frac{\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}\left(3d^4e+d^2e^3x\right)}{x}\,dx, x, x^2\right)}{2d^2} + \left(2de^2(4-p)\right)\int \left(d^2-e^2x^2\right)^{-3+}$$

$$= -\frac{2e\left(d^2-e^2x^2\right)^{-2+p}}{2-p} - \frac{d\left(d^2-e^2x^2\right)^{-2+p}}{x} - \frac{1}{2}(3e)\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx, x, x^2\right) + \frac{\left(2e^2(4-\right.}{}$$

$$= -\frac{2e\left(d^2-e^2x^2\right)^{-2+p}}{2-p} - \frac{d\left(d^2-e^2x^2\right)^{-2+p}}{x} + \frac{2e^2(4-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\right.}{d^5}$$

Mathematica [C] time = 0.135839, size = 83, normalized size = 0.5

$$\frac{\left(1-\frac{d^2}{e^2x^2}\right)^{-p}(d-ex)^p(d+ex)^pF_1\left(4-2p;-p,3-p;5-2p;\frac{d}{ex},-\frac{d}{ex}\right)}{2e^3(p-2)x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] ((d - e*x)^p*(d + e*x)^p*AppellF1[4 - 2*p, -p, 3 - p, 5 - 2*p, d/(e*x), -(d
/(e*x))])/(2*e^3*(-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4)
```

Maple [F] time = 0.661, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x^2\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^3x^5+3\,de^2x^4+3\,d^2ex^3+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^2(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)
```

3.293 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$

Optimal. Leaf size=173

$$\frac{e^2(6-p)\left(d^2-e^2x^2\right)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)}-\frac{2e^3(8-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6}+$$

```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(2*x^2) + (3*e*(d^2 - e^2*x^2)^(-2 + p))/x -
(2*e^3*(8 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^
2*x^2)/d^2])/(d^6*(1 - (e^2*x^2)/d^2)^p) + (e^2*(6 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d*(2 - p)
)
```

Rubi [A] time = 0.265752, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 266, 65, 246, 245}

$$\frac{e^2(6-p)\left(d^2-e^2x^2\right)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)}-\frac{2e^3(8-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6}+$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(2*x^2) + (3*e*(d^2 - e^2*x^2)^(-2 + p))/x -
(2*e^3*(8 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^
2*x^2)/d^2])/(d^6*(1 - (e^2*x^2)/d^2)^p) + (e^2*(6 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d*(2 - p)
)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^3(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^3}\,dx$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(6d^4e-2d^3e^2(6-p)x+2d^2e^3x^2\right)}{x^2}\,dx}{2d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{x} + \frac{\int \frac{\left(2d^5e^2(6-p)-4d^4e^3(8-3p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx}{2d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{x} - \left(2e^3(8-3p)\right)\int \left(d^2-e^2x^2\right)^{-3+p}\,dx + \left(de^2(6-p)\right)\int \frac{\left(d^2\right.}{}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{x} + \frac{1}{2}\left(de^2(6-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x}\,dx, x, x^2\right) - \frac{\left(2e\right.}{}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{x} - \frac{2e^3(8-3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2}{d}\right.}{d^6}$$

Mathematica [A] time = 0.680982, size = 341, normalized size = 1.97

$$\frac{\left(d^2-e^2x^2\right)^p\left(\frac{4d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} + \frac{24de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{24d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} + \frac{3e^22^{p+3}(d-ex)}{}\right.}{8d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((24*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (4*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(3 + p)*e^2*(d
- e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hy
pergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (24*d*e^2*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 -
d^2/(e^2*x^2))^p)))/(8*d^6)
```

Maple [F] time = 0.68, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2} e^{2}+d^{2}\right)^{p}}{x^{3} \left(e x+d\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x+d\right)^{3} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{e^{3} x^{6}+3\, d e^{2} x^{5}+3\, d^{2} e x^{4}+d^{3} x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+e x\right)\left(d+e x\right)\right)^{p}}{x^{3}\left(d+e x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^3), x)
```

3.294 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$

Optimal. Leaf size=179

$$-\frac{e^3(10-3p)\left(d^2-e^2x^2\right)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}-\frac{2e^2(8-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^5x}$$

```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(3*x^3) + (3*e*(d^2 - e^2*x^2)^(-2 + p))/(2*x
^2) - (2*e^2*(8 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2,
(e^2*x^2)/d^2])/(3*d^5*x*(1 - (e^2*x^2)/d^2)^p) - (e^3*(10 - 3*p)*(d^2 - e^
2*x^2)^(-2 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2
*d^2*(2 - p))
```

Rubi [A] time = 0.274783, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 365, 364, 266, 65}

$$-\frac{e^3(10-3p)\left(d^2-e^2x^2\right)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}-\frac{2e^2(8-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^5x}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```


```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(3*x^3) + (3*e*(d^2 - e^2*x^2)^(-2 + p))/(2*x
^2) - (2*e^2*(8 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2,
(e^2*x^2)/d^2])/(3*d^5*x*(1 - (e^2*x^2)/d^2)^p) - (e^3*(10 - 3*p)*(d^2 - e^
2*x^2)^(-2 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2
*d^2*(2 - p))
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^4}\,dx$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(9d^4e-2d^3e^2(8-p)x+3d^2e^3x^2\right)}{x^3}\,dx}{3d^2}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} + \frac{\int \frac{\left(4d^5e^2(8-p)-6d^4e^3(10-3p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx}{6d^4}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} - \left(e^3(10-3p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx + \frac{1}{3}\left(2de^2(8-p)\right)\int$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} - \frac{1}{2}\left(e^3(10-3p)\right)\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x}\,dx, x, x^2\right) + \frac{(2}{}$$
$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{3x^3} + \frac{3e\left(d^2-e^2x^2\right)^{-2+p}}{2x^2} - \frac{2e^2(8-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right.}{3d^5x}$$

Mathematica [B] time = 0.544901, size = 393, normalized size = 2.2

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{36d^3e\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} - \frac{120de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} - \frac{8d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3} - \frac{144d^2e^2(1}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-8*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^2
])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (144*d^2*e^2*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (36*d^3*e*Hypergeometric2F1
[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (
15*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x
)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeom
etric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) +
 (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*
d)])/((1 + p)*(1 + (e*x)/d)^p) - (120*d*e^3*Hypergeometric2F1[-p, -p, 1 - p
, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(24*d^7)
```

Maple [F] time = 0.711, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2}e^{2}+d^{2}\right)^{p}}{x^{4}\left(ex+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{3}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{e^{3}x^{7}+3\,de^{2}x^{6}+3\,d^{2}ex^{5}+d^{3}x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x^{4}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)
```

3.295 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3} \, dx$

Optimal. Leaf size=174

$$\frac{e^4(10-p)\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(2,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{4d^3(2-p)}+\frac{2e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^6x}+$$

```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(4*x^4) + (e*(d^2 - e^2*x^2)^(-2 + p))/x^3 +
(2*e^3*(4 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x
^2)/d^2])/(d^6*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(10 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[2, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(4*d^3*(2 -
p))
```

Rubi [A] time = 0.27686, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 266, 65, 365, 364}

$$\frac{e^4(10-p)\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(2,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{4d^3(2-p)}+\frac{2e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^6x}+$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(-2 + p))/(4*x^4) + (e*(d^2 - e^2*x^2)^(-2 + p))/x^3 +
(2*e^3*(4 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x
^2)/d^2])/(d^6*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(10 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[2, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(4*d^3*(2 -
p))
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^5(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^5}\,dx$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(12d^4e-2d^3e^2(10-p)x+4d^2e^3x^2\right)}{x^4}\,dx}{4d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} + \frac{\int \frac{\left(6d^5e^2(10-p)-24d^4e^3(4-p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x^3}\,dx}{12d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} - \left(2e^3(4-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx + \frac{1}{2}\left(de^2(10-p)\right)\int \frac{(d}{}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} + \frac{1}{4}\left(de^2(10-p)\right)\operatorname{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x^2}\,dx, x, x^2\right) - \frac{(2e^3}{}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{-2+p}}{4x^4} + \frac{e\left(d^2-e^2x^2\right)^{-2+p}}{x^3} + \frac{2e^3(4-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^6x}$$

Mathematica [B] time = 0.624467, size = 446, normalized size = 2.56

$$\left(d^2-e^2x^2\right)^p\left(\frac{24d^3e^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} + \frac{4d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right)}{(p-2)x^4} + \frac{60de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{8d^4e\left(1-\frac{e^2}{}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (80*d^2*e^3*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (24*d^3*e^2*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
 (15*2^(2 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (4*d^5*Hypergeometric2F1[2 - p, -p,
3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (5*2^(1 + p)*
e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
+ p)*(1 + (e*x)/d)^p) + (2^p*e^4*(d - e*x)*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (60*d*e^4*Hypergeometr
ic2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^8)
```

Maple [F] time = 0.693, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x^5\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^3x^8+3\,de^2x^7+3\,d^2ex^6+d^3x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x^5\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^5), x)
```

3.296 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4} dx$

Optimal. Leaf size=265

$$-\frac{4\left(p^2+15p+16\right)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{5}{2},4-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4\left(1-4p^2\right)}+\frac{d^2(12p+13)x^5\left(d^2-e^2x^2\right)^{p-3}}{1-4p^2}-\frac{e^2x^7\left(d^2-e^2x\right.}{2p+1}$$

```
[Out] (-4*d^7*(d^2 - e^2*x^2)^(-3 + p))/(e^5*(3 - p)) + (d^2*(13 + 12*p)*x^5*(d^2
 - e^2*x^2)^(-3 + p))/(1 - 4*p^2) - (e^2*x^7*(d^2 - e^2*x^2)^(-3 + p))/(1 +
 2*p) + (10*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (8*d^3*(d^2 - e^2
*x^2)^(-1 + p))/(e^5*(1 - p)) - (2*d*(d^2 - e^2*x^2)^p)/(e^5*p) - (4*(16 +
15*p + p^2)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 4 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^4*(1 - 4*p^2)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.305566, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {852, 1652, 1267, 459, 365, 364, 446, 77}

$$-\frac{4\left(p^2+15p+16\right)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p {}_2F_1\left(\frac{5}{2},4-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4\left(1-4p^2\right)}+\frac{d^2(12p+13)x^5\left(d^2-e^2x^2\right)^{p-3}}{1-4p^2}-\frac{e^2x^7\left(d^2-e^2x\right.}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```


```
[Out] (-4*d^7*(d^2 - e^2*x^2)^(-3 + p))/(e^5*(3 - p)) + (d^2*(13 + 12*p)*x^5*(d^2
 - e^2*x^2)^(-3 + p))/(1 - 4*p^2) - (e^2*x^7*(d^2 - e^2*x^2)^(-3 + p))/(1 +
 2*p) + (10*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (8*d^3*(d^2 - e^2
*x^2)^(-1 + p))/(e^5*(1 - p)) - (2*d*(d^2 - e^2*x^2)^p)/(e^5*p) - (4*(16 +
15*p + p^2)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 4 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^4*(1 - 4*p^2)*(1 - (e^2*x^2)/d^2)^p)
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
```

```
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^4\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx = \int x^4(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}\,dx$$
$$= \int x^5\left(d^2-e^2x^2\right)^{-4+p}\left(-4d^3e-4de^3x^2\right)dx + \int x^4\left(d^2-e^2x^2\right)^{-4+p}\left(d^4+6d^2e^2x^2+e^4x^4\right)dx$$
$$= -\frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-4+p}\left(-4d^3e-4de^3x\right)dx,x,x^2\right) - \frac{\int x^4\left(d^2\right.}{}$$
$$= \frac{d^2(13+12p)x^5\left(d^2-e^2x^2\right)^{-3+p}}{1-4p^2} - \frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int\left(-\frac{8d^7\left(d^2-e^2x\right)^{-4+p}}{e^3}+\right.\right.$$
$$= -\frac{4d^7\left(d^2-e^2x^2\right)^{-3+p}}{e^5(3-p)} + \frac{d^2(13+12p)x^5\left(d^2-e^2x^2\right)^{-3+p}}{1-4p^2} - \frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p} + \frac{10d^5\left(d^2-e\right.}{e^5(2-}$$
$$= -\frac{4d^7\left(d^2-e^2x^2\right)^{-3+p}}{e^5(3-p)} + \frac{d^2(13+12p)x^5\left(d^2-e^2x^2\right)^{-3+p}}{1-4p^2} - \frac{e^2x^7\left(d^2-e^2x^2\right)^{-3+p}}{1+2p} + \frac{10d^5\left(d^2-e\right.}{e^5(2-}$$

Mathematica [C] time = 0.15664, size = 66, normalized size = 0.25

$$\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(5;-p,4-p;6;\frac{ex}{d},-\frac{ex}{d}\right)}{5d^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 4 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d^4*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.722, size = 0, normalized size = 0.

$$\int \frac{x^4\left(-x^2e^2+d^2\right)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^4}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2 + d^2\right)^p x^4}{e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2 + 4\,d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^4/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*
d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(-\left(-d + ex\right)\left(d + ex\right)\right)^p}{\left(d + ex\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p x^4}{\left(ex + d\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)
```

3.297 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=211

$$\frac{3\,2^{p-2}(p+2)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4p(d+ex)^2}-\frac{d(2p+1)\left(d^2-e^2x^2\right)^{p+1}}{e^4(1-2p)p(d+ex)^3}+$$

```
[Out] (d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(3 - p)*(d + e*x)^4) - (d*(1 + 2*p)*(d
^2 - e^2*x^2)^(1 + p))/(e^4*(1 - 2*p)*p*(d + e*x)^3) - (d^2 - e^2*x^2)^(1 +
 p)/(2*e^4*p*(d + e*x)^2) + (3*2^(-2 + p)*(2 + p)*(1 + (e*x)/d)^(-1 - p)*(d
^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d
)])/(d^2*e^4*(1 - 2*p)*(3 - p)*p*(1 + p))
```

Rubi [A] time = 0.375452, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1639, 793, 678, 69}

$$\frac{3\,2^{p-2}(p+2)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4p(d+ex)^2}-\frac{d(2p+1)\left(d^2-e^2x^2\right)^{p+1}}{e^4(1-2p)p(d+ex)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(3 - p)*(d + e*x)^4) - (d*(1 + 2*p)*(d
^2 - e^2*x^2)^(1 + p))/(e^4*(1 - 2*p)*p*(d + e*x)^3) - (d^2 - e^2*x^2)^(1 +
 p)/(2*e^4*p*(d + e*x)^2) + (3*2^(-2 + p)*(2 + p)*(1 + (e*x)/d)^(-1 - p)*(d
^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d
)])/(d^2*e^4*(1 - 2*p)*(3 - p)*p*(1 + p))
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
```

```
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(2d^3e^2+2d^2e^3(2+p)x+2de^4(1+2p)x^2\right)}{(d+ex)^4}\,dx}{2e^5p}$$

$$= -\frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\int \frac{\left(8d^3e^6(1+p)+2d^2e^7\left(4+3p+2p^2\right)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx}{2e^9(1-2p)p}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\left(6d^2(2+p)\right)\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx}{e^3(1-2p)(3-p)p}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} - \frac{\left(6(2+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)\right.}{d^2}$$

$$= \frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^4(3-p)(d+ex)^4} - \frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^4p(d+ex)^2} + \frac{3\ 2^{-2+p}(2+p)\left(1+\frac{ex}{d}\right)^{-1-p}\left(d\right.}{d^2e^4(1-}$$

Mathematica [C] time = 0.120592, size = 66, normalized size = 0.31

$$\frac{x^4(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(4;-p,4-p;5;\frac{ex}{d},-\frac{ex}{d}\right)}{4d^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (x^4*(d - e*x)^p*(d + e*x)^p*AppellF1[4, -p, 4 - p, 5, (e*x)/d, -((e*x)/d)]
)/(4*d^4*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.732, size = 0, normalized size = 0.

$$\int \frac{x^3\left(-x^2e^2+d^2\right)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p x^3}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^3/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*
d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p x^3}{\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^4, x)
```

3.298 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} dx$

Optimal. Leaf size=163

$$-\frac{2^{p-3}(p+7)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^3(1-2p)(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^3(3-p)(d+ex)^4}$$

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(3 - p)*(d + e*x)^4) + (d^2 - e^2*x^2)^
(1 + p)/(e^3*(1 - 2*p)*(d + e*x)^3) - (2^(-3 + p)*(7 + p)*(1 + (e*x)/d)^(-1
 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)])/(d^3*e^3*(1 - 2*p)*(3 - p)*(1 + p))
```

Rubi [A] time = 0.1906, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1639, 793, 678, 69}

$$-\frac{2^{p-3}(p+7)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(p+1)}+\frac{\left(d^2-e^2x^2\right)^{p+1}}{e^3(1-2p)(d+ex)^3}-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^3(3-p)(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] -(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(3 - p)*(d + e*x)^4) + (d^2 - e^2*x^2)^
(1 + p)/(e^3*(1 - 2*p)*(d + e*x)^3) - (2^(-3 + p)*(7 + p)*(1 + (e*x)/d)^(-1
 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)])/(d^3*e^3*(1 - 2*p)*(3 - p)*(1 + p))
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
```

```
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{x^2\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx = \frac{\left(d^2-e^2x^2\right)^{1+p}}{e^3(1-2p)(d+ex)^3}+\frac{\int \frac{\left(3d^2e^2+2de^3(1+p)x\right)\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx}{e^4(1-2p)}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(3-p)(d+ex)^4}+\frac{\left(d^2-e^2x^2\right)^{1+p}}{e^3(1-2p)(d+ex)^3}+\frac{(d(7+p))\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx}{e^2(1-2p)(3-p)}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(3-p)(d+ex)^4}+\frac{\left(d^2-e^2x^2\right)^{1+p}}{e^3(1-2p)(d+ex)^3}+\frac{\left((7+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right.}{d^3e^2(1-2p)(3-p)}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^3(3-p)(d+ex)^4}+\frac{\left(d^2-e^2x^2\right)^{1+p}}{e^3(1-2p)(d+ex)^3}-\frac{2^{-3+p}(7+p)\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(3\right.}{d^3e^3(1-2p)(3-p)(1+p)}$$

Mathematica [A] time = 0.135175, size = 130, normalized size = 0.8

$$-\frac{2^{p-4}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^p\left(4\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)-4\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)+{}_2F_1\left(4-p,p\right.\right.}{d^2e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[2 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d^2
*e^3*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.742, size = 0, normalized size = 0.

$$\int \frac{x^2\left(-x^2e^2+d^2\right)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^2}{(ex + d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^2}{e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2 + 4\,d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x^2/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*
d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,(-(-d + ex)\,(d + ex))^p}{(d + ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^2}{(ex + d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)
```

3.299 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=118

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(3-p)(d+ex)^4}-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(p+1)}$$

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(3 - p)*(d + e*x)^4) - (2^(-2 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^4*e^2*(3 - p)*(1 + p))
```

Rubi [A] time = 0.0535491, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {793, 678, 69}

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(3-p)(d+ex)^4}-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(3 - p)*(d + e*x)^4) - (2^(-2 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^4*e^2*(3 - p)*(1 + p))
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} + \frac{2\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx}{e(3-p)}$$
$$= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} + \frac{\left(2(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-3+p}dx}{d^4e(3-p)}$$
$$= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} - \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(1+p)}$$

Mathematica [A] time = 0.0904859, size = 102, normalized size = 0.86

$$\frac{2^{p-4}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}(d^2-e^2x^2)^p\left({}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)-2\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)\right)}{d^3e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[3 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d^3*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F] time = 0.706, size = 0, normalized size = 0.

$$\int \frac{x(-x^2e^2+d^2)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2+d^2)^p x}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(-e^2x^2+d^2)^p x}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*x/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^
3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}x}{\left(ex+d\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

3.300 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4} \, dx$

Optimal. Leaf size=73

$$-\frac{2^{p-4}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^5e(p+1)}$$

```
[Out] -((2^(-4 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^5*e*(1 + p)))
```

Rubi [A] time = 0.0310373, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {678, 69}

$$-\frac{2^{p-4}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^5e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^5*e*(1 + p)))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d^2-e^2x^2\right)^p}{(d+ex)^4}\,dx &= \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-4+p}\,dx}{d^5}\\ &= -\frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^5e(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0452202, size = 75, normalized size = 1.03

$$-\frac{2^{p-4}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}\left(d^2-e^2x^2\right)^{p}\,{}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[4 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F] time = 0.688, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*
e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2 + d^2\right)^p}{\left(ex + d\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

3.301 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$

Optimal. Leaf size=204

$$\frac{(d^2-e^2x^2)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2(2-p)}-\frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}-\frac{4dex\left(d^2-e\right.}{5-2}$$

```
[Out] (4*d^2*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (4*d*e*x*(d^2 - e^2*x^2)^(-3 + p
))/(5 - 2*p) - (d^2 - e^2*x^2)^(-2 + p)/(2*(2 - p)) - (8*e*(2 - p)*x*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^5*(5 - 2*
p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2)^(-2 + p)*Hypergeometric2F1[1,
-2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*(2 - p))
```

Rubi [A] time = 0.208253, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {852, 1652, 1251, 951, 79, 65, 388, 246, 245}

$$\frac{(d^2-e^2x^2)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2(2-p)}-\frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}-\frac{4dex\left(d^2-e\right.}{5-2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```


```
[Out] (4*d^2*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (4*d*e*x*(d^2 - e^2*x^2)^(-3 + p
))/(5 - 2*p) - (d^2 - e^2*x^2)^(-2 + p)/(2*(2 - p)) - (8*e*(2 - p)*x*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^5*(5 - 2*
p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2)^(-2 + p)*Hypergeometric2F1[1,
-2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*(2 - p))
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x(d+ex)^4}\,dx = \int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}}{x}\,dx$$
$$= \int \left(d^2-e^2x^2\right)^{-4+p}\left(-4d^3e-4de^3x^2\right)dx + \int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(d^4+6d^2e^2x^2+e^4x^4\right)}{x}\,dx$$
$$= -\frac{4dex\left(d^2-e^2x^2\right)^{-3+p}}{5-2p} + \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-4+p}\left(d^4+6d^2e^2x+e^4x^2\right)}{x}\,dx, x, x^2\right) - \frac{\left(8d^3e(2-\right.}{}$$
$$= -\frac{4dex\left(d^2-e^2x^2\right)^{-3+p}}{5-2p} - \frac{\left(d^2-e^2x^2\right)^{-2+p}}{2(2-p)} - \frac{\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-4+p}\left(-d^4e^4(2-p)-7d^2e^6(2-p)x\right)}{x}\,dx, x, x^2\right)}{2e^4(2-p)} -$$
$$= \frac{4d^2\left(d^2-e^2x^2\right)^{-3+p}}{3-p} - \frac{4dex\left(d^2-e^2x^2\right)^{-3+p}}{5-2p} - \frac{\left(d^2-e^2x^2\right)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right.}{d^5(5-2}$$
$$= \frac{4d^2\left(d^2-e^2x^2\right)^{-3+p}}{3-p} - \frac{4dex\left(d^2-e^2x^2\right)^{-3+p}}{5-2p} - \frac{\left(d^2-e^2x^2\right)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right.}{d^5(5-2}$$

Mathematica [C] time = 0.11495, size = 83, normalized size = 0.41

$$\frac{\left(1-\frac{d^2}{e^2x^2}\right)^{-p}(d-ex)^p(d+ex)^pF_1\left(4-2p;-p,4-p;5-2p;\frac{d}{ex},-\frac{d}{ex}\right)}{2e^4(p-2)x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```

```
[Out] ((d - e*x)^p*(d + e*x)^p*AppellF1[4 - 2*p, -p, 4 - p, 5 - 2*p, d/(e*x), -(d
/(e*x))])/(2*e^4*(-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4)
```

Maple [F] time = 0.674, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^5+4\,de^3x^4+6\,d^2e^2x^3+4\,d^3ex^2+d^4x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^5 + 4*d*e^3*x^4 + 6*d^2*e^2*x^3 + 4*d^3*
e*x^2 + d^4*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)
```

3.302 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$

Optimal. Leaf size=207

$$\frac{4e^2(p^2-9p+16)\,x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)}-\frac{2e(d^2-e^2x^2)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{d(2-p)}+$$

```
[Out] (-4*d*e*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (d^2*(d^2 - e^2*x^2)^(-3 + p))/
x + (e^2*x*(d^2 - e^2*x^2)^(-3 + p))/(5 - 2*p) + (4*e^2*(16 - 9*p + p^2)*x*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(
5 - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(d^2 - e^2*x^2)^(-2 + p)*Hypergeomet
ric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d*(2 - p))
```

Rubi [A] time = 0.277891, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {852, 1807, 1652, 446, 79, 65, 388, 246, 245}

$$\frac{4e^2(p^2-9p+16)\,x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)}-\frac{2e(d^2-e^2x^2)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{d(2-p)}+$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```


```
[Out] (-4*d*e*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (d^2*(d^2 - e^2*x^2)^(-3 + p))/
x + (e^2*x*(d^2 - e^2*x^2)^(-3 + p))/(5 - 2*p) + (4*e^2*(16 - 9*p + p^2)*x*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(
5 - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(d^2 - e^2*x^2)^(-2 + p)*Hypergeomet
ric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d*(2 - p))
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
```

```
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^2(d+ex)^4}\,dx=\int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}}{x^2}\,dx$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{x}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(4d^5e-d^4e^2(13-2p)x+4d^3e^3x^2-d^2e^4x^3\right)}{x}\,dx}{d^2}$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{x}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(4d^5e+4d^3e^3x^2\right)}{x}\,dx}{d^2}-\frac{\int \left(d^2-e^2x^2\right)^{-4+p}\left(-d^4e^2(13-2p)-d^2e^4x\right.}{d^2}$$
$$=-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{x}+\frac{e^2x\left(d^2-e^2x^2\right)^{-3+p}}{5-2p}-\frac{\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-4+p}\left(4d^5e+4d^3e^3x\right)}{x}\,dx,x,x^2\right)}{2d^2}+\frac{\left(4d^2e^2\right.}{}$$
$$=-\frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{3-p}-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{x}+\frac{e^2x\left(d^2-e^2x^2\right)^{-3+p}}{5-2p}-(2de)\,\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3}}{x}\right.$$
$$=-\frac{4de\left(d^2-e^2x^2\right)^{-3+p}}{3-p}-\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{x}+\frac{e^2x\left(d^2-e^2x^2\right)^{-3+p}}{5-2p}+\frac{4e^2\left(16-9p+p^2\right)x\left(d^2-e^2\right.}{}$$

Mathematica [C] time = 0.190724, size = 82, normalized size = 0.4

$$\frac{\left(1-\frac{d^2}{e^2x^2}\right)^{-p}(d-ex)^p(d+ex)^pF_1\left(5-2p;-p,4-p;6-2p;\frac{d}{ex},-\frac{d}{ex}\right)}{e^4(2p-5)x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

```
[Out] ((d - e*x)^p*(d + e*x)^p*AppellF1[5 - 2*p, -p, 4 - p, 6 - 2*p, d/(e*x), -(d
/(e*x))])/(e^4*(-5 + 2*p)*(1 - d^2/(e^2*x^2))^p*x^5)
```

Maple [F] time = 0.679, size = 0, normalized size = 0.

$$\int \frac{\left(-x^2e^2+d^2\right)^p}{x^2\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^6+4\,de^3x^5+6\,d^2e^2x^4+4\,d^3ex^3+d^4x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^6 + 4*d*e^3*x^5 + 6*d^2*e^2*x^4 + 4*d^3*
e*x^3 + d^4*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x^2\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)
```

3.303 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=211

$$\frac{e^2(10-p)\left(d^2-e^2x^2\right)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^7}+$$

```
[Out] (e^2*(11 - p)*(d^2 - e^2*x^2)^(-3 + p))/(2*(3 - p)) - (d^2*(d^2 - e^2*x^2)^
(-3 + p))/(2*x^2) + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/x - (8*e^3*(4 - p)*x*(
d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^7*(1
 - (e^2*x^2)/d^2)^p) + (e^2*(10 - p)*(d^2 - e^2*x^2)^(-2 + p)*Hypergeometri
c2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(2 - p))
```

Rubi [A] time = 0.367507, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {852, 1807, 1652, 446, 79, 65, 12, 246, 245}

$$\frac{e^2(10-p)\left(d^2-e^2x^2\right)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^7}+$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

```
[Out] (e^2*(11 - p)*(d^2 - e^2*x^2)^(-3 + p))/(2*(3 - p)) - (d^2*(d^2 - e^2*x^2)^
(-3 + p))/(2*x^2) + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/x - (8*e^3*(4 - p)*x*(
d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^7*(1
 - (e^2*x^2)/d^2)^p) + (e^2*(10 - p)*(d^2 - e^2*x^2)^(-2 + p)*Hypergeometri
c2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(2 - p))
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
```

```
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx = \int \frac{(d-ex)^4(d^2-e^2x^2)^{-4+p}}{x^3}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(8d^5e-2d^4e^2(10-p)x+8d^3e^3x^2-2d^2e^4x^3)}{x^2}\,dx}{2d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} + \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(2d^6e^2(10-p)-16d^5e^3(4-p)x+2d^4e^4x^2)}{x}\,dx}{2d^4}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} + \frac{\int -16d^5e^3(4-p)(d^2-e^2x^2)^{-4+p}\,dx}{2d^4} + \frac{\int \frac{(d^2-e^2x^2)^{-}}{}}{}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} + \frac{\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}(2d^6e^2(10-p)+2d^4e^4x)}{x}\,dx, x, x^2\right)}{4d^4} - ($$

$$= \frac{e^2(11-p)(d^2-e^2x^2)^{-3+p}}{2(3-p)} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} + \frac{1}{2}\left(e^2(10-p)\right)\text{Subst}\Bigg(\int$$

$$= \frac{e^2(11-p)(d^2-e^2x^2)^{-3+p}}{2(3-p)} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} - \frac{8e^3(4-p)x(d^2-e^2x^2)^{p}}{}$$

Mathematica [A] time = 0.856493, size = 399, normalized size = 1.89

$$\frac{(d^2-e^2x^2)^p\left(\frac{8d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} + \frac{80de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{64d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} + \frac{5e^22^{p+4}(d}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((64*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (8*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (5*2^(4 + p)*e^2*(d
- e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) + (3*2^(3 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d -
 e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
+ (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (80*d*e^2*Hypergeometric2F1[-p
, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(16*d^7)
```

Maple [F] time = 0.704, size = 0, normalized size = 0.

$$\int \frac{(-x^2e^2+d^2)^p}{x^3(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p}{e^4x^7+4\,de^3x^6+6\,d^2e^2x^5+4\,d^3ex^4+d^4x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^7 + 4*d*e^3*x^6 + 6*d^2*e^2*x^5 + 4*d^3*
e*x^4 + d^4*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{x^3\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)
```

3.304 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=210

$$\frac{4e^4(p^2-17p+48)\,x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{3d^8}-\frac{2e^3(5-p)(d^2-e^2x^2)^{p-3}\,{}_2F_1(1,p-3;p-2;1\cdots}{d(3-p)}$$

```
[Out] -(d^2*(d^2 - e^2*x^2)^(-3 + p))/(3*x^3) + (2*d*e*(d^2 - e^2*x^2)^(-3 + p))/
x^2 - (e^2*(27 - 2*p)*(d^2 - e^2*x^2)^(-3 + p))/(3*x) + (4*e^4*(48 - 17*p +
 p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2]
)/(3*d^8*(1 - (e^2*x^2)/d^2)^p) - (2*e^3*(5 - p)*(d^2 - e^2*x^2)^(-3 + p)*H
ypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d*(3 - p))
```

Rubi [A] time = 0.390951, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 266, 65, 246, 245}

$$\frac{4e^4(p^2-17p+48)\,x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{3d^8}-\frac{2e^3(5-p)(d^2-e^2x^2)^{p-3}\,{}_2F_1(1,p-3;p-2;1\cdots}{d(3-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```


```
[Out] -(d^2*(d^2 - e^2*x^2)^(-3 + p))/(3*x^3) + (2*d*e*(d^2 - e^2*x^2)^(-3 + p))/
x^2 - (e^2*(27 - 2*p)*(d^2 - e^2*x^2)^(-3 + p))/(3*x) + (4*e^4*(48 - 17*p +
 p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2]
)/(3*d^8*(1 - (e^2*x^2)/d^2)^p) - (2*e^3*(5 - p)*(d^2 - e^2*x^2)^(-3 + p)*H
ypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d*(3 - p))
```


Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(d^2-e^2x^2\right)^p}{x^4(d+ex)^4}\,dx = \int \frac{(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}}{x^4}\,dx$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(12d^5e-d^4e^2(27-2p)x+12d^3e^3x^2-3d^2e^4x^3\right)}{x^3}\,dx}{3d^2}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2} + \frac{\int \frac{\left(d^2-e^2x^2\right)^{-4+p}\left(2d^6e^2(27-2p)-24d^5e^3(5-p)x+6d^4e^4x^2\right)}{x^2}\,dx}{6d^4}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2} - \frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x} - \frac{\int \frac{\left(24d^7e^3(5-p)-8d^6e^4(48-17p\right.}{x}}{6d^6}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2} - \frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x} - \left(4de^3(5-p)\right)\int \frac{\left(d^2-e\right.}{}$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2} - \frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x} - \left(2de^3(5-p)\right)\mathrm{Subst}\left(\int\right.$$

$$= -\frac{d^2\left(d^2-e^2x^2\right)^{-3+p}}{3x^3} + \frac{2de\left(d^2-e^2x^2\right)^{-3+p}}{x^2} - \frac{e^2(27-2p)\left(d^2-e^2x^2\right)^{-3+p}}{3x} + \frac{4e^4\left(48-17p+p^2\right)x\left(d\right.}{}$$

Mathematica [B] time = 0.674403, size = 452, normalized size = 2.15

$$\left(d^2-e^2x^2\right)^p\left(-\frac{96d^3e\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} - \frac{480de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} - \frac{16d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3} - \frac{480d^2e^2\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-16*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (480*d^2*e^2*Hypergeometric2F1[-1/2, -p,
1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (96*d^3*e*Hypergeometric2F
1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
(15*2^(5 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(3 + p)*e^3*(-d + e*x)*Hyperge
ometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p)
 + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometr
ic2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (4
80*d*e^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*
x^2))^p)))/(48*d^8)
```

Maple [F] time = 0.677, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2}e^{2}+d^{2}\right)^{p}}{x^{4}\left(ex+d\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{4}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{e^{4}x^{8}+4\,de^{3}x^{7}+6\,d^{2}e^{2}x^{6}+4\,d^{3}ex^{5}+d^{4}x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^8 + 4*d*e^3*x^7 + 6*d^2*e^2*x^6 + 4*d^3*
e*x^5 + d^4*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x^{4}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{4}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4), x)
```

3.305 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$

Optimal. Leaf size=216

$$\frac{e^4(p^2-21p+70)(d^2-e^2x^2)^{p-3}\,{}_2F_1\left(1,p-3;p-2;1-\frac{e^2x^2}{d^2}\right)}{4d^2(3-p)}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{1}{2},4-p;\frac{1}{2};\right.}{3d^7x}$$

```
[Out] -(d^2*(d^2 - e^2*x^2)^(-3 + p))/(4*x^4) + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/
(3*x^3) - (e^2*(17 - p)*(d^2 - e^2*x^2)^(-3 + p))/(4*x^2) + (8*e^3*(6 - p)*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 4 - p, 1/2, (e^2*x^2)/d^2])/(3*d^
7*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(70 - 21*p + p^2)*(d^2 - e^2*x^2)^(-3 + p
)*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(4*d^2*(3 - p))
```

Rubi [A] time = 0.414663, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {852, 1807, 764, 365, 364, 266, 65}

$$\frac{e^4(p^2-21p+70)(d^2-e^2x^2)^{p-3}\,{}_2F_1\left(1,p-3;p-2;1-\frac{e^2x^2}{d^2}\right)}{4d^2(3-p)}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{1}{2},4-p;\frac{1}{2};\right.}{3d^7x}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

```
[Out] -(d^2*(d^2 - e^2*x^2)^(-3 + p))/(4*x^4) + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/
(3*x^3) - (e^2*(17 - p)*(d^2 - e^2*x^2)^(-3 + p))/(4*x^2) + (8*e^3*(6 - p)*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 4 - p, 1/2, (e^2*x^2)/d^2])/(3*d^
7*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(70 - 21*p + p^2)*(d^2 - e^2*x^2)^(-3 + p
)*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(4*d^2*(3 - p))
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx = \int \frac{(d-ex)^4(d^2-e^2x^2)^{-4+p}}{x^5}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(16d^5e-2d^4e^2(17-p)x+16d^3e^3x^2-4d^2e^4x^3)}{x^4}\,dx}{4d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(6d^6e^2(17-p)-32d^5e^3(6-p)x+12d^4e^4x^2)}{x^3}\,dx}{12d^4}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)(d^2-e^2x^2)^{-3+p}}{4x^2} - \frac{\int \frac{(64d^7e^3(6-p)-12d^6e^4(70-21p}{x^2}}{24d^6}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)(d^2-e^2x^2)^{-3+p}}{4x^2} - \frac{1}{3}(8de^3(6-p))\int \frac{(d^2-e}{}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)(d^2-e^2x^2)^{-3+p}}{4x^2} + \frac{1}{4}(e^4(70-21p+p^2))\,\mathrm{Su}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)(d^2-e^2x^2)^{-3+p}}{4x^2} + \frac{8e^3(6-p)(d^2-e^2x^2)^p(1}{}$$

Mathematica [B] time = 0.833583, size = 505, normalized size = 2.34

$$\frac{(d^2-e^2x^2)^p\left(\frac{240d^3e^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(p-1)x^2} + \frac{24d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right)}{(p-2)x^4} + \frac{840de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p} + \frac{64d^4e(}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((64*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d
^2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (960*d^2*e^3*Hypergeometric2F1[-1/2, -p,
 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (240*d^3*e^2*Hypergeometr
ic2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2
) + (105*2^(3 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d
- e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (24*d^5*Hypergeometric2F1[2 - p,
 -p, 3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (45*2^(2
 + p)*e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
)/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1
[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*
e^4*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
+ p)*(1 + (e*x)/d)^p) + (840*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^
2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(48*d^9)
```

Maple [F] time = 0.681, size = 0, normalized size = 0.

$$\int \frac{\left(-x^{2}e^{2}+d^{2}\right)^{p}}{x^{5}\left(ex+d\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{\left(ex+d\right)^{4}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{p}}{e^{4}x^{9}+4\,de^{3}x^{8}+6\,d^{2}e^{2}x^{7}+4\,d^{3}ex^{6}+d^{4}x^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p/(e^4*x^9 + 4*d*e^3*x^8 + 6*d^2*e^2*x^7 + 4*d^3*
e*x^6 + d^4*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^5(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^4x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)
```

3.306 $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=264

$$\frac{2d^2e(2m+3p+7)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+2p+4)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}+\frac{2d^3(2m+p}{}$$

```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(1 + p))/(g^2*(4 + m + 2*p)) + (2*d^3*(3 + 2*m + p)*
(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
 (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (2*d^2*e
*(7 + 2*m + 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/
2, -p, (4 + m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(4 + m + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```

Rubi [A] time = 0.372519, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1809, 808, 365, 364}

$$\frac{2d^2e(2m+3p+7)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+2p+4)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}+\frac{2d^3(2m+p}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```


```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(1 + p))/(g^2*(4 + m + 2*p)) + (2*d^3*(3 + 2*m + p)*
(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
 (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (2*d^2*e
*(7 + 2*m + 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/
2, -p, (4 + m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(4 + m + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```


Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
```

```
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)}-\frac{\int (gx)^m\left(d^2-e^2x^2\right)^p\left(-d^3e^2(4+m+2p)-2d^2e^3(7+2m\right.}{e^2(4+m+2p)}$$

$$=-\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)}+\frac{\int (gx)^m\left(2d^3e^4(3+2m+p)(\right.}{}$$

$$=-\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)}+\frac{\left(2d^3(3+2m+p)\right)\int (gx)^m\left(\right.}{3+m+2p}$$

$$=-\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)}+\frac{\left(2d^3(3+2m+p)\left(d^2-e^2x^2\right.\right.}{3}$$

$$=-\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)}+\frac{2d^3(3+2m+p)(gx)^{1+m}\left(d^2\right.}{g(}$$

Mathematica [A] time = 0.174636, size = 194, normalized size = 0.73

$$x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(\frac{d^3\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{m+1}+ex\left(\frac{3d^2\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{m+2}+ex\left(\frac{3d\,{}_2F_1\left(\frac{m+3}{2},-p;\frac{m}{}\right.}{m+3}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p,
(4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)/2,
 -p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m)/2,
 -p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(1 - (e^2*x^2)/d^2)^p
```

Maple [F] time = 0.682, size = 0, normalized size = 0.

$$\int (gx)^m(ex+d)^3\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(-e^2x^2+d^2\right)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p*(g*x)
^m, x)
```

Sympy [C] time = 61.7982, size = 262, normalized size = 0.99

$$\frac{d^3d^{2p}g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{3d^2d^{2p}eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{3dd^{2p}e^2g^mx^3x^m\Gamma}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**2*d**(2*
p)*e*g**m*x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x*
*2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) + 3*d*d**(2*p)*e**2*g**m*x**3
*x**m*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/(2*gamma(m/2 + 5/2)) + d**(2*p)*e**3*g**m*x**4*x**m*gamm
a(m/2 + 2)*hyper((-p, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/(2*gamma(m/2 + 3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(-e^2x^2+d^2\right)^p (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.307 $\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=206

$$\frac{2de(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{2d^2(m+p+2)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2}\right.}{g(m+1)(m+2p+3)}$$

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p))) + (2*d^2*(2 +
m + p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3
+ m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (
2*d*e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 +
 m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.168222, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1809, 808, 365, 364}

$$\frac{2de(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{2d^2(m+p+2)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2}\right.}{g(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p))) + (2*d^2*(2 +
m + p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3
+ m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (
2*d*e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 +
 m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx = -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{\int (gx)^m\left(-2d^2e^2(2+m+p)-2de^3(3+m+2p)x\right)\left(d^2 \cdots\right.}{e^2(3+m+2p)}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} + \frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p\,dx}{g} + \frac{\left(2d^2(2+m+p)\right)\int (\cdots}{3+m \cdots}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} + \frac{\left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx \cdots}{g}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} + \frac{2d^2(2+m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\cdots\right.}{g(1+m)(3+m+2p)}$$

Mathematica [A] time = 0.0942459, size = 169, normalized size = 0.82

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2}\cdots\right.\right.\right.}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)
/2, -p, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric
2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeometric2F
1[(3 + m)/2, -p, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 + m)*(1 -
 (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.639, size = 0, normalized size = 0.

$$\int (gx)^m(ex+d)^2\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2\,dex + d^2\right)\left(-e^2x^2 + d^2\right)^p \left(gx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Sympy [C] time = 33.3851, size = 192, normalized size = 0.93

$$\frac{d^2d^{2p}g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{dd^{2p}eg^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{d^{2p}e^2g^m x^3 x^m \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d*d**(2*p)*e*
g**m*x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2*ex
p_polar(2*I*pi)/d**2)/gamma(m/2 + 2) + d**(2*p)*e**2*g**m*x**3*x**m*gamma(m
/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/
d**2)/(2*gamma(m/2 + 5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p \left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.308 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=153

$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{d(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

```
[Out] (d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) + (e*(g*x)^(2 + m)*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2])
/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0676501, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {808, 365, 364}

$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{d(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) + (e*(g*x)^(2 + m)*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2])
/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int (gx)^m\left(d^2-e^2x^2\right)^p\,dx + \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p\,dx}{g}$$

$$= \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx + \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int}{g}$$

$$= \frac{d(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1\right.}{}$$

Mathematica [A] time = 0.0363992, size = 116, normalized size = 0.76

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d(m+2)\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)+e(m+1)x\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3
 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4 +
m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.465, size = 0, normalized size = 0.

$$\int (gx)^m(ex+d)\left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex+d)\left(-e^2x^2+d^2\right)^p(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Sympy [C] time = 19.3651, size = 122, normalized size = 0.8

$$\frac{dd^{2p}g^m xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{d^{2p}eg^m x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**(2*p)*e*g**m*
x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_pol
ar(2*I*pi)/d**2)/(2*gamma(m/2 + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p (gx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.309 $\int (gx)^m \left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2
, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.0202574, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {365, 364}

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2
, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (gx)^m\left(d^2-e^2x^2\right)^p\,dx&=\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\&=\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0098816, size = 73, normalized size = 0.97

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+1}{2}+1;\frac{e^2x^2}{d^2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2
, (e^2*x^2)/d^2])/((1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.466, size = 0, normalized size = 0.

$$\int (gx)^m \left(-x^2e^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^p (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-e^2x^2+d^2\right)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

Sympy [C] time = 5.76504, size = 61, normalized size = 0.81

$$\frac{d^{2p}g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,),
e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^p \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

3.310 $\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{d+ex}\,dx$

Optimal. Leaf size=163

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{dg(m+1)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m+4}{2};\right.}{d^2g^2(m+2)}$$

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m
)/2, (e^2*x^2)/d^2])/(d*g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) - (e*(g*x)^(2 + m)
*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2, (e^2*x^2)
/d^2])/(d^2*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.135751, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {892, 82, 126, 365, 364}

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{dg(m+1)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m+4}{2};\right.}{d^2g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m
)/2, (e^2*x^2)/d^2])/(d*g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) - (e*(g*x)^(2 + m)
*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2, (e^2*x^2)
/d^2])/(d^2*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
```

```
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{d+ex}\,dx = \left((d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^p(d+ex)^{-1+p}\,dx$$

$$= \left(d(d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^{-1+p}(d+ex)^{-1+p}\,dx - \frac{\left(e(d-ex)^{-p}(d+ex\right.}{}$$

$$= d\int (gx)^m\left(d^2-e^2x^2\right)^{-1+p}\,dx - \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}\,dx}{g}$$

$$= \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d} - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}}{d^2g}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{dg(1+m)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right.}{d^2g^2(2}$$

Mathematica [A] time = 0.0536167, size = 124, normalized size = 0.76

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d(m+2)\,{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\,{}_2F_1\left(\frac{m}{2}+1,1-p;\frac{m}{2}+2;\frac{e^2x^2}{d^2}\right)\right)}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(-(e*(1 + m)*x*Hypergeometric2F1[1 + m/2, 1 -
p, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p,
 (3 + m)/2, (e^2*x^2)/d^2]))/(d^2*(1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.674, size = 0, normalized size = 0.

$$\int \frac{(gx)^m\left(-x^2e^2+d^2\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p (gx)^m}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p (gx)^m}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

Sympy [C] time = 11.8517, size = 337, normalized size = 2.07

$$-\frac{0^p d d^{2p} g^m m x^m \Phi\left(\frac{d^2}{e^2x^2}, 1, \frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4e^2x\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^p d d^{2p} g^m x^m \Phi\left(\frac{d^2}{e^2x^2}, 1, \frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4e^2x\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^p d^{2p} g^m m x^m \Phi\left(\frac{d^2}{e^2x^2}, 1, \frac{me^{i\pi}}{2}\right.}{4e\Gamma\left(1-\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] -0**p*d*d**(2*p)*g**m*m*x**m*lerchphi(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma
(1/2 - m/2)/(4*e**2*x*gamma(3/2 - m/2)) + 0**p*d*d**(2*p)*g**m*x**m*lerchph
i(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*e**2*x*gamma(3/2 - m/
2)) + 0**p*d**(2*p)*g**m*m*x**m*lerchphi(d**2/(e**2*x**2), 1, m*exp_polar(I
*pi)/2)*gamma(-m/2)/(4*e*gamma(1 - m/2)) + d*e**(2*p)*g**m*p*x**m*x**(2*p)*
exp(I*pi*p)*gamma(p)*gamma(-m/2 - p + 1/2)*hyper((1 - p, -m/2 - p + 1/2), (
-m/2 - p + 3/2,), d**2/(e**2*x**2))/(2*e**2*x*gamma(p + 1)*gamma(-m/2 - p +
 3/2)) - e**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p)
*hyper((1 - p, -m/2 - p), (-m/2 - p + 1,), d**2/(e**2*x**2))/(2*e*gamma(p +
 1)*gamma(-m/2 - p + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p (gx)^m}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d), x)
```

3.311 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=214

$$-\frac{2e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},2-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^3g^2(m+2)}-\frac{2(m+p)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2}\right.}{d^2g(m+1)(-m-2p+1)}$$

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^(-1 + p))/(g*(1 - m - 2*p)) - (2*(m + p)*(g*
x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 2 - p, (3 + m)/2,
 (e^2*x^2)/d^2])/(d^2*g*(1 + m)*(1 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e
*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 2 - p, (4 + m
)/2, (e^2*x^2)/d^2])/(d^3*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rubi [A] time = 0.223438, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1809, 808, 365, 364}

$$-\frac{2e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},2-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^3g^2(m+2)}-\frac{2(m+p)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2}\right.}{d^2g(m+1)(-m-2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^(-1 + p))/(g*(1 - m - 2*p)) - (2*(m + p)*(g*
x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 2 - p, (3 + m)/2,
 (e^2*x^2)/d^2])/(d^2*g*(1 + m)*(1 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e
*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 2 - p, (4 + m
)/2, (e^2*x^2)/d^2])/(d^3*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx = \int (gx)^m(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} + \frac{\int (gx)^m\left(-2d^2e^2(m+p)-2de^3(1-m-2p)x\right)\left(d^2-e^2x^2\right)^{-2+p}\,dx}{e^2(1-m-2p)}$$
$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}\,dx}{g} - \frac{\left(2d^2(m+p)\right)\int (gx)^m\left(d^2-e^2\right.}{1-m-2p}$$
$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{-2+p}\,dx}{d^3g} - \frac{\left(2(m\right.}{}$$
$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)} - \frac{2(m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2},2-p;\frac{3+m}{2};\frac{e^2x}{d^2}\right.}{d^2g(1+m)(1-m-2p)}$$

Mathematica [A] time = 0.110775, size = 180, normalized size = 0.84

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{m+1}{2},2-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)-e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{m+2}{2},2-p;\frac{m+}{2}\right.\right.\right.}{d^4(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)
/2, 2 - p, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeomet
ric2F1[(2 + m)/2, 2 - p, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hypergeome
tric2F1[(3 + m)/2, 2 - p, (5 + m)/2, (e^2*x^2)/d^2])))/(d^4*(1 + m)*(2 + m)
*(3 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F] time = 0.694, size = 0, normalized size = 0.

$$\int \frac{(gx)^m\left(-x^2e^2+d^2\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p\left(gx\right)^m}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-e^2x^2+d^2\right)^p\left(gx\right)^m}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx\right)^m\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p\left(gx\right)^m}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)
```

3.312 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=275

$$-\frac{2e(-2m-3p+2)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},3-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^4g^2(m+2)(-m-2p+2)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}-\frac{2(2m+p)(g\cdots}{\cdots}$$

```
[Out] (3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g*(3 - m - 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g^2*(2 - m - 2*p)) - (2*(2*m + p)*(g*x)^(
1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 + m)/2, (e^
2*x^2)/d^2])/(d^3*g*(1 + m)*(3 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(2
- 2*m - 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3
 - p, (4 + m)/2, (e^2*x^2)/d^2])/(d^4*g^2*(2 + m)*(2 - m - 2*p)*(1 - (e^2*x
^2)/d^2)^p)
```

Rubi [A] time = 0.441517, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {852, 1809, 808, 365, 364}

$$-\frac{2e(-2m-3p+2)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},3-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^4g^2(m+2)(-m-2p+2)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}-\frac{2(2m+p)(g\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g*(3 - m - 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g^2*(2 - m - 2*p)) - (2*(2*m + p)*(g*x)^(
1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 + m)/2, (e^
2*x^2)/d^2])/(d^3*g*(1 + m)*(3 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(2
- 2*m - 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3
 - p, (4 + m)/2, (e^2*x^2)/d^2])/(d^4*g^2*(2 + m)*(2 - m - 2*p)*(1 - (e^2*x
^2)/d^2)^p)
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx = \int (gx)^m(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx$$

$$= -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} + \frac{\int (gx)^m\left(d^2-e^2x^2\right)^{-3+p}\left(d^3e^2(2-m-2p)-2d^2e^3(2-2m-3\right.}{e^2(2-m-2p)}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} + \frac{\int (gx)^m\left(-2d^3e^4(2-m-2p)(2m\right.}{}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{\left(2d^2e(2-2m-3p)\right)\int (gx)^{1+m}\left(d\right.}{g(2-m-2p)}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{\left(2e(2-2m-3p)\left(d^2-e^2x^2\right)^p\left(1\right.\right.}{d^4g(2}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{2(2m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1\right.}{d^3g(1+m}$$

Mathematica [A] time = 0.203333, size = 206, normalized size = 0.75

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(\frac{d^3\,{}_2F_1\left(\frac{m+1}{2},3-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{m+1}+ex\left(ex\left(\frac{3d\,{}_2F_1\left(\frac{m+3}{2},3-p;\frac{m+5}{2};\frac{e^2x^2}{d^2}\right)}{m+3}-\frac{ex\,{}_2F_1\left(\frac{m+4}{2},3-p;\frac{m+6}{2};\frac{e^2x^2}{d^2}\right)}{m+4}\right)-\frac{3d^2}{}\right.\right.}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 +
 m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[(2 + m)/2,
3 - p, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3
+ m)/2, 3 - p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric2F1[
(4 + m)/2, 3 - p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*(1 - (e^2*x^2
)/d^2)^p)
```

Maple [F] time = 0.714, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (-x^2e^2 + d^2)^p}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p (gx)^m}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (-(-d + ex)(d + ex))^p}{(d + ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-e^2x^2+d^2\right)^p(gx)^m}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^3, x)
```

3.313 $\int \frac{(gx)^m(1-a^2x^2)^p}{1+ax}\,dx$

Optimal. Leaf size=89

$$\frac{(gx)^{m+1}\,{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};a^2x^2\right)}{g(m+1)}-\frac{a(gx)^{m+2}\,{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m+4}{2};a^2x^2\right)}{g^2(m+2)}$$

```
[Out] ((g*x)^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2])/(g*
(1 + m)) - (a*(g*x)^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2,
a^2*x^2])/(g^2*(2 + m))
```

Rubi [A] time = 0.0555549, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {890, 82, 125, 364}

$$\frac{(gx)^{m+1}\,{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};a^2x^2\right)}{g(m+1)}-\frac{a(gx)^{m+2}\,{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m+4}{2};a^2x^2\right)}{g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
```

```
[Out] ((g*x)^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2])/(g*
(1 + m)) - (a*(g*x)^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2,
a^2*x^2])/(g^2*(2 + m))
```

Rule 890

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x
] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m, 0] &&  !
IGtQ[n, 0]
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(gx)^m\left(1-a^2x^2\right)^p}{1+ax}\,dx &= \int (gx)^m(1-ax)^p(1+ax)^{-1+p}\,dx\\ &= -\frac{a\int (gx)^{1+m}(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx}{g}+\int (gx)^m(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx\\ &= -\frac{a\int (gx)^{1+m}\left(1-a^2x^2\right)^{-1+p}\,dx}{g}+\int (gx)^m\left(1-a^2x^2\right)^{-1+p}\,dx\\ &= \frac{(gx)^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};a^2x^2\right)}{g(1+m)}-\frac{a(gx)^{2+m}\,{}_2F_1\left(\frac{2+m}{2},1-p;\frac{4+m}{2};a^2x^2\right)}{g^2(2+m)}\end{aligned}$$

Mathematica [A] time = 0.0456823, size = 77, normalized size = 0.87

$$x(gx)^m\left(\frac{{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax\,{}_2F_1\left(\frac{m}{2}+1,1-p;\frac{m}{2}+2;a^2x^2\right)}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
```

```
[Out] x*(g*x)^m*(-((a*x*Hypergeometric2F1[1 + m/2, 1 - p, 2 + m/2, a^2*x^2])/(2 +
 m)) + Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2]/(1 + m))
```

Maple [F] time = 0.674, size = 0, normalized size = 0.

$$\int \frac{\left(gx\right)^m\left(-a^2x^2+1\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
```

```
[Out] int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^p\left(gx\right)^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-a^2x^2+1\right)^p (gx)^m}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

Sympy [C] time = 10.2144, size = 308, normalized size = 3.46

$$\frac{0^p g^m m x^m \Phi\left(\frac{1}{a^2x^2},1,\frac{me^{i\pi}}{2}\right)\Gamma\left(-\frac{m}{2}\right)}{4a\Gamma\left(1-\frac{m}{2}\right)} - \frac{0^p g^m m x^m \Phi\left(\frac{1}{a^2x^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4a^2x\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^p g^m x^m \Phi\left(\frac{1}{a^2x^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4a^2x\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} - \frac{a^{2p}g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-a**2*x**2+1)**p/(a*x+1),x)
```

```
[Out] 0**p*g**m*m*x**m*lerchphi(1/(a**2*x**2), 1, m*exp_polar(I*pi)/2)*gamma(-m/2
)/(4*a*gamma(1 - m/2)) - 0**p*g**m*m*x**m*lerchphi(1/(a**2*x**2), 1, 1/2 -
m/2)*gamma(1/2 - m/2)/(4*a**2*x*gamma(3/2 - m/2)) + 0**p*g**m*x**m*lerchphi
(1/(a**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*a**2*x*gamma(3/2 - m/2))
- a**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p)*hyper(
(1 - p, -m/2 - p), (-m/2 - p + 1,), 1/(a**2*x**2))/(2*a*gamma(p + 1)*gamma(
-m/2 - p + 1)) + a**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
m/2 - p + 1/2)*hyper((1 - p, -m/2 - p + 1/2), (-m/2 - p + 3/2,), 1/(a**2*x*
*2))/(2*a**2*x*gamma(p + 1)*gamma(-m/2 - p + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^p (gx)^m}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

3.314 $\int (gx)^m (d+ex)^n \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=96

$$\frac{(gx)^{m+1}(d+ex)^n \left(1-\frac{ex}{d}\right)^{-p} \left(d^2-e^2x^2\right)^p \left(\frac{ex}{d}+1\right)^{-n-p} F_1\left(m+1;-p,-n-p;m+2;\frac{ex}{d},-\frac{ex}{d}\right)}{g(m+1)}$$

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*(1 + (e*x)/d)^(-n - p)*(d^2 - e^2*x^2)^p*AppellF
1[1 + m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/(g*(1 + m)*(1 - (e*x)/d)^
p)
```

Rubi [A] time = 0.0829346, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {892, 135, 133}

$$\frac{(gx)^{m+1}(d+ex)^n \left(1-\frac{ex}{d}\right)^{-p} \left(d^2-e^2x^2\right)^p \left(\frac{ex}{d}+1\right)^{-n-p} F_1\left(m+1;-p,-n-p;m+2;\frac{ex}{d},-\frac{ex}{d}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```


```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*(1 + (e*x)/d)^(-n - p)*(d^2 - e^2*x^2)^p*AppellF
1[1 + m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/(g*(1 + m)*(1 - (e*x)/d)^
p)
```


Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)^n\left(d^2-e^2x^2\right)^p\,dx = \left((d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^p(d+ex)^{n+p}\,dx$$
$$= \left((d+ex)^{-p}\left(1-\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d+ex)^{n+p}\left(1-\frac{ex}{d}\right)^p dx$$
$$= \left((d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m\left(1-\frac{ex}{d}\right)^p\left(1+\frac{ex}{d}\right)^{n+p} dx$$
$$= \frac{(gx)^{1+m}(d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^p F_1\left(1+m;-p,-n-p;2+m;\frac{ex}{d},\right.}{g(1+m)}$$

Mathematica [A] time = 0.110527, size = 90, normalized size = 0.94

$$\frac{x(gx)^m(d-ex)^p\left(\frac{d-ex}{d}\right)^{-p}(d+ex)^{n+p}\left(\frac{d+ex}{d}\right)^{-n-p}F_1\left(m+1;-p,-n-p;m+2;\frac{ex}{d},-\frac{ex}{d}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d - e*x)^p*(d + e*x)^(n + p)*((d + e*x)/d)^(-n - p)*AppellF1[1
+ m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/((1 + m)*((d - e*x)/d)^p)
```

Maple [F] time = 0.729, size = 0, normalized size = 0.

$$\int (gx)^m (ex+d)^n \left(-x^2e^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2+d^2\right)^p(ex+d)^n (gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-e^2x^2+d^2\right)^p(ex+d)^n (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(-e**2*x**2+d**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^2x^2 + d^2\right)^p (ex + d)^n \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*(e*x + d)^n*(g*x)^m, x)
```

3.315 $\int \frac{x\sqrt{1+x}}{1+x^2} dx$

Optimal. Leaf size=214

$$2\sqrt{x+1}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}\right.$$

```
[Out] 2*Sqrt[1 + x] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 +
 Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] - ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[
1 + x])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] + (Sqrt[(1 + Sqrt[2])
/2]*Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2 - (Sqrt[(1
+ Sqrt[2])/2]*Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2
```

Rubi [A] time = 0.246522, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {825, 827, 1169, 634, 618, 204, 628}

$$2\sqrt{x+1}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 +
 Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] - ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[
1 + x])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] + (Sqrt[(1 + Sqrt[2])
/2]*Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2 - (Sqrt[(1
+ Sqrt[2])/2]*Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{1+x}}{1+x^2}\,dx &= 2\sqrt{1+x}+\int \frac{-1+x}{\sqrt{1+x}\left(1+x^2\right)}\,dx \\
&= 2\sqrt{1+x}+2\,\text{Subst}\left(\int \frac{-2+x^2}{2-2x^2+x^4}\,dx,x,\sqrt{1+x}\right) \\
&= 2\sqrt{1+x}+\frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}-\left(-2-\sqrt{2}\right)x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}+\left(-2-\sqrt{2}\right)x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+\right.}{2\sqrt{1+\sqrt{2}}} \\
&= 2\sqrt{1+x}-\frac{1}{2}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx,x,\sqrt{1+x}\right)-\frac{1}{2}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int\right. \\
&= 2\sqrt{1+x}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(1+\sqrt{2}+\right. \\
&= 2\sqrt{1+x}+\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)-\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+}{\sqrt{2\left(-1+\right.}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.0433481, size = 60, normalized size = 0.28

$$
2\sqrt{x+1}-\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)-\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] - Sqrt[1 - I]*ArcTanh[Sqrt[1 + x]/Sqrt[1 - I]] - Sqrt[1 + I]*
ArcTanh[Sqrt[1 + x]/Sqrt[1 + I]]
```

Maple [A] time = 0.077, size = 240, normalized size = 1.1

$$2\sqrt{1+x}-\frac{\sqrt{2+2\sqrt{2}}}{4}\ln\left(1+x+\sqrt{2}+\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)+\frac{1}{\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}+\sqrt{2+2\sqrt{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2)/(x^2+1),x)
```

```
[Out] 2*(1+x)^(1/2)-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1
/2))^(1/2)+1/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2)
)/(-2+2*2^(1/2))^(1/2))-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+(2+2*2
^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)-(1+x)^(1/2)
*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*
(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/(-2+2*2^(1
/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x+1}x}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)*x/(x^2 + 1), x)
```

Fricas [A] time = 1.67516, size = 1048, normalized size = 4.9

$$-\frac{1}{8}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}\log\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+x+\sqrt{2}+1\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*2^(1/4)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4)*log(1/2*2^(1/4)*sqrt(x + 1)
*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + x + sqrt(2) + 1) + 1/8*2^(1/4)*(sqrt(
2) + 2)*sqrt(-2*sqrt(2) + 4)*log(-1/2*2^(1/4)*sqrt(x + 1)*(sqrt(2) + 2)*sqr
t(-2*sqrt(2) + 4) + x + sqrt(2) + 1) + 1/2*2^(3/4)*sqrt(-2*sqrt(2) + 4)*arc
tan(1/4*2^(3/4)*sqrt(2^(1/4)*sqrt(x + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4)
 + 2*x + 2*sqrt(2) + 2)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) - 1/2*2^(3/4)*sq
rt(x + 1)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) - sqrt(2) - 1) + 1/2*2^(3/4)*s
```

```
qrt(-2*sqrt(2) + 4)*arctan(1/4*2^(3/4)*sqrt(-2^(1/4)*sqrt(x + 1)*(sqrt(2) +
 2)*sqrt(-2*sqrt(2) + 4) + 2*x + 2*sqrt(2) + 2)*(sqrt(2) + 2)*sqrt(-2*sqrt(
2) + 4) - 1/2*2^(3/4)*sqrt(x + 1)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + sqrt
(2) + 1) + 2*sqrt(x + 1)
```

Sympy [A] time = 8.44238, size = 68, normalized size = 0.32

$$2\sqrt{x+1} - 4\,\mathrm{RootSum}\left(512t^4 + 32t^2 + 1, \left(t \mapsto t\log\left(-128t^3 + \sqrt{x+1}\right)\right)\right) + 2\,\mathrm{RootSum}\left(128t^4 + 16t^2 + 1, \left(t \mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*sqrt(x + 1) - 4*RootSum(512*_t**4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*
_t**3 + sqrt(x + 1)))) + 2*RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*
log(64*_t**3 + 4*_t + sqrt(x + 1))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}x}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + 1)*x/(x^2 + 1), x)
```

3.316 $\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=255

$$-\frac{d\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6}+\frac{\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{60c^2e^3}+\frac{d\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{8ce^5}-\frac{d^4}{}$$

```
[Out] (d*(8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^5) + ((47*c*d^
2 - 8*a*e^2)*(a + c*x^2)^(3/2))/(60*c^2*e^3) - (13*d*(d + e*x)*(a + c*x^2)^
(3/2))/(20*c*e^3) + ((d + e*x)^2*(a + c*x^2)^(3/2))/(5*c*e^3) - (d*(8*c^2*d
^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3
/2)*e^6) - (d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/e^6
```

Rubi [A] time = 0.629224, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 815, 844, 217, 206, 725}

$$-\frac{d\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6}+\frac{\left(a+cx^2\right)^{3/2}\left(47cd^2-8ae^2\right)}{60c^2e^3}+\frac{d\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{8ce^5}-\frac{d^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^5) + ((47*c*d^
2 - 8*a*e^2)*(a + c*x^2)^(3/2))/(60*c^2*e^3) - (13*d*(d + e*x)*(a + c*x^2)^
(3/2))/(20*c*e^3) + ((d + e*x)^2*(a + c*x^2)^(3/2))/(5*c*e^3) - (d*(8*c^2*d
^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3
/2)*e^6) - (d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/e^6
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \frac{\int \frac{\sqrt{a+cx^2}\left(-2ad^2e^2-de\left(3cd^2+4ae^2\right)x-e^2\left(11cd^2+2ae^2\right)x^2-13cde^3x^3\right)}{d+ex}\,dx}{5ce^4}$$

$$= -\frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \frac{\int \frac{\sqrt{a+cx^2}\left(5acd^2e^5+3cde^4\left(9cd^2-ae^2\right)x+ce^5\left(47cd^2-8ae^2\right)x\right.}{d+ex}}{20c^2e^7}$$

$$= \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \frac{(d+ex)^2\left(a+cx^2\right)^{3/2}}{5ce^3} + \frac{\int \frac{\left(15ac^2d^2e^7-15c^2\right.}{}}{}$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \cdots$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \cdots$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \cdots$$

$$= \frac{d\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^5} + \frac{\left(47cd^2-8ae^2\right)\left(a+cx^2\right)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\left(a+cx^2\right)^{3/2}}{20ce^3} + \cdots$$

Mathematica [A] time = 0.668256, size = 259, normalized size = 1.02

$$\frac{e\sqrt{a+cx^2}\left(-16a^2e^4+ace^2\left(40d^2-15dex+8e^2x^2\right)+2c^2\left(20d^2e^2x^2-30d^3ex+60d^4-15de^3x^3+12e^4x^4\right)\right)-120c^2d^4\sqrt{\cdots}}{120c^2e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(-16*a^2*e^4 + a*c*e^2*(40*d^2 - 15*d*e*x + 8*e^2*x^2) +
 2*c^2*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4))
+ (15*Sqrt[a]*Sqrt[c]*d*e^2*(-4*c*d^2 + a*e^2)*Sqrt[a + c*x^2]*ArcSinh[(Sqr
t[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a] - 120*c^(5/2)*d^5*ArcTanh[(Sqrt[c]*x)
/Sqrt[a + c*x^2]] - 120*c^2*d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(
Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(120*c^2*e^6)
```

Maple [B] time = 0.278, size = 560, normalized size = 2.2

$$\frac{x^2}{5ce}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{2a}{15c^2e}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{dx}{4ce^2}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{adx}{8ce^2}\sqrt{cx^2+a}+\frac{a^2d}{8e^2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+\frac{d^2}{3e^3c}\left(cx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/5/e*x^2*(c*x^2+a)^(3/2)/c-2/15/e*a/c^2*(c*x^2+a)^(3/2)-1/4*d/e^2*x*(c*x^2
+a)^(3/2)/c+1/8*d/e^2*a/c*x*(c*x^2+a)^(1/2)+1/8*d/e^2*a^2/c^(3/2)*ln(x*c^(1
/2)+(c*x^2+a)^(1/2))+1/3*d^2/e^3*(c*x^2+a)^(3/2)/c-1/2*d^3/e^4*x*(c*x^2+a)^
(1/2)-1/2*d^3/e^4*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+d^4/e^5*((d/e+x)^
2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-d^5/e^6*c^(1/2)*ln((-c*d/e+(d/
e+x)*c)/c^(1/2)+((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-d^4/
e^5/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a
*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2
))/(d/e+x))*a-d^6/e^7/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c
*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^
2+c*d^2)/e^2)^(1/2))/(d/e+x))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 40.7479, size = 2407, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/240*(120*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
```

```
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^
2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(24*c^
2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5
+ 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(
c*x^2 + a))/(c^2*e^6), -1/240*(240*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log
(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(24*c^2*e^5*x^4 - 30*c^2*d
*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^2*e^3 +
 a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^6
), 1/120*(60*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (24*c^2*e^5*x^4 - 3
0*c^2*d*e^4*x^3 + 120*c^2*d^4*e + 40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^
2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(
c^2*e^6), -1/120*(120*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e
^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*
x^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)
*x/sqrt(c*x^2 + a)) - (24*c^2*e^5*x^4 - 30*c^2*d*e^4*x^3 + 120*c^2*d^4*e +
40*a*c*d^2*e^3 - 16*a^2*e^5 + 8*(5*c^2*d^2*e^3 + a*c*e^5)*x^2 - 15*(4*c^2*d
^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A] time = 1.1966, size = 340, normalized size = 1.33

$$\frac{2\left(cd^6+ad^4e^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-6)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4xe^{(-1)}-5de^{(-2)}\right)x+\frac{4\left(5c^3d^2e^{18}+ac^2e^{20}\right)e}{c^3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^6 + a*d^4*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-6)/sqrt(-c*d^2 - a*e^2) + 1/120*sqrt(c*x^2 + a)*
((2*(3*(4*x*e^(-1) - 5*d*e^(-2))*x + 4*(5*c^3*d^2*e^18 + a*c^2*e^20)*e^(-21
)/c^3)*x - 15*(4*c^3*d^3*e^17 + a*c^2*d*e^19)*e^(-21)/c^3)*x + 8*(15*c^3*d^
4*e^16 + 5*a*c^2*d^2*e^18 - 2*a^2*c*e^20)*e^(-21)/c^3) + 1/8*(8*c^(5/2)*d^5
 + 4*a*c^(3/2)*d^3*e^2 - a^2*sqrt(c)*d*e^4)*e^(-6)*log(abs(-sqrt(c)*x + sqr
t(c*x^2 + a)))/c^2
```

3.317 $\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=211

$$\frac{\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}-\frac{\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{8ce^4}+\frac{d^3\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5}$$

```
[Out] -((8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^4) - (7*d*(a +
c*x^2)^(3/2))/(12*c*e^2) + ((d + e*x)*(a + c*x^2)^(3/2))/(4*c*e^2) + ((8*c^
2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c
^(3/2)*e^5) + (d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 +
a*e^2]*Sqrt[a + c*x^2])])/e^5
```

Rubi [A] time = 0.390428, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 815, 844, 217, 206, 725}

$$\frac{\left(-a^2e^4+4acd^2e^2+8c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}-\frac{\sqrt{a+cx^2}\left(8cd^3-ex\left(4cd^2-ae^2\right)\right)}{8ce^4}+\frac{d^3\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -((8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^4) - (7*d*(a +
c*x^2)^(3/2))/(12*c*e^2) + ((d + e*x)*(a + c*x^2)^(3/2))/(4*c*e^2) + ((8*c^
2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c
^(3/2)*e^5) + (d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 +
a*e^2]*Sqrt[a + c*x^2])])/e^5
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+cx^2}\left(-ade^2-e\left(3cd^2+ae^2\right)x-7cde^2x^2\right)}{d+ex}\,dx}{4ce^3}$$

$$= -\frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\left(-3acde^4+3ce^3\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{d+ex}\,dx}{12c^2e^5}$$

$$= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{-3ac^2de^4\left(4cd^2+ae^2 \cdots\right.}{(\cdots}}{\cdots}$$

$$= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} - \frac{\left(d^3\left(cd^2+ae^2\right)\right)\cdots}{e^5\cdots}$$

$$= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\left(d^3\left(cd^2+ae^2\right)\right)\cdots}{\cdots}$$

$$= -\frac{\left(8cd^3-e\left(4cd^2-ae^2\right)x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\left(8c^2d^4+4acd^2e^2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.465301, size = 225, normalized size = 1.07

$$\frac{24c^{3/2}d^4\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+e\sqrt{a+cx^2}\left(ae^2(3ex-8d)+c\left(12d^2ex-24d^3-8de^2x^2+6e^3x^3\right)\right)+24cd^3\sqrt{ae^2+cd^2}\tanh^{\cdots}}{24ce^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -(Sqrt[a]*(-4*c*d^2 + a*e^2)*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/
(8*c^(3/2)*e^3*Sqrt[1 + (c*x^2)/a]) + (e*Sqrt[a + c*x^2]*(a*e^2*(-8*d + 3*e
*x) + c*(-24*d^3 + 12*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3)) + 24*c^(3/2)*d^4*
```

```
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 24*c*d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(24*c*e^5)
```

Maple [B] time = 0.237, size = 515, normalized size = 2.4

$$\frac{x}{4ce}\left(cx^2+a\right)^{\frac{3}{2}}-\frac{ax}{8ce}\sqrt{cx^2+a}-\frac{a^2}{8e}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}-\frac{d}{3ce^2}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{d^2x}{2e^3}\sqrt{cx^2+a}+\frac{ad^2}{2e^3}\ln\left(x\sqrt{c}+\sqrt{cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/4/e*x*(c*x^2+a)^(3/2)/c-1/8/e*a/c*x*(c*x^2+a)^(1/2)-1/8/e*a^2/c^(3/2)*ln(
x*c^(1/2)+(c*x^2+a)^(1/2))-1/3*d*(c*x^2+a)^(3/2)/c/e^2+1/2*d^2/e^3*x*(c*x^2
+a)^(1/2)+1/2*d^2/e^3*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-d^3/e^4*((d/e
+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+d^4/e^5*c^(1/2)*ln((-c*d/e
+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+
d^3/e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2
*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^
(1/2))/(d/e+x))*a+d^5/e^6/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2
-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(
a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 37.417, size = 2076, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^
2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e
^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*
x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c
*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/48*(48*sqrt(-c*d^2 - a*e^2)*c^2*d^3*
arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^
2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqr
t(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*e^4*x^3 - 8
*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*x
```

```
)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(12*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a
*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 +
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(8*c^
2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
) + (6*c^2*e^4*x^3 - 8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^
2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^2*e^5), 1/24*(24*sqrt(-c*d^2 -
a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*e^4*x^3 -
8*c^2*d*e^3*x^2 - 24*c^2*d^3*e - 8*a*c*d*e^3 + 3*(4*c^2*d^2*e^2 + a*c*e^4)*
x)*sqrt(c*x^2 + a))/(c^2*e^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A] time = 1.25477, size = 271, normalized size = 1.28

$$-\frac{2\left(cd^5+ad^3e^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-5)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(3\,xe^{(-1)}-4\,de^{(-2)}\right)x+\frac{3\left(4\,c^2d^2e^{12}+ace^{14}\right)e^{(-15}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^5 + a*d^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))*e^(-5)/sqrt(-c*d^2 - a*e^2) + 1/24*sqrt(c*x^2 + a)*
((2*(3*x*e^(-1) - 4*d*e^(-2))*x + 3*(4*c^2*d^2*e^12 + a*c*e^14)*e^(-15)/c^2
)*x - 8*(3*c^2*d^3*e^11 + a*c*d*e^13)*e^(-15)/c^2) - 1/8*(8*c^2*d^4 + 4*a*c
*d^2*e^2 - a^2*e^4)*e^(-5)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.318 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=153

$$-\frac{d^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{d\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^4}+\frac{d\sqrt{a+cx^2}(2d-ex)}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}$$

```
[Out] (d*(2*d - e*x)*Sqrt[a + c*x^2])/(2*e^3) + (a + c*x^2)^(3/2)/(3*c*e) - (d*(2
*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^4) - (d^
2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2])])/e^4
```

Rubi [A] time = 0.210684, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1654, 12, 815, 844, 217, 206, 725}

$$-\frac{d^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{d\left(ae^2+2cd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^4}+\frac{d\sqrt{a+cx^2}(2d-ex)}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```


```
[Out] (d*(2*d - e*x)*Sqrt[a + c*x^2])/(2*e^3) + (a + c*x^2)^(3/2)/(3*c*e) - (d*(2
*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^4) - (d^
2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2])])/e^4
```


Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
```

```
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx &= \frac{\left(a+cx^2\right)^{3/2}}{3ce}+\frac{\int -\frac{3cdex\sqrt{a+cx^2}}{d+ex}\,dx}{3ce^2}\\
&= \frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx}{e}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\int \frac{-acde+c\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}+\frac{\left(d^2\left(cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4}-\frac{\left(d\left(2cd^2+ae^2\right)\right)\int \frac{1}{\sqrt{a+c}}}{2e^4}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{\left(d^2\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^4}-\frac{\left(d\left(2c\right.\right.}{}\\
&= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{\left(a+cx^2\right)^{3/2}}{3ce}-\frac{d\left(2cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^4}-\frac{d^2\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{}{\sqrt{c}}\right.}{e^4}
\end{aligned}$$

Mathematica [A] time = 0.348231, size = 193, normalized size = 1.26

$$\frac{-6c^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+e\sqrt{a+cx^2}\left(2ae^2+c\left(6d^2-3dex+2e^2x^2\right)\right)-6cd^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)-\frac{3}{}}{6ce^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(2*a*e^2 + c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) - (3*Sqrt[a]
*Sqrt[c]*d*e^2*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^
2)/a] - 6*c^(3/2)*d^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - 6*c*d^2*Sqrt[c
*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(6*c*e^4)
```

Maple [B] time = 0.237, size = 448, normalized size = 2.9

$$\frac{1}{3\,ce}\left(cx^{2}+a\right)^{\frac{3}{2}}-\frac{dx}{2\,e^{2}}\sqrt{cx^{2}+a}-\frac{ad}{2\,e^{2}}\ln\left(x\sqrt{c}+\sqrt{cx^{2}+a}\right)\frac{1}{\sqrt{c}}+\frac{d^{2}}{e^{3}}\sqrt{\left(\frac{d}{e}+x\right)^{2}c-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^{2}+cd^{2}}{e^{2}}}-\frac{d^{3}}{e^{4}}\sqrt{c}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/3*(c*x^2+a)^(3/2)/c/e-1/2*d/e^2*x*(c*x^2+a)^(1/2)-1/2/e^2*d*a/c^(1/2)*ln(
x*c^(1/2)+(c*x^2+a)^(1/2))+d^2/e^3*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^
2)/e^2)^(1/2)-d^3/e^4*c^(1/2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c-2*
c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-d^2/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)
^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a-d^4/e^5/((a*e^2+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e
^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.84752, size = 1710, normalized size = 11.18

$$\left[\frac{6\,\sqrt{cd^{2}+ae^{2}}cd^{2}\log\left(\frac{2\,acdex-acd^{2}-2\,a^{2}e^{2}-\left(2\,c^{2}d^{2}+ace^{2}\right)x^{2}-2\,\sqrt{cd^{2}+ae^{2}}(cdx-ae)\sqrt{cx^{2}+a}}{e^{2}x^{2}+2\,dex+d^{2}}\right)+3\left(2\,cd^{3}+ade^{2}\right)\sqrt{c}\log\left(-2\,cx^{2}+2\,\sqrt{cx^{2}+a}\right.}{12\,ce^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c
*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -1/12*(12*sqrt(-c*d^2 - a*e^2)*
c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
```

```
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6
*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), 1/6*(3*sqrt(c*d^2 + a*e^2)*c*
d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*
sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2
)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2
*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4), -
1/6*(6*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c*d^
3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c*e^3*x^2 - 3
*c*d*e^2*x + 6*c*d^2*e + 2*a*e^3)*sqrt(c*x^2 + a))/(c*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a + cx^2}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A] time = 1.19474, size = 212, normalized size = 1.39

$$\frac{2\left(cd^4 + ad^2e^2\right)\arctan\left(-\frac{\left(\sqrt{c}x - \sqrt{cx^2+a}\right)e + \sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2 - ae^2}} + \frac{\left(2\,cd^3 + ade^2\right)e^{(-4)}\log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right)}{2\sqrt{c}} + \frac{1}{6}\sqrt{cx^2 + a}\left(2\,xe^{(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^4 + a*d^2*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/2*(2*c*d^3 + a*d*e^2
)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/6*sqrt(c*x^2 +
a)*((2*x*e^(-1) - 3*d*e^(-2))*x + 2*(3*c*d^2*e^7 + a*e^9)*e^(-10)/c)
```

3.319 $\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=127

$$\frac{(ae^2+2cd^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^3}+\frac{d\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

```
[Out] -((2*d - e*x)*Sqrt[a + c*x^2])/(2*e^2) + ((2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^3) + (d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*
e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^3
```

Rubi [A] time = 0.105262, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {815, 844, 217, 206, 725}

$$\frac{(ae^2+2cd^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^3}+\frac{d\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3}-\frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -((2*d - e*x)*Sqrt[a + c*x^2])/(2*e^2) + ((2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^3) + (d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*
e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^3
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx &= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}+\frac{\int \frac{-acde+c\left(2cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^2}\\ &= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}-\frac{\left(d\left(cd^2+ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3}+\frac{\left(2cd^2+ae^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2e^3}\\ &= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}+\frac{\left(d\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3}+\frac{\left(2cd^2+ae^2\right)\text{Subst}\left(\int\right.}{2e^3}\\ &= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}+\frac{\left(2cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}e^3}+\frac{d\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.303128, size = 175, normalized size = 1.38

$$\frac{\frac{a^{3/2}e^2\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{a+cx^2}}+2d\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)+2\sqrt{c}d^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)-2de\sqrt{a+cx^2}+e^2x\sqrt{a+cx}}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (-2*d*e*Sqrt[a + c*x^2] + e^2*x*Sqrt[a + c*x^2] + (a^(3/2)*e^2*Sqrt[1 + (c*
x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[a + c*x^2]) + 2*Sqrt[c]
*d^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 2*d*Sqrt[c*d^2 + a*e^2]*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(2*e^3)
```

Maple [B] time = 0.24, size = 423, normalized size = 3.3

$$\frac{x}{2e}\sqrt{cx^2+a}+\frac{a}{2e}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{d}{e^2}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}+\frac{d^2}{e^3}\sqrt{c}\ln\left(\left(-\frac{cd}{e}+\left(\frac{d}{e}+x\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*x*(c*x^2+a)^(1/2)+1/2/e*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-d/e^2
*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+d^2/e^3*c^(1/2)*ln((
-c*d/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
1/2))+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+
x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2))/(d/e+x))*a+d^3/e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)
```

```
/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+
x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.46924, size = 1507, normalized size = 11.87

$$\left[\frac{2\sqrt{cd^2+ae^2}cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(2\,cd^2+ae^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}\right.}{4\,ce^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (2*c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c
*e^3), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
 a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (2*c
*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(
c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(sqrt(c*d^2 + a*e^2)*c*d*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) -
(2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c*e^2*x -
2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d*arctan(s
qrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2
*d^2 + a*c*e^2)*x^2)) - (2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (c*e^2*x - 2*c*d*e)*sqrt(c*x^2 + a))/(c*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A] time = 1.18797, size = 182, normalized size = 1.43

$$-\frac{2\left(cd^{3}+ade^{2}\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^{2}+a}\right)e+\sqrt{c}d}{\sqrt{-cd^{2}-ae^{2}}}\right)e^{(-3)}}{\sqrt{-cd^{2}-ae^{2}}}-\frac{\left(2\,c^{\frac{3}{2}}d^{2}+a\sqrt{c}e^{2}\right)e^{(-3)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^{2}+a}\right|\right)}{2\,c}+\frac{1}{2}\,\sqrt{cx^{2}+a}\left(xe^{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^3 + a*d*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))*e^(-3)/sqrt(-c*d^2 - a*e^2) - 1/2*(2*c^(3/2)*d^2 + a*
sqrt(c)*e^2)*e^(-3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c + 1/2*sqrt(c*x
^2 + a)*(x*e^(-1) - 2*d*e^(-2))
```

3.320 $\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

```
[Out] Sqrt[a + c*x^2]/e - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^2 -
(Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/e^2
```

Rubi [A] time = 0.0709663, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {735, 844, 217, 206, 725}

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^2]/e - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^2 -
(Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/e^2
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx = \frac{\sqrt{a+cx^2}}{e} + \frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}$$
$$= \frac{\sqrt{a+cx^2}}{e} + \left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx - \frac{(cd)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}$$
$$= \frac{\sqrt{a+cx^2}}{e} + \left(-a-\frac{cd^2}{e^2}\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right) - \frac{(cd)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+}}\right.}{e^2}$$
$$= \frac{\sqrt{a+cx^2}}{e} - \frac{\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e^2} - \frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2}$$

Mathematica [A] time = 0.029866, size = 99, normalized size = 0.96

$$\frac{-\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right) - \sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) + e\sqrt{a+cx^2}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2] - Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[
c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]
)/e^2
```

Maple [B] time = 0.233, size = 381, normalized size = 3.7

$$\frac{1}{e}\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}} - \frac{d}{e^2}\sqrt{c}\ln\left(\left(-\frac{cd}{e}+\left(\frac{d}{e}+x\right)c\right)\frac{1}{\sqrt{c}} + \sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+c}{e^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-1/e^2*c^(1/2)*d*l
n((-c*d/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2
)^(1/2))-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e
+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2))/(d/e+x))*a-1/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/
e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x
)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.31, size = 1274, normalized size = 12.37

$$\left[\frac{\sqrt{c}d\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\,\sqrt{cx^2+a}e+\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{c}}{e^2x^2+2\,dex+d^2}\right.}{2\,e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*
x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (
2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/s
qrt(c*x^2 + a)) + 2*sqrt(c*x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*
x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/e^2, 1/2*(sqrt(
c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*x^2 + a)*e
- 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x
^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2, (sqrt(-c)*d*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqrt(c*x^2 + a)*e - sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)))/e^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A] time = 1.14324, size = 147, normalized size = 1.43

$$\sqrt{c}de^{(-2)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)+\frac{2\left(cd^2+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}}+\sqrt{cx^2+a}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] sqrt(c)*d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a))) + 2*(c*d^2 + a*e^2)
*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2)
)*e^(-2)/sqrt(-c*d^2 - a*e^2) + sqrt(c*x^2 + a)*e^(-1)
```

3.321 $\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}$$

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e + (Sqrt[c*d^2 + a*e^2]*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*e) - (Sqrt[a]
*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rubi [A] time = 0.0996047, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {896, 266, 63, 208, 844, 217, 206, 725}

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e + (Sqrt[c*d^2 + a*e^2]*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*e) - (Sqrt[a]
*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = -\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d} + \frac{a\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d}$$

$$= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e} - \left(\frac{cd}{e}+\frac{ae}{d}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx$$

$$= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e} - \left(-\frac{cd}{e}-\frac{ae}{d}\right)\mathrm{Subst}\left(\int \frac{1}{cd^2+\ldots}\right.$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{e} + \frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{de} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Mathematica [A] time = 0.055872, size = 113, normalized size = 0.97

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)+\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)-\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[c*d^2 + a*e^2]*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])] - Sqrt[a]*e*ArcTanh
[Sqrt[a + c*x^2]/Sqrt[a]])/(d*e)
```

Maple [B] time = 0.243, size = 420, normalized size = 3.6

$$-\frac{1}{d}\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)+\frac{1}{d}\sqrt{cx^2+a}-\frac{1}{d}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}+\frac{1}{e}\sqrt{c}\ln\Bigg(\Big(-\frac{cd}{e}+\Big(\frac{d}{e}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x/(e*x+d),x)
```

```
[Out] -1/d*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+1/d*(c*x^2+a)^(1/2)-1/d*
((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+c^(1/2)/e*ln((-c*d/e+
(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+1
/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*
e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)
)/(d/e+x))*a+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/
e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c
*d^2)/e^2)^(1/2))/(d/e+x))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.19537, size = 2957, normalized size = 25.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a)*e
*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(c*d^2 + a*e^2)*
log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt
(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/
(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*lo
g(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - sqrt(c*d^2 + a*e^2)*log
((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*
d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*
e), 1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a
)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(-c*d^2 - a
*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-c)*d*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a)
+ 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e),
1/2*(2*sqrt(-a)*e*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(c)*d*log(-2*c*x^2
 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
```

```
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sq
rt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - 2*sqrt(-a)*e*arctan(sqrt(-a)/
sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-a)*e*arctan(sqrt
(-a)/sqrt(c*x^2 + a)) + sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) + 2*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e), -(sqr
t(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(-a)*e*arctan(sqrt(-a)/sqr
t(c*x^2 + a)) - sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)))/(d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{x\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.322 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^2 + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/d^2
```

Rubi [A] time = 0.166599, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {961, 277, 217, 206, 266, 50, 63, 208, 735, 844, 725}

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^2 + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/d^2
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^2} - \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e^2\sqrt{a+cx^2}}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^2}dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x}dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{d+ex}dx}{d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{d^2} - \frac{\sqrt{a+cx^2}}{dx} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}dx}{d} - \frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}dx, x, x^2\right)}{2d^2} + \frac{e\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{c\int \frac{1}{\sqrt{a+cx^2}}dx}{d} + \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}dx, x, x^2\right)}{2d^2} + \Big(c +$$

$$= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}dx, x, \sqrt{a\cdots}\right.}{cd^2}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

Mathematica [A] time = 0.249484, size = 178, normalized size = 1.7

$$\frac{-\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right) - \frac{d\sqrt{a+cx^2}}{x} + \frac{\sqrt{a}\sqrt{c}d\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a+cx^2}} - \sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) + \sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] (-((d*Sqrt[a + c*x^2])/x) + (Sqrt[a]*Sqrt[c]*d*Sqrt[1 + (c*x^2)/a]*ArcSinh[
(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a + c*x^2] - Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[
a + c*x^2]] - Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])] + Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

Maple [B] time = 0.24, size = 486, normalized size = 4.6

$$\frac{e}{d^2}\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right) - \frac{e}{d^2}\sqrt{cx^2+a} + \frac{e}{d^2}\sqrt{\left(\frac{d}{e}+x\right)^2c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}} - \frac{1}{d}\sqrt{c}\ln\left(\left(-\frac{cd}{e}+\left(\frac{d}{e}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] e/d^2*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-e/d^2*(c*x^2+a)^(1/2)+e
/d^2*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-1/d*c^(1/2)*ln((
-c*d/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
1/2))-e/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+
x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2))/(d/e+x))*a-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2
-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(
a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*c-1/d/a/x*(c*x^2+a)^(3/2)+1/d*c/a*x*(c*x^
2+a)^(1/2)+1/d*c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^2), x)
```

Fricas [A] time = 2.13652, size = 1326, normalized size = 12.63

$$\left[\frac{\sqrt{a}ex\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)+\sqrt{cd^2+ae^2}x\log\left(\frac{2acdex-acd^2-2a^2e^2-\left(2c^2d^2+ace^2\right)x^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)-2\sqrt{cx^2+a}}{2d^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*e*x*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqr
t(c*d^2 + a*e^2)*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), 1/2*(sqrt(a)*e*x*log(-(c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*x*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), -1/2*(2*sqrt(-a)*
e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*x*log((2*a*c*d*e
*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2
)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(c*x^2
+ a)*d)/(d^2*x), -(sqrt(-a)*e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + sqrt(-c*
d^2 - a*e^2)*x*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a
*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + sqrt(c*x^2 + a)*d)/(d^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{x^2\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x)), x)
```

Giac [A] time = 1.19735, size = 196, normalized size = 1.87

$$-\frac{2\,a\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}d^2}+\frac{2\,a\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)d}+\frac{2\left(cd^2+ae^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*a*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*d^2) + 2*a
*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*d) + 2*(c*d^2 + a*e^2)*arct
an(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/(sq
rt(-c*d^2 - a*e^2)*d^2)
```

3.323 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{a}e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (e*Sqrt[c*d^2 +
a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^3 -
(c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*e^2*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rubi [A] time = 0.21066, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {961, 266, 47, 63, 208, 277, 217, 206, 50, 735, 844, 725}

$$-\frac{\sqrt{a}e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (e*Sqrt[c*d^2 +
a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^3 -
(c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*e^2*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

[{a, c, d, e}, x]

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx = \int\left(\frac{\sqrt{a+cx^2}}{dx^3} - \frac{e\sqrt{a+cx^2}}{d^2x^2} + \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{e^3\sqrt{a+cx^2}}{d^3(d+ex)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^3}$$

$$= -\frac{e^2\sqrt{a+cx^2}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx\right.}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d} + \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(ce)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,d\right.}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d} + \frac{(ce)\,\text{Subst}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a}e^2\tanh}{}$$

Mathematica [A] time = 0.424999, size = 283, normalized size = 1.77

$$-\frac{-2ex^2\sqrt{a+cx^2}\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right) + cd^2x^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right) + 2\sqrt{a}\sqrt{c}dex^2\sqrt{\frac{cx^2}{a}+1}\sinh}{2d^3x^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -(a*d^2 - 2*a*d*e*x + c*d^2*x^2 - 2*c*d*e*x^3 + 2*Sqrt[a]*Sqrt[c]*d*e*x^2*S
qrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] - 2*Sqrt[c]*d*e*x^2*Sqrt[a
+ c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - 2*e*Sqrt[c*d^2 + a*e^2]*x^2
*Sqrt[a + c*x^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]
)] + 2*Sqrt[a]*e^2*x^2*Sqrt[a + c*x^2]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]] + c
*d^2*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x^2)/a]])/(2*d^3*x^2*Sqrt[
a + c*x^2])
```

Maple [B] time = 0.239, size = 567, normalized size = 3.5

$$-\frac{e^2}{d^3}\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right) + \frac{e^2}{d^3}\sqrt{cx^2+a} - \frac{e^2}{d^3}\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}} + \frac{e}{d^2}\sqrt{c}\ln\left(\left(-\frac{cd}{e}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] -e^2/d^3*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+e^2/d^3*(c*x^2+a)^(1
/2)-e^2/d^3*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+e/d^2*c^(
1/2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^
```

```
2)/e^2)^(1/2))+e^2/d^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*
c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e
^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+1/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/
e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*c-1/2/d/a/x^2*(c*x^2+a)^(3/2)-
1/2/d*c/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+1/2/d*c/a*(c*x^2+a)^(
1/2)+e/d^2/a/x*(c*x^2+a)^(3/2)-e/d^2*c/a*x*(c*x^2+a)^(1/2)-e/d^2*c^(1/2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^3), x)
```

Fricas [A] time = 2.25257, size = 1577, normalized size = 9.86

$$\left[\frac{2\sqrt{cd^2+ae^2}\,aex^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2+2\,ae^2\right)\sqrt{a}x^2\log\left(-\frac{cx^2-2\,\sqrt{cx^2+a}\sqrt{a}+2\,a}{x^2}\right.}{4\,ad^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*
x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*
x^2 + a))/(a*d^3*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a
*c*e^2)*x^2)) + (c*d^2 + 2*a*e^2)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)
, 1/2*(sqrt(c*d^2 + a*e^2)*a*e*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
 (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(s
qrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)
, 1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e*x^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^
2 + 2*a*e^2)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*e*x - a
*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x)), x)
```

Giac [A] time = 1.1892, size = 311, normalized size = 1.94

$$-\frac{2\left(cd^2e+ae^3\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^3}+\frac{\left(cd^2+2\,ae^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d^3}+\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3cd-2\left(\sqrt{c}x\right.}{\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^3) + (c*d^2 + 2*a*e^2)*arctan
(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^3) + ((sqrt(c)*x - sq
rt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*sqrt(c)*e + (sqr
t(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^
2 + a))^2 - a)^2*d^2)
```

3.324 $\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=191

$$-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{\sqrt{a}e^3\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}-\frac{e^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}+\frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^2}-\frac{(a \ldots}{}$$

```
[Out] (e*Sqrt[a + c*x^2])/(2*d^2*x^2) - (e^2*Sqrt[a + c*x^2])/(d^3*x) - (a + c*x^
2)^(3/2)/(3*a*d*x^3) - (e^2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^4 + (c*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[
a]])/(2*Sqrt[a]*d^2) + (Sqrt[a]*e^3*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^4
```

Rubi [A] time = 0.231933, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {961, 264, 266, 47, 63, 208, 277, 217, 206, 50, 735, 844, 725}

$$-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{\sqrt{a}e^3\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}-\frac{e^2\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}+\frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^2}-\frac{(a \ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (e*Sqrt[a + c*x^2])/(2*d^2*x^2) - (e^2*Sqrt[a + c*x^2])/(d^3*x) - (a + c*x^
2)^(3/2)/(3*a*d*x^3) - (e^2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^4 + (c*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[
a]])/(2*Sqrt[a]*d^2) + (Sqrt[a]*e^3*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^4
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^4} - \frac{e\sqrt{a+cx^2}}{d^2x^3} + \frac{e^2\sqrt{a+cx^2}}{d^3x^2} - \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e^4\sqrt{a+cx^2}}{d^4(d+ex)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^4} + \frac{e^4\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^4}$$

$$= \frac{e^3\sqrt{a+cx^2}}{d^4} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d^2} + \frac{(ce^2)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^3} - \frac{e^3\,\text{Subs}}{}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{(ce)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d^2} - \frac{(ce^2)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^3} + \frac{(ce^2)}{}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} + \frac{\sqrt{c}e^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^3} - \frac{e\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{e^2\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4} + \frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d^2}$$

Mathematica [A] time = 1.17276, size = 301, normalized size = 1.58

$$-\frac{6e^2\left(\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)+\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\right)-\frac{3d^2e\left(cx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)+a+cx^2\right)}{x^2\sqrt{a+cx^2}}+\frac{2d^3(a+cx^2)^{3/2}}{ax^3}+}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -(-6*e^3*Sqrt[a + c*x^2] + (2*d^3*(a + c*x^2)^(3/2))/(a*x^3) + (6*d*e^2*(a
+ c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]
]))/(x*Sqrt[a + c*x^2]) + 6*e^2*(Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]] + Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[
a + c*x^2])]) + 6*e^3*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sq
rt[a]]) - (3*d^2*e*(a + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 +
(c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]))/(6*d^4)
```

Maple [B] time = 0.272, size = 600, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] e^3/d^4*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-e^3/d^4*(c*x^2+a)^(1/
2)-1/3*(c*x^2+a)^(3/2)/a/d/x^3+e^3/d^4*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+
c*d^2)/e^2)^(1/2)-e^2/d^3*c^(1/2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*
c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))-e^3/d^4/((a*e^2+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/
e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a-e/d^2/((a*e^2
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)
/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
*c+1/2*e/d^2/a/x^2*(c*x^2+a)^(3/2)+1/2*e/d^2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(c
*x^2+a)^(1/2))/x)-1/2*e/d^2*c/a*(c*x^2+a)^(1/2)-e^2/d^3/a/x*(c*x^2+a)^(3/2)
+e^2/d^3*c/a*x*(c*x^2+a)^(1/2)+e^2/d^3*c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^4), x)
```

Fricas [A] time = 2.2252, size = 1791, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(a)*x^3*l
og(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*a*d^2*e*x - 2*a*d
^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/12*(12*sqr
t(-c*d^2 - a*e^2)*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c*d^2*e + 2*
a*e^3)*sqrt(a)*x^3*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*
(3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4
*x^3), 1/6*(3*sqrt(c*d^2 + a*e^2)*a*e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c*d^2*e + 2*a*e^3)*sqrt(-a)
*x^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 +
 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3), -1/6*(6*sqrt(-c*d^2 - a*e^2)
*a*e^2*x^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d
^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(c*d^2*e + 2*a*e^3)*sqrt(-a)*x
^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3
*a*d*e^2)*x^2)*sqrt(c*x^2 + a))/(a*d^4*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**4*(d + e*x)), x)
```

Giac [A] time = 1.21167, size = 417, normalized size = 2.18

$$\frac{2\left(cd^2e^2+ae^4\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^4}-\frac{\left(cd^2e+2\,ae^3\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d^4}-\frac{3\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^5cde-6\left(\sqrt{c}x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^2*e^2 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^4) - (c*d^2*e + 2*a*e^3)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^4) - 1/3*(3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*c*d*e - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(3/2)
*d^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*sqrt(c)*e^2 - 3*(sqrt(c)*x - sqr
t(c*x^2 + a))*a^2*c*d*e - 2*a^2*c^(3/2)*d^2 + 12*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a^2*sqrt(c)*e^2 - 6*a^3*sqrt(c)*e^2)/(((sqrt(c)*x - sqrt(c*x^2 + a))^
2 - a)^3*d^3)
```

3.325 $\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=274

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}+\frac{e^3\sqrt{a+cx^2}}{d^4x}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}-\frac{\sqrt{a}e^4\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}+\frac{e^3\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5}-c$$

```
[Out] -Sqrt[a + c*x^2]/(4*d*x^4) - (c*Sqrt[a + c*x^2])/(8*a*d*x^2) - (e^2*Sqrt[a
+ c*x^2])/(2*d^3*x^2) + (e^3*Sqrt[a + c*x^2])/(d^4*x) + (e*(a + c*x^2)^(3/2
))/(3*a*d^2*x^3) + (e^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d
^2 + a*e^2]*Sqrt[a + c*x^2])])/d^5 + (c^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])
/(8*a^(3/2)*d) - (c*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d^3) -
 (Sqrt[a]*e^4*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^5
```

Rubi [A] time = 0.29656, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {961, 266, 47, 51, 63, 208, 264, 277, 217, 206, 50, 735, 844, 725}

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}+\frac{e^3\sqrt{a+cx^2}}{d^4x}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}-\frac{\sqrt{a}e^4\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}+\frac{e^3\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5}-c$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(4*d*x^4) - (c*Sqrt[a + c*x^2])/(8*a*d*x^2) - (e^2*Sqrt[a
+ c*x^2])/(2*d^3*x^2) + (e^3*Sqrt[a + c*x^2])/(d^4*x) + (e*(a + c*x^2)^(3/2
))/(3*a*d^2*x^3) + (e^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d
^2 + a*e^2]*Sqrt[a + c*x^2])])/d^5 + (c^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])
/(8*a^(3/2)*d) - (c*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d^3) -
 (Sqrt[a]*e^4*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^5
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^5} - \frac{e\sqrt{a+cx^2}}{d^2x^4} + \frac{e^2\sqrt{a+cx^2}}{d^3x^3} - \frac{e^3\sqrt{a+cx^2}}{d^4x^2} + \frac{e^4\sqrt{a+cx^2}}{d^5x} - \frac{e^5\sqrt{a+cx^2}}{d^5(d+ex)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^5}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^4} + \frac{e^4\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^5} - \frac{e^5\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^5}$$

$$= -\frac{e^4\sqrt{a+cx^2}}{d^5} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^3}\,dx,x,x^2\right)}{2d} + \frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx,x,x^2\right)}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{c\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx,x,x^2\right)}{8d} + \frac{\left(ce^2\right)\text{S}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} - \frac{\sqrt{c}e^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^4} - $$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\tanh^{-1}\left(\right.}{d^5}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e\left(a+cx^2\right)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2+ae^2}\tanh^{-1}\left(\right.}{d^5}$$

Mathematica [C] time = 1.16158, size = 344, normalized size = 1.26

$$\frac{-\frac{2c^2d^4\left(a+cx^2\right)^{3/2}{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{a}+1\right)}{a^3} - \frac{3d^2e^2\left(cx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)+a+cx^2\right)}{x^2\sqrt{a+cx^2}} + 6e^3\left(\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right) + \sqrt{cd}\,\text{t}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (-6*e^4*Sqrt[a + c*x^2] + (2*d^3*e*(a + c*x^2)^(3/2))/(a*x^3) + (6*d*e^3*(a
 + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a
```

```
]]))/(x*Sqrt[a + c*x^2]) + 6*e^3*(Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*
x^2]] + Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt
[a + c*x^2])]) + 6*e^4*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/S
qrt[a]]) - (3*d^2*e^2*(a + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1
 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]) - (2*c^2*d^4*(a + c*x^2)^(3/2)*Hyper
geometric2F1[3/2, 3, 5/2, 1 + (c*x^2)/a])/a^3)/(6*d^5)
```

Maple [B] time = 0.24, size = 703, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^5/(e*x+d),x)
```

```
[Out] -e^4/d^5*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+e^4/d^5*(c*x^2+a)^(1
/2)+1/3*e*(c*x^2+a)^(3/2)/a/d^2/x^3-e^4/d^5*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a
*e^2+c*d^2)/e^2)^(1/2)+e^3/d^4*c^(1/2)*ln((-c*d/e+(d/e+x)*c)/c^(1/2)+((d/e+
x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))+e^4/d^5/((a*e^2+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)
*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))*a+e^2/d^3/
((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2
+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(
d/e+x))*c-1/4/d/a/x^4*(c*x^2+a)^(3/2)+1/8/d*c/a^2/x^2*(c*x^2+a)^(3/2)+1/8/d
*c^2/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-1/8/d*c^2/a^2*(c*x^2+a)^
(1/2)-1/2*e^2/d^3/a/x^2*(c*x^2+a)^(3/2)-1/2*e^2/d^3*c/a^(1/2)*ln((2*a+2*a^(
1/2)*(c*x^2+a)^(1/2))/x)+1/2*e^2/d^3*c/a*(c*x^2+a)^(1/2)+e^3/d^4/a/x*(c*x^2
+a)^(3/2)-e^3/d^4*c/a*x*(c*x^2+a)^(1/2)-e^3/d^4*c^(1/2)*ln(x*c^(1/2)+(c*x^2
+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^5), x)
```

Fricas [A] time = 2.70868, size = 2152, normalized size = 7.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a
```

```
^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2
*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4
+ 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/48*(48*sqrt(-c*d^2
- a*e^2)*a^2*e^3*x^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2
*e^2 - 8*a^2*e^4)*sqrt(a)*x^4*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3
*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), 1/24*(12*sq
rt(c*d^2 + a*e^2)*a^2*e^3*x^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt
(-a)*x^4*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6*a^2*d^4 + 8*
(a*c*d^3*e + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2
 + a))/(a^2*d^5*x^4), 1/24*(24*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^4*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) - 3*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*sqrt(-a)*x^4*a
rctan(sqrt(-a)/sqrt(c*x^2 + a)) + (8*a^2*d^3*e*x - 6*a^2*d^4 + 8*(a*c*d^3*e
 + 3*a^2*d*e^3)*x^3 - 3*(a*c*d^4 + 4*a^2*d^2*e^2)*x^2)*sqrt(c*x^2 + a))/(a^
2*d^5*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{x^5\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**5*(d + e*x)), x)
```

Giac [B] time = 1.23568, size = 805, normalized size = 2.94

$$-\frac{2\left(cd^2e^3+ae^5\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^5}-\frac{\left(c^2d^4-4\,acd^2e^2-8\,a^2e^4\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}ad^5}+\frac{3\left(\sqrt{c}x-\sqrt{cx^2+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e^3 + a*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^5) - 1/4*(c^2*d^4 - 4*a*c*d
^2*e^2 - 8*a^2*e^4)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-
a)*a*d^5) + 1/12*(3*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^2*d^3 - 24*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*a*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a*c^2*d^3 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c*d*e^2 + 24*(sqrt(c)*x -
sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a
^2*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*d*e^2 - 24*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*a^2*sqrt(c)*e^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a
^3*c^(3/2)*d^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d^3 - 12*(sqrt(c
)*x - sqrt(c*x^2 + a))^3*a^3*c*d*e^2 + 72*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a
^3*sqrt(c)*e^3 + 8*a^4*c^(3/2)*d^2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4
```

```
*c*d*e^2 - 72*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*e^3 + 24*a^5*sqrt
(c)*e^3)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^4*a*d^4)
```

3.326 $\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{6c^2e^3}-\frac{d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}}-\frac{7d\sqrt{a+cx^2}(d+ex)}{6ce^3}+\frac{\sqrt{a}}{}$$

```
[Out] ((11*c*d^2 - 4*a*e^2)*Sqrt[a + c*x^2])/(6*c^2*e^3) - (7*d*(d + e*x)*Sqrt[a
+ c*x^2])/(6*c*e^3) + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^3) - (d*(2*c*d^2
 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*Sqrt[c*d^2 +
 a*e^2])
```

Rubi [A] time = 0.481779, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1654, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(11cd^2-4ae^2\right)}{6c^2e^3}-\frac{d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}}-\frac{7d\sqrt{a+cx^2}(d+ex)}{6ce^3}+\frac{\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((11*c*d^2 - 4*a*e^2)*Sqrt[a + c*x^2])/(6*c^2*e^3) - (7*d*(d + e*x)*Sqrt[a
+ c*x^2])/(6*c*e^3) + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^3) - (d*(2*c*d^2
 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*Sqrt[c*d^2 +
 a*e^2])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{-2ad^2e^2-de\left(cd^2+4ae^2\right)x-e^2\left(5cd^2+2ae^2\right)x^2-7cde^3x^3}{(d+ex)\sqrt{a+cx^2}}\,dx}{3ce^4}$$

$$= -\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{3acd^2e^5+cde^4\left(5cd^2-ae^2\right)x+ce^5\left(11cd^2-4ae^2\right)x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{6c^2e^7}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{3ac^2d^2e^7-3c^2de^6\left(2cd^2-ae^2 \right.}{(d+ex)\sqrt{a+cx^2}}}{6c^3e^9}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{d^4\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4} - $$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d^4\operatorname{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\right.}{e^4}$$

$$= \frac{\left(11cd^2-4ae^2\right)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\right.}{2c^{3/2}e^4}$$

Mathematica [A] time = 0.266167, size = 149, normalized size = 0.76

$$\frac{\frac{e\sqrt{a+cx^2}\left(-4ae^2+6cd^2-3cdex+2ce^2x^2\right)}{c^2} - \frac{3d\left(2cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}} - \frac{6d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(6*c*d^2 - 4*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2))/c^2 - (3
*d*(2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/c^(3/2) - (6*d^4
*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[c*d^2 +
 a*e^2])/(6*e^4)
```

Maple [A] time = 0.237, size = 260, normalized size = 1.3

$$\frac{x^2}{3\,ce}\sqrt{cx^2+a} - \frac{2\,a}{3\,c^2e}\sqrt{cx^2+a} - \frac{dx}{2\,ce^2}\sqrt{cx^2+a} + \frac{ad}{2\,e^2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}} + \frac{d^2}{e^3c}\sqrt{cx^2+a} - \frac{d^3}{e^4}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3/e*x^2/c*(c*x^2+a)^(1/2)-2/3/e*a/c^2*(c*x^2+a)^(1/2)-1/2*d/e^2*x/c*(c*x^
2+a)^(1/2)+1/2*d/e^2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+d^2/e^3/c*(c*x
^2+a)^(1/2)-d^3/e^4*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-d^4/e^5/((a*e^2+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e
^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.2924, size = 2218, normalized size = 11.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x
^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^2 - a^2*d*e^
4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*d^4*e
 + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e
^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/12*(12*sq
rt(-c*d^2 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 3*(2*c^2*d^5 + a
*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
 - a) - 2*(6*c^2*d^4*e + 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^
5)*x^2 - 3*(c^2*d^3*e^2 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c
^2*e^6), 1/6*(3*sqrt(c*d^2 + a*e^2)*c^2*d^4*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 3*(2*c^2*d^5 + a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*c^2*d^4*e + 2*a*
c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2 + a*
c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), -1/6*(6*sqrt(-c*d^2
 - a*e^2)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^
2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (6*c^2*d^4*e +
 2*a*c*d^2*e^3 - 4*a^2*e^5 + 2*(c^2*d^2*e^3 + a*c*e^5)*x^2 - 3*(c^2*d^3*e^2
 + a*c*d*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a + cx^2}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A] time = 1.23821, size = 220, normalized size = 1.13

$$\frac{2\,d^4\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{6}\sqrt{cx^2+a}\left(x\left(\frac{2\,xe^{(-1)}}{c}-\frac{3\,de^{(-2)}}{c}\right)+\frac{2\left(3\,c^2d^2e^7-2\,ace^9\right)e^{(-10)}}{c^3}\right)+\frac{\left(2\,c^{\frac{3}{2}}d^3-a\sqrt{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/6*sqrt(c*x^2 + a)*(x*(2*x*e^(-1)/c -
 3*d*e^(-2)/c) + 2*(3*c^2*d^2*e^7 - 2*a*c*e^9)*e^(-10)/c^3) + 1/2*(2*c^(3/2
)*d^3 - a*sqrt(c)*d*e^2)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^2
```

3.327 $\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=152

$$\frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}}-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}$$

```
[Out] (-3*d*Sqrt[a + c*x^2])/(2*c*e^2) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^2) +
((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^3) +
(d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^3*Sqr
t[c*d^2 + a*e^2])
```

Rubi [A] time = 0.273506, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1654, 844, 217, 206, 725}

$$\frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}}-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-3*d*Sqrt[a + c*x^2])/(2*c*e^2) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^2) +
((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^3) +
(d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^3*Sqr
t[c*d^2 + a*e^2])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx &= \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\int \frac{-ade^2-e(cd^2+ae^2)x-3cde^2x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3} \\
&= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{\int \frac{-acde^4+ce^3(2cd^2-ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2c^2e^5} \\
&= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} - \frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3} + \frac{(2cd^2-ae^2)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2ce^3} \\
&= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3} + \frac{(2cd^2-ae^2)\,\text{Subs}}{}\\
&= -\frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2} + \frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3} + \frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3\sqrt{cd^2+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.236951, size = 131, normalized size = 0.86

$$\frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+\sqrt{c}\left(\frac{2cd^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}+e\sqrt{a+cx^2}(ex-2d)\right)}{2c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[c]*(e*(-2*d
+ e*x)*Sqrt[a + c*x^2] + (2*c*d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2]])])/Sqrt[c*d^2 + a*e^2]))/(2*c^(3/2)*e^3)
```

Maple [A] time = 0.261, size = 217, normalized size = 1.4

$$\frac{x}{2ce}\sqrt{cx^2+a} - \frac{a}{2e}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}} - \frac{d}{ce^2}\sqrt{cx^2+a} + \frac{d^2}{e^3}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}} + \frac{d^3}{e^4}\ln\left(\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e*x/c*(c*x^2+a)^(1/2)-1/2/e*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-d*(
c*x^2+a)^(1/2)/c/e^2+d^2/e^3*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d^3/e^4/
```

```
((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2
+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(
d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.4052, size = 1917, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c^2*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sq
rt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*
a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^
2*e^5), 1/4*(4*sqrt(-c*d^2 - a*e^2)*c^2*d^3*arctan(sqrt(-c*d^2 - a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) -
 (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) - 2*(2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*
x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(sqrt(c*d^2 + a*e^2)*c^2
*d^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2*e^2 + a*c*e^4)*x)*sqrt(c
*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c^2*d^3*a
rctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2
 + (c^2*d^2 + a*c*e^2)*x^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)
*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^3*e + 2*a*c*d*e^3 - (c^2*d^2
*e^2 + a*c*e^4)*x)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a + cx^2}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A] time = 1.22742, size = 174, normalized size = 1.14

$$-\frac{2\,d^3\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-3)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{2}\sqrt{cx^2+a}\left(\frac{xe^{(-1)}}{c}-\frac{2\,de^{(-2)}}{c}\right)-\frac{\left(2\,cd^2-ae^2\right)e^{(-3)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*d^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))*e^(-3)/sqrt(-c*d^2 - a*e^2) + 1/2*sqrt(c*x^2 + a)*(x*e^(-1)/c - 2*d
*e^(-2)/c) - 1/2*(2*c*d^2 - a*e^2)*e^(-3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/c^(3/2)
```

3.328 $\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=109

$$-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}+\frac{\sqrt{a+cx^2}}{ce}$$

```
[Out] Sqrt[a + c*x^2]/(c*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e
^2) - (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e
^2*Sqrt[c*d^2 + a*e^2])
```

Rubi [A] time = 0.127501, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1654, 12, 844, 217, 206, 725}

$$-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}+\frac{\sqrt{a+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e
^2) - (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e
^2*Sqrt[c*d^2 + a*e^2])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx &= \frac{\sqrt{a+cx^2}}{ce}-\frac{\int \frac{cdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}+\frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e^2}-\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0845403, size = 105, normalized size = 0.96

$$
\frac{-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}+\frac{e\sqrt{a+cx^2}}{c}}{e^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2])/c - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c] -
 (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[c*
d^2 + a*e^2])/e^2
```

Maple [A] time = 0.264, size = 172, normalized size = 1.6

$$
\frac{1}{ce}\sqrt{cx^2+a}-\frac{d}{e^2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{d^2}{e^3}\ln\left(\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+\right.}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/e-1/e^2*d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-d^2/e^3/(
(a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+
c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d
/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.92065, size = 1590, normalized size = 14.59

$$\left[\frac{\sqrt{cd^2+ae^2}cd^2\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^3+ade^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{}\right.}{2\left(c^2d^2e^2+ace^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2
*d^2*e^2 + a*c*e^4), -1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2
- a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*
e^2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqr
t(c)*x - a) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4),
 1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2
*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-
c)*x/sqrt(c*x^2 + a)) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 +
 a*c*e^4), -(sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*
d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*d^2*e + a*e
^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A] time = 1.15939, size = 142, normalized size = 1.3

$$\frac{2\,d^2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}}+\frac{de^{(-2)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}e^{(-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))*e^(-2)/sqrt(-c*d^2 - a*e^2) + d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x
^2 + a)))/sqrt(c) + sqrt(c*x^2 + a)*e^(-1)/c
```

3.329 $\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}$$

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

Rubi [A] time = 0.0439798, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {844, 217, 206, 725}

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e} - \frac{d\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e} + \frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\sqrt{cd^2+ae^2}}$$

Mathematica [A] time = 0.0267371, size = 86, normalized size = 1.

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

Maple [B] time = 0.232, size = 151, normalized size = 1.8

$$\frac{1}{e}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}} + \frac{d}{e^2}\ln\left(\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)
^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.68189, size = 1353, normalized size = 15.73

$$\left[\frac{\sqrt{cd^2+ae^2}cd\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2+ae^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}\right.}{2\left(c^2d^2e+ace^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(2*sqrt(-c*d^2 - a*
e^2)*c*d*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2
 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x
^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(sqrt(c*d
^2 + a*e^2)*c*d*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e
^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2
*d*e*x + d^2)) - 2*(c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 +
a)))/(c^2*d^2*e + a*c*e^3), (sqrt(-c*d^2 - a*e^2)*c*d*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/(c^
2*d^2*e + a*c*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A] time = 1.22846, size = 119, normalized size = 1.38

$$-\frac{2\,d\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^{(-1)}}{\sqrt{-cd^2-ae^2}}-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*d*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))*e^(-1)/sqrt(-c*d^2 - a*e^2) - e^(-1)*log(abs(-sqrt(c)*x + sqrt(c*x^2
+ a)))/sqrt(c)
```

3.330 $\int \frac{1}{(d+ex)\sqrt{a+cx^2}} dx$

Optimal. Leaf size=54

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Rubi [A] time = 0.0172949, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {725, 206}

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+cx^2}} dx &= -\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2} dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.0067823, size = 54, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Maple [B] time = 0.223, size = 127, normalized size = 2.4

$$-\frac{1}{e}\ln\left(\left(2\,\frac{ae^2+cd^2}{e^2}-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\,\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}\right)\left(\frac{d}{e}+x\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((
a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/
2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09186, size = 435, normalized size = 8.06

$$\left[\frac{\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2+ae^2}},-\frac{\sqrt{-cd^2-ae^2}\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{acd^2+a^2e^2+\left(c^2d^2+ace^2\right)x^2}\right)}{cd^2+ae^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2
*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^
2))/sqrt(c*d^2 + a*e^2), -sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)
)/(c*d^2 + a*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A] time = 1.19766, size = 80, normalized size = 1.48

$$\frac{2 \arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2
))/sqrt(-c*d^2 - a*e^2)
```

3.331 $\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rubi [A] time = 0.0830174, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {961, 266, 63, 208, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{dx\sqrt{a+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+cx^2}}\right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.0481575, size = 86, normalized size = 1.

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Maple [B] time = 0.24, size = 158, normalized size = 1.8

$$-\frac{1}{d}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)\frac{1}{\sqrt{a}} + \frac{1}{d}\ln\left(\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+1/d/((a*e^2+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(
```

```
(d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x), x)
```

Fricas [A] time = 2.24347, size = 1358, normalized size = 15.79

$$\left[\frac{\sqrt{cd^2+ae^2}\,ae\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2+ae^2\right)\sqrt{a}\log\left(-\frac{cx^2-2\,\sqrt{cx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{2\left(acd^3+a^2de^2\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c
^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)
)/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(2*sqrt(-c*d^2
- a*e^2)*a*e*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c
*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/2*(
sqrt(c*d^2 + a*e^2)*a*e*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2
 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2
*x^2 + 2*d*e*x + d^2)) + 2*(c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*
x^2 + a)))/(a*c*d^3 + a^2*d*e^2), (sqrt(-c*d^2 - a*e^2)*a*e*arctan(sqrt(-c*
d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 +
a*c*e^2)*x^2)) + (c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)))
/(a*c*d^3 + a^2*d*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.332 $\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=111

$$-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2 + a*e^2]) + (e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rubi [A] time = 0.0953832, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {961, 264, 266, 63, 208, 725, 206}

$$-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2 + a*e^2]) + (e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx &= \int\left(\frac{1}{dx^2\sqrt{a+cx^2}}-\frac{e}{d^2x\sqrt{a+cx^2}}+\frac{e^2}{d^2(d+ex)\sqrt{a+cx^2}}\right)dx\\
&= \frac{\int\frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d}-\frac{e\int\frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2}+\frac{e^2\int\frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{a+cx^2}}{adx}-\frac{e\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d^2}-\frac{e^2\,\mathrm{Subst}\left(\int\frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^2}\\
&= -\frac{\sqrt{a+cx^2}}{adx}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}-\frac{e\,\mathrm{Subst}\left(\int\frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd^2}\\
&= -\frac{\sqrt{a+cx^2}}{adx}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}\end{aligned}$$

Mathematica [A] time = 0.0899508, size = 107, normalized size = 0.96

$$\frac{-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}-\frac{d\sqrt{a+cx^2}}{ax}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-((d*Sqrt[a + c*x^2])/(a*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*
e^2]*Sqrt[a + c*x^2])])/Sqrt[c*d^2 + a*e^2] + (e*ArcTanh[Sqrt[a + c*x^2]/Sq
rt[a]])/Sqrt[a])/d^2
```

Maple [A] time = 0.236, size = 180, normalized size = 1.6

$$\frac{e}{d^2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{e}{d^2}\ln\left(\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] e/d^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-e/d^2/((a*e^2+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2
)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-(c*x^2+a)
^(1/2)/a/d/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^2), x)
```

Fricas [A] time = 2.42056, size = 1639, normalized size = 14.77

$$\left[\frac{\sqrt{cd^2+ae^2}ae^2x\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)+\left(cd^2e+ae^3\right)\sqrt{a}x\log\left(-\frac{cx^2+2\,\sqrt{cx^2+a}\sqrt{a}+2\,a}{x^2}\right.}{2\left(acd^4+a^2d^2e^2\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 -
(2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a))/(e^2*x^2 + 2*d*e*x + d^2)) + (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2
+ 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 +
a))/((a*c*d^4 + a^2*d^2*e^2)*x), -1/2*(2*sqrt(-c*d^2 - a*e^2)*a*e^2*x*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - (c*d^2*e + a*e^3)*sqrt(a)*x*log(-(c*x^2 + 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*
c*d^4 + a^2*d^2*e^2)*x), 1/2*(sqrt(c*d^2 + a*e^2)*a*e^2*x*log((2*a*c*d*e*x
- a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e
^3)*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - 2*(c*d^3 + a*d*e^2)*sqrt(
c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^2)*x), -(sqrt(-c*d^2 - a*e^2)*a*e^2*x*arc
tan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
 (c^2*d^2 + a*c*e^2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(-a)*x*arctan(sqrt(-a)/s
qrt(c*x^2 + a)) + (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/((a*c*d^4 + a^2*d^2*e^
2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A] time = 1.14147, size = 192, normalized size = 1.73

$$2\,c\left(\frac{\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^2}{\sqrt{-cd^2-ae^2}cd^2}-\frac{\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}cd^2}+\frac{1}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)\sqrt{c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*(arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))*e^2/(sqrt(-c*d^2 - a*e^2)*c*d^2) - arctan(-(sqrt(c)*x - sqrt(c*x^2 +
a))/sqrt(-a))*e/(sqrt(-a)*c*d^2) + 1/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)
*sqrt(c)*d))
```

3.333 $\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}} dx$

Optimal. Leaf size=168

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (e^3*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*
e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - (e^2*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rubi [A] time = 0.140467, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {961, 266, 51, 63, 208, 264, 725, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (e^3*ArcTanh
[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*
e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - (e^2*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{dx^3\sqrt{a+cx^2}} - \frac{e}{d^2x^2\sqrt{a+cx^2}} + \frac{e^2}{d^3x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)\sqrt{a+cx^2}}\right)dx$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^3} + \frac{e^3\operatorname{Subst}\left(\int \frac{}{cd^2+\cdots}\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4ad} + \frac{e^2\operatorname{S}\cdots}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}\cdots}\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} - \frac{e^2\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{}\right.}{\sqrt{a}d^3}$$

Mathematica [A] time = 0.716823, size = 163, normalized size = 0.97

$$\frac{\frac{2e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}} + \frac{d\left(cdx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right) - (a+cx^2)(d-2ex)\right)}{ax^2\sqrt{a+cx^2}} - \frac{2e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/Sqrt[
c*d^2 + a*e^2] - (2*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a] + (d*(-((
d - 2*e*x)*(a + c*x^2)) + c*d*x^2*Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x
^2)/a]]))/(a*x^2*Sqrt[a + c*x^2]))/(2*d^3)
```

Maple [A] time = 0.246, size = 236, normalized size = 1.4

$$-\frac{e^2}{d^3}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{e^2}{d^3}\ln\left(\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -e^2/d^3/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+e^2/d^3/((a*e^2+c*d^
2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)
^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/2*
(c*x^2+a)^(1/2)/a/d/x^2+1/2/d*c/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/
x)+e*(c*x^2+a)^(1/2)/a/d^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^3), x)
```

Fricas [A] time = 2.80162, size = 1987, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*
e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4
)*sqrt(a)*x^2*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*
d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d
^5 + a^3*d^3*e^2)*x^2), 1/4*(4*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
```

```
2 + a*c*e^2)*x^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(a)*x^2*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*d^4 + a^2*d^2*e^2 -
2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^
2), 1/2*(sqrt(c*d^2 + a*e^2)*a^2*e^3*x^2*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2
*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^
4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 -
 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x
^2), 1/2*(2*sqrt(-c*d^2 - a*e^2)*a^2*e^3*x^2*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- (c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(c*x
^2 + a)) - (a*c*d^4 + a^2*d^2*e^2 - 2*(a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*x^2
 + a))/((a^2*c*d^5 + a^3*d^3*e^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A] time = 1.17297, size = 323, normalized size = 1.92

$$-c^{\frac{3}{2}}\left(\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^3}{\sqrt{-cd^2-ae^2}\,c^{\frac{3}{2}}d^3}+\frac{\left(cd^2-2\,ae^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ac^{\frac{3}{2}}d^3}-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3\sqrt{c}d-2\left(\sqrt{c}x-\sqrt{cx^2\right.}}{\left(\left(\sqrt{c}x-\sqrt{cx^2\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -c^(3/2)*(2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d
^2 - a*e^2))*e^3/(sqrt(-c*d^2 - a*e^2)*c^(3/2)*d^3) + (c*d^2 - 2*a*e^2)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^(3/2)*d^3) - ((s
qrt(c)*x - sqrt(c*x^2 + a))^3*sqrt(c)*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*sqrt(c)*d + 2*a^2*e)/(((sqrt(c)*x -
sqrt(c*x^2 + a))^2 - a)^2*a*c*d^2))
```

3.334 $\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2}+\frac{\sqrt{a+cx^2}}{c^2e}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] (a*(a*e + c*d*x))/(c^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + Sqrt[a + c*x^2]/(
c^2*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^2) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e
^2)^(3/2))
```

Rubi [A] time = 0.312321, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1647, 1654, 844, 217, 206, 725}

$$\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2}+\frac{\sqrt{a+cx^2}}{c^2e}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a*e + c*d*x))/(c^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + Sqrt[a + c*x^2]/(
c^2*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^2) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e
^2)^(3/2))
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{\frac{a^2d^2}{cd^2+ae^2}-ax^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{\int \frac{\frac{a^2cd^2e^2}{cd^2+ae^2}+acdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac^2e^2}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce^2} + \frac{d^4\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2\left(cd^2+ae^2\right)}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce^2} - \frac{d^4\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2 \ldots}\right.}{e^2\left(cd^2 \ldots\right.}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} - \frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.514877, size = 179, normalized size = 1.23

$$\frac{\frac{e\left(2a^2e^2+ac\left(d^2+dex+e^2x^2\right)+c^2d^2x^2\right)}{c^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{\sqrt{a}d\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}\sqrt{a+cx^2}} - \frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] ((e*(2*a^2*e^2 + c^2*d^2*x^2 + a*c*(d^2 + d*e*x + e^2*x^2)))/(c^2*(c*d^2 +
a*e^2)*Sqrt[a + c*x^2]) - (Sqrt[a]*d*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x
)/Sqrt[a]])/(c^(3/2)*Sqrt[a + c*x^2]) - (d^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*
```

```
d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2))/e^2
```

Maple [B] time = 0.239, size = 396, normalized size = 2.7

$$\frac{x^2}{ce}\frac{1}{\sqrt{cx^2+a}}+2\frac{a}{c^2e\sqrt{cx^2+a}}+\frac{dx}{ce^2}\frac{1}{\sqrt{cx^2+a}}-\frac{d}{e^2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}-\frac{d^2}{e^3c}\frac{1}{\sqrt{cx^2+a}}-\frac{d^3x}{e^4a}\frac{1}{\sqrt{cx^2+a}}+\frac{d}{e^3\left(ae^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/e*x^2/c/(c*x^2+a)^(1/2)+2/e*a/c^2/(c*x^2+a)^(1/2)+d/e^2*x/c/(c*x^2+a)^(1/
2)-d/e^2/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-d^2/e^3/c/(c*x^2+a)^(1/2)-d^
3/e^4*x/a/(c*x^2+a)^(1/2)+d^4/e^3/(a*e^2+c*d^2)/((d/e+x)^2*c-2*c*d/e*(d/e+x
)+(a*e^2+c*d^2)/e^2)^(1/2)+d^5/e^4/(a*e^2+c*d^2)/a/((d/e+x)^2*c-2*c*d/e*(d/
e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x*c-d^4/e^3/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*
((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 31.8025, size = 3092, normalized size = 21.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2
 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^4*e + 3*a^2*c
*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c
^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^
2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 + a^2*c^3*e^6)*x^2), -
1/2*(2*(c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d^5 + 2*a*c^2*d^
3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^
```

```
2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 +
a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c
^4*d^2*e^4 + a^2*c^3*e^6)*x^2), 1/2*(2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d
*e^4 + (c^3*d^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log(
(2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d
^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(
a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c^2*d^2*e^3 +
a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d
^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*c^4*d^2*e^4 +
 a^2*c^3*e^6)*x^2), -((c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan
(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c
^2*d^2 + a*c*e^2)*x^2)) - (a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (c^3*d
^5 + 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) - (a*c^2*d^4*e + 3*a^2*c*d^2*e^3 + 2*a^3*e^5 + (c^3*d^4*e + 2*a*c
^2*d^2*e^3 + a^2*c*e^5)*x^2 + (a*c^2*d^3*e^2 + a^2*c*d*e^4)*x)*sqrt(c*x^2 +
 a))/(a*c^4*d^4*e^2 + 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6 + (c^5*d^4*e^2 + 2*a*
c^4*d^2*e^4 + a^2*c^3*e^6)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a + cx^2\right)^{\frac{3}{2}} (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [B] time = 1.22742, size = 404, normalized size = 2.77

$$\frac{2\, d^4 \arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2e^2 + ae^4\right)\sqrt{-cd^2 - ae^2}} + \frac{de^{(-2)} \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right)}{c^{\frac{3}{2}}} + \frac{\left(\frac{\left(c^4d^4e^5+2\,ac^3d^2e^7+a^2c^2e^9\right)x}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}} + \frac{ac^3d^3e^6+a^2c^2de^8}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}}\right)x +}{\sqrt{cx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2*e^2 + a*e^4)*sqrt(-c*d^2 - a*e^2)) + d*e^(-2)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2) + (((c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c
^2*e^9)*x/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^10) + (a*c^3*d^3*e^6 +
 a^2*c^2*d*e^8)/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^10))*x + (a*c^3*
d^4*e^5 + 3*a^2*c^2*d^2*e^7 + 2*a^3*c*e^9)/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 +
 a^2*c^3*e^10))/sqrt(c*x^2 + a)
```

3.335 $\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] (a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]]/(c^(3/2)*e) + (d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/(e*(c*d^2 + a*e^2)^(3/2))
```

Rubi [A] time = 0.164837, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1647, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```


```
[Out] (a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]]/(c^(3/2)*e) + (d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/(e*(c*d^2 + a*e^2)^(3/2))
```


Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{-\frac{a^2de}{cd^2+ae^2}-ax}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce} - \frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e\left(cd^2+ae^2\right)}$$

$$= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e\left(cd^2+ae^2\right)}$$

$$= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e} + \frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.317333, size = 153, normalized size = 1.24

$$\frac{\frac{\sqrt{c}\left(cd^3\sqrt{a+cx^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)+ae(d-ex)\sqrt{ae^2+cd^2}\right)}{\left(ae^2+cd^2\right)^{3/2}}+\sqrt{a}\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}e\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] + (Sqrt[c]*(a*e*S
qrt[c*d^2 + a*e^2]*(d - e*x) + c*d^3*Sqrt[a + c*x^2]*ArcTanh[(a*e - c*d*x)/
(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]))/(c*d^2 + a*e^2)^(3/2))/(c^(3/2)*e*
Sqrt[a + c*x^2])
```

Maple [B] time = 0.242, size = 354, normalized size = 2.9

$$-\frac{x}{ce}\frac{1}{\sqrt{cx^2+a}}+\frac{1}{e}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+\frac{d}{ce^2}\frac{1}{\sqrt{cx^2+a}}+\frac{d^2x}{ae^3}\frac{1}{\sqrt{cx^2+a}}-\frac{d^3}{e^2\left(ae^2+cd^2\right)}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/e*x/c/(c*x^2+a)^(1/2)+1/e/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+d/e^2/c/
(c*x^2+a)^(1/2)+d^2/e^3*x/a/(c*x^2+a)^(1/2)-d^3/e^2/(a*e^2+c*d^2)/((d/e+x)^
```

```
2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-d^4/e^3/(a*e^2+c*d^2)/a/((d/e+
x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x*c+d^3/e^2/(a*e^2+c*d^2)/(
(a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+
c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d
/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 32.7009, size = 2691, normalized size = 21.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^3*e + a^2*c*d*e^3
 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3
*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2),
1/2*(2*(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*
e^2 + a^2*c*e^4)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x -
a) + 2*(a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x
^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*
c^4*d^2*e^3 + a^2*c^3*e^5)*x^2), -1/2*(2*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log((2
*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2
 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*
c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(c*x^2 + a))/(
a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^
3 + a^2*c^3*e^5)*x^2), ((c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arct
an(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 +
(c^2*d^2 + a*c*e^2)*x^2)) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d
^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^
2 + a)) + (a*c^2*d^3*e + a^2*c*d*e^3 - (a*c^2*d^2*e^2 + a^2*c*e^4)*x)*sqrt(
c*x^2 + a))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2
*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a + cx^2\right)^{\frac{3}{2}} (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.19655, size = 296, normalized size = 2.41

$$-\frac{2\, d^3 \arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2e + ae^3\right)\sqrt{-cd^2 - ae^2}} - \frac{\frac{\left(ac^2d^2e^3+a^2ce^5\right)x}{c^4d^4e^2+2\,ac^3d^2e^4+a^2c^2e^6} - \frac{ac^2d^3e^2+a^2cde^4}{c^4d^4e^2+2\,ac^3d^2e^4+a^2c^2e^6}}{\sqrt{cx^2 + a}} - \frac{e^{(-1)} \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*d^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))/((c*d^2*e + a*e^3)*sqrt(-c*d^2 - a*e^2)) - ((a*c^2*d^2*e^3 + a^2*c*
e^5)*x/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6) - (a*c^2*d^3*e^2 + a^2
*c*d*e^4)/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6))/sqrt(c*x^2 + a) -
e^(-1)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.336 $\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rubi [A] time = 0.111139, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1647, 12, 725, 206}

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{acd^2}{\left(cd^2+ae^2\right)(d+ex)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$

$$= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.0970718, size = 95, normalized size = 1.

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e -
c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Maple [B] time = 0.234, size = 311, normalized size = 3.3

$$-\frac{1}{ce}\frac{1}{\sqrt{cx^2+a}} - \frac{dx}{ae^2}\frac{1}{\sqrt{cx^2+a}} + \frac{d^2}{e\left(ae^2+cd^2\right)}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}}} + \frac{d^3xc}{e^2\left(ae^2+cd^2\right)a}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/e/c/(c*x^2+a)^(1/2)-1/e^2*d*x/a/(c*x^2+a)^(1/2)+d^2/e/(a*e^2+c*d^2)/((d/
e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+d^3/e^2/(a*e^2+c*d^2)/a/(
(d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x*c-d^2/e/(a*e^2+c*d^2
)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e
^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))
/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.47039, size = 913, normalized size = 9.61

$$\left[\frac{\left(c^2d^2x^2+acd^2\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(acd^2e+a^2e^3+\left(c^2d^3+acd\right.\right.}{2\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^2*x^2 + a*c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 +
 a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2), -((c^2*d^2*x^2
 + a*c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.14283, size = 235, normalized size = 2.47

$$-\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}}-\frac{\frac{\left(c^2d^3+acde^2\right)x}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}+\frac{acd^2e+a^2e^3}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*d^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) - ((c^2*d^3 + a*c*d*e^2)*x/(c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4) + (a*c*d^2*e + a^2*e^3)/(c^3*d^4 + 2*
a*c^2*d^2*e^2 + a^2*c*e^4))/sqrt(c*x^2 + a)
```

3.337 $\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{d-ex}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)}$$

```
[Out] -((d - e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rubi [A] time = 0.0519365, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {823, 12, 725, 206}

$$\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{d-ex}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((d - e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{acde}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac\left(cd^2+ae^2\right)}$$
$$= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{(de)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$
$$= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{(de)\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$
$$= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.0583673, size = 88, normalized size = 1.

$$\frac{ex-d}{\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-d + e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (d*e*ArcTanh[(a*e - c*d*x)/(
Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])])/(c*d^2 + a*e^2)^(3/2)
```

Maple [B] time = 0.23, size = 283, normalized size = 3.2

$$\frac{x}{ae}\frac{1}{\sqrt{cx^2+a}} - \frac{d}{ae^2+cd^2}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}}} - \frac{d^2xc}{e\left(ae^2+cd^2\right)a}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/e*x/a/(c*x^2+a)^(1/2)-d/(a*e^2+c*d^2)/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2
+c*d^2)/e^2)^(1/2)-d^2/e/(a*e^2+c*d^2)/a/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^
2+c*d^2)/e^2)^(1/2)*x*c+d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c
*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.5926, size = 863, normalized size = 9.81

$$\left[\frac{\left(cdex^2+ade\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2+2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(cd^3+ade^2-\left(cd^2e+ae^3\right)x\right)\sqrt{cx^2+a}}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4+\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x^2 + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^3 + a*d*e^2 - (c*d^2*
e + a*e^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^
3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2), ((c*d*e*x^2 + a*d*e)*sqrt(-c*d^2
 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^
2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (c*d^3 + a*d*e^2 - (c*d^2*e + a*e
^3)*x)*sqrt(c*x^2 + a))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 +
 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [B] time = 1.14151, size = 219, normalized size = 2.49

$$\frac{2\,d\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}}+\frac{\frac{\left(cd^2e+ae^3\right)x}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{cd^3+ade^2}{c^2d^4+2\,acd^2e^2+a^2e^4}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^
2))*e/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + ((c*d^2*e + a*e^3)*x/(c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4) - (c*d^3 + a*d*e^2)/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4))/sqrt(c*x^2 + a)
```

3.338 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rubi [A] time = 0.0461382, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {741, 12, 725, 206}

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e^2\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$

$$= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.0501925, size = 94, normalized size = 1.

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} - \frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Maple [B] time = 0.225, size = 260, normalized size = 2.8

$$\frac{e}{ae^2+cd^2}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}} + \frac{cdx}{\left(ae^2+cd^2\right)a}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}} - \frac{e}{ae^2+cd^2}\ln\left(\left(2\frac{ae}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] e/(a*e^2+c*d^2)/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+d/(a*
e^2+c*d^2)/a/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x*c-e/(a
*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+
x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.67039, size = 913, normalized size = 9.71

$$\left[\frac{\left(ace^2x^2 + a^2e^2\right)\sqrt{cd^2 + ae^2}\log\left(\frac{2\,acdex - acd^2 - 2\,a^2e^2 - \left(2\,c^2d^2 + ace^2\right)x^2 - 2\,\sqrt{cd^2 + ae^2}(cdx - ae)\sqrt{cx^2 + a}}{e^2x^2 + 2\,dex + d^2}\right) + 2\left(acd^2e + a^2e^3 + \left(c^2d^3 + acd\right.\right.}{2\left(a^2c^2d^4 + 2\,a^3cd^2e^2 + a^4e^4 + \left(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*e^2*x^2 + a^2*e^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c*d^2*e + a^2*e^3 +
 (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2), -((a*c*e^2*x^2
 + a^2*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - (a*c*d^2*e
 + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(a^2*c^2*d^4 + 2*a^3
*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^2\right)^{\frac{3}{2}}(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.17332, size = 232, normalized size = 2.47

$$\frac{\frac{\left(c^2d^3 + acde^2\right)x}{ac^2d^4 + 2\,a^2cd^2e^2 + a^3e^4} + \frac{acd^2e + a^2e^3}{ac^2d^4 + 2\,a^2cd^2e^2 + a^3e^4}}{\sqrt{cx^2 + a}} - \frac{2\,\arctan\left(\frac{\left(\sqrt{c}x - \sqrt{cx^2 + a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 - ae^2}}\right)e^2}{\left(cd^2 + ae^2\right)\sqrt{-cd^2 - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^2*d^3 + a*c*d*e^2)*x/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) + (a*c*d^2
*e + a^2*e^3)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))/sqrt(c*x^2 + a) - 2*
arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*
e^2/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2))
```

3.339 $\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{e(ae+cdx)}{ad\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{1}{ad\sqrt{a+cx^2}}$$

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (e*(a*e + c*d*x))/(a*d*(c*d^2 + a*e^2)*Sqrt[a + c
*x^2]) + (e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(d*(c*d^2 + a*e^2)^(3/2)) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rubi [A] time = 0.134966, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {961, 266, 51, 63, 208, 741, 12, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{e(ae+cdx)}{ad\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{1}{ad\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (e*(a*e + c*d*x))/(a*d*(c*d^2 + a*e^2)*Sqrt[a + c
*x^2]) + (e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(d*(c*d^2 + a*e^2)^(3/2)) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= \int \left(\frac{1}{dx\left(a+cx^2\right)^{3/2}} - \frac{e}{d(d+ex)\left(a+cx^2\right)^{3/2}}\right)dx\\
&= \frac{\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d} - \frac{e\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx}{d}\\
&= -\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx, x, x^2\right)}{2d} - \frac{e\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ad\left(cd^2+ae^2\right)}\\
&= \frac{1}{ad\sqrt{a+cx^2}} - \frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2ad} - \frac{e^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}}{d\left(cd^2+ae^2\right)}\\
&= \frac{1}{ad\sqrt{a+cx^2}} - \frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{acd} + \frac{e^3\,\text{Subst}}{}\\
&= \frac{1}{ad\sqrt{a+cx^2}} - \frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}
\end{aligned}$$

Mathematica [C] time = 0.191429, size = 132, normalized size = 0.9

$$\frac{-\frac{e(ae+cdx)}{a\sqrt{a+cx^2}(ae^2+cd^2)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((e*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (e^3*ArcTanh[(a
*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) +
 Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a]/(a*Sqrt[a + c*x^2]))/d
```

Maple [B] time = 0.236, size = 318, normalized size = 2.2

$$\frac{1}{ad}\frac{1}{\sqrt{cx^2+a}}-\frac{1}{d}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{e^2}{d\left(ae^2+cd^2\right)}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}}-\frac{cex}{\left(ae^2+cd^2\right)a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/a/d/(c*x^2+a)^(1/2)-1/d/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-1/d
/(a*e^2+c*d^2)*e^2/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-e/
(a*e^2+c*d^2)/a/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)*x*c+1
/d/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*
d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2
+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x), x)
```

Fricas [B] time = 4.00096, size = 2695, normalized size = 18.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*
d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log
(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2
*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4
*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^
2), 1/2*(2*(a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*
d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) +
 2*a)/x^2) + 2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*
sqrt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2), 1/2*(2*(a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-a)*arctan(
sqrt(-a)/sqrt(c*x^2 + a)) + (a^2*c*e^3*x^2 + a^3*e^3)*sqrt(c*d^2 + a*e^2)*l
og((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(
c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) +
2*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sqrt(c*x^2 +
a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d
^3*e^2 + a^4*c*d*e^4)*x^2), ((a^2*c*e^3*x^2 + a^3*e^3)*sqrt(-c*d^2 - a*e^2)
*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e
^2 + (c^2*d^2 + a*c*e^2)*x^2)) + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c
*x^2 + a)) + (a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x)*sq
rt(c*x^2 + a))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 +
2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.340 $\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2
]) - (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^
2*(c*d^2 + a*e^2)^(3/2)) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^
2)
```

Rubi [A] time = 0.166561, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {961, 271, 191, 266, 51, 63, 208, 741, 12, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2
]) - (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^
2*(c*d^2 + a*e^2)^(3/2)) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^
2)
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \left(\frac{1}{dx^2\left(a+cx^2\right)^{3/2}} - \frac{e}{d^2x\left(a+cx^2\right)^{3/2}} + \frac{e^2}{d^2(d+ex)\left(a+cx^2\right)^{3/2}}\right)dx$$

$$= \frac{\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}}\,dx}{d} - \frac{e\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx}{d^2}$$

$$= -\frac{1}{adx\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{(2c)\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{ad} - \frac{e\,\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx\right.}{2d^2}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e\,\text{Subst}\left(\int \right.}{2}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{e^4\tanh^{-1}\left(\right.}{d^2\left(cd^2\right.}$$

Mathematica [C] time = 0.447135, size = 163, normalized size = 0.84

$$-\frac{\frac{d\left(a+2cx^2\right)}{a^2x\sqrt{a+cx^2}} - \frac{e^2(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{e\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((-((e^2*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*(a + 2*c
*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Hypergeometric2F1[-1/2,
 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c*x^2]))/d^2)
```

Maple [B] time = 0.24, size = 363, normalized size = 1.9

$$-\frac{e}{ad^2}\frac{1}{\sqrt{cx^2+a}} + \frac{e}{d^2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)a^{-\frac{3}{2}} + \frac{e^3}{d^2\left(ae^2+cd^2\right)}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}} + \frac{e^2xc}{d\left(ae^2+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -e/a/d^2/(c*x^2+a)^(1/2)+e/d^2/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x
)+e^3/d^2/(a*e^2+c*d^2)/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/
2)+e^2/d/(a*e^2+c*d^2)/a/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2)*x*c-e^3/d^2/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)
/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+
x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/a/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(
```

```
c*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^2), x)
```

Fricas [B] time = 4.43378, size = 3152, normalized size = 16.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*
c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + ((c^3*d^4*e + 2*a*c^
2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*s
qrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c^2*d^5 +
 2*a^2*c*d^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*
x^2 + (a*c^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a
^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*
d^2*e^4)*x), -1/2*(2*(a^2*c*e^4*x^3 + a^3*e^4*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 +
 (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(
c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4
 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d
^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e
^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x), -1/2*(2*((c^3*d
^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 +
a^3*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^2*c*e^4*x^3 + a^
3*e^4*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^
2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))
/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4 +
(2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c^2*d^4*e + a^2*c*d^2*
e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)
*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x), -((a^2*c*e^4*x^3 +
 a^3*e^4*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + ((c^3*d^4*
e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^3 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3
*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a*c^2*d^5 + 2*a^2*c*d
^3*e^2 + a^3*d*e^4 + (2*c^3*d^5 + 3*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^2 + (a*c
^2*d^4*e + a^2*c*d^2*e^3)*x)*sqrt(c*x^2 + a))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4
*e^2 + a^4*c*d^2*e^4)*x^3 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a+cx^2\right)^{\frac{3}{2}} (d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.22314, size = 359, normalized size = 1.85

$$-\frac{\frac{(ac^3d^3+a^2c^2de^2)x}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}+\frac{a^2c^2d^2e+a^3ce^3}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}}{\sqrt{cx^2+a}}-\frac{2\,\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^4}{\left(cd^4+ad^2e^2\right)\sqrt{-cd^2-ae^2}}-\frac{2\,\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}ad^2}+\frac{2\cdots}{\left(\left(\sqrt{c}x-\sqrt{cx^2\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a*c^3*d^3 + a^2*c^2*d*e^2)*x/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4) +
 (a^2*c^2*d^2*e + a^3*c*e^3)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))/sqr
t(c*x^2 + a) - 2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(
-c*d^2 - a*e^2))*e^4/((c*d^4 + a*d^2*e^2)*sqrt(-c*d^2 - a*e^2)) - 2*arctan(
-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*a*d^2) + 2*sqrt(c)/(((
sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*a*d)
```

3.341 $\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=276

$$-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^2}{ad^3\sqrt{a+c}}$$

```
[Out] (-3*c)/(2*a^2*d*Sqrt[a + c*x^2]) + e^2/(a*d^3*Sqrt[a + c*x^2]) - 1/(2*a*d*x
^2*Sqrt[a + c*x^2]) + e/(a*d^2*x*Sqrt[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*Sqrt
[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a*d^3*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])
+ (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*(
c*d^2 + a*e^2)^(3/2)) + (3*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(5/2)*d
) - (e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^3)
```

Rubi [A] time = 0.235688, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {961, 266, 51, 63, 208, 271, 191, 741, 12, 725, 206}

$$-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{3\sqrt{a+cx^2}}{2a^2dx^2}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^2}{ad^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] e^2/(a*d^3*Sqrt[a + c*x^2]) + 1/(a*d*x^2*Sqrt[a + c*x^2]) + e/(a*d^2*x*Sqrt
[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*Sqrt[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a
*d^3*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (3*Sqrt[a + c*x^2])/(2*a^2*d*x^2) +
 (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*(c
*d^2 + a*e^2)^(3/2)) + (3*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(5/2)*d)
 - (e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^3)
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^{3/2}}dx = \int \left(\frac{1}{dx^3\left(a+cx^2\right)^{3/2}} - \frac{e}{d^2x^2\left(a+cx^2\right)^{3/2}} + \frac{e^2}{d^3x\left(a+cx^2\right)^{3/2}} - \frac{e^3}{d^3(d+ex)\left(a+cx^2\right)^{3/2}}\right)$$

$$= \frac{\int \frac{1}{x^3\left(a+cx^2\right)^{3/2}}dx}{d} - \frac{e\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}}dx}{d^2} + \frac{e^2\int \frac{1}{x\left(a+cx^2\right)^{3/2}}dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}dx}{d^3}$$

$$= \frac{e}{ad^2x\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x^2(a+cx)^{3/2}}dx, x, x^2\right)}{2d} + \frac{(2ce)\int}{a}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a}}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a}}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a}}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a}}$$

Mathematica [C] time = 0.382267, size = 203, normalized size = 0.74

$$\frac{-\frac{cd^2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a^2\sqrt{a+cx^2}} + \frac{de\left(a+2cx^2\right)}{a^2x\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)} + \frac{e^5\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{e^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{a}+1\right)}{a\sqrt{a+cx^2}}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((e^3*(a*e + c*d*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*(a + 2*c
*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a
*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e^2*Hypergeometric2F1[-1/
2, 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c*x^2]) - (c*d^2*Hypergeometric2F1[-
1/2, 2, 1/2, 1 + (c*x^2)/a])/(a^2*Sqrt[a + c*x^2]))/d^3
```

Maple [A] time = 0.247, size = 439, normalized size = 1.6

$$\frac{e^2}{ad^3}\frac{1}{\sqrt{cx^2+a}} - \frac{e^2}{d^3}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{cx^2+a}\right)\right)a^{-\frac{3}{2}} - \frac{e^4}{d^3\left(ae^2+cd^2\right)}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2+cd^2}{e^2}}} - \frac{e^3x}{d^2\left(ae^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] e^2/a/d^3/(c*x^2+a)^(1/2)-e^2/d^3/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2)
)/x)-e^4/d^3/(a*e^2+c*d^2)/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^
(1/2)-e^3/d^2/(a*e^2+c*d^2)/a/((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^
2)^(1/2)*x*c+e^4/d^3/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c
*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*
(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/2/a/d/x^2/(c*x^2+a)^(1/2)-3/2*
c/a^2/d/(c*x^2+a)^(1/2)+3/2/d*c/a^(5/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/
x)+e/a/d^2/x/(c*x^2+a)^(1/2)+2*c*e*x/a^2/d^2/(c*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^3), x)
```

Fricas [A] time = 6.96821, size = 3868, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
 - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(
c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - ((3*c^4*d^6 + 4*
a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c
^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c
*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*
e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3*d^
6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*d^3
*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2 + a
^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2), 1/4
*(4*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e
^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^6)*
x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)*sq
rt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a^2*c^2*d^6
+ 2*a^3*c*d^4*e^2 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^
3*c*d*e^5)*x^3 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*
(a^2*c^2*d^5*e + 2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3
*d^7 + 2*a^4*c^2*d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*
e^2 + a^6*d^3*e^4)*x^2), -1/2*(((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*
e^4 - 2*a^3*c*e^6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 -
 2*a^4*e^6)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^3*c*e^5*x^4
 + a^4*e^5*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2
- (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d
^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 + (3*a*c^3
```

```
*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e + 2*a^3*c*
d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*d^5*e^2
+ a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4)*x^2),
1/2*(2*(a^3*c*e^5*x^4 + a^4*e^5*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^
2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*
c*e^2)*x^2)) - ((3*c^4*d^6 + 4*a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 - 2*a^3*c*e^
6)*x^4 + (3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4 - 2*a^4*e^6)*x^2)
*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a^2*c^2*d^6 + 2*a^3*c*d^4*e^2
 + a^4*d^2*e^4 - 2*(2*a*c^3*d^5*e + 3*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^3 +
(3*a*c^3*d^6 + 4*a^2*c^2*d^4*e^2 + a^3*c*d^2*e^4)*x^2 - 2*(a^2*c^2*d^5*e +
2*a^3*c*d^3*e^3 + a^4*d*e^5)*x)*sqrt(c*x^2 + a))/((a^3*c^3*d^7 + 2*a^4*c^2*
d^5*e^2 + a^5*c*d^3*e^4)*x^4 + (a^4*c^2*d^7 + 2*a^5*c*d^5*e^2 + a^6*d^3*e^4
)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+cx^2\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A] time = 1.25382, size = 483, normalized size = 1.75

$$\frac{\frac{\left(a^2c^3d^2e+a^3c^2e^3\right)x}{a^4c^2d^4+2\,a^5cd^2e^2+a^6e^4}-\frac{a^2c^3d^3+a^3c^2de^2}{a^4c^2d^4+2\,a^5cd^2e^2+a^6e^4}}{\sqrt{cx^2+a}}-\frac{2\,\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^5}{\left(cd^5+ad^3e^2\right)\sqrt{-cd^2-ae^2}}-\frac{\left(3\,cd^2-2\,ae^2\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2d^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^2*c^3*d^2*e + a^3*c^2*e^3)*x/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4)
- (a^2*c^3*d^3 + a^3*c^2*d*e^2)/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4))/
sqrt(c*x^2 + a) - 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/s
qrt(-c*d^2 - a*e^2))*e^5/((c*d^5 + a*d^3*e^2)*sqrt(-c*d^2 - a*e^2)) - (3*c*
d^2 - 2*a*e^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^
2*d^3) + ((sqrt(c)*x - sqrt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a*sqrt(c)*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)
/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^2*a^2*d^2)
```

3.342 $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=244

$$\frac{\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)}{3c^2e^4}-\frac{d\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}-\frac{d^4\left(5ae^2+4cd^2\right)\tanh^{-1}\left(\frac{ae-cd\ldots}{\sqrt{a+cx^2}\sqrt{ae\ldots}}\right)}{e^5\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] ((13*c*d^2 - 2*a*e^2)*Sqrt[a + c*x^2])/(3*c^2*e^4) + (d^5*Sqrt[a + c*x^2])/
(e^4*(c*d^2 + a*e^2)*(d + e*x)) - (5*d*(d + e*x)*Sqrt[a + c*x^2])/(3*c*e^4)
 + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^4) - (d*(4*c*d^2 - a*e^2)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^5) - (d^4*(4*c*d^2 + 5*a*e^2)*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^5*(c*d^2 + a*e^
2)^(3/2))
```

Rubi [A] time = 0.889832, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(13cd^2-2ae^2\right)}{3c^2e^4}-\frac{d\left(4cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)\left(ae^2+cd^2\right)}-\frac{d^4\left(5ae^2+4cd^2\right)\tanh^{-1}\left(\frac{ae-cd\ldots}{\sqrt{a+cx^2}\sqrt{ae\ldots}}\right)}{e^5\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```



```
[Out] ((13*c*d^2 - 2*a*e^2)*Sqrt[a + c*x^2])/(3*c^2*e^4) + (d^5*Sqrt[a + c*x^2])/
(e^4*(c*d^2 + a*e^2)*(d + e*x)) - (5*d*(d + e*x)*Sqrt[a + c*x^2])/(3*c*e^4)
 + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^4) - (d*(4*c*d^2 - a*e^2)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^5) - (d^4*(4*c*d^2 + 5*a*e^2)*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^5*(c*d^2 + a*e^
2)^(3/2))
```


Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-\frac{ad^4}{e^3}+\frac{d^3\left(cd^2+ae^2\right)x}{e^4}-\frac{d^2\left(cd^2+ae^2\right)x^2}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^3-\frac{\left(cd^2+ae^2\right)x^4}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{\int \frac{-ad^2e\left(cd^2-2ae^2\right)+4d\left(cd^2+ae^2\right)^2x+2e\left(cd^2+ae^2\right)^2x^2+10\cdots}{(d+ex)\sqrt{a+cx^2}}}{3ce^4\left(cd^2+ae^2\right)}$$

$$= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{\int \frac{-6acd^2e^4\left(2cd^2+ae^2\right)-2c\cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+\cdots}}{3ce^4}$$

$$= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+\cdots}}{3ce^4}$$

$$= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+\cdots}}{3ce^4}$$

$$= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+\cdots}}{3ce^4}$$

Mathematica [A] time = 0.571505, size = 230, normalized size = 0.94

$$\frac{e\sqrt{a+cx^2}\left(-\frac{2ae^2}{c^2}+\frac{3d^5}{(d+ex)\left(ae^2+cd^2\right)}+\frac{9d^2-3dex+e^2x^2}{c}\right)-\frac{3d\left(4cd^2-ae^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}}-\frac{3d^4\left(5ae^2+4cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}+}{3e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*((-2*a*e^2)/c^2 + (3*d^5)/((c*d^2 + a*e^2)*(d + e*x)) +
(9*d^2 - 3*d*e*x + e^2*x^2)/c) + (3*d^4*(4*c*d^2 + 5*a*e^2)*Log[d + e*x])/(
c*d^2 + a*e^2)^(3/2) - (3*d*(4*c*d^2 - a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*
x^2]])/c^(3/2) - (3*d^4*(4*c*d^2 + 5*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 +
a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/(3*e^5)
```

Maple [B] time = 0.253, size = 474, normalized size = 1.9

$$\frac{x^2}{3\,ce^2}\sqrt{cx^2+a}-\frac{2\,a}{3\,c^2e^2}\sqrt{cx^2+a}-\frac{dx}{e^3c}\sqrt{cx^2+a}+\frac{ad}{e^3}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)c^{-\frac{3}{2}}+3\,\frac{d^2\sqrt{cx^2+a}}{e^4c}-4\,\frac{d^3\ln\left(x\sqrt{c}+\sqrt{cx^2+\ldots}\right.}{e^5\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3/e^2*x^2/c*(c*x^2+a)^(1/2)-2/3/e^2*a/c^2*(c*x^2+a)^(1/2)-d/e^3*x/c*(c*x^
2+a)^(1/2)+d/e^3*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))+3*d^2/e^4/c*(c*x^2
+a)^(1/2)-4*d^3/e^5*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-5/e^6*d^4/((a*e^2
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)
/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
+d^5/e^5/(a*e^2+c*d^2)/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e
^2)^(1/2)+d^6/e^6*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*
d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(
d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.343 $\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{(6cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}+\frac{d^3\left(4ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\left(ae^2+cd^2\right)^{3/2}}-\frac{5d\sqrt{a+cx^2}}{2ce^3}+\sqrt{a+}$$

```
[Out] (-5*d*Sqrt[a + c*x^2])/(2*c*e^3) - (d^4*Sqrt[a + c*x^2])/(e^3*(c*d^2 + a*e^
2)*(d + e*x)) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^3) + ((6*c*d^2 - a*e^2)*
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) + (d^3*(3*c*d^2 + 4*a
*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*(c
*d^2 + a*e^2)^(3/2))
```

Rubi [A] time = 0.523424, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{(6cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}+\frac{d^3\left(4ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\left(ae^2+cd^2\right)^{3/2}}-\frac{5d\sqrt{a+cx^2}}{2ce^3}+\sqrt{a+}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-5*d*Sqrt[a + c*x^2])/(2*c*e^3) - (d^4*Sqrt[a + c*x^2])/(e^3*(c*d^2 + a*e^
2)*(d + e*x)) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^3) + ((6*c*d^2 - a*e^2)*
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) + (d^3*(3*c*d^2 + 4*a
*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*(c
*d^2 + a*e^2)^(3/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\frac{ad^3}{e^2}-\frac{d^2\left(cd^2+ae^2\right)x}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^2-\frac{\left(cd^2+ae^2\right)x^3}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} - \frac{\int \frac{ade\left(3cd^2+ae^2\right)-\left(c^2d^4-a^2e^4\right)x+5cde\left(cd^2+ae^2\right)x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3\left(cd^2+ae^2\right)}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} - \frac{\int \frac{acde^3\left(3cd^2+ae^2\right)-ce^2\left(6cd^2-ae^2\right)(c}{(d+ex)\sqrt{a+cx^2}}}{2c^2e^5\left(cd^2+ae^2\right)}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2ce^4} - $$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\right.}{2ce^4}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right.}{2c^{3/2}e^4}$$

Mathematica [A] time = 0.429782, size = 208, normalized size = 1.02

$$\frac{\frac{\left(6cd^2-ae^2\right)\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}} + e\sqrt{a+cx^2}\left(\frac{ex-4d}{c}-\frac{2d^4}{(d+ex)\left(ae^2+cd^2\right)}\right) + \frac{2d^3\left(4ae^2+3cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{2d^3\left(4ae^2+3cd^2\right)\log}{\left(ae^2+cd^2\right)^{3/2}}}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*((-4*d + e*x)/c - (2*d^4)/((c*d^2 + a*e^2)*(d + e*x))) -
 (2*d^3*(3*c*d^2 + 4*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) + ((6*c*d^2
 - a*e^2)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/c^(3/2) + (2*d^3*(3*c*d^2 + 4
*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*
e^2)^(3/2))/(2*e^4)
```

Maple [B] time = 0.255, size = 435, normalized size = 2.1

$$\frac{x}{2\,ce^{2}}\sqrt{cx^{2}+a}-\frac{a}{2\,e^{2}}\ln\left(x\sqrt{c}+\sqrt{cx^{2}+a}\right)c^{-\frac{3}{2}}-2\,\frac{d\sqrt{cx^{2}+a}}{ce^{3}}+3\,\frac{d^{2}\ln\left(x\sqrt{c}+\sqrt{cx^{2}+a}\right)}{e^{4}\sqrt{c}}+4\,\frac{d^{3}}{e^{5}}\ln\left(\left(2\,\frac{ae^{2}+cd^{2}}{e^{2}}-2\,\frac{cd}{e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e^2*x/c*(c*x^2+a)^(1/2)-1/2/e^2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
-2*d*(c*x^2+a)^(1/2)/c/e^3+3*d^2/e^4*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+
4/e^5*d^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)
+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2
)^(1/2))/(d/e+x))-d^4/e^4/(a*e^2+c*d^2)/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x
)+(a*e^2+c*d^2)/e^2)^(1/2)-d^5/e^5*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2
)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e
+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\sqrt{a+cx^{2}}\,(d+ex)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.344 $\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=160

$$\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}-\frac{d^2\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^{3/2}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3}+\frac{\sqrt{a+cx^2}}{ce^2}$$

```
[Out] Sqrt[a + c*x^2]/(c*e^2) + (d^3*Sqrt[a + c*x^2])/(e^2*(c*d^2 + a*e^2)*(d + e
*x)) - (2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e^3) - (d^2*(2*c
*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])
])/(e^3*(c*d^2 + a*e^2)^(3/2))
```

Rubi [A] time = 0.326939, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1651, 1654, 844, 217, 206, 725}

$$\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}-\frac{d^2\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^{3/2}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3}+\frac{\sqrt{a+cx^2}}{ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e^2) + (d^3*Sqrt[a + c*x^2])/(e^2*(c*d^2 + a*e^2)*(d + e
*x)) - (2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e^3) - (d^2*(2*c
*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])
])/(e^3*(c*d^2 + a*e^2)^(3/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-\frac{ad^2}{e}+d\left(a+\frac{cd^2}{e^2}\right)x-\frac{\left(cd^2+ae^2\right)x^2}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-acd^2e+2cd\left(cd^2+ae^2\right)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2\left(cd^2+ae^2\right)}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{(2d)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^3} + \frac{\left(d^2\left(2cd^2+3ae^2\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}}{e^3\left(cd^2+ae^2\right)}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^3} - \left(d^2\left(2cd^2+3ae\right.\right.$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^3} - \frac{d^2\left(2cd^2+3ae^2\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{cd^2\cdots}}\right.}{e^3\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.317033, size = 184, normalized size = 1.15

$$\frac{e\sqrt{a+cx^2}\left(\frac{d^3}{(d+ex)\left(ae^2+cd^2\right)}+\frac{1}{c}\right)-\frac{d^2\left(3ae^2+2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{d^2\left(3ae^2+2cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{2d\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(c^(-1) + d^3/((c*d^2 + a*e^2)*(d + e*x))) + (d^2*(2*c*d
^2 + 3*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) - (2*d*Log[c*x + Sqrt[c]*
Sqrt[a + c*x^2]])/Sqrt[c] - (d^2*(2*c*d^2 + 3*a*e^2)*Log[a*e - c*d*x + Sqrt
```

```
[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/e^3
```

Maple [B] time = 0.247, size = 386, normalized size = 2.4

$$\frac{1}{ce^2}\sqrt{cx^2+a}-2\,\frac{d\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{e^3\sqrt{c}}-3\,\frac{d^2}{e^4}\ln\left(\left(2\,\frac{ae^2+cd^2}{e^2}-2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\,\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\,\frac{cd}{e}\left(\frac{d}{e}+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/e^2-2/e^3*d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-3/e^4*d
^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*
e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)
)/(d/e+x))+d^3/e^3/(a*e^2+c*d^2)/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^
2+c*d^2)/e^2)^(1/2)+d^4/e^4*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c
-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 51.1241, size = 2989, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3
 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 +
a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 -
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2)) + 2*(2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d
^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*
c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*d^5 + 3*
a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(
sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^
2*d^2 + a*c*e^2)*x^2)) - (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*
e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - (2*c^2*d^5*e + 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 +
2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5
```

```
 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), 1/2*(4*(c^
2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^
5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c^2*d^5 + 3*a*c*d^3*
e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x
 - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(2*c^2*d^5*e
+ 3*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sq
rt(c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4
+ 2*a*c^2*d^2*e^6 + a^2*c*e^8)*x), -((2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^
4*e + 3*a*c*d^2*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2))
- 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a
^2*d*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*d^5*e + 3
*a*c*d^3*e^3 + a^2*d*e^5 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x)*sqrt(
c*x^2 + a))/(c^3*d^5*e^3 + 2*a*c^2*d^3*e^5 + a^2*c*d*e^7 + (c^3*d^4*e^4 + 2
*a*c^2*d^2*e^6 + a^2*c*e^8)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.345 $\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}} \, dx$

Optimal. Leaf size=137

$$-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)}+\frac{d\left(2ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}$$

```
[Out] -((d^2*Sqrt[a + c*x^2])/(e*(c*d^2 + a*e^2)*(d + e*x))) + ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]]/(Sqrt[c]*e^2) + (d*(c*d^2 + 2*a*e^2)*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e^2)^(3/2))
```

Rubi [A] time = 0.168101, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1651, 844, 217, 206, 725}

$$-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)}+\frac{d\left(2ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((d^2*Sqrt[a + c*x^2])/(e*(c*d^2 + a*e^2)*(d + e*x))) + ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]]/(Sqrt[c]*e^2) + (d*(c*d^2 + 2*a*e^2)*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e^2)^(3/2))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{ad-\frac{\left(cd^2+ae^2\right)x}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2} - \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-}\right.}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2} + \frac{d\left(2a+\frac{cd^2}{e^2}\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.371877, size = 172, normalized size = 1.26

$$\frac{d\left(\frac{\left(2ae^2+cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{de\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}\right)-\frac{\left(2ade^2+cd^3\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}}+\frac{\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-(((c*d^3 + 2*a*d*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2)) + Log[c*x + Sq
rt[c]*Sqrt[a + c*x^2]]/Sqrt[c] + d*(-((d*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2
)*(d + e*x))) + ((c*d^2 + 2*a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sq
rt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2)))/e^2
```

Maple [B] time = 0.245, size = 368, normalized size = 2.7

$$\frac{1}{e^2}\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}+2\frac{d}{e^3}\ln\left(\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/e^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+2*d/e^3/((a*e^2+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/
e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-d^2/e^2/(a*e^2+
c*d^2)/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)-d^3/e^
3*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e
*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*
```

```
d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 46.1452, size = 2615, normalized size = 19.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a
^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + (c^2*d
^4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((
2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c
^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a
^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), 1/2*(2*(c^2*d^
4 + 2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arcta
n(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (
c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*
e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 +
2*a*c^2*d^3*e^4 + a^2*c*d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)
*x), -1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*
e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^4 +
2*a*c*d^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(c*d^2 + a*e^2)*log((2*a*c
*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a
*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^
4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*
d*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x), ((c^2*d^4 + 2*a*c*d
^2*e^2 + (c^2*d^3*e + 2*a*c*d*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d
^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a
*c*e^2)*x^2)) - (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d
^2*e^3 + a^2*e^5)*x)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^4
*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^5*e^2 + 2*a*c^2*d^3*e^4 + a^2*c*d
*e^6 + (c^3*d^4*e^3 + 2*a*c^2*d^2*e^5 + a^2*c*e^7)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + cx^2}\,(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.346 $\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rubi [A] time = 0.0384235, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {807, 725, 206}

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{(ae)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{(ae)\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$

$$= \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

Mathematica [A] time = 0.0569128, size = 90, normalized size = 1.

$$\frac{d\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} - \frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Maple [B] time = 0.247, size = 340, normalized size = 3.8

$$-\frac{1}{e^2}\ln\left(\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(\frac{d}{e}+x\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*
((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(
1/2))/(d/e+x))+d/e/(a*e^2+c*d^2)/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^
2+c*d^2)/e^2)^(1/2)+d^2/e^2*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c
-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.59371, size = 783, normalized size = 8.7

$$\left[\frac{\left(ae^{2}x+ade\right)\sqrt{cd^{2}+ae^{2}}\log\left(\frac{2\,acdex-acd^{2}-2\,a^{2}e^{2}-\left(2\,c^{2}d^{2}+ace^{2}\right)x^{2}-2\,\sqrt{cd^{2}+ae^{2}}(cdx-ae)\sqrt{cx^{2}+a}}{e^{2}x^{2}+2\,dex+d^{2}}\right)+2\left(cd^{3}+ade^{2}\right)\sqrt{cx^{2}+a}}{2\left(c^{2}d^{5}+2\,acd^{3}e^{2}+a^{2}de^{4}+\left(c^{2}d^{4}e+2\,acd^{2}e^{3}+a^{2}e^{5}\right)x\right)},-\frac{\left(ae^{2}x+a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*e^2*x + a*d*e)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((a*e^2*x + a*d*e)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) - (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^
2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+cx^{2}}\,(d+ex)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.347 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=91

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rubi [A] time = 0.0338812, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {731, 725, 206}

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{(cd)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$
$$= -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{(cd)\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$
$$= -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

Mathematica [A] time = 0.0819869, size = 115, normalized size = 1.26

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} - \frac{cd\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{(ae^2+cd^2)^{3/2}} + \frac{cd\log(d+ex)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + (c*d*Log[d + e*x])/(c*
d^2 + a*e^2)^(3/2) - (c*d*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2]])/(c*d^2 + a*e^2)^(3/2)
```

Maple [B] time = 0.303, size = 210, normalized size = 2.3

$$-\frac{1}{ae^2+cd^2}\sqrt{\left(\frac{d}{e}+x\right)^2 c - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2+cd^2}{e^2}}\left(\frac{d}{e}+x\right)^{-1} - \frac{cd}{e(ae^2+cd^2)}\ln\left(\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(\frac{d}{e}+x\right) + 2\sqrt{\frac{ae^2}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/(a*e^2+c*d^2)/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1
/2)-1/e*c*d/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2
-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(
a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56502, size = 783, normalized size = 8.6

$$\left[\frac{\left(cdex+cd^2\right)\sqrt{cd^2+ae^2}\log\left(\frac{2\,acdex-acd^2-2\,a^2e^2-\left(2\,c^2d^2+ace^2\right)x^2-2\,\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}}{e^2x^2+2\,dex+d^2}\right)-2\left(cd^2e+ae^3\right)\sqrt{cx^2+a}}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4+\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)x\right)},-\frac{\left(cdex\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x + c*d^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*
a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^2
 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
a^2*e^5)*x), -((c*d*e*x + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^
2)*x^2)) + (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^
2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [B] time = 4.84051, size = 439, normalized size = 4.82

$$-\frac{\sqrt{cd^2+ae^2}cde\log\left(\left|-\sqrt{cd^2+ae^2}cd+\left(cd^2+ae^2\right)\left(\sqrt{c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}}+\frac{\sqrt{cd^2e^2+ae^4}e^{(-1)}}{xe+d}\right)\right|\right)}{\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}+\frac{\left(c^{\frac{3}{2}}d^2+\sqrt{cd^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c*d^2 + a*e^2)*c*d*e*log(abs(-sqrt(c*d^2 + a*e^2)*c*d + (c*d^2 + a*e^
2)*(sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2) + sqr
t(c*d^2*e^2 + a*e^4)*e^(-1)/(x*e + d))))/((c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*
e^5)*sgn(1/(x*e + d))) + (c^(3/2)*d^2 + sqrt(c*d^2 + a*e^2)*c*d*log(abs(c^(
3/2)*d^2 - sqrt(c*d^2 + a*e^2)*c*d + a*sqrt(c)*e^2)) + a*sqrt(c)*e^2)*sgn(1
/(x*e + d))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - sqrt(c - 2*c*d/(x*e + d)
+ c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c*d^2*sgn(1/(x*e + d)) + a*e^2*sg
n(1/(x*e + d)))
```

3.348 $\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=179

$$\frac{e^2\sqrt{a+cx^2}}{d(d+ex)\left(ae^2+cd^2\right)}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

```
[Out] (e^2*Sqrt[a + c*x^2])/(d*(c*d^2 + a*e^2)*(d + e*x)) + (c*e*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Ar
cTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2
 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d^2)
```

Rubi [A] time = 0.137258, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {961, 266, 63, 208, 731, 725, 206}

$$\frac{e^2\sqrt{a+cx^2}}{d(d+ex)\left(ae^2+cd^2\right)}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```


```
[Out] (e^2*Sqrt[a + c*x^2])/(d*(c*d^2 + a*e^2)*(d + e*x)) + (c*e*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Ar
cTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2
 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d^2)
```


Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{d^2x\sqrt{a+cx^2}} - \frac{e}{d(d+ex)^2\sqrt{a+cx^2}} - \frac{e}{d^2(d+ex)\sqrt{a+cx^2}}\right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2} - \frac{e\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{d} \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} + \frac{e\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d^2} \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} + \\
&= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}} + \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} - \tanh^{-}
\end{aligned}
$$

Mathematica [A] time = 0.251733, size = 178, normalized size = 0.99

$$
\frac{\frac{de^2\sqrt{a+cx^2}}{(d+ex)\left(ae^2+cd^2\right)} + \frac{e\left(ae^2+2cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{e\left(ae^2+2cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} - \frac{\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{\sqrt{a}} + \frac{\log(x)}{\sqrt{a}}}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((d*e^2*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) + Log[x]/Sqrt[a] - (e*
(2*c*d^2 + a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) - Log[a + Sqrt[a]*Sqr
t[a + c*x^2]]/Sqrt[a] + (e*(2*c*d^2 + a*e^2)*Log[a*e - c*d*x + Sqrt[c*d^2 +
 a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/d^2
```

Maple [B] time = 0.256, size = 364, normalized size = 2.

$$-\frac{1}{d^2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{1}{d^2}\ln\left(\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)\cdot\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/d^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+1/d^2/((a*e^2+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/
2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+e/d/(a*e
^2+c*d^2)/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2)+c/(
a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e
+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2))/(d/e+x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^2+a}(ex+d)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x), x)
```

Fricas [A] time = 4.89092, size = 2619, normalized size = 14.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((2*a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 +
a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 +
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x +
d^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 +
 a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) +
2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 +
 a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), 1/2*(2*(2*
a*c*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 - a*e^2)*a
rctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2
 + (c^2*d^2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*
d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*
d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*
d^2*e^5)*x), 1/2*(2*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*c
```

```
*d^3*e + a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(c*d^2 + a*e^2)*log((
2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^
2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(a
*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3
*d^3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x), ((2*a*c*d^3*e
+ a^2*d*e^3 + (2*a*c*d^2*e^2 + a^2*e^4)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^
2 + a*c*e^2)*x^2)) + (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*
a*c*d^2*e^3 + a^2*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (a*c*
d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^
3*e^4 + (a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.349 $\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=212

$$-\frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)\left(ae^2+cd^2\right)}-\frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}-\frac{\sqrt{a+cx^2}}{ad^2x}$$

```
[Out] -(Sqrt[a + c*x^2]/(a*d^2*x)) - (e^3*Sqrt[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*(
d + e*x)) - (c*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^
2])])/(d*(c*d^2 + a*e^2)^(3/2)) - (2*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*e^2]) + (2*e*ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rubi [A] time = 0.16785, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {961, 264, 266, 63, 208, 731, 725, 206}

$$-\frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)\left(ae^2+cd^2\right)}-\frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}-\frac{\sqrt{a+cx^2}}{ad^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```


```
[Out] -(Sqrt[a + c*x^2]/(a*d^2*x)) - (e^3*Sqrt[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*(
d + e*x)) - (c*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^
2])])/(d*(c*d^2 + a*e^2)^(3/2)) - (2*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*e^2]) + (2*e*ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```


Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{d^2x^2\sqrt{a+cx^2}} - \frac{2e}{d^3x\sqrt{a+cx^2}} + \frac{e^2}{d^2(d+ex)^2\sqrt{a+cx^2}} + \frac{2e^2}{d^3(d+ex)\sqrt{a+cx^2}}\right)dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} + \frac{\left(2e^2\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3} + \frac{e^2\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{d^3} - \frac{\left(2e^2\right)\mathrm{Subst}\left(\int \frac{}{cd}\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{c}+}\right.}{c}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}} - \frac{2e^2\tanh^{-1}\left(\frac{ae}{\sqrt{cd^2+}}\right.}{d^3\sqrt{cd^2+a}}$$

Mathematica [A] time = 0.389819, size = 197, normalized size = 0.93

$$\frac{-d\sqrt{a+cx^2}\left(\frac{e^3}{(d+ex)\left(ae^2+cd^2\right)}+\frac{1}{ax}\right) - \frac{e^2\left(2ae^2+3cd^2\right)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{e^2\left(2ae^2+3cd^2\right)\log(d+ex)}{\left(ae^2+cd^2\right)^{3/2}} + \frac{2e\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{\sqrt{a}} - 2e}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-(d*Sqrt[a + c*x^2]*(1/(a*x) + e^3/((c*d^2 + a*e^2)*(d + e*x)))) - (2*e*Lo
g[x])/Sqrt[a] + (e^2*(3*c*d^2 + 2*a*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2
) + (2*e*Log[a + Sqrt[a]*Sqrt[a + c*x^2]])/Sqrt[a] - (e^2*(3*c*d^2 + 2*a*e^
2)*Log[a*e - c*d*x + Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^
(3/2))/d^3
```

Maple [B] time = 0.238, size = 395, normalized size = 1.9

$$2\frac{e}{d^3\sqrt{a}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{cx^2+a}}{x}\right)-2\frac{e}{d^3}\ln\left(\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c-2\frac{cd}{e}\left(\frac{d}{e}+x\right)+\frac{a}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] 2/d^3*e/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-2*e/d^3/((a*e^2+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^
(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/d^2
/(a*e^2+c*d^2)*e^2/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^
(1/2)-1/d*c*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e
^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)
+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-(c*x^2+a)^(1/2)/a/d^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^2), x)
```

Fricas [A] time = 4.6874, size = 3093, normalized size = 14.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2
 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*
c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2
+ 2*d*e*x + d^2)) + 2*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d
^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5
```

```
*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c^2*d^7*e + 2*a^2
*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^2 + a^3*d^4*e^4)
*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e
^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
 a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^2*d^4*e^2 + 2*a*c*
d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(a)
*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + (c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^
2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2
*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x), -1/2*(4*((c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a
^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)*sqrt(-a)*arctan(sq
rt(-a)/sqrt(c*x^2 + a)) - sqrt(c*d^2 + a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*
x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^
2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^2 + a))/((a*c
^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2*a^2*c*d^6*e^
2 + a^3*d^4*e^4)*x), -(sqrt(-c*d^2 - a*e^2)*((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^
2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) + 2*((c^
2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2
*d*e^5)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (c^2*d^6 + 2*a*c*d^4
*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 3*a*c*d^3*e^3 + 2*a^2*d*e^5)*x)*sqrt(c*x^
2 + a))/((a*c^2*d^7*e + 2*a^2*c*d^5*e^3 + a^3*d^3*e^5)*x^2 + (a*c^2*d^8 + 2
*a^2*c*d^6*e^2 + a^3*d^4*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.350 $\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=268

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}+\frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)\left(ae^2+cd^2\right)}+\frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}}+\frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{3e^2\tanh^{-1}\left(\cdots\right.}{\sqrt{a}d^4}$$

```
[Out] -Sqrt[a + c*x^2]/(2*a*d^2*x^2) + (2*e*Sqrt[a + c*x^2])/(a*d^3*x) + (e^4*Sqr
t[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2)) + (3*
e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^4*Sqrt
[c*d^2 + a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d^2) - (
3*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^4)
```

Rubi [A] time = 0.222282, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {961, 266, 51, 63, 208, 264, 731, 725, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}+\frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)\left(ae^2+cd^2\right)}+\frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}}+\frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{3e^2\tanh^{-1}\left(\cdots\right.}{\sqrt{a}d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d^2*x^2) + (2*e*Sqrt[a + c*x^2])/(a*d^3*x) + (e^4*Sqr
t[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2)) + (3*
e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^4*Sqrt
[c*d^2 + a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d^2) - (
3*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^4)
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}dx = \int \left(\frac{1}{d^2x^3\sqrt{a+cx^2}} - \frac{2e}{d^3x^2\sqrt{a+cx^2}} + \frac{3e^2}{d^4x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)^2\sqrt{a+cx^2}} - \frac{3e^3}{d^4(d+ex)\sqrt{a}}\right.$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}dx}{d^2} - \frac{(2e)\int \frac{1}{x^2\sqrt{a+cx^2}}dx}{d^3} + \frac{(3e^2)\int \frac{1}{x\sqrt{a+cx^2}}dx}{d^4} - \frac{(3e^3)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}dx}{d^4} - \frac{e^3\int \frac{}{(d+}}{}$$

$$= \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}dx, x, x^2\right)}{2d^2} + \frac{(3e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+}}\right.}{2d^4}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4\sqrt{cd^2+ae^2}} - \frac{c\,\text{Subs}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2(cd^2+ae^2)^{3/2}} + \frac{3e^3\text{ta}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2(cd^2+ae^2)^{3/2}} + \frac{3e^3\text{ta}}{}$$

Mathematica [A] time = 0.530588, size = 229, normalized size = 0.85

$$\frac{\frac{(cd^2-6ae^2)\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{a^{3/2}} + \frac{\log(x)(6ae^2-cd^2)}{a^{3/2}} + d\sqrt{a+cx^2}\left(\frac{2e^4}{(d+ex)(ae^2+cd^2)} - \frac{d-4ex}{ax^2}\right) + \frac{2e^3(3ae^2+4cd^2)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{(ae^2+cd^2)^{3/2}} - 2e^3}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2]*(-((d - 4*e*x)/(a*x^2)) + (2*e^4)/((c*d^2 + a*e^2)*(d +
e*x))) + ((-(c*d^2) + 6*a*e^2)*Log[x])/a^(3/2) - (2*e^3*(4*c*d^2 + 3*a*e^2)
*Log[d + e*x])/(c*d^2 + a*e^2)^(3/2) + ((c*d^2 - 6*a*e^2)*Log[a + Sqrt[a]*S
qrt[a + c*x^2]])/a^(3/2) + (2*e^3*(4*c*d^2 + 3*a*e^2)*Log[a*e - c*d*x + Sqr
t[c*d^2 + a*e^2]*Sqrt[a + c*x^2]])/(c*d^2 + a*e^2)^(3/2))/(2*d^4)
```

Maple [A] time = 0.249, size = 452, normalized size = 1.7

$$-3\,\frac{e^2}{d^4\sqrt{a}}\ln\left(\frac{2\,a+2\,\sqrt{a}\sqrt{cx^2+a}}{x}\right) + 3\,\frac{e^2}{d^4}\ln\left(\left(2\,\frac{ae^2+cd^2}{e^2} - 2\,\frac{cd}{e}\left(\frac{d}{e}+x\right) + 2\,\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c - 2\,\frac{cd}{e}\left(\frac{d}{e}+x\right)+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] -3/d^4*e^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+3*e^2/d^4/((a*e^2+
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/
e^2)^(1/2)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))+
1/d^3*e^3/(a*e^2+c*d^2)/(d/e+x)*((d/e+x)^2*c-2*c*d/e*(d/e+x)+(a*e^2+c*d^2)/
e^2)^(1/2)+1/d^2*e^2*c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
+c*d^2)/e^2-2*c*d/e*(d/e+x)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c-2*c*d/
```

```
e*(d/e+x)+(a*e^2+c*d^2)/e^2)^(1/2))/(d/e+x))-1/2*(c*x^2+a)^(1/2)/a/d^2/x^2+
1/2/d^2*c/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+2*e*(c*x^2+a)^(1/2)
/a/d^3/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^3), x)
```

Fricas [A] time = 9.00832, size = 3771, normalized size = 14.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5
)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*
d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(
e^2*x^2 + 2*d*e*x + d^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^
5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*
d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*
d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5
)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3
+ (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), 1/4*(4*((4*a^2*c*d^2*
e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt(-c*d^2 - a
*e^2)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 +
a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^
2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^
4 - 6*a^3*d*e^6)*x^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2
+ 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a
^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4
*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*e^4)*x^2), -1/2*(((c^3
*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c^3*d^7 - 4
*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(-a)*arctan(sqrt(
-a)/sqrt(c*x^2 + a)) - ((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d^3*e^
3 + 3*a^3*d*e^5)*x^2)*sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^
2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqr
t(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a
^3*d^3*e^4 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a
*c^2*d^6*e + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d
^8*e + 2*a^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2
+ a^4*d^5*e^4)*x^2), 1/2*(2*((4*a^2*c*d^2*e^4 + 3*a^3*e^6)*x^3 + (4*a^2*c*d
^3*e^3 + 3*a^3*d*e^5)*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c*d^2 - a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a)/(a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2
)) - ((c^3*d^6*e - 4*a*c^2*d^4*e^3 - 11*a^2*c*d^2*e^5 - 6*a^3*e^7)*x^3 + (c
```

```
^3*d^7 - 4*a*c^2*d^5*e^2 - 11*a^2*c*d^3*e^4 - 6*a^3*d*e^6)*x^2)*sqrt(-a)*ar
ctan(sqrt(-a)/sqrt(c*x^2 + a)) - (a*c^2*d^7 + 2*a^2*c*d^5*e^2 + a^3*d^3*e^4
 - 2*(2*a*c^2*d^5*e^2 + 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - 3*(a*c^2*d^6*e
 + 2*a^2*c*d^4*e^3 + a^3*d^2*e^5)*x)*sqrt(c*x^2 + a))/((a^2*c^2*d^8*e + 2*a
^3*c*d^6*e^3 + a^4*d^4*e^5)*x^3 + (a^2*c^2*d^9 + 2*a^3*c*d^7*e^2 + a^4*d^5*
e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.351 $\int x^2(a+bx)^n\left(c+dx^2\right)dx$

Optimal. Leaf size=135

$$\frac{a^2\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^5(n+1)}-\frac{2a\left(2a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{\left(6a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

```
[Out] (a^2*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (2*a*(b^2*c + 2*a^2
*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + ((b^2*c + 6*a^2*d)*(a + b*x)^(3 + n)
)/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5
 + n))/(b^5*(5 + n))
```

Rubi [A] time = 0.0815451, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {948}

$$\frac{a^2\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^5(n+1)}-\frac{2a\left(2a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{\left(6a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] (a^2*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (2*a*(b^2*c + 2*a^2
*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + ((b^2*c + 6*a^2*d)*(a + b*x)^(3 + n)
)/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5
 + n))/(b^5*(5 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)dx&=\int\left(\frac{\left(a^2b^2c+a^4d\right)(a+bx)^n}{b^4}-\frac{2\left(ab^2c+2a^3d\right)(a+bx)^{1+n}}{b^4}+\frac{\left(b^2c+6a^2d\right)(a+bx)^{2+n}}{b^4}-\right.\\&=\frac{a^2\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^5(1+n)}-\frac{2a\left(b^2c+2a^2d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{\left(b^2c+6a^2d\right)(a+bx)^{3+n}}{b^5(3+n)}-\end{aligned}$$

Mathematica [A] time = 0.100699, size = 114, normalized size = 0.84

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)^2\left(6a^2d+b^2c\right)}{n+3}-\frac{2a(a+bx)\left(2a^2d+b^2c\right)}{n+2}+\frac{a^2b^2c+a^4d}{n+1}+\frac{d(a+bx)^4}{n+5}-\frac{4ad(a+bx)^3}{n+4}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^2*c + a^4*d)/(1 + n) - (2*a*(b^2*c + 2*a^2*d)*(a
 + b*x))/(2 + n) + ((b^2*c + 6*a^2*d)*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*
x)^3)/(4 + n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

Maple [B] time = 0.049, size = 328, normalized size = 2.4

$$\frac{(bx+a)^{1+n}\left(b^4dn^4x^4+10\,b^4dn^3x^4-4\,ab^3dn^3x^3+b^4cn^4x^2+35\,b^4dn^2x^4-24\,ab^3dn^2x^3+12\,b^4cn^3x^2+50\,b^4dnx^4+12\,a^2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c),x)
```

```
[Out] (b*x+a)^(1+n)*(b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+b^4*c*n^4*x
^2+35*b^4*d*n^2*x^4-24*a*b^3*d*n^2*x^3+12*b^4*c*n^3*x^2+50*b^4*d*n*x^4+12*a
^2*b^2*d*n^2*x^2-2*a*b^3*c*n^3*x-44*a*b^3*d*n*x^3+49*b^4*c*n^2*x^2+24*b^4*d
*x^4+36*a^2*b^2*d*n*x^2-20*a*b^3*c*n^2*x-24*a*b^3*d*x^3+78*b^4*c*n*x^2-24*a
^3*b*d*n*x+2*a^2*b^2*c*n^2+24*a^2*b^2*d*x^2-58*a*b^3*c*n*x+40*b^4*c*x^2-24*
a^3*b*d*x+18*a^2*b^2*c*n-40*a*b^3*c*x+24*a^4*d+40*a^2*b^2*c)/b^5/(n^5+15*n^
4+85*n^3+225*n^2+274*n+120)
```

Maxima [A] time = 1.0251, size = 284, normalized size = 2.1

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^nc}{\left(n^3+6\,n^2+11\,n+6\right)b^3}+\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\left(n^4+6\,n^3+\cdots\right.\right.}{\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n +
24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n
)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)
^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [B] time = 1.93335, size = 786, normalized size = 5.82

$$\frac{\left(2\,a^3b^2cn^2+18\,a^3b^2cn+40\,a^3b^2c+24\,a^5d+\left(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24\,b^5d\right)x^5+\left(ab^4dn^4+6\,ab^4dn^3\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^2*c*n^2 + 18*a^3*b^2*c*n + 40*a^3*b^2*c + 24*a^5*d + (b^5*d*n^4 +
10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5 + (a*b^4*d*n^4 + 6
*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + (b^5*c*n^4 + 40*b^5*c +
4*(3*b^5*c - a^2*b^3*d)*n^3 + (49*b^5*c - 12*a^2*b^3*d)*n^2 + 2*(39*b^5*c -
 4*a^2*b^3*d)*n)*x^3 + (a*b^4*c*n^4 + 10*a*b^4*c*n^3 + (29*a*b^4*c + 12*a^3
*b^2*d)*n^2 + 4*(5*a*b^4*c + 3*a^3*b^2*d)*n)*x^2 - 2*(a^2*b^3*c*n^3 + 9*a^2
```

```
*b^3*c*n^2 + 4*(5*a^2*b^3*c + 3*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^
5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [A] time = 10.4669, size = 4078, normalized size = 30.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**5/5), Eq(b, 0)), (12*a**6*d*log(a/b + x)/(
12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*
a**2*b**9*x**4) + 7*a**6*d/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x*
*2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 48*a**5*b*d*x*log(a/b + x)/(1
2*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a
**2*b**9*x**4) + 16*a**5*b*d*x/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**
7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 72*a**4*b**2*d*x**2*log(a
/b + x)/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x
**3 + 12*a**2*b**9*x**4) + 48*a**3*b**3*d*x**3*log(a/b + x)/(12*a**6*b**5 +
 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4
) - 24*a**3*b**3*d*x**3/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2
+ 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 12*a**2*b**4*d*x**4*log(a/b + x)
/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 1
2*a**2*b**9*x**4) - 18*a**2*b**4*d*x**4/(12*a**6*b**5 + 48*a**5*b**6*x + 72
*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 4*a*b**5*c*x**3/
(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12
*a**2*b**9*x**4) + b**6*c*x**4/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**
7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4), Eq(n, -5)), (-12*a**5*d*lo
g(a/b + x)/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**3)
 - 10*a**5*d/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**
3) - 36*a**4*b*d*x*log(a/b + x)/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*
x**2 + 3*a*b**8*x**3) - 18*a**4*b*d*x/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2
*b**7*x**2 + 3*a*b**8*x**3) - 36*a**3*b**2*d*x**2*log(a/b + x)/(3*a**4*b**5
 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**3) - 12*a**2*b**3*d*x**3*
log(a/b + x)/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**
3) + 12*a**2*b**3*d*x**3/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 +
3*a*b**8*x**3) + 3*a*b**4*d*x**4/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7
*x**2 + 3*a*b**8*x**3) + b**5*c*x**3/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*
b**7*x**2 + 3*a*b**8*x**3), Eq(n, -4)), (12*a**4*d*log(a/b + x)/(2*a**2*b**
5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2*b**5 + 4*a*b**6*x + 2*b**
7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**
2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 2*a**2*b**2*c
*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 3*a**2*b**2*c/(2*a
**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b**2*d*x**2*log(a/b + x)/(2*
a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*c*x*log(a/b + x)/(2*a**2*b
**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*c*x/(2*a**2*b**5 + 4*a*b**6*x +
2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 2
*b**4*c*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + b**4*d
*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -3)), (-12*a**4*d*log
(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*b**5 + 3*b**6*x) - 12*a**3
*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**2*b**2*c*log(a/b + x)/(3*a
*b**5 + 3*b**6*x) - 6*a**2*b**2*c/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d*x**
2/(3*a*b**5 + 3*b**6*x) - 6*a*b**3*c*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) -
 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x) + 3*b**4*c*x**2/(3*a*b**5 + 3*b**6*x
) + b**4*d*x**4/(3*a*b**5 + 3*b**6*x), Eq(n, -2)), (a**4*d*log(a/b + x)/b**
5 - a**3*d*x/b**4 + a**2*c*log(a/b + x)/b**3 + a**2*d*x**2/(2*b**3) - a*c*x
```

```
/b**2 - a*d*x**3/(3*b**2) + c*x**2/(2*b) + d*x**4/(4*b), Eq(n, -1)), (24*a*
*5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 2*a**3*b**
2*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n
**2 + 274*b**5*n + 120*b**5) + 18*a**3*b**2*c*n*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 40*a*
*3*b**2*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5
) + 12*a**3*b**2*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 2*a**2*b**3*c*n**3*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) - 18*a**2*b**3*c*n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 40*a**2*b**3*c*
n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3*(a + b*x)**n/(b**5*n**5 +
 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*
a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*d*n*x**3*(a + b*x)*
*n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
120*b**5) + a*b**4*c*n**4*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 10*a*b**4*c*n**3*x**2*
(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274
*b**5*n + 120*b**5) + 29*a*b**4*c*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 20*a*b**4
*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**4*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*
a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**4*d*n**2*x**4*(a + b*x)**n
/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 12
0*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**
5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*c*n**4*x**3*(a + b*x
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
+ 120*b**5) + 12*b**5*c*n**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 49*b**5*c*n**2*x**3
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27
4*b**5*n + 120*b**5) + 78*b**5*c*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n
**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 40*b**5*c*x**
3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d*n*
*3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5*(a + b*x)**n/(b**5*n**5 +
 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 50*
b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b
**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**5*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), Tru
e))
```

Giac [B] time = 1.23197, size = 842, normalized size = 6.24

$$\frac{(bx+a)^n b^5 dn^4 x^5 + (bx+a)^n ab^4 dn^4 x^4 + 10\,(bx+a)^n b^5 dn^3 x^5 + (bx+a)^n b^5 cn^4 x^3 + 6\,(bx+a)^n ab^4 dn^3 x^4 + 35\,(bx+a)^n b^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + (b*x + a)^n*b^5*c*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 +
35*(b*x + a)^n*b^5*d*n^2*x^5 + (b*x + a)^n*a*b^4*c*n^4*x^2 + 12*(b*x + a)^n
*b^5*c*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x + a)^n*a*b^4*d*n
^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + 10*(b*x + a)^n*a*b^4*c*n^3*x^2 + 49*(
b*x + a)^n*b^5*c*n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b*x + a)^n
*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 - 2*(b*x + a)^n*a^2*b^3*c*n^3*x +
 29*(b*x + a)^n*a*b^4*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 + 78*(b*
x + a)^n*b^5*c*n*x^3 - 8*(b*x + a)^n*a^2*b^3*d*n*x^3 - 18*(b*x + a)^n*a^2*b
^3*c*n^2*x + 20*(b*x + a)^n*a*b^4*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^2
+ 40*(b*x + a)^n*b^5*c*x^3 + 2*(b*x + a)^n*a^3*b^2*c*n^2 - 40*(b*x + a)^n*a
^2*b^3*c*n*x - 24*(b*x + a)^n*a^4*b*d*n*x + 18*(b*x + a)^n*a^3*b^2*c*n + 40
*(b*x + a)^n*a^3*b^2*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*b
^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.352 $\int x(a+bx)^n \left(c+dx^2\right) dx$

Optimal. Leaf size=102

$$-\frac{a\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

```
[Out] -((a*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + ((b^2*c + 3*a^2*d)
*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n))
 + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

Rubi [A] time = 0.052293, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {772}

$$-\frac{a\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] -((a*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + ((b^2*c + 3*a^2*d)
*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n))
 + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\left(c+dx^2\right)dx &= \int\left(\frac{a\left(-b^2c-a^2d\right)(a+bx)^n}{b^3}+\frac{\left(b^2c+3a^2d\right)(a+bx)^{1+n}}{b^3}-\frac{3ad(a+bx)^{2+n}}{b^3}+\frac{d(a+bx)^{3+n}}{b^3}\right)\\ &= -\frac{a\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)}+\frac{\left(b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.118785, size = 109, normalized size = 1.07

$$\frac{(a+bx)^{n+1}\left(6a^2bd(n+1)x-6a^3d-ab^2\left(c\left(n^2+7n+12\right)+3d\left(n^2+3n+2\right)x^2\right)+b^3\left(n^2+4n+3\right)x\left(c(n+4)+d(n+\right.\right.}{b^4(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(-6*a^3*d + 6*a^2*b*d*(1 + n)*x + b^3*(3 + 4*n + n^2)*x*
(c*(4 + n) + d*(2 + n)*x^2) - a*b^2*(c*(12 + 7*n + n^2) + 3*d*(2 + 3*n + n^
2)*x^2)))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [A] time = 0.046, size = 195, normalized size = 1.9

$$-\frac{(bx+a)^{1+n}\left(-b^3dn^3x^3-6\,b^3dn^2x^3+3\,ab^2dn^2x^2-b^3cn^3x-11\,b^3dnx^3+9\,ab^2dnx^2-8\,b^3cn^2x-6\,dx^3b^3-6\,a^2bdnx\right.}{b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c),x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-b^3*c*n^3*
x-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-8*b^3*c*n^2*x-6*b^3*d*x^3-6*a^2*b*d*n*x+a*
b^2*c*n^2+6*a*b^2*d*x^2-19*b^3*c*n*x-6*a^2*b*d*x+7*a*b^2*c*n-12*b^3*c*x+6*a
^3*d+12*a*b^2*c)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A] time = 1.02795, size = 197, normalized size = 1.93

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n c}{\left(n^2+3\,n+2\right)b^2}+\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a\right.}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n
)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2
+ 50*n + 24)*b^4)
```

Fricas [B] time = 1.88853, size = 516, normalized size = 5.06

$$-\frac{\left(a^2b^2cn^2+7\,a^2b^2cn+12\,a^2b^2c+6\,a^4d-\left(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d\right)x^4-\left(ab^3dn^3+3\,ab^3dn^2+2\,ab^3dn\right)x^3\right.}{b^4n^4+10\,b^4n^3+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^2*c*n^2 + 7*a^2*b^2*c*n + 12*a^2*b^2*c + 6*a^4*d - (b^4*d*n^3 + 6*b
^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 - (a*b^3*d*n^3 + 3*a*b^3*d*n^2 + 2*a*b
^3*d*n)*x^3 - (b^4*c*n^3 + 12*b^4*c + (8*b^4*c - 3*a^2*b^2*d)*n^2 + (19*b^4
*c - 3*a^2*b^2*d)*n)*x^2 - (a*b^3*c*n^3 + 7*a*b^3*c*n^2 + 6*(2*a*b^3*c + a^
3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24
*b^4)
```

Sympy [A] time = 4.73363, size = 2179, normalized size = 21.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6
*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 5*a**3*d/(6*a
**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*l
og(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 9*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
 - a*b**2*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x*
*2 + 6*b**7*x**3) - 3*b**3*c*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x*
*2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*
x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*x**3/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*b*
*2*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*x**2*log(a/b + x
)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*c*x/(2*a**2*b**4 + 4*a*
b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x*
*2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d/(2
*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a
*b**2*c*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c/(2*a*b**4 + 2*b**5*
x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*x*log(a/b + x)/(2*a*b
**4 + 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*d*l
og(a/b + x)/b**4 + a**2*d*x/b**3 - a*c*log(a/b + x)/b**2 - a*d*x**2/(2*b**2
) + c*x/b + d*x**3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a + b*
x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - a**
2*b**2*c*n**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b*
*4*n + 24*b**4) - 7*a**2*b**2*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
35*b**4*n**2 + 50*b**4*n + 24*b**4) - 12*a**2*b**2*c*(a + b*x)**n/(b**4*n**
4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n**2
*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*
b**4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*x*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 7*a*b**3*c*n**2*x*(a
 + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
+ 12*a*b**3*c*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*
n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(a + b*x)
**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b
**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**
4*n + 24*b**4) + b**4*c*n**3*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
35*b**4*n**2 + 50*b**4*n + 24*b**4) + 8*b**4*c*n**2*x**2*(a + b*x)**n/(b**4
*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 19*b**4*c*n*x*
*2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b
**4) + 12*b**4*c*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
 + 50*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**
4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a + b*x)
**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b*
*4*d*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4
*n + 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b
**4*n**2 + 50*b**4*n + 24*b**4), True))
```

Giac [B] time = 1.13034, size = 554, normalized size = 5.43

$$\frac{(bx+a)^n b^4 d n^3 x^4 + (bx+a)^n a b^3 d n^3 x^3 + 6(bx+a)^n b^4 d n^2 x^4 + (bx+a)^n b^4 c n^3 x^2 + 3(bx+a)^n a b^3 d n^2 x^3 + 11(bx+a)^n b^4 d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + (b*x + a)^n*b^4*c*n^3*x^2 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 1
1*(b*x + a)^n*b^4*d*n*x^4 + (b*x + a)^n*a*b^3*c*n^3*x + 8*(b*x + a)^n*b^4*c
*n^2*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a*b^3*d*n*x^3 +
6*(b*x + a)^n*b^4*d*x^4 + 7*(b*x + a)^n*a*b^3*c*n^2*x + 19*(b*x + a)^n*b^4*
c*n*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 - (b*x + a)^n*a^2*b^2*c*n^2 + 12*(b
*x + a)^n*a*b^3*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 12*(b*x + a)^n*b^4*c*x^
2 - 7*(b*x + a)^n*a^2*b^2*c*n - 12*(b*x + a)^n*a^2*b^2*c - 6*(b*x + a)^n*a^
4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.353 $\int (a+bx)^n \left(c+dx^2\right) dx$

Optimal. Leaf size=70

$$\frac{\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

```
[Out] ((b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n
))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n))
```

Rubi [A] time = 0.031032, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$\frac{\left(a^2d+b^2c\right)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n
))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n \left(c+dx^2\right) dx &= \int \left(\frac{\left(b^2c+a^2d\right)(a+bx)^n}{b^2}-\frac{2ad(a+bx)^{1+n}}{b^2}+\frac{d(a+bx)^{2+n}}{b^2}\right) dx\\ &= \frac{\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0470745, size = 65, normalized size = 0.93

$$\frac{(a+bx)^{n+1}\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) +
d*(1 + n)*x^2)))/(b^3*(1 + n)*(2 + n)*(3 + n))
```

Maple [A] time = 0.049, size = 100, normalized size = 1.4

$$\frac{(bx+a)^{1+n}\left(b^2dn^2x^2+3\,b^2dnx^2-2\,abdnx+b^2cn^2+2\,dx^2b^2-2\,adxb+5\,b^2cn+2\,a^2d+6\,b^2c\right)}{b^3\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c),x)
```

```
[Out] (b*x+a)^(1+n)*(b^2*d*n^2*x^2+3*b^2*d*n*x^2-2*a*b*d*n*x+b^2*c*n^2+2*b^2*d*x^
2-2*a*b*d*x+5*b^2*c*n+2*a^2*d+6*b^2*c)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90208, size = 308, normalized size = 4.4

$$\frac{\left(ab^2cn^2+5\,ab^2cn+6\,ab^2c+2\,a^3d+\left(b^3dn^2+3\,b^3dn+2\,b^3d\right)x^3+\left(ab^2dn^2+ab^2dn\right)x^2+\left(b^3cn^2+6\,b^3c+\left(5\,b^3c-2\right.\right.\right.}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (a*b^2*c*n^2 + 5*a*b^2*c*n + 6*a*b^2*c + 2*a^3*d + (b^3*d*n^2 + 3*b^3*d*n +
 2*b^3*d)*x^3 + (a*b^2*d*n^2 + a*b^2*d*n)*x^2 + (b^3*c*n^2 + 6*b^3*c + (5*b
^3*c - 2*a^2*b*d)*n)*x)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3
)
```

Sympy [A] time = 2.31337, size = 978, normalized size = 13.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**3/3), Eq(b, 0)), (2*a**2*d*log(a/b + x)/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d/(2*a**2*b**3 + 4*a*b**4*x + 2*
b**5*x**2) + 4*a*b*d*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2
) + 4*a*b*d*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**3 + 4
*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**3*d*log(a/b + x)/(a**2*b**3 +
a*b**4*x) - 2*a**2*b*d*x*log(a/b + x)/(a**2*b**3 + a*b**4*x) + 2*a**2*b*d*x
```

```
/(a**2*b**3 + a*b**4*x) + a*b**2*d*x**2/(a**2*b**3 + a*b**4*x) + b**3*c*x/(
a**2*b**3 + a*b**4*x), Eq(n, -2)), (a**2*d*log(a/b + x)/b**3 - a*d*x/b**2 +
 c*log(a/b + x)/b + d*x**2/(2*b), Eq(n, -1)), (2*a**3*d*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*d*n*x*(a + b*x)**n/(b**
3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*c*n**2*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 5*a*b**2*c*n*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*a*b**2*c*(a + b*x)**n/(b**3
*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*d*n**2*x**2*(a + b*x)**n
/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*d*n*x**2*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*c*n**2*x*(a + b*x
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 5*b**3*c*n*x*(a + b*x
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**3*c*x*(a + b*x)*
*n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*d*n**2*x**3*(a + b
*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*d*n*x**3*(a
+ b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*d*x**3*(a
 + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True))
```

Giac [B] time = 1.20527, size = 320, normalized size = 4.57

$$\frac{(bx+a)^n b^3dn^2x^3 + (bx+a)^n ab^2dn^2x^2 + 3\,(bx+a)^n b^3dnx^3 + (bx+a)^n b^3cn^2x + (bx+a)^n ab^2dnx^2 + 2\,(bx+a)^n b^3dx^3 + (}{b^3n^3 + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*d*n^2*x^3 + (b*x + a)^n*a*b^2*d*n^2*x^2 + 3*(b*x + a)^n*b^
3*d*n*x^3 + (b*x + a)^n*b^3*c*n^2*x + (b*x + a)^n*a*b^2*d*n*x^2 + 2*(b*x +
a)^n*b^3*d*x^3 + (b*x + a)^n*a*b^2*c*n^2 + 5*(b*x + a)^n*b^3*c*n*x - 2*(b*x
 + a)^n*a^2*b*d*n*x + 5*(b*x + a)^n*a*b^2*c*n + 6*(b*x + a)^n*b^3*c*x + 6*(
b*x + a)^n*a*b^2*c + 2*(b*x + a)^n*a^3*d)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n +
 6*b^3)
```

3.354 $\int \frac{(a+bx)^n(c+dx^2)}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)}+\frac{d(a+bx)^{n+2}}{b^2(n+2)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[Out] -((a*d*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (d*(a + b*x)^(2 + n))/(b^2*(2 +
n)) - (c*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])
/(a*(1 + n))
```

Rubi [A] time = 0.0508392, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {952, 80, 65}

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)}+\frac{d(a+bx)^{n+2}}{b^2(n+2)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2))/x,x]
```

```
[Out] -((a*d*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (d*(a + b*x)^(2 + n))/(b^2*(2 +
n)) - (c*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])
/(a*(1 + n))
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n\left(c+dx^2\right)}{x}\,dx = \frac{d(a+bx)^{2+n}}{b^2(2+n)} + \frac{\int \frac{(a+bx)^n\left(b^2c(2+n)-abd(2+n)x\right)}{x}\,dx}{b^2(2+n)}$$
$$= -\frac{ad(a+bx)^{1+n}}{b^2(1+n)} + \frac{d(a+bx)^{2+n}}{b^2(2+n)} + c\int \frac{(a+bx)^n}{x}\,dx$$
$$= -\frac{ad(a+bx)^{1+n}}{b^2(1+n)} + \frac{d(a+bx)^{2+n}}{b^2(2+n)} - \frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A] time = 0.0449481, size = 64, normalized size = 0.83

$$-\frac{(a+bx)^{n+1}\left(b^2c(n+2)\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)+ad(a-b(n+1)x)\right)}{ab^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2))/x,x]
```

```
[Out] -(((a + b*x)^(1 + n)*(a*d*(a - b*(1 + n)*x) + b^2*c*(2 + n)*Hypergeometric2
F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/(a*b^2*(1 + n)*(2 + n)))
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n\left(dx^2+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^2+c)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^2+c\right)(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*(b*x + a)^n/x, x)
```

Sympy [B] time = 6.48357, size = 347, normalized size = 4.51

$$-\frac{b^{n}cn\left(\frac{a}{b}+x\right)^{n}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma(n+2)}-\frac{b^{n}c\left(\frac{a}{b}+x\right)^{n}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma(n+2)}+d\begin{cases}\frac{a^{n}x^{2}}{2} \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^{2}+b^{3}x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^{2}+b^{3}x} \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^{2}}+\frac{x}{b} \\ -\frac{a^{2}(a+bx)^{n}}{b^{2}n^{2}+3b^{2}n+2b^{2}}+\frac{abnx(}{b^{2}n^{2}+3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)/x,x)
```

```
[Out] -b**n*c*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(n +
 2) - b**n*c*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) + d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**
3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x) - b*x/(a*b**2 + b**3*x), Eq(n, -2
)), (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2
 + 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2
) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a
 + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) - b*b**n*c*n*x*(a/b + x)
**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c*
x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(dx^{2}+c\right)(bx+a)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

3.355 $\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=232

$$\frac{\left(12a^2b^2cd+15a^4d^2+b^4c^2\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{a^2\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{2a\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{4ad\,(5$$

```
[Out] (a^2*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (2*a*(b^2*c + a^2
*d)*(b^2*c + 3*a^2*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) + ((b^4*c^2 + 12*a^2
*b^2*c*d + 15*a^4*d^2)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d*(2*b^2*c +
 5*a^2*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (d*(2*b^2*c + 15*a^2*d)*(a + b
*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d
^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rubi [A] time = 0.139042, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {948}

$$\frac{\left(12a^2b^2cd+15a^4d^2+b^4c^2\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{a^2\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{2a\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{4ad\,(5$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (2*a*(b^2*c + a^2
*d)*(b^2*c + 3*a^2*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) + ((b^4*c^2 + 12*a^2
*b^2*c*d + 15*a^4*d^2)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d*(2*b^2*c +
 5*a^2*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (d*(2*b^2*c + 15*a^2*d)*(a + b
*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d
^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx=\int\left(\frac{\left(ab^2c+a^3d\right)^2(a+bx)^n}{b^6}+\frac{2a\left(-b^2c-3a^2d\right)\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^6}+\frac{\left(b^4c^2+12a^2b^2c\right.}{}\right.$$

$$=\frac{a^2\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^7(1+n)}-\frac{2a\left(b^2c+a^2d\right)\left(b^2c+3a^2d\right)(a+bx)^{2+n}}{b^7(2+n)}+\frac{\left(b^4c^2+12a^2b^2cd\right.}{b^7}$$

Mathematica [A] time = 0.198186, size = 199, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)^2\left(12a^2b^2cd+15a^4d^2+b^4c^2\right)}{n+3}+\frac{d(a+bx)^4\left(15a^2d+2b^2c\right)}{n+5}-\frac{4ad(a+bx)^3\left(5a^2d+2b^2c\right)}{n+4}-\frac{2a(a+bx)\left(a^2d+b^2c\right)\left(3a^2d+b^2c\right)}{n+2}+\frac{\left(a^3d+ab^2c\right)^2}{n+1}+\right.}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^2*c + a^3*d)^2/(1 + n) - (2*a*(b^2*c + a^2*d)*(b^2
*c + 3*a^2*d)*(a + b*x))/(2 + n) + ((b^4*c^2 + 12*a^2*b^2*c*d + 15*a^4*d^2)
*(a + b*x)^2)/(3 + n) - (4*a*d*(2*b^2*c + 5*a^2*d)*(a + b*x)^3)/(4 + n) + (
d*(2*b^2*c + 15*a^2*d)*(a + b*x)^4)/(5 + n) - (6*a*d^2*(a + b*x)^5)/(6 + n)
 + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

Maple [B] time = 0.054, size = 1000, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] (b*x+a)^(1+n)*(b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5*x^5+2*b^6
*c*d*n^6*x^4+175*b^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+46*b^6*c*d*n^5*x^4+73
5*b^6*d^2*n^3*x^6+30*a^2*b^4*d^2*n^4*x^4-8*a*b^5*c*d*n^5*x^3-510*a*b^5*d^2*
n^3*x^5+b^6*c^2*n^6*x^2+414*b^6*c*d*n^4*x^4+1624*b^6*d^2*n^2*x^6+300*a^2*b^
4*d^2*n^3*x^4-152*a*b^5*c*d*n^4*x^3-1350*a*b^5*d^2*n^2*x^5+25*b^6*c^2*n^5*x
^2+1850*b^6*c*d*n^3*x^4+1764*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+24*a^2*b
^4*c*d*n^4*x^2+1050*a^2*b^4*d^2*n^2*x^4-2*a*b^5*c^2*n^5*x-1048*a*b^5*c*d*n^
3*x^3-1644*a*b^5*d^2*n*x^5+247*b^6*c^2*n^4*x^2+4288*b^6*c*d*n^2*x^4+720*b^6
*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+384*a^2*b^4*c*d*n^3*x^2+1500*a^2*b^4*d^2*n
*x^4-46*a*b^5*c^2*n^4*x-3208*a*b^5*c*d*n^2*x^3-720*a*b^5*d^2*x^5+1219*b^6*c
^2*n^3*x^2+4824*b^6*c*d*n*x^4+360*a^4*b^2*d^2*n^2*x^2-48*a^3*b^3*c*d*n^3*x-
1320*a^3*b^3*d^2*n*x^3+2*a^2*b^4*c^2*n^4+1992*a^2*b^4*c*d*n^2*x^2+720*a^2*b
^4*d^2*x^4-402*a*b^5*c^2*n^3*x-4320*a*b^5*c*d*n*x^3+3112*b^6*c^2*n^2*x^2+20
16*b^6*c*d*x^4+1080*a^4*b^2*d^2*n*x^2-672*a^3*b^3*c*d*n^2*x-720*a^3*b^3*d^2
*x^3+44*a^2*b^4*c^2*n^3+3648*a^2*b^4*c*d*n*x^2-1634*a*b^5*c^2*n^2*x-2016*a*
b^5*c*d*x^3+3796*b^6*c^2*n*x^2-720*a^5*b*d^2*n*x+48*a^4*b^2*c*d*n^2+720*a^4
*b^2*d^2*x^2-2640*a^3*b^3*c*d*n*x+358*a^2*b^4*c^2*n^2+2016*a^2*b^4*c*d*x^2-
2956*a*b^5*c^2*n*x+1680*b^6*c^2*x^2-720*a^5*b*d^2*x+624*a^4*b^2*c*d*n-2016*
a^3*b^3*c*d*x+1276*a^2*b^4*c^2*n-1680*a*b^5*c^2*x+720*a^6*d^2+2016*a^4*b^2*
c*d+1680*a^2*b^4*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+1
3068*n+5040)
```

Maxima [A] time = 1.0588, size = 603, normalized size = 2.6

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^nc^2}{\left(n^3+6\,n^2+11\,n+6\right)b^3}+\frac{2\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\left(n^4+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
```

```
+ a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 +
6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

Fricas [B] time = 1.99366, size = 2246, normalized size = 9.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^4*c^2*n^4 + 44*a^3*b^4*c^2*n^3 + 1680*a^3*b^4*c^2 + 2016*a^5*b^2*c
*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*n^4 + 735*b^
7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^7 + (a*b^6*d
^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^3 + 274*a*b^
6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 + 2*(b^7*c*d*n^6 + 1008*b^7*c*d + (23*b^7*
c*d - 3*a^2*b^5*d^2)*n^5 + 3*(69*b^7*c*d - 10*a^2*b^5*d^2)*n^4 + 5*(185*b^7
*c*d - 21*a^2*b^5*d^2)*n^3 + 2*(1072*b^7*c*d - 75*a^2*b^5*d^2)*n^2 + 36*(67
*b^7*c*d - 2*a^2*b^5*d^2)*n)*x^5 + 2*(a*b^6*c*d*n^6 + 19*a*b^6*c*d*n^5 + (1
31*a*b^6*c*d + 15*a^3*b^4*d^2)*n^4 + (401*a*b^6*c*d + 90*a^3*b^4*d^2)*n^3 +
 15*(36*a*b^6*c*d + 11*a^3*b^4*d^2)*n^2 + 18*(14*a*b^6*c*d + 5*a^3*b^4*d^2)
*n)*x^4 + (b^7*c^2*n^6 + 1680*b^7*c^2 + (25*b^7*c^2 - 8*a^2*b^5*c*d)*n^5 +
(247*b^7*c^2 - 128*a^2*b^5*c*d)*n^4 + (1219*b^7*c^2 - 664*a^2*b^5*c*d - 120
*a^4*b^3*d^2)*n^3 + 8*(389*b^7*c^2 - 152*a^2*b^5*c*d - 45*a^4*b^3*d^2)*n^2
+ 4*(949*b^7*c^2 - 168*a^2*b^5*c*d - 60*a^4*b^3*d^2)*n)*x^3 + 2*(179*a^3*b^
4*c^2 + 24*a^5*b^2*c*d)*n^2 + (a*b^6*c^2*n^6 + 23*a*b^6*c^2*n^5 + 3*(67*a*b
^6*c^2 + 8*a^3*b^4*c*d)*n^4 + (817*a*b^6*c^2 + 336*a^3*b^4*c*d)*n^3 + 2*(73
9*a*b^6*c^2 + 660*a^3*b^4*c*d + 180*a^5*b^2*d^2)*n^2 + 24*(35*a*b^6*c^2 + 4
2*a^3*b^4*c*d + 15*a^5*b^2*d^2)*n)*x^2 + 4*(319*a^3*b^4*c^2 + 156*a^5*b^2*c
*d)*n - 2*(a^2*b^5*c^2*n^5 + 22*a^2*b^5*c^2*n^4 + (179*a^2*b^5*c^2 + 24*a^4
*b^3*c*d)*n^3 + 2*(319*a^2*b^5*c^2 + 156*a^4*b^3*c*d)*n^2 + 24*(35*a^2*b^5*
c^2 + 42*a^4*b^3*c*d + 15*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*n^7 + 28*b^7*n^
6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n
 + 5040*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15821, size = 2363, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^5 + 15*(b*x + a)^n*a*b^6*d
^2*n^5*x^6 + 175*(b*x + a)^n*b^7*d^2*n^4*x^7 + 2*(b*x + a)^n*a*b^6*c*d*n^6*
x^4 + 46*(b*x + a)^n*b^7*c*d*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^2*n^5*x^5 +
85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7 + (b*x +
 a)^n*b^7*c^2*n^6*x^3 + 38*(b*x + a)^n*a*b^6*c*d*n^5*x^4 + 414*(b*x + a)^n*
b^7*c*d*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^
6*d^2*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^2*n^2*x^7 + (b*x + a)^n*a*b^6*c^2*n^
6*x^2 + 25*(b*x + a)^n*b^7*c^2*n^5*x^3 - 8*(b*x + a)^n*a^2*b^5*c*d*n^5*x^3
+ 262*(b*x + a)^n*a*b^6*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 +
1850*(b*x + a)^n*b^7*c*d*n^3*x^5 - 210*(b*x + a)^n*a^2*b^5*d^2*n^3*x^5 + 27
4*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b^7*d^2*n*x^7 + 23*(b*x
+ a)^n*a*b^6*c^2*n^5*x^2 + 247*(b*x + a)^n*b^7*c^2*n^4*x^3 - 128*(b*x + a)^
n*a^2*b^5*c*d*n^4*x^3 + 802*(b*x + a)^n*a*b^6*c*d*n^3*x^4 + 180*(b*x + a)^n
*a^3*b^4*d^2*n^3*x^4 + 4288*(b*x + a)^n*b^7*c*d*n^2*x^5 - 300*(b*x + a)^n*a
^2*b^5*d^2*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*
d^2*x^7 - 2*(b*x + a)^n*a^2*b^5*c^2*n^5*x + 201*(b*x + a)^n*a*b^6*c^2*n^4*x
^2 + 24*(b*x + a)^n*a^3*b^4*c*d*n^4*x^2 + 1219*(b*x + a)^n*b^7*c^2*n^3*x^3
- 664*(b*x + a)^n*a^2*b^5*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^2*n^3*x^3
 + 1080*(b*x + a)^n*a*b^6*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*n^2*x^4
 + 4824*(b*x + a)^n*b^7*c*d*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 - 44*
(b*x + a)^n*a^2*b^5*c^2*n^4*x + 817*(b*x + a)^n*a*b^6*c^2*n^3*x^2 + 336*(b*
x + a)^n*a^3*b^4*c*d*n^3*x^2 + 3112*(b*x + a)^n*b^7*c^2*n^2*x^3 - 1216*(b*x
 + a)^n*a^2*b^5*c*d*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 504*(b*
x + a)^n*a*b^6*c*d*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^2*n*x^4 + 2016*(b*x +
a)^n*b^7*c*d*x^5 + 2*(b*x + a)^n*a^3*b^4*c^2*n^4 - 358*(b*x + a)^n*a^2*b^5*
c^2*n^3*x - 48*(b*x + a)^n*a^4*b^3*c*d*n^3*x + 1478*(b*x + a)^n*a*b^6*c^2*n
^2*x^2 + 1320*(b*x + a)^n*a^3*b^4*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2
*n^2*x^2 + 3796*(b*x + a)^n*b^7*c^2*n*x^3 - 672*(b*x + a)^n*a^2*b^5*c*d*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 44*(b*x + a)^n*a^3*b^4*c^2*n^3 - 1
276*(b*x + a)^n*a^2*b^5*c^2*n^2*x - 624*(b*x + a)^n*a^4*b^3*c*d*n^2*x + 840
*(b*x + a)^n*a*b^6*c^2*n*x^2 + 1008*(b*x + a)^n*a^3*b^4*c*d*n*x^2 + 360*(b*
x + a)^n*a^5*b^2*d^2*n*x^2 + 1680*(b*x + a)^n*b^7*c^2*x^3 + 358*(b*x + a)^n
*a^3*b^4*c^2*n^2 + 48*(b*x + a)^n*a^5*b^2*c*d*n^2 - 1680*(b*x + a)^n*a^2*b^
5*c^2*n*x - 2016*(b*x + a)^n*a^4*b^3*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 1276*(b*x + a)^n*a^3*b^4*c^2*n + 624*(b*x + a)^n*a^5*b^2*c*d*n + 1680*(
b*x + a)^n*a^3*b^4*c^2 + 2016*(b*x + a)^n*a^5*b^2*c*d + 720*(b*x + a)^n*a^7
*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 1
3132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

3.356 $\int x(a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=185

$$-\frac{a\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^6(n+1)}+\frac{\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{2ad\left(5a^2d+3b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{2d\left(5a^2d+b^2c\right)(a}{b^6(n+4)}$$

```
[Out] -((a*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^6*(1 + n))) + ((b^2*c + a^2*d)
*(b^2*c + 5*a^2*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) - (2*a*d*(3*b^2*c + 5*a
^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (2*d*(b^2*c + 5*a^2*d)*(a + b*x)^(
4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a
 + b*x)^(6 + n))/(b^6*(6 + n))
```

Rubi [A] time = 0.100492, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {772}

$$-\frac{a\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^6(n+1)}+\frac{\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{2ad\left(5a^2d+3b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{2d\left(5a^2d+b^2c\right)(a}{b^6(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^2,x]
```


```
[Out] -((a*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^6*(1 + n))) + ((b^2*c + a^2*d)
*(b^2*c + 5*a^2*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) - (2*a*d*(3*b^2*c + 5*a
^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (2*d*(b^2*c + 5*a^2*d)*(a + b*x)^(
4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a
 + b*x)^(6 + n))/(b^6*(6 + n))
```


Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^2\right)^2\,dx=\int\left(-\frac{a\left(b^2c+a^2d\right)^2(a+bx)^n}{b^5}+\frac{\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a+bx)^{1+n}}{b^5}-\frac{2ad\left(3b^2c+5a^2d\right)}{b^5}\right.$$

$$=-\frac{a\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^6(1+n)}+\frac{\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a+bx)^{2+n}}{b^6(2+n)}-\frac{2ad\left(3b^2c+5a^2d\right)(a}{b^6(3+n)}$$

Mathematica [A] time = 0.555637, size = 323, normalized size = 1.75

$$\frac{(a+bx)^{n+1}\left(-a(n+6)\left(4(n+4)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)-4ad(n+1)(a+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(a + b*x)*(
c + d*x^2)^2 - a*(6 + n)*(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2
 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*
(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 +
 n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2))) + 4*(1 + n)*(a + b*x)*((b
^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n)
 + d*(2 + n)*x^2)) - a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b
^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(5 + n)*(6 + n))
```

Maple [B] time = 0.052, size = 677, normalized size = 3.7

$$-\frac{(bx+a)^{1+n}\left(-b^5d^2n^5x^5-15\,b^5d^2n^4x^5+5\,ab^4d^2n^4x^4-2\,b^5cdn^5x^3-85\,b^5d^2n^3x^5+50\,ab^4d^2n^3x^4-34\,b^5cdn^4x^3-22\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^5*d^2*n^5*x^5-15*b^5*d^2*n^4*x^5+5*a*b^4*d^2*n^4*x^4-2*b
^5*c*d*n^5*x^3-85*b^5*d^2*n^3*x^5+50*a*b^4*d^2*n^3*x^4-34*b^5*c*d*n^4*x^3-2
25*b^5*d^2*n^2*x^5-20*a^2*b^3*d^2*n^3*x^3+6*a*b^4*c*d*n^4*x^2+175*a*b^4*d^2
*n^2*x^4-b^5*c^2*n^5*x-214*b^5*c*d*n^3*x^3-274*b^5*d^2*n*x^5-120*a^2*b^3*d^
2*n^2*x^3+84*a*b^4*c*d*n^3*x^2+250*a*b^4*d^2*n*x^4-19*b^5*c^2*n^4*x-614*b^5
*c*d*n^2*x^3-120*b^5*d^2*x^5+60*a^3*b^2*d^2*n^2*x^2-12*a^2*b^3*c*d*n^3*x-22
0*a^2*b^3*d^2*n*x^3+a*b^4*c^2*n^4+390*a*b^4*c*d*n^2*x^2+120*a*b^4*d^2*x^4-1
37*b^5*c^2*n^3*x-792*b^5*c*d*n*x^3+180*a^3*b^2*d^2*n*x^2-144*a^2*b^3*c*d*n^
2*x-120*a^2*b^3*d^2*x^3+18*a*b^4*c^2*n^3+672*a*b^4*c*d*n*x^2-461*b^5*c^2*n^
2*x-360*b^5*c*d*x^3-120*a^4*b*d^2*n*x+12*a^3*b^2*c*d*n^2+120*a^3*b^2*d^2*x^
2-492*a^2*b^3*c*d*n*x+119*a*b^4*c^2*n^2+360*a*b^4*c*d*x^2-702*b^5*c^2*n*x-1
20*a^4*b*d^2*x+132*a^3*b^2*c*d*n-360*a^2*b^3*c*d*x+342*a*b^4*c^2*n-360*b^5*
c^2*x+120*a^5*d^2+360*a^3*b^2*c*d+360*a*b^4*c^2)/b^6/(n^6+21*n^5+175*n^4+73
5*n^3+1624*n^2+1764*n+720)
```

Maxima [A] time = 1.04189, size = 452, normalized size = 2.44

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n c^2}{\left(n^2+3\,n+2\right)b^2}+\frac{2\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+\right.}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 3
5*n^2 + 50*n + 24)*b^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*
b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^
3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^
2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d^2/((n^6 + 21*n^
```

```
5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

Fricas [B] time = 2.31646, size = 1612, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^4*c^2*n^4 + 18*a^2*b^4*c^2*n^3 + 360*a^2*b^4*c^2 + 360*a^4*b^2*c*d
+ 120*a^6*d^2 - (b^6*d^2*n^5 + 15*b^6*d^2*n^4 + 85*b^6*d^2*n^3 + 225*b^6*d^
2*n^2 + 274*b^6*d^2*n + 120*b^6*d^2)*x^6 - (a*b^5*d^2*n^5 + 10*a*b^5*d^2*n^
4 + 35*a*b^5*d^2*n^3 + 50*a*b^5*d^2*n^2 + 24*a*b^5*d^2*n)*x^5 - (2*b^6*c*d*
n^5 + 360*b^6*c*d + (34*b^6*c*d - 5*a^2*b^4*d^2)*n^4 + 2*(107*b^6*c*d - 15*
a^2*b^4*d^2)*n^3 + (614*b^6*c*d - 55*a^2*b^4*d^2)*n^2 + 6*(132*b^6*c*d - 5*
a^2*b^4*d^2)*n)*x^4 - 2*(a*b^5*c*d*n^5 + 14*a*b^5*c*d*n^4 + 5*(13*a*b^5*c*d
 + 2*a^3*b^3*d^2)*n^3 + 2*(56*a*b^5*c*d + 15*a^3*b^3*d^2)*n^2 + 20*(3*a*b^5
*c*d + a^3*b^3*d^2)*n)*x^3 + (119*a^2*b^4*c^2 + 12*a^4*b^2*c*d)*n^2 - (b^6*
c^2*n^5 + 360*b^6*c^2 + (19*b^6*c^2 - 6*a^2*b^4*c*d)*n^4 + (137*b^6*c^2 - 7
2*a^2*b^4*c*d)*n^3 + (461*b^6*c^2 - 246*a^2*b^4*c*d - 60*a^4*b^2*d^2)*n^2 +
 6*(117*b^6*c^2 - 30*a^2*b^4*c*d - 10*a^4*b^2*d^2)*n)*x^2 + 6*(57*a^2*b^4*c
^2 + 22*a^4*b^2*c*d)*n - (a*b^5*c^2*n^5 + 18*a*b^5*c^2*n^4 + (119*a*b^5*c^2
 + 12*a^3*b^3*c*d)*n^3 + 6*(57*a*b^5*c^2 + 22*a^3*b^3*c*d)*n^2 + 120*(3*a*b
^5*c^2 + 3*a^3*b^3*c*d + a^5*b*d^2)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5
 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Sympy [A] time = 17.4636, size = 8803, normalized size = 47.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**2/2 + c*d*x**4/2 + d**2*x**6/6), Eq(b, 0)), (60*a*
*5*d**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 +
 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d**2/(60
*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 30
0*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*d**2*x*log(a/b + x)/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 625*a**4*b*d**2*x/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
**5) - 6*a**3*b**2*c*d/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d
**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d
**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b*
*9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 30*a**2*b**3*c*d*x/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 600*a**2*b**3*d**2*x**3*log(a/b + x)/(60*a**5*b*
*6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**1
0*x**4 + 60*b**11*x**5) + 900*a**2*b**3*d**2*x**3/(60*a**5*b**6 + 300*a**4*
b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b*
```

```
*11*x**5) - 3*a*b**4*c**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x
**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 60*a*b**4*c*
d*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4*log(a/b +
x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**
3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4/(60*a**5*b**6
+ 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x
**4 + 60*b**11*x**5) - 15*b**5*c**2*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600
*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) -
60*b**5*c*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600
*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d**2*x**5*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**6*d**2*lo
g(a/b + x)/(12*a**5*b**6 + 48*a**4*b**7*x + 72*a**3*b**8*x**2 + 48*a**2*b**
9*x**3 + 12*a*b**10*x**4) - 35*a**6*d**2/(12*a**5*b**6 + 48*a**4*b**7*x + 7
2*a**3*b**8*x**2 + 48*a**2*b**9*x**3 + 12*a*b**10*x**4) - 240*a**5*b*d**2*x
*log(a/b + x)/(12*a**5*b**6 + 48*a**4*b**7*x + 72*a**3*b**8*x**2 + 48*a**2*
b**9*x**3 + 12*a*b**10*x**4) - 80*a**5*b*d**2*x/(12*a**5*b**6 + 48*a**4*b**
7*x + 72*a**3*b**8*x**2 + 48*a**2*b**9*x**3 + 12*a*b**10*x**4) - 360*a**4*b
**2*d**2*x**2*log(a/b + x)/(12*a**5*b**6 + 48*a**4*b**7*x + 72*a**3*b**8*x*
*2 + 48*a**2*b**9*x**3 + 12*a*b**10*x**4) - 240*a**3*b**3*d**2*x**3*log(a/b
 + x)/(12*a**5*b**6 + 48*a**4*b**7*x + 72*a**3*b**8*x**2 + 48*a**2*b**9*x**
3 + 12*a*b**10*x**4) + 120*a**3*b**3*d**2*x**3/(12*a**5*b**6 + 48*a**4*b**7
*x + 72*a**3*b**8*x**2 + 48*a**2*b**9*x**3 + 12*a*b**10*x**4) - a**2*b**4*c
**2/(12*a**5*b**6 + 48*a**4*b**7*x + 72*a**3*b**8*x**2 + 48*a**2*b**9*x**3
+ 12*a*b**10*x**4) - 60*a**2*b**4*d**2*x**4*log(a/b + x)/(12*a**5*b**6 + 48
*a**4*b**7*x + 72*a**3*b**8*x**2 + 48*a**2*b**9*x**3 + 12*a*b**10*x**4) + 9
0*a**2*b**4*d**2*x**4/(12*a**5*b**6 + 48*a**4*b**7*x + 72*a**3*b**8*x**2 +
48*a**2*b**9*x**3 + 12*a*b**10*x**4) - 4*a*b**5*c**2*x/(12*a**5*b**6 + 48*a
**4*b**7*x + 72*a**3*b**8*x**2 + 48*a**2*b**9*x**3 + 12*a*b**10*x**4) + 12*
a*b**5*d**2*x**5/(12*a**5*b**6 + 48*a**4*b**7*x + 72*a**3*b**8*x**2 + 48*a*
*2*b**9*x**3 + 12*a*b**10*x**4) + 6*b**6*c*d*x**4/(12*a**5*b**6 + 48*a**4*b
**7*x + 72*a**3*b**8*x**2 + 48*a**2*b**9*x**3 + 12*a*b**10*x**4), Eq(n, -5)
), (60*a**5*d**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**
2 + 6*b**9*x**3) + 50*a**5*d**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x
**2 + 6*b**9*x**3) + 180*a**4*b*d**2*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*
b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 90*a**4*b*d**2*x/(6*a**3*b**6 + 18
*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 12*a**3*b**2*c*d*log(a/b + x
)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 10*a**3*b
**2*c*d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180
*a**3*b**2*d**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8
*x**2 + 6*b**9*x**3) + 36*a**2*b**3*c*d*x*log(a/b + x)/(6*a**3*b**6 + 18*a*
*2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 18*a**2*b**3*c*d*x/(6*a**3*b**6
 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d**2*x**3*
log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
- 60*a**2*b**3*d**2*x**3/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6
*b**9*x**3) - a*b**4*c**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 +
6*b**9*x**3) + 36*a*b**4*c*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*
x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*d**2*x**4/(6*a**3*b**6 + 18*a
**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 3*b**5*c**2*x/(6*a**3*b**6 + 1
8*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 12*b**5*c*d*x**3*log(a/b +
x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 12*b**5*
c*d*x**3/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*
b**5*d**2*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3
), Eq(n, -4)), (-60*a**5*d**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b
**8*x**2) - 90*a**5*d**2/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a*
*4*b*d**2*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a*
*4*b*d**2*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 36*a**3*b**2*c*d*lo
g(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 54*a**3*b**2*c*d/(6*
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a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d**2*x**2*log(a/b + x
)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 72*a**2*b**3*c*d*x*log(a/b +
x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 72*a**2*b**3*c*d*x/(6*a**2*b
6 + 12*a*b7*x + 6*b**8*x**2) + 20*a**2*b**3*d**2*x**3/(6*a**2*b**6 + 12
*a*b**7*x + 6*b**8*x**2) - 3*a*b**4*c**2/(6*a**2*b**6 + 12*a*b**7*x + 6*b**
8*x**2) - 36*a*b**4*c*d*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b*
*8*x**2) - 5*a*b**4*d**2*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 6
*b**5*c**2*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 12*b**5*c*d*x**3/(
6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d**2*x**5/(6*a**2*b**6 +
12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d**2*log(a/b + x)/(12*a*b*
*6 + 12*b**7*x) + 60*a**5*d**2/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*d**2*x*l
og(a/b + x)/(12*a*b**6 + 12*b**7*x) + 72*a**3*b**2*c*d*log(a/b + x)/(12*a*b
6 + 12*b7*x) + 72*a**3*b**2*c*d/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*
d**2*x**2/(12*a*b**6 + 12*b**7*x) + 72*a**2*b**3*c*d*x*log(a/b + x)/(12*a*b
6 + 12*b7*x) + 10*a**2*b**3*d**2*x**3/(12*a*b**6 + 12*b**7*x) + 12*a*b*
*4*c**2*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 12*a*b**4*c**2/(12*a*b**6 +
12*b**7*x) - 36*a*b**4*c*d*x**2/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*d**2*x**
4/(12*a*b**6 + 12*b**7*x) + 12*b**5*c**2*x*log(a/b + x)/(12*a*b**6 + 12*b**
7*x) + 12*b**5*c*d*x**3/(12*a*b**6 + 12*b**7*x) + 3*b**5*d**2*x**5/(12*a*b*
*6 + 12*b**7*x), Eq(n, -2)), (-a**5*d**2*log(a/b + x)/b**6 + a**4*d**2*x/b*
*5 - 2*a**3*c*d*log(a/b + x)/b**4 - a**3*d**2*x**2/(2*b**4) + 2*a**2*c*d*x/
b**3 + a**2*d**2*x**3/(3*b**3) - a*c**2*log(a/b + x)/b**2 - a*c*d*x**2/b**2
 - a*d**2*x**4/(4*b**2) + c**2*x/b + 2*c*d*x**3/(3*b) + d**2*x**5/(5*b), Eq
(n, -1)), (-120*a**6*d**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6
*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5
*b*d**2*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b*
*6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 12*a**4*b**2*c*d*n**2*
(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 16
24*b**6*n**2 + 1764*b**6*n + 720*b**6) - 132*a**4*b**2*c*d*n*(a + b*x)**n/(
b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
 1764*b**6*n + 720*b**6) - 360*a**4*b**2*c*d*(a + b*x)**n/(b**6*n**6 + 21*b
6*n5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) - 60*a**4*b**2*d**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n*
*5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**
6) - 60*a**4*b**2*d**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*
b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 12*a
3*b3*c*d*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
+ 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 132*a**3*b**3*
c*d*n**2*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**
6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 360*a**3*b**3*c*d*n*x*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**3*d**2*n**3*x**3*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 60*a**3*b**3*d**2*n**2*x**3*(a + b*x)**n/
(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
+ 1764*b**6*n + 720*b**6) + 40*a**3*b**3*d**2*n*x**3*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) - a**2*b**4*c**2*n**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n*
*5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**
6) - 18*a**2*b**4*c**2*n**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 119*a*
*2*b**4*c**2*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 342*a**2*b**4*c*
*2*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 360*a**2*b**4*c**2*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
*2 + 1764*b**6*n + 720*b**6) - 6*a**2*b**4*c*d*n**4*x**2*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 176
4*b**6*n + 720*b**6) - 72*a**2*b**4*c*d*n**3*x**2*(a + b*x)**n/(b**6*n**6 +

21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) - 246*a**2*b**4*c*d*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) - 180*a**2*b**4*c*d*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) -
5*a**2*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**
2*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 55*a**2*b**4
*d**2*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*d**2*
n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*c**2*n**5*x*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + 18*a*b**5*c**2*n**4*x*(a + b*x)**n/(b**6*n
6 + 21*b6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 119*a*b**5*c**2*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) + 342*a*b**5*c**2*n**2*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 3
60*a*b**5*c**2*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 2*a*b**5*c*d*n*
*5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
3 + 1624*b6*n**2 + 1764*b**6*n + 720*b**6) + 28*a*b**5*c*d*n**4*x**3*(a
+ b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624
*b**6*n**2 + 1764*b**6*n + 720*b**6) + 130*a*b**5*c*d*n**3*x**3*(a + b*x)**
n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**
2 + 1764*b**6*n + 720*b**6) + 224*a*b**5*c*d*n**2*x**3*(a + b*x)**n/(b**6*n
6 + 21*b6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 120*a*b**5*c*d*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) + a*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*
a*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35*a*b**5*d*
*2*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
6*n3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*d**2*n**2*x
5*(a + b*x)n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b**5*d**2*n*x**5*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + b**6*c**2*n**5*x**2*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) + 19*b**6*c**2*n**4*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 137*b**6*c**2*n**3*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 46
1*b**6*c**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 702*b**6*c**
2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*
n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 360*b**6*c**2*x**2*(a + b
*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6) + 2*b**6*c*d*n**5*x**4*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 34*b**6*c*d*n**4*x**4*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) + 214*b**6*c*d*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 6
14*b**6*c*d*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 792*b**6*c*d

```
*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 360*b**6*c*d*x**4*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + b**6*d**2*n**5*x**6*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) + 15*b**6*d**2*n**4*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 85*b**6*d**2*n**3*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 225
*b**6*d**2*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*d**2
*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*d**2*x**6*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6), True))
```

Giac [B] time = 1.13746, size = 1709, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d^2*n^5*x^6 + (b*x + a)^n*a*b^5*d^2*n^5*x^5 + 15*(b*x + a)
^n*b^6*d^2*n^4*x^6 + 2*(b*x + a)^n*b^6*c*d*n^5*x^4 + 10*(b*x + a)^n*a*b^5*d
^2*n^4*x^5 + 85*(b*x + a)^n*b^6*d^2*n^3*x^6 + 2*(b*x + a)^n*a*b^5*c*d*n^5*x
^3 + 34*(b*x + a)^n*b^6*c*d*n^4*x^4 - 5*(b*x + a)^n*a^2*b^4*d^2*n^4*x^4 + 3
5*(b*x + a)^n*a*b^5*d^2*n^3*x^5 + 225*(b*x + a)^n*b^6*d^2*n^2*x^6 + (b*x +
a)^n*b^6*c^2*n^5*x^2 + 28*(b*x + a)^n*a*b^5*c*d*n^4*x^3 + 214*(b*x + a)^n*b
^6*c*d*n^3*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n^3*x^4 + 50*(b*x + a)^n*a*b^5*
d^2*n^2*x^5 + 274*(b*x + a)^n*b^6*d^2*n*x^6 + (b*x + a)^n*a*b^5*c^2*n^5*x +
 19*(b*x + a)^n*b^6*c^2*n^4*x^2 - 6*(b*x + a)^n*a^2*b^4*c*d*n^4*x^2 + 130*(
b*x + a)^n*a*b^5*c*d*n^3*x^3 + 20*(b*x + a)^n*a^3*b^3*d^2*n^3*x^3 + 614*(b*
x + a)^n*b^6*c*d*n^2*x^4 - 55*(b*x + a)^n*a^2*b^4*d^2*n^2*x^4 + 24*(b*x + a
)^n*a*b^5*d^2*n*x^5 + 120*(b*x + a)^n*b^6*d^2*x^6 + 18*(b*x + a)^n*a*b^5*c^
2*n^4*x + 137*(b*x + a)^n*b^6*c^2*n^3*x^2 - 72*(b*x + a)^n*a^2*b^4*c*d*n^3*
x^2 + 224*(b*x + a)^n*a*b^5*c*d*n^2*x^3 + 60*(b*x + a)^n*a^3*b^3*d^2*n^2*x^
3 + 792*(b*x + a)^n*b^6*c*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n*x^4 - (b*x
 + a)^n*a^2*b^4*c^2*n^4 + 119*(b*x + a)^n*a*b^5*c^2*n^3*x + 12*(b*x + a)^n*
a^3*b^3*c*d*n^3*x + 461*(b*x + a)^n*b^6*c^2*n^2*x^2 - 246*(b*x + a)^n*a^2*b
^4*c*d*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n^2*x^2 + 120*(b*x + a)^n*a*b^5
*c*d*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d^2*n*x^3 + 360*(b*x + a)^n*b^6*c*d*x^4
 - 18*(b*x + a)^n*a^2*b^4*c^2*n^3 + 342*(b*x + a)^n*a*b^5*c^2*n^2*x + 132*(
b*x + a)^n*a^3*b^3*c*d*n^2*x + 702*(b*x + a)^n*b^6*c^2*n*x^2 - 180*(b*x + a
)^n*a^2*b^4*c*d*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n*x^2 - 119*(b*x + a)^n*
a^2*b^4*c^2*n^2 - 12*(b*x + a)^n*a^4*b^2*c*d*n^2 + 360*(b*x + a)^n*a*b^5*c^
2*n*x + 360*(b*x + a)^n*a^3*b^3*c*d*n*x + 120*(b*x + a)^n*a^5*b*d^2*n*x + 3
60*(b*x + a)^n*b^6*c^2*x^2 - 342*(b*x + a)^n*a^2*b^4*c^2*n - 132*(b*x + a)^
n*a^4*b^2*c*d*n - 360*(b*x + a)^n*a^2*b^4*c^2 - 360*(b*x + a)^n*a^4*b^2*c*d
 - 120*(b*x + a)^n*a^6*d^2)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n
^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

3.357 $\int (a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=140

$$\frac{\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^5(n+1)}-\frac{4ad\left(a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{2d\left(3a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)}+\frac{d^2(a+bx)}{b^5(n+5}$$

```
[Out] ((b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (4*a*d*(b^2*c + a^2*d
)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (2*d*(b^2*c + 3*a^2*d)*(a + b*x)^(3 +
n))/(b^5*(3 + n)) - (4*a*d^2*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d^2*(a + b
*x)^(5 + n))/(b^5*(5 + n))
```

Rubi [A] time = 0.0680572, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{\left(a^2d+b^2c\right)^2(a+bx)^{n+1}}{b^5(n+1)}-\frac{4ad\left(a^2d+b^2c\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{2d\left(3a^2d+b^2c\right)(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)}+\frac{d^2(a+bx)}{b^5(n+5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (4*a*d*(b^2*c + a^2*d
)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (2*d*(b^2*c + 3*a^2*d)*(a + b*x)^(3 +
n))/(b^5*(3 + n)) - (4*a*d^2*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d^2*(a + b
*x)^(5 + n))/(b^5*(5 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx)^n\left(c+dx^2\right)^2\,dx=\int \left(\frac{\left(b^2c+a^2d\right)^2(a+bx)^n}{b^4}-\frac{4ad\left(b^2c+a^2d\right)(a+bx)^{1+n}}{b^4}+\frac{2d\left(b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4}-\right.$$

$$=\frac{\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^5(1+n)}-\frac{4ad\left(b^2c+a^2d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{2d\left(b^2c+3a^2d\right)(a+bx)^{3+n}}{b^5(3+n)}-$$

Mathematica [A] time = 0.19372, size = 160, normalized size = 1.14

$$\frac{(a+bx)^{n+1}\left(\frac{4\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)}{b^4(n+1)(n+2)(n+3)}-\frac{4ad(a+bx)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)+d(n+2)x^2\right)\right)}{b^4(n+2)(n+3)(n+4)}+\left(c+dx^2\right.\right.}{b(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((c + d*x^2)^2 + (4*(b^2*c + a^2*d)*(2*a^2*d - 2*a*b*d*(
1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)))/(b^4*(1 + n)*(2 + n)*(
3 + n)) - (4*a*d*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4
 + n) + d*(2 + n)*x^2)))/(b^4*(2 + n)*(3 + n)*(4 + n))))/(b*(5 + n))
```

Maple [B] time = 0.054, size = 420, normalized size = 3.

$$\frac{(bx+a)^{1+n}\left(b^4d^2n^4x^4+10\,b^4d^2n^3x^4-4\,ab^3d^2n^3x^3+2\,b^4cdn^4x^2+35\,b^4d^2n^2x^4-24\,ab^3d^2n^2x^3+24\,b^4cdn^3x^2+50\,b^4d^2n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^2,x)
```

```
[Out] (b*x+a)^(1+n)*(b^4*d^2*n^4*x^4+10*b^4*d^2*n^3*x^4-4*a*b^3*d^2*n^3*x^3+2*b^4
*c*d*n^4*x^2+35*b^4*d^2*n^2*x^4-24*a*b^3*d^2*n^2*x^3+24*b^4*c*d*n^3*x^2+50*
b^4*d^2*n*x^4+12*a^2*b^2*d^2*n^2*x^2-4*a*b^3*c*d*n^3*x-44*a*b^3*d^2*n*x^3+b
^4*c^2*n^4+98*b^4*c*d*n^2*x^2+24*b^4*d^2*x^4+36*a^2*b^2*d^2*n*x^2-40*a*b^3*
c*d*n^2*x-24*a*b^3*d^2*x^3+14*b^4*c^2*n^3+156*b^4*c*d*n*x^2-24*a^3*b*d^2*n*
x+4*a^2*b^2*c*d*n^2+24*a^2*b^2*d^2*x^2-116*a*b^3*c*d*n*x+71*b^4*c^2*n^2+80*
b^4*c*d*x^2-24*a^3*b*d^2*x+36*a^2*b^2*c*d*n-80*a*b^3*c*d*x+154*b^4*c^2*n+24
*a^4*d^2+80*a^2*b^2*c*d+120*b^4*c^2)/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+1
20)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15156, size = 1092, normalized size = 7.8

$$\frac{\left(ab^4c^2n^4+14\,ab^4c^2n^3+120\,ab^4c^2+80\,a^3b^2cd+24\,a^5d^2+\left(b^5d^2n^4+10\,b^5d^2n^3+35\,b^5d^2n^2+50\,b^5d^2n+24\,b^5d^2\right)x^5+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^4*c^2*n^4 + 14*a*b^4*c^2*n^3 + 120*a*b^4*c^2 + 80*a^3*b^2*c*d + 24*a^5
*d^2 + (b^5*d^2*n^4 + 10*b^5*d^2*n^3 + 35*b^5*d^2*n^2 + 50*b^5*d^2*n + 24*b
^5*d^2)*x^5 + (a*b^4*d^2*n^4 + 6*a*b^4*d^2*n^3 + 11*a*b^4*d^2*n^2 + 6*a*b^4
*d^2*n)*x^4 + 2*(b^5*c*d*n^4 + 40*b^5*c*d + 2*(6*b^5*c*d - a^2*b^3*d^2)*n^3
 + (49*b^5*c*d - 6*a^2*b^3*d^2)*n^2 + 2*(39*b^5*c*d - 2*a^2*b^3*d^2)*n)*x^3
 + (71*a*b^4*c^2 + 4*a^3*b^2*c*d)*n^2 + 2*(a*b^4*c*d*n^4 + 10*a*b^4*c*d*n^3
 + (29*a*b^4*c*d + 6*a^3*b^2*d^2)*n^2 + 2*(10*a*b^4*c*d + 3*a^3*b^2*d^2)*n)
*x^2 + 2*(77*a*b^4*c^2 + 18*a^3*b^2*c*d)*n + (b^5*c^2*n^4 + 120*b^5*c^2 + 2
```

```
*(7*b^5*c^2 - 2*a^2*b^3*c*d)*n^3 + (71*b^5*c^2 - 36*a^2*b^3*c*d)*n^2 + 2*(7
7*b^5*c^2 - 40*a^2*b^3*c*d - 12*a^4*b*d^2)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*
b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [A] time = 33.4862, size = 5049, normalized size = 36.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5), Eq(b, 0)), (12*a**6*
d**2*log(a/b + x)/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a
**3*b**8*x**3 + 12*a**2*b**9*x**4) + 7*a**6*d**2/(12*a**6*b**5 + 48*a**5*b*
*6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 48*a**5
*b*d**2*x*log(a/b + x)/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 +
 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 16*a**5*b*d**2*x/(12*a**6*b**5 +
48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4)
 + 72*a**4*b**2*d**2*x**2*log(a/b + x)/(12*a**6*b**5 + 48*a**5*b**6*x + 72*
a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 48*a**3*b**3*d**2
*x**3*log(a/b + x)/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*
a**3*b**8*x**3 + 12*a**2*b**9*x**4) - 24*a**3*b**3*d**2*x**3/(12*a**6*b**5
+ 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**
4) - 3*a**2*b**4*c**2/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 +
48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 12*a**2*b**4*d**2*x**4*log(a/b + x
)/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 +
12*a**2*b**9*x**4) - 18*a**2*b**4*d**2*x**4/(12*a**6*b**5 + 48*a**5*b**6*x
+ 72*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4) + 8*a*b**5*c*d
*x**3/(12*a**6*b**5 + 48*a**5*b**6*x + 72*a**4*b**7*x**2 + 48*a**3*b**8*x**
3 + 12*a**2*b**9*x**4) + 2*b**6*c*d*x**4/(12*a**6*b**5 + 48*a**5*b**6*x + 7
2*a**4*b**7*x**2 + 48*a**3*b**8*x**3 + 12*a**2*b**9*x**4), Eq(n, -5)), (-12
*a**5*d**2*log(a/b + x)/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3
*a*b**8*x**3) - 10*a**5*d**2/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**
2 + 3*a*b**8*x**3) - 36*a**4*b*d**2*x*log(a/b + x)/(3*a**4*b**5 + 9*a**3*b*
*6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**3) - 18*a**4*b*d**2*x/(3*a**4*b**5 +
9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**3) - 36*a**3*b**2*d**2*x**2*
log(a/b + x)/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**
3) - 12*a**2*b**3*d**2*x**3*log(a/b + x)/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a
**2*b**7*x**2 + 3*a*b**8*x**3) + 12*a**2*b**3*d**2*x**3/(3*a**4*b**5 + 9*a*
*3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**3) - a*b**4*c**2/(3*a**4*b**5 +
9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**3) + 3*a*b**4*d**2*x**4/(3*a
**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**3) + 2*b**5*c*d*x
**3/(3*a**4*b**5 + 9*a**3*b**6*x + 9*a**2*b**7*x**2 + 3*a*b**8*x**3), Eq(n,
 -4)), (12*a**4*d**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2)
+ 18*a**4*d**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d**2*x*
log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d**2*x/(2
*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a**2*b**2*c*d*log(a/b + x)/(2*a*
*2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 6*a**2*b**2*c*d/(2*a**2*b**5 + 4*a*b*
*6*x + 2*b**7*x**2) + 12*a**2*b**2*d**2*x**2*log(a/b + x)/(2*a**2*b**5 + 4*
a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b*
*6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**
2) - 4*a*b**3*d**2*x**3/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - b**4*c**
2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*b**4*c*d*x**2*log(a/b + x)/(
2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + b**4*d**2*x**4/(2*a**2*b**5 + 4*a
*b**6*x + 2*b**7*x**2), Eq(n, -3)), (-12*a**4*d**2*log(a/b + x)/(3*a*b**5 +
 3*b**6*x) - 12*a**4*d**2/(3*a*b**5 + 3*b**6*x) - 12*a**3*b*d**2*x*log(a/b
```

```
+ x)/(3*a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d*log(a/b + x)/(3*a*b**5 + 3*b*
*6*x) - 12*a**2*b**2*c*d/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d**2*x**2/(3*a
*b**5 + 3*b**6*x) - 12*a*b**3*c*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 2*
a*b**3*d**2*x**3/(3*a*b**5 + 3*b**6*x) - 3*b**4*c**2/(3*a*b**5 + 3*b**6*x)
+ 6*b**4*c*d*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d**2*x**4/(3*a*b**5 + 3*b**6
*x), Eq(n, -2)), (a**4*d**2*log(a/b + x)/b**5 - a**3*d**2*x/b**4 + 2*a**2*c
*d*log(a/b + x)/b**3 + a**2*d**2*x**2/(2*b**3) - 2*a*c*d*x/b**2 - a*d**2*x*
*3/(3*b**2) + c**2*log(a/b + x)/b + c*d*x**2/b + d**2*x**4/(4*b), Eq(n, -1)
), (24*a**5*d**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 22
5*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d**2*n*x*(a + b*x)**n/(b**
5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**
5) + 4*a**3*b**2*c*d*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 36*a**3*b**2*c*d*n*(a + b*x
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
+ 120*b**5) + 80*a**3*b**2*c*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d**2*n**2
*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d**2*n*x**2*(a + b*x)**n/(b**5*n**
5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) -
4*a**2*b**3*c*d*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**
3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 36*a**2*b**3*c*d*n**2*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) - 80*a**2*b**3*c*d*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d**2
*n**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5
*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*d**2*n**2*x**3*(a + b*x)**n/(
b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*
b**5) - 8*a**2*b**3*d**2*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85
*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c**2*n**4*(a +
 b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**
5*n + 120*b**5) + 14*a*b**4*c**2*n**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 71*a*b**4*c**2*
n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 154*a*b**4*c**2*n*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 120*a*b*
*4*c**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 2*a*b**4*c*d*n**4*x**2*(a + b*x)**n/(b**5*n**
5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
20*a*b**4*c*d*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n*
*3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 58*a*b**4*c*d*n**2*x**2*(a +
b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) + 40*a*b**4*c*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**
4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d**2*n**
4*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**5*n**
5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
11*a*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n
**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d**2*n*x**4*(a + b*
x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
 + 120*b**5) + b**5*c**2*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85
*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 14*b**5*c**2*n**3*x*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) + 71*b**5*c**2*n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 154*b**5*c**
2*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
 + 274*b**5*n + 120*b**5) + 120*b**5*c**2*x*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 2*b**5*c*
d*n**4*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**
5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*c*d*n**3*x**3*(a + b*x)**n/(b**5*
```

```
n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
 + 98*b**5*c*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n
**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 156*b**5*c*d*n*x**3*(a + b*x
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
+ 120*b**5) + 80*b**5*c*d*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d**2*n**4*x**5*(a
 + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b
**5*n + 120*b**5) + 10*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*
*2*n**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b*
*5*n**2 + 274*b**5*n + 120*b**5) + 50*b**5*d**2*n*x**5*(a + b*x)**n/(b**5*n
**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
+ 24*b**5*d**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5), True))
```

Giac [B] time = 1.13712, size = 1149, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d^2*n^4*x^5 + (b*x + a)^n*a*b^4*d^2*n^4*x^4 + 10*(b*x + a)
^n*b^5*d^2*n^3*x^5 + 2*(b*x + a)^n*b^5*c*d*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d^
2*n^3*x^4 + 35*(b*x + a)^n*b^5*d^2*n^2*x^5 + 2*(b*x + a)^n*a*b^4*c*d*n^4*x^
2 + 24*(b*x + a)^n*b^5*c*d*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d^2*n^3*x^3 + 11
*(b*x + a)^n*a*b^4*d^2*n^2*x^4 + 50*(b*x + a)^n*b^5*d^2*n*x^5 + (b*x + a)^n
*b^5*c^2*n^4*x + 20*(b*x + a)^n*a*b^4*c*d*n^3*x^2 + 98*(b*x + a)^n*b^5*c*d*
n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d^2*n^2*x^3 + 6*(b*x + a)^n*a*b^4*d^2*n*x^
4 + 24*(b*x + a)^n*b^5*d^2*x^5 + (b*x + a)^n*a*b^4*c^2*n^4 + 14*(b*x + a)^n
*b^5*c^2*n^3*x - 4*(b*x + a)^n*a^2*b^3*c*d*n^3*x + 58*(b*x + a)^n*a*b^4*c*d
*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n^2*x^2 + 156*(b*x + a)^n*b^5*c*d*n*x
^3 - 8*(b*x + a)^n*a^2*b^3*d^2*n*x^3 + 14*(b*x + a)^n*a*b^4*c^2*n^3 + 71*(b
*x + a)^n*b^5*c^2*n^2*x - 36*(b*x + a)^n*a^2*b^3*c*d*n^2*x + 40*(b*x + a)^n
*a*b^4*c*d*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n*x^2 + 80*(b*x + a)^n*b^5*c*
d*x^3 + 71*(b*x + a)^n*a*b^4*c^2*n^2 + 4*(b*x + a)^n*a^3*b^2*c*d*n^2 + 154*
(b*x + a)^n*b^5*c^2*n*x - 80*(b*x + a)^n*a^2*b^3*c*d*n*x - 24*(b*x + a)^n*a
^4*b*d^2*n*x + 154*(b*x + a)^n*a*b^4*c^2*n + 36*(b*x + a)^n*a^3*b^2*c*d*n +
 120*(b*x + a)^n*b^5*c^2*x + 120*(b*x + a)^n*a*b^4*c^2 + 80*(b*x + a)^n*a^3
*b^2*c*d + 24*(b*x + a)^n*a^5*d^2)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225
*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.358 $\int \frac{(a+bx)^n(c+dx^2)^2}{x}\,dx$

Optimal. Leaf size=148

$$-\frac{ad\left(a^2d+2b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{d\left(3a^2d+2b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)}+\frac{d^2(a+bx)^{n+4}}{b^4(n+4)}-\frac{c^2(a+bx)^{n+1}\,{}_2F_1\left(1,n\right.}{a(n+}$$

```
[Out] -((a*d*(2*b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (d*(2*b^2*c +
3*a^2*d)*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^(3 + n))/(b^
4*(3 + n)) + (d^2*(a + b*x)^(4 + n))/(b^4*(4 + n)) - (c^2*(a + b*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rubi [A] time = 0.209861, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {952, 1620, 65}

$$-\frac{ad\left(a^2d+2b^2c\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{d\left(3a^2d+2b^2c\right)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)}+\frac{d^2(a+bx)^{n+4}}{b^4(n+4)}-\frac{c^2(a+bx)^{n+1}\,{}_2F_1\left(1,n\right.}{a(n+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2)^2)/x,x]
```

```
[Out] -((a*d*(2*b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (d*(2*b^2*c +
3*a^2*d)*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^(3 + n))/(b^
4*(3 + n)) + (d^2*(a + b*x)^(4 + n))/(b^4*(4 + n)) - (c^2*(a + b*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n \left(c+dx^2\right)^2}{x} dx = \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + \frac{\int \frac{(a+bx)^n \left(b^4c^2(4+n)-a^3bd^2(4+n)x+b^2d\left(2b^2c-3a^2d\right)(4+n)x^2-3ab^3d^2(4+n)x^3\right)}{x} dx}{b^4(4+n)}$$

$$= \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + \frac{\int \left(-abd\left(2b^2c+a^2d\right)(4+n)(a+bx)^n + \frac{\left(4b^4c^2+b^4c^2n\right)(a+bx)^n}{x} + bd\left(2b^2c+\right.\right.}{b^4(4+n)}$$

$$= -\frac{ad\left(2b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)} + \frac{d\left(2b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)} + \frac{d^2(a+}{b^4(4}$$

$$= -\frac{ad\left(2b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)} + \frac{d\left(2b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)} + \frac{d^2(a+}{b^4(4}$$

Mathematica [A] time = 0.142298, size = 132, normalized size = 0.89

$$(a+bx)^{n+1}\left(\frac{d(a+bx)\left(3a^2d+2b^2c\right)}{b^4(n+2)} - \frac{ad\left(a^2d+2b^2c\right)}{b^4(n+1)} + \frac{d^2(a+bx)^3}{b^4(n+4)} - \frac{3ad^2(a+bx)^2}{b^4(n+3)} - \frac{c^2\,{}_2F_1\left(1,n+1;n+2;\frac{a+bx}{a}\right)}{an+a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2)^2)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*(-((a*d*(2*b^2*c + a^2*d))/(b^4*(1 + n))) + (d*(2*b^2*c +
 3*a^2*d)*(a + b*x))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^2)/(b^4*(3 + n)) +
(d^2*(a + b*x)^3)/(b^4*(4 + n)) - (c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*x)/a])/(a + a*n))
```

Maple [F] time = 0.53, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n \left(dx^2+c\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^2/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^2+c)^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^2 (bx+a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^4 + 2\,cdx^2 + c^2\right)(bx + a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x + a)^n/x, x)
```

Sympy [B] time = 11.2243, size = 1678, normalized size = 11.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2/x,x)
```

```
[Out] -b**n*c**2*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**2*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 2*c*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*
b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x) - b*x/(a*b**2 + b**3*x)
, Eq(n, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/
(b**2*n**2 + 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*
n + 2*b**2) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b*
*2*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + d**2*Piecewi
se((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**
5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 5*a**3/(6*a**3*b**4 + 18*a**2*b**5*x
+ 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 9*a**2*b*x/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a/b + x)
/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*x**
3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3
) - 6*b**3*x**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**
3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x*
*2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x*log(a/b
 + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a**2*b**4 +
 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*
b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2
), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3/(2*a*b**
4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*x*
*2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a
**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b), Eq(n, -
1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b*
*4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*
b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b*x)**n/(b**4
*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n*
x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24
*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n*x**3*(a + b
*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + b*
*4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**
4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)**n/(b**4*n**
```

```
4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*x**4*(a + b
*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4), Tru
e)) - b*b**n*c**2*n*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n +
1)/(a*gamma(n + 2)) - b*b**n*c**2*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n +
 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^2(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

3.359 $\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=343

$$\frac{\left(a^2d+b^2c\right)\left(17a^2b^2cd+28a^4d^2+b^4c^2\right)(a+bx)^{n+3}}{b^9(n+3)}-\frac{4ad\left(15a^2b^2cd+14a^4d^2+3b^4c^2\right)(a+bx)^{n+4}}{b^9(n+4)}+\frac{d\left(45a^2b^2cd+70a\right.}{b^9}$$

```
[Out] (a^2*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^2*c + a^2
*d)^2*(b^2*c + 4*a^2*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^2*c + a^2*d)
*(b^4*c^2 + 17*a^2*b^2*c*d + 28*a^4*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) -
 (4*a*d*(3*b^4*c^2 + 15*a^2*b^2*c*d + 14*a^4*d^2)*(a + b*x)^(4 + n))/(b^9*(
4 + n)) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d + 70*a^4*d^2)*(a + b*x)^(5 + n))/(
b^9*(5 + n)) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)^(6 + n))/(b^9*(6 + n
)) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*
(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rubi [A] time = 0.210556, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {948}

$$\frac{\left(a^2d+b^2c\right)\left(17a^2b^2cd+28a^4d^2+b^4c^2\right)(a+bx)^{n+3}}{b^9(n+3)}-\frac{4ad\left(15a^2b^2cd+14a^4d^2+3b^4c^2\right)(a+bx)^{n+4}}{b^9(n+4)}+\frac{d\left(45a^2b^2cd+70a\right.}{b^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^2*c + a^2
*d)^2*(b^2*c + 4*a^2*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^2*c + a^2*d)
*(b^4*c^2 + 17*a^2*b^2*c*d + 28*a^4*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) -
 (4*a*d*(3*b^4*c^2 + 15*a^2*b^2*c*d + 14*a^4*d^2)*(a + b*x)^(4 + n))/(b^9*(
4 + n)) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d + 70*a^4*d^2)*(a + b*x)^(5 + n))/(
b^9*(5 + n)) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)^(6 + n))/(b^9*(6 + n
)) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*
(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx=\int\left(\frac{a^2\left(b^2c+a^2d\right)^3(a+bx)^n}{b^8}-\frac{2a\left(b^2c+a^2d\right)^2\left(b^2c+4a^2d\right)(a+bx)^{1+n}}{b^8}+\frac{\left(b^2c+a^2d\right)\left(b\right.}{}\right.$$

$$=\frac{a^2\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a\left(b^2c+a^2d\right)^2\left(b^2c+4a^2d\right)(a+bx)^{2+n}}{b^9(2+n)}+\frac{\left(b^2c+a^2d\right)\left(b^4c\right.}{}$$

Mathematica [A] time = 0.272227, size = 302, normalized size = 0.88

$$\frac{(a+bx)^{n+1}\left(\frac{d(a+bx)^4\left(45a^2b^2cd+70a^4d^2+3b^4c^2\right)}{n+5}-\frac{4ad(a+bx)^3\left(15a^2b^2cd+14a^4d^2+3b^4c^2\right)}{n+4}+\frac{(a+bx)^2\left(a^2d+b^2c\right)\left(17a^2b^2cd+28a^4d^2+b^4c^2\right)}{n+3}+\frac{d^2(a+bx)}{}\right.}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^2*c + a^2*d)^3)/(1 + n) - (2*a*(b^2*c + a^2*d)^
2*(b^2*c + 4*a^2*d)*(a + b*x))/(2 + n) + ((b^2*c + a^2*d)*(b^4*c^2 + 17*a^2
*b^2*c*d + 28*a^4*d^2)*(a + b*x)^2)/(3 + n) - (4*a*d*(3*b^4*c^2 + 15*a^2*b^
2*c*d + 14*a^4*d^2)*(a + b*x)^3)/(4 + n) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d +
 70*a^4*d^2)*(a + b*x)^4)/(5 + n) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)
^5)/(6 + n) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^6)/(7 + n) - (8*a*d^3*(a
+ b*x)^7)/(8 + n) + (d^3*(a + b*x)^8)/(9 + n)))/b^9
```

Maple [B] time = 0.065, size = 2232, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] (b*x+a)^(1+n)*(b^8*d^3*n^8*x^8+36*b^8*d^3*n^7*x^8-8*a*b^7*d^3*n^7*x^7+3*b^8
*c*d^2*n^8*x^6+546*b^8*d^3*n^6*x^8-224*a*b^7*d^3*n^6*x^7+114*b^8*c*d^2*n^7*
x^6+4536*b^8*d^3*n^5*x^8+56*a^2*b^6*d^3*n^6*x^6-18*a*b^7*c*d^2*n^7*x^5-2576
*a*b^7*d^3*n^5*x^7+3*b^8*c^2*d*n^8*x^4+1812*b^8*c*d^2*n^6*x^6+22449*b^8*d^3
*n^4*x^8+1176*a^2*b^6*d^3*n^5*x^6-576*a*b^7*c*d^2*n^6*x^5-15680*a*b^7*d^3*n
^4*x^7+120*b^8*c^2*d*n^7*x^4+15666*b^8*c*d^2*n^5*x^6+67284*b^8*d^3*n^3*x^8-
336*a^3*b^5*d^3*n^5*x^5+90*a^2*b^6*c*d^2*n^6*x^4+9800*a^2*b^6*d^3*n^4*x^6-1
2*a*b^7*c^2*d*n^7*x^3-7416*a*b^7*c*d^2*n^5*x^5-54152*a*b^7*d^3*n^3*x^7+b^8*
c^3*n^8*x^2+2010*b^8*c^2*d*n^6*x^4+80157*b^8*c*d^2*n^4*x^6+118124*b^8*d^3*n
^2*x^8-5040*a^3*b^5*d^3*n^4*x^5+2430*a^2*b^6*c*d^2*n^5*x^4+41160*a^2*b^6*d^
3*n^3*x^6-432*a*b^7*c^2*d*n^6*x^3-49500*a*b^7*c*d^2*n^4*x^5-105056*a*b^7*d^
3*n^2*x^7+42*b^8*c^3*n^7*x^2+18300*b^8*c^2*d*n^5*x^4+246876*b^8*c*d^2*n^3*x
^6+109584*b^8*d^3*n*x^8+1680*a^4*b^4*d^3*n^4*x^4-360*a^3*b^5*c*d^2*n^5*x^3-
28560*a^3*b^5*d^3*n^3*x^5+36*a^2*b^6*c^2*d*n^6*x^2+24930*a^2*b^6*c*d^2*n^4*
x^4+90944*a^2*b^6*d^3*n^2*x^6-2*a*b^7*c^3*n^7*x-6312*a*b^7*c^2*d*n^5*x^3-18
3942*a*b^7*c*d^2*n^3*x^5-104544*a*b^7*d^3*n*x^7+744*b^8*c^3*n^6*x^2+98319*b
^8*c^2*d*n^4*x^4+442908*b^8*c*d^2*n^2*x^6+40320*b^8*d^3*x^8+16800*a^4*b^4*d
^3*n^3*x^4-8280*a^3*b^5*c*d^2*n^4*x^3-75600*a^3*b^5*d^3*n^2*x^5+1188*a^2*b^
6*c^2*d*n^5*x^2+122850*a^2*b^6*c*d^2*n^3*x^4+98784*a^2*b^6*d^3*n*x^6-80*a*b
^7*c^3*n^6*x-47952*a*b^7*c^2*d*n^4*x^3-377604*a*b^7*c*d^2*n^2*x^5-40320*a*b
^7*d^3*x^7+7218*b^8*c^3*n^5*x^2+316380*b^8*c^2*d*n^3*x^4+417744*b^8*c*d^2*n
*x^6-6720*a^5*b^3*d^3*n^3*x^3+1080*a^4*b^4*c*d^2*n^4*x^2+58800*a^4*b^4*d^3*
n^2*x^4-72*a^3*b^5*c^2*d*n^5*x-66600*a^3*b^5*c*d^2*n^3*x^3-92064*a^3*b^5*d^
3*n*x^5+2*a^2*b^6*c^3*n^6+15372*a^2*b^6*c^2*d*n^4*x^2+305460*a^2*b^6*c*d^2*
n^2*x^4+40320*a^2*b^6*d^3*x^6-1328*a*b^7*c^3*n^5*x-201468*a*b^7*c^2*d*n^3*x
^3-391824*a*b^7*c*d^2*n*x^5+41619*b^8*c^3*n^4*x^2+589140*b^8*c^2*d*n^2*x^4+
155520*b^8*c*d^2*x^6-40320*a^5*b^3*d^3*n^2*x^3+21600*a^4*b^4*c*d^2*n^3*x^2+
84000*a^4*b^4*d^3*n*x^4-2232*a^3*b^5*c^2*d*n^4*x-225000*a^3*b^5*c*d^2*n^2*x
^3-40320*a^3*b^5*d^3*x^5+78*a^2*b^6*c^3*n^5+97740*a^2*b^6*c^2*d*n^3*x^2+360
720*a^2*b^6*c*d^2*n*x^4-11780*a*b^7*c^3*n^4*x-459648*a*b^7*c^2*d*n^2*x^3-15
```

```
5520*a*b^7*c*d^2*x^5+144468*b^8*c^3*n^3*x^2+572400*b^8*c^2*d*n*x^4+20160*a^
6*b^2*d^3*n^2*x^2-2160*a^5*b^3*c*d^2*n^3*x-73920*a^5*b^3*d^3*n*x^3+72*a^4*b
^4*c^2*d*n^4+135000*a^4*b^4*c*d^2*n^2*x^2+40320*a^4*b^4*d^3*x^4-26280*a^3*b
^5*c^2*d*n^3*x-321840*a^3*b^5*c*d^2*n*x^3+1250*a^2*b^6*c^3*n^4+311184*a^2*b
^6*c^2*d*n^2*x^2+155520*a^2*b^6*c*d^2*x^4-59678*a*b^7*c^3*n^3*x-517968*a*b^
7*c^2*d*n*x^3+290276*b^8*c^3*n^2*x^2+217728*b^8*c^2*d*x^4+60480*a^6*b^2*d^3
*n*x^2-38880*a^5*b^3*c*d^2*n^2*x-40320*a^5*b^3*d^3*x^3+2160*a^4*b^4*c^2*d*n
^3+270000*a^4*b^4*c*d^2*n*x^2-142920*a^3*b^5*c^2*d*n^2*x-155520*a^3*b^5*c*d
^2*x^3+10530*a^2*b^6*c^3*n^3+445392*a^2*b^6*c^2*d*n*x^2-169580*a*b^7*c^3*n^
2*x-217728*a*b^7*c^2*d*x^3+301872*b^8*c^3*n*x^2-40320*a^7*b*d^3*n*x+2160*a^
6*b^2*c*d^2*n^2+40320*a^6*b^2*d^3*x^2-192240*a^5*b^3*c*d^2*n*x+24120*a^4*b^
4*c^2*d*n^2+155520*a^4*b^4*c*d^2*x^2-336528*a^3*b^5*c^2*d*n*x+49148*a^2*b^6
*c^3*n^2+217728*a^2*b^6*c^2*d*x^2-241392*a*b^7*c^3*n*x+120960*b^8*c^3*x^2-4
0320*a^7*b*d^3*x+36720*a^6*b^2*c*d^2*n-155520*a^5*b^3*c*d^2*x+118800*a^4*b^
4*c^2*d*n-217728*a^3*b^5*c^2*d*x+120432*a^2*b^6*c^3*n-120960*a*b^7*c^3*x+40
320*a^8*d^3+155520*a^6*b^2*c*d^2+217728*a^4*b^4*c^2*d+120960*a^2*b^6*c^3)/b
^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+723680*n^3+1172700*n^2
+1026576*n+362880)
```

Maxima [B] time = 1.0823, size = 1073, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + 3*((n^
6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 +
15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 +
35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b
^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 -
720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*
n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^8 + 36*n^7 + 546*n^
6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^9*x
^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2
+ 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764
*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x
^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*
n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)
*(b*x + a)^n*d^3/((n^9 + 45*n^8 + 870*n^7 + 9450*n^6 + 63273*n^5 + 269325*n
^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)*b^9)
```

Fricas [B] time = 2.2272, size = 4895, normalized size = 14.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^3*n^6 + 78*a^3*b^6*c^3*n^5 + 120960*a^3*b^6*c^3 + 217728*a^5*b
^4*c^2*d + 155520*a^7*b^2*c*d^2 + 40320*a^9*d^3 + (b^9*d^3*n^8 + 36*b^9*d^3
*n^7 + 546*b^9*d^3*n^6 + 4536*b^9*d^3*n^5 + 22449*b^9*d^3*n^4 + 67284*b^9*d
^3*n^3 + 118124*b^9*d^3*n^2 + 109584*b^9*d^3*n + 40320*b^9*d^3)*x^9 + (a*b^
8*d^3*n^8 + 28*a*b^8*d^3*n^7 + 322*a*b^8*d^3*n^6 + 1960*a*b^8*d^3*n^5 + 676
9*a*b^8*d^3*n^4 + 13132*a*b^8*d^3*n^3 + 13068*a*b^8*d^3*n^2 + 5040*a*b^8*d^
3*n)*x^8 + (3*b^9*c*d^2*n^8 + 155520*b^9*c*d^2 + 2*(57*b^9*c*d^2 - 4*a^2*b^
7*d^3)*n^7 + 12*(151*b^9*c*d^2 - 14*a^2*b^7*d^3)*n^6 + 14*(1119*b^9*c*d^2 -
 100*a^2*b^7*d^3)*n^5 + 21*(3817*b^9*c*d^2 - 280*a^2*b^7*d^3)*n^4 + 28*(881
7*b^9*c*d^2 - 464*a^2*b^7*d^3)*n^3 + 36*(12303*b^9*c*d^2 - 392*a^2*b^7*d^3)
*n^2 + 144*(2901*b^9*c*d^2 - 40*a^2*b^7*d^3)*n)*x^7 + (3*a*b^8*c*d^2*n^8 +
96*a*b^8*c*d^2*n^7 + 4*(309*a*b^8*c*d^2 + 14*a^3*b^6*d^3)*n^6 + 30*(275*a*b
^8*c*d^2 + 28*a^3*b^6*d^3)*n^5 + (30657*a*b^8*c*d^2 + 4760*a^3*b^6*d^3)*n^4
 + 6*(10489*a*b^8*c*d^2 + 2100*a^3*b^6*d^3)*n^3 + 8*(8163*a*b^8*c*d^2 + 191
8*a^3*b^6*d^3)*n^2 + 960*(27*a*b^8*c*d^2 + 7*a^3*b^6*d^3)*n)*x^6 + 3*(b^9*c
^2*d*n^8 + 72576*b^9*c^2*d + 2*(20*b^9*c^2*d - 3*a^2*b^7*c*d^2)*n^7 + 2*(33
5*b^9*c^2*d - 81*a^2*b^7*c*d^2)*n^6 + 2*(3050*b^9*c^2*d - 831*a^2*b^7*c*d^2
 - 56*a^4*b^5*d^3)*n^5 + (32773*b^9*c^2*d - 8190*a^2*b^7*c*d^2 - 1120*a^4*b
^5*d^3)*n^4 + 4*(26365*b^9*c^2*d - 5091*a^2*b^7*c*d^2 - 980*a^4*b^5*d^3)*n^
3 + 4*(49095*b^9*c^2*d - 6012*a^2*b^7*c*d^2 - 1400*a^4*b^5*d^3)*n^2 + 48*(3
975*b^9*c^2*d - 216*a^2*b^7*c*d^2 - 56*a^4*b^5*d^3)*n)*x^5 + 2*(625*a^3*b^6
*c^3 + 36*a^5*b^4*c^2*d)*n^4 + 3*(a*b^8*c^2*d*n^8 + 36*a*b^8*c^2*d*n^7 + 2*
(263*a*b^8*c^2*d + 15*a^3*b^6*c*d^2)*n^6 + 6*(666*a*b^8*c^2*d + 115*a^3*b^6
*c*d^2)*n^5 + (16789*a*b^8*c^2*d + 5550*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^
4 + 6*(6384*a*b^8*c^2*d + 3125*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^3 + 4*(10
791*a*b^8*c^2*d + 6705*a^3*b^6*c*d^2 + 1540*a^5*b^4*d^3)*n^2 + 96*(189*a*b^
8*c^2*d + 135*a^3*b^6*c*d^2 + 35*a^5*b^4*d^3)*n)*x^4 + 270*(39*a^3*b^6*c^3
+ 8*a^5*b^4*c^2*d)*n^3 + (b^9*c^3*n^8 + 120960*b^9*c^3 + 6*(7*b^9*c^3 - 2*a
^2*b^7*c^2*d)*n^7 + 12*(62*b^9*c^3 - 33*a^2*b^7*c^2*d)*n^6 + 6*(1203*b^9*c^
3 - 854*a^2*b^7*c^2*d - 60*a^4*b^5*c*d^2)*n^5 + 3*(13873*b^9*c^3 - 10860*a^
2*b^7*c^2*d - 2400*a^4*b^5*c*d^2)*n^4 + 12*(12039*b^9*c^3 - 8644*a^2*b^7*c^
2*d - 3750*a^4*b^5*c*d^2 - 560*a^6*b^3*d^3)*n^3 + 4*(72569*b^9*c^3 - 37116*
a^2*b^7*c^2*d - 22500*a^4*b^5*c*d^2 - 5040*a^6*b^3*d^3)*n^2 + 48*(6289*b^9*
c^3 - 1512*a^2*b^7*c^2*d - 1080*a^4*b^5*c*d^2 - 280*a^6*b^3*d^3)*n)*x^3 + 4
*(12287*a^3*b^6*c^3 + 6030*a^5*b^4*c^2*d + 540*a^7*b^2*c*d^2)*n^2 + (a*b^8*
c^3*n^8 + 40*a*b^8*c^3*n^7 + 4*(166*a*b^8*c^3 + 9*a^3*b^6*c^2*d)*n^6 + 62*(
95*a*b^8*c^3 + 18*a^3*b^6*c^2*d)*n^5 + (29839*a*b^8*c^3 + 13140*a^3*b^6*c^2
*d + 1080*a^5*b^4*c*d^2)*n^4 + 10*(8479*a*b^8*c^3 + 7146*a^3*b^6*c^2*d + 19
44*a^5*b^4*c*d^2)*n^3 + 24*(5029*a*b^8*c^3 + 7011*a^3*b^6*c^2*d + 4005*a^5*
b^4*c*d^2 + 840*a^7*b^2*d^3)*n^2 + 576*(105*a*b^8*c^3 + 189*a^3*b^6*c^2*d +
 135*a^5*b^4*c*d^2 + 35*a^7*b^2*d^3)*n)*x^2 + 48*(2509*a^3*b^6*c^3 + 2475*a
^5*b^4*c^2*d + 765*a^7*b^2*c*d^2)*n - 2*(a^2*b^7*c^3*n^7 + 39*a^2*b^7*c^3*n
^6 + (625*a^2*b^7*c^3 + 36*a^4*b^5*c^2*d)*n^5 + 135*(39*a^2*b^7*c^3 + 8*a^4
*b^5*c^2*d)*n^4 + 2*(12287*a^2*b^7*c^3 + 6030*a^4*b^5*c^2*d + 540*a^6*b^3*c
*d^2)*n^3 + 24*(2509*a^2*b^7*c^3 + 2475*a^4*b^5*c^2*d + 765*a^6*b^3*c*d^2)*
n^2 + 576*(105*a^2*b^7*c^3 + 189*a^4*b^5*c^2*d + 135*a^6*b^3*c*d^2 + 35*a^8
*b*d^3)*n)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^
6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 102
6576*b^9*n + 362880*b^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25576, size = 5013, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^3*n^8*x^9 + (b*x + a)^n*a*b^8*d^3*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^3*n^7*x^9 + 3*(b*x + a)^n*b^9*c*d^2*n^8*x^7 + 28*(b*x + a)^n*a*b^8
*d^3*n^7*x^8 + 546*(b*x + a)^n*b^9*d^3*n^6*x^9 + 3*(b*x + a)^n*a*b^8*c*d^2*
n^8*x^6 + 114*(b*x + a)^n*b^9*c*d^2*n^7*x^7 - 8*(b*x + a)^n*a^2*b^7*d^3*n^7
*x^7 + 322*(b*x + a)^n*a*b^8*d^3*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^3*n^5*x^9
 + 3*(b*x + a)^n*b^9*c^2*d*n^8*x^5 + 96*(b*x + a)^n*a*b^8*c*d^2*n^7*x^6 + 1
812*(b*x + a)^n*b^9*c*d^2*n^6*x^7 - 168*(b*x + a)^n*a^2*b^7*d^3*n^6*x^7 + 1
960*(b*x + a)^n*a*b^8*d^3*n^5*x^8 + 22449*(b*x + a)^n*b^9*d^3*n^4*x^9 + 3*(
b*x + a)^n*a*b^8*c^2*d*n^8*x^4 + 120*(b*x + a)^n*b^9*c^2*d*n^7*x^5 - 18*(b*
x + a)^n*a^2*b^7*c*d^2*n^7*x^5 + 1236*(b*x + a)^n*a*b^8*c*d^2*n^6*x^6 + 56*
(b*x + a)^n*a^3*b^6*d^3*n^6*x^6 + 15666*(b*x + a)^n*b^9*c*d^2*n^5*x^7 - 140
0*(b*x + a)^n*a^2*b^7*d^3*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^3*n^4*x^8 + 67
284*(b*x + a)^n*b^9*d^3*n^3*x^9 + (b*x + a)^n*b^9*c^3*n^8*x^3 + 108*(b*x +
a)^n*a*b^8*c^2*d*n^7*x^4 + 2010*(b*x + a)^n*b^9*c^2*d*n^6*x^5 - 486*(b*x +
a)^n*a^2*b^7*c*d^2*n^6*x^5 + 8250*(b*x + a)^n*a*b^8*c*d^2*n^5*x^6 + 840*(b*
x + a)^n*a^3*b^6*d^3*n^5*x^6 + 80157*(b*x + a)^n*b^9*c*d^2*n^4*x^7 - 5880*(
b*x + a)^n*a^2*b^7*d^3*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^3*n^3*x^8 + 1181
24*(b*x + a)^n*b^9*d^3*n^2*x^9 + (b*x + a)^n*a*b^8*c^3*n^8*x^2 + 42*(b*x +
a)^n*b^9*c^3*n^7*x^3 - 12*(b*x + a)^n*a^2*b^7*c^2*d*n^7*x^3 + 1578*(b*x + a
)^n*a*b^8*c^2*d*n^6*x^4 + 90*(b*x + a)^n*a^3*b^6*c*d^2*n^6*x^4 + 18300*(b*x
 + a)^n*b^9*c^2*d*n^5*x^5 - 4986*(b*x + a)^n*a^2*b^7*c*d^2*n^5*x^5 - 336*(b
*x + a)^n*a^4*b^5*d^3*n^5*x^5 + 30657*(b*x + a)^n*a*b^8*c*d^2*n^4*x^6 + 476
0*(b*x + a)^n*a^3*b^6*d^3*n^4*x^6 + 246876*(b*x + a)^n*b^9*c*d^2*n^3*x^7 -
12992*(b*x + a)^n*a^2*b^7*d^3*n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^3*n^2*x^8
 + 109584*(b*x + a)^n*b^9*d^3*n*x^9 + 40*(b*x + a)^n*a*b^8*c^3*n^7*x^2 + 74
4*(b*x + a)^n*b^9*c^3*n^6*x^3 - 396*(b*x + a)^n*a^2*b^7*c^2*d*n^6*x^3 + 119
88*(b*x + a)^n*a*b^8*c^2*d*n^5*x^4 + 2070*(b*x + a)^n*a^3*b^6*c*d^2*n^5*x^4
 + 98319*(b*x + a)^n*b^9*c^2*d*n^4*x^5 - 24570*(b*x + a)^n*a^2*b^7*c*d^2*n^
4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^3*n^4*x^5 + 62934*(b*x + a)^n*a*b^8*c*d^
2*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^3*n^3*x^6 + 442908*(b*x + a)^n*b^9*
c*d^2*n^2*x^7 - 14112*(b*x + a)^n*a^2*b^7*d^3*n^2*x^7 + 5040*(b*x + a)^n*a*
b^8*d^3*n*x^8 + 40320*(b*x + a)^n*b^9*d^3*x^9 - 2*(b*x + a)^n*a^2*b^7*c^3*n
^7*x + 664*(b*x + a)^n*a*b^8*c^3*n^6*x^2 + 36*(b*x + a)^n*a^3*b^6*c^2*d*n^6
*x^2 + 7218*(b*x + a)^n*b^9*c^3*n^5*x^3 - 5124*(b*x + a)^n*a^2*b^7*c^2*d*n^
5*x^3 - 360*(b*x + a)^n*a^4*b^5*c*d^2*n^5*x^3 + 50367*(b*x + a)^n*a*b^8*c^2
*d*n^4*x^4 + 16650*(b*x + a)^n*a^3*b^6*c*d^2*n^4*x^4 + 1680*(b*x + a)^n*a^5
*b^4*d^3*n^4*x^4 + 316380*(b*x + a)^n*b^9*c^2*d*n^3*x^5 - 61092*(b*x + a)^n
*a^2*b^7*c*d^2*n^3*x^5 - 11760*(b*x + a)^n*a^4*b^5*d^3*n^3*x^5 + 65304*(b*x
 + a)^n*a*b^8*c*d^2*n^2*x^6 + 15344*(b*x + a)^n*a^3*b^6*d^3*n^2*x^6 + 41774
4*(b*x + a)^n*b^9*c*d^2*n*x^7 - 5760*(b*x + a)^n*a^2*b^7*d^3*n*x^7 - 78*(b*
x + a)^n*a^2*b^7*c^3*n^6*x + 5890*(b*x + a)^n*a*b^8*c^3*n^5*x^2 + 1116*(b*x
 + a)^n*a^3*b^6*c^2*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^3*n^4*x^3 - 32580*(
b*x + a)^n*a^2*b^7*c^2*d*n^4*x^3 - 7200*(b*x + a)^n*a^4*b^5*c*d^2*n^4*x^3 +
 114912*(b*x + a)^n*a*b^8*c^2*d*n^3*x^4 + 56250*(b*x + a)^n*a^3*b^6*c*d^2*n
^3*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n^3*x^4 + 589140*(b*x + a)^n*b^9*c^2
*d*n^2*x^5 - 72144*(b*x + a)^n*a^2*b^7*c*d^2*n^2*x^5 - 16800*(b*x + a)^n*a^
4*b^5*d^3*n^2*x^5 + 25920*(b*x + a)^n*a*b^8*c*d^2*n*x^6 + 6720*(b*x + a)^n*
```

a^3*b^6*d^3*n*x^6 + 155520*(b*x + a)^n*b^9*c*d^2*x^7 + 2*(b*x + a)^n*a^3*b^
6*c^3*n^6 - 1250*(b*x + a)^n*a^2*b^7*c^3*n^5*x - 72*(b*x + a)^n*a^4*b^5*c^2
*d*n^5*x + 29839*(b*x + a)^n*a*b^8*c^3*n^4*x^2 + 13140*(b*x + a)^n*a^3*b^6*
c^2*d*n^4*x^2 + 1080*(b*x + a)^n*a^5*b^4*c*d^2*n^4*x^2 + 144468*(b*x + a)^n
*b^9*c^3*n^3*x^3 - 103728*(b*x + a)^n*a^2*b^7*c^2*d*n^3*x^3 - 45000*(b*x +
a)^n*a^4*b^5*c*d^2*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^3*n^3*x^3 + 129492*
(b*x + a)^n*a*b^8*c^2*d*n^2*x^4 + 80460*(b*x + a)^n*a^3*b^6*c*d^2*n^2*x^4 +
 18480*(b*x + a)^n*a^5*b^4*d^3*n^2*x^4 + 572400*(b*x + a)^n*b^9*c^2*d*n*x^5
 - 31104*(b*x + a)^n*a^2*b^7*c*d^2*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^3*n*x
^5 + 78*(b*x + a)^n*a^3*b^6*c^3*n^5 - 10530*(b*x + a)^n*a^2*b^7*c^3*n^4*x -
 2160*(b*x + a)^n*a^4*b^5*c^2*d*n^4*x + 84790*(b*x + a)^n*a*b^8*c^3*n^3*x^2
 + 71460*(b*x + a)^n*a^3*b^6*c^2*d*n^3*x^2 + 19440*(b*x + a)^n*a^5*b^4*c*d^
2*n^3*x^2 + 290276*(b*x + a)^n*b^9*c^3*n^2*x^3 - 148464*(b*x + a)^n*a^2*b^7
*c^2*d*n^2*x^3 - 90000*(b*x + a)^n*a^4*b^5*c*d^2*n^2*x^3 - 20160*(b*x + a)^
n*a^6*b^3*d^3*n^2*x^3 + 54432*(b*x + a)^n*a*b^8*c^2*d*n*x^4 + 38880*(b*x +
a)^n*a^3*b^6*c*d^2*n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n*x^4 + 217728*(b*
x + a)^n*b^9*c^2*d*x^5 + 1250*(b*x + a)^n*a^3*b^6*c^3*n^4 + 72*(b*x + a)^n*
a^5*b^4*c^2*d*n^4 - 49148*(b*x + a)^n*a^2*b^7*c^3*n^3*x - 24120*(b*x + a)^n
*a^4*b^5*c^2*d*n^3*x - 2160*(b*x + a)^n*a^6*b^3*c*d^2*n^3*x + 120696*(b*x +
 a)^n*a*b^8*c^3*n^2*x^2 + 168264*(b*x + a)^n*a^3*b^6*c^2*d*n^2*x^2 + 96120*
(b*x + a)^n*a^5*b^4*c*d^2*n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^3*n^2*x^2 +
 301872*(b*x + a)^n*b^9*c^3*n*x^3 - 72576*(b*x + a)^n*a^2*b^7*c^2*d*n*x^3 -
 51840*(b*x + a)^n*a^4*b^5*c*d^2*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d^3*n*x^
3 + 10530*(b*x + a)^n*a^3*b^6*c^3*n^3 + 2160*(b*x + a)^n*a^5*b^4*c^2*d*n^3
 - 120432*(b*x + a)^n*a^2*b^7*c^3*n^2*x - 118800*(b*x + a)^n*a^4*b^5*c^2*d*n
^2*x - 36720*(b*x + a)^n*a^6*b^3*c*d^2*n^2*x + 60480*(b*x + a)^n*a*b^8*c^3*
n*x^2 + 108864*(b*x + a)^n*a^3*b^6*c^2*d*n*x^2 + 77760*(b*x + a)^n*a^5*b^4*
c*d^2*n*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^3*n*x^2 + 120960*(b*x + a)^n*b^9*
c^3*x^3 + 49148*(b*x + a)^n*a^3*b^6*c^3*n^2 + 24120*(b*x + a)^n*a^5*b^4*c^2
*d*n^2 + 2160*(b*x + a)^n*a^7*b^2*c*d^2*n^2 - 120960*(b*x + a)^n*a^2*b^7*c^
3*n*x - 217728*(b*x + a)^n*a^4*b^5*c^2*d*n*x - 155520*(b*x + a)^n*a^6*b^3*c
*d^2*n*x - 40320*(b*x + a)^n*a^8*b*d^3*n*x + 120432*(b*x + a)^n*a^3*b^6*c^3
n + 118800(b*x + a)^n*a^5*b^4*c^2*d*n + 36720*(b*x + a)^n*a^7*b^2*c*d^2*n
 + 120960*(b*x + a)^n*a^3*b^6*c^3 + 217728*(b*x + a)^n*a^5*b^4*c^2*d + 1555
20*(b*x + a)^n*a^7*b^2*c*d^2 + 40320*(b*x + a)^n*a^9*d^3)/(b^9*n^9 + 45*b^9
*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680
*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)

3.360 $\int x(a+bx)^n \left(c+dx^2\right)^3 dx$

Optimal. Leaf size=282

$$\frac{d\left(30a^2b^2cd+35a^4d^2+3b^4c^2\right)(a+bx)^{n+4}}{b^8(n+4)}-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+6}}{b^8(n+6)}-\frac{a\left(a^2d+b\right.}{b}$$

```
[Out] -((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d)
^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a^2
*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2 +
30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^2*(3
*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7*a^2*
d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^8*(7 +
 n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rubi [A] time = 0.167943, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {772}

$$\frac{d\left(30a^2b^2cd+35a^4d^2+3b^4c^2\right)(a+bx)^{n+4}}{b^8(n+4)}-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+6}}{b^8(n+6)}-\frac{a\left(a^2d+b\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d)
^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a^2
*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2 +
30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^2*(3
*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7*a^2*
d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^8*(7 +
 n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\left(c+dx^2\right)^3 dx &= \int \left(-\frac{a\left(b^2c+a^2d\right)^3(a+bx)^n}{b^7}+\frac{\left(b^2c+a^2d\right)^2\left(b^2c+7a^2d\right)(a+bx)^{1+n}}{b^7}+\frac{3ad\left(-3b^2c-7a^2\right.}{}\right.\\ &= -\frac{a\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^8(1+n)}+\frac{\left(b^2c+a^2d\right)^2\left(b^2c+7a^2d\right)(a+bx)^{2+n}}{b^8(2+n)}-\frac{3ad\left(b^2c+a^2d\right)(3b}{b^8(}\end{aligned}$$

Mathematica [B] time = 1.65199, size = 709, normalized size = 2.51

$$\frac{(a+bx)^{n+1}\left(-a(n+8)\left(6(n+6)\left(a^2d+b^2c\right)\left(4(n+4)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x\right.\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7
+ n)*(a + b*x)*(c + d*x^2)^3 - a*(8 + n)*(b^6*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*(5 + n)*(6 + n)*(c + d*x^2)^3 + 6*(b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n)*(
2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*d -
 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1 +
n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(2 +
 n)*x^2))) - 6*a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(
c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2
*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d -
2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))) + 6*(1 + n)*
(a + b*x)*((b^2*c + a^2*d)*(7 + n)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(c
+ d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(
3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d - 2*
a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2))) - a*d*(2 + n)*(
a + b*x)*(b^4*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^
2*d)*(6 + n)*(2*a^2*d - 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 +
 n)*x^2)) - 4*a*d*(3 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(4 + n)*x + b^2*(5 +
 n)*(c*(6 + n) + d*(4 + n)*x^2))))))/(b^8*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(
5 + n)*(6 + n)*(7 + n)*(8 + n))
```

Maple [B] time = 0.059, size = 1639, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^7*d^3*n^7*x^7-28*b^7*d^3*n^6*x^7+7*a*b^6*d^3*n^6*x^6-3*b
^7*c*d^2*n^7*x^5-322*b^7*d^3*n^5*x^7+147*a*b^6*d^3*n^5*x^6-90*b^7*c*d^2*n^6
*x^5-1960*b^7*d^3*n^4*x^7-42*a^2*b^5*d^3*n^5*x^5+15*a*b^6*c*d^2*n^6*x^4+122
5*a*b^6*d^3*n^4*x^6-3*b^7*c^2*d*n^7*x^3-1098*b^7*c*d^2*n^5*x^5-6769*b^7*d^3
*n^3*x^7-630*a^2*b^5*d^3*n^4*x^5+375*a*b^6*c*d^2*n^5*x^4+5145*a*b^6*d^3*n^3
*x^6-96*b^7*c^2*d*n^6*x^3-7020*b^7*c*d^2*n^4*x^5-13132*b^7*d^3*n^2*x^7+210*
a^3*b^4*d^3*n^4*x^4-60*a^2*b^5*c*d^2*n^5*x^3-3570*a^2*b^5*d^3*n^3*x^5+9*a*b
^6*c^2*d*n^6*x^2+3615*a*b^6*c*d^2*n^4*x^4+11368*a*b^6*d^3*n^2*x^6-b^7*c^3*n
^7*x-1254*b^7*c^2*d*n^5*x^3-25227*b^7*c*d^2*n^3*x^5-13068*b^7*d^3*n*x^7+210
0*a^3*b^4*d^3*n^3*x^4-1260*a^2*b^5*c*d^2*n^4*x^3-9450*a^2*b^5*d^3*n^2*x^5+2
61*a*b^6*c^2*d*n^5*x^2+17025*a*b^6*c*d^2*n^3*x^4+12348*a*b^6*d^3*n*x^6-34*b
^7*c^3*n^6*x-8592*b^7*c^2*d*n^4*x^3-50490*b^7*c*d^2*n^2*x^5-5040*b^7*d^3*x^
7-840*a^4*b^3*d^3*n^3*x^3+180*a^3*b^4*c*d^2*n^4*x^2+7350*a^3*b^4*d^3*n^2*x^
4-18*a^2*b^5*c^2*d*n^5*x-9420*a^2*b^5*c*d^2*n^3*x^3-11508*a^2*b^5*d^3*n*x^5
+a*b^6*c^3*n^6+2979*a*b^6*c^2*d*n^4*x^2+41010*a*b^6*c*d^2*n^2*x^4+5040*a*b^
6*d^3*x^6-478*b^7*c^3*n^5*x-32979*b^7*c^2*d*n^3*x^3-51432*b^7*c*d^2*n*x^5-5
040*a^4*b^3*d^3*n^2*x^3+3240*a^3*b^4*c*d^2*n^3*x^2+10500*a^3*b^4*d^3*n*x^4-
486*a^2*b^5*c^2*d*n^4*x-30420*a^2*b^5*c*d^2*n^2*x^3-5040*a^2*b^5*d^3*x^5+33
*a*b^6*c^3*n^5+16839*a*b^6*c^2*d*n^3*x^2+47400*a*b^6*c*d^2*n*x^4-3580*b^7*c
^3*n^4*x-69936*b^7*c^2*d*n^2*x^3-20160*b^7*c*d^2*x^5+2520*a^5*b^2*d^3*n^2*x
^2-360*a^4*b^3*c*d^2*n^3*x-9240*a^4*b^3*d^3*n*x^3+18*a^3*b^4*c^2*d*n^4+1854
0*a^3*b^4*c*d^2*n^2*x^2+5040*a^3*b^4*d^3*x^4-4986*a^2*b^5*c^2*d*n^3*x-42360
*a^2*b^5*c*d^2*n*x^3+445*a*b^6*c^3*n^4+48420*a*b^6*c^2*d*n^2*x^2+20160*a*b^
6*c*d^2*x^4-15289*b^7*c^3*n^3*x-74628*b^7*c^2*d*n*x^3+7560*a^5*b^2*d^3*n*x^
2-5760*a^4*b^3*c*d^2*n^2*x-5040*a^4*b^3*d^3*x^3+468*a^3*b^4*c^2*d*n^3+35640
```

```
*a^3*b^4*c*d^2*n*x^2-23706*a^2*b^5*c^2*d*n^2*x-20160*a^2*b^5*c*d^2*x^3+3135
*a*b^6*c^3*n^3+64548*a*b^6*c^2*d*n*x^2-36706*b^7*c^3*n^2*x-30240*b^7*c^2*d*
x^3-5040*a^6*b*d^3*n*x+360*a^5*b^2*c*d^2*n^2+5040*a^5*b^2*d^3*x^2-25560*a^4
*b^3*c*d^2*n*x+4518*a^3*b^4*c^2*d*n^2+20160*a^3*b^4*c*d^2*x^2-49428*a^2*b^5
*c^2*d*n*x+12154*a*b^6*c^3*n^2+30240*a*b^6*c^2*d*x^2-44712*b^7*c^3*n*x-5040
*a^6*b*d^3*x+5400*a^5*b^2*c*d^2*n-20160*a^4*b^3*c*d^2*x+19188*a^3*b^4*c^2*d
*n-30240*a^2*b^5*c^2*d*x+24552*a*b^6*c^3*n-20160*b^7*c^3*x+5040*a^7*d^3+201
60*a^5*b^2*c*d^2+30240*a^3*b^4*c^2*d+20160*a*b^6*c^3)/b^8/(n^8+36*n^7+546*n
^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)
```

Maxima [B] time = 1.08426, size = 844, normalized size = 2.99

$$\frac{\left(b^2(n+1)x^2 + abnx - a^2\right)(bx+a)^n c^3}{\left(n^2+3n+2\right)b^2} + \frac{3\left(\left(n^3+6n^2+11n+6\right)b^4x^4 + \left(n^3+3n^2+2n\right)ab^3x^3 - 3\left(n^2+n\right)a^2b^2x^2 + 6a^3\right.}{\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 +
 35*n^2 + 50*n + 24)*b^4) + 3*((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 1
20)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 +
6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60
*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c*d^2/((n^6 +
 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + ((n^7 + 28*n^
6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (
n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 -
 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^
5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^
2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)
*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*d^3/((n^8 + 36*n^7 +
546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)
*b^8)
```

Fricas [B] time = 2.24505, size = 3702, normalized size = 13.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^3*n^6 + 33*a^2*b^6*c^3*n^5 + 20160*a^2*b^6*c^3 + 30240*a^4*b^4*
c^2*d + 20160*a^6*b^2*c*d^2 + 5040*a^8*d^3 - (b^8*d^3*n^7 + 28*b^8*d^3*n^6
+ 322*b^8*d^3*n^5 + 1960*b^8*d^3*n^4 + 6769*b^8*d^3*n^3 + 13132*b^8*d^3*n^2
 + 13068*b^8*d^3*n + 5040*b^8*d^3)*x^8 - (a*b^7*d^3*n^7 + 21*a*b^7*d^3*n^6
+ 175*a*b^7*d^3*n^5 + 735*a*b^7*d^3*n^4 + 1624*a*b^7*d^3*n^3 + 1764*a*b^7*d
^3*n^2 + 720*a*b^7*d^3*n)*x^7 - (3*b^8*c*d^2*n^7 + 20160*b^8*c*d^2 + (90*b^
8*c*d^2 - 7*a^2*b^6*d^3)*n^6 + 3*(366*b^8*c*d^2 - 35*a^2*b^6*d^3)*n^5 + 5*(
1404*b^8*c*d^2 - 119*a^2*b^6*d^3)*n^4 + 9*(2803*b^8*c*d^2 - 175*a^2*b^6*d^3
)*n^3 + 2*(25245*b^8*c*d^2 - 959*a^2*b^6*d^3)*n^2 + 24*(2143*b^8*c*d^2 - 35
*a^2*b^6*d^3)*n)*x^6 - 3*(a*b^7*c*d^2*n^7 + 25*a*b^7*c*d^2*n^6 + (241*a*b^7
```

```
*c*d^2 + 14*a^3*b^5*d^3)*n^5 + 5*(227*a*b^7*c*d^2 + 28*a^3*b^5*d^3)*n^4 + 2
*(1367*a*b^7*c*d^2 + 245*a^3*b^5*d^3)*n^3 + 20*(158*a*b^7*c*d^2 + 35*a^3*b^
5*d^3)*n^2 + 336*(4*a*b^7*c*d^2 + a^3*b^5*d^3)*n)*x^5 + (445*a^2*b^6*c^3 +
18*a^4*b^4*c^2*d)*n^4 - 3*(b^8*c^2*d*n^7 + 10080*b^8*c^2*d + (32*b^8*c^2*d
- 5*a^2*b^6*c*d^2)*n^6 + (418*b^8*c^2*d - 105*a^2*b^6*c*d^2)*n^5 + (2864*b^
8*c^2*d - 785*a^2*b^6*c*d^2 - 70*a^4*b^4*d^3)*n^4 + (10993*b^8*c^2*d - 2535
*a^2*b^6*c*d^2 - 420*a^4*b^4*d^3)*n^3 + 2*(11656*b^8*c^2*d - 1765*a^2*b^6*c
*d^2 - 385*a^4*b^4*d^3)*n^2 + 12*(2073*b^8*c^2*d - 140*a^2*b^6*c*d^2 - 35*a
^4*b^4*d^3)*n)*x^4 + 3*(1045*a^2*b^6*c^3 + 156*a^4*b^4*c^2*d)*n^3 - 3*(a*b^
7*c^2*d*n^7 + 29*a*b^7*c^2*d*n^6 + (331*a*b^7*c^2*d + 20*a^3*b^5*c*d^2)*n^5
 + (1871*a*b^7*c^2*d + 360*a^3*b^5*c*d^2)*n^4 + 20*(269*a*b^7*c^2*d + 103*a
^3*b^5*c*d^2 + 14*a^5*b^3*d^3)*n^3 + 4*(1793*a*b^7*c^2*d + 990*a^3*b^5*c*d^
2 + 210*a^5*b^3*d^3)*n^2 + 560*(6*a*b^7*c^2*d + 4*a^3*b^5*c*d^2 + a^5*b^3*d
^3)*n)*x^3 + 2*(6077*a^2*b^6*c^3 + 2259*a^4*b^4*c^2*d + 180*a^6*b^2*c*d^2)*
n^2 - (b^8*c^3*n^7 + 20160*b^8*c^3 + (34*b^8*c^3 - 9*a^2*b^6*c^2*d)*n^6 + (
478*b^8*c^3 - 243*a^2*b^6*c^2*d)*n^5 + (3580*b^8*c^3 - 2493*a^2*b^6*c^2*d -
 180*a^4*b^4*c*d^2)*n^4 + (15289*b^8*c^3 - 11853*a^2*b^6*c^2*d - 2880*a^4*b
^4*c*d^2)*n^3 + 2*(18353*b^8*c^3 - 12357*a^2*b^6*c^2*d - 6390*a^4*b^4*c*d^2
 - 1260*a^6*b^2*d^3)*n^2 + 72*(621*b^8*c^3 - 210*a^2*b^6*c^2*d - 140*a^4*b^
4*c*d^2 - 35*a^6*b^2*d^3)*n)*x^2 + 36*(682*a^2*b^6*c^3 + 533*a^4*b^4*c^2*d
+ 150*a^6*b^2*c*d^2)*n - (a*b^7*c^3*n^7 + 33*a*b^7*c^3*n^6 + (445*a*b^7*c^3
 + 18*a^3*b^5*c^2*d)*n^5 + 3*(1045*a*b^7*c^3 + 156*a^3*b^5*c^2*d)*n^4 + 2*(
6077*a*b^7*c^3 + 2259*a^3*b^5*c^2*d + 180*a^5*b^3*c*d^2)*n^3 + 36*(682*a*b^
7*c^3 + 533*a^3*b^5*c^2*d + 150*a^5*b^3*c*d^2)*n^2 + 5040*(4*a*b^7*c^3 + 6*
a^3*b^5*c^2*d + 4*a^5*b^3*c*d^2 + a^7*b*d^3)*n)*x)*(b*x + a)^n/(b^8*n^8 + 3
6*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 11
8124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26606, size = 3849, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^3*n^7*x^8 + (b*x + a)^n*a*b^7*d^3*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^3*n^6*x^8 + 3*(b*x + a)^n*b^8*c*d^2*n^7*x^6 + 21*(b*x + a)^n*a*b^7
*d^3*n^6*x^7 + 322*(b*x + a)^n*b^8*d^3*n^5*x^8 + 3*(b*x + a)^n*a*b^7*c*d^2*
n^7*x^5 + 90*(b*x + a)^n*b^8*c*d^2*n^6*x^6 - 7*(b*x + a)^n*a^2*b^6*d^3*n^6*
x^6 + 175*(b*x + a)^n*a*b^7*d^3*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^3*n^4*x^8
+ 3*(b*x + a)^n*b^8*c^2*d*n^7*x^4 + 75*(b*x + a)^n*a*b^7*c*d^2*n^6*x^5 + 10
98*(b*x + a)^n*b^8*c*d^2*n^5*x^6 - 105*(b*x + a)^n*a^2*b^6*d^3*n^5*x^6 + 73
5*(b*x + a)^n*a*b^7*d^3*n^4*x^7 + 6769*(b*x + a)^n*b^8*d^3*n^3*x^8 + 3*(b*x
```

+ a)^n*a*b^7*c^2*d*n^7*x^3 + 96*(b*x + a)^n*b^8*c^2*d*n^6*x^4 - 15*(b*x + a)^n*a^2*b^6*c*d^2*n^6*x^4 + 723*(b*x + a)^n*a*b^7*c*d^2*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^3*n^5*x^5 + 7020*(b*x + a)^n*b^8*c*d^2*n^4*x^6 - 595*(b*x + a)^n*a^2*b^6*d^3*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^3*n^3*x^7 + 13132*(b*x + a)^n*b^8*d^3*n^2*x^8 + (b*x + a)^n*b^8*c^3*n^7*x^2 + 87*(b*x + a)^n*a*b^7*c^2*d*n^6*x^3 + 1254*(b*x + a)^n*b^8*c^2*d*n^5*x^4 - 315*(b*x + a)^n*a^2*b^6*c*d^2*n^5*x^4 + 3405*(b*x + a)^n*a*b^7*c*d^2*n^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^3*n^4*x^5 + 25227*(b*x + a)^n*b^8*c*d^2*n^3*x^6 - 1575*(b*x + a)^n*a^2*b^6*d^3*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^3*n^2*x^7 + 13068*(b*x + a)^n*b^8*d^3*n*x^8 + (b*x + a)^n*a*b^7*c^3*n^7*x + 34*(b*x + a)^n*b^8*c^3*n^6*x^2 - 9*(b*x + a)^n*a^2*b^6*c^2*d*n^6*x^2 + 993*(b*x + a)^n*a*b^7*c^2*d*n^5*x^3 + 60*(b*x + a)^n*a^3*b^5*c*d^2*n^5*x^3 + 8592*(b*x + a)^n*b^8*c^2*d*n^4*x^4 - 2355*(b*x + a)^n*a^2*b^6*c*d^2*n^4*x^4 - 210*(b*x + a)^n*a^4*b^4*d^3*n^4*x^4 + 8202*(b*x + a)^n*a*b^7*c*d^2*n^3*x^5 + 1470*(b*x + a)^n*a^3*b^5*d^3*n^3*x^5 + 50490*(b*x + a)^n*b^8*c*d^2*n^2*x^6 - 1918*(b*x + a)^n*a^2*b^6*d^3*n^2*x^6 + 720*(b*x + a)^n*a*b^7*d^3*n*x^7 + 5040*(b*x + a)^n*b^8*d^3*x^8 + 33*(b*x + a)^n*a*b^7*c^3*n^6*x + 478*(b*x + a)^n*b^8*c^3*n^5*x^2 - 243*(b*x + a)^n*a^2*b^6*c^2*d*n^5*x^2 + 5613*(b*x + a)^n*a*b^7*c^2*d*n^4*x^3 + 1080*(b*x + a)^n*a^3*b^5*c*d^2*n^4*x^3 + 32979*(b*x + a)^n*b^8*c^2*d*n^3*x^4 - 7605*(b*x + a)^n*a^2*b^6*c*d^2*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^3*n^3*x^4 + 9480*(b*x + a)^n*a*b^7*c*d^2*n^2*x^5 + 2100*(b*x + a)^n*a^3*b^5*d^3*n^2*x^5 + 51432*(b*x + a)^n*b^8*c*d^2*n*x^6 - 840*(b*x + a)^n*a^2*b^6*d^3*n*x^6 - (b*x + a)^n*a^2*b^6*c^3*n^6 + 445*(b*x + a)^n*a*b^7*c^3*n^5*x + 18*(b*x + a)^n*a^3*b^5*c^2*d*n^5*x + 3580*(b*x + a)^n*b^8*c^3*n^4*x^2 - 2493*(b*x + a)^n*a^2*b^6*c^2*d*n^4*x^2 - 180*(b*x + a)^n*a^4*b^4*c*d^2*n^4*x^2 + 16140*(b*x + a)^n*a*b^7*c^2*d*n^3*x^3 + 6180*(b*x + a)^n*a^3*b^5*c*d^2*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^3*n^3*x^3 + 69936*(b*x + a)^n*b^8*c^2*d*n^2*x^4 - 10590*(b*x + a)^n*a^2*b^6*c*d^2*n^2*x^4 - 2310*(b*x + a)^n*a^4*b^4*d^3*n^2*x^4 + 4032*(b*x + a)^n*a*b^7*c*d^2*n*x^5 + 1008*(b*x + a)^n*a^3*b^5*d^3*n*x^5 + 20160*(b*x + a)^n*b^8*c*d^2*x^6 - 33*(b*x + a)^n*a^2*b^6*c^3*n^5 + 3135*(b*x + a)^n*a*b^7*c^3*n^4*x + 468*(b*x + a)^n*a^3*b^5*c^2*d*n^4*x + 15289*(b*x + a)^n*b^8*c^3*n^3*x^2 - 11853*(b*x + a)^n*a^2*b^6*c^2*d*n^3*x^2 - 2880*(b*x + a)^n*a^4*b^4*c*d^2*n^3*x^2 + 21516*(b*x + a)^n*a*b^7*c^2*d*n^2*x^3 + 11880*(b*x + a)^n*a^3*b^5*c*d^2*n^2*x^3 + 2520*(b*x + a)^n*a^5*b^3*d^3*n^2*x^3 + 74628*(b*x + a)^n*b^8*c^2*d*n*x^4 - 5040*(b*x + a)^n*a^2*b^6*c*d^2*n*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^3*n*x^4 - 445*(b*x + a)^n*a^2*b^6*c^3*n^4 - 18*(b*x + a)^n*a^4*b^4*c^2*d*n^4 + 12154*(b*x + a)^n*a*b^7*c^3*n^3*x + 4518*(b*x + a)^n*a^3*b^5*c^2*d*n^3*x + 360*(b*x + a)^n*a^5*b^3*c*d^2*n^3*x + 36706*(b*x + a)^n*b^8*c^3*n^2*x^2 - 24714*(b*x + a)^n*a^2*b^6*c^2*d*n^2*x^2 - 12780*(b*x + a)^n*a^4*b^4*c*d^2*n^2*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^3*n^2*x^2 + 10080*(b*x + a)^n*a*b^7*c^2*d*n*x^3 + 6720*(b*x + a)^n*a^3*b^5*c*d^2*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^3*n*x^3 + 30240*(b*x + a)^n*b^8*c^2*d*x^4 - 3135*(b*x + a)^n*a^2*b^6*c^3*n^3 - 468*(b*x + a)^n*a^4*b^4*c^2*d*n^3 + 24552*(b*x + a)^n*a*b^7*c^3*n^2*x + 19188*(b*x + a)^n*a^3*b^5*c^2*d*n^2*x + 5400*(b*x + a)^n*a^5*b^3*c*d^2*n^2*x + 44712*(b*x + a)^n*b^8*c^3*n*x^2 - 15120*(b*x + a)^n*a^2*b^6*c^2*d*n*x^2 - 10080*(b*x + a)^n*a^4*b^4*c*d^2*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^3*n*x^2 - 12154*(b*x + a)^n*a^2*b^6*c^3*n^2 - 4518*(b*x + a)^n*a^4*b^4*c^2*d*n^2 - 360*(b*x + a)^n*a^6*b^2*c*d^2*n^2 + 20160*(b*x + a)^n*a*b^7*c^3*n*x + 30240*(b*x + a)^n*a^3*b^5*c^2*d*n*x + 20160*(b*x + a)^n*a^5*b^3*c*d^2*n*x + 5040*(b*x + a)^n*a^7*b*d^3*n*x + 20160*(b*x + a)^n*b^8*c^3*x^2 - 24552*(b*x + a)^n*a^2*b^6*c^3*n - 19188*(b*x + a)^n*a^4*b^4*c^2*d*n - 5400*(b*x + a)^n*a^6*b^2*c*d^2*n - 20160*(b*x + a)^n*a^2*b^6*c^3 - 30240*(b*x + a)^n*a^4*b^4*c^2*d - 20160*(b*x + a)^n*a^6*b^2*c*d^2 - 5040*(b*x + a)^n*a^8*d^3)/(b^8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)

3.361 $\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=223

$$-\frac{4ad^2\left(5a^2d+3b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{3d^2\left(5a^2d+b^2c\right)(a+bx)^{n+5}}{b^7(n+5)}+\frac{\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^7(n+1)}-\frac{6ad\left(a^2d+b^2c\right)^2(a+b\ldots}{b^7(n+2)}$$

```
[Out] ((b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (6*a*d*(b^2*c + a^2*d
)^2*(a + b*x)^(2 + n))/(b^7*(2 + n)) + (3*d*(b^2*c + a^2*d)*(b^2*c + 5*a^2*
d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d^2*(3*b^2*c + 5*a^2*d)*(a + b*x
)^(4 + n))/(b^7*(4 + n)) + (3*d^2*(b^2*c + 5*a^2*d)*(a + b*x)^(5 + n))/(b^7
*(5 + n)) - (6*a*d^3*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d^3*(a + b*x)^(7 +
 n))/(b^7*(7 + n))
```

Rubi [A] time = 0.125572, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{4ad^2\left(5a^2d+3b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{3d^2\left(5a^2d+b^2c\right)(a+bx)^{n+5}}{b^7(n+5)}+\frac{\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^7(n+1)}-\frac{6ad\left(a^2d+b^2c\right)^2(a+b\ldots}{b^7(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (6*a*d*(b^2*c + a^2*d
)^2*(a + b*x)^(2 + n))/(b^7*(2 + n)) + (3*d*(b^2*c + a^2*d)*(b^2*c + 5*a^2*
d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d^2*(3*b^2*c + 5*a^2*d)*(a + b*x
)^(4 + n))/(b^7*(4 + n)) + (3*d^2*(b^2*c + 5*a^2*d)*(a + b*x)^(5 + n))/(b^7
*(5 + n)) - (6*a*d^3*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d^3*(a + b*x)^(7 +
 n))/(b^7*(7 + n))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx)^n\left(c+dx^2\right)^3 dx = \int \left(\frac{\left(b^2c+a^2d\right)^3(a+bx)^n}{b^6}-\frac{6ad\left(b^2c+a^2d\right)^2(a+bx)^{1+n}}{b^6}+\frac{3d\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(\ldots}{b^6}\right.$$

$$=\frac{\left(b^2c+a^2d\right)^3(a+bx)^{1+n}}{b^7(1+n)}-\frac{6ad\left(b^2c+a^2d\right)^2(a+bx)^{2+n}}{b^7(2+n)}+\frac{3d\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a\ldots}{b^7(3+n)}$$

Mathematica [A] time = 0.529307, size = 347, normalized size = 1.56

$$\frac{(a+bx)^{n+1}\left(\frac{6\left((n+6)\left(a^2d+b^2c\right)\left(4(n+4)\left(a^2d+b^2c\right)\left(2a^2d-2abd(n+1)x+b^2(n+2)\left(c(n+3)+d(n+1)x^2\right)\right)-4ad(n+1)(a+bx)\left(2a^2d-2abd(n+2)x+b^2(n+3)\left(c(n+4)\ldots\right.\right.\right.\right.}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((c + d*x^2)^3 + (6*((b^2*c + a^2*d)*(6 + n)*(b^4*(1 + n
)*(2 + n)*(3 + n)*(4 + n)*(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(4 + n)*(2*a^2*
d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) + d*(1 + n)*x^2)) - 4*a*d*(1
 + n)*(a + b*x)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^2*(3 + n)*(c*(4 + n) + d*(
2 + n)*x^2))) - a*d*(1 + n)*(a + b*x)*(b^4*(2 + n)*(3 + n)*(4 + n)*(5 + n)*
(c + d*x^2)^2 + 4*(b^2*c + a^2*d)*(5 + n)*(2*a^2*d - 2*a*b*d*(2 + n)*x + b^
2*(3 + n)*(c*(4 + n) + d*(2 + n)*x^2)) - 4*a*d*(2 + n)*(a + b*x)*(2*a^2*d -
 2*a*b*d*(3 + n)*x + b^2*(4 + n)*(c*(5 + n) + d*(3 + n)*x^2)))))/(b^6*(1 +
n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))))/(b*(7 + n))
```

Maple [B] time = 0.058, size = 1140, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] (b*x+a)^(1+n)*(b^6*d^3*n^6*x^6+21*b^6*d^3*n^5*x^6-6*a*b^5*d^3*n^5*x^5+3*b^6
*c*d^2*n^6*x^4+175*b^6*d^3*n^4*x^6-90*a*b^5*d^3*n^4*x^5+69*b^6*c*d^2*n^5*x^
4+735*b^6*d^3*n^3*x^6+30*a^2*b^4*d^3*n^4*x^4-12*a*b^5*c*d^2*n^5*x^3-510*a*b
^5*d^3*n^3*x^5+3*b^6*c^2*d*n^6*x^2+621*b^6*c*d^2*n^4*x^4+1624*b^6*d^3*n^2*x
^6+300*a^2*b^4*d^3*n^3*x^4-228*a*b^5*c*d^2*n^4*x^3-1350*a*b^5*d^3*n^2*x^5+7
5*b^6*c^2*d*n^5*x^2+2775*b^6*c*d^2*n^3*x^4+1764*b^6*d^3*n*x^6-120*a^3*b^3*d
^3*n^3*x^3+36*a^2*b^4*c*d^2*n^4*x^2+1050*a^2*b^4*d^3*n^2*x^4-6*a*b^5*c^2*d*
n^5*x-1572*a*b^5*c*d^2*n^3*x^3-1644*a*b^5*d^3*n*x^5+b^6*c^3*n^6+741*b^6*c^2
*d*n^4*x^2+6432*b^6*c*d^2*n^2*x^4+720*b^6*d^3*x^6-720*a^3*b^3*d^3*n^2*x^3+5
76*a^2*b^4*c*d^2*n^3*x^2+1500*a^2*b^4*d^3*n*x^4-138*a*b^5*c^2*d*n^4*x-4812*
a*b^5*c*d^2*n^2*x^3-720*a*b^5*d^3*x^5+27*b^6*c^3*n^5+3657*b^6*c^2*d*n^3*x^2
+7236*b^6*c*d^2*n*x^4+360*a^4*b^2*d^3*n^2*x^2-72*a^3*b^3*c*d^2*n^3*x-1320*a
^3*b^3*d^3*n*x^3+6*a^2*b^4*c^2*d*n^4+2988*a^2*b^4*c*d^2*n^2*x^2+720*a^2*b^4
*d^3*x^4-1206*a*b^5*c^2*d*n^3*x-6480*a*b^5*c*d^2*n*x^3+295*b^6*c^3*n^4+9336
*b^6*c^2*d*n^2*x^2+3024*b^6*c*d^2*x^4+1080*a^4*b^2*d^3*n*x^2-1008*a^3*b^3*c
*d^2*n^2*x-720*a^3*b^3*d^3*x^3+132*a^2*b^4*c^2*d*n^3+5472*a^2*b^4*c*d^2*n*x
^2-4902*a*b^5*c^2*d*n^2*x-3024*a*b^5*c*d^2*x^3+1665*b^6*c^3*n^3+11388*b^6*c
^2*d*n*x^2-720*a^5*b*d^3*n*x+72*a^4*b^2*c*d^2*n^2+720*a^4*b^2*d^3*x^2-3960*
a^3*b^3*c*d^2*n*x+1074*a^2*b^4*c^2*d*n^2+3024*a^2*b^4*c*d^2*x^2-8868*a*b^5*
c^2*d*n*x+5104*b^6*c^3*n^2+5040*b^6*c^2*d*x^2-720*a^5*b*d^3*x+936*a^4*b^2*c
*d^2*n-3024*a^3*b^3*c*d^2*x+3828*a^2*b^4*c^2*d*n-5040*a*b^5*c^2*d*x+8028*b^
6*c^3*n+720*a^6*d^3+3024*a^4*b^2*c*d^2+5040*a^2*b^4*c^2*d+5040*b^6*c^3)/b^7
/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15924, size = 2692, normalized size = 12.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^3*n^6 + 27*a*b^6*c^3*n^5 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3
024*a^5*b^2*c*d^2 + 720*a^7*d^3 + (b^7*d^3*n^6 + 21*b^7*d^3*n^5 + 175*b^7*d
^3*n^4 + 735*b^7*d^3*n^3 + 1624*b^7*d^3*n^2 + 1764*b^7*d^3*n + 720*b^7*d^3)
*x^7 + (a*b^6*d^3*n^6 + 15*a*b^6*d^3*n^5 + 85*a*b^6*d^3*n^4 + 225*a*b^6*d^3
*n^3 + 274*a*b^6*d^3*n^2 + 120*a*b^6*d^3*n)*x^6 + 3*(b^7*c*d^2*n^6 + 1008*b
^7*c*d^2 + (23*b^7*c*d^2 - 2*a^2*b^5*d^3)*n^5 + (207*b^7*c*d^2 - 20*a^2*b^5
*d^3)*n^4 + 5*(185*b^7*c*d^2 - 14*a^2*b^5*d^3)*n^3 + 4*(536*b^7*c*d^2 - 25*
a^2*b^5*d^3)*n^2 + 12*(201*b^7*c*d^2 - 4*a^2*b^5*d^3)*n)*x^5 + (295*a*b^6*c
^3 + 6*a^3*b^4*c^2*d)*n^4 + 3*(a*b^6*c*d^2*n^6 + 19*a*b^6*c*d^2*n^5 + (131*
a*b^6*c*d^2 + 10*a^3*b^4*d^3)*n^4 + (401*a*b^6*c*d^2 + 60*a^3*b^4*d^3)*n^3
+ 10*(54*a*b^6*c*d^2 + 11*a^3*b^4*d^3)*n^2 + 12*(21*a*b^6*c*d^2 + 5*a^3*b^4
*d^3)*n)*x^4 + 3*(555*a*b^6*c^3 + 44*a^3*b^4*c^2*d)*n^3 + 3*(b^7*c^2*d*n^6
+ 1680*b^7*c^2*d + (25*b^7*c^2*d - 4*a^2*b^5*c*d^2)*n^5 + (247*b^7*c^2*d -
64*a^2*b^5*c*d^2)*n^4 + (1219*b^7*c^2*d - 332*a^2*b^5*c*d^2 - 40*a^4*b^3*d^
3)*n^3 + 8*(389*b^7*c^2*d - 76*a^2*b^5*c*d^2 - 15*a^4*b^3*d^3)*n^2 + 4*(949
*b^7*c^2*d - 84*a^2*b^5*c*d^2 - 20*a^4*b^3*d^3)*n)*x^3 + 2*(2552*a*b^6*c^3
+ 537*a^3*b^4*c^2*d + 36*a^5*b^2*c*d^2)*n^2 + 3*(a*b^6*c^2*d*n^6 + 23*a*b^6
*c^2*d*n^5 + 3*(67*a*b^6*c^2*d + 4*a^3*b^4*c*d^2)*n^4 + (817*a*b^6*c^2*d +
168*a^3*b^4*c*d^2)*n^3 + 2*(739*a*b^6*c^2*d + 330*a^3*b^4*c*d^2 + 60*a^5*b^
2*d^3)*n^2 + 24*(35*a*b^6*c^2*d + 21*a^3*b^4*c*d^2 + 5*a^5*b^2*d^3)*n)*x^2
+ 12*(669*a*b^6*c^3 + 319*a^3*b^4*c^2*d + 78*a^5*b^2*c*d^2)*n + (b^7*c^3*n^
6 + 5040*b^7*c^3 + 3*(9*b^7*c^3 - 2*a^2*b^5*c^2*d)*n^5 + (295*b^7*c^3 - 132
*a^2*b^5*c^2*d)*n^4 + 3*(555*b^7*c^3 - 358*a^2*b^5*c^2*d - 24*a^4*b^3*c*d^2
)*n^3 + 4*(1276*b^7*c^3 - 957*a^2*b^5*c^2*d - 234*a^4*b^3*c*d^2)*n^2 + 36*(
223*b^7*c^3 - 140*a^2*b^5*c^2*d - 84*a^4*b^3*c*d^2 - 20*a^6*b*d^3)*n)*x)*(b
*x + a)^n/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3
 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2109, size = 2815, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")

[Out] ((b*x + a)^n*b^7*d^3*n^6*x^7 + (b*x + a)^n*a*b^6*d^3*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^3*n^5*x^7 + 3*(b*x + a)^n*b^7*c*d^2*n^6*x^5 + 15*(b*x + a)^n*a*b^6
*d^3*n^5*x^6 + 175*(b*x + a)^n*b^7*d^3*n^4*x^7 + 3*(b*x + a)^n*a*b^6*c*d^2*
n^6*x^4 + 69*(b*x + a)^n*b^7*c*d^2*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^3*n^5*
x^5 + 85*(b*x + a)^n*a*b^6*d^3*n^4*x^6 + 735*(b*x + a)^n*b^7*d^3*n^3*x^7 +
3*(b*x + a)^n*b^7*c^2*d*n^6*x^3 + 57*(b*x + a)^n*a*b^6*c*d^2*n^5*x^4 + 621*
(b*x + a)^n*b^7*c*d^2*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^3*n^4*x^5 + 225*(b
*x + a)^n*a*b^6*d^3*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^3*n^2*x^7 + 3*(b*x + a
)^n*a*b^6*c^2*d*n^6*x^2 + 75*(b*x + a)^n*b^7*c^2*d*n^5*x^3 - 12*(b*x + a)^n
*a^2*b^5*c*d^2*n^5*x^3 + 393*(b*x + a)^n*a*b^6*c*d^2*n^4*x^4 + 30*(b*x + a)
^n*a^3*b^4*d^3*n^4*x^4 + 2775*(b*x + a)^n*b^7*c*d^2*n^3*x^5 - 210*(b*x + a)
^n*a^2*b^5*d^3*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^3*n^2*x^6 + 1764*(b*x + a)
^n*b^7*d^3*n*x^7 + (b*x + a)^n*b^7*c^3*n^6*x + 69*(b*x + a)^n*a*b^6*c^2*d*n
^5*x^2 + 741*(b*x + a)^n*b^7*c^2*d*n^4*x^3 - 192*(b*x + a)^n*a^2*b^5*c*d^2*
n^4*x^3 + 1203*(b*x + a)^n*a*b^6*c*d^2*n^3*x^4 + 180*(b*x + a)^n*a^3*b^4*d^
3*n^3*x^4 + 6432*(b*x + a)^n*b^7*c*d^2*n^2*x^5 - 300*(b*x + a)^n*a^2*b^5*d^
3*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^3*n*x^6 + 720*(b*x + a)^n*b^7*d^3*x^7 +
 (b*x + a)^n*a*b^6*c^3*n^6 + 27*(b*x + a)^n*b^7*c^3*n^5*x - 6*(b*x + a)^n*a
^2*b^5*c^2*d*n^5*x + 603*(b*x + a)^n*a*b^6*c^2*d*n^4*x^2 + 36*(b*x + a)^n*a
^3*b^4*c*d^2*n^4*x^2 + 3657*(b*x + a)^n*b^7*c^2*d*n^3*x^3 - 996*(b*x + a)^n
*a^2*b^5*c*d^2*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^3*n^3*x^3 + 1620*(b*x +
a)^n*a*b^6*c*d^2*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^3*n^2*x^4 + 7236*(b*x
+ a)^n*b^7*c*d^2*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^3*n*x^5 + 27*(b*x + a)^n
*a*b^6*c^3*n^5 + 295*(b*x + a)^n*b^7*c^3*n^4*x - 132*(b*x + a)^n*a^2*b^5*c^
2*d*n^4*x + 2451*(b*x + a)^n*a*b^6*c^2*d*n^3*x^2 + 504*(b*x + a)^n*a^3*b^4*
c*d^2*n^3*x^2 + 9336*(b*x + a)^n*b^7*c^2*d*n^2*x^3 - 1824*(b*x + a)^n*a^2*b
^5*c*d^2*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^3*n^2*x^3 + 756*(b*x + a)^n*a*
b^6*c*d^2*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^3*n*x^4 + 3024*(b*x + a)^n*b^7*
c*d^2*x^5 + 295*(b*x + a)^n*a*b^6*c^3*n^4 + 6*(b*x + a)^n*a^3*b^4*c^2*d*n^4
 + 1665*(b*x + a)^n*b^7*c^3*n^3*x - 1074*(b*x + a)^n*a^2*b^5*c^2*d*n^3*x -
72*(b*x + a)^n*a^4*b^3*c*d^2*n^3*x + 4434*(b*x + a)^n*a*b^6*c^2*d*n^2*x^2 +
 1980*(b*x + a)^n*a^3*b^4*c*d^2*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^3*n^2*x
^2 + 11388*(b*x + a)^n*b^7*c^2*d*n*x^3 - 1008*(b*x + a)^n*a^2*b^5*c*d^2*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^3*n*x^3 + 1665*(b*x + a)^n*a*b^6*c^3*n^3 + 1
32*(b*x + a)^n*a^3*b^4*c^2*d*n^3 + 5104*(b*x + a)^n*b^7*c^3*n^2*x - 3828*(b
*x + a)^n*a^2*b^5*c^2*d*n^2*x - 936*(b*x + a)^n*a^4*b^3*c*d^2*n^2*x + 2520*
(b*x + a)^n*a*b^6*c^2*d*n*x^2 + 1512*(b*x + a)^n*a^3*b^4*c*d^2*n*x^2 + 360*
(b*x + a)^n*a^5*b^2*d^3*n*x^2 + 5040*(b*x + a)^n*b^7*c^2*d*x^3 + 5104*(b*x
+ a)^n*a*b^6*c^3*n^2 + 1074*(b*x + a)^n*a^3*b^4*c^2*d*n^2 + 72*(b*x + a)^n*
a^5*b^2*c*d^2*n^2 + 8028*(b*x + a)^n*b^7*c^3*n*x - 5040*(b*x + a)^n*a^2*b^5
*c^2*d*n*x - 3024*(b*x + a)^n*a^4*b^3*c*d^2*n*x - 720*(b*x + a)^n*a^6*b*d^3
*n*x + 8028*(b*x + a)^n*a*b^6*c^3*n + 3828*(b*x + a)^n*a^3*b^4*c^2*d*n + 93
6*(b*x + a)^n*a^5*b^2*c*d^2*n + 5040*(b*x + a)^n*b^7*c^3*x + 5040*(b*x + a)
^n*a*b^6*c^3 + 5040*(b*x + a)^n*a^3*b^4*c^2*d + 3024*(b*x + a)^n*a^5*b^2*c*
d^2 + 720*(b*x + a)^n*a^7*d^3)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b
^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

3.362 $\int \frac{(a+bx)^n (c+dx^2)^3}{x} \, dx$

Optimal. Leaf size=246

$$-\frac{ad\left(3a^2b^2cd+a^4d^2+3b^4c^2\right)(a+bx)^{n+1}}{b^6(n+1)}+\frac{d\left(9a^2b^2cd+5a^4d^2+3b^4c^2\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{ad^2\left(10a^2d+9b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}$$

```
[Out] -((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2)*(a + b*x)^(1 + n))/(b^6*(1 + n
))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x)^(2 + n))/(b^6*(2
 + n)) - (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (d^
2*(3*b^2*c + 10*a^2*d)*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d^3*(a + b*x
)^(5 + n))/(b^6*(5 + n)) + (d^3*(a + b*x)^(6 + n))/(b^6*(6 + n)) - (c^3*(a
+ b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rubi [A] time = 0.344616, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {952, 1620, 65}

$$-\frac{ad\left(3a^2b^2cd+a^4d^2+3b^4c^2\right)(a+bx)^{n+1}}{b^6(n+1)}+\frac{d\left(9a^2b^2cd+5a^4d^2+3b^4c^2\right)(a+bx)^{n+2}}{b^6(n+2)}-\frac{ad^2\left(10a^2d+9b^2c\right)(a+bx)^{n+3}}{b^6(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

```
[Out] -((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2)*(a + b*x)^(1 + n))/(b^6*(1 + n
))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x)^(2 + n))/(b^6*(2
 + n)) - (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (d^
2*(3*b^2*c + 10*a^2*d)*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d^3*(a + b*x
)^(5 + n))/(b^6*(5 + n)) + (d^3*(a + b*x)^(6 + n))/(b^6*(6 + n)) - (c^3*(a
+ b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
```

```
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n\left(c+dx^2\right)^3}{x}\,dx = \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + \frac{\int \frac{(a+bx)^n\left(b^6c^3(6+n)-a^5bd^3(6+n)x+b^2d\left(3b^4c^2-5a^4d^2\right)(6+n)x^2-10a^3b^3d^3(6+n)x^3+b^4d^2\left(3b^2c-10a\right.}{x}}{b^6(6+n)}$$

$$= \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + \frac{\int \left(-abd\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(6+n)(a+bx)^n + \frac{\left(6b^6c^3+b^6c^3n\right)(a+bx)^n}{x} + b\right.}{}$$

$$= -\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx)^{2+n}}{b^6(2+n)} - \frac{ad^2}{}$$

$$= -\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx)^{2+n}}{b^6(2+n)} - \frac{ad^2}{}$$

Mathematica [A] time = 0.21327, size = 226, normalized size = 0.92

$$(a+bx)^{n+1}\left(\frac{d(a+bx)\left(9a^2b^2cd+5a^4d^2+3b^4c^2\right)}{b^6(n+2)} - \frac{ad\left(3a^2b^2cd+a^4d^2+3b^4c^2\right)}{b^6(n+1)} + \frac{d^2(a+bx)^3\left(10a^2d+3b^2c\right)}{b^6(n+4)} - \frac{ad^2(a+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*(-((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2))/(b^6*(1 +
n))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x))/(b^6*(2 + n))
- (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^2)/(b^6*(3 + n)) + (d^2*(3*b^2*c +
10*a^2*d)*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^3*(a + b*x)^4)/(b^6*(5 + n))
+ (d^3*(a + b*x)^5)/(b^6*(6 + n)) - (c^3*Hypergeometric2F1[1, 1 + n, 2 + n,
 (a + b*x)/a])/(a + a*n))
```

Maple [F] time = 0.526, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n\left(dx^2+c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^3(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3\right)(bx + a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x + a)^n/x, x)
```

Sympy [B] time = 20.6092, size = 5690, normalized size = 23.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3/x,x)
```

```
[Out] -b**n*c**3*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**3*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 3*c**2*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/
(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x) - b*x/(a*b**2 + b**3
*x), Eq(n, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)*
*n/(b**2*n**2 + 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b*
*2*n + 2*b**2) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) +
 b**2*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + 3*c*d**2*
Piecewise((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a
**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 5*a**3/(6*a**3*b**4 + 18*a**2*
b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 9*a**2*b*x/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b
**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b*
*7*x**3) - 6*b**3*x**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b
**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*
b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x*
log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a**2
*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b**4
 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b*
*6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3/(
2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*
b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2
)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b),
Eq(n, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
+ 50*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b*x)**
n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*
b**2*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4
*n + 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 3
```

```
5*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n*x**3
*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**
4) + b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
 50*b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*
n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)**n/(b
**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*x**4
*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**
4), True)) + d**3*Piecewise((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/
(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7
*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*
x**5) + 300*a**4*b*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**
3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 625*
a**4*b*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b*
*9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*x**2*log(a/b +
x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**
3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*x**2/(60*a**5*b**6 +
 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x*
*4 + 60*b**11*x**5) + 600*a**2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a
**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 6
0*b**11*x**5) + 900*a**2*b**3*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a*
*3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300
*a*b**4*x**4*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x
**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x
**4/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x*
*3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*a**5
*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b
**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*log(a/b + x)/(12*a**4*b**
6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
35*a**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3
 + 12*b**10*x**4) - 240*a**4*b*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*
x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 80*a**4*b*x/(12*a
**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x
**4) - 360*a**3*b**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*
a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3*log(a
/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3
 + 12*b**10*x**4) + 120*a**2*b**3*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*
a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*x**4*log(a/b +
 x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 1
2*b**10*x**4) + 90*a*b**4*x**4/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**
8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*x**5/(12*a**4*b**6 + 48*
a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5
)), (60*a**5*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 +
6*b**9*x**3) + 50*a**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b
**9*x**3) + 180*a**4*b*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*
b**8*x**2 + 6*b**9*x**3) + 90*a**4*b*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a
*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2*log(a/b + x)/(6*a**3*b**6 +
18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*x**3*log(a/b
+ x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 60*a**
2*b**3*x**3/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) -
 15*a*b**4*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**
3) + 3*b**5*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x*
*3), Eq(n, -4)), (-60*a**5*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8
*x**2) - 90*a**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x*l
og(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x/(6*a**
2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**
2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*x**3/(6*a**2*b**6 + 12*a
*b**7*x + 6*b**8*x**2) - 5*a*b**4*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*
```

```
x**2) + 2*b**5*x**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)),
(60*a**5*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 60*a**5/(12*a*b**6 + 12*b**
7*x) + 60*a**4*b*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*x**2
/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*x**3/(12*a*b**6 + 12*b**7*x) - 5*a*
b**4*x**4/(12*a*b**6 + 12*b**7*x) + 3*b**5*x**5/(12*a*b**6 + 12*b**7*x), Eq
(n, -2)), (-a**5*log(a/b + x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**
2*x**3/(3*b**3) - a*x**4/(4*b**2) + x**5/(5*b), Eq(n, -1)), (-120*a**6*(a +
 b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b
**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*n*x*(a + b*x)**n/(b**6*n**6
 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) - 60*a**4*b**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) - 60*a**4*b**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**
6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3
*b**3*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 60*a**3*b**3*n**2*
x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*n*x**3*(a + b*x)
**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n
**2 + 1764*b**6*n + 720*b**6) - 5*a**2*b**4*n**4*x**4*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) - 30*a**2*b**4*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) - 55*a**2*b**4*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30
*a**2*b**4*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*n**5*x**5
*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1
624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**5*n**4*x**5*(a + b*x)**n/
(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
+ 1764*b**6*n + 720*b**6) + 35*a*b**5*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 2
1*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
+ 720*b**6) + 50*a*b**5*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) +
24*a*b**5*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 7
35*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*n**5*x**6*(a
 + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624
*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*n**4*x**6*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 176
4*b**6*n + 720*b**6) + 85*b**6*n**3*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) + 225*b**6*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6
*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6
*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*x**6*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6), True)) - b*b**n*c**3*n*x*(a/b + x)**n*lerchphi(
1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c**3*x*(a/b + x
)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^3(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

3.363 $\int \frac{x^4(d+ex)^n}{a+cx^2} \, dx$

Optimal. Leaf size=250

$$\frac{\left(cd^2-ae^2\right)(d+ex)^{n+1}}{c^2e^3(n+1)}+\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{}}{\sqrt{c}}\right.}{2c^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[Out] ((c*d^2 - a*e^2)*(d + e*x)^(1 + n))/(c^2*e^3*(1 + n)) - (2*d*(d + e*x)^(2 +
 n))/(c*e^3*(2 + n)) + (d + e*x)^(3 + n)/(c*e^3*(3 + n)) + ((-a)^(3/2)*(d +
 e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[
c]*d - Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - ((-a)^(3/2)
*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rubi [A] time = 0.403131, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1629, 712, 68}

$$\frac{\left(cd^2-ae^2\right)(d+ex)^{n+1}}{c^2e^3(n+1)}+\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{}}{\sqrt{c}}\right.}{2c^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((c*d^2 - a*e^2)*(d + e*x)^(1 + n))/(c^2*e^3*(1 + n)) - (2*d*(d + e*x)^(2 +
 n))/(c*e^3*(2 + n)) + (d + e*x)^(3 + n)/(c*e^3*(3 + n)) + ((-a)^(3/2)*(d +
 e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[
c]*d - Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - ((-a)^(3/2)
*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{\left(cd^2-ae^2\right)(d+ex)^n}{c^2e^2} - \frac{2d(d+ex)^{1+n}}{ce^2} + \frac{(d+ex)^{2+n}}{ce^2} + \frac{a^2(d+ex)^n}{c^2\left(a+cx^2\right)}\right)dx$$
$$= \frac{\left(cd^2-ae^2\right)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{a^2\int \frac{(d+ex)^n}{a+cx^2}\,dx}{c^2}$$
$$= \frac{\left(cd^2-ae^2\right)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{a^2\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{c^2}$$
$$= \frac{\left(cd^2-ae^2\right)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} - \frac{(-a)^{3/2}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2c^2} - \frac{(-a)^{3/2}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2c^2}$$
$$= \frac{\left(cd^2-ae^2\right)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{(-a)^{3/2}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}$$

Mathematica [A] time = 0.519903, size = 217, normalized size = 0.87

$$\frac{(d+ex)^{n+1}\left(\frac{2\left(cd^2-ae^2\right)}{e^3(n+1)} + \frac{(-a)^{3/2}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} + \frac{\sqrt{-a}a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} + \frac{2c(d+ex)^2}{e^3(n+3)} - \frac{4cd(d+ex)}{e^3(n+2)}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*(c*d^2 - a*e^2))/(e^3*(1 + n)) - (4*c*d*(d + e*x))/(
e^3*(2 + n)) + (2*c*(d + e*x)^2)/(e^3*(3 + n)) + ((-a)^(3/2)*Hypergeometric
2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[
c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt[-a]*a*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*
(1 + n))))/(2*c^2)
```

Maple [F] time = 0.692, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^4}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x^4}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^4/(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (d+ex)^n}{a+cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x**4*(d + e*x)**n/(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^4}{cx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a), x)
```

3.364 $\int \frac{x^3(d+ex)^n}{a+cx^2} dx$

Optimal. Leaf size=209

$$\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}-\frac{d(d+ex)^{n+1}}{ce^2(n+1)}+\frac{(d+ex)^{n+2}}{ce^2(n+2)}$$

```
[Out] -((d*(d + e*x)^(1 + n))/(c*e^2*(1 + n))) + (d + e*x)^(2 + n)/(c*e^2*(2 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)
)
```

Rubi [A] time = 0.225294, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1629, 831, 68}

$$\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}-\frac{d(d+ex)^{n+1}}{ce^2(n+1)}+\frac{(d+ex)^{n+2}}{ce^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -((d*(d + e*x)^(1 + n))/(c*e^2*(1 + n))) + (d + e*x)^(2 + n)/(c*e^2*(2 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)
)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \left(-\frac{d(d+ex)^n}{ce} + \frac{(d+ex)^{1+n}}{ce} - \frac{ax(d+ex)^n}{c\left(a+cx^2\right)}\right) dx$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} - \frac{a\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{c}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} - \frac{a\int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}x\right)}\right) dx}{c}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} + \frac{a\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2c^{3/2}} - \frac{a\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2c^{3/2}}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)} + \frac{(d+ex)^{2+n}}{ce^2(2+n)} + \frac{a(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{a(d+ex)^{1+n}\,{}_2F_1\left(1,\right.}{2c^{3/2}\left(\sqrt{c}d+\right.}$$

Mathematica [A] time = 0.230813, size = 168, normalized size = 0.8

$$\frac{(d+ex)^{n+1}\left(\frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e} + \frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}e+\sqrt{c}d} - \frac{2\sqrt{c}(d-e(n+1)x)}{e^2(n+2)}\right)}{2c^{3/2}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*Sqrt[c]*(d - e*(1 + n)*x))/(e^2*(2 + n)) + (a*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
)/(Sqrt[c]*d - Sqrt[-a]*e) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]
*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(2*c^(3/2
)*(1 + n))
```

Maple [F] time = 0.724, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^3}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x^3}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^3/(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (d+ex)^n}{a+cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x**3*(d + e*x)**n/(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^3}{cx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a), x)
```

3.365 $\int \frac{x^2(d+ex)^n}{a+cx^2} dx$

Optimal. Leaf size=194

$$\frac{\sqrt{-a}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{\sqrt{-a}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}+\frac{(d+ex)^{n+1}}{ce(n+1)}$$

```
[Out] (d + e*x)^(1 + n)/(c*e*(1 + n)) + (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c*(
Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c*
(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rubi [A] time = 0.226158, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1629, 712, 68}

$$\frac{\sqrt{-a}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{\sqrt{-a}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2c(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}+\frac{(d+ex)^{n+1}}{ce(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] (d + e*x)^(1 + n)/(c*e*(1 + n)) + (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c*(
Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c*
(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{(d+ex)^n}{c} - \frac{a(d+ex)^n}{c\left(a+cx^2\right)}\right)dx$$
$$= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{a\int \frac{(d+ex)^n}{a+cx^2}\,dx}{c}$$
$$= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{a\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{c}$$
$$= \frac{(d+ex)^{1+n}}{ce(1+n)} - \frac{\sqrt{-a}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2c} - \frac{\sqrt{-a}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2c}$$
$$= \frac{(d+ex)^{1+n}}{ce(1+n)} + \frac{\sqrt{-a}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2c\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{\sqrt{-a}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{2c\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

Mathematica [A] time = 0.146173, size = 170, normalized size = 0.88

$$\frac{(d+ex)^{n+1}\left(2\left(ae^2+cd^2\right)+e\left(\sqrt{-a}\sqrt{c}d-ae\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)-e\left(\sqrt{-a}\sqrt{c}d+ae\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d}{\sqrt{c}d+}\right.\right.}{2ce(n+1)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*(2*(c*d^2 + a*e^2) + e*(Sqrt[-a]*Sqrt[c]*d - a*e)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
- e*(Sqrt[-a]*Sqrt[c]*d + a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*
(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]))/(2*c*e*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F] time = 0.737, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^2}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x^2}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^2/(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (d+ex)^n}{a+cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x**2*(d + e*x)**n/(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^2}{cx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a), x)
```

3.366 $\int \frac{x(d+ex)^n}{a+cx^2} dx$

Optimal. Leaf size=163

$$-\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[Out] -((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (
(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rubi [A] time = 0.0942188, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {831, 68}

$$-\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (
(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex)^n}{a+cx^2} dx &= \int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}x\right)}\right) dx \\
&= -\frac{\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x} dx}{2\sqrt{c}}+\frac{\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x} dx}{2\sqrt{c}} \\
&= -\frac{(d+ex)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{(d+ex)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.0671825, size = 151, normalized size = 0.93

$$-\frac{(d+ex)^{n+1}\left(\left(\sqrt{-a}e+\sqrt{c}d\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(\sqrt{c}d-\sqrt{-a}e\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)\right)}{2\sqrt{c}(n+1)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -((d + e*x)^(1 + n)*((Sqrt[c]*d + Sqrt[-a]*e)*Hypergeometric2F1[1, 1 + n, 2
 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (Sqrt[c]*d - Sqrt[-a]
*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqr
t[-a]*e)]))/(2*Sqrt[c]*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F] time = 0.768, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x/(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x/(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(d + ex\right)^{n}}{a + cx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x*(d + e*x)**n/(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{n}x}{cx^{2} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x/(c*x^2 + a), x)
```

3.367 $\int \frac{(d+ex)^n}{a+cx^2} \, dx$

Optimal. Leaf size=167

$$\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

```
[Out] ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (
(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rubi [A] time = 0.0912727, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {712, 68}

$$\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (
(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^n}{a+cx^2} \, dx &= \int \left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right) dx\\
&= -\frac{\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x} \, dx}{2\sqrt{-a}}-\frac{\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x} \, dx}{2\sqrt{-a}}\\
&= \frac{(d+ex)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{(d+ex)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.0744895, size = 145, normalized size = 0.87

$$\frac{(d+ex)^{n+1}\left(\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}e+\sqrt{c}d}\right)}{2\sqrt{-a}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)]/(Sqrt[c]*d - Sqrt[-a]*e) - Hypergeometric2F1[1, 1
 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[c]*d + Sqr
t[-a]*e)))/(2*Sqrt[-a]*(1 + n))
```

Maple [F] time = 0.739, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int((e*x+d)^n/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n}{cx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**n/(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a), x)
```

3.368 $\int \frac{(d+ex)^n}{x\left(a+cx^2\right)} dx$

Optimal. Leaf size=207

$$\frac{\sqrt{c}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{\sqrt{c}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n\right.}{ad(n+1)}$$

```
[Out] (Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) +
(Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) -
((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e*x)/d])/(a*d*(1
 + n))
```

Rubi [A] time = 0.183317, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {961, 65, 831, 68}

$$\frac{\sqrt{c}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{\sqrt{c}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n\right.}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x*(a + c*x^2)),x]
```

```
[Out] (Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) +
(Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) -
((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e*x)/d])/(a*d*(1
 + n))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^n}{x\left(a+cx^2\right)}dx &= \int \left(\frac{(d+ex)^n}{ax} - \frac{cx(d+ex)^n}{a\left(a+cx^2\right)}\right)dx \\
&= \frac{\int \frac{(d+ex)^n}{x}dx}{a} - \frac{c\int \frac{x(d+ex)^n}{a+cx^2}dx}{a} \\
&= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)} - \frac{c\int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{a} \\
&= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)} + \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}dx}{2a} - \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}dx}{2a} \\
&= \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2a\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)} - \frac{(d
\end{aligned}$$

Mathematica [A] time = 0.130955, size = 189, normalized size = 0.91

$$\frac{(d+ex)^{n+1}\left(-2\left(ae^2+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{ex}{d}+1\right)+\left(\sqrt{-a}\sqrt{c}de+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(cd^2-\sqrt{\cdot}\right.}{2ad(n+1)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x*(a + c*x^2)),x]
```

```
[Out] ((d + e*x)^(1 + n)*((c*d^2 + Sqrt[-a]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (c*d^2 - Sqrt[-a
]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)] - 2*(c*d^2 + a*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n,
 1 + (e*x)/d]))/(2*a*d*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F] time = 0.742, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{x\left(cx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x/(c*x^2+a),x)
```

```
[Out] int((e*x+d)^n/x/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{(cx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)^n}{cx^3+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c*x^3 + a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^n}{x(a+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**n/(x*(a + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{(cx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x), x)
```

3.369 $\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx$

Optimal. Leaf size=207

$$\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1\left(2,n+1;\right.}{ad^2(n+1)}$$

```
[Out] (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*
x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 +
n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])
/(a*d^2*(1 + n))
```

Rubi [A] time = 0.220608, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {961, 65, 712, 68}

$$\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1\left(2,n+1;\right.}{ad^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

```
[Out] (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*
x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 +
n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])
/(a*d^2*(1 + n))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)}\,dx = \int \left(\frac{(d+ex)^n}{ax^2}-\frac{c(d+ex)^n}{a\left(a+cx^2\right)}\right)dx$$
$$=\frac{\int \frac{(d+ex)^n}{x^2}\,dx}{a}-\frac{c\int \frac{(d+ex)^n}{a+cx^2}\,dx}{a}$$
$$=\frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)}-\frac{c\int \left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{a}$$
$$=\frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)}-\frac{c\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2(-a)^{3/2}}-\frac{c\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2(-a)^{3/2}}$$
$$=\frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2(-a)^{3/2}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}+\frac{e(d+ex)}{}$$

Mathematica [A] time = 0.290434, size = 167, normalized size = 0.81

$$\frac{(d+ex)^{n+1}\left(-\frac{c\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d+ae}+\frac{c\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}\sqrt{c}d-ae}+\frac{2e\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{d^2}\right)}{2a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

```
[Out] ((d + e*x)^(1 + n)*(-((c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[c]*d + a*e)) + (c*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sq
rt[-a]*Sqrt[c]*d - a*e) + (2*e*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)
/d])/d^2))/(2*a*(1 + n))
```

Maple [F] time = 0.752, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{x^2\left(cx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x^2/(c*x^2+a),x)
```

```
[Out] int((e*x+d)^n/x^2/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{(cx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n}{cx^4+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c*x^4 + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x**2/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**n/(x**2*(a + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{(cx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)
```

3.370 $\int \frac{x^4(d+ex)^n}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=332

$$\frac{(d+ex)^{n+1}\left(3\sqrt{-a}cd^2+a\sqrt{c}den+\sqrt{-a}ae^2(n+3)\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(3\sqrt{-a}cd^2-a\sqrt{c}den+\sqrt{\right.}{4c^2(n+1)\left(\sqrt{\right.}$$

```
[Out] (d + e*x)^(1 + n)/(c^2*e*(1 + n)) + (a*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*
c^2*(c*d^2 + a*e^2)*(a + c*x^2)) + ((3*Sqrt[-a]*c*d^2 + a*Sqrt[c]*d*e*n + S
qrt[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d - Sqrt[-a
]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((3*Sqrt[-a]*c*d^2 - a*Sqrt[c]*d*e*n + Sqrt
[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d + Sqrt[-a]*e
)*(c*d^2 + a*e^2)*(1 + n))
```

Rubi [A] time = 0.450803, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1649, 1629, 68}

$$\frac{(d+ex)^{n+1}\left(3\sqrt{-a}cd^2+a\sqrt{c}den+\sqrt{-a}ae^2(n+3)\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(3\sqrt{-a}cd^2-a\sqrt{c}den+\sqrt{\right.}{4c^2(n+1)\left(\sqrt{\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] (d + e*x)^(1 + n)/(c^2*e*(1 + n)) + (a*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*
c^2*(c*d^2 + a*e^2)*(a + c*x^2)) + ((3*Sqrt[-a]*c*d^2 + a*Sqrt[c]*d*e*n + S
qrt[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d - Sqrt[-a
]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((3*Sqrt[-a]*c*d^2 - a*Sqrt[c]*d*e*n + Sqrt
[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d + Sqrt[-a]*e
)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
 + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
 x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
 && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(\frac{a^2\left(cd^2+ae^2(1+n)\right)}{c^2}+\frac{a^2denx}{c}-2a\left(d^2+\frac{ae^2}{c}\right)x^2\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(-\frac{2a\left(cd^2+ae^2\right)(d+ex)^n}{c^2}+\frac{\left(-\frac{a^3den}{c^{3/2}}+\sqrt{-a}\left(\frac{3a^2d^2}{c}+\frac{3a^3e^2}{c^2}+\frac{a^3e^2n}{c^2}\right)\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\left(\frac{a^3den}{c^{3/2}}+\sqrt{-}\right.}{}\right.}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(3\sqrt{-a}cd^2-a\sqrt{c}den+\sqrt{-a}ae^2(3+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4c^2\left(cd^2+ae^2\right)} -$$

$$= \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(3\sqrt{-a}cd^2+a\sqrt{c}den+\sqrt{-a}ae^2(3+n)\right)(d+ex)^{1+n}\,{}_2F_1}{4c^2\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1}$$

Mathematica [A] time = 0.788456, size = 413, normalized size = 1.24

$$\frac{(d+ex)^{n+1}\left(\frac{a\left(\frac{\left(\sqrt{-a}\sqrt{c}den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}-\frac{\left(-\sqrt{-a}\sqrt{c}den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}e+\sqrt{c}d}\right)}{\sqrt{-a}(n+1)\left(ae^2+cd^2\right)}+\frac{2a(ae+cdx)}{\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\frac{4\sqrt{-a}\,{}_2F_1}{(n}\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*(4/(e + e*n) + (2*a*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a +
 c*x^2)) + (4*Sqrt[-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (4*Sqrt[
-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqr
t[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n
) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e
^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(Sq
rt[-a]*(c*d^2 + a*e^2)*(1 + n))))/(4*c^2)
```

Maple [F] time = 0.716, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^4}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^4}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x^4}{c^2x^4+2\,acx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^4/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^4}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

3.371 $\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=297

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(n+2)+2cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(\sqrt{-a}den-\frac{ae^2(n+2)+2cd^2}{\sqrt{c}}\right)}{4c(n+1)\left(\sqrt{c}d-\sqrt{-a}\right.}$$

```
[Out] (a*(d - e*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((Sqrt[
-a]*d*e*n - (2*c*d^2 + a*e^2*(2 + n))/Sqrt[c])*(d + e*x)^(1 + n)*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*
c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((2*c*d^2 + Sqrt[-a]*
Sqrt[c]*d*e*n + a*e^2*(2 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^(3/2)*(Sqrt[c]
*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rubi [A] time = 0.410105, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1649, 831, 68}

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(n+2)+2cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4c^{3/2}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(\sqrt{-a}den-\frac{ae^2(n+2)+2cd^2}{\sqrt{c}}\right)}{4c(n+1)\left(\sqrt{c}d-\sqrt{-a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] (a*(d - e*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((Sqrt[
-a]*d*e*n - (2*c*d^2 + a*e^2*(2 + n))/Sqrt[c])*(d + e*x)^(1 + n)*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*
c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((2*c*d^2 + Sqrt[-a]*
Sqrt[c]*d*e*n + a*e^2*(2 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^(3/2)*(Sqrt[c]
*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
 + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
 x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
 && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(\frac{a^2den}{c}-\frac{a\left(2cd^2+ae^2(2+n)\right)x}{c}\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(\frac{\sqrt{-a}a^2den}{c}+\frac{a^2\left(2cd^2+ae^2(2+n)\right)}{c^{3/2}}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\left(\frac{\sqrt{-a}a^2den}{c}-\frac{a^2\left(2cd^2+ae^2(2+n)\right)}{c^{3/2}}\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(2cd^2+\sqrt{-a}\sqrt{c}den+ae^2(2+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4c^{3/2}\left(cd^2+ae^2\right)} - \frac{\left(\sqrt{-a}den-\frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right.}{4c\left(cd^2+a\right.}$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(\sqrt{-a}den-\frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4c\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)} - \frac{\left(2c\right.}{}$$

Mathematica [A] time = 0.845067, size = 247, normalized size = 0.83

$$-\frac{(d+ex)^{n+1}\left(\frac{\left(-\sqrt{-a}\sqrt{c}den+ae^2(n+2)+2cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{\left(\sqrt{-a}\sqrt{c}den+ae^2(n+2)+2cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}+\frac{2a\sqrt{c}(ex-d)}{a+cx^2}\right)}{4c^{3/2}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -((d + e*x)^(1 + n)*((2*a*Sqrt[c]*(-d + e*x))/(a + c*x^2) + ((2*c*d^2 - Sqr
t[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, (Sq
rt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 +
n)) + ((2*c*d^2 + Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometric2F1
[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*
d + Sqrt[-a]*e)*(1 + n))))/(4*c^(3/2)*(c*d^2 + a*e^2))
```

Maple [F] time = 0.769, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^3}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^3}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x^3}{c^2x^4+2\,acx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^3/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^3}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)
```

3.372 $\int \frac{x^2(d+ex)^n}{\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=306

$$\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2 \ldots\right.}{4\sqrt{-a}c(n+1)\left(\sqrt{-a}e+\ldots\right.}$$

```
[Out] -((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((c*
d^2 - Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*S
qrt[-a]*c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((c*d^2 + Sqr
t[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*Sqrt[-a]*c
*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rubi [A] time = 0.530149, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1649, 831, 68}

$$\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}c(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2 \ldots\right.}{4\sqrt{-a}c(n+1)\left(\sqrt{-a}e+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((c*
d^2 - Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*S
qrt[-a]*c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((c*d^2 + Sqr
t[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*Sqrt[-a]*c
*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
 + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m},
 x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m, 0]
 && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-\frac{a\left(cd^2+ae^2(1+n)\right)}{c}-adenx\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(\frac{a^2den}{\sqrt{c}}-\frac{\sqrt{-a}a\left(cd^2+ae^2(1+n)\right)}{c}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\left(-\frac{a^2den}{\sqrt{c}}-\frac{\sqrt{-a}a\left(cd^2+ae^2(1+n)\right)}{c}\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(cd^2-\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4\sqrt{-a}c\left(cd^2+ae^2\right)} - \frac{\left(cd^2+\sqrt{-a}\sqrt{c}den+\ldots\right.}{4\sqrt{-a}c\ldots}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(cd^2-\sqrt{-a}\sqrt{c}den+ae^2(1+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d\ldots}{\sqrt{c}d-\ldots}\right.}{4\sqrt{-a}c\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)}$$

Mathematica [A] time = 0.648254, size = 403, normalized size = 1.32

$$\frac{(d+ex)^{n+1}\left(\frac{a\left(\frac{\left(\sqrt{-a}\sqrt{c}den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{c}d-\sqrt{-a}e}-\frac{\left(-\sqrt{-a}\sqrt{c}den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}e+\sqrt{c}d}\right)}{(-a)^{3/2}(n+1)\left(ae^2+cd^2\right)} - \frac{2(ae+cdx)}{\left(a+cx^2\right)\left(ae^2+cd^2\right)} + \frac{2\,{}_2F_1\left(1,n\ldots\right.}{\sqrt{-a}(n\ldots}\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a + c*x^2)) + (2*H
ypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]
*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (2*Hypergeometric2F1[1,
 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a]*(Sq
rt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n) + Sqrt[-a]*Sq
rt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c
]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e^2*(-1 + n) - S
qrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/((-a)^(3/2)*(c*d^2
 + a*e^2)*(1 + n))))/(4*c)
```

Maple [F] time = 0.764, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^2}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x^2}{c^2x^4+2\,acx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x^2/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^2}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^2/(c*x^2 + a)^2, x)
```

3.373 $\int \frac{x(d+ex)^n}{\left(a+cx^2\right)^2} dx$

Optimal. Leaf size=279

$$\frac{en\left(\sqrt{-a}e+\sqrt{c}d\right)(d+ex)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{en\left(\sqrt{-a}\sqrt{c}d+ae\right)(d+ex)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}$$

```
[Out] -((d - e*x)*(d + e*x)^(1 + n))/(2*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(Sqrt[c
]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*Sqrt[-a]*Sqrt[c]*(Sqrt[c]*d
 - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(Sqrt[-a]*Sqrt[c]*d + a*e)*n*(
d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sq
rt[c]*d + Sqrt[-a]*e)])/(4*a*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^
2)*(1 + n))
```

Rubi [A] time = 0.296966, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {823, 831, 68}

$$\frac{en\left(\sqrt{-a}e+\sqrt{c}d\right)(d+ex)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{c}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{en\left(\sqrt{-a}\sqrt{c}d+ae\right)(d+ex)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a\sqrt{c}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -((d - e*x)*(d + e*x)^(1 + n))/(2*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(Sqrt[c
]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*Sqrt[-a]*Sqrt[c]*(Sqrt[c]*d
 - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(Sqrt[-a]*Sqrt[c]*d + a*e)*n*(
d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sq
rt[c]*d + Sqrt[-a]*e)])/(4*a*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^
2)*(1 + n))
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-acden+ace^2nx\right)}{a+cx^2}\,dx}{2ac\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(-\sqrt{-a}acden-a^2\sqrt{c}e^2n\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\left(-\sqrt{-a}acden+a^2\sqrt{c}e^2n\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{2ac\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(e\left(\sqrt{-a}\sqrt{c}d-ae\right)n\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{4a\sqrt{c}\left(cd^2+ae^2\right)} + \frac{\left(e\left(\sqrt{-a}d+\frac{ae}{\sqrt{c}}\right)n\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4a\left(cd^2+ae^2\right)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(\sqrt{c}d+\sqrt{-a}e\right)n(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)} + \frac{e\left(\sqrt{-a}\sqrt{c}d\right.}{4}$$

Mathematica [A] time = 0.370314, size = 230, normalized size = 0.82

$$\frac{(d+ex)^{n+1}\left(-\frac{\left(\sqrt{-a}cden-a\sqrt{c}e^2n\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} + \frac{\left(\sqrt{-a}cden+a\sqrt{c}e^2n\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} - \frac{2ac(d-ex)}{a+cx^2}\right)}{4ac\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*a*c*(d - e*x))/(a + c*x^2) - ((Sqrt[-a]*c*d*e*n - a
*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqr
t[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + ((Sqrt[-a]*c*d*
e*n + a*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*c*(
c*d^2 + a*e^2))
```

Maple [F] time = 0.727, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)^n x}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*x/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)
```

3.374 $\int \frac{(d+ex)^n}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=304

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(1-n)+c\right.}{4(-a)^{3/2}(n+1)\left(\sqrt{-a}e+\right.}$$

```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) - ((c*d
^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-
a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + ((c*d^2 + a*e^
2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1,
1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*(-a)^(3/2)*
(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rubi [A] time = 0.416526, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {741, 831, 68}

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}den+ae^2(1-n)+c\right.}{4(-a)^{3/2}(n+1)\left(\sqrt{-a}e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(a + c*x^2)^2,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) - ((c*d
^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-
a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + ((c*d^2 + a*e^
2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1,
1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*(-a)^(3/2)*
(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^n}{\left(a+cx^2\right)^2}\,dx = \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-cd^2-ae^2(1-n)+cdenx\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(\sqrt{-a}\left(-cd^2-ae^2(1-n)\right)-a\sqrt{c}den\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\left(\sqrt{-a}\left(-cd^2-ae^2(1-n)\right)+a\sqrt{c}den\right)(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right) d}{2a\left(cd^2+ae^2\right)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(cd^2+ae^2(1-n)-\sqrt{-a}\sqrt{c}den\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{4(-a)^{3/2}\left(cd^2+ae^2\right)} + \frac{\left(cd^2+ae^2(1-n)+\sqrt{\right.}{4(-a)^{3/2}\left(\right.}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(cd^2+ae^2(1-n)+\sqrt{-a}\sqrt{c}den\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}}\right.}{4(-a)^{3/2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+n)}$$

Mathematica [A] time = 0.323153, size = 253, normalized size = 0.83

$$\frac{(d+ex)^{n+1}\left(\frac{\left(\sqrt{-a}\sqrt{c}den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)} + \frac{\left(\sqrt{-a}\sqrt{c}den+ae^2(n-1)-cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{\sqrt{-a}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)} + \frac{2(ae+cdx)}{a+cx^2}\right)}{4a\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*(a*e + c*d*x))/(a + c*x^2) + ((c*d^2 - a*e^2*(-1 + n
) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 +
n)) + ((-(c*d^2) + a*e^2*(-1 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2
F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a
]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*(c*d^2 + a*e^2))
```

Maple [F] time = 0.738, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int((e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n}{c^2x^4+2\,acx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a)^2, x)
```

3.375 $\int \frac{(d+ex)^n}{x(a+cx^2)^2} \, dx$

Optimal. Leaf size=489

$$-\frac{\sqrt{c}en\left(\sqrt{-a}\sqrt{c}d+ae\right)(d+ex)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}(d+ex)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\ldots$$

```
[Out] (c*(d - e*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (Sqrt[c
]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt
[c]*e*(Sqrt[c]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-a)^(3/2)*(Sq
rt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (Sqrt[c]*(d + e*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)])/(2*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) - (Sqrt[c]*e*(Sqrt[-a]*Sqr
t[c]*d + a*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(c
*d^2 + a*e^2)*(1 + n)) - ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, 1 + (e*x)/d])/(a^2*d*(1 + n))
```

Rubi [A] time = 0.604872, antiderivative size = 489, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {961, 65, 823, 831, 68}

$$-\frac{\sqrt{c}en\left(\sqrt{-a}\sqrt{c}d+ae\right)(d+ex)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{4a^2(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}(d+ex)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```


```
[Out] (c*(d - e*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (Sqrt[c
]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt
[c]*e*(Sqrt[c]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-a)^(3/2)*(Sq
rt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (Sqrt[c]*(d + e*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)])/(2*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) - (Sqrt[c]*e*(Sqrt[-a]*Sqr
t[c]*d + a*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(c
*d^2 + a*e^2)*(1 + n)) - ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, 1 + (e*x)/d])/(a^2*d*(1 + n))
```


Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx = \int \left(\frac{(d+ex)^n}{a^2x} - \frac{cx(d+ex)^n}{a\left(a+cx^2\right)^2} - \frac{cx(d+ex)^n}{a^2\left(a+cx^2\right)}\right) dx$$

$$= \frac{\int \frac{(d+ex)^n}{x}\,dx}{a^2} - \frac{c\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{a^2} - \frac{c\int \frac{x(d+ex)^n}{\left(a+cx^2\right)^2}\,dx}{a}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} - \frac{c\int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\cdots\right.}\cdots\right.}{a^2}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} + \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2a^2} - \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}+\cdots}}{2a^2}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+\cdots\right.}{2a^2\left(\sqrt{c}d+\sqrt{-\cdots}\right.}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\sqrt{c}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a^2\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{\sqrt{c}e\left(\sqrt{-a}\sqrt{c}d-ae\right)n(d\cdots}{4a^2\left(\sqrt{c}d-\cdots\right.}$$

Mathematica [A] time = 0.934426, size = 391, normalized size = 0.8

$$\frac{(d+ex)^{n+1}\left(\frac{\sqrt{c}en\left(\left(\sqrt{-a}cd^2-2a\sqrt{c}de+(-a)^{3/2}e^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)+\left(-\sqrt{-a}cd^2-2a\sqrt{c}de+\sqrt{-a}ae^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)\right)}{(n+1)\left(ae^2+cd^2\right)^2} + \frac{2ac(d-ex)}{\left(a+cx^2\right)\left(ae^2+cd^2\right)\cdots}\cdots\right.}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*a*c*(d - e*x))/((c*d^2 + a*e^2)*(a + c*x^2)) - (4*Hy
pergeometric2F1[1, 1 + n, 2 + n, (d + e*x)/d])/(d + d*n) + (2*Sqrt[c]*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
)/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (2*Sqrt[c]*Hypergeometric2F1[1, 1 +
n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt
[-a]*e)*(1 + n)) + (Sqrt[c]*e*n*((Sqrt[-a]*c*d^2 - 2*a*Sqrt[c]*d*e + (-a)^(
3/2)*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d
 - Sqrt[-a]*e)] + (-(Sqrt[-a]*c*d^2) - 2*a*Sqrt[c]*d*e + Sqrt[-a]*a*e^2)*Hy
pergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*
e)]))/((c*d^2 + a*e^2)^2*(1 + n))))/(4*a^2)
```

Maple [F] time = 0.73, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{x\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x/(c*x^2+a)^2,x)
```

```
[Out] int((e*x+d)^n/x/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n}{c^2x^5+2\,acx^3+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c^2*x^5 + 2*a*c*x^3 + a^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)
```

3.376 $\int \frac{(d+ex)^n}{x^2(a+cx^2)^2}\,dx$

Optimal. Leaf size=513

$$-\frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{a^2d^2(n+1)}-\frac{c(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)}{4(-a)^{5/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right.}$$

```
[Out] -(c*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a^2*(c*d^2 + a*e^2)*(a + c*x^2)) -
(c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - (c*(c*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hy
pergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*
e)])/(4*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (c*(
d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sq
rt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (
c*(c*d^2 + a*e^2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])
/(4*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(d +
e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d^2*(1 +
 n))
```

Rubi [A] time = 0.698169, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {961, 65, 741, 831, 68, 712}

$$-\frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{a^2d^2(n+1)}-\frac{c(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}den+ae^2(1-n)+cd^2\right)}{4(-a)^{5/2}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

```
[Out] -(c*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a^2*(c*d^2 + a*e^2)*(a + c*x^2)) -
(c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - (c*(c*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hy
pergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*
e)])/(4*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (c*(
d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sq
rt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (
c*(c*d^2 + a*e^2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])
/(4*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(d +
e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d^2*(1 +
 n))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx = \int \left(\frac{(d+ex)^n}{a^2x^2} - \frac{c(d+ex)^n}{a\left(a+cx^2\right)^2} - \frac{c(d+ex)^n}{a^2\left(a+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{(d+ex)^n}{x^2}\,dx}{a^2} - \frac{c\int \frac{(d+ex)^n}{a+cx^2}\,dx}{a^2} - \frac{c\int \frac{(d+ex)^n}{\left(a+cx^2\right)^2}\,dx}{a}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} - \frac{c\int\left(\frac{\sqrt{-a}(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\sqrt{-a}(d+e}{2a\left(\sqrt{-a}+\right.}\right.}{a^2}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} + \frac{c\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2(-a)^{5/2}} + \frac{c\int \frac{(d+e}{\sqrt{-a}+}}{2(-a}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{5/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} + \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+\right.}{2(-a)^{5/2}\left(\sqrt{c}d+\sqrt{\right.}}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2(-a)^{5/2}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)} - \frac{c\left(cd^2+ae^2(1-n)+\sqrt{\right.}}{4(-a)^{5/}}$$

Mathematica [A] time = 0.760214, size = 437, normalized size = 0.85

$$\frac{1}{4}(d+ex)^{n+1}\left(-\frac{2c(ae+cdx)}{a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\frac{4e\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{a^2d^2(n+1)}+\frac{ac\left(\frac{\left(\sqrt{-a}\sqrt{c}den-ae^2(n-1)+cd^2\right){}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+}{\sqrt{c}d-\sqrt{}}\right.}{\sqrt{c}d-\sqrt{-a}e}\right.}{(-a)^{7/2}(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*c*(a*e + c*d*x))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2))
+ (2*c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d -
Sqrt[-a]*e)])/((-a)^(5/2)*(-(Sqrt[c]*d) + Sqrt[-a]*e)*(1 + n)) + (2*c*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]
)/((-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*c*(((c*d^2 - a*e^2*(-1
 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]
*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 -
 a*e^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))
/((-a)^(7/2)*(c*d^2 + a*e^2)*(1 + n)) + (4*e*Hypergeometric2F1[2, 1 + n, 2
+ n, 1 + (e*x)/d])/(a^2*d^2*(1 + n))))/4
```

Maple [F] time = 0.753, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{x^2\left(cx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
```

```
[Out] int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n}{c^2x^6+2\,acx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n/(c^2*x^6 + 2*a*c*x^4 + a^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x**2/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{\left(cx^2+a\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)
```

3.377 $\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx$

Optimal. Leaf size=399

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(a^2e^4(m+n+2)(m+n+3)(m+n+4)(m+n+5)+cd^2(m+1)(m+2)\left(2ae^2\left(m^2+m(\right.\right.\right.}{e^4g(m+1)(m+n+2)(m+n+3)(m+n+4)(m}$$

```
[Out] -((c*d*(2 + m)*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*
(9 + 2*n)))*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^4*g*(2 + m + n)*(3 + m + n)
*(4 + m + n)*(5 + m + n))) + (c*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2
 + 9*n + n^2 + m*(9 + 2*n)))*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e^3*g^2*(3 +
 m + n)*(4 + m + n)*(5 + m + n)) - (c^2*d*(4 + m)*(g*x)^(3 + m)*(d + e*x)^(
1 + n))/(e^2*g^3*(4 + m + n)*(5 + m + n)) + (c^2*(g*x)^(4 + m)*(d + e*x)^(1
 + n))/(e*g^4*(5 + m + n)) + ((a^2*e^4*(2 + m + n)*(3 + m + n)*(4 + m + n)*
(5 + m + n) + c*d^2*(1 + m)*(2 + m)*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 +
 m^2 + 9*n + n^2 + m*(9 + 2*n))))*(g*x)^(1 + m)*(d + e*x)^n*Hypergeometric2
F1[1 + m, -n, 2 + m, -((e*x)/d)])/(e^4*g*(1 + m)*(2 + m + n)*(3 + m + n)*(4
 + m + n)*(5 + m + n)*(1 + (e*x)/d)^n)
```

Rubi [A] time = 0.764657, antiderivative size = 377, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {952, 1623, 80, 66, 64}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(\frac{a^2}{m+1}+\frac{cd^2(m+2)\left(2ae^2\left(m^2+m(2n+9)+n^2+9n+20\right)+cd^2\left(m^2+7m+12\right)\right)}{e^4(m+n+2)(m+n+3)(m+n+4)(m+n+5)}\right){}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)}{g}+\frac{c(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

```
[Out] -((c*d*(2 + m)*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*
(9 + 2*n)))*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^4*g*(2 + m + n)*(3 + m + n)
*(4 + m + n)*(5 + m + n))) + (c*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2
 + 9*n + n^2 + m*(9 + 2*n)))*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e^3*g^2*(3 +
 m + n)*(4 + m + n)*(5 + m + n)) - (c^2*d*(4 + m)*(g*x)^(3 + m)*(d + e*x)^(
1 + n))/(e^2*g^3*(4 + m + n)*(5 + m + n)) + (c^2*(g*x)^(4 + m)*(d + e*x)^(1
 + n))/(e*g^4*(5 + m + n)) + ((a^2/(1 + m) + (c*d^2*(2 + m)*(c*d^2*(12 + 7*
m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*(9 + 2*n))))/(e^4*(2 + m + n)*
(3 + m + n)*(4 + m + n)*(5 + m + n)))*(g*x)^(1 + m)*(d + e*x)^n*Hypergeomet
ric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + (e*x)/d)^n)
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k*(a + b*
```

```
x)^(m + q)*(c + d*x)^(n + 1))/(d*b^q*(m + n + q + 1)), x] + Dist[1/(d*b^q*(
m + n + q + 1)), Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q +
 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)^
(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
x] && PolyQ[Px, x] && GtQ[Expon[Px, x], 2]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)^2\,dx = \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(a^2eg^4(5+m+n)+2aceg^4(5+m+n)x^2-c\right.}{eg^4(5+m+n)}$$

$$= -\frac{c^2d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2g^3(4+m+n)(5+m+n)} + \frac{c^2(gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(a^2e^2g^7(4\right.}{}$$

$$= \frac{c\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)(gx)^{2+m}(d+ex)^{1+n}}{e^3g^2(3+m+n)(4+m+n)(5+m+n)} -$$

$$= -\frac{cd(2+m)\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)(gx)^{1+m}(d}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)}$$

$$= -\frac{cd(2+m)\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)(gx)^{1+m}(d}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)}$$

$$= -\frac{cd(2+m)\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)(gx)^{1+m}(d}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)}$$

Mathematica [A] time = 0.14715, size = 275, normalized size = 0.69

$$\frac{x(gx)^m(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(a^2e^4\,{}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)+2acd^2e^2\,{}_2F_1\left(m+1,-n-2;m+2;-\frac{ex}{d}\right)-4acd^2e^2\,{}_2F_1\left(m\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

```
[Out] (x*(g*x)^m*(d + e*x)^n*(c^2*d^4*Hypergeometric2F1[1 + m, -4 - n, 2 + m, -((
e*x)/d)] - 4*c^2*d^4*Hypergeometric2F1[1 + m, -3 - n, 2 + m, -((e*x)/d)] +
6*c^2*d^4*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] + 2*a*c*d^2*e
^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] - 4*c^2*d^4*Hypergeo
metric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] - 4*a*c*d^2*e^2*Hypergeometric2
F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + c^2*d^4*Hypergeometric2F1[1 + m, -n,
 2 + m, -((e*x)/d)] + 2*a*c*d^2*e^2*Hypergeometric2F1[1 + m, -n, 2 + m, -((
e*x)/d)] + a^2*e^4*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^4*(
1 + m)*(1 + (e*x)/d)^n)
```

Maple [F] time = 0.628, size = 0, normalized size = 0.

$$\int (gx)^m (ex+d)^n (cx^2+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^2+a)^2 (ex+d)^n (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2x^4+2\,acx^2+a^2\right)(ex+d)^n\,(gx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(e*x + d)^n*(g*x)^m, x)
```

Sympy [C] time = 116.433, size = 131, normalized size = 0.33

$$\frac{a^2d^ng^mxx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-n,m+1\\m+2\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)}+\frac{2acd^ng^mx^3x^m\Gamma(m+3)\,{}_2F_1\left(\begin{matrix}-n,m+3\\m+4\end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)}+\frac{c^2d^ng^mx^5x^m\Gamma(m+5)\,{}_2F\ldots}{\Gamma(m+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(c*x**2+a)**2,x)
```

```
[Out] a**2*d**n*g**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp_pol
ar(I*pi)/d)/gamma(m + 2) + 2*a*c*d**n*g**m*x**3*x**m*gamma(m + 3)*hyper((-n
, m + 3), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4) + c**2*d**n*g**m*x*
*5*x**m*gamma(m + 5)*hyper((-n, m + 5), (m + 6,), e*x*exp_polar(I*pi)/d)/ga
mma(m + 6)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^2 (ex+d)^n \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)
```

3.378 $\int (gx)^m (d+ex)^n \left(a+cx^2\right) dx$

Optimal. Leaf size=164

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d}+1\right)^{-n} \left(ae^2(m+n+2)(m+n+3)+cd^2(m+1)(m+2)\right) {}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)}{e^2g(m+1)(m+n+2)(m+n+3)} - \frac{cd(m+2}{e^2g(m+}$$

```
[Out] -((c*d*(2 + m)*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^2*g*(2 + m + n)*(3 + m +
 n))) + (c*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e*g^2*(3 + m + n)) + ((c*d^2*(
1 + m)*(2 + m) + a*e^2*(2 + m + n)*(3 + m + n))*(g*x)^(1 + m)*(d + e*x)^n*H
ypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(e^2*g*(1 + m)*(2 + m + n)*
(3 + m + n)*(1 + (e*x)/d)^n)
```

Rubi [A] time = 0.133066, antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {952, 80, 66, 64}

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d}+1\right)^{-n} \left(\frac{a}{m+1}+\frac{cd^2(m+2)}{e^2(m+n+2)(m+n+3)}\right) {}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)}{g} - \frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}}{e^2g(m+n+2)(m+n+3)} + \frac{c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

```
[Out] -((c*d*(2 + m)*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^2*g*(2 + m + n)*(3 + m +
 n))) + (c*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e*g^2*(3 + m + n)) + ((a/(1 +
m) + (c*d^2*(2 + m))/(e^2*(2 + m + n)*(3 + m + n)))*(g*x)^(1 + m)*(d + e*x)
^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + (e*x)/d)^n)
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)dx = \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(aeg^2(3+m+n)-cdg^2(2+m)x\right)dx}{eg^2(3+m+n)}$$

$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \left(a+\frac{cd^2(1+m)(2+m)}{e^2(2+m+n)(3+m+n)}\right.$$

$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \left(\left(a+\frac{cd^2(1+m)(2+m)}{e^2(2+m+n)(3+m+n)}\right.\right.$$

$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \frac{\left(a+\frac{cd^2(1+m)(2+m)}{e^2(2+m+n)(3+m+n)}\right)(gx)^{1+m}}{}$$

Mathematica [A] time = 0.0691794, size = 113, normalized size = 0.69

$$\frac{x(gx)^m(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(\left(ae^2+cd^2\right){}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)+cd^2\,{}_2F_1\left(m+1,-n-2;m+2;-\frac{ex}{d}\right)-2cd^2\,{}_2F_1\left(m+\right.\right.}{e^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

```
[Out] (x*(g*x)^m*(d + e*x)^n*(c*d^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*
x)/d)] - 2*c*d^2*Hypergeometric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + (c*d
^2 + a*e^2)*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^2*(1 + m)*
(1 + (e*x)/d)^n)
```

Maple [F] time = 0.428, size = 0, normalized size = 0.

$$\int \left(gx\right)^m(ex+d)^n\left(cx^2+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)(ex+d)^n\left(gx\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + a\right)(ex + d)^n \left(gx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

Sympy [C] time = 18.9112, size = 82, normalized size = 0.5

$$\frac{ad^n g^m x x^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)} + \frac{cd^n g^m x^3 x^m \Gamma(m+3)\, {}_2F_1\left(\begin{matrix} -n, m+3 \\ m+4 \end{matrix}\middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(c*x**2+a),x)
```

```
[Out] a*d**n*g**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp_polar(
I*pi)/d)/gamma(m + 2) + c*d**n*g**m*x**3*x**m*gamma(m + 3)*hyper((-n, m + 3
), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + a\right)(ex + d)^n \left(gx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)
```

3.379 $\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=148

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\right.}{2ag(m+1)}$$

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(2*a*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e
*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(2*a
*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rubi [A] time = 0.176814, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {912, 135, 133}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\right.}{2ag(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(2*a*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e
*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(2*a
*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{\sqrt{-a}(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}x\right)} + \frac{\sqrt{-a}(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx$$

$$= -\frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2\sqrt{-a}} - \frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\left((d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}-\sqrt{c}x}\,dx}{2\sqrt{-a}} - \frac{\left((d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}+\sqrt{c}x}\,dx}{2\sqrt{-a}}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{2ag(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}}{2a}$$

Mathematica [F] time = 0.116095, size = 0, normalized size = 0.

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2), x]
```

Maple [F] time = 0.752, size = 0, normalized size = 0.

$$\int \frac{(gx)^m(ex+d)^n}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n(gx)^m}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n(gx)^m}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n (gx)^m}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

3.380 $\int \frac{(gx)^m(d+ex)^n}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=295

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;\right.}{4a^2g(m+1)}$$

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d +
 e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4
*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m
, -n, 2, 2 + m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 +
 (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 2, 2 + m, -((
e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rubi [A] time = 0.417643, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {961, 135, 133, 912}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;\right.}{4a^2g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d +
 e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4
*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m
, -n, 2, 2 + m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 +
 (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 2, 2 + m, -((
e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \int \left(-\frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\sqrt{c}-cx\right)^2} - \frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\sqrt{c}+cx\right)^2} - \frac{c(gx)^m(d+ex)^n}{2a\left(-ac-c^2x^2\right)}\right)dx$$

$$= -\frac{c\int \frac{(gx)^m(d+ex)^n}{\left(\sqrt{-a}\sqrt{c}-cx\right)^2}\,dx}{4a} - \frac{c\int \frac{(gx)^m(d+ex)^n}{\left(\sqrt{-a}\sqrt{c}+cx\right)^2}\,dx}{4a} - \frac{c\int \frac{(gx)^m(d+ex)^n}{-ac-c^2x^2}\,dx}{2a}$$

$$= -\frac{c\int \left(-\frac{\sqrt{-a}(gx)^m(d+ex)^n}{2ac\left(\sqrt{-a}-\sqrt{c}x\right)} - \frac{\sqrt{-a}(gx)^m(d+ex)^n}{2ac\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx}{2a} - \frac{\left(c(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\left(\sqrt{-a}\sqrt{c}-cx\right)^2}\,dx}{4a} - \frac{\left(c(d+e\right.}{}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1}{4a^2g(}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1}{4a^2g(}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1}{4a^2g(}$$

Mathematica [F] time = 0.222908, size = 0, normalized size = 0.

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x]
```

Maple [F] time = 0.73, size = 0, normalized size = 0.

$$\int \frac{(gx)^m(ex+d)^n}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n (gx)^m}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n (gx)^m}{c^2x^4+2\,acx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^n*(g*x)^m/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n (gx)^m}{(cx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)
```

3.381 $\int x^5(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{a^2d\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{ad\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{d\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (a^2*d*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*d*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (d*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*x^7*(a + b*x^2)^p
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0840607, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 266, 43, 365, 364}

$$\frac{a^2d\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{ad\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{d\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*d*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (d*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*x^7*(a + b*x^2)^p
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx = d\int x^5\left(a+bx^2\right)^p\,dx + e\int x^6\left(a+bx^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int x^2(a+bx)^p\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^6\left(1+\frac{bx^2}{a}\right)^p\,dx$$
$$= \frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right) + \frac{1}{2}d\,\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2} - \frac{2a(a$$
$$= \frac{a^2d\left(a+bx^2\right)^{1+p}}{2b^3(1+p)} - \frac{ad\left(a+bx^2\right)^{2+p}}{b^3(2+p)} + \frac{d\left(a+bx^2\right)^{3+p}}{2b^3(3+p)} + \frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F$$

Mathematica [A] time = 0.0884401, size = 112, normalized size = 0.9

$$\frac{1}{14}\left(a+bx^2\right)^p\left(\frac{7d\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)} + 2ex^7\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((7*d*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
+ p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*e*x^7*Hypergeometric2F1[7/2
, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

Maple [F] time = 0.439, size = 0, normalized size = 0.

$$\int x^5(ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e\int\left(bx^2+a\right)^p x^6\,dx + \frac{\left(\left(p^2+3p+2\right)b^3x^6+\left(p^2+p\right)ab^2x^4-2a^2bpx^2+2a^3\right)\left(bx^2+a\right)^p d}{2\left(p^3+6p^2+11p+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^6, x) + 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p
)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)*(b*x^2 + a)^p*d/((p^3 + 6*p^2 + 11*p +
```

```
6)*b^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^6+dx^5\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^6 + d*x^5)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 59.9262, size = 1010, normalized size = 8.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d*Piecew
ise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*
b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(
4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2/(4*a**2*b**3 + 8*a*b**4*x
**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3
+ 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4
*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt
(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*
sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) - 2*b**2
*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log
(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*
sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*
x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a
*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b*
*3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**4/(
4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*
b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*
p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3
 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b
**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2
)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a
+ b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x*
*6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(bx^2+a\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")

[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^5, x)
```

3.382 $\int x^4(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{a^2e\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{ae\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{e\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (a^2*e*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*e*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (e*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*x^5*(a + b*x^2)^p
*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0848504, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 365, 364, 266, 43}

$$\frac{a^2e\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{ae\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{e\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (a^2*e*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*e*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (e*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*x^5*(a + b*x^2)^p
*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(d+ex)\left(a+bx^2\right)^p\,dx &= d\int x^4\left(a+bx^2\right)^p\,dx+e\int x^5\left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}e\,\text{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^2\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{1}{5}dx^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+\frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a\right.\right.\\ &= \frac{a^2e\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}-\frac{ae\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{e\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\end{aligned}$$

Mathematica [A] time = 0.0714254, size = 112, normalized size = 0.9

$$\frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5e\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+2dx^5\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
+ p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeometric2F1[5/2
, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$\int x^4(ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(ex+d)\left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^5+dx^4\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^5 + d*x^4)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 37.9123, size = 1010, normalized size = 8.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + e*Piecew
ise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*
b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(
4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2/(4*a**2*b**3 + 8*a*b**4*x
**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3
+ 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4
*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt
(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*
sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) - 2*b**2
*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log
(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*
sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*
x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a
*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b*
*3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**4/(
4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*
b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*
p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3
 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b
**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2
)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a
+ b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x*
*6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^4, x)
```

3.383 $\int x^3(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=100

$$-\frac{ad\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{d\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] -(a*d*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (d*(a + b*x^2)^(2 + p))/(2*b^2
*(2 + p)) + (e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/
a)])/(5*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.064525, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 266, 43, 365, 364}

$$-\frac{ad\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{d\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] -(a*d*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (d*(a + b*x^2)^(2 + p))/(2*b^2
*(2 + p)) + (e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/
a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^3(d+ex)\left(a+bx^2\right)^p\,dx = d\int x^3\left(a+bx^2\right)^p\,dx + e\int x^4\left(a+bx^2\right)^p\,dx$$
$$= \frac{1}{2}d\,\text{Subst}\left(\int x(a+bx)^p\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^p\,dx$$
$$= \frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right) + \frac{1}{2}d\,\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b} + \frac{(a+bx}{b}\right.\right. \ldots$$
$$= -\frac{ad\left(a+bx^2\right)^{1+p}}{2b^2(1+p)} + \frac{d\left(a+bx^2\right)^{2+p}}{2b^2(2+p)} + \frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.0751153, size = 87, normalized size = 0.87

$$\frac{1}{10}\left(a+bx^2\right)^p\left(2ex^5\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right) - \frac{5d\left(a+bx^2\right)\left(a-b(p+1)x^2\right)}{b^2(p+1)(p+2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p)
) + (2*e*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)
^p))/10
```

Maple [F] time = 0.411, size = 0, normalized size = 0.

$$\int x^3(ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e\int\left(bx^2+a\right)^p x^4\,dx + \frac{\left(b^2(p+1)x^4 + abpx^2 - a^2\right)\left(bx^2+a\right)^p d}{2\left(p^2+3p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^4, x) + 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)
*(b*x^2 + a)^p*d/((p^2 + 3*p + 2)*b^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^4+dx^3\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 28.6126, size = 394, normalized size = 3.94

$$\frac{a^p e x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+d\begin{cases}\frac{a^px^4}{4} & \text{for}\\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} & \text{for}\\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2}+\frac{x^2}{2b} & \text{for}\\ -\frac{a^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2} & \text{oth}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d*Piecew
ise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2
*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(
2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 +
2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
, Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*
sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2
*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 +
 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p +
 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(ex+d)\left(bx^2+a\right)^px^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^3, x)
```

3.384 $\int x^2(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=100

$$-\frac{ae\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] -(a*e*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e*(a + b*x^2)^(2 + p))/(2*b^2
*(2 + p)) + (d*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/
a)])/(3*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0590601, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 365, 364, 266, 43}

$$-\frac{ae\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] -(a*e*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e*(a + b*x^2)^(2 + p))/(2*b^2
*(2 + p)) + (d*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/
a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(d+ex)\left(a+bx^2\right)^p\,dx &= d\int x^2\left(a+bx^2\right)^p\,dx+e\int x^3\left(a+bx^2\right)^p\,dx\\ &=\frac{1}{2}e\,\text{Subst}\left(\int x(a+bx)^p\,dx,x,x^2\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &=\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+\frac{1}{2}e\,\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+\right.\right.\\ &=-\frac{ae\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0708861, size = 87, normalized size = 0.87

$$\frac{1}{6}\left(a+bx^2\right)^p\left(2dx^3\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)-\frac{3e\left(a+bx^2\right)\left(a-b(p+1)x^2\right)}{b^2(p+1)(p+2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((-3*e*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p)
) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)
^p))/6
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int x^2\,(ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^2*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^3 + dx^2\right)\left(bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 18.6494, size = 394, normalized size = 3.94

$$\frac{a^p d x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + e\begin{cases} \frac{a^p x^4}{4} & \text{for } b = \\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} & \text{for } p = \\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2} + \frac{x^2}{2b} & \text{for } p = \\ -\frac{a^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + e*Piecew
ise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2
*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(
2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 +
2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
, Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*
sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2
*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 +
 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p +
 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

3.385 $\int x(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e*x^3*(a + b*x^2)^p*Hypergeometric
2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0326914, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {764, 261, 365, 364}

$$\frac{d\left(a+bx^2\right)^{p+1}}{2b(p+1)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e*x^3*(a + b*x^2)^p*Hypergeometric
2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x(d+ex)\left(a+bx^2\right)^p\,dx = d\int x\left(a+bx^2\right)^p\,dx + e\int x^2\left(a+bx^2\right)^p\,dx$$
$$= \frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)} + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\,dx$$
$$= \frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)} + \frac{1}{3}ex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.0343881, size = 71, normalized size = 0.95

$$\frac{1}{6}\left(a+bx^2\right)^p\left(\frac{3d\left(a+bx^2\right)}{b(p+1)}+2ex^3\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((3*d*(a + b*x^2))/(b*(1 + p)) + (2*e*x^3*Hypergeometric2F1[
3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/6
```

Maple [F] time = 0.405, size = 0, normalized size = 0.

$$\int x\,(ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ex^2+dx\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x)*(b*x^2 + a)^p, x)
```

Sympy [A] time = 14.3415, size = 65, normalized size = 0.87

$$\frac{a^p e x^3 {}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2 e^{i\pi}}{a}\right)}{3}+d\left(\begin{cases}\frac{a^p x^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d*Piecew
ise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(bx^2+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x, x)
```

3.386 $\int (d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=70

$$dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

```
[Out] (e*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (d*x*(a + b*x^2)^p*Hypergeometric2F
1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rubi [A] time = 0.0197929, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {641, 246, 245}

$$dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (e*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (d*x*(a + b*x^2)^p*Hypergeometric2F
1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+bx^2\right)^p\,dx &= \frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+d\int \left(a+bx^2\right)^p\,dx\\ &= \frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+dx\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0740842, size = 98, normalized size = 1.4

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2bd(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+bex^2\left(\frac{bx^2}{a}+1\right)^p+ae\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(b*e*x^2*(1 + (b*x^2)/a)^p + a*e*(-1 + (1 + (b*x^2)/a)^p) +
2*b*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(2*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.406, size = 0, normalized size = 0.

$$\int (ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(b*x^2 + a)^p, x)
```

Sympy [A] time = 9.82081, size = 61, normalized size = 0.87

$$a^p dx\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+e\left(\begin{cases}\frac{a^px^2}{2} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{\left(a+bx^2\right)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + e*Piecewise((
a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(a + b*x**2), True))/(2*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p, x)
```

3.387 $\int \frac{(d+ex)\left(a+bx^2\right)^p}{x} dx$

Optimal. Leaf size=88

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

```
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

Rubi [A] time = 0.0528735, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 266, 65, 246, 245}

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x,x]
```

```
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx &= d\int \frac{\left(a+bx^2\right)^p}{x}\,dx + e\int \left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx\\ &= ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right) - \frac{d\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0551568, size = 88, normalized size = 1.

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right) - \frac{d\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x,x]
```

```
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

Maple [F] time = 0.399, size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)\left(bx^2+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(b*x^2 + a)^p/x, x)
```

Sympy [C] time = 15.9876, size = 65, normalized size = 0.74

$$a^pex_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)-\frac{b^pdx^{2p}\Gamma\left(-p\right)_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x,x)
```

```
[Out] a**p*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*x**(2*
p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma
(1 - p))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

3.388 $\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^2} dx$

Optimal. Leaf size=91

$$-\frac{d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{e\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

```
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rubi [A] time = 0.0527079, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 365, 364, 266, 65}

$$-\frac{d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{e\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^2}\,dx = d\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx + e\int \frac{\left(a+bx^2\right)^p}{x}\,dx$$
$$= \frac{1}{2}e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx$$
$$= -\frac{d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{b}{}\right.}{2a(1+p)}$$

Mathematica [A] time = 0.0511335, size = 91, normalized size = 1.

$$-\frac{d\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

Maple [F] time = 0.42, size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)\left(bx^2+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

Sympy [C] time = 19.3732, size = 68, normalized size = 0.75

$$-\frac{a^p d\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}-\frac{b^p e x^{2p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*e*x**(
2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gam
ma(1 - p))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^2, x)
```

3.389 $\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=92

$$\frac{bd\left(a+bx^2\right)^{p+1}{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

```
[Out] -((e*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) + (b*d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + (b*x^2)/a])/(2*a^2*(1 + p))
```

Rubi [A] time = 0.0529163, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {764, 266, 65, 365, 364}

$$\frac{bd\left(a+bx^2\right)^{p+1}{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{e\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -((e*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) + (b*d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + (b*x^2)/a])/(2*a^2*(1 + p))
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx = d\int \frac{\left(a+bx^2\right)^p}{x^3}\,dx + e\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{x^2}\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx$$

$$= -\frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} + \frac{bd\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

Mathematica [A] time = 0.047315, size = 89, normalized size = 0.97

$$\frac{1}{2}\left(a+bx^2\right)^p\left(\frac{bd\left(a+bx^2\right){}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{a^2(p+1)} - \frac{2e\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x^3,x]
```

```
[Out] ((a + b*x^2)^p*((-2*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1
 + (b*x^2)/a)^p) + (b*d*(a + b*x^2)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x^2)/a])/(a^2*(1 + p))))/2
```

Maple [F] time = 0.428, size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x^3,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)\left(bx^2+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

Sympy [C] time = 27.9828, size = 71, normalized size = 0.77

$$-\frac{a^p e\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p d x^{2p}\Gamma\left(1-p\right) {}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma\left(2-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x**3,x)
```

```
[Out] -a**p*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d*x**(
2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**2))/
(2*x**2*gamma(2 - p))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

3.390 $\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=188

$$\frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{2}{7}dex^7\left(a+b\right.$$

```
[Out] (a^2*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) - (a*(2*b*d^2 - 3
*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^4*(2 + p)) + ((b*d^2 - 3*a*e^2)*(a + b*x^
2)^(3 + p))/(2*b^4*(3 + p)) + (e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (
2*d*e*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(
1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.179971, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1652, 446, 77, 12, 365, 364}

$$\frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{2}{7}dex^7\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (a^2*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) - (a*(2*b*d^2 - 3
*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^4*(2 + p)) + ((b*d^2 - 3*a*e^2)*(a + b*x^
2)^(3 + p))/(2*b^4*(3 + p)) + (e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (
2*d*e*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(
1 + (b*x^2)/a)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^6\left(a+bx^2\right)^p\,dx+\int x^5\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^6\left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^2\left(-bd^2+ae^2\right)(a+bx)^p}{b^3}+\frac{a\left(-2bd^2+3ae^2\right)(a+bx)^{1+p}}{b^3}+\frac{\left(bd^2-3ae\right.}{}\right.\right.\\ &= \frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}\end{aligned}$$

Mathematica [A] time = 0.21493, size = 205, normalized size = 1.09

$$\frac{1}{14}\left(a+bx^2\right)^p\left(\frac{7d^2\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{7e^2\left(a+bx^2\right)\left(6a^2b(p+1)x^2-6a^3-3ab^2\left(\right.\right.}{b^4(p+1)(p+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((7*d^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*
p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*e^2*(a + b*x^2)*(-6*a^3 +
 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6*p^2
+ p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (4*d*e*x^7*Hypergeomet
ric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

Maple [F] time = 0.614, size = 0, normalized size = 0.

$$\int x^5\,(ex+d)^2\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(p^2+3\,p+2\right)b^3x^6+\left(p^2+p\right)ab^2x^4-2\,a^2bpx^2+2\,a^3\right)\left(bx^2+a\right)^pd^2}{2\left(p^3+6\,p^2+11\,p+6\right)b^3}+\int\left(e^2x^7+2\,dex^6\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((e^2*x^7 + 2*
d*e*x^6)*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^7+2\,dex^6+d^2x^5\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^7 + 2*d*e*x^6 + d^2*x^5)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 82.4758, size = 3043, normalized size = 16.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**2
*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(
4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b)
 + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**
2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b)
+ x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(
a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4
*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) -
 2*b**2*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a
**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*s
qrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*
x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x*
*4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x
)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) +
 x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**
2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 1
```

```
2*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b*
*3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)*
*p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a +
 b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x
**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*
b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3
), True)) + e**2*Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a)*
sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7
*x**6) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x
**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 2*a**3/(12*a**3*b**4 + 36*a**2*b**5*
x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqrt(
1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6
) + 18*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*sqrt(a)*sqr
t(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x*
*6) + 18*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b
**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) - 9*a*b**2*x**4/(12*a**3*b**4 + 3
6*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(-I*sqrt
(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*
b**7*x**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a*
*2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) - 9*b**3*x**6/(12*a**3*b**4 +
 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*l
og(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) -
6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x
**4) - 3*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*
log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) -
 12*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 +
 4*b**6*x**4) - 6*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 +
8*a*b**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(
4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) + 6*a*b**2*x**4/(4*a**2*b**4 + 8
*a*b**5*x**2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*
b**6*x**4), Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4
*b**5*x**2) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2)
+ 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/
(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2) + b**3*x*
*6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(-I*sqrt(a)*sqrt(1/b) +
x)/(2*b**4) - a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x**2/(2*b**
3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/(2*
b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3*b*
p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**
4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b*
*4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a + b*
x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4)
 + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p
**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4*p*
*4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x**6*
(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 4
8*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b
**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2*b**4
*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*x**
8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p +
 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b*
*4*p**2 + 100*b**4*p + 48*b**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(bx^2+a\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^5, x)
```

3.391 $\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=177

$$\frac{a^2de\left(a+bx^2\right)^{p+1}}{b^3(p+1)}-\frac{2ade\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{de\left(a+bx^2\right)^{p+3}}{b^3(p+3)}-\frac{x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(5ae^2-bd^2(2p+7)\right)\,{}_2F_1\left(\frac{5}{2},-p;\right.}{5b(2p+7)}$$

```
[Out] (a^2*d*e*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (e^2*x^5*(a + b*x^2)^(1 + p))
/(b*(7 + 2*p)) - (2*a*d*e*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (d*e*(a + b*
x^2)^(3 + p))/(b^3*(3 + p)) - ((5*a*e^2 - b*d^2*(7 + 2*p))*x^5*(a + b*x^2)^
p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*b*(7 + 2*p)*(1 + (b*x^2
)/a)^p)
```

Rubi [A] time = 0.162716, antiderivative size = 169, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1652, 459, 365, 364, 12, 266, 43}

$$\frac{a^2de\left(a+bx^2\right)^{p+1}}{b^3(p+1)}-\frac{2ade\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{de\left(a+bx^2\right)^{p+3}}{b^3(p+3)}+\frac{1}{5}x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{5ae^2}{2bp+7b}\right){}_2F_1\left(\frac{5}{2},-p;\frac{7}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*e*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (e^2*x^5*(a + b*x^2)^(1 + p))
/(b*(7 + 2*p)) - (2*a*d*e*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (d*e*(a + b*
x^2)^(3 + p))/(b^3*(3 + p)) + ((d^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x
^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^5\left(a+bx^2\right)^p\,dx+\int x^4\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+(2de)\int x^5\left(a+bx^2\right)^p\,dx-\left(-d^2+\frac{5ae^2}{7b+2bp}\right)\int x^4\left(a+bx^2\right)^p\,dx\\ &= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+(de)\,\mathrm{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^2\right)-\left(\left(-d^2+\frac{5ae^2}{7b+2bp}\right)\left(a+bx^2\right)^p\right.\\ &= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{5}\left(d^2-\frac{5ae^2}{7b+2bp}\right)x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+\\ &= \frac{a^2de\left(a+bx^2\right)^{1+p}}{b^3(1+p)}+\frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}-\frac{2ade\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{de\left(a+bx^2\right)^{3+p}}{b^3(3+p)}+\frac{1}{5}\left(d^2-\right.\end{aligned}$$

Mathematica [A] time = 0.169891, size = 156, normalized size = 0.88

$$\frac{1}{35}\left(a+bx^2\right)^p\left(\frac{35de\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+7d^2x^5\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+5e\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((35*d*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*d^2*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (5*e^2*x^7*Hypergeometri
c2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/35
```

Maple [F] time = 0.598, size = 0, normalized size = 0.

$$\int x^4(ex+d)^2\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^6+2\,dex^5+d^2x^4\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^6 + 2*d*e*x^5 + d^2*x^4)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 84.8486, size = 1046, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*
e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 2*d*e*Piec
ewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**
2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2/(4*a**2*b**3 + 8*a*b**4
*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/
(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sq
rt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(
I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) - 2*b*
*2*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*l
og(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a
)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2)
 - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*
b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2
*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*
```

```
b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**4
/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 2
2*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**
3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p*
*3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2
*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x*
*2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(
a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*
x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), Tr
ue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 (bx^2+a)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^4, x)
```

3.392 $\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=149

$$-\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p\right.$$

```
[Out] -(a*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + ((b*d^2 - 2*a*e^
2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(2*b^3*
(3 + p)) + (2*d*e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^
2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.138046, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1652, 446, 77, 12, 365, 364}

$$-\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```


```
[Out] -(a*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + ((b*d^2 - 2*a*e^
2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(2*b^3*
(3 + p)) + (2*d*e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^
2)/a)])/(5*(1 + (b*x^2)/a)^p)
```


Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^4\left(a+bx^2\right)^p\,dx+\int x^3\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)dx\\ &=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(d^2+e^2x\right)dx,x,x^2\right)+(2de)\int x^4\left(a+bx^2\right)^p\,dx\\ &=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a\left(-bd^2+ae^2\right)(a+bx)^p}{b^2}+\frac{\left(bd^2-2ae^2\right)(a+bx)^{1+p}}{b^2}+\frac{e^2(a+bx)^{2+p}}{b^2}\right)dx,x,\right.\\ &=-\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{e^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{2}{5}dex^5\left(a+b\right.\end{aligned}$$

Mathematica [A] time = 0.150886, size = 152, normalized size = 1.02

$$\frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5e^2\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{5d^2\left(a+bx^2\right)\left(b(p+1)x^2-a\right)}{b^2(p+1)(p+2)}+4dex^5\left(\frac{bx^2}{a}+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*d^2*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
p)) + (5*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x
^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (4*d*e*x^5*Hypergeometric2F1[5/2, -p,
7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

Maple [F] time = 0.62, size = 0, normalized size = 0.

$$\int x^3\,(ex+d)^2\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2(p+1)x^4+abpx^2-a^2\right)\left(bx^2+a\right)^p d^2}{2\left(p^2+3p+2\right)b^2}+\int\left(e^2x^5+2\,dex^4\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^2/((p^2 + 3*p + 2)*
b^2) + integrate((e^2*x^5 + 2*d*e*x^4)*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^5+2\,dex^4+d^2x^3\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^5 + 2*d*e*x^4 + d^2*x^3)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 44.1619, size = 1384, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**2
*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b
**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2
) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*
b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**
3*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*s
qrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2
)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2
*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b
**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**
2), True)) + e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)
*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*
sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2/(
4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt
(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*s
qrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*
x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2
 + 4*b**5*x**4) - 2*b**2*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4),
Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2)
 - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2
```

```
*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3
 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**
4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt
(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) -
a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p
**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p
/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a +
 b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x
**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b*
*3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b
**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*
p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22
*b**3*p + 12*b**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(bx^2+a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^3, x)
```

3.393 $\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=152

$$-\frac{ade\left(a+bx^2\right)^{p+1}}{b^2(p+1)}+\frac{de\left(a+bx^2\right)^{p+2}}{b^2(p+2)}-\frac{x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2-bd^2(2p+5)\right){}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)}{3b(2p+5)}+\frac{e^2x^3\left(a+\right.}{b(2p}$$

```
[Out] -((a*d*e*(a + b*x^2)^(1 + p))/(b^2*(1 + p))) + (e^2*x^3*(a + b*x^2)^(1 + p)
)/(b*(5 + 2*p)) + (d*e*(a + b*x^2)^(2 + p))/(b^2*(2 + p)) - ((3*a*e^2 - b*d
^2*(5 + 2*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a
)])/(3*b*(5 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.136522, antiderivative size = 144, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1652, 459, 365, 364, 12, 266, 43}

$$-\frac{ade\left(a+bx^2\right)^{p+1}}{b^2(p+1)}+\frac{de\left(a+bx^2\right)^{p+2}}{b^2(p+2)}+\frac{1}{3}x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{3ae^2}{2bp+5b}\right){}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+\frac{e^2x^3\left(a+\right.}{b(2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] -((a*d*e*(a + b*x^2)^(1 + p))/(b^2*(1 + p))) + (e^2*x^3*(a + b*x^2)^(1 + p)
)/(b*(5 + 2*p)) + (d*e*(a + b*x^2)^(2 + p))/(b^2*(2 + p)) + ((d^2 - (3*a*e^
2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^
2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^3\left(a+bx^2\right)^p\,dx + \int x^2\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx \\
&= \frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)} + (2de)\int x^3\left(a+bx^2\right)^p\,dx - \left(-d^2+\frac{3ae^2}{5b+2bp}\right)\int x^2\left(a+bx^2\right)^p\,dx \\
&= \frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)} + (de)\,\text{Subst}\left(\int x(a+bx)^p\,dx, x, x^2\right) - \left(\left(-d^2+\frac{3ae^2}{5b+2bp}\right)\left(a+bx^2\right)^p\right. \\
&= \frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)} + \frac{1}{3}\left(d^2-\frac{3ae^2}{5b+2bp}\right)x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+ \\
&= -\frac{ade\left(a+bx^2\right)^{1+p}}{b^2(1+p)} + \frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)} + \frac{de\left(a+bx^2\right)^{2+p}}{b^2(2+p)} + \frac{1}{3}\left(d^2-\frac{3ae^2}{5b+2bp}\right)x^3\left(a+bx^2\right.
\end{aligned}$$

Mathematica [A] time = 0.159155, size = 139, normalized size = 0.91

$$\frac{1}{15}\left(a+bx^2\right)^p\left(\frac{3e\left(e\left(p^2+3p+2\right)x^5\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)-\frac{5d\left(a+bx^2\right)\left(a-b(p+1)x^2\right)}{b^2}\right)}{(p+1)(p+2)}+5d^2x^3\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*d^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^p + (3*e*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/b^2 + (e*(2
 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p))/((1 + p)*(2 + p))))/15
```

Maple [F] time = 0.519, size = 0, normalized size = 0.

$$\int x^2 (ex+d)^2 \left(bx^2+a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(bx^2+a\right)^p x^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^4+2\,dex^3+d^2x^2\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 39.5501, size = 430, normalized size = 2.83

$$\frac{a^p d^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + \frac{a^p e^2 x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + 2de\left(\begin{cases} \frac{a^p x^4}{4} \\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(-i\sqrt{a} \cdots\right.}{2ab^2+2b \cdots} \\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2} + \frac{x^2}{2b} \\ -\frac{a^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4\left(a+bx^2\right)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4\left(a+ \cdots\right.}{2b^2p^2+6b \cdots} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p*
e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 2*d*e*Piec
ewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 +
 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a
/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2
```

```
+ 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**
2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a
)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/
(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2
 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p
 + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), T
rue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(bx^2+a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

3.394 $\int x(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=113

$$\frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^2*(a + b*x^2)^(2
 + p))/(2*b^2*(2 + p)) + (2*d*e*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p
, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0970589, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1652, 444, 43, 12, 365, 364}

$$\frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{e^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^2*(a + b*x^2)^(2
 + p))/(2*b^2*(2 + p)) + (2*d*e*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p
, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^2\left(a+bx^2\right)^p\,dx+\int x\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^2\left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{\left(bd^2-ae^2\right)(a+bx)^p}{b}+\frac{e^2(a+bx)^{1+p}}{b}\right)dx,x,x^2\right)+\left(2de\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right.\right.\\ &= \frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\right.\end{aligned}$$

Mathematica [A] time = 0.191503, size = 184, normalized size = 1.63

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(-3a^2e^2\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)+3b^2x^2\left(\frac{bx^2}{a}+1\right)^p\left(d^2(p+2)+e^2(p+1)x^2\right)+4b^2de\left(p^2+3p+2\right)x^3\,{}_2F_1\right.}{6b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(3*b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + e^2*(1 + p)*x^2)
 - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + 3*a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p
+ d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 4*b^2*d*e*(2 + 3*p + p^2)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)]))/(6*b^2*(1 + p)*(2 + p)*(1 + (b
*x^2)/a)^p)
```

Maple [F] time = 0.498, size = 0, normalized size = 0.

$$\int x\,(ex+d)^2\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^{2}x^{3}+2\,dex^{2}+d^{2}x\right)\left(bx^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^3 + 2*d*e*x^2 + d^2*x)*(b*x^2 + a)^p, x)
```

Sympy [A] time = 21.2082, size = 439, normalized size = 3.88

$$\frac{2a^{p}dex^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3}+d^{2}\left(\begin{cases}\frac{a^{p}x^{2}}{2} & \text{for}\ b=0\\\frac{\begin{cases}\frac{(a+bx^{2})^{p+1}}{p+1} & \text{for}\ p\neq -1\\\log\left(a+bx^{2}\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)+e^{2}\left(\begin{cases}\frac{a^{p}x^{4}}{4} & \\\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}} & \\-\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^{2}}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^{2}} & \\-\frac{a^{2}(a+bx^{2})^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{abpx^{2}(a+bx^{2})^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+ & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d**2
*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p +
1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**2*Piecewise((a*
*p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x
**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**
2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*
x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p,
 -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/
b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p
**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2
*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2
) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(ex+d)^{2}\left(bx^{2}+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x, x)
```

3.395 $\int (d+ex)^2 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=133

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2-bd^2(2p+3)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(2p+3)}+\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}+\frac{de(p+2)\left(a+bx^2\right)^{p+1}}{b(p+1)(2p+3)}$$

```
[Out] (d*e*(2 + p)*(a + b*x^2)^(1 + p))/(b*(1 + p)*(3 + 2*p)) + (e*(d + e*x)*(a +
 b*x^2)^(1 + p))/(b*(3 + 2*p)) - ((a*e^2 - b*d^2*(3 + 2*p))*x*(a + b*x^2)^p
*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b*(3 + 2*p)*(1 + (b*x^2)/a
)^p)
```

Rubi [A] time = 0.0774444, antiderivative size = 125, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {743, 641, 246, 245}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+3b}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}+\frac{de(p+2)\left(a+bx^2\right)^{p+1}}{b(p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (d*e*(2 + p)*(a + b*x^2)^(1 + p))/(b*(1 + p)*(3 + 2*p)) + (e*(d + e*x)*(a +
 b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \frac{\int \left(-ae^2+bd^2(3+2p)+2bde(2+p)x\right)\left(a+bx^2\right)^p \, dx}{b(3+2p)}$$

$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(d^2 - \frac{ae^2}{3b+2bp}\right)\int \left(a+bx^2\right)^p \, dx$$

$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(\left(d^2 - \frac{ae^2}{3b+2bp}\right)\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right)$$

$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(d^2 - \frac{ae^2}{3b+2bp}\right)x\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}$$

Mathematica [A] time = 0.110959, size = 133, normalized size = 1.

$$\frac{\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(3bd^2(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+e\left(3d\left(bx^2\left(\frac{bx^2}{a}+1\right)^p+a\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)\right)+be(p+1)x^3\,{}_2F_1\left(\right.\right.\right.}{3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(3*b*d^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)] + e*(3*d*(b*x^2*(1 + (b*x^2)/a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + b*e*
(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(3*b*(1 + p)*(
1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.502, size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \left(bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p, x)
```

Sympy [A] time = 18.8585, size = 97, normalized size = 0.73

$$a^pd^2x{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+\frac{a^pe^2x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+2de\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{\left(a+bx^2\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e**2*
x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 2*d*e*Piecewise
((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

3.396 $\int \frac{(d+ex)^2(a+bx^2)^p}{x} dx$

Optimal. Leaf size=118

$$-\frac{d^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+2dex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e^2\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

```
[Out] (e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (2*d*e*x*(a + b*x^2)^p*Hypergeome
tric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rubi [A] time = 0.093041, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1652, 446, 80, 65, 12, 246, 245}

$$-\frac{d^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+2dex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e^2\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

```
[Out] (e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (2*d*e*x*(a + b*x^2)^p*Hypergeome
tric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
```

```
m] || GtQ[-(d/(b*c)), 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x}\,dx &= \int 2de\left(a+bx^2\right)^p\,dx+\int \frac{\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)}{x}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p\left(d^2+e^2x\right)}{x}\,dx,x,x^2\right)+(2de)\int \left(a+bx^2\right)^p\,dx\\ &= \frac{e^2\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\frac{1}{2}d^2\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\left(2de\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \Big(\\ &= \frac{e^2\left(a+bx^2\right)^{1+p}}{2b(1+p)}+2dex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d^2\left(a+bx^2\right)^{1+p}\,{}_2}{}\end{aligned}$$

Mathematica [A] time = 0.101324, size = 101, normalized size = 0.86

$$\frac{1}{2}\left(a+bx^2\right)^p\left(\frac{\left(a+bx^2\right)\left(ae^2-bd^2\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)\right)}{ab(p+1)}+4dex\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

```
[Out] ((a + b*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p + ((a + b*x^2)*(a*e^2 - b*d^2*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 + (b*x^2)/a]))/(a*b*(1 + p))))/2
```

Maple [F] time = 0.492, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(bx^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\left(bx^2+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x, x)
```

Sympy [A] time = 19.2026, size = 109, normalized size = 0.92

$$2a^pdex\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)-\frac{b^pd^2x^{2p}\Gamma(-p)\,{}_2F_1\left(\left.\begin{matrix}-p,-p\\1-p\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}+e^2\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p/x,x)
```

```
[Out] 2*a**p*d*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**2
*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(
2*gamma(1 - p)) + e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a +
b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(bx^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x, x)
```

3.397 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^2} dx$

Optimal. Leaf size=127

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(2p+1)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{d^2\left(a+bx^2\right)^{p+1}}{ax}-\frac{de\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\right.}{a(p+1)}$$

```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(a*x)) + ((a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) -
 (d*e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a]
)/(a*(1 + p))
```

Rubi [A] time = 0.12257, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1807, 764, 266, 65, 246, 245}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(2p+1)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{d^2\left(a+bx^2\right)^{p+1}}{ax}-\frac{de\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\right.}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```


```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(a*x)) + ((a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) -
 (d*e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a]
)/(a*(1 + p))
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
```

```
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x^2}\,dx = -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax} - \frac{\int \frac{\left(-2ade-\left(ae^2+bd^2(1+2p)\right)x\right)\left(a+bx^2\right)^p}{x}\,dx}{a}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax} + (2de)\int \frac{\left(a+bx^2\right)^p}{x}\,dx + \frac{\left(ae^2+bd^2(1+2p)\right)\int \left(a+bx^2\right)^p\,dx}{a}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax} + (de)\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \frac{\left(\left(ae^2+bd^2(1+2p)\right)\left(a+bx^2\right)^p \right.}{a}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax} + \frac{\left(ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a} - \ldots$$

Mathematica [A] time = 0.080191, size = 134, normalized size = 1.06

$$-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ad^2(p+1)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)+ex\left(d\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)-a\ldots\right.\right.}{a(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(a*d^2*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/
a)] + e*x*(-(a*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]) +
 d*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*
x^2)/a])))/(a*(1 + p)*x*(1 + (b*x^2)/a)^p))
```

Maple [F] time = 0.516, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(bx^2+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2+2\,dex+d^2)(bx^2+a)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x^2, x)
```

Sympy [C] time = 25.3617, size = 95, normalized size = 0.75

$$-\frac{a^p d^2 {}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}+a^pe^2x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)-\frac{b^pdex^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d**2*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p*e**
2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*e*x**(2*p)*
gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/gamma(1 - p
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(bx^2+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^2, x)
```

3.398 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{d^2(a+bx^2)^{p+1}}{2ax^2}-\frac{2de(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2}\right.}{x}$$

```
[Out] -(d^2*(a + b*x^2)^(1 + p))/(2*a*x^2) - (2*d*e*(a + b*x^2)^p*Hypergeometric2
F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - ((a*e^2 + b*d^2*p)
*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*
a^2*(1 + p))
```

Rubi [A] time = 0.12457, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1807, 764, 365, 364, 266, 65}

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{d^2(a+bx^2)^{p+1}}{2ax^2}-\frac{2de(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2}\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -(d^2*(a + b*x^2)^(1 + p))/(2*a*x^2) - (2*d*e*(a + b*x^2)^p*Hypergeometric2
F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - ((a*e^2 + b*d^2*p)
*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*
a^2*(1 + p))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x^3}\,dx &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2ax^2}-\frac{\int \frac{\left(-4ade-2\left(ae^2+bd^2p\right)x\right)\left(a+bx^2\right)^p}{x^2}\,dx}{2a}\\ &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2ax^2}+(2de)\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx+\frac{\left(ae^2+bd^2p\right)\int \frac{\left(a+bx^2\right)^p}{x}\,dx}{a}\\ &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2ax^2}+\frac{\left(ae^2+bd^2p\right)\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2a}+\left(2de\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\\ &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{2ax^2}-\frac{2de\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.0956011, size = 119, normalized size = 0.94

$$\frac{1}{2}\left(a+bx^2\right)^p\left(-\frac{\left(a+bx^2\right)\left(ae^2\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)-bd^2\,{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)\right)}{a^2(p+1)}-\frac{4de\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},\right.}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

```
[Out] ((a + b*x^2)^p*((-4*d*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^p) - ((a + b*x^2)*(a*e^2*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a]))
/(a^2*(1 + p))))/2
```

Maple [F] time = 0.509, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x^3,x)
```

[Out] int((e*x+d)^2*(b*x^2+a)^p/x^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2 (bx^2+a)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="maxima")

[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2+2\,dex+d^2)(bx^2+a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="fricas")

[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(b*x^2 + a)^p/x^3, x)

Sympy [C] time = 32.8256, size = 119, normalized size = 0.94

$$-\frac{2a^p de\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p d^2 x^{2p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma(2-p)} - \frac{b^p e^2 x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(b*x**2+a)**p/x**3,x)

[Out] -2*a**p*d*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d*
*2*x**(2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*
x**2))/(2*x**2*gamma(2 - p)) - b**p*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p),
 (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2 (bx^2+a)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="giac")

[Out] integrate((e*x + d)^2*(b*x^2 + a)^p/x^3, x)

3.399 $\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=247

$$\frac{a^2d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{ad\left(2bd^2-9ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{d\left(bd^2-9ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{3de^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}-\frac{ex^7\left(\right.}{}$$

```
[Out] (a^2*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (e^3*x^7*(a
 + b*x^2)^(1 + p))/(b*(9 + 2*p)) - (a*d*(2*b*d^2 - 9*a*e^2)*(a + b*x^2)^(2
+ p))/(2*b^4*(2 + p)) + (d*(b*d^2 - 9*a*e^2)*(a + b*x^2)^(3 + p))/(2*b^4*(3
 + p)) + (3*d*e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) - (e*(7*a*e^2 - 3*b*
d^2*(9 + 2*p))*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/
a)])/(7*b*(9 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.246802, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1652, 446, 77, 459, 365, 364}

$$\frac{a^2d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{ad\left(2bd^2-9ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{d\left(bd^2-9ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{3de^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{1}{7}ex^7$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (e^3*x^7*(a
 + b*x^2)^(1 + p))/(b*(9 + 2*p)) - (a*d*(2*b*d^2 - 9*a*e^2)*(a + b*x^2)^(2
+ p))/(2*b^4*(2 + p)) + (d*(b*d^2 - 9*a*e^2)*(a + b*x^2)^(3 + p))/(2*b^4*(3
 + p)) + (3*d*e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (e*(3*d^2 - (7*a*e
^2)/(9*b + 2*b*p))*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x
^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^5\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^6\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}+\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\left(e\left(3d^2-\frac{7ae}{9b+2}\right.\right.$$

$$=\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2d\left(-bd^2+3ae^2\right)(a+bx)^p}{b^3}+\frac{ad\left(-2bd^2+9ae^2\right)(}{b^3}\right.\right.$$

$$=\frac{a^2d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}-\frac{ad\left(2bd^2-9ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+$$

Mathematica [A] time = 0.233041, size = 249, normalized size = 1.01

$$\frac{1}{126}\left(a+bx^2\right)^p\left(\frac{63d^3\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{189de^2\left(a+bx^2\right)\left(6a^2b(p+1)x^2-6a^3-\right.}{b^4(p+1)(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((63*d^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (189*d*e^2*(a + b*x^2)*(-6*
a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6
*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (54*d^2*e*x^7*Hyp
ergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (14*e^3*x^9
*Hypergeometric2F1[9/2, -p, 11/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/126
```

Maple [F] time = 0.539, size = 0, normalized size = 0.

$$\int x^5(ex+d)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\left(p^2+3\,p+2\right)b^3x^6+\left(p^2+p\right)ab^2x^4-2\,a^2bpx^2+2\,a^3\right)\left(bx^2+a\right)^p d^3}{2\left(p^3+6\,p^2+11\,p+6\right)b^3}+\int\left(e^3x^8+3\,de^2x^7+3\,d^2ex^6\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((e^3*x^8 + 3*
d*e^2*x^7 + 3*d^2*e*x^6)*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^3x^8+3\,de^2x^7+3\,d^2ex^6+d^3x^5\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(b*x^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex+d\right)^3\left(bx^2+a\right)^p x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^5, x)
```

3.400 $\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=249

$$\frac{a^2e\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{3ae\left(2bd^2-ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{3e\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^3\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}-\frac{dx^5\left(\right.}{}$$

```
[Out] (a^2*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*d*e^2*x^
5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) - (3*a*e*(2*b*d^2 - a*e^2)*(a + b*x^2)
^(2 + p))/(2*b^4*(2 + p)) + (3*e*(b*d^2 - a*e^2)*(a + b*x^2)^(3 + p))/(2*b^
4*(3 + p)) + (e^3*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) - (d*(15*a*e^2 - b*d
^2*(7 + 2*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a
)])/(5*b*(7 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.235465, antiderivative size = 241, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1652, 459, 365, 364, 446, 77}

$$\frac{a^2e\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{3ae\left(2bd^2-ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{3e\left(bd^2-ae^2\right)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^3\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{1}{5}dx^5$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (a^2*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*d*e^2*x^
5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) - (3*a*e*(2*b*d^2 - a*e^2)*(a + b*x^2)
^(2 + p))/(2*b^4*(2 + p)) + (3*e*(b*d^2 - a*e^2)*(a + b*x^2)^(3 + p))/(2*b^
4*(3 + p)) + (e^3*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (d*(d^2 - (15*a*e^
2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^
2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx = \int x^4\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx + \int x^5\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$
$$= \frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} + \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\left(3d^2e+e^3x\right)dx, x, x^2\right) + \left(d\left(d^2 - \frac{15ae^2}{7b+2b\ldots}\right.\right.$$
$$= \frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} + \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2e\left(-3bd^2+ae^2\right)(a+bx)^p}{b^3} + \frac{3ae\left(-2bd^2+ae^2\right)(a\ldots}{b^3}\right.\right.$$
$$= \frac{a^2e\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)} + \frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} - \frac{3ae\left(2bd^2-ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)} + \ldots$$

Mathematica [A] time = 0.215936, size = 249, normalized size = 1.

$$\frac{1}{70}\left(a+bx^2\right)^p\left(\frac{105d^2e\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)} + \frac{35e^3\left(a+bx^2\right)\left(6a^2b(p+1)x^2-6a^3-3ab^2\ldots\right)}{b^4(p+1)(p+\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((105*d^2*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2
+ 3*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (35*e^3*(a + b*x^2)*(-6*
a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6
*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyper
geometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (30*d*e^2*x^7
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

Maple [F] time = 0.516, size = 0, normalized size = 0.

$$\int x^4(ex+d)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 (bx^2+a)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^7+3\,de^2x^6+3\,d^2ex^5+d^3x^4\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 87.2607, size = 3079, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*a**
p*d*e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 3*d**2
*e*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/
b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2/(4*a**2*b**3 + 8
*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a
**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b
) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqr
t(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x*
*4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
 - 2*b**2*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2
*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I
*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**
4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2)
 - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*
x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) +
```

```
x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2)
 + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p
**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 +
 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*
b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2
)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p
*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) +
2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b*
*3), True)) + e**3*Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a
)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b*
*7*x**6) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5
*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 2*a**3/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqr
t(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x*
*6) + 18*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b
**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*sqrt(a)*s
qrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*
x**6) + 18*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2
*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) - 9*a*b**2*x**4/(12*a**3*b**4 +
 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(-I*sq
rt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 1
2*b**7*x**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*
a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) - 9*b**3*x**6/(12*a**3*b**4
 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3
*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4)
- 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6
*x**4) - 3*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**
2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4)
 - 12*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2
 + 4*b**6*x**4) - 6*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4
+ 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)
/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) + 6*a*b**2*x**4/(4*a**2*b**4 +
 8*a*b**5*x**2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 +
4*b**6*x**4), Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 +
 4*b**5*x**2) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2
) + 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b
) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x
)/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2) + b**3*
x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(2*b**4) - a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x**2/(2*b
**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/(
2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3*
b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b
**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*
b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a +
b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**
4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4
*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4*
p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x**
6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p +
 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70
*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2*b*
*4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*x
**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p
 + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*
b**4*p**2 + 100*b**4*p + 48*b**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(bx^2+a\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)
```

3.401 $\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=207

$$-\frac{ad\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{d\left(bd^2-6ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{3de^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}-\frac{ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(5ae^2-3b\right.}{5b(2p+}$$

```
[Out] -(a*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (e^3*x^5*(a
+ b*x^2)^(1 + p))/(b*(7 + 2*p)) + (d*(b*d^2 - 6*a*e^2)*(a + b*x^2)^(2 + p))
/(2*b^3*(2 + p)) + (3*d*e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) - (e*(5*a*
e^2 - 3*b*d^2*(7 + 2*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2,
-((b*x^2)/a)])/(5*b*(7 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.200835, antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1652, 446, 77, 459, 365, 364}

$$-\frac{ad\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{d\left(bd^2-6ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{3de^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] -(a*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (e^3*x^5*(a
+ b*x^2)^(1 + p))/(b*(7 + 2*p)) + (d*(b*d^2 - 6*a*e^2)*(a + b*x^2)^(2 + p))
/(2*b^3*(2 + p)) + (3*d*e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*(3*d^
2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7
/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^3\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^4\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$

$$=\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\left(e\left(3d^2-\frac{5ae^2}{7b+2\right.\right.$$

$$=\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{ad\left(-bd^2+3ae^2\right)(a+bx)^p}{b^2}+\frac{\left(bd^3-6ade^2\right)(a+bx)}{b^2}\right.\right.$$

$$=-\frac{ad\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{d\left(bd^2-6ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{3d}{}$$

Mathematica [A] time = 0.162284, size = 196, normalized size = 0.95

$$\frac{1}{70}\left(a+bx^2\right)^p\left(\frac{105de^2\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{35d^3\left(a+bx^2\right)\left(b(p+1)x^2-a\right)}{b^2(p+1)(p+2)}+42d^2e\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((35*d^3*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
 p)) + (105*d*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p
^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (42*d^2*e*x^5*Hypergeometric2F1[5
/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (10*e^3*x^7*Hypergeometric2
F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

Maple [F] time = 0.579, size = 0, normalized size = 0.

$$\int x^3\left(ex+d\right)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)

[Out] int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2(p+1)x^4+abpx^2-a^2\right)\left(bx^2+a\right)^p d^3}{2\left(p^2+3p+2\right)b^2}+\int\left(e^3x^6+3\,de^2x^5+3\,d^2ex^4\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")

[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^3/((p^2 + 3*p + 2)*b^2) + integrate((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4)*(b*x^2 + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^3x^6+3\,de^2x^5+3\,d^2ex^4+d^3x^3\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")

[Out] integral((e^3*x^6 + 3*d*e^2*x^5 + 3*d^2*e*x^4 + d^3*x^3)*(b*x^2 + a)^p, x)

Sympy [C] time = 76.0631, size = 1420, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x+d)**3*(b*x**2+a)**p,x)

[Out] 3*a**p*d**2*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*e**3*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**3*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True)) + 3*d*e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I

```
*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**
2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x*
*2 + 4*b**5*x**4) - 2*b**2*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
, Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**
2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/
(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b*
*3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b
**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sq
rt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3)
- a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3
*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)*
*p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p
*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) +
b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**
3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
22*b**3*p + 12*b**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 (bx^2+a)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^3, x)
```

3.402 $\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=210

$$-\frac{ae\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{e\left(3bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^3\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}-\frac{dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(9ae^2-bd^2(\right.}{3b(2p+5)}$$

```
[Out] -(a*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (3*d*e^2*x^3
*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (e*(3*b*d^2 - 2*a*e^2)*(a + b*x^2)^(2
 + p))/(2*b^3*(2 + p)) + (e^3*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) - (d*(9*
a*e^2 - b*d^2*(5 + 2*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2,
-((b*x^2)/a)])/(3*b*(5 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.203982, antiderivative size = 202, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1652, 459, 365, 364, 446, 77}

$$-\frac{ae\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{e\left(3bd^2-2ae^2\right)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^3\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{9}{2bp}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] -(a*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (3*d*e^2*x^3
*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (e*(3*b*d^2 - 2*a*e^2)*(a + b*x^2)^(2
 + p))/(2*b^3*(2 + p)) + (e^3*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*(d^
2 - (9*a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5
/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^2\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^3\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx$$
$$=\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(3d^2e+e^3x\right)dx,x,x^2\right)+\left(d\left(d^2-\frac{9ae}{5b+}\right.\right.$$
$$=\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{ae\left(-3bd^2+ae^2\right)(a+bx)^p}{b^2}+\frac{\left(3bd^2e-2ae^3\right)(a+}{b^2}\right.\right.$$
$$=-\frac{ae\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{e\left(3bd^2-2ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+$$

Mathematica [A] time = 0.153075, size = 196, normalized size = 0.93

$$\frac{1}{30}\left(a+bx^2\right)^p\left(\frac{15e^3\left(a+bx^2\right)\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{b^3(p+1)(p+2)(p+3)}+\frac{45d^2e\left(a+bx^2\right)\left(b(p+1)x^2-a\right)}{b^2(p+1)(p+2)}+10d^3x^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((45*d^2*e*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2
 + p)) + (15*e^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^
2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (10*d^3*x^3*Hypergeometric2F1[3/2,
 -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (18*d*e^2*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/30
```

Maple [F] time = 0.555, size = 0, normalized size = 0.

$$\int x^2\left(ex+d\right)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)

[Out] int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(bx^2+a\right)^p x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")

[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^5 + 3\,de^2x^4 + 3\,d^2ex^3 + d^3x^2\right)\left(bx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")

[Out] integral((e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 44.4356, size = 1420, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(b*x**2+a)**p,x)

[Out] a**p*d**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 3*a**
p*d*e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*d**2
*e*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a
*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x*
*2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*
a*b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b
**3*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I
*sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x*
*2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6
*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b
**2), True)) + e**3*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(
a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(
I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + a**2
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sq
rt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I
*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**
2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
```

```
x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x*
*2 + 4*b**5*x**4) - 2*b**2*x**4/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
, Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**
2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/
(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b*
*3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b
**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sq
rt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3)
- a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3
*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)*
*p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p
*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) +
b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**
3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
22*b**3*p + 12*b**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(bx^2+a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x^2, x)
```

3.403 $\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=167

$$\frac{d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{3de^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}-\frac{ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2-bd^2(2p+5)\right){}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)}{b(2p+5)}+$$

```
[Out] (d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^3*x^3*(a + b
*x^2)^(1 + p))/(b*(5 + 2*p)) + (3*d*e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)
) - (e*(a*e^2 - b*d^2*(5 + 2*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -
p, 5/2, -((b*x^2)/a)])/(b*(5 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.149759, antiderivative size = 159, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1652, 444, 43, 459, 365, 364}

$$\frac{d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{3de^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+5b}\right){}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^3*x^3*(a + b
*x^2)^(1 + p))/(b*(5 + 2*p)) + (3*d*e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)
) + (e*(d^2 - (a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/
2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x(d+ex)^3\left(a+bx^2\right)^p\,dx &= \int x\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^2\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx\\ &= \frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^p\left(d^3+3de^2x\right)dx,x,x^2\right)+\left(3e\left(d^2-\frac{ae^2}{5b+2bp}\right.\right.\\ &= \frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{\left(bd^3-3ade^2\right)(a+bx)^p}{b}+\frac{3de^2(a+bx)^{1+p}}{b}\right)dx,x,x^2\right.\\ &= \frac{d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{3de^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+e\left(d^2-\frac{ae^2}{5b+2bp}\right.\end{aligned}$$

Mathematica [A] time = 0.244, size = 228, normalized size = 1.37

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(5d\left(-3a^2e^2\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)+b^2x^2\left(\frac{bx^2}{a}+1\right)^p\left(d^2(p+2)+3e^2(p+1)x^2\right)+ab\left(d^2(p+2)\left(\left(\frac{bx^2}{a}\right.\right.\right.\right.}{10b^2(}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(5*d*(b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + 3*e^2*(1 + p)
*x^2) - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(3*e^2*p*x^2*(1 + (b*x^2)/
a)^p + d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p))) + 10*b^2*d^2*e*(2 + 3*p + p^2
)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 2*b^2*e^3*(2 + 3*p +
p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)]))/(10*b^2*(1 + p)*(2
 + p)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.536, size = 0, normalized size = 0.

$$\int x\left(ex+d\right)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^4 + 3\,de^2x^3 + 3\,d^2ex^2 + d^3x\right)\left(bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x)*(b*x^2 + a)^p, x)
```

Sympy [A] time = 33.897, size = 471, normalized size = 2.82

$$a^p d^2 e x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^3 x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + d^3\left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\ b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(a + bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + 3de^2\left(\begin{cases} \frac{a^p x^4}{4} \\ \frac{a\log(-\dots}{2ab\dots} \\ -\frac{a\log\dots}{\dots} \\ -\frac{a^2(\dots}{2b^2p^2\dots} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + a**p*
e**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**3*Piece
wise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne
(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + 3*d*e**2*Piecewise((a**p
*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**
2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2
+ 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x*
*2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -
2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/b)
 + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**
2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p
 + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2)
```

```
+ b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \left(bx^2+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p*x, x)
```

3.404 $\int (d+ex)^3 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=176

$$-\frac{e\left(a+bx^2\right)^{p+1}\left(ae^2-3bd^2(p+2)\right)}{2b^2(p+1)(p+2)}-\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2-bd^2(2p+3)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(2p+3)}+\frac{3de^2x\left(a+bx^2\right)}{b(2p+3)}$$

```
[Out] -(e*(a*e^2 - 3*b*d^2*(2 + p))*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)*(2 + p))
+ (3*d*e^2*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + (e^3*x^2*(a + b*x^2)^(1 +
 p))/(2*b*(2 + p)) - (d*(3*a*e^2 - b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hyperge
ometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b*(3 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.149465, antiderivative size = 169, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {743, 780, 246, 245}

$$-\frac{e\left(a+bx^2\right)^{p+1}\left((2p+3)\left(ae^2-bd^2(2p+5)\right)-2bde(p+1)(p+3)x\right)}{2b^2(p+2)\left(2p^2+5p+3\right)}+dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{3ae^2}{2bp+3b}\right){}_2F_1\left(\frac{1}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (e*(d + e*x)^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) - (e*((3 + 2*p)*(a*e^2 -
b*d^2*(5 + 2*p)) - 2*b*d*e*(1 + p)*(3 + p)*x)*(a + b*x^2)^(1 + p))/(2*b^2*(
2 + p)*(3 + 5*p + 2*p^2)) + (d*(d^2 - (3*a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2
)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (d+ex)^3\left(a+bx^2\right)^p\,dx = \frac{e(d+ex)^2\left(a+bx^2\right)^{1+p}}{2b(2+p)} + \frac{\int (d+ex)\left(-2\left(ae^2-bd^2(2+p)\right)+2bde(3+p)x\right)\left(a+bx^2\right)^p\,dx}{2b(2+p)}$$

$$= \frac{e(d+ex)^2\left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e\left((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x\right)\left(a+bx^2\right)}{2b^2(2+p)\left(3+5p+2p^2\right)}$$

$$= \frac{e(d+ex)^2\left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e\left((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x\right)\left(a+bx^2\right)}{2b^2(2+p)\left(3+5p+2p^2\right)}$$

$$= \frac{e(d+ex)^2\left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e\left((3+2p)\left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x\right)\left(a+bx^2\right)}{2b^2(2+p)\left(3+5p+2p^2\right)}$$

Mathematica [A] time = 0.218668, size = 223, normalized size = 1.27

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(e\left(-a^2e^2\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)+b^2x^2\left(\frac{bx^2}{a}+1\right)^p\left(3d^2(p+2)+e^2(p+1)x^2\right)+2b^2de\left(p^2+3p+2\right)x^3\right.\right.}{2b^2(p+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(2*b^2*d^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*x^2)/a)] + e*(b^2*x^2*(1 + (b*x^2)/a)^p*(3*d^2*(2 + p) + e^2*(1 + p)*
x^2) - a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p
+ 3*d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 2*b^2*d*e*(2 + 3*p + p^2)*x^3*H
ypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(2*b^2*(1 + p)*(2 + p)*(1 +
 (b*x^2)/a)^p)
```

Maple [F] time = 0.535, size = 0, normalized size = 0.

$$\int (ex+d)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p, x)
```

Sympy [C] time = 21.1549, size = 468, normalized size = 2.66

$$a^pd^3x{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+a^pde^2x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+3d^2e\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}\frac{a^px^4}{4} & \\ \frac{a\log\left(-\right.}{2ab^2} & \\ -\frac{a\log}{} & \\ -\frac{a^2(}{2b^2p^2} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*e**
2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*d**2*e*Piecew
ise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**3*Piecewise((a**p*x**4
/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) +
a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b
**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) +
 b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)),
(-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x)
/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6
*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*
b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**
2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p, x)
```

3.405 $\int \frac{(d+ex)^3(a+bx^2)^p}{x}\,dx$

Optimal. Leaf size=171

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2-3bd^2(2p+3)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(2p+3)}-\frac{d^3\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+\frac{3de}{}$$

```
[Out] (3*d*e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e^3*x*(a + b*x^2)^(1 + p))/(
b*(3 + 2*p)) - (e*(a*e^2 - 3*b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b*(3 + 2*p)*(1 + (b*x^2)/a)^p) - (d^3*(a
 + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(
1 + p))
```

Rubi [A] time = 0.132944, antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1652, 446, 80, 65, 388, 246, 245}

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{ae^2}{2bp+3b}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d^3\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+\frac{3de^2(}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

```
[Out] (3*d*e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e^3*x*(a + b*x^2)^(1 + p))/(
b*(3 + 2*p)) + (e*(3*d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^3*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3 \left(a+bx^2\right)^p}{x}\,dx &= \int \frac{\left(a+bx^2\right)^p \left(d^3+3de^2x^2\right)}{x}\,dx + \int \left(a+bx^2\right)^p \left(3d^2e+e^3x^2\right)\,dx \\ &= \frac{e^3x\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^p \left(d^3+3de^2x\right)}{x}\,dx, x, x^2\right) + \left(e\left(3d^2-\frac{ae^2}{3b+2bp}\right)\right) \\ &= \frac{3de^2\left(a+bx^2\right)^{1+p}}{2b(1+p)} + \frac{e^3x\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \frac{1}{2}d^3\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(e\left(3d^2-\frac{}{3}\right.\right. \\ &= \frac{3de^2\left(a+bx^2\right)^{1+p}}{2b(1+p)} + \frac{e^3x\left(a+bx^2\right)^{1+p}}{b(3+2p)} + e\left(3d^2-\frac{ae^2}{3b+2bp}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\end{aligned}$$

Mathematica [A] time = 0.13129, size = 170, normalized size = 0.99

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(18abd^2e(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-3bd^3\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)+\right.}{6ab(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

```
[Out] ((a + b*x^2)^p*(18*a*b*d^2*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b
*x^2)/a)] - 3*b*d^3*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*x^2)/a] + a*e^2*(9*d*(a + b*x^2)*(1 + (b*x^2)/a)^p + 2*b*e
*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(6*a*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.513, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 \left(bx^2+a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 \left(bx^2+a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(bx^2+a\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x, x)
```

Sympy [A] time = 28.2809, size = 144, normalized size = 0.84

$$3a^pd^2ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+\frac{a^pe^3x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}-\frac{b^pd^3x^{2p}\Gamma\left(-p\right)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}+3de^2\left(\begin{cases}\frac{a^px^2}{2} & \\ \frac{\begin{cases}\frac{\left(a+bx^2\right)^{p+1}}{p+1} & \text{for } p \\ \log\left(a+bx^2\right) & \text{other}\end{cases}}{2b} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x,x)
```

```
[Out] 3*a**p*d**2*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e
**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 - b**p*d**3*x
**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*
gamma(1 - p)) + 3*d*e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a
```

```
+ b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 (bx^2+a)^p}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x, x)
```

3.406 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^2} dx$

Optimal. Leaf size=159

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+1)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{3d^2e\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}-\frac{d^3}{}$$

```
[Out] (e^3*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) - (d^3*(a + b*x^2)^(1 + p))/(a*x) +
 (d*(3*a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (3*d^2*e*(a + b*x^2)^(1 + p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rubi [A] time = 0.189275, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1807, 1652, 446, 80, 65, 12, 246, 245}

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+1)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{3d^2e\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}-\frac{d^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

```
[Out] (e^3*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) - (d^3*(a + b*x^2)^(1 + p))/(a*x) +
 (d*(3*a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (3*d^2*e*(a + b*x^2)^(1 + p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x^2}\,dx &= -\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}-\frac{\int \frac{\left(a+bx^2\right)^p\left(-3ad^2e-d\left(3ae^2+bd^2(1+2p)\right)x-ae^3x^2\right)}{x}\,dx}{a}\\
&= -\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}+\frac{\int d\left(3ae^2+bd^2(1+2p)\right)\left(a+bx^2\right)^p\,dx}{a}-\frac{\int \frac{\left(a+bx^2\right)^p\left(-3ad^2e-ae^3x^2\right)}{x}\,dx}{a}\\
&= -\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}-\frac{\text{Subst}\left(\int \frac{(a+bx)^p\left(-3ad^2e-ae^3x\right)}{x}\,dx,x,x^2\right)}{2a}+\frac{\left(d\left(3ae^2+bd^2(1+2p)\right)\right)\int (\cdots}{a}\\
&= \frac{e^3\left(a+bx^2\right)^{1+p}}{2b(1+p)}-\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}+\frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\frac{\left(d\left(3ae^2+\cdots\right.\right.}{\cdots}\\
&= \frac{e^3\left(a+bx^2\right)^{1+p}}{2b(1+p)}-\frac{d^3\left(a+bx^2\right)^{1+p}}{ax}+\frac{d\left(3ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1(\cdots}{a}\end{aligned}$$

Mathematica [A] time = 0.123825, size = 154, normalized size = 0.97

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ex\left(\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p\left(ae^2-3bd^2\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)\right)+6abde(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\cdots\right.\right.\right.}{2ab(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

```
[Out] ((a + b*x^2)^p*(-2*a*b*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^
2)/a)] + e*x*(6*a*b*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)] + (a + b*x^2)*(1 + (b*x^2)/a)^p*(a*e^2 - 3*b*d^2*Hypergeometric2F1[1,
1 + p, 2 + p, 1 + (b*x^2)/a]))))/(2*a*b*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.515, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 \left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 \left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(bx^2+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x^2, x)
```

Sympy [A] time = 29.6593, size = 143, normalized size = 0.9

$$-\frac{a^p d^3 {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x} + 3a^p de^2 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right) - \frac{3b^p d^2 e x^{2p}\Gamma\left(-p\right) {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)} + e^3\left(\left\{\begin{matrix} \frac{a^p x^2}{2} \\ \frac{\begin{cases} \frac{\left(a+bx^2\right)^{p+1}}{p+1} & \text{for } p \\ \log\left(a+bx^2\right) & \text{othe} \end{cases}}{2b} \end{matrix}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d**3*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + 3*a**p*d
*e**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - 3*b**p*d**2*e*
x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2
*gamma(1 - p)) + e**3*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b
*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 (bx^2+a)^p}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)
```

3.407 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx$

Optimal. Leaf size=168

$$-\frac{d\left(a+bx^2\right)^{p+1}\left(3ae^2+bd^2p\right){}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}+\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+3bd^2(2p+1)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\right.}{a}$$

```
[Out] -(d^3*(a + b*x^2)^(1 + p))/(2*a*x^2) - (3*d^2*e*(a + b*x^2)^(1 + p))/(a*x)
+ (e*(a*e^2 + 3*b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p,
 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (d*(3*a*e^2 + b*d^2*p)*(a + b*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*(1 +
 p))
```

Rubi [A] time = 0.2215, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1807, 764, 266, 65, 246, 245}

$$-\frac{d\left(a+bx^2\right)^{p+1}\left(3ae^2+bd^2p\right){}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}+\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+3bd^2(2p+1)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```


```
[Out] -(d^3*(a + b*x^2)^(1 + p))/(2*a*x^2) - (3*d^2*e*(a + b*x^2)^(1 + p))/(a*x)
+ (e*(a*e^2 + 3*b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p,
 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (d*(3*a*e^2 + b*d^2*p)*(a + b*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*(1 +
 p))
```


Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
```

```
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x^3}\,dx = -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2}-\frac{\int \frac{\left(a+bx^2\right)^p\left(-6ad^2e-2d\left(3ae^2+bd^2p\right)x-2ae^3x^2\right)}{x^2}\,dx}{2a}$$

$$=-\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2}-\frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax}+\frac{\int \frac{\left(2ad\left(3ae^2+bd^2p\right)+2ae\left(ae^2+3bd^2(1+2p)\right)x\right)\left(a+bx^2\right)^p}{x}\,dx}{2a^2}$$

$$=-\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2}-\frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax}+\frac{\left(d\left(3ae^2+bd^2p\right)\right)\int \frac{\left(a+bx^2\right)^p}{x}\,dx}{a}+\frac{\left(e\left(ae^2+3bd^2\right.\right.}{}$$

$$=-\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2}-\frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax}+\frac{\left(d\left(3ae^2+bd^2p\right)\right)\mathrm{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2a}+$$

$$=-\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2}-\frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax}+\frac{e\left(ae^2+3bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\ 2}{a}$$

Mathematica [A] time = 0.130246, size = 174, normalized size = 1.04

$$-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(x\left(d\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^p\left(3ae^2\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)-bd^2\,{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)\right.\right.\right.}{2a^2(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -((a + b*x^2)^p*(6*a^2*d^2*e*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*
x^2)/a)] + x*(-2*a^2*e^3*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2
)/a)] + d*(a + b*x^2)*(1 + (b*x^2)/a)^p*(3*a*e^2*Hypergeometric2F1[1, 1 + p
, 2 + p, 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x
^2)/a]))))/(2*a^2*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.526, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x^3,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(bx^2+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(b*x^2 + a)^p/x^3, x)
```

Sympy [C] time = 36.8128, size = 150, normalized size = 0.89

$$-\frac{3a^pd^2e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}+a^pe^3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)-\frac{b^pd^3x^{2p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix}-p,1-p\\ 2-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma\left(2-p\right)}-\frac{3b^pde^2x^{2p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,\\ 1-\end{matrix}\right.}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x**3,x)
```

```
[Out] -3*a**p*d**2*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p
*e**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**3*x**(
2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**2))/
(2*x**2*gamma(2 - p)) - 3*b**p*d*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p), (1
 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)
```

3.408 $\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=199

$$\frac{x^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}+\frac{(bd^2-ae^2)(a+bx^2)^{p+1}}{2b^2e^3(p+1)}+\frac{(a+bx^2)^{p+2}}{2b^2e(p+2)}-\frac{d^4(a+bx^2)^{p+1}\,{}_2F_1\left(1,p\right.}{2e^3(p+1)\left(a\right.}$$

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*e^3*(1 + p)) + (a + b*x^2)^(2
+ p)/(2*b^2*e*(2 + p)) + (x^5*(a + b*x^2)^p*AppellF1[5/2, -p, 1, 7/2, -((b*
x^2)/a), (e^2*x^2)/d^2])/(5*d*(1 + (b*x^2)/a)^p) - (d^4*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
e^3*(b*d^2 + a*e^2)*(1 + p))
```

Rubi [A] time = 0.226079, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {959, 511, 510, 446, 88, 68}

$$\frac{x^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}+\frac{(bd^2-ae^2)(a+bx^2)^{p+1}}{2b^2e^3(p+1)}+\frac{(a+bx^2)^{p+2}}{2b^2e(p+2)}-\frac{d^4(a+bx^2)^{p+1}\,{}_2F_1\left(1,p\right.}{2e^3(p+1)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*e^3*(1 + p)) + (a + b*x^2)^(2
+ p)/(2*b^2*e*(2 + p)) + (x^5*(a + b*x^2)^p*AppellF1[5/2, -p, 1, 7/2, -((b*
x^2)/a), (e^2*x^2)/d^2])/(5*d*(1 + (b*x^2)/a)^p) - (d^4*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
e^3*(b*d^2 + a*e^2)*(1 + p))
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(a+bx^2\right)^p}{d+ex}\,dx &= d\int \frac{x^4\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^5\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx \\
&= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{x^2(a+bx)^p}{d^2-e^2x}\,dx,x,x^2\right)\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx \\
&= \frac{x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}-\frac{1}{2}e\,\text{Subst}\left(\int\left(\frac{\left(-bd^2+ae^2\right)(a+bx)^p}{be^4}-\right.\right. \\
&= \frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2e^3(1+p)}+\frac{\left(a+bx^2\right)^{2+p}}{2b^2e(2+p)}+\frac{x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} \\
&= \frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2e^3(1+p)}+\frac{\left(a+bx^2\right)^{2+p}}{2b^2e(2+p)}+\frac{x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}
\end{aligned}
$$

Mathematica [F] time = 0.673851, size = 0, normalized size = 0.

$$\int \frac{x^4\left(a+bx^2\right)^p}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x), x]
```

Maple [F] time = 0.658, size = 0, normalized size = 0.

$$\int \frac{x^4 \left(b x^2 + a\right)^p}{e x + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x^4*(b*x^2+a)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^2 + a\right)^p x^4}{e x + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b x^2 + a\right)^p x^4}{e x + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^4/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^2 + a\right)^p x^4}{e x + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d), x)
```

3.409 $\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=163

$$-\frac{ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2}+\frac{d^3\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)\left(ae^2+bd^2\right)}-\frac{d\left(a+bx^2\right)^{p+1}}{2be^2(p+1)}$$

```
[Out] -(d*(a + b*x^2)^(1 + p))/(2*b*e^2*(1 + p)) - (e*x^5*(a + b*x^2)^p*AppellF1[
5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(5*d^2*(1 + (b*x^2)/a)^p) +
(d^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2
))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^2)*(1 + p))
```

Rubi [A] time = 0.147575, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {959, 446, 80, 68, 511, 510}

$$-\frac{ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2}+\frac{d^3\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)\left(ae^2+bd^2\right)}-\frac{d\left(a+bx^2\right)^{p+1}}{2be^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] -(d*(a + b*x^2)^(1 + p))/(2*b*e^2*(1 + p)) - (e*x^5*(a + b*x^2)^p*AppellF1[
5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(5*d^2*(1 + (b*x^2)/a)^p) +
(d^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2
))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^2)*(1 + p))
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)^p}{d+ex}\,dx = d\int \frac{x^3\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^4\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx$$

$$= \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{x(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right) - \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx$$

$$= -\frac{d\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} - \frac{ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3\,\mathrm{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx\right.}{2e^2}$$

$$= -\frac{d\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} - \frac{ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3\left(a+bx^2\right)^{1+p}{}_2F_1\left(1\right.}{2e^2\left(bd^2+\right.}$$

Mathematica [A] time = 0.355292, size = 260, normalized size = 1.6

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(\frac{bx^2}{a}+1\right)^{-p}\left(6bd^2(p+1)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+e\left(2be(p+1)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)-3d\left(bx^2\left(\frac{bx^2}{a}+1\right)^p+a\left(\left(\frac{bx^2}{a}+1\right)^p-1\right)\right)\right)\right)}{b(p+1)} - \frac{3d^3\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\right.}{}\right.}{6e^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*((-3*d^3*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(
d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) + (e*(6*b*d^2*(1 + p)*x*H
ypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + e*(-3*d*(b*x^2*(1 + (b*x^2)/
a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + 2*b*e*(1 + p)*x^3*Hypergeometric2F1[3/
2, -p, 5/2, -((b*x^2)/a)])))/(b*(1 + p)*(1 + (b*x^2)/a)^p)))/(6*e^4)
```

Maple [F] time = 0.648, size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{p} x^{3}}{e x+d}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x^3*(b*x^2+a)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{p} x^{3}}{e x+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b x^{2}+a\right)^{p} x^{3}}{e x+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^3/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{p} x^{3}}{e x+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d), x)
```

3.410 $\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=161

$$\frac{x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)\left(ae^2+bd^2\right)}+\frac{\left(a+bx^2\right)^{p+1}}{2be(p+1)}$$

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e*(1 + p)) + (x^3*(a + b*x^2)^p*AppellF1[3/2, -p,
1, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d*(1 + (b*x^2)/a)^p) - (d^2*(a + b
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
 a*e^2)])/(2*e*(b*d^2 + a*e^2)*(1 + p))
```

Rubi [A] time = 0.148172, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {959, 511, 510, 446, 80, 68}

$$\frac{x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)\left(ae^2+bd^2\right)}+\frac{\left(a+bx^2\right)^{p+1}}{2be(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e*(1 + p)) + (x^3*(a + b*x^2)^p*AppellF1[3/2, -p,
1, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d*(1 + (b*x^2)/a)^p) - (d^2*(a + b
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
 a*e^2)])/(2*e*(b*d^2 + a*e^2)*(1 + p))
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^p}{d+ex}\,dx = d\int \frac{x^2\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^3\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx$$

$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{x(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)\right) + \left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be(1+p)} + \frac{x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d} - \frac{d^2\,\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx, x, \ldots\right.}{2e}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be(1+p)} + \frac{x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d} - \frac{d^2\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1\ldots\right.}{2e\left(bd^2+ae^2\right)\ldots}$$

Mathematica [A] time = 0.253906, size = 227, normalized size = 1.41

$$\frac{\left(a+bx^2\right)^p\left(bd^2(p+1)\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)-2bdep(p+1)x\left(\frac{bx^2}{a}+1\right)^{-p}\ldots\right.}{2be^3p(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*(a*e^2*p + b*e^2*p*x^2 - (a*e^2*p)/(1 + (b*x^2)/a)^p + (b*d^
2*(1 + p)*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (
d + Sqrt[-(a/b)]*e)/(d + e*x)])/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*
(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*b*d*e*p*(1 + p)*x*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*b*e^3*p*(1 + p))
```

Maple [F] time = 0.652, size = 0, normalized size = 0.

$$\int \frac{x^2\left(bx^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x^2*(b*x^2+a)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^p x^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d), x)
```

3.411 $\int \frac{x(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=173

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e}+\frac{d\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}\right.}{}$$

```
[Out] -((x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(e*(1 + (b*x^2)/a)^p)) + (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -
((b*x^2)/a)])/(e*(1 + (b*x^2)/a)^p) + (d*(a + b*x^2)^(1 + p)*Hypergeometric
2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)
*(1 + p))
```

Rubi [A] time = 0.148441, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e}+\frac{d\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] -((x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(e*(1 + (b*x^2)/a)^p)) + (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -
((b*x^2)/a)])/(e*(1 + (b*x^2)/a)^p) + (d*(a + b*x^2)^(1 + p)*Hypergeometric
2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)
*(1 + p))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+bx^2\right)^p}{d+ex}\,dx &= \frac{\int \left(a+bx^2\right)^p\,dx}{e} - \frac{d\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{e}\\
&= -\frac{d\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e} + \frac{\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx}{e}\\
&= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} - d\int \frac{x\left(a+bx^2\right)^p}{-d^2+e^2x^2}\,dx - \frac{d^2\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx}{e}\\
&= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} - \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right) - \frac{\left(d^2\left(a+bx^2\right)^p\right.}{}\\
&= -\frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e} + \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e}
\end{aligned}
$$

Mathematica [A] time = 0.21198, size = 172, normalized size = 0.99

$$\frac{\left(a+bx^2\right)^p\left(2ex\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}\right)}{2e^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*(-((d*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(
d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d +
 e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*e*x*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^2)
```

Maple [F] time = 0.635, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+bx^2\right)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*x**2)**p/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d), x)
```

3.412 $\int \frac{(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=125

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}$$

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))
```

Rubi [A] time = 0.103611, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {757, 430, 429, 444, 68}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(d + e*x),x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx &= \int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx\\
&= d\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx+e\int \frac{x\left(a+bx^2\right)^p}{-d^2+e^2x^2}\,dx\\
&= \frac{1}{2}e\,\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\
&= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{e^2\left(a+bx^2\right)}{bd^2+ae^2}\right)}{2\left(bd^2+ae^2\right)(1+p)}
\end{aligned}
$$

Mathematica [A] time = 0.0532645, size = 131, normalized size = 1.05

$$
\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{2ep}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*
x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*p*((e*(-Sqrt[-(a/b)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)
```

Maple [F] time = 0.63, size = 0, normalized size = 0.

$$
\int \frac{\left(bx^2+a\right)^p}{ex+d}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(bx^2+a\right)^p}{ex+d}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d),x)
```

```
[Out] Integral((a + b*x**2)**p/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(e*x + d), x)
```

3.413 $\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx$

Optimal. Leaf size=176

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d(p+1)\left(ae^2+bd^2\right)}-\frac{\left(a+bx^2\right)^{p+1}{}_2F_1\left(\right.}{2a}$$

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^2*(1 + (b*x^2)/a)^p)) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1,
1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d*(b*d^2 + a*e^2)*(1 +
 p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/
a])/(2*a*d*(1 + p))
```

Rubi [A] time = 0.148597, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {959, 446, 86, 65, 68, 430, 429}

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d(p+1)\left(ae^2+bd^2\right)}-\frac{\left(a+bx^2\right)^{p+1}{}_2F_1\left(\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^2*(1 + (b*x^2)/a)^p)) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1,
1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d*(b*d^2 + a*e^2)*(1 +
 p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/
a])/(2*a*d*(1 + p))
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{x(d+ex)}\,dx &= d\int \frac{\left(a+bx^2\right)^p}{x\left(d^2-e^2x^2\right)}\,dx - e\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx\\ &= \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{x\left(d^2-e^2x\right)}\,dx,x,x^2\right)-\left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\ &= -\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2d}+\frac{e^2\,\text{Subst}\left(\int \right.}{}\\ &= -\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2\left(a+bx^2\right)}{bd^2+ae^2}\right.}{2d\left(bd^2+ae^2\right)(1+p)}\end{aligned}$$

Mathematica [A] time = 0.203014, size = 170, normalized size = 0.97

$$\frac{\left(a+bx^2\right)^p\left(\left(\frac{a}{bx^2}+1\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)-\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)\right.}{2dp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*(-(AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))
^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p
```

```
, -(a/(b*x^2))]/(1 + a/(b*x^2))^p))/(2*d*p)
```

Maple [F] time = 0.645, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x (ex + d)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/x/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{ex^2 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e*x^2 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^p}{x (d + ex)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d),x)
```

```
[Out] Integral((a + b*x**2)**p/(x*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x), x)
```

3.414 $\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx$

Optimal. Leaf size=178

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx}-\frac{e^3(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)}+\frac{e(a+bx^2)^{p+1}\,{}_2F_1(\ldots}{2ad\ldots}$$

```
[Out] -(((a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(
d*x*(1 + (b*x^2)/a)^p)) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 +
 p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 +
p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)
/a])/(2*a*d^2*(1 + p))
```

Rubi [A] time = 0.168589, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {959, 511, 510, 446, 86, 65, 68}

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx}-\frac{e^3(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)}+\frac{e(a+bx^2)^{p+1}\,{}_2F_1(\ldots}{2ad\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```


```
[Out] -(((a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(
d*x*(1 + (b*x^2)/a)^p)) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 +
 p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 +
p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)
/a])/(2*a*d^2*(1 + p))
```


Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = d\int \frac{(a+bx^2)^p}{x^2(d^2-e^2x^2)}\,dx - e\int \frac{(a+bx^2)^p}{x(d^2-e^2x^2)}\,dx$$
$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x(d^2-e^2x)}\,dx, x, x^2\right)\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2(d^2-e^2x^2)}\,dx$$
$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^2} - \frac{e^3\,\text{Subst}\left(\int \right.}{2}$$
$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e^3(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right.}{2d^2(bd^2+ae^2)(1+p)}$$

Mathematica [A] time = 0.379426, size = 214, normalized size = 1.2

$$\frac{(a+bx^2)^p\left(\frac{e\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} - \frac{2d\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e\left(\frac{a}{bx^2}+1\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\right.}{p}\right.}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*((e*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d
+ e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*d*Hypergeometric2F1[-1/2,
 -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-p, -
p, 1 - p, -(a/(b*x^2))])/(p*(1 + a/(b*x^2))^p)))/(2*d^2)
```

Maple [F] time = 0.652, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^2\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/x^2/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e*x^3 + d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^p}{x^2\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d),x)
```

```
[Out] Integral((a + b*x**2)**p/(x**2*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)
```

3.415 $\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx$

Optimal. Leaf size=213

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2d^3(p+1)}+\frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x}+\frac{e^4(a+b}{}$$

```
[Out] -(a + b*x^2)^(1 + p)/(2*a*d*x^2) + (e*(a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1
/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^2*x*(1 + (b*x^2)/a)^p) + (e^4*(a + b*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a
*e^2)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a*e^2 + b*d^2*p)*(a + b*x^2)^(1
 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*d^3*(1 + p)
)
```

Rubi [A] time = 0.242437, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {959, 446, 103, 156, 65, 68, 511, 510}

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2d^3(p+1)}+\frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x}+\frac{e^4(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] -(a + b*x^2)^(1 + p)/(2*a*d*x^2) + (e*(a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1
/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^2*x*(1 + (b*x^2)/a)^p) + (e^4*(a + b*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a
*e^2)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a*e^2 + b*d^2*p)*(a + b*x^2)^(1
 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*d^3*(1 + p)
)
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = d\int \frac{(a+bx^2)^p}{x^3\left(d^2-e^2x^2\right)}\,dx - e\int \frac{(a+bx^2)^p}{x^2\left(d^2-e^2x^2\right)}\,dx$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{x^2\left(d^2-e^2x\right)}\,dx, x, x^2\right) - \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2\left(d^2-e^2x^2\right)}\,dx$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p\left(-ae^2-bd^2p+be^2p\right.}{x\left(d^2-e^2x\right)}\right.}{2ad}$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4\,\text{Subst}\left(\int \frac{(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)}{2d^3}$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;\right.}{2d^3\left(bd^2+ae^2\right)(1}$$

Mathematica [A] time = 0.297581, size = 256, normalized size = 1.2

$$\frac{\left(a+bx^2\right)^p\left(-\frac{e^2\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} + \left(\frac{a}{bx^2}+1\right)^{-p}\left(\frac{d^2\,{}_2F_1\left(1-p,-p;2-p;-\frac{a}{bx^2}\right)}{(p-1)x^2} + \frac{e^2\,{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)}{p}\right.\right.}{2d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*(-((e^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)
/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d
 + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*d*e*Hypergeometric2F
1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) + ((d^2*Hypergeometri
c2F1[1 - p, -p, 2 - p, -(a/(b*x^2))])/((-1 + p)*x^2) + (e^2*Hypergeometric2
F1[-p, -p, 1 - p, -(a/(b*x^2))])/p)/(1 + a/(b*x^2))^p))/(2*d^3)
```

Maple [F] time = 0.746, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^3\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^3/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/x^3/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{ex^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)
```

3.416 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=392

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{be^4(2p+3)\left(ae^2+bd^2\right)}-\frac{2d^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right.}{}$$

```
[Out] -((d*(4 + 3*p)*(a + b*x^2)^(1 + p))/(b*e^3*(1 + p)*(3 + 2*p))) - (d^4*(a +
b*x^2)^(1 + p))/(e^3*(b*d^2 + a*e^2)*(d + e*x)) + ((d + e*x)*(a + b*x^2)^(1
 + p))/(b*e^3*(3 + 2*p)) - (2*d^2*(2*a*e^2 + b*d^2*(2 + p))*x*(a + b*x^2)^p
*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^
2)*(1 + (b*x^2)/a)^p) - ((a^2*e^4 - 2*a*b*d^2*e^2*(4 + 3*p) - 2*b^2*d^4*(6
+ 7*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b*e^4*(b*d^2 + a*e^2)*(3 + 2*p)*(1 + (b*x^2)/a)^p) + (d^3*(2*a*e^2 + b
*d^2*(2 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(e^3*(b*d^2 + a*e^2)^2*(1 + p))
```

Rubi [A] time = 0.888099, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1651, 1654, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{be^4(2p+3)\left(ae^2+bd^2\right)}-\frac{2d^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
```


```
[Out] -((d*(4 + 3*p)*(a + b*x^2)^(1 + p))/(b*e^3*(1 + p)*(3 + 2*p))) - (d^4*(a +
b*x^2)^(1 + p))/(e^3*(b*d^2 + a*e^2)*(d + e*x)) + ((d + e*x)*(a + b*x^2)^(1
 + p))/(b*e^3*(3 + 2*p)) - (2*d^2*(2*a*e^2 + b*d^2*(2 + p))*x*(a + b*x^2)^p
*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^
2)*(1 + (b*x^2)/a)^p) - ((a^2*e^4 - 2*a*b*d^2*e^2*(4 + 3*p) - 2*b^2*d^4*(6
+ 7*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b*e^4*(b*d^2 + a*e^2)*(3 + 2*p)*(1 + (b*x^2)/a)^p) + (d^3*(2*a*e^2 + b
*d^2*(2 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(e^3*(b*d^2 + a*e^2)^2*(1 + p))
```


Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^2}\,dx = -\frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\left(a+bx^2\right)^p\left(\frac{ad^3}{e^2}-\frac{d^2\left(ae^2+2bd^2(1+p)\right)x}{e^3}+d\left(a+\frac{bd^2}{e^2}\right)x^2-\frac{\left(bd^2+ae^2\right)x^3}{e}\right)}{d+ex}\,dx}{bd^2+ae^2}$$

$$= -\frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\int \frac{\left(a+bx^2\right)^p\left(ade\left(ae^2+2bd^2(2+p)\right)+\left(a^2e^4-4b^2d^4(1+p)^2\right)x+2bde}{d+ex}}{be^3\left(bd^2+ae^2\right)(3+2p)}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \int \frac{\left(2abde^3(1+p)\left(ae^2+2bd^2(2\right.\right.}{}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(2d^3\left(2ae^2+bd^2(2+\right.\right.}{e^4\left(bd^2+\right.}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(2d^3\left(2ae^2+bd^2(2+\right.\right.}{e^4}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(a^2e^4-2abd^2e^2(4+3\right.}{}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{\left(a^2e^4-2abd^2e^2(4+3\right.}{}$$

$$= -\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)} - \frac{2d^2\left(2ae^2+bd^2(2+p\right.}{}$$

Mathematica [F] time = 0.726362, size = 0, normalized size = 0.

$$\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]

[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2, x]
```

Maple [F] time = 0.664, size = 0, normalized size = 0.

$$\int \frac{x^4\left(bx^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p x^4}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p x^4}{e^2x^2+2\,dex+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^4/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p x^4}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^2, x)
```

3.417 $\int \frac{x^3\left(a+bx^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=321

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+3)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)}-\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(2p+3)\right)}{e^3\left(ae^2+bd^2\right)}$$

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^2*(1 + p)) + (d^3*(a + b*x^2)^(1 + p))/(e^2*(b*d
^2 + a*e^2)*(d + e*x)) + (d*(3*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*App
ellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2 + a*e^2)*(
1 + (b*x^2)/a)^p) - (d*(2*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^3*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)
^p) - (d^2*(3*a*e^2 + b*d^2*(3 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^
2)^2*(1 + p))
```

Rubi [A] time = 0.545407, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1651, 1654, 844, 246, 245, 757, 430, 429, 444, 68}

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+3)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)}-\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(2p+3)\right)}{e^3\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^2*(1 + p)) + (d^3*(a + b*x^2)^(1 + p))/(e^2*(b*d
^2 + a*e^2)*(d + e*x)) + (d*(3*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*App
ellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2 + a*e^2)*(
1 + (b*x^2)/a)^p) - (d*(2*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^3*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)
^p) - (d^2*(3*a*e^2 + b*d^2*(3 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^
2)^2*(1 + p))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
```

```
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
```

```
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)^p}{(d+ex)^2}\,dx = \frac{d^3\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\left(a+bx^2\right)^p\left(-\frac{ad^2}{e}+d\left(a+\frac{2bd^2(1+p)}{e^2}\right)x-\frac{\left(bd^2+ae^2\right)x^2}{e}\right)}{d+ex}\,dx}{bd^2+ae^2}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\left(-2abd^2e(1+p)+2bd(1+p)\left(2ae^2+bd^2(3+2p)\right)x\right)\left(a+bx^2\right)^p}{d+ex}\,dx}{2be^2\left(bd^2+ae^2\right)(1+p)}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)(d+ex)} - \frac{\left(d\left(2ae^2+bd^2(3+2p)\right)\right)\int \left(a+bx^2\right)^p\,dx}{e^3\left(bd^2+ae^2\right)} + \frac{\left(d^2\left(3ae^2+b\right.\right.}{e}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(d^2\left(3ae^2+bd^2(3+2p)\right)\right)\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e^3\left(bd^2+ae^2\right)} -$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)(d+ex)} - \frac{d\left(2ae^2+bd^2(3+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p\right.}{e^3\left(bd^2+ae^2\right)}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)(d+ex)} - \frac{d\left(2ae^2+bd^2(3+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p\right.}{e^3\left(bd^2+ae^2\right)}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^2(1+p)} + \frac{d^3\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)(d+ex)} + \frac{d\left(3ae^2+bd^2(3+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,\right.}{e^3\left(bd^2+ae^2\right)}$$

Mathematica [A] time = 0.734757, size = 343, normalized size = 1.07

$$\frac{\left(a+bx^2\right)^p\left(-\frac{2d^3\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(2p-1)(d+ex)} + \frac{3d^2\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{\cdot}}{d+\cdot}\right.}{p}\right.}{2e^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*((-2*d^3*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)
]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a
/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (
3*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d +
Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(S
qrt[-(a/b)] + x))/(d + e*x))^p) + e*((e*(a + b*x^2 - a/(1 + (b*x^2)/a)^p))/
(b + b*p) - (4*d*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p)))/(2*e^4)
```

Maple [F] time = 0.65, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x^3}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x^3}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x^3}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x^3}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)
```

3.418 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=281

$$-\frac{2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2\left(ae^2+bd^2\right)}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+2bd^2(p+1)\right)}{e^2\left(ae^2+bd^2\right)}$$

```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x))) - (2*(a*e^2 + b*
d^2*(1 + p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e^2*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) + ((a*e^2 + 2*b*d^2*(1 +
p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^2*(b*
d^2 + a*e^2)*(1 + (b*x^2)/a)^p) + (d*(a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1
 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)]
)/(e*(b*d^2 + a*e^2)^2*(1 + p))
```

Rubi [A] time = 0.331684, antiderivative size = 277, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {1651, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2\left(ae^2+bd^2\right)}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a+\frac{2bd^2(p+1)}{e^2}\right){}_2F_1}{ae^2+bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x))) - (2*(a*e^2 + b*
d^2*(1 + p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e^2*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) + ((a + (2*b*d^2*(1 + p))
/e^2)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/((b*d^
2 + a*e^2)*(1 + (b*x^2)/a)^p) + (d*(a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 +
 p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/
(e*(b*d^2 + a*e^2)^2*(1 + p))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^p}{(d+ex)^2}\,dx = -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\left(ad-\frac{\left(ae^2+2bd^2(1+p)\right)x}{e}\right)\left(a+bx^2\right)^p}{d+ex}\,dx}{bd^2+ae^2}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{\left(2d\left(ae^2+bd^2(1+p)\right)\right)\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{e^2\left(bd^2+ae^2\right)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)\int \left(a+bx^2\right)^p\,dx}{bd^2+ae^2}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{\left(2d\left(ae^2+bd^2(1+p)\right)\right)\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e^2\left(bd^2+ae^2\right)} + \frac{\left(\left(a+\frac{2bd^2(1+p)}{e^2}\right.\right.}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{bd^2+ae^2} - \frac{\left(2d^2\left(a\right.\right.}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{bd^2+ae^2} - \frac{\left(d\left(ae^2\right.\right.}{}$$

$$= -\frac{d^2\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{2\left(ae^2+bd^2(1+p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2\left(bd^2+ae^2\right)}$$

Mathematica [A] time = 0.33191, size = 300, normalized size = 1.07

$$\frac{\left(a+bx^2\right)^p\left(\frac{d^2\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(2p-1)(d+ex)} - \frac{d\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-}}{d+ex}\right.}{p}\right.}{e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*((d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)
] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d*A
ppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(
a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a
/b)] + x))/(d + e*x))^p) + (e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(1 + (b*x^2)/a)^p))/e^3
```

Maple [F] time = 0.648, size = 0, normalized size = 0.

$$\int \frac{x^2\left(bx^2+a\right)^p}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p x^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p x^2}{e^2x^2+2\,dex+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p x^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)
```

3.419 $\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=273

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(2p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\left(ae^2+bd^2\right)}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p\right.}{e\left(ae^2+bd^2\right)}$$

```
[Out] (d*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d + e*x)) + ((a*e^2 + b*d^2*(1 +
2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2
])/(d*e*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*x*(a + b*x^2)^p
*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(b*d^2 + a*e^2)*(1 + (b*
x^2)/a)^p) - ((a*e^2 + b*d^2*(1 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2
F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^
2*(1 + p))
```

Rubi [A] time = 0.23065, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {835, 844, 246, 245, 757, 430, 429, 444, 68}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2(2p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\left(ae^2+bd^2\right)}-\frac{bd(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p\right.}{e\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```


```
[Out] (d*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d + e*x)) + ((a*e^2 + b*d^2*(1 +
2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2
])/(d*e*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*x*(a + b*x^2)^p
*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(b*d^2 + a*e^2)*(1 + (b*
x^2)/a)^p) - ((a*e^2 + b*d^2*(1 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2
F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^
2*(1 + p))
```


Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^p}{(d+ex)^2}\,dx = \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} - \frac{\int \frac{(-ae+bd(1+2p)x)\left(a+bx^2\right)^p}{d+ex}\,dx}{bd^2+ae^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} - \frac{(bd(1+2p))\int\left(a+bx^2\right)^p\,dx}{e\left(bd^2+ae^2\right)} + \frac{\left(ae^2+bd^2(1+2p)\right)\int\frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{e\left(bd^2+ae^2\right)}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(ae^2+bd^2(1+2p)\right)\int\left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{e\left(bd^2+ae^2\right)} - \frac{\left(bd(1+2p)\left(a+bx^2\right)^p\right.}{e\left(b\right.}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} - \frac{bd(1+2p)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e\left(bd^2+ae^2\right)} + \frac{\left(ae^2+bd^2(1+\right.}{bd^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} - \frac{bd(1+2p)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e\left(bd^2+ae^2\right)} + \frac{\left(ae^2+bd^2(1+\right.}{}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)(d+ex)} + \frac{\left(ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\left(bd^2+ae^2\right)} - \cdots$$

Mathematica [A] time = 0.173315, size = 223, normalized size = 0.82

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left((2p-1)(d+ex)F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)-2dpF_1\left(1-2p;-p,-\right.\right.}{2e^2p(2p-1)(d+ex)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*(-2*d*p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + (-1 + 2*p)*(d + e*x)*Appel
lF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)
]*e)/(d + e*x)]))/(2*e^2*p*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p
*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

Maple [F] time = 0.652, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p x}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx^2+a)^p x}{e^2x^2+2\,dex+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p x}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)
```

3.420 $\int \frac{(a+bx^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=244

$$-\frac{2bpx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{ae^2+bd^2}+\frac{b(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{ae^2+bd^2}-\frac{bde}{}$$

```
[Out] (e^2*x*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d^2 - e^2*x^2)) - (2*b*p*x*(a
 + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/((b*d^2
 + a*e^2)*(1 + (b*x^2)/a)^p) + (b*(1 + 2*p)*x*(a + b*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, -((b*x^2)/a)])/((b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) - (b*d*
e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/
(b*d^2 + a*e^2)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rubi [A] time = 0.179792, antiderivative size = 191, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {757, 430, 429, 444, 68, 511, 510}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^4}-\frac{bde\left(a+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^2)^p/(d + e*x)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
^2*(1 + (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -(
(b*x^2)/a), (e^2*x^2)/d^2])/(3*d^4*(1 + (b*x^2)/a)^p) - (b*d*e*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2
)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^p}{(d+ex)^2}\,dx &= \int \left(\frac{d^2\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}-\frac{2dex\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}+\frac{e^2x^2\left(a+bx^2\right)^p}{\left(-d^2+e^2x^2\right)^2}\right)dx\\
&= d^2\int \frac{\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx-(2de)\int \frac{x\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx+e^2\int \frac{x^2\left(a+bx^2\right)^p}{\left(-d^2+e^2x^2\right)^2}\,dx\\
&= -\left((de)\,\text{Subst}\left(\int \frac{(a+bx)^p}{\left(d^2-e^2x\right)^2}\,dx,x,x^2\right)\right)+\left(d^2\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx+\left(e^2\left(a+\right.\right.\\
&= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\right.}{3d^4}
\end{aligned}
$$

Mathematica [A] time = 0.0874415, size = 141, normalized size = 0.58

$$
\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{e(2p-1)(d+ex)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(e*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

Maple [F] time = 0.658, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int((b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(e*x + d)^2, x)
```

3.421 $\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{e^3x^3\left(a+\right.}{}$$

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^3*(1 + (b*x^2)/a)^p)) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -
((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)
^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^5*(1 + (b*x
^2)/a)^p) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^2*(1
 + p)) + (b*e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2
*(a + b*x^2))/(b*d^2 + a*e^2)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rubi [A] time = 0.427372, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {961, 266, 65, 757, 430, 429, 444, 68, 511, 510}

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{e^3x^3\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^3*(1 + (b*x^2)/a)^p)) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -
((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)
^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^5*(1 + (b*x
^2)/a)^p) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^2*(1
 + p)) + (b*e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2
*(a + b*x^2))/(b*d^2 + a*e^2)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}dx = \int \left(\frac{(a+bx^2)^p}{d^2x} - \frac{e(a+bx^2)^p}{d(d+ex)^2} - \frac{e(a+bx^2)^p}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x}dx}{d^2} - \frac{e\int \frac{(a+bx^2)^p}{d+ex}dx}{d^2} - \frac{e\int \frac{(a+bx^2)^p}{(d+ex)^2}dx}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}dx, x, x^2\right)}{2d^2} - \frac{e\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{d^2} - \frac{e\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+bx^2)^p}{(d^2-e^2x^2)^2} + \frac{e^2x^2(a+}{(-d^2+e}\right.}{d}$$

$$= -\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} - \frac{e\int \frac{(a+bx^2)^p}{d^2-e^2x^2}dx}{d} - (de)\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2}dx + (2e^2)\int \frac{x}{(}$$

$$= -\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} + e^2\,\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^2}dx, x, x^2\right) - \frac{e^2\,\text{Subst}\left(\int \frac{(a}{-d}\right.}{2d}$$

$$= -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{b}{}\right.}{d^3}$$

Mathematica [A] time = 0.372258, size = 303, normalized size = 0.82

$$\frac{(a+bx^2)^p\left(\frac{\left(\frac{a}{bx^2}+1\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)-\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} - \frac{2d\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1(}{(2p-}\right)}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] ((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b
)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (-(
AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-
(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b
)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))]/(1
 + a/(b*x^2))^p)/p))/(2*d^2)
```

Maple [F] time = 0.646, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{x(ex+d)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d)^2,x)
```

```
[Out] int((b*x^2+a)^p/x/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(ex+d\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{e^2x^3+2\,dex^2+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(ex+d\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)
```

3.422 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx$

Optimal. Leaf size=421

$$\frac{2e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e^4x^3\left(a\right.}{}$$

```
[Out] (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^
2])/(d^4*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2
, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x
^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^6*(1 + (
b*x^2)/a)^p) - ((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)
])/(d^2*x*(1 + (b*x^2)/a)^p) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1
, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^3*(b*d^2 + a*e^2)*(1
 + p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x
^2)/a])/(a*d^3*(1 + p)) - (b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1
 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d*(b*d^2 + a*e^2)^2*(1 +
p))
```

Rubi [A] time = 0.446496, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {961, 365, 364, 266, 65, 757, 430, 429, 444, 68, 511, 510}

$$\frac{2e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e^4x^3\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```


```
[Out] (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^
2])/(d^4*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2
, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x
^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^6*(1 + (
b*x^2)/a)^p) - ((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)
])/(d^2*x*(1 + (b*x^2)/a)^p) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1
, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^3*(b*d^2 + a*e^2)*(1
 + p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x
^2)/a])/(a*d^3*(1 + p)) - (b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1
 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d*(b*d^2 + a*e^2)^2*(1 +
p))
```


Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
```

```
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \left(\frac{(a+bx^2)^p}{d^2x^2} - \frac{2e(a+bx^2)^p}{d^3x} + \frac{e^2(a+bx^2)^p}{d^2(d+ex)^2} + \frac{2e^2(a+bx^2)^p}{d^3(d+ex)}\right)dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x^2}\,dx}{d^2} - \frac{(2e)\int \frac{(a+bx^2)^p}{x}\,dx}{d^3} + \frac{(2e^2)\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^3} + \frac{e^2\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d^2}$$

$$= -\frac{e\,\mathrm{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{d^3} + \frac{(2e^2)\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{d^3} + \frac{e^2\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+bx^2)^{p}}{(d^2-e^2x^2)^2}\right.}{d^2}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x} + \frac{e(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{ad^3(1+p)} + e^2$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x} + \frac{e(a+bx^2)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{ad^3(1+p)} + \frac{e^3}{}$$

$$= \frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} + \frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};\right.}{d^4}$$

Mathematica [F] time = 0.13013, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^2), x]
```

Maple [F] time = 0.64, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{x^2\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
```

```
[Out] int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(ex+d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^p}{e^2x^4+2\,dex^3+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(ex+d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)
```

3.423 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^3} \, dx$

Optimal. Leaf size=449

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\left(ae^2+bd^2\right)^2}-\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}\right.}{}$$

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^3*(1 + p)) - (d^4*(a + b*x^2)^(1 + p))/(2*e^3*(b
*d^2 + a*e^2)*(d + e*x)^2) + (d^3*(4*a*e^2 + b*d^2*(3 + p))*(a + b*x^2)^(1
+ p))/(e^3*(b*d^2 + a*e^2)^2*(d + e*x)) + (d*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4
+ 3*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/
2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p)
- (d*(3*a^2*e^4 + 2*a*b*d^2*e^2*(5 + 4*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a
 + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^4*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - (d^2*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2
*d^4*(6 + 7*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^3*(b*d^2 + a*e^2)^3*(1 + p))
```

Rubi [A] time = 0.938452, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1651, 1654, 844, 246, 245, 757, 430, 429, 444, 68}

$$\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4\left(2p^2+7p+6\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\left(ae^2+bd^2\right)^2}-\frac{dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^3*(1 + p)) - (d^4*(a + b*x^2)^(1 + p))/(2*e^3*(b
*d^2 + a*e^2)*(d + e*x)^2) + (d^3*(4*a*e^2 + b*d^2*(3 + p))*(a + b*x^2)^(1
+ p))/(e^3*(b*d^2 + a*e^2)^2*(d + e*x)) + (d*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4
+ 3*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/
2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p)
- (d*(3*a^2*e^4 + 2*a*b*d^2*e^2*(5 + 4*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a
 + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^4*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - (d^2*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2
*d^4*(6 + 7*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^3*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^3}\,dx = -\frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{\left(a+bx^2\right)^p\left(\frac{2ad^3}{e^2}-\frac{2d^2\left(ae^2+bd^2(1+p)\right)x}{e^3}+2d\left(a+\frac{bd^2}{e^2}\right)x^2-2\left(\frac{bd^2}{e}+ae\right)x^3\right)}{(d+ex)^2}\,dx}{2\left(bd^2+ae^2\right)}$$

$$= -\frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\int \frac{\left(a+bx^2\right)^p\left(2ad^2\left(3a+\frac{bd^2(2+p)}{e^2}\right)-\frac{2d\left(2a^2\cdots\right.}{\cdots}\right.}{\cdots}}{2\left(\cdots\right.}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\int \frac{\left(4abd^2(1+p)\left(3ae^2\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(d^2\left(6a^2e^4+3ab\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(d^2\left(6a^2e^4+3ab\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} - \frac{d\left(3a^2e^4+2abd^2\cdots\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} - \frac{d\left(3a^2e^4+2abd^2\cdots\right.}{\cdots}$$

$$= \frac{\left(a+bx^2\right)^{1+p}}{2be^3(1+p)} - \frac{d^4\left(a+bx^2\right)^{1+p}}{2e^3\left(bd^2+ae^2\right)(d+ex)^2} + \frac{d^3\left(4ae^2+bd^2(3+p)\right)\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)^2(d+ex)} + \frac{d\left(6a^2e^4+3abd^2\cdots\right.}{\cdots}$$

Mathematica [F] time = 0.901414, size = 0, normalized size = 0.

$$\int \frac{x^4\left(a+bx^2\right)^p}{(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^3, x]
```

Maple [F] time = 0.688, size = 0, normalized size = 0.

$$\int \frac{x^4 \left(b x^2+a\right)^p}{\left(e x+d\right)^3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^2+a\right)^p x^4}{\left(e x+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b x^2+a\right)^p x^4}{e^3 x^3+3\,d e^2 x^2+3\,d^2 e x+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^2+a\right)^p x^4}{\left(e x+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)
```

3.424 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=416

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)^2}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx}{a}\right.}{}$$

```
[Out] (d^3*(a + b*x^2)^(1 + p))/(2*e^2*(b*d^2 + a*e^2)*(d + e*x)^2) - (d^2*(3*a*e
^2 + b*d^2*(2 + p))*(a + b*x^2)^(1 + p))/(e^2*(b*d^2 + a*e^2)^2*(d + e*x))
- ((3*a^2*e^4 + a*b*d^2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*x*(a + b
*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2
 + a*e^2)^2*(1 + (b*x^2)/a)^p) + ((a^2*e^4 + a*b*d^2*e^2*(5 + 6*p) + b^2*d^
4*(3 + 5*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x
^2)/a)])/(e^3*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (d*(3*a^2*e^4 + a*b*d^
2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeome
tric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2
+ a*e^2)^3*(1 + p))
```

Rubi [A] time = 0.591918, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {1651, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2e^4+abd^2e^2(7p+6)+b^2d^4\left(2p^2+5p+3\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)^2}+\frac{x\left(a+bx^2\right)^p\left(\frac{bx}{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```


```
[Out] (d^3*(a + b*x^2)^(1 + p))/(2*e^2*(b*d^2 + a*e^2)*(d + e*x)^2) - (d^2*(3*a*e
^2 + b*d^2*(2 + p))*(a + b*x^2)^(1 + p))/(e^2*(b*d^2 + a*e^2)^2*(d + e*x))
- ((3*a^2*e^4 + a*b*d^2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*x*(a + b
*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2
 + a*e^2)^2*(1 + (b*x^2)/a)^p) + ((a^2*e^4 + a*b*d^2*e^2*(5 + 6*p) + b^2*d^
4*(3 + 5*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x
^2)/a)])/(e^3*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (d*(3*a^2*e^4 + a*b*d^
2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeome
tric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2
+ a*e^2)^3*(1 + p))
```


Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3 \left(a+bx^2\right)^p}{(d+ex)^3}\,dx = \frac{d^3 \left(a+bx^2\right)^{1+p}}{2e^2 \left(bd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{\left(a+bx^2\right)^p \left(-\frac{2ad^2}{e}+2d\left(a+\frac{bd^2(1+p)}{e^2}\right)x-2\left(\frac{bd^2}{e}+ae\right)x^2\right)}{(d+ex)^2}\,dx}{2\left(bd^2+ae^2\right)}$$

$$= \frac{d^3 \left(a+bx^2\right)^{1+p}}{2e^2 \left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\int \frac{\left(-\frac{2ad\left(2ae^2+bd^2(1+p)\right)}{e}+\frac{2\left(a^2e^4+abd^2e^2 \cdots\right.}{\cdots}\right.}{d+e \cdots}}{2\left(bd^2 \cdots\right.}$$

$$= \frac{d^3 \left(a+bx^2\right)^{1+p}}{2e^2 \left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(a^2e^4+abd^2e^2(5+6p)+b^2d \cdots\right.}{e^3\left(bc \cdots\right.}$$

$$= \frac{d^3 \left(a+bx^2\right)^{1+p}}{2e^2 \left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} - \frac{\left(d\left(3a^2e^4+abd^2e^2(6+7p)+ \cdots\right.\right.}{\cdots}$$

$$= \frac{d^3 \left(a+bx^2\right)^{1+p}}{2e^2 \left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(a^2e^4+abd^2e^2(5+6p)+b^2d \cdots\right.}{\cdots}$$

$$= \frac{d^3 \left(a+bx^2\right)^{1+p}}{2e^2 \left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(a^2e^4+abd^2e^2(5+6p)+b^2d \cdots\right.}{\cdots}$$

$$= \frac{d^3 \left(a+bx^2\right)^{1+p}}{2e^2 \left(bd^2+ae^2\right)(d+ex)^2} - \frac{d^2\left(3ae^2+bd^2(2+p)\right)\left(a+bx^2\right)^{1+p}}{e^2\left(bd^2+ae^2\right)^2(d+ex)} - \frac{\left(3a^2e^4+abd^2e^2(6+7p)+b^2 \cdots\right.}{\cdots}$$

Mathematica [F] time = 0.633978, size = 0, normalized size = 0.

$$\int \frac{x^3 \left(a+bx^2\right)^p}{(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^3, x]
```

Maple [F] time = 0.695, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x^3}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x^3}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p x^3}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x^3}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)
```

3.425 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=396

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de^2\left(ae^2+bd^2\right)^2}-\frac{\left(a+bx^2\right)^{p+1}\left(a^2e^4\right.}{}$$

```
[Out] -(d^2*(a + b*x^2)^(1 + p))/(2*e*(b*d^2 + a*e^2)*(d + e*x)^2) + (d*(2*a*e^2
+ b*d^2*(1 + p))*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)^2*(d + e*x)) + ((a
^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 + 3*p + 2*p^2))*x*(a + b*x^2)^p
*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*e^2*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*(2*a*e^2 + b*d^2*(1 + p))*x*(a +
 b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^2*(b*d^2 + a*e^
2)^2*(1 + (b*x^2)/a)^p) - ((a^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 +
3*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(2*e*(b*d^2 + a*e^2)^3*(1 + p))
```

Rubi [A] time = 0.569918, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1651, 835, 844, 246, 245, 757, 430, 429, 444, 68}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4+abd^2e^2(5p+2)+b^2d^4\left(2p^2+3p+1\right)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de^2\left(ae^2+bd^2\right)^2}-\frac{\left(a+bx^2\right)^{p+1}\left(a^2e^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```


```
[Out] -(d^2*(a + b*x^2)^(1 + p))/(2*e*(b*d^2 + a*e^2)*(d + e*x)^2) + (d*(2*a*e^2
+ b*d^2*(1 + p))*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)^2*(d + e*x)) + ((a
^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 + 3*p + 2*p^2))*x*(a + b*x^2)^p
*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*e^2*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*(2*a*e^2 + b*d^2*(1 + p))*x*(a +
 b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^2*(b*d^2 + a*e^
2)^2*(1 + (b*x^2)/a)^p) - ((a^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 +
3*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(2*e*(b*d^2 + a*e^2)^3*(1 + p))
```


Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
```

```
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))]])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^p}{(d+ex)^3}\,dx=-\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2}-\frac{\int \frac{\left(2ad-\frac{2\left(ae^2+bd^2(1+p)\right)x}{e}\right)\left(a+bx^2\right)^p}{(d+ex)^2}\,dx}{2\left(bd^2+ae^2\right)}$$

$$=-\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2}+\frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)}+\frac{\int \frac{\left(2a\left(ae^2+bd^2p\right)-\frac{2bd(1+2p)\left(2ae^2+bd^2(1\cdots\right.}{e}\cdots}{d+ex}}{2\left(bd^2+ae^2\right)^2}$$

$$=-\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2}+\frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)}-\frac{\left(bd(1+2p)\left(2ae^2+bd^2(1+p\cdots\right.\right.}{e^2\left(bd^2+ae^2\cdots\right.}$$

$$=-\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2}+\frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)}+\frac{\left(a^2e^4+abd^2e^2(2+5p)+b^2d^4\cdots\right.}{\cdots}$$

$$=-\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2}+\frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)}-\frac{bd(1+2p)\left(2ae^2+bd^2(1+p)\cdots\right.}{\cdots}$$

$$=-\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2}+\frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)}-\frac{bd(1+2p)\left(2ae^2+bd^2(1+p)\cdots\right.}{\cdots}$$

$$=-\frac{d^2\left(a+bx^2\right)^{1+p}}{2e\left(bd^2+ae^2\right)(d+ex)^2}+\frac{d\left(2ae^2+bd^2(1+p)\right)\left(a+bx^2\right)^{1+p}}{e\left(bd^2+ae^2\right)^2(d+ex)}+\frac{\left(a^2e^4+abd^2e^2(2+5p)+b^2d^4\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.260409, size = 290, normalized size = 0.73

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{d^2F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(p-1)(d+ex)^2}-\frac{4dF_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(2p-1)(d+ex)}+\frac{F_1\left(-2p;-p,\cdots\right.}{\cdots}\right.}{2e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((-4*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*(d + e*x)) + (d^
2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + S
qrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*(d + e*x)^2) + AppellF1[-2*p, -p, -p,
1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/p)
)/(2*e^3*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d +
 e*x))^p)
```

Maple [F] time = 0.69, size = 0, normalized size = 0.

$$\int \frac{x^{2} \left(b\,x^{2}+a\right)^{p}}{\left(e x +d \right)^{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b x^{2} + a\right)}^{p} x^{2}}{{\left(e x + d\right)}^{3}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(b x^{2} + a\right)}^{p} x^{2}}{e^{3} x^{3} + 3 \, d e^{2} x^{2} + 3 \, d^{2} e x + d^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b x^{2} + a\right)}^{p} x^{2}}{{\left(e x + d\right)}^{3}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)
```

3.426 $\int \frac{x(a+bx^2)^p}{(d+ex)^3} \, dx$

Optimal. Leaf size=336

$$-\frac{bpx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e\left(ae^2+bd^2\right)^2}+\frac{b(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2\right.}{e\left(ae^2+bd^2\right)^2}$$

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d + e*x)^2) - ((a*e^2 + b*d^2*p
)*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)^2*(d + e*x)) - (b*p*(3*a*e^2 + b*d^
2*(1 + 2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*(1 + 2*p)*(a*e^2 + b
*d^2*p)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(
b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*d*p*(3*a*e^2 + b*d^2*(1 + 2*p))*(a
 + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d
^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rubi [A] time = 0.40036, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {835, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{bpx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3ae^2+bd^2(2p+1)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e\left(ae^2+bd^2\right)^2}+\frac{b(2p+1)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(ae^2+bd^2\right.}{e\left(ae^2+bd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d + e*x)^2) - ((a*e^2 + b*d^2*p
)*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)^2*(d + e*x)) - (b*p*(3*a*e^2 + b*d^
2*(1 + 2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*(1 + 2*p)*(a*e^2 + b
*d^2*p)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(
b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*d*p*(3*a*e^2 + b*d^2*(1 + 2*p))*(a
 + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d
^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^p}{(d+ex)^3}\,dx = \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{(-2ae+2bdpx)\left(a+bx^2\right)^p}{(d+ex)^2}\,dx}{2\left(bd^2+ae^2\right)}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\int \frac{\left(2abde(1-p)+2b(1+2p)\left(ae^2+bd^2p\right)x\right)\left(a+bx^2\right)^p}{d+ex}\,dx}{2\left(bd^2+ae^2\right)^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{\left(b(1+2p)\left(ae^2+bd^2p\right)\right)\int \left(a+bx^2\right)^p\,dx}{e\left(bd^2+ae^2\right)^2} - \ldots$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} - \frac{\left(bdp\left(3ae^2+bd^2(1+2p)\right)\right)\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2} + \frac{ex(a\ldots}{-d^2\ldots}\right.}{e\left(bd^2+ae^2\right)^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{b(1+2p)\left(ae^2+bd^2p\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)\ldots}{e\left(bd^2+ae^2\right)^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} + \frac{b(1+2p)\left(ae^2+bd^2p\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)\ldots}{e\left(bd^2+ae^2\right)^2}$$

$$= \frac{d\left(a+bx^2\right)^{1+p}}{2\left(bd^2+ae^2\right)(d+ex)^2} - \frac{\left(ae^2+bd^2p\right)\left(a+bx^2\right)^{1+p}}{\left(bd^2+ae^2\right)^2(d+ex)} - \frac{bp\left(3ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)\ldots}{e\left(bd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.223992, size = 229, normalized size = 0.68

$$\frac{\left(a+bx^2\right)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(2(p-1)(d+ex)F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)+d(1-2p)F_1\left(2-2p;\ldots\right.\right.}{2e^2(p-1)(2p-1)(d+ex)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*(2*(-1 + p)*(d + e*x)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d
- Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + d*(1 - 2*p)*
AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqr
t[-(a/b)]*e)/(d + e*x)]))/(2*e^2*(-1 + p)*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x
))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

Maple [F] time = 0.687, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p x}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p x}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p x}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p x}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x/(e*x + d)^3, x)
```

3.427 $\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=322

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}+\frac{e^2x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{3b^2d^2e\left(a+\right.}{}$$

```
[Out] -(d^2*e*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (x*(a
+ b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1
+ (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((b*x^2
)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (b*e*(2*a*e^2 + b*d^2*(1 +
p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2)
)/(b*d^2 + a*e^2)])/(4*(b*d^2 + a*e^2)^3*(1 + p)) - (3*b^2*d^2*e*(a + b*x^2
)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e
^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rubi [A] time = 0.329286, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {757, 430, 429, 444, 68, 511, 510, 446, 78}

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}+\frac{e^2x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{3b^2d^2e\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -(d^2*e*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (x*(a
+ b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1
+ (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((b*x^2
)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (b*e*(2*a*e^2 + b*d^2*(1 +
p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2)
)/(b*d^2 + a*e^2)])/(4*(b*d^2 + a*e^2)^3*(1 + p)) - (3*b^2*d^2*e*(a + b*x^2
)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e
^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx = \int\left(\frac{d^3(a+bx^2)^p}{(d^2-e^2x^2)^3} - \frac{3d^2ex(a+bx^2)^p}{(d^2-e^2x^2)^3} + \frac{3de^2x^2(a+bx^2)^p}{(d^2-e^2x^2)^3} + \frac{e^3x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}\right)dx$$

$$= d^3\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx - (3d^2e)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx + (3de^2)\int \frac{x^2(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx + e^3\int \frac{x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}$$

$$= -\left(\frac{1}{2}(3d^2e)\,\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^3}\,dx, x, x^2\right)\right) + \frac{1}{2}e^3\,\text{Subst}\left(\int \frac{x(a+bx)^p}{(-d^2+e^2x)^3}\,dx, x, x^2\right) + \left(d^3(a+bx^2\right.$$

$$= -\frac{d^2e(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2x^3(a+bx^2)^p(1}{}$$

$$= -\frac{d^2e(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2x^3(a+bx^2)^p(1}{}$$

Mathematica [A] time = 0.119855, size = 142, normalized size = 0.44

$$\frac{(a+bx^2)^p\left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} F_1\left(2-2p; -p, -p; 3-2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{2e(p-1)(d+ex)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*(-1 + p)*((e*(-Sqrt[-(a/b)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

Maple [F] time = 0.679, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int((b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2 + a\right)^p}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^p}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p/(e*x + d)^3, x)
```

3.428 $\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$

Optimal. Leaf size=700

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\right)}{}$$

```
[Out] (d*e^2*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) - (e*x*(a
 + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]])/(d^4*(1
 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a
), (e^2*x^2)/d^2]])/(d^4*(1 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/
2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]])/(d^4*(1 + (b*x^2)/a)^p) - (e^3
*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2]])/
(3*d^6*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2
, -((b*x^2)/a), (e^2*x^2)/d^2]])/(d^6*(1 + (b*x^2)/a)^p) + (e^2*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2
)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^3*(1 + p)) + (b*e^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)
])/(d*(b*d^2 + a*e^2)^2*(1 + p)) - (b*e^2*(2*a*e^2 + b*d^2*(1 + p))*(a + b*
x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
a*e^2)])/(4*d*(b*d^2 + a*e^2)^3*(1 + p)) + (3*b^2*d*e^2*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
(b*d^2 + a*e^2)^3*(1 + p))
```

Rubi [A] time = 0.818105, antiderivative size = 700, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {961, 266, 65, 757, 430, 429, 444, 68, 511, 510, 446, 78}

$$-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (d*e^2*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) - (e*x*(a
 + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]])/(d^4*(1
 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a
), (e^2*x^2)/d^2]])/(d^4*(1 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/
2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]])/(d^4*(1 + (b*x^2)/a)^p) - (e^3
*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2]])/
(3*d^6*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2
, -((b*x^2)/a), (e^2*x^2)/d^2]])/(d^6*(1 + (b*x^2)/a)^p) + (e^2*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2
)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^3*(1 + p)) + (b*e^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)
])/(d*(b*d^2 + a*e^2)^2*(1 + p)) - (b*e^2*(2*a*e^2 + b*d^2*(1 + p))*(a + b*
x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
a*e^2)])/(4*d*(b*d^2 + a*e^2)^3*(1 + p)) + (3*b^2*d*e^2*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
```

```
(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^p}{x(d+ex)^3}\,dx &= \int \left(\frac{\left(a+bx^2\right)^p}{d^3x}-\frac{e\left(a+bx^2\right)^p}{d(d+ex)^3}-\frac{e\left(a+bx^2\right)^p}{d^2(d+ex)^2}-\frac{e\left(a+bx^2\right)^p}{d^3(d+ex)}\right)dx\\
&= \frac{\int \frac{\left(a+bx^2\right)^p}{x}\,dx}{d^3}-\frac{e\int \frac{\left(a+bx^2\right)^p}{d+ex}\,dx}{d^3}-\frac{e\int \frac{\left(a+bx^2\right)^p}{(d+ex)^2}\,dx}{d^2}-\frac{e\int \frac{\left(a+bx^2\right)^p}{(d+ex)^3}\,dx}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)}{2d^3}-\frac{e\int \left(\frac{d\left(a+bx^2\right)^p}{d^2-e^2x^2}+\frac{ex\left(a+bx^2\right)^p}{-d^2+e^2x^2}\right)dx}{d^3}-\frac{e\int \left(\frac{d^2\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}-\frac{2dex\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}+\frac{e^2x^2\left(a+bx^2\right.}{\left(-d^2+e^2x^2\right.}\right.}{d^2}\\
&= -\frac{\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}-e\int \frac{\left(a+bx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx-\frac{e\int \frac{\left(a+bx^2\right)^p}{d^2-e^2x^2}\,dx}{d^2}-\left(d^2e\right)\int \frac{\left(a+b\right.}{\left(d^2-e\right.}\\
&= -\frac{\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)}-\frac{e^2\operatorname{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)}{2d^3}+\frac{e^2\operatorname{Subst}\left(\int \frac{(a+bx)^p}{\left(d^2-e^2x\right)^2}\,dx\right.}{d}\\
&= \frac{de^2\left(a+bx^2\right)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2}-\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\right)^p\left(1+\right.}{}\\
&= \frac{de^2\left(a+bx^2\right)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2}-\frac{ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex\left(a+bx^2\right)^p\left(1+\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.120365, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] Integrate[(a + b*x^2)^p/(x*(d + e*x)^3), x]
```

Maple [F] time = 0.645, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{x(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d)^3,x)
```

```
[Out] int((b*x^2+a)^p/x/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{(ex+d)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^p}{e^3x^4+3\,de^2x^3+3\,d^2ex^2+d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(ex+d\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)
```

3.429 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$

Optimal. Leaf size=754

$$\frac{3e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{2e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{e^2x\left(a\right.}{}$$

```
[Out] -(e^3*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (3*e^2*x
*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5
*(1 + (b*x^2)/a)^p) + (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b
*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*Ap
pellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^
p) + (2*e^4*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*
x^2)/d^2])/(3*d^7*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x^2)^p*AppellF1[3/2,
 -p, 3, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^7*(1 + (b*x^2)/a)^p) - ((a +
b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(d^3*x*(1 + (b*x^2
)/a)^p) - (3*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^4*(b*d^2 + a*e^2)*(1 + p)) + (3*e*(a
+ b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^
4*(1 + p)) - (2*b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^2)^2*(1 + p)) + (b*
e^3*(2*a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 +
p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(4*d^2*(b*d^2 + a*e^2)^3*(1 +
 p)) - (3*b^2*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e
^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rubi [A] time = 0.859768, antiderivative size = 754, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {961, 365, 364, 266, 65, 757, 430, 429, 444, 68, 511, 510, 446, 78}

$$\frac{3e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{2e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{e^2x\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] -(e^3*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (3*e^2*x
*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5
*(1 + (b*x^2)/a)^p) + (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b
*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*Ap
pellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^
p) + (2*e^4*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*
x^2)/d^2])/(3*d^7*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x^2)^p*AppellF1[3/2,
 -p, 3, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^7*(1 + (b*x^2)/a)^p) - ((a +
b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(d^3*x*(1 + (b*x^2
)/a)^p) - (3*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^4*(b*d^2 + a*e^2)*(1 + p)) + (3*e*(a
+ b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^
4*(1 + p)) - (2*b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^2)^2*(1 + p)) + (b*
e^3*(2*a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 +
p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(4*d^2*(b*d^2 + a*e^2)^3*(1 +
```

```
 p)) - (3*b^2*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e
^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \int \left(\frac{(a+bx^2)^p}{d^3x^2} - \frac{3e(a+bx^2)^p}{d^4x} + \frac{e^2(a+bx^2)^p}{d^2(d+ex)^3} + \frac{2e^2(a+bx^2)^p}{d^3(d+ex)^2} + \frac{3e^2(a+bx^2)^p}{d^4(d+ex)}\right)dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x^2}\,dx}{d^3} - \frac{(3e)\int \frac{(a+bx^2)^p}{x}\,dx}{d^4} + \frac{(3e^2)\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^4} + \frac{(2e^2)\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d^3} + \frac{e^2\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx}{d^2}$$

$$= -\frac{(3e)\,\mathrm{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^4} + \frac{(3e^2)\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{d^4} + \frac{(2e^2)\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+b}{(d^2-e^2x}\right.}{d^3}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} + \frac{3e(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^4(1+p)} + \frac{(3e^2)}{}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} + \frac{3e(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^4(1+p)} - \frac{1}{2}(3e$$

$$= -\frac{e^3(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \frac{2e^2x(a+bx^2)}{}$$

$$= -\frac{e^3(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \frac{2e^2x(a+bx^2)}{}$$

Mathematica [F] time = 0.17533, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^3), x]
```

Maple [F] time = 0.793, size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{x^2(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

```
[Out] int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^p}{(ex+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{e^3x^5+3\,de^2x^4+3\,d^2ex^3+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(ex+d\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)
```

3.430 $\int (gx)^m (d+ex)^3 \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=276

$$-\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ae^2(m+2)-3cd^2(m+2p+4)\right){}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{cg^2(m+2)(m+2p+4)}-\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+\right.}{}$$

```
[Out] (3*d*e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + (e^3*(g*x
)^(2 + m)*(a + c*x^2)^(1 + p))/(c*g^2*(4 + m + 2*p)) - (d*(3*a*e^2*(1 + m)
- c*d^2*(3 + m + 2*p))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m
)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(c*g*(1 + m)*(3 + m + 2*p)*(1 + (c*x^2)/
a)^p) - (e*(a*e^2*(2 + m) - 3*c*d^2*(4 + m + 2*p))*(g*x)^(2 + m)*(a + c*x^2
)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(c*g^2*(2 +
m)*(4 + m + 2*p)*(1 + (c*x^2)/a)^p)
```

Rubi [A] time = 0.468204, antiderivative size = 254, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1809, 808, 365, 364}

$$\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{3d^2}{m+2}-\frac{ae^2}{c(m+2p+4)}\right){}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{g^2}+\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{d^2}{m+1}-\frac{3}{c(m}\right.}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] (3*d*e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + (e^3*(g*x
)^(2 + m)*(a + c*x^2)^(1 + p))/(c*g^2*(4 + m + 2*p)) + (d*(d^2/(1 + m) - (3
*a*e^2)/(c*(3 + m + 2*p)))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1
 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + (c*x^2)/a)^p) + (e*((3*d^2)/
(2 + m) - (a*e^2)/(c*(4 + m + 2*p)))*(g*x)^(2 + m)*(a + c*x^2)^p*Hypergeome
tric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(1 + (c*x^2)/a)^p)
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx = \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{\int (gx)^m\left(a+cx^2\right)^p\left(cd^3(4+m+2p)-e\left(ae^2(2+m)-3cd\right.\right.}{c(4+m+2p)}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{\int (gx)^m\left(-cd(4+m+2p)\left(3a\right.\right.}{}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \left(d\left(d^2-\frac{3ae^2(1+m)}{c(3+m+2p)}\right)\right)\int (\ldots$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \left(d\left(d^2-\frac{3ae^2(1+m)}{c(3+m+2p)}\right)\left(a+\ldots\right.\right.$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{d\left(d^2-\frac{3ae^2(1+m)}{c(3+m+2p)}\right)(gx)^{1+m}\left(a\ldots\right.}{}$$

Mathematica [A] time = 0.203106, size = 182, normalized size = 0.66

$$x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ex\left(\frac{3d^2\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{m+2}+ex\left(\frac{3d\,{}_2F_1\left(\frac{m+3}{2},-p;\frac{m+5}{2};-\frac{cx^2}{a}\right)}{m+3}+\frac{ex\,{}_2F_1\left(\frac{m+4}{2},-p;\ldots\right)}{m+4\ldots}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p, (4 +
m)/2, -((c*x^2)/a)])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)/2, -p, (
5 + m)/2, -((c*x^2)/a)])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m)/2, -p, (6
 + m)/2, -((c*x^2)/a)])/(4 + m)))))/(1 + (c*x^2)/a)^p
```

Maple [F] time = 0.557, size = 0, normalized size = 0.

$$\int (gx)^m (ex+d)^3 \left(cx^2+a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 (cx^2+a)^p (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\left(cx^2+a\right)^p(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(c*x^2 + a)^p*(g*x)^m, x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 (cx^2+a)^p (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

3.431 $\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=205

$$-\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(ae^2(m+1)-cd^2(m+2p+3)\right){}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{cg(m+1)(m+2p+3)}+\frac{2de(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx}{a}\cdots\right.}{g\cdots}$$

```
[Out] (e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) - ((a*e^2*(1 +
m) - c*d^2*(3 + m + 2*p))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1
+ m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(c*g*(1 + m)*(3 + m + 2*p)*(1 + (c*x^
2)/a)^p) + (2*d*e*(g*x)^(2 + m)*(a + c*x^2)^p*Hypergeometric2F1[(2 + m)/2,
-p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rubi [A] time = 0.196665, antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1809, 808, 365, 364}

$$\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{d^2}{m+1}-\frac{ae^2}{c(m+2p+3)}\right){}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g}+\frac{2de(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\cdots\right.}{g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + ((d^2/(1 + m)
 - (a*e^2)/(c*(3 + m + 2*p)))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1
[(1 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + (c*x^2)/a)^p) + (2*d*e*(g
*x)^(2 + m)*(a + c*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*
x^2)/a)])/(g^2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx &= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\int (gx)^m\left(-ae^2(1+m)+cd^2(3+m+2p)+2cde(3+m+2p)x\right.}{c(3+m+2p)}\\
&= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{(2de)\int (gx)^{1+m}\left(a+cx^2\right)^p\,dx}{g}+\left(d^2-\frac{ae^2(1+m)}{c(3+m+2p)}\right)\int (gx)\\
&= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\left(2de\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^{1+m}\left(1+\frac{cx^2}{a}\right)^p\,dx}{g}+\left(\left(d^2-\right.\right.\\
&= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\left(d^2-\frac{ae^2(1+m)}{c(3+m+2p)}\right)(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\right.}{g(1+m)}\end{aligned}$$

Mathematica [A] time = 0.117482, size = 158, normalized size = 0.77

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d^2\left(m^2+5m+6\right){}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)+e(m+1)x\left(2d(m+3)\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right.\right.\right.}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)/2,
-p, (3 + m)/2, -((c*x^2)/a)] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric2F1[(
2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)] + e*(2 + m)*x*Hypergeometric2F1[(3 +
 m)/2, -p, (5 + m)/2, -((c*x^2)/a)])))/((1 + m)*(2 + m)*(3 + m)*(1 + (c*x^2
)/a)^p)
```

Maple [F] time = 0.548, size = 0, normalized size = 0.

$$\int \left(gx\right)^m(ex+d)^2\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2\left(cx^2+a\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\left(cx^2+a\right)^p\left(gx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(ex+d)^2\left(cx^2+a\right)^p\left(gx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

3.432 $\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=135

$$\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}+\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{g^2(m+2)}$$

```
[Out] (d*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p) + (e*(g*x)^(2 + m)*(a + c*x^2)
^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(2 + m)*
(1 + (c*x^2)/a)^p)
```

Rubi [A] time = 0.0626675, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {808, 365, 364}

$$\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}+\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (d*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p) + (e*(g*x)^(2 + m)*(a + c*x^2)
^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(2 + m)*
(1 + (c*x^2)/a)^p)
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p dx = d\int (gx)^m\left(a+cx^2\right)^p dx + \frac{e\int (gx)^{1+m}\left(a+cx^2\right)^p dx}{g}$$
$$= \left(d\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p dx + \frac{\left(e\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)}{g}$$
$$= \frac{d(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\right.}{g}$$

Mathematica [A] time = 0.03681, size = 106, normalized size = 0.79

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d(m+2)\,{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)+e(m+1)x\,{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3 + m
)/2, -((c*x^2)/a)] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2
, -((c*x^2)/a)]))/((1 + m)*(2 + m)*(1 + (c*x^2)/a)^p)
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int (gx)^m (ex+d)\left(cx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+a\right)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex+d)\left(cx^2+a\right)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)\left(cx^2+a\right)^p (gx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

3.433 $\int (gx)^m \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=66

$$\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}$$

```
[Out] ((g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p)
```

Rubi [A] time = 0.0187254, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(a + c*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p)
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (gx)^m\left(a+cx^2\right)^p\,dx &= \left(\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p\,dx\\ &= \frac{(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0087121, size = 64, normalized size = 0.97

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+1}{2}+1;-\frac{cx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(c*x^2)/a)])/((1 + m)*(1 + (c*x^2)/a)^p)
```

Maple [F] time = 0.397, size = 0, normalized size = 0.

$$\int \left(gx\right)^{m} \left(cx^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2}+a\right)^{p} \left(gx\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^{2}+a\right)^{p} \left(gx\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [C] time = 34.8471, size = 54, normalized size = 0.82

$$\frac{a^{p} g^{m} x x^{m} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\right| \frac{c x^{2} e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p,x)
```

```
[Out] a**p*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), c*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p (gx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m, x)
```

3.434 $\int \frac{(gx)^m(a+cx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=157

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(m+1)}-\frac{ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,1;\frac{m+4}{2};-\frac{cx^2}{a},\right.}{d^2(m+2)}$$

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d*(1 + m)*(1 + (c*x^2)/a)^p) - (e*x^2*(g*x)^m*(a + c*x^2
)^p*AppellF1[(2 + m)/2, -p, 1, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^
2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rubi [A] time = 0.143294, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 511, 510}

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(m+1)}-\frac{ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,1;\frac{m+4}{2};-\frac{cx^2}{a},\right.}{d^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d*(1 + m)*(1 + (c*x^2)/a)^p) - (e*x^2*(g*x)^m*(a + c*x^2
)^p*AppellF1[(2 + m)/2, -p, 1, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^
2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(gx)^m \left(a+cx^2\right)^p}{d+ex}\,dx = \left(dx^{-m}(gx)^m\right)\int \frac{x^m \left(a+cx^2\right)^p}{d^2-e^2x^2}\,dx - \left(ex^{-m}(gx)^m\right)\int \frac{x^{1+m}\left(a+cx^2\right)^p}{d^2-e^2x^2}\,dx$$

$$= \left(dx^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{d^2-e^2x^2}\,dx - \left(ex^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-}\right.$$

$$= \frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,1;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(1+m)} - \frac{ex^2(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)}{d^2}$$

Mathematica [F] time = 0.0967025, size = 0, normalized size = 0.

$$\int \frac{(gx)^m \left(a+cx^2\right)^p}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
```

```
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x), x]
```

Maple [F] time = 0.677, size = 0, normalized size = 0.

$$\int \frac{(gx)^m \left(cx^2+a\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
```

```
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p (gx)^m}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^p (gx)^m}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + a\right)^p \left(gx\right)^m}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

3.435 $\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=238

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,2;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+1)}-\frac{2ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,2;\frac{m+4}{2};-\right.}{d^3(m+2)}$$

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 2, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^2*(1 + m)*(1 + (c*x^2)/a)^p) - (2*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 2, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^3*(2 + m)*(1 + (c*x^2)/a)^p) + (e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF1[
(3 + m)/2, -p, 2, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^4*(3 + m)*(1
+ (c*x^2)/a)^p)
```

Rubi [A] time = 0.278811, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {962, 511, 510}

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,2;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+1)}-\frac{2ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,2;\frac{m+4}{2};-\right.}{d^3(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 2, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^2*(1 + m)*(1 + (c*x^2)/a)^p) - (2*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 2, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^3*(2 + m)*(1 + (c*x^2)/a)^p) + (e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF1[
(3 + m)/2, -p, 2, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^4*(3 + m)*(1
+ (c*x^2)/a)^p)
```

Rule 962

```
Int[((g_.)*(x_))^(n_.)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_),
 x_Symbol] :> Dist[(g*x)^n/x^n, Int[ExpandIntegrand[x^n*(a + c*x^2)^p, (d/(
d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(-m), x], x], x] /; FreeQ[{a, c, d,
 e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[m, 0] &&  !IntegerQ[p] &&
  !IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(gx)^m \left(a+cx^2\right)^p}{(d+ex)^2}\,dx = \left(x^{-m}(gx)^m\right)\int \left(\frac{d^2x^m\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2} - \frac{2dex^{1+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2} + \frac{e^2x^{2+m}\left(a+cx^2\right)^p}{\left(-d^2+e^2x^2\right)^2}\right)dx$$

$$= \left(d^2x^{-m}(gx)^m\right)\int \frac{x^m\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx - \left(2dex^{-m}(gx)^m\right)\int \frac{x^{1+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx + \left(e^2x^{-m}(gx)^m\right)\int \frac{x}{(}$$

$$= \left(d^2x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{\left(d^2-e^2x^2\right)^2}\,dx - \left(2dex^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)\right.$$

$$= \frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,2;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(1+m)} - \frac{2ex^2(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-}}{d^3(2}$$

Mathematica [F] time = 0.0936035, size = 0, normalized size = 0.

$$\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x]
```

Maple [F] time = 0.638, size = 0, normalized size = 0.

$$\int \frac{(gx)^m\left(cx^2+a\right)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p(gx)^m}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^p\left(gx\right)^m}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2+a\right)^p\left(gx\right)^m}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^2, x)
```

3.436 $\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=321

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,3;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(m+1)}-\frac{3ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,3;\frac{m+4}{2};-\frac{cx^2}{a}\right.}{d^4(m+2)}$$

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 3, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^3*(1 + m)*(1 + (c*x^2)/a)^p) - (3*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 3, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^4*(2 + m)*(1 + (c*x^2)/a)^p) + (3*e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF
1[(3 + m)/2, -p, 3, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^5*(3 + m)*(
1 + (c*x^2)/a)^p) - (e^3*x^4*(g*x)^m*(a + c*x^2)^p*AppellF1[(4 + m)/2, -p,
3, (6 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^6*(4 + m)*(1 + (c*x^2)/a)^p)
```

Rubi [A] time = 0.373623, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {962, 511, 510}

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,3;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(m+1)}-\frac{3ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,3;\frac{m+4}{2};-\frac{cx^2}{a}\right.}{d^4(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 3, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^3*(1 + m)*(1 + (c*x^2)/a)^p) - (3*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 3, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^4*(2 + m)*(1 + (c*x^2)/a)^p) + (3*e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF
1[(3 + m)/2, -p, 3, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^5*(3 + m)*(
1 + (c*x^2)/a)^p) - (e^3*x^4*(g*x)^m*(a + c*x^2)^p*AppellF1[(4 + m)/2, -p,
3, (6 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^6*(4 + m)*(1 + (c*x^2)/a)^p)
```

Rule 962

```
Int[((g_.)*(x_))^(n_.)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_),
 x_Symbol] :> Dist[(g*x)^n/x^n, Int[ExpandIntegrand[x^n*(a + c*x^2)^p, (d/(
d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(-m), x], x], x] /; FreeQ[{a, c, d,
 e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[m, 0] &&  !IntegerQ[p] &&
  !IntegerQ[n]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^3}\,dx = \left(x^{-m}(gx)^m\right)\int\left(\frac{d^3x^m\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}-\frac{3d^2ex^{1+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}+\frac{3de^2x^{2+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}+\frac{e^3x^{3+m}\left(a\right.}{\left(-d^2+e\right.}$$

$$=\left(d^3x^{-m}(gx)^m\right)\int\frac{x^m\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx-\left(3d^2ex^{-m}(gx)^m\right)\int\frac{x^{1+m}\left(a+cx^2\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx+\left(3de^2x^{-m}(gx)^{m}\right.$$

$$=\left(d^3x^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{\left(d^2-e^2x^2\right)^3}\,dx-\left(3d^2ex^{-m}(gx)^m\left(a+cx^2\right)^p\left(1+\frac{c}{}\right.\right.$$

$$=\frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,3;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(1+m)}-\frac{3ex^2(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right.}{d^4}$$

Mathematica [F] time = 0.201937, size = 0, normalized size = 0.

$$\int \frac{(gx)^m\left(a+cx^2\right)^p}{(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]

[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x]
```

Maple [F] time = 0.699, size = 0, normalized size = 0.

$$\int \frac{(gx)^m\left(cx^2+a\right)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)

[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p(gx)^m}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+a\right)^p\left(gx\right)^m}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^p\left(gx\right)^m}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)
```

3.437 $\int \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(-2cdex\left(-5a^2e^4-6acd^2e^2+35c^2d^4\right)-17a^2cd^2e^4-15a^3e^6-25ac^2d^4e^2+105c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192c^3d^3e^4}+$$

```
[Out] ((a/(c*d) - (7*d)/e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/24
+ (x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((105*c^3*d^6 -
 25*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6 - 2*c*d*e*(35*c^2*d^4 - 6
*a*c*d^2*e^2 - 5*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
192*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*
c*d^2*e^4 + 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(7/2)*d^(7
/2)*e^(9/2))
```

Rubi [A] time = 0.506088, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 832, 779, 621, 206}

$$-\frac{\left(-2cdex\left(-5a^2e^4-6acd^2e^2+35c^2d^4\right)-17a^2cd^2e^4-15a^3e^6-25ac^2d^4e^2+105c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192c^3d^3e^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```


```
[Out] ((a/(c*d) - (7*d)/e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/24
+ (x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((105*c^3*d^6 -
 25*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6 - 2*c*d*e*(35*c^2*d^4 - 6
*a*c*d^2*e^2 - 5*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
192*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*
c*d^2*e^4 + 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(7/2)*d^(7
/2)*e^(9/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \int \frac{x^3(ae+cdx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e} + \frac{\int \frac{x^2\left(-3acd^2e-\frac{1}{2}cd\left(7cd^2-ae^2\right)x\right)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4cde}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{4e}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{4e}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{4e}$$

$$= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{4e}$$

Mathematica [A] time = 1.76984, size = 304, normalized size = 0.88

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\sqrt{cd}\sqrt{cd^2-ae^2}\left(9a^2cd^2e^4+5a^3e^6+15ac^2d^4e^2+35c^3d^6\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}} - \sqrt{c}\sqrt{d}\sqrt{e}\left(a^2cde^4(10ex-17d)-15a^3e\right.}{192c^{7/2}d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^3*e^6 + a^
2*c*d*e^4*(-17*d + 10*e*x) + a*c^2*d^2*e^2*(-25*d^2 + 12*d*e*x - 8*e^2*x^2)
 + c^3*d^3*(105*d^3 - 70*d^2*e*x + 56*d*e^2*x^2 - 48*e^3*x^3))) + (3*Sqrt[c
*d]*Sqrt[c*d^2 - a*e^2]*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 +
5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*S
qrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)])))/(192*c^(7/2)*d^(7/2)*e^(9/2))
```

Maple [B] time = 0.07, size = 946, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

```
[Out] 5/32*e/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-5/128*e^4/d^3/
c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4-d^3/e^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(
d/e+x))^(1/2)-13/24/e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+29/64/e^4
*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*d^3/e^2*ln((1/2*a*e^2-1/2*
c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^
(1/2))/(d*e*c)^(1/2)*a+1/2*d^5/e^4*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(
d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*c
-3/64*d/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+1/4/e^2*x*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)/d/c-5/24/e/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*
a+7/16/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+5/64*e^2/d^3/c^3*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+43/64/e^2*d/c*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)*a-29/128/e^4*d^5*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*
e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+11/32/e^2
*a*d^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+29/32/e^3*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)*x+19/64/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-1/32
*e^2*a^3/c^2/d*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60576, size = 1438, normalized size = 4.17

$$\left[-\frac{3\left(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4*e^
4*x^3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3*c*d*
e^7 - 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a*c^3*d
^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^4*d^4*e^5), -1/384*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^
4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2
*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4*d^4*e^4*x^
3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7
- 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a*c^3*d^4*e
^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
4*d^4*e^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.438 $\int \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=251

$$-\frac{\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5c\right.\right.}{24c}$$

```
[Out] (x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) + (((5*c*d^2 - 3*a*
e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(24*c^2*d^2*e^3) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(5/2)*d^(5/
2)*e^(7/2))
```

Rubi [A] time = 0.259928, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 832, 779, 621, 206}

$$-\frac{\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5c\right.\right.}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) + (((5*c*d^2 - 3*a*
e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(24*c^2*d^2*e^3) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(5/2)*d^(5/
2)*e^(7/2))
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \int \frac{x^2(ae+cdx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e} + \frac{\int \frac{x\left(-2acd^2e-\frac{1}{2}cd\left(5cd^2-ae^2\right)x\right)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3cde}$$

$$= \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e} + \frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2cde\left(5cd^2-a\right.\right.}{24c^2d^2e^3}$$

$$= \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e} + \frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2cde\left(5cd^2-a\right.\right.}{24c^2d^2e^3}$$

$$= \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e} + \frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)-2cde\left(5cd^2-a\right.\right.}{24c^2d^2e^3}$$

Mathematica [A] time = 0.896443, size = 245, normalized size = 0.98

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(-3a^2e^4+2acde^2(ex-2d)+c^2d^2\left(15d^2-10dex+8e^2x^2\right)\right)-\frac{3\sqrt{cd}\sqrt{cd^2-ae^2}\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\cdots}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+\cdots}{cd^2-a\cdots}}}\right.}{24c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4 + 2*a*c
*d*e^2*(-2*d + e*x) + c^2*d^2*(15*d^2 - 10*d*e*x + 8*e^2*x^2)) - (3*Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcSinh[(Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sq
rt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(24*c^(5/2)*d^(5/2
```

```
)*e^(7/2))
```

Maple [B] time = 0.062, size = 713, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

```
[Out] 1/3/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/c-1/4/d/c*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)*x*a-3/4/e^2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)*x-1/8*e/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-1/2/e/c*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-3/8/e^3*d^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/16*e^3/d^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3+1/16*e/c*ln
((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/(d*e*c)^(1/2)*a^2-5/16/e*d^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d
*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+3/16/e
^3*d^4*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+d^2/e^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(1/2)+1/2*d^2/e*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/
2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*a-1/2*d^4/e
^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05408, size = 1126, normalized size = 4.49

$$\left[-\frac{3\left(5c^3d^6-3ac^2d^4e^2-a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
```

```
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 4*a*c
^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/48*(3*(5*c^3*d^6 - 3*a
*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*
e^3*x^2 + 15*c^3*d^5*e - 4*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2
 - a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*
e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{(d+ex)(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.439 $\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=207

$$\frac{(cd^2-ae^2)(ae^2+3cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2cde(d+ex)}-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{x(}$$

```
[Out] -((a/(c*d) + (3*d)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/4 + (a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*c*d*e*(d + e*x)) + ((c*d^2 -
 a*e^2)*(3*c*d^2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(
3/2)*e^(5/2))
```

Rubi [A] time = 0.191597, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {794, 664, 621, 206}

$$\frac{(cd^2-ae^2)(ae^2+3cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2cde(d+ex)}-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{x(}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```


```
[Out] -((a/(c*d) + (3*d)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/4 + (a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*c*d*e*(d + e*x)) + ((c*d^2 -
 a*e^2)*(3*c*d^2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(
3/2)*e^(5/2))
```


Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2cde(d+ex)} + \frac{1}{4}\left(-\frac{3d}{e}-\frac{ae}{cd}\right)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2cde(d+ex)}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2cde(d+ex)}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2cde(d+ex)}$$

Mathematica [A] time = 0.70049, size = 197, normalized size = 0.95

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{cd}\sqrt{cd^2-ae^2}\left(ae^2+3cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}+\sqrt{c}\sqrt{d}\sqrt{e}\left(ae^2+cd(2ex-3d)\right)\right)}{4c^{3/2}d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2 + c*d*(-3*d
+ 2*e*x)) + (Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*(3*c*d^2 + a*e^2)*ArcSinh[(Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sq
rt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(4*c^(3/2)*d^(3/2)
*e^(5/2))
```

Maple [B] time = 0.056, size = 516, normalized size = 2.5

$$\frac{x}{2e}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}+\frac{a}{4cd}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}+\frac{d}{4e^2}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}-\frac{a^2e^2}{8cd}\ln\left(\left(\frac{ae}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/4/d/c*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)*a+1/4/e^2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/
```

```
8*e^2/d/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+1/4*d*ln((1/2*a*e^2+1/2*c*d^2+c*d*
e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a
-1/8/e^2*d^3*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-d/e^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d
^2)*(d/e+x))^(1/2)-1/2*d*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/
2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*a+1/2*d^3/e
^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08766, size = 902, normalized size = 4.36

$$\left[-\frac{\left(3\,c^2d^4-2\,acd^2e^2-a^2e^4\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdex+c\right.\right.}{16\,c^2d^2e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/16*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2
*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*
d*e^3)*x) - 4*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/8*((3*c^2*d^4 - 2*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^
2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*
e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{(d+ex)(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.440 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rubi [A] time = 0.0703544, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {664, 621, 206}

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2e^2}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{}{\sqrt{a}}\right.}{e^2}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e} - \frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

Mathematica [A] time = 0.808852, size = 155, normalized size = 1.18

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{e}-\frac{c^{3/2}d^{3/2}\sqrt{cd^2-ae^2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}\right)}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] - (c^(3/2)*d^(3/2)*Sqrt[c*d^2 - a*e
^2]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d
^2 - a*e^2])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 -
 a*e^2)])))/e^(3/2)
```

Maple [A] time = 0.057, size = 205, normalized size = 1.6

$$\frac{1}{e}\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right)\left(\frac{d}{e}+x\right)}+\frac{ae}{2}\ln\left(\left(\frac{ae^2}{2}-\frac{cd^2}{2}+\left(\frac{d}{e}+x\right)cde\right)\frac{1}{\sqrt{dec}}+\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right)\left(\frac{d}{e}+x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/2*e*ln((1/2*a*e^2-1/2*c
*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
1/2))/(d*e*c)^(1/2)*a-1/2/e*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^
(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94667, size = 725, normalized size = 5.53

$$\left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}cde-(cd^2-ae^2)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{cdex^2+ade+(c\right.}{4cde^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)
*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sq
rt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)))/(c
*d*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(d+ex)(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError
```

3.441 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

```
[Out] (Sqrt[c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (Sqrt[a]*Sqrt
[e]*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[d]
```

Rubi [A] time = 0.166877, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 843, 621, 206, 724}

$$\frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (Sqrt[a]*Sqrt
[e]*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[d]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x(d+ex)}\,dx = \int \frac{ae+cdx}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= (cd)\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx + (ae)\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= (2cd)\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right) - (2ae)\,\text{Subst}$$

$$= \frac{\sqrt{c}\sqrt{d}\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{2ade+\left(cd^2+a\right.}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2\right.}}\right)}{\sqrt{d}}$$

Mathematica [A] time = 0.188225, size = 210, normalized size = 1.25

$$-\frac{2\sqrt{ae+cdx}\left(\sqrt{a}\sqrt{ce}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)-\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[a*e + c*d*x]*(-(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))
/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt
[c*d]*Sqrt[c*d^2 - a*e^2])]) + Sqrt[a]*Sqrt[c]*e*Sqrt[d + e*x]*ArcTanh[(Sqr
t[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(Sqrt[c]*Sqrt[d]
*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.059, size = 439, normalized size = 2.6

$$\frac{1}{d}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}+\frac{ae^2}{2d}\ln\left(\left(\frac{ae^2}{2}+\frac{cd^2}{2}+cdex\right)\frac{1}{\sqrt{dec}}+\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}\right)\frac{1}{\sqrt{dec}}+\frac{cd}{2}\ln\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x)
```

```
[Out] 1/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2/d*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2
)*a*e^2+1/2*d*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*c-a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a
*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-1
/d*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/2/d*ln((1/2*a*e^2-1/2*c*
d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2))/(d*e*c)^(1/2)*a*e^2+1/2*d*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*
c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.20476, size = 2012, normalized size = 11.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/2*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
+ 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 1/2*sqrt(a*e/d)*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*
(a*c*d^3*e + a^2*d*e^3)*x)/x^2), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^
2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 1/2*sqrt(a*e/d)*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*
(a*c*d^3*e + a^2*d*e^3)*x)/x^2), sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d
*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 1/2*sqrt(c*d/e)*log(8*c^
2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*
e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 +
 a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d
)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

3.442 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^2(d+ex)} dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}$$

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*Arc
Tanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.142182, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {849, 806, 724, 206}

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*Arc
Tanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^2(d+ex)}\,dx=\int \frac{ae+cdx}{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}-\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x}}}{2ade}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}+\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{4ade-x^2}\right.}{ade}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{dx}-\frac{\left(cd^2-ae^2\right)\tanh^{-1}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+ca}}\right.}{2\sqrt{a}d^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.144192, size = 117, normalized size = 0.85

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\left(ae^2-cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}}-\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[d]/x) + ((-(c*d^2) + a*e^2)*ArcTanh[
(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt
[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/d^(3/2)
```

Maple [B] time = 0.061, size = 594, normalized size = 4.3

$$-\frac{ae^3}{2d^2}\ln\left(\left(\frac{ae^2}{2}+\frac{cd^2}{2}+cdex\right)\frac{1}{\sqrt{dec}}+\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}\right)\frac{1}{\sqrt{dec}}+\frac{ce}{2}\ln\left(\left(\frac{ae^2}{2}+\frac{cd^2}{2}+cdex\right)\frac{1}{\sqrt{dec}}+\sqrt{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x)
```

```
[Out] -1/2*e^3/d^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+1/2*e*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*
c+1/2*e^2/d*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+e/d^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^
2)*(d/e+x))^(1/2)+1/2*e^3/d^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c
)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*a-1/2*
e*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*
```

```
e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*c-1/d^2/a/e/x*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)+1/a/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c-1/2*d/(
a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/x)*c+1/d*c/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^2), x)
```

Fricas [A] time = 2.56704, size = 755, normalized size = 5.51

$$\left[-\frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,xade + (cd^2 - ae^2)\sqrt{ade}\,x\log\left(\frac{8a^2d^2e^2 + (c^2d^4 + 6acd^2e^2 + a^2e^4)x^2 + 4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,x(2ade + (cd^2 + a\cdots}{x^2}\right.}{4ad^2ex}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e + (c*d^2 - a*e^2
)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^
2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2))/(a*d^2*e*x), -1/2*(2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e - (c*d^2 - a*e^2)*sqrt(-a
*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e
+ a^2*d*e^3)*x)))/(a*d^2*e*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.443 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=202

$$\frac{(cd^2-ae^2)(3ae^2+cd^2)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4x}-\frac{\sqrt{x(ae^2+cd^2}}{2d}$$

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*d*x^2) - ((c/(a*e) - (3*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*x) + ((c*d^2 - a*e^2)*
(c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(5/2)*e
^(3/2))
```

Rubi [A] time = 0.275852, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 834, 806, 724, 206}

$$\frac{(cd^2-ae^2)(3ae^2+cd^2)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4x}-\frac{\sqrt{x(ae^2+cd^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*d*x^2) - ((c/(a*e) - (3*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*x) + ((c*d^2 - a*e^2)*
(c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(5/2)*e
^(3/2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
```

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3(d+ex)}\,dx=\int \frac{ae+cdx}{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2}-\frac{\int \frac{-\frac{1}{2}ae\left(cd^2-3ae^2\right)+acde^2x}{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2ade}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}-$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}+$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x}+$$

Mathematica [A] time = 0.17332, size = 162, normalized size = 0.8

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\left(-3a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}+\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(ae(3ex-2d)-cd^2x\right)}{x^2}\right)}{4a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c*d^2*x) + a*e*
(-2*d + 3*e*x)))/x^2 + ((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(Sqrt
[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]
*Sqrt[d + e*x])))/(4*a^(3/2)*d^(5/2)*e^(3/2))
```

Maple [B] time = 0.063, size = 882, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x)
```

```
[Out] -1/4*e^2/d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*e^4/d^3*ln((1/2*a*
e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(d*e*c)^(1/2)*a-1/2*e^2/d*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*c-3/8*e^3/d^2*a/(a*
d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/x)-e^2/d^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/
2)-1/2*e^4/d^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*
(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*a+1/2*e^2/d*ln((1/2*a
*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(1/2))/(d*e*c)^(1/2)*c-1/2/d^2/a/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+5/4/d^3/a/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/4/d/a^2/
e^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c-1/4*d/a^2/e^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2+1/4*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2
)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c+1/8*d^2/a
/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2-5/4/d^2/a*e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)*x-1/4/a^2/e*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/d/
a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{(ex + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^3), x)
```

Fricas [A] time = 5.46581, size = 945, normalized size = 4.68

$$\left[-\frac{\left(c^2d^4 + 2\,acd^2e^2 - 3\,a^2e^4\right)\sqrt{ade}x^2\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2-4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+8\,\left(acd^3e+a^2de^3\right)x}{x^2}\right.}{16\,a^2d^3e^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/16*((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^
2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e
```

```
+ (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3
*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^2), -1/8*((c^2*d
^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^2*d^2*e^
2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(a^2*d^3*e^2*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.444 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=286

$$-\frac{(cd^2-ae^2)(5a^2e^4+2acd^2e^2+c^2d^4)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}}+\frac{(3cd^2-5ae^2)(3ae^2+cd^2)\sqrt{x(ae^2+cd^2)}}{24a^2d^3e^2x}$$

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*d*x^3) - ((c/(a*e) - (5*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*x^2) + ((3*c*d^2 - 5*
a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*a
^2*d^3*e^2*x) - ((c*d^2 - a*e^2)*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcT
anh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^(5/2)*d^(7/2)*e^(5/2))
```

Rubi [A] time = 0.404314, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 834, 806, 724, 206}

$$-\frac{(cd^2-ae^2)(5a^2e^4+2acd^2e^2+c^2d^4)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}}+\frac{(3cd^2-5ae^2)(3ae^2+cd^2)\sqrt{x(ae^2+cd^2)}}{24a^2d^3e^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*d*x^3) - ((c/(a*e) - (5*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*x^2) + ((3*c*d^2 - 5*
a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*a
^2*d^3*e^2*x) - ((c*d^2 - a*e^2)*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcT
anh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4(d+ex)}\,dx=\int \frac{ae+cdx}{x^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3}-\frac{\int \frac{-\frac{1}{2}ae\left(cd^2-5ae^2\right)+2acde^2x}{x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3ade}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12x^2}+\frac{\int}{}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12x^2}+\frac{(3}{}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12x^2}+\frac{(3}{}$$

$$=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12x^2}+\frac{(3}{}$$

Mathematica [A] time = 0.256527, size = 210, normalized size = 0.73

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^2\left(-8d^2+10dex-15e^2x^2\right)-2acd^2ex(d-2ex)+3c^2d^4x^2\right)}{x^3}-\frac{3\left(3a^2cd^2e^4-5a^3e^6+ac^2d^4e^2+c^3d^6\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}\right)}{24a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 - 2
*a*c*d^2*e*x*(d - 2*e*x) + a^2*e^2*(-8*d^2 + 10*d*e*x - 15*e^2*x^2)))/x^3 -
```

```
(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(Sqrt[d
]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*S
qrt[d + e*x])))/(24*a^(5/2)*d^(7/2)*e^(5/2))
```

Maple [B] time = 0.069, size = 1165, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x)
```

```
[Out] 1/4/d/a^2/e^2/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c-1/2/d^2/a^2/e/x
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c-1/16*d^3/a^2/e^2/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/x)*c^3+11/8/d^3/a*e^2*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+
1/8*d/a^3/e^2*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/2*e^5/d^4*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(d*e*c)^(1/2)*a+1/2*e^3/d^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e
*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*c+5/16*e^4
/d^3*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+e^3/d^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2)+3/8*e^3/d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*e^5/d^
4*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*
e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*a-1/2*e^3/d^2*ln((1/2*a*e^2-1/2*c*
d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2))/(d*e*c)^(1/2)*c-1/8/a^3/e^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*
c^2+9/8/d^2/a*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c+1/8*d^2/a^3/e^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3-3/16/d*e^2/(a*d*e)^(1/2)*ln((2*a
*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/x)*c-1/16*d/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2+1/2/d/a^2*c^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/3/d^2/a/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)-11/8/d^4/a*e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/4/d^3/a/
x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/8/a^2/e*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{(ex + d)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^4), x)
```

Fricas [A] time = 15.7016, size = 1169, normalized size = 4.09

$$\left[-\frac{3\left(c^3d^6+ac^2d^4e^2+3a^2cd^2e^4-5a^3e^6\right)\sqrt{ade}x^3\log\left(\frac{8a^2d^2e^2+\left(c^2d^4+6acd^2e^2+a^2e^4\right)x^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{}}{x^2}\right.}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(a*d*
e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 - (3*a*c^2*d
^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e
^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3), 1/48
*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-a*d*e)*x^
3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*
d*e^3)*x)) - 2*(8*a^3*d^3*e^3 - (3*a*c^2*d^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d
*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.445 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=389

$$-\frac{(25a^2cd^2e^4-105a^3e^6+17ac^2d^4e^2+15c^3d^6)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{192a^3d^4e^3x}+\frac{(-35a^2e^4+6acd^2e^2+5c^2d^4)\sqrt{x(ae^2+c}}{96a^2d^3e^2x^2}$$

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*d*x^4) - ((c/(a*e) - (7*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*x^3) + ((5*c^2*d^4 +
6*a*c*d^2*e^2 - 35*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(9
6*a^2*d^3*e^2*x^2) - ((15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*e^4 - 1
05*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192*a^3*d^4*e^3*x
) + ((c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 35*a
^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(7/2)*d^(9/2)*e^(7/2))
```

Rubi [A] time = 0.593513, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 834, 806, 724, 206}

$$-\frac{(25a^2cd^2e^4-105a^3e^6+17ac^2d^4e^2+15c^3d^6)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{192a^3d^4e^3x}+\frac{(-35a^2e^4+6acd^2e^2+5c^2d^4)\sqrt{x(ae^2+c}}{96a^2d^3e^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```


```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*d*x^4) - ((c/(a*e) - (7*e)/
d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*x^3) + ((5*c^2*d^4 +
6*a*c*d^2*e^2 - 35*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(9
6*a^2*d^3*e^2*x^2) - ((15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*e^4 - 1
05*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192*a^3*d^4*e^3*x
) + ((c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 35*a
^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(7/2)*d^(9/2)*e^(7/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
```

```
IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5(d+ex)}\,dx = \int \frac{ae+cdx}{x^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\int \frac{-\frac{1}{2}ae\left(cd^2-7ae^2\right)+3acde^2x}{x^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4ade}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3} + \frac{(5 \cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3} + \frac{(5 \cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3} + \frac{(5 \cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24x^3} + \frac{(5 \cdots}{\cdots}$$

Mathematica [A] time = 0.377932, size = 273, normalized size = 0.7

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2cd^2e^2x\left(-8d^2+12dex-25e^2x^2\right)+a^3e^3\left(56d^2ex-48d^3-70de^2x^2+105e^3x^3\right)+ac^2d^4ex^2(10d-17ex)-15c^3d^6x^3\right)}{x^4} + \frac{3\left(6a^2c^2d \cdots\right.}{\cdots}\right.}{192a^{7/2}d^{9/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-15*c^3*d^6*x^3 +
 a*c^2*d^4*e*x^2*(10*d - 17*e*x) + a^2*c*d^2*e^2*x*(-8*d^2 + 12*d*e*x - 25*
e^2*x^2) + a^3*e^3*(-48*d^3 + 56*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))/x^
4 + (3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6
- 35*a^4*e^8)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d +
 e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*a^(7/2)*d^(9/2)*e^(7/2))
```

Maple [B] time = 0.073, size = 1494, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x)
```

```
[Out] 13/24/d^3/a/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-17/32/d/a^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2-1/2*e^4/d^3*ln((1/2*a*e^2+1/2*c*d^2+c*
d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
*c-35/128*e^5/d^4*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+93/64*e^2/d^5/a/x*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+43/64/d^3/a^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)*c-39/32*e^2/d^3/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c-29/3
2*e/d^4/a/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-7/32/e^2*d/a^3*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3+5/32*e^3/d^2/(a*d*e)^(1/2)*ln((2*a*d*
e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
x)*c+3/64*e/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2-19/64/e/a^3*c^3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)*x-e^4/d^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))
^(1/2)-29/64*e^4/d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*e^6/d^5*ln
((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)*a+1/2*e^4/d^3*ln((1/2*a*e^2-1/2*c*d^2+
(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))
/(d*e*c)^(1/2)*c-1/4/d^2/a/e/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-5/
64*d^3/a^4/e^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^4-5/32/a^3/e^3/x^2
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+1/2*e^6/d^5*ln((1/2*a*e^2+1/2*
c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*
c)^(1/2)*a-7/16/e/d^2/a^2/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c+19/
64/e^2/d/a^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+1/32/e*d^2/a^2/(
a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/x)*c^3-93/64*e^3/d^4/a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)*x-43/64*e/d^2/a^2*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
x+5/24/d/a^2/e^2/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c-5/64*d^2/a^4
/e^3*c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+5/64*d/a^4/e^4/x*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^3+5/128*d^4/a^3/e^3/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/x)*c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{(ex + d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^5), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.446 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=449

$$-\frac{\left(-6cdex\left(-7a^2e^4-6acd^2e^2+21c^2d^4\right)-33a^2cd^2e^4-35a^3e^6-21ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960c^3d^3e^4}+$$

```
[Out] ((21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a^3*c*d^2*e^6 - 7*a^4*e^8)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*c^4*d^4*
e^5) + ((a/(c*d) - (3*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/20 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - ((10
5*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e*(21*
c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(960*c^3*d^3*e^4) - ((c*d^2 - a*e^2)^3*(21*c^3*d^6 + 21*a*c^2*d
^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
1024*c^(9/2)*d^(9/2)*e^(11/2))
```

Rubi [A] time = 0.569861, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 832, 779, 612, 621, 206}

$$-\frac{\left(-6cdex\left(-7a^2e^4-6acd^2e^2+21c^2d^4\right)-33a^2cd^2e^4-35a^3e^6-21ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960c^3d^3e^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```


```
[Out] ((21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a^3*c*d^2*e^6 - 7*a^4*e^8)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*c^4*d^4*
e^5) + ((a/(c*d) - (3*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/20 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - ((10
5*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e*(21*
c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(960*c^3*d^3*e^4) - ((c*d^2 - a*e^2)^3*(21*c^3*d^6 + 21*a*c^2*d
^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
1024*c^(9/2)*d^(9/2)*e^(11/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
```

```
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int x^3(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$$

$$=\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e}+\frac{\int x^2\left(-3acd^2e-\frac{3}{2}cd\left(3cd^2-ae^2\right)x\right)\sqrt{a}}{6cde}$$

$$=\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+\right.}{6e}$$

$$=\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+\right.}{6e}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+a\right.}}{512c^4d^4e^5}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+a\right.}}{512c^4d^4e^5}$$

$$=\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+a\right.}}{512c^4d^4e^5}$$

Mathematica [A] time = 2.60505, size = 425, normalized size = 0.95

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(2a^3c^2d^2e^6\left(27d^2-16dex-28e^2x^2\right)+6a^2c^3d^3e^4\left(-6d^2ex+13d^3+4de^2x^2+8e^3x^3\right)+5a^4c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10 + 5*
a^4*c*d*e^8*(11*d + 14*e*x) + 2*a^3*c^2*d^2*e^6*(27*d^2 - 16*d*e*x - 28*e^2
*x^2) + 6*a^2*c^3*d^3*e^4*(13*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 + 8*e^3*x^3) +
a*c^4*d^4*e^2*(-525*d^4 + 336*d^3*e*x - 264*d^2*e^2*x^2 + 224*d*e^3*x^3 + 1
664*e^4*x^4) + c^5*d^5*(315*d^5 - 210*d^4*e*x + 168*d^3*e^2*x^2 - 144*d^2*e
^3*x^3 + 128*d*e^4*x^4 + 1280*e^5*x^5)) - (15*Sqrt[c*d]*(c*d^2 - a*e^2)^(5/
2)*(21*c^3*d^6 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcSinh[(
Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]
)/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(7680*c^(9/2)
*d^(9/2)*e^(11/2))
```

Maple [B] time = 0.065, size = 1883, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

```
[Out] -7/512*e^5/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+65/192/e^2*d
/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-1/3*d^3/e^4*(c*d*e*(d/e+x)^2+(
a*e^2-c*d^2)*(d/e+x))^(3/2)+43/192/e^4*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)-19/60/e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-3/16*d^4/e*a^2*
ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^
2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/4*d^5/e^4*c*(c*d*e*(d/e+x)^2+(a*e^
2-c*d^2)*(d/e+x))^(1/2)*x-1/16*d^8/e^5*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*
c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)
^(1/2)-43/256/e^4*d^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3/1024*e^
3/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4+177/1024/e*d^4*ln((1/2*a*e^2+1/2*c*d^2
+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1
/2)*a^2+43/1024/e^5*d^8*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+7/192*e^2/d^3/c^3*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-15/512*e^3/d^2/c^3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+29/256/e*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)*a^2+1/4/e^2*a*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+11/48
/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a+1/6/e^2*x*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(5/2)/d/c-1/4*d^3/e^2*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2)*x-1/8*d^2/e*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2
)-3/128*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-7/60/e/d^2/c^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a+1/8*d^6/e^5*c*(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2)+43/96/e^3*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)*x+21/512/e^3*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-7/256*e/c^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-43/512/e^5*d^6*c*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+29/192/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)*a^2+7/1024*e^7/d^4/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^6+7/96*e/d^2/c^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^2-3/512*e^5/d^2/c^3*ln((1/2*a*e^2
+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
(d*e*c)^(1/2)*a^5-1/32*e^2*a^3/c^2/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)*x-17/256*e*d^2/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3-7/256*e^4/d^3/c^3*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-75/512/e^3*d^6*c*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c
)^(1/2)*a+1/16*d^2*e*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(
1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/16*d^6/
e^3*a*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)
^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33069, size = 2233, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 4*a^3
*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e -
525*a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^
3*e^9 - 105*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 - 16*(9
*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^6*d^9*e^
3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2 - 2*(105*
c^6*d^10*e^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^3*d^4*e^8
- 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5
*d^5*e^6), 1/15360*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^
4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*
sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 2*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11*e - 525*a*c^5*
d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^3*e^9 - 10
5*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4 - 16*(9*c^6*d^8*e
^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^6*d^9*e^3 - 33*a*c
^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2 - 2*(105*c^6*d^10*e
^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^3*d^4*e^8 - 35*a^4*c
^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.447 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=352

$$-\frac{\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^3d^3e^4}+\frac{\left(-15a^2e^4-6cdex\left(7cd^2-\right.\right.}{}$$

```
[Out] -((c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(c*d^2 + a*e^2 +
2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*c^3*d^3*e^4) +
 (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) + ((35*c^2*d^4 -
 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(240*c^2*d^2*e^3) + ((c*d^2 - a*e^2)^3*(7
*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(
2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
56*c^(7/2)*d^(7/2)*e^(9/2))
```

Rubi [A] time = 0.328369, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {851, 832, 779, 612, 621, 206}

$$-\frac{\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^3d^3e^4}+\frac{\left(-15a^2e^4-6cdex\left(7cd^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -((c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(c*d^2 + a*e^2 +
2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*c^3*d^3*e^4) +
 (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) + ((35*c^2*d^4 -
 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(240*c^2*d^2*e^3) + ((c*d^2 - a*e^2)^3*(7
*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(
2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
56*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int x^2(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}+\frac{\int x\left(-2acd^2e-\frac{1}{2}cd\left(7cd^2-3ae^2\right)x\right)\sqrt{\ldots}}{5cde}$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}+\frac{\left(35c^2d^4-12acd^2e^2-15a^2e^4-6cde\left(7c\ldots\right.\right.}{24\ldots}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae\ldots\right.}}{128c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae\ldots\right.}}{128c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae\ldots\right.}}{128c^3d^3e^4}$$

Mathematica [A] time = 3.00127, size = 497, normalized size = 1.41

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\frac{5\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(8c^3d^3e^3\sqrt{cd}\sqrt{cd^2-ae^2}(ae+cdx)^3\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}-cd\left(cd^2-ae^2\right)\left(-3c^{5/2}d^{5/2}\sqrt{e}\left(cd^2-ae^2\right)^2\sqrt{ae+cdx}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-2e\cdots\right.\right.}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}}{384c^5d^5e^4(ae+cdx\cdots}\right.}{5cde}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(x*(a*e + c*d*x)^2*(d + e*x) + (-48*c^4*d^4*
e^3*(7*c*d^2 + 5*a*e^2)*(a*e + c*d*x)^3*(d + e*x) + (5*(7*c^2*d^4 + 6*a*c*d
^2*e^2 + 3*a^2*e^4)*(8*c^3*d^3*Sqrt[c*d]*e^3*Sqrt[c*d^2 - a*e^2]*(a*e + c*d
*x)^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - c*d*(c*d^2 - a*e^2)*(3*(c*d)^
(5/2)*e*(c*d^2 - a*e^2)^(3/2)*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a
*e^2)] - 2*(c*d)^(5/2)*e^2*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)^2*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)] - 3*c^(5/2)*d^(5/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*Sqr
t[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2])])))/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e
*x))/(c*d^2 - a*e^2)]))/(384*c^5*d^5*e^4*(a*e + c*d*x))))/(5*c*d*e)
```

Maple [B] time = 0.061, size = 1560, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

```
[Out] 1/8*d*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+3/16*d^3*a^2*ln((
1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1/8*d^5/e^4*c*(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2)-1/4/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a+9/128
/e^4*d^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/32*d/c*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)*a^2-21/128*d^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d
*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-3/64
/e^2*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+1/5/e^2*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(5/2)/d/c-3/8/e^2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)*x+1/4*d^2/e*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/4*d^4/e^
3*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/8/d/c*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(3/2)*x*a+3/64*e^2/d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)*a^3-9/256/e^4*d^7*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/16*e/d^2/c^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+9/64/e^3*d^4*c*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)*x+3/64*e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*
a^2-15/64/e*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+1/16*d^7/e^4*c^
2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*
e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/128*e^4/d^3/c^3*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*a^4+1/3*d^2/e^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e
+x))^(3/2)-3/16/e^3*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+33/256/e^2*
d^5*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+9/128*e^2*d/c*ln((1/2*a*e^2+1/2*c*d^2+c*
d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
```

```
*a^3+3/64*e^3/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3-3/256*e
^6/d^3/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5+3/256*e^4/d/c^2*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e
*c)^(1/2)*a^4-1/16*d*e^2*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*
c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/16*
d^5/e^2*a*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92423, size = 1797, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2
*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2
+ c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3
)*x) - 4*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*
c^3*d^5*e^5 - 30*a^3*c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*
c^4*d^4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)
*x^2 + 2*(35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^
2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5), -
1/3840*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d
^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c
^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(384*c^5*d^5
*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*c^3*d^5*e^5 - 30*a^3*
c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*c^4*d^4*e^6)*x^3 - 8*
(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*x^2 + 2*(35*c^5*d^8*
e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^2*d^2*e^8)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.448 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=295

$$\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^2d^2e^3}-\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{}{2\sqrt{c}\sqrt{d}\sqrt{}}\right.}{128c^{5/2}d^{5/2}e^{7/2}}$$

```
[Out] ((c*d^2 - a*e^2)*(5*c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c^2*d^2*e^3) - (((3*a)/(c*d) + (5*d)
/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/24 + (a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2)/(4*c*d*e*(d + e*x)) - ((c*d^2 - a*e^2)^3*(5*c*
d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(5/2)*d^(5/2)*e^(7
/2))
```

Rubi [A] time = 0.282479, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {794, 664, 612, 621, 206}

$$\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^2d^2e^3}-\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{}{2\sqrt{c}\sqrt{d}\sqrt{}}\right.}{128c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)*(5*c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c^2*d^2*e^3) - (((3*a)/(c*d) + (5*d)
/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/24 + (a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2)/(4*c*d*e*(d + e*x)) - ((c*d^2 - a*e^2)^3*(5*c*
d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(5/2)*d^(5/2)*e^(7
/2))
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4cde(d+ex)} + \frac{1}{8}\left(-\frac{5d}{e}-\frac{3ae}{cd}\right)\int \frac{\left(ade+\left(cd^2+ae^2\right)x\right.}{d+ex}$$

$$= -\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+c\right.}{4cde(d+ex)}$$

$$= \frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2e^3}$$

$$= \frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2e^3}$$

$$= \frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2e^3}$$

Mathematica [A] time = 1.30039, size = 276, normalized size = 0.94

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(3a^2cde^4(3d+2ex)-9a^3e^6+ac^2d^2e^2\left(-31d^2+20dex+72e^2x^2\right)+c^3d^3\left(-10d^2ex+15d\right.\right.\right.}{192c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-9*a^3*e^6 + 3*a^2
*c*d*e^4*(3*d + 2*e*x) + a*c^2*d^2*e^2*(-31*d^2 + 20*d*e*x + 72*e^2*x^2) +
c^3*d^3*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - (3*Sqrt[c*d]*(c
*d^2 - a*e^2)^(5/2)*(5*c*d^2 + 3*a*e^2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sq
rt[a*e + c*d*x]])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[
(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(192*c^(5/2)*d^(5/2)*e^(7/2))
```

Maple [B] time = 0.064, size = 1279, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

```
[Out] 1/4/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/8/d/c*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)*a+1/8/e^2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-3/
32*e^2/d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+3/16*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a-3/32/e^2*d^3*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)*x-3/64*e^3/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3
+3/64*e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+3/64/e*d^2*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-3/64/e^3*d^4*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)+3/128*e^5/d^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4-3/32*e^3/c*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(d*e*c)^(1/2)*a^3+9/64*e*d^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*
e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-3/32/
e*d^4*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+3/128/e^3*d^6*c^2*ln((1/2*a*e^2+1/2*c*
d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)
^(1/2)-1/3*d/e^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-1/4*d*a*(c*d
*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/8*e*a^2/c*(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2)+1/16*e^3*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c
*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^
(1/2)-3/16*d^2*e*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(
c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/16*d^4/e*a*c*
ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^
2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/4*d^3/e^2*c*(c*d*e*(d/e+x)^2+(a*e^
2-c*d^2)*(d/e+x))^(1/2)*x+1/8*d^4/e^3*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e
+x))^(1/2)-1/16*d^6/e^3*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^
(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85972, size = 1426, normalized size = 4.83

$$\left[-\frac{3\left(5c^4d^8-12ac^3d^6e^2+6a^2c^2d^4e^4+4a^3cd^2e^6-3a^4e^8\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{cdex^2+ad\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*
e^6 - 3*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*
d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4*e^4
*x^3 + 15*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7
+ 8*(c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d^4*e
^4 - 3*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^
3*d^3*e^4), 1/384*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e^4*x^3 + 15
*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7 + 8*(c^4*
d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d^4*e^4 - 3*a^
2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.449 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

<u>**Optimal**</u>. Leaf size=201

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cdex\right)$$

```
[Out] ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/8 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) +
((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(3/2)*d^(3/2)*e
^(5/2))
```

<u>**Rubi**</u> **[A]** time = 0.117427, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {664, 612, 621, 206}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cdex\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/8 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) +
((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(3/2)*d^(3/2)*e
^(5/2))
```

<u>Rule 664</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

<u>Rule 612</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

<u>Rule 621</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int\sqrt{ade+\left(cd^2+a\right.}}{2e^2}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2\right.\right.}{}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2\right.\right.}{}$$

$$= \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2} + \frac{\left(ade+\left(cd^2\right.\right.}{}$$

Mathematica [A] time = 0.707945, size = 264, normalized size = 1.31

$$\frac{\sqrt{c}\sqrt{d}\left(3\left(cd^2-ae^2\right)^{7/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(-a^2cde^3(8d+17ex)-3a^3e\right.\right.}{24e^{5/2}(cd)^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-3*a^3*e^5
 - a^2*c*d*e^3*(8*d + 17*e*x) + a*c^2*d^2*e*(3*d^2 - 10*d*e*x - 22*e^2*x^2)
 + c^3*d^3*x*(3*d^2 - 2*d*e*x - 8*e^2*x^2))) + 3*(c*d^2 - a*e^2)^(7/2)*Sqrt
[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d
]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2)])]))/(24*(c*d)^(
5/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.054, size = 566, normalized size = 2.8

$$\frac{1}{3e}\left(cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right)\left(\frac{d}{e}+x\right)\right)^{\frac{3}{2}}+\frac{aex}{4}\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right)\left(\frac{d}{e}+x\right)}+\frac{a^2e^2}{8cd}\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+1/4*e*a*(c*d*e*(d/e+x)^
2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/8*e^2*a^2/d/c*(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2)-1/16*e^4*a^3/d/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)
/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
+3/16*e^2*a^2*d*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e
```

```
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/16*a*d^3*c*ln((1/2
*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2
)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1/4/e*c*d^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(1/2)*x-1/8/e^2*c*d^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2
)+1/16/e^2*c^2*d^5*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*
d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86472, size = 1125, normalized size = 5.6

$$\left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\right(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^
2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 - 3*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(
c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8*c^3*d^3*
e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 +
7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e
^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError
```

3.450 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=251

$$-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}e^{3/2}}-a^{3/2}\sqrt{d}e^{3/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+\dots}}\right.$$

```
[Out] ((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(4*e) - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(8*Sqrt[c]*Sqrt[d]*e^(3/2)) - a^(3/2)*Sqrt[d]*e^(3/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rubi [A] time = 0.276345, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 814, 843, 621, 206, 724}

$$-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}e^{3/2}}-a^{3/2}\sqrt{d}e^{3/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+\dots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] ((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(4*e) - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(8*Sqrt[c]*Sqrt[d]*e^(3/2)) - a^(3/2)*Sqrt[d]*e^(3/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}\,dx$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\frac{\int \frac{-4a^2cd^2e^3+\frac{1}{2}cd\left(c^2d^4-6acd\right.}{x\sqrt{ade+\left(cd^2+ae^2\right)x}}}{4cde}$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}+\left(a^2de^2\right)\int \frac{}{x\sqrt{ade+\left(c\right.}}$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\left(2a^2de^2\right)\mathrm{Subst}\left(\int \frac{}{4a}\right.$$

$$=\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\frac{\left(c^2d^4-6acd^2e^2-3a^2e\right.}{}$$

Mathematica [A] time = 0.889789, size = 275, normalized size = 1.1

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(-\frac{\sqrt{c}\sqrt{d}\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\frac{8a^{3/2}\sqrt{d}e^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}+\sqrt{e}\sqrt{ae+cdx}\left(5ae^2+cd(d\right.\right.}{4e^{3/2}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(5*a*e^2 + c*d*(d
 + 2*e*x)) - (Sqrt[c]*Sqrt[d]*(c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcSinh
[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]
)])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) -
 (8*a^(3/2)*Sqrt[d]*e^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/Sqrt[d + e*x]))/(4*e^(3/2)*Sqrt[a*e + c*d*x])
```

Maple [B] time = 0.065, size = 1130, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x)
```

```
[Out] 1/3/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/4/d*a*e^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)*x+1/8/d^2*a^2*e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+5/4*a*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/16/d^2*a^3*e^5/c*
ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/(d*e*c)^(1/2)+9/16*a^2*e^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d
*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+9/16*d^2
*a*e*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+1/4*d*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)*x+1/8*d^2*c/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/16*d^4*c^2/e
*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2))/(d*e*c)^(1/2)-d*a^2*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-1/3/d*(c*d
*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-1/4/d*a*e^2*(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2)*x-1/8/d^2*a^2*e^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)
*(d/e+x))^(1/2)+1/16/d^2*a^3*e^5/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(
d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3
/16*a^2*e^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/16*d^2*a*e*c*ln((1/2*a
*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/4*d*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x
))^(1/2)*x+1/8*d^2*c/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/16*d
^4*c^2/e*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x
)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 26.2505, size = 2855, normalized size = 11.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/16*(8*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - (c^2*
d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4
*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c*d*e^2), 1/8*(4*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 +
 (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^
2*d*e^3)*x)/x^2) + (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^
2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)
) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c*d*e^2), 1/16*(16*sqrt(-a*d*e)*a*c*d*e^3*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - (
c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(c*d*e^2), 1/8*(8*sqrt(-a*d*e)*a*c*d*e^3*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + (c^2
*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^
2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x
+ c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*
d*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

3.451 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=240

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(ae-cdx)}{x}+\frac{\sqrt{c}\sqrt{d}\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\left(ae^2+3\right.}{}$$

```
[Out] -(((a*e - c*d*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (Sqrt[c]
*Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*S
qrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[e]) -
 (Sqrt[a]*Sqrt[e]*(3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(
2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
*Sqrt[d])
```

Rubi [A] time = 0.273474, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(ae-cdx)}{x}+\frac{\sqrt{c}\sqrt{d}\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\left(ae^2+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] -(((a*e - c*d*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (Sqrt[c]
*Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*S
qrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[e]) -
 (Sqrt[a]*Sqrt[e]*(3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(
2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
*Sqrt[d])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^2}\,dx$$

$$=-\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}-\frac{1}{2}\int \frac{-ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3\cdots\right.}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cd\cdots}}$$

$$=-\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}+\frac{1}{2}\left(ae\left(3cd^2+ae^2\right)\right)\int \frac{}{x\sqrt{ade+\left(cd\cdots\right.}}$$

$$=-\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}-\left(ae\left(3cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4ade-\cdots}\right.$$

$$=-\frac{(ae-cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x}+\frac{\sqrt{c}\sqrt{d}\left(cd^2+3ae^2\right)\tanh^{-1}\left(\frac{\cdots}{2\sqrt{c}\sqrt{d}\sqrt{\cdots}}\right.}{2\sqrt{e}}$$

Mathematica [A] time = 1.37625, size = 263, normalized size = 1.1

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{cd}\sqrt{cd}\left(3ae^2+cd^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\frac{\sqrt{a}\sqrt{e}\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}}+\frac{\sqrt{d}\sqrt{ae+cdx}(cdx-ae)}{x}\right)}{\sqrt{d}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[d]*(-(a*e) + c*d*x)*Sqrt[a*e + c*d*x]
)/x + (Sqrt[c]*d*Sqrt[c*d]*(c*d^2 + 3*a*e^2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[e]*Sqrt[c*d^2
 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (Sqrt[a]*Sqrt[e]*(3*c*d^
2 + a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*
x])])/Sqrt[d + e*x]))/(Sqrt[d]*Sqrt[a*e + c*d*x])
```

Maple [B] time = 0.068, size = 1310, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x)
```

```
[Out] 1/a/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c+1/d*c/a*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)*x-1/d^2/a/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-
1/8*e^4/d^3*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/16*e^4/d*a^2*ln
((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/(d*e*c)^(1/2)+27/16*e^2*d*a*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(
d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/4*e^3
/d^2*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/4*e*c*(c*d*e*(d/e+x)^2+(
a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/16*d^3*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*
c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)
^(1/2)+1/4*e^3/d^2*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/8*e^
4/d^3*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+3/16*e^4/d*a^2*ln
((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+e^2/d*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+5/4*e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+7/16*d^3*c^2*ln
((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/(d*e*c)^(1/2)-1/2*e^3*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2
)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+2/3*e/d^2*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+17/8*d*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)+1/3*e/d^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-1/8*d*
c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/16*e^6/d^3*a^3/c*ln((1/2*
a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)
*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/16*e^2*d*a*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+
x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e
*c)^(1/2)+1/16*e^6/d^3*a^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-3/2*d^2*a*e/(a*d*e)
^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/x)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^2), x)
```

Fricas [A] time = 10.4746, size = 2589, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/4*((c*d^2 + 3*a*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c
*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e +
(c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/4*(2*(c*d^2
+ 3*a*e^2)*sqrt(-c*d/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e +
 (c^2*d^3 + a*c*d*e^2)*x)) - (3*c*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2
*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(c*d*x - a*e))/x, 1/4*(2*(3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arctan(1/2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a
*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + (c*d^2 + 3*a
*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/2*((c*d^2 + 3*a*e^2)*sqrt(-c*d
/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^
2)*x)) - (3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2
*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.452 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=256

$$-\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{a}d^{3/2}\sqrt{e}}+c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade-}}\right.$$

```
[Out] -((2*a*d*e + (5*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*d*x^2) + c^(3/2)*d^(3/2)*Sqrt[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])] -
 ((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*
x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(8*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.282725, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 810, 843, 621, 206, 724}

$$-\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{a}d^{3/2}\sqrt{e}}+c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade-}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] -((2*a*d*e + (5*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*d*x^2) + c^(3/2)*d^(3/2)*Sqrt[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])] -
 ((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*
x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(8*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
```

```
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3}\,dx$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}-\frac{\int \frac{-\frac{1}{2}ae\left(3c^2d^4+6acd^2e^2\right.}{x\sqrt{ade+\left(cd^2+\right.}}}{4ac}$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+\left(c^2d^2e\right)\int \frac{}{\sqrt{ade+}}$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+\left(2c^2d^2e\right)\mathrm{Subst}\left(\int\right.$$

$$=-\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2}+c^{3/2}d^{3/2}\sqrt{e}\tanh^{-1}$$

Mathematica [A] time = 2.55466, size = 285, normalized size = 1.11

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d+ex}\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}+\frac{8e(cd)^{5/2}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{c^{3/2}}-\frac{\sqrt{d}\sqrt{e}(d+ex)\sqrt{ae+cdx}\left(ae(2d\right.}{x^2}\right.}{4d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(5*c*d^2
*x + a*e*(2*d + e*x)))/x^2) + (8*(c*d)^(5/2)*e*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*
d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c
*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/c^(3/2) - ((3*c^2*d^4 + 6*a*c*d^2*
e^2 - a^2*e^4)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*S
qrt[e]*Sqrt[d + e*x])])/Sqrt[a]))/(4*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

Maple [B] time = 0.073, size = 1604, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x)
```

```
[Out] -1/16*e^7/d^4*a^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/4/d/a^2/e^2/x*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(5/2)*c-3/4/d^2/a*e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)*x-3/4*d*a*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c+1/16*e^7/d^4*a^3/c*ln((
1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/8*e^4/d*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)
-1/2/d^2/a/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/4*d^2/a/e*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2+1/4/a^2/e*c^2*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(3/2)*x-1/4*e^4/d^3*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2)*x-1/8*e^5/d^4*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-3/16*
e^5/d^2*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/16*e^3*a*c*ln((1/2*a*e
^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2))/(d*e*c)^(1/2)+1/4*e^2/d*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e
+x))^(1/2)*x-1/16*e*d^2*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^
(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/4*e^4/
d^3*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/8*e^5/d^4*a^2/c*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/16*e^5/d^2*a^2*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-
3/16*e^3*a*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/2*e^2/d*c*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)*x+17/16*e*d^2*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)
^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+1/4*d/a^2/e^2
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+3/4*d/a*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)*x*c^2-1/3*e^2/d^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))
^(3/2)+1/8*e*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-5/12*e^2/d^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+7/8*e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
```

```
*x^2)^(1/2)-1/4*e^3/d^2*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/4/d^3/a
/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-3/8*d^3/(a*d*e)^(1/2)*ln((2*a*d*
e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
x)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^3), x)
```

Fricas [A] time = 13.7635, size = 2934, normalized size = 11.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(8*sqrt(c*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*
x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - (3*c^2*d^4
+ 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 +
6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1/16*(16*sqrt(-c*d*e)*a*c*d^3*e*x^
2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*
e^3)*x)) + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2
*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), 1/8*(4*sqrt(c
*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c^2*d^4 + 6*a*c*d^2*e^2
 - a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*
d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a
^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1
/8*(8*sqrt(-c*d*e)*a*c*d^3*e*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*
c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*
e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2
 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^
```

```
3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"giac")

[Out] Exception raised: TypeError
```

3.453 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=211

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+2ad\right.}{8}$$

```
[Out] -((c/(a*e) - e/d^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(8*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
/(3*d*x^3) + ((c*d^2 - a*e^2)^3*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sq
rt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^
(3/2)*d^(5/2)*e^(3/2))
```

Rubi [A] time = 0.236498, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 806, 720, 724, 206}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+2ad\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] -((c/(a*e) - e/d^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(8*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
/(3*d*x^3) + ((c*d^2 - a*e^2)^3*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sq
rt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^
(3/2)*d^(5/2)*e^(3/2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
```

```
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx = \int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3dx^3} - \frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x}}{x^3}}{2ade}$$

$$= -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2} - \frac{\left(ade+\left(cd^2+\right.\right.}{}$$

$$= -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2} - \frac{\left(ade+\left(cd^2+\right.\right.}{}$$

$$= -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2} - \frac{\left(ade+\left(cd^2+\right.\right.}{}$$

Mathematica [A] time = 0.283088, size = 188, normalized size = 0.89

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{ae+cdx}}-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^2\left(8d^2+2dex-3e^2x^2\right)+2acd^2ex(7d+4ex)+3c^2d^4x^2\right)}{x^3}\right)}{24a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 +
 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*x - 3*e^2*x^2)))/x^3)
 + (3*(c*d^2 - a*e^2)^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*a^(3/2)*d^(5/2)*
e^(3/2))
```

Maple [B] time = 0.077, size = 1945, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x)
```

```
[Out] -1/8*d^3/a^2/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3+11/24/d^2/a*e*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c+3/16*a*e^3/(a*d*e)^(1/2)*ln((2*a*
d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/x)*c+1/24/a^3/e^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^2-1/3/d^2/a
/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/3/d/a^2*c^2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)*x-3/16*d^2*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d
^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2-1/16*
e^2*d*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/16*e^5/d^2*a^2/(a*d*e)^(1/2)*ln((2*a
*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/x)-1/4*e^5/d^4*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/8*e^6/d^5*a
^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/3*e^3/d^4*(c*d*e*(d/e+x)^2+(
a*e^2-c*d^2)*(d/e+x))^(3/2)+3/8*e^3/d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)-3/16*e^6/d^3*a^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3/8*e^3/d^2*c*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/4*e^5/d^4*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(1/2)*x+1/8*e^6/d^5*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^
(1/2)+3/16*e^6/d^3*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)
+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1/4*e^3/d^2*c
*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/16*e^2*d*c^2*ln((1/2*a*e
^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2))/(d*e*c)^(1/2)-17/24/d^4/a*e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(5/2)-1/24*d^2/a^3/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^3+5/24/
a^2/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+1/8*d/a*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)*c^2+7/12/d^3/a/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(5/2)-1/8*e^2/d*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/8*e^4/d^
3*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/16*e^8/d^5*a^3/c*ln((1/2*a*e^
2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(d*e*c)^(1/2)+3/16*e^4/d*a*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/16*e^8/d^5*a^3/c
*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/16*e^4/d*a*c*ln((1/2*a*e^2-1/2*c*
d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2))/(d*e*c)^(1/2)+1/16*d^4/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^3+17/24/d^3/a*
e^2*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-1/24*d/a^3/e^2*c^3*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/12/d/a^2/e^2/x^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)*c-1/3/d^2/a^2/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
*c-1/8*d^2/a^2/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^4), x)
```

Fricas [A] time = 10.3689, size = 1168, normalized size = 5.54

$$\left[-\frac{3\left(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6\right)\sqrt{ade}x^3\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2-4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade}+}{x^2}\right.}{96\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d*
e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 + (3*a*c^2*d
^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^
4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3), -1/48
*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^
3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*
d*e^3)*x)) + 2*(8*a^3*d^3*e^3 + (3*a*c^2*d^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*
e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.454 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=295

$$\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64a^2d^3e^2x^2}-\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{}{2}\right.}{128a^{5/2}d^{7/2}e}$$

```
[Out] ((c*d^2 - a*e^2)*(3*c*d^2 + 5*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*a^2*d^3*e^2*x^2) - (a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*d*x^4) - (((3*c)/(a*e) - (5*e)/d^2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*x^3) - ((c*d^2 - a*e^2)^3*(3*
c*d^2 + 5*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(5/2)*d^(7/2)*
e^(5/2))
```

Rubi [A] time = 0.385999, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 834, 806, 720, 724, 206}

$$\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64a^2d^3e^2x^2}-\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{}{2}\right.}{128a^{5/2}d^{7/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)*(3*c*d^2 + 5*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*a^2*d^3*e^2*x^2) - (a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*d*x^4) - (((3*c)/(a*e) - (5*e)/d^2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*x^3) - ((c*d^2 - a*e^2)^3*(3*
c*d^2 + 5*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(5/2)*d^(7/2)*
e^(5/2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx = \int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(3cd^2-5ae^2\right)+acde^2x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4}}{4ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24x^3} -$$

$$= \frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64a^2d^3e^2x^2} -$$

$$= \frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64a^2d^3e^2x^2} -$$

$$= \frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64a^2d^3e^2x^2} -$$

Mathematica [A] time = 0.313529, size = 253, normalized size = 0.86

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{x(5ae^2+3cd^2)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2(8d^2+2dex-3e^2x^2)+2acd^2ex(7d+4ex)+3c^2d^4x^2\right)-3x^3(cd^2-ae^2)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right.\cdots\right)}{a^{3/2}d^{5/2}e^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}}\cdots\right)}{192adex^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*(a*e + c*d*x)^2*(d + e*x) + ((3*c*d^2 +
 5*a*e^2)*x*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(3*c^2
*d^4*x^2 + 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*x - 3*e^2*x
^2)) - 3*(c*d^2 - a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]
*Sqrt[e]*Sqrt[d + e*x])]))/(a^(3/2)*d^(5/2)*e^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[
d + e*x])))/(192*a*d*e*x^4)
```

Maple [B] time = 0.09, size = 2427, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x)
```

```
[Out] -29/96/d/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+11/24/d^3/a/x^3*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/8*e^3/d^2*c*(c*d*e*(d/e+x)^2+(a*e^2
-c*d^2)*(d/e+x))^(1/2)-1/16*e^3*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/
(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-
5/64*e^5/d^4*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/32*e^3/d^2*c*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/16*e^3*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+
1/64*d^3/a^4/e^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^4+3/64*d^4/a^3/e
^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^4-1/4/d^2/a/e/x^4*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(5/2)-1/32/a^3/e^3/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(5/2)*c^2+1/4*e^6/d^5*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/8*e
^7/d^6*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/16*e^7/d^4*a^2*ln((1
/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/(d*e*c)^(1/2)-21/64*e^4/d^3*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)*x+5/128*e^6/d^3*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-5/96/e^2*d/a^3*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^3+1/32/e*d^2/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*c^3+133/192*e^2/d^5/a/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2)-59/96*e/d^4/a/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-53/96*e^2/d^3/
a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c+3/64*e^2*d/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/x)*c^2-19/192/e/a^3*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+5/64*
d/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3-1/4*e^6/d^5*a*(c*d*e*(d
/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/8*e^7/d^6*a^2/c*(c*d*e*(d/e+x)^2+(
a*e^2-c*d^2)*(d/e+x))^(1/2)-3/16*e^7/d^4*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x
)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*
c)^(1/2)+1/4*e^4/d^3*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/3*
e^4/d^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-23/64*e^4/d^5*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+91/192/d^3/a^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(5/2)*c+1/32*d^3/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^3-1/32*e/a*(a*d*e+
```

```
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2-1/16*e^9/d^6*a^3/c*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c
)^(1/2)-3/16*e^5/d^2*a*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3/64*d^3/a^3/e^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^4-3/128*d^5/a^2/e^2/(a*d*e)^(1/2)*ln(
(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/x)*c^4+1/8/d/a^2/e^2/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c+1
/64*d^2/a^4/e^3*c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-1/64*d/a^4/e^
4/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^3+1/16*e^9/d^6*a^3/c*ln((1/2*
a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)
*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/16*e^5/d^2*a*c*ln((1/2*a*e^2-1/2*c*d^2+(d/
e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d
*e*c)^(1/2)-3/64*e^2/d/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^2+19/1
92/e^2/d/a^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^2-13/48/e/d^2/a^2/
x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c-3/32*e^4/d*a/(a*d*e)^(1/2)*ln
((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/x)*c-133/192*e^3/d^4/a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-
91/192*e/d^2/a^2*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^5), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.455 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=395

$$-\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^3d^4e^3x^2}+\frac{\left(-35a^2e^4+12acd^2e^2+15\right.}{}$$

```
[Out] -((c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*a^3*d^4*e^3*
x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(5*d*x^5) - (((3*c)/(a
*e) - (7*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(40*x^4) +
((15*c^2*d^4 + 12*a*c*d^2*e^2 - 35*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(240*a^2*d^3*e^2*x^3) + ((c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a
*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(7/2)*d
^(9/2)*e^(7/2))
```

Rubi [A] time = 0.513995, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^3d^4e^3x^2}+\frac{\left(-35a^2e^4+12acd^2e^2+15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```


```
[Out] -((c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*a^3*d^4*e^3*
x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(5*d*x^5) - (((3*c)/(a
*e) - (7*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(40*x^4) +
((15*c^2*d^4 + 12*a*c*d^2*e^2 - 35*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(240*a^2*d^3*e^2*x^3) + ((c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a
*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(7/2)*d
^(9/2)*e^(7/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
```

```
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^6}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5}-\frac{\int \frac{\left(-\frac{1}{2}ae\left(3cd^2-7ae^2\right)+2acde^2x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5}}{5ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5}-\frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{40x^4}+$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5}-\frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{40x^4}+$$

$$=-\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae\right.}}{128a^3d^4e^3x^2}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae\right.}}{128a^3d^4e^3x^2}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae\right.}}{128a^3d^4e^3x^2}$$

Mathematica [A] time = 0.523731, size = 310, normalized size = 0.78

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{5x^2\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2\left(-8d^2-2dex+3e^2x^2\right)-2acd^2ex(7d+4ex)-3c^2d^4x^2\right)+3x^3\left(cd^2-ae^2\right)^3\tanh^{-1}\right.}{a^{5/2}d^{7/2}e^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}}\right.}{1920adex^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-384*(a*e + c*d*x)^2*(d + e*x) + (48*(5*c*d
^2 + 7*a*e^2)*x*(a*e + c*d*x)^2*(d + e*x))/(a*d*e) + (5*(3*c^2*d^4 + 6*a*c*
d^2*e^2 + 7*a^2*e^4)*x^2*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d
+ e*x]*(-3*c^2*d^4*x^2 - 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(-8*d^2 - 2*
d*e*x + 3*e^2*x^2)) + 3*(c*d^2 - a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c
*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(a^(5/2)*d^(7/2)*e^(5/2)*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x])))/(1920*a*d*e*x^5)
```

Maple [B] time = 0.102, size = 2888, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x)
```

```
[Out] 3/16*e^8/d^5*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*
e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1/4*e^5/d^4*c*(c*d*
e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/16*e^4/d*c^2*ln((1/2*a*e^2-1/2
```

```
*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))
^(1/2))/(d*e*c)^(1/2)-1/8*e^8/d^7*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)-3/16*e^8/d^5*a^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+39/128*e^5/d^4*c*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/16*e^4/d*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-
7/256*e^7/d^4*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-1/4*e^7/d^6*a*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*x-263/384*e^3/d^6/a/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(5/2)+1/32*e^2/d/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2+19/48*e
/d^2/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+121/192*e^2/d^5/a/x^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+227/384*e^3/d^4/a*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)*c-25/48*e/d^4/a/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)-1/32*e/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3+73/192/d^3
/a^2/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c+23/96/d/a^3*c^3*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-1/16/a^3/e^3/x^3*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(5/2)*c^2-3/128*d^3/a^3/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)*c^4-1/5/d^2/a/e/x^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-3/128*d^5/a^
4/e^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^5+3/128*d^2/a^4/e^3*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^4-1/128*d^4/a^5/e^5*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)*c^5-3/64/a^4/e^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2)*c^3+7/128*e^6/d^5*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/128*e^4/
d^3*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/8*e^4/d^3*c*(c*d*e*(d/e+x)^
2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+3/8/d^3/a/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(5/2)-1/32*d/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3+7/48/e/a^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^3-3/128*e^3/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/x)*c^2+1/4*e^7/d^6*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/
8*e^8/d^7*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/3*e^5/d^6*(
c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+45/128*e^5/d^6*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(3/2)-1/4/d^2/a^2/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)*c-3/256*d^4/a^2/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*
e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^4+1/8/d/a^2/e^2/x^4*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c+9/64/d/a^3/e^2/x^2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(5/2)*c^2+3/64*d/a^4/e^2*c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)*x-23/96/e/d^2/a^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c
^2+3/64*e^3/d^2/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^2+15/256*e^5/
d^2*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c+263/384*e^4/d^5/a*c*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)*x+103/192*e^2/d^3/a^2*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)*x-103/192*e/d^4/a^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c-
1/128*e*d^2/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^3-1/16*e^10/d^7*a^3/c*ln((1/2*a*
e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(
d/e+x))^(1/2))/(d*e*c)^(1/2)-3/64/e*d^2/a^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)*x*c^4-3/16*e^6/d^3*a*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e
*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/16
*e^10/d^7*a^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3/16*e^6/d^3*a*c*ln((1/2*a*e^2+1
/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d
*e*c)^(1/2)-3/128*d^4/a^4/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^5
+3/256*d^6/a^3/e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^5+1/128*d^2/a^5/e^5/x*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^4+1/64*d/a^4/e^4/x^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(5/2)*c^3-1/128*d^3/a^5/e^4*c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.456 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=498

$$-\frac{\left(21a^2cd^2e^4-105a^3e^6+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3}+\frac{\left(-21a^2e^4+6acd^2e^2+7c^2d^4\right)\left(x\left(ae^2\right.\right.}{160a^2d^3e^2x^4}$$

```
[Out] ((7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^4*e^8)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^4*d^
5*e^4*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(6*d*x^6) - ((c/
(a*e) - (3*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(20*x^5)
+ ((7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(160*a^2*d^3*e^2*x^4) - ((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 2
1*a^2*c*d^2*e^4 - 105*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
))/(960*a^3*d^4*e^3*x^3) - ((c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2
 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*
Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(102
4*a^(9/2)*d^(11/2)*e^(9/2))
```

Rubi [A] time = 0.724514, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(21a^2cd^2e^4-105a^3e^6+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3}+\frac{\left(-21a^2e^4+6acd^2e^2+7c^2d^4\right)\left(x\left(ae^2\right.\right.}{160a^2d^3e^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] ((7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^4*e^8)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^4*d^
5*e^4*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(6*d*x^6) - ((c/
(a*e) - (3*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(20*x^5)
+ ((7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(160*a^2*d^3*e^2*x^4) - ((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 2
1*a^2*c*d^2*e^4 - 105*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
))/(960*a^3*d^4*e^3*x^3) - ((c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2
 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*
Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(102
4*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx = \int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^7}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\int \frac{\left(-\frac{3}{2}ae\left(cd^2-3ae^2\right)+3acde^2x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{x^6}}{6ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{20x^5}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{20x^5}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{20x^5}$$

$$= \frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2\right.}}{512a^4d^5e^4x^2}$$

$$= \frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2\right.}}{512a^4d^5e^4x^2}$$

$$= \frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2\right.}}{512a^4d^5e^4x^2}$$

Mathematica [A] time = 0.88486, size = 380, normalized size = 0.76

$$-\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{16x^2(d+ex)\left(63a^2e^4+54acd^2e^2+35c^2d^4\right)(ae+cdx)^2}{a^2d^2e^2}+\frac{5x^3\left(21a^2cd^2e^4+21a^3e^6+15ac^2d^4e^2+7c^3d^6\right)\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e\right.\right.}{a^{7/2}d^{9}}\right.}{76}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] -(Sqrt[(a*e + c*d*x)*(d + e*x)]*(1280*(a*e + c*d*x)^2*(d + e*x) - (128*(7*c
*d^2 + 9*a*e^2)*x*(a*e + c*d*x)^2*(d + e*x))/(a*d*e) + (16*(35*c^2*d^4 + 54
*a*c*d^2*e^2 + 63*a^2*e^4)*x^2*(a*e + c*d*x)^2*(d + e*x))/(a^2*d^2*e^2) + (
5*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*x^3*(Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c^2*d^4*x^2 - 2*a*c*
d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(-8*d^2 - 2*d*e*x + 3*e^2*x^2)) + 3*(c*d^2
- a*e^2)^3*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d
+ e*x])])))/(a^(7/2)*d^(9/2)*e^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(768
0*a*d*e*x^6)
```

Maple [B] time = 0.108, size = 3387, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x)
```

[Out] -533/1536*e^6/d^7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/3*e^6/d^7*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+7/1536*d^5/a^6/e^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^6+7/512*d^6/a^5/e^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^6-21/512*e^7/d^6*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/8*e^5/d^4*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-23/96/d/a^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^3+1/64*e/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3+19/60/d^3/a/x^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/8*e^5/d^4*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/6/d^2/a/e/x^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-7/96/a^3/e^3/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^2-1/4*e^8/d^7*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/8*e^9/d^8*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-3/16*e^9/d^6*a^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/4*e^6/d^5*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/16*e^5/d^2*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+107/192*e^2/d^5/a/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-15/512*e^3/d^2/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2-235/384*e^4/d^5/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c+1045/1536*e^4/d^7/a/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-703/1536*e^2/d^3/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2-491/768*e^3/d^6/a/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+15/1024*e^4/d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2+257/768/d^3/a^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^2+1/4*e^8/d^7*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/8*e^9/d^8*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/16*e^9/d^6*a^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-149/512*e^6/d^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/16*e^5/d^2*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+21/1024*e^8/d^5*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-43/96*e/d^4/a/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+13/512/e*d^2/a^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^4-131/1536/e^2*d/a^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^4-5/384/e^4*d^3/a^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^5+1/64/e^3*d^4/a^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^5+65/192/d^3/a^2/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c+3/1024*d^3/a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^4-109/768/e/a^4*c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-1/12/e^3/a^4/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^3+7/256*d/a^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^4+1/16*e^11/d^8*a^3/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/16*e^7/d^4*a*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-21/512*e^4/d^3/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^2+877/1536*e^2/d^5/a^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c+3/256*e^2/d/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3-21/512*e^6/d^3*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c-43/96*e/d^4/a^2/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c-1045/1536*e^5/d^6/a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-877/1536*e^3/d^4/a^2*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-257/768*e/d^2/a^3*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/256*e^2*d/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^3-1/16*e^11/d^8*a^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-3/16*e^7/d^4*a*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+7/512*d^5/a^5/e^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^6-7/1536*d^3/a^6/e^6/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^5+7/60/d/a^2/e^2/x^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c-7/768*d^2/a^5/e^5/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^4+7/1536*d^4/a^6/e^5*c^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-7/1024*d^7

```
/a^4/e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^6+7/192*d/a^4/e^4/x^3*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(5/2)*c^3-41/1536/e^3*d^2/a^5*c^5*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)*x+3/512/e^2*d^5/a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^5+29/192
/e^2/d/a^3/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^2+109/768/e^2/d/a^
4/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^3+15/512/e^2*d^3/a^4*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^5+41/1536/e^4*d/a^5/x*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(5/2)*c^4-11/48/e/d^2/a^2/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(5/2)*c-91/384/e/d^2/a^3/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*
c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^7), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**7/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.457 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=574

$$-\frac{3\left(35a^2cd^2e^4+15a^3e^6+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^5d^5e^6}+\frac{(35a^2\ldots}{}$$

```
[Out] (-3*(c*d^2 - a*e^2)^3*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 1
5*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(16384*c^5*d^5*e^6) + ((c*d^2 - a*e^2)*(33*c^3*d^6 + 45*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^4*d^4*e^5) + (((5*a)/(c*d) - (
11*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/112 + (x^3*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*e) - ((231*c^3*d^6 - 15*a*c
^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*c*d*e*(33*c^2*d^4 - 10*a*c
*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4
480*c^3*d^3*e^4) + (3*(c*d^2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35
*a^2*c*d^2*e^4 + 15*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(1
1/2)*d^(11/2)*e^(13/2))
```

Rubi [A] time = 0.694762, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 832, 779, 612, 621, 206}

$$-\frac{3\left(35a^2cd^2e^4+15a^3e^6+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^5d^5e^6}+\frac{(35a^2\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```


```
[Out] (-3*(c*d^2 - a*e^2)^3*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 1
5*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(16384*c^5*d^5*e^6) + ((c*d^2 - a*e^2)*(33*c^3*d^6 + 45*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^4*d^4*e^5) + (((5*a)/(c*d) - (
11*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/112 + (x^3*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*e) - ((231*c^3*d^6 - 15*a*c
^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*c*d*e*(33*c^2*d^4 - 10*a*c
*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4
480*c^3*d^3*e^4) + (3*(c*d^2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35
*a^2*c*d^2*e^4 + 15*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(1
1/2)*d^(11/2)*e^(13/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
```

```
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\int x^3(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$$

$$=\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8e}+\frac{\int x^2\left(-3acd^2e-\frac{1}{2}cd\left(11cd^2-5ae^2\right)\right.}{8}$$

$$=\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right.\right.}{8e}$$

$$=\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right.\right.}{8e}$$

$$=\frac{\left(cd^2-ae^2\right)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right.}{2048c^4d^4e^5}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2\right.}{16384c^5d^5e^6}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2\right.}{16384c^5d^5e^6}$$

$$=-\frac{3\left(cd^2-ae^2\right)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2\right.}{16384c^5d^5e^6}$$

Mathematica [A] time = 4.2967, size = 681, normalized size = 1.19

$$\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{e}\left(35a^6c^2d^2e^{11}\left(29d^2-37dex-6e^2x^2\right)+5a^5c^3d^3e^9\left(93d^2ex+185d^3+100de^2x^2+24e^3x^3\right)+5a^4c^4d^4e^7\left(-30d^2e^2x^2+65d^3ex+265d^4-56\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*(1575*a^8*e^15 - 525*a^7*c*d*e^13*
(7*d - e*x) + 35*a^6*c^2*d^2*e^11*(29*d^2 - 37*d*e*x - 6*e^2*x^2) + 5*a^5*c
^3*d^3*e^9*(185*d^3 + 93*d^2*e*x + 100*d*e^2*x^2 + 24*e^3*x^3) + 5*a^4*c^4*
d^4*e^7*(265*d^4 + 65*d^3*e*x - 30*d^2*e^2*x^2 - 56*d*e^3*x^3 - 16*e^4*x^4)
 + a^3*c^5*d^5*e^5*(-11193*d^5 + 8359*d^4*e*x - 6088*d^3*e^2*x^2 + 5040*d^2
*e^3*x^3 + 139200*d*e^4*x^4 + 104320*e^5*x^5) + a^2*c^6*d^6*e^3*(11445*d^6
- 18669*d^5*e*x + 12962*d^4*e^2*x^2 - 10544*d^3*e^3*x^3 + 9120*d^2*e^4*x^4
+ 350080*d*e^5*x^5 + 272640*e^6*x^6) + c^8*d^8*x*(-3465*d^7 + 2310*d^6*e*x
- 1848*d^5*e^2*x^2 + 1584*d^4*e^3*x^3 - 1408*d^3*e^4*x^4 + 1280*d^2*e^5*x^5
 + 87040*d*e^6*x^6 + 71680*e^7*x^7) + a*c^7*d^7*e*(-3465*d^7 + 13755*d^6*e*
x - 9324*d^5*e^2*x^2 + 7512*d^4*e^3*x^3 - 6464*d^3*e^4*x^4 + 5760*d^2*e^5*x
^5 + 299520*d*e^6*x^6 + 240640*e^7*x^7)))/(a*e + c*d*x) + (105*Sqrt[c]*Sqrt
[d]*(c*d^2 - a*e^2)^(9/2)*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4
 + 15*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*
d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))
/(c*d^2 - a*e^2)])))/(573440*c^5*d^5*e^(13/2))
```

Maple [B] time = 0.074, size = 3178, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

```
[Out] -95/1024/e^4*d^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+285/8192/e^5*d
^8*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+45/16384*e^8/d^5/c^5*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^7-9/112/e/d^2/c^2*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(7/2)*a-285/32768/e^6*d^11*c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*
x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-705
/16384/e^4*d^7*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-15/2048*e^5/d^4/
c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^5-1/16*d^2/e*a^2/c*(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-9/64*d^4/e*a^2*(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(1/2)*x+3/128*d*e^2*a^4/c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(1/2)+3/64*d^7/e^4*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/
2)-3/64*d^8/e^5*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/8*d^3
/e^2*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+1/8*d^5/e^4*c*(c*d*e
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+3/256*d^11/e^6*c^3*ln((1/2*a*e^2-
1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+
x))^(1/2))/(d*e*c)^(1/2)-75/16384*e^4/d/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)*a^5+65/1024/e*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2-46
5/16384*e^2*d/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+29/128/e^2*d/
c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a-5/512*d/c*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)*x*a^2+1155/8192/e*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)*x*a^2-45/8192*e^3/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4+3
5/256/e^2*a*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+5/32/e/c*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x*a+1/8/e^2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(7/2)/d/c+3/128*e^2/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a^
3-1/5*d^3/e^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)+19/128/e^4*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-25/112/e^3/c*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(7/2)-35/2048*e^3/d^2/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)*a^4+15/16384*e^6/d^3/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6-15/
128*d^5*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/64*d^3*a^3/c*(c*d*e*(d
/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/16*d^6/e^5*c*(c*d*e*(d/e+x)^2+(a*e^2
-c*d^2)*(d/e+x))^(3/2)-3/128*d^9/e^6*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/
e+x))^(1/2)+465/4096*d^5*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^3+735/16384*d^3/c*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+13/128/d/c^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(5/2)*a^2+19/64/e^3*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2
)*x+45/2048/e^3*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-15/1024*e/c^2
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3+165/16384/e^2*d^5*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-95/2048/e^5*d^6*c*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(3/2)+285/16384/e^6*d^9*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-975/16384*e^2*d^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4-855/8192/e^2*d^7*c*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(d*e*c)^(1/2)*a^2-45/32768*e^10/d^5/c^5*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2
)*a^8+15/4096*e^8/d^3/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^7-15/8192*e^6/d/c^3*
ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/(d*e*c)^(1/2)*a^6-105/2048*e*d^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)*x*a^3+3/64*e/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x
*a^2-15/1024*e^4/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^4+15/2
56*d^3*e^2*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*
```

```
e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/64*d^2*e*a^3/c*(c
*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+9/64*d^6/e^3*a*c*(c*d*e*(d/e+
x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/256*d*e^4*a^5/c^2*ln((1/2*a*e^2-1/2*c
*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
1/2))/(d*e*c)^(1/2)+15/128*d^7/e^2*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*
d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(
1/2)-15/256*d^9/e^4*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1
/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+45/8192*e^
7/d^4/c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^6-15/4096*e^5/d^2/c^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5-5/256*e^2*a^3/c^2/d*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+195/4096/e^4*d^9*c^2*ln((1/2*a*e^2+1/2*c*
d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)
^(1/2)*a-495/4096/e^3*d^6*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+45/
4096*e^4*d/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.52865, size = 3413, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2293760*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 -
40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*
d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x
^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*
e^3)*x) + 4*(71680*c^8*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^13*e^3
 - 11193*a^2*c^6*d^11*e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e^9 + 10
15*a^5*c^3*d^5*e^11 - 3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5120*(17*
c^8*d^9*e^7 + 33*a*c^7*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c^7*d^8*e^
8 + 81*a^2*c^6*d^6*e^10)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^9*e^7 - 10
75*a^2*c^6*d^7*e^9 - 5*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e^4 - 316*a*
c^7*d^10*e^6 + 290*a^2*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*a^4*c^4*d^4*
e^12)*x^3 - 8*(231*c^8*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2*c^6*d^9*e^7
- 50*a^3*c^5*d^7*e^9 + 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3*e^13)*x^2 + 2
*(1155*c^8*d^14*e^2 - 3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^10*e^6 - 300*a^3
*c^5*d^8*e^8 - 275*a^4*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12 - 525*a^6*c^2*d
^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^7), -1/
1146880*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 - 40*
a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*d^4
```

```
*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/
(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(71680*c^8
*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d^13*e^3 - 11193*a^2*c^6*d^11*
e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*e^9 + 1015*a^5*c^3*d^5*e^11 -
3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 + 5120*(17*c^8*d^9*e^7 + 33*a*c^7
*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*c^7*d^8*e^8 + 81*a^2*c^6*d^6*e^1
0)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d^9*e^7 - 1075*a^2*c^6*d^7*e^9 - 5
*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*e^4 - 316*a*c^7*d^10*e^6 + 290*a^2
*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45*a^4*c^4*d^4*e^12)*x^3 - 8*(231*c^8
*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^2*c^6*d^9*e^7 - 50*a^3*c^5*d^7*e^9 +
 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^3*e^13)*x^2 + 2*(1155*c^8*d^14*e^2 -
3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^10*e^6 - 300*a^3*c^5*d^8*e^8 - 275*a^4
*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^12 - 525*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.458 $\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=452

$$\frac{(5a^2e^4+10acd^2e^2+9c^2d^4)(cd^2-ae^2)^3(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{1024c^4d^4e^5}-\frac{(5a^2e^4+10acd^2e^2+9c^2d^4)}{}$$

```
[Out] ((c*d^2 - a*e^2)^3*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*c^4*d^4*e^5
) - ((c*d^2 - a*e^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*c^3*d^3*
e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*e) + ((63*c^2
*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(840*c^2*d^2*e^3) - ((c*d^2 - a*e^
2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(2048*c^(9/2)*d^(9/2)*e^(11/2))
```

Rubi [A] time = 0.410182, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {851, 832, 779, 612, 621, 206}

$$\frac{(5a^2e^4+10acd^2e^2+9c^2d^4)(cd^2-ae^2)^3(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{1024c^4d^4e^5}-\frac{(5a^2e^4+10acd^2e^2+9c^2d^4)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```


```
[Out] ((c*d^2 - a*e^2)^3*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*c^4*d^4*e^5
) - ((c*d^2 - a*e^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*c^3*d^3*
e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*e) + ((63*c^2
*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(840*c^2*d^2*e^3) - ((c*d^2 - a*e^
2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(2048*c^(9/2)*d^(9/2)*e^(11/2))
```


Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
```

```
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \int x^2(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$$

$$= \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7e} + \frac{\int x\left(-2acd^2e-\frac{1}{2}cd\left(9cd^2-5ae^2\right)x\right)\left(a\right.}{7cde}$$

$$= \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7e} + \frac{\left(63c^2d^4-20acd^2e^2-35a^2e^4-10cde\left(9\right.\right.}{8}$$

$$= -\frac{\left(cd^2-ae^2\right)\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae\right.\right.}{384c^3d^3e^4}$$

$$= \frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+a\right.}}{1024c^4d^4e^5}$$

$$= \frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+a\right.}}{1024c^4d^4e^5}$$

$$= \frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+a\right.}}{1024c^4d^4e^5}$$

Mathematica [B] time = 6.14201, size = 1221, normalized size = 2.7

$$\frac{x(d+ex)((ae+cdx)(d+ex))^{3/2}(ae+cdx)^2}{7cde}+\frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\left(-9cd^2-7ae^2\right)(d+ex)^{5/2}(ae+cdx)^{7/2}}{12cde}+\frac{\left(cd^2-ae^2\right)\left(-6acd^2e^2-\frac{1}{2}\left(-\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (x*(a*e + c*d*x)^2*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(3/2))/(7*c*d*e) + (
((a*e + c*d*x)*(d + e*x))^(3/2)*(((-9*c*d^2 - 7*a*e^2)*(a*e + c*d*x)^(7/2)*
(d + e*x)^(5/2))/(12*c*d*e) + ((c*d^2 - a*e^2)*(-6*a*c*d^2*e^2 - ((-9*c*d^2
 - 7*a*e^2)*((7*c*d^2)/2 + (5*a*e^2)/2))/2)*(a*e + c*d*x)^(7/2)*Sqrt[d + e*
x]*(1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) -
 (a*c*d*e^2)/(c*d^2 - a*e^2))))^(5/2)*((7*(3/(8*(1 + (c*d*e*(a*e + c*d*x))/
((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))
)^2) + (1 + (c*d*e*(a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^
2) - (a*c*d*e^2)/(c*d^2 - a*e^2))))^(-1)))/10 + (21*(c*d^2 - a*e^2)^4*((c^2
*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^4*((2*c*d*e*(a*e + c*d
*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e
^2))) - (4*c^2*d^2*e^2*(a*e + c*d*x)^2)/(3*(c*d^2 - a*e^2)^2*((c^2*d^3)/(c*
d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2))^2) + (16*c^3*d^3*e^3*(a*e + c*d
*x)^3)/(15*(c*d^2 - a*e^2)^3*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^
2 - a*e^2))^3) - (2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/
(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)])])/(Sqrt[c*d^2 - a*e^2]*Sqrt
[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)]*Sqrt[1 + (c*d*e*(
a*e + c*d*x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*
d^2 - a*e^2))]])))/(512*c^4*d^4*e^4*(a*e + c*d*x)^4*(1 + (c*d*e*(a*e + c*d*
x))/((c*d^2 - a*e^2)*((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^
2))))^2)))/(21*c^3*d^3*e*((c*d)/((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c
*d^2 - a*e^2)))^(3/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(7*c*d*e*(a*
e + c*d*x)^(3/2)*(d + e*x)^(3/2))
```

Maple [B] time = 0.072, size = 2731, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

```
[Out] 5/384*e^4/d^3/c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^4+5/128/e^3*d^6
*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+5/192*e/c*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(3/2)*x*a^2-1/24*e/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(5/2)*a^2-15/512/e^4*d^7*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+15/2
048/e^5*d^10*c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+5/192*e^2/d/c^2*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(3/2)*a^3-1/12/d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2)*x*a+5/64/e^3*d^4*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-5/128*e*d^2
/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-5/1024*e^7/d^4/c^4*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^6-25/192/e*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)*x*a-15/2048*e^5/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5+3/64*d^7/e^
4*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/256*d^10/e^5*c^3*ln
((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/8*d^2/e*a*(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(3/2)*x+3/64*d^2*e*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+
x))^(1/2)-1/8*d^4/e^3*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+3/2
56*e^5*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*
(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-225/2048*e*d^4*ln((1/
2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(d*e*c)^(1/2)*a^3-3/64*d^6/e^3*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2)+15/128*d^4*e*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c
)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/5*d^
2/e^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)-1/8/e^3*d^2*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/16*d*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(3/2)+9/64*d^3*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3
/128*e^3*a^4/c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/16*d^5/e^4
*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+3/128*d^8/e^5*c^2*(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-15/1024/e^5*d^8*c^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)+5/128/e^4*d^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)-1/6/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a-1/4/e^2*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x-15/1024/e*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)*a^2+35/1024*e^3/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+1
/7/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/d/c-5/192/e^2*d^3*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a-35/256*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)*x*a^2-5/96*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2-9/64*d^5
/e^2*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-15/128*d^6/e*a^2*c
*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+15/256*d^8/e^3*a*c^2*ln((1/2*a*e^2-
1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+
x))^(1/2))/(d*e*c)^(1/2)+125/2048*e^3*d^2/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x
)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^4+
195/2048/e*d^6*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2-15/2048*e^7/d^2/c^3*ln((1/2
*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(d*e*c)^(1/2)*a^6+15/256*e^2*d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)*x*a^3+5/2048*e^9/d^4/c^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^7-5/512*e^6/d^3/c
^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^5+5/512*e^4/d/c^2*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-85/2048/e^3*d^8*c^2*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c
```

```
)^(1/2)*a+55/512/e^2*d^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+5/19
2*e^3/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^3-15/256*d^2*e^3*
a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2
+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/64*d*e^2*a^3/c*(c*d*e*(d/e+x
)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.17263, size = 2803, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/430080*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*
a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^1
2 - 5*a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d
^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(15360*c^7*d^7*e
^7*x^6 + 945*c^7*d^13*e - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*
a^3*c^4*d^7*e^7 - 525*a^4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*a^6*c*d
*e^13 + 1280*(15*c^7*d^8*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d^9*e^5 +
 380*a*c^6*d^7*e^7 + 185*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^4 - 93*a*
c^6*d^8*e^6 - 2095*a^2*c^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8*(63*c^7*d
^11*e^3 - 218*a*c^6*d^9*e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d^5*e^9 - 35
*a^4*c^3*d^3*e^11)*x^2 - 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*e^4 + 1166*a
^2*c^5*d^8*e^6 - 150*a^3*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 - 175*a^5*c^2*d
^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6), 1/2
15040*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*a^3*c
^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^12 - 5
*a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2
 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15360*c^7*d^7*e^7*x^6 + 945*c^7*d^13*e
- 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*a^3*c^4*d^7*e^7 - 525*a^
4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*a^6*c*d*e^13 + 1280*(15*c^7*d^8
*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d^9*e^5 + 380*a*c^6*d^7*e^7 + 185
*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^4 - 93*a*c^6*d^8*e^6 - 2095*a^2*c
^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8*(63*c^7*d^11*e^3 - 218*a*c^6*d^9*
e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d^5*e^9 - 35*a^4*c^3*d^3*e^11)*x^2 -
 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*e^4 + 1166*a^2*c^5*d^8*e^6 - 150*a^3
*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 - 175*a^5*c^2*d^2*e^12)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.459 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=381

$$-\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^3d^3e^4}+\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2\right.}{19}$$

```
[Out] -((c*d^2 - a*e^2)^3*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(512*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*
(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(192*c^2*d^2*e^3) - (((5*a)/(c*d) + (7*d)/e^2)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/60 + (a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2)/(6*c*d*e*(d + e*x)) + ((c*d^2 - a*e^2)^5*(7*c*d^2 + 5*a*e^2)
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(1024*c^(7/2)*d^(7/2)*e^(9/2))
```

Rubi [A] time = 0.386024, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {794, 664, 612, 621, 206}

$$-\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^3d^3e^4}+\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2\right.}{19}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```


```
[Out] -((c*d^2 - a*e^2)^3*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(512*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*
(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(192*c^2*d^2*e^3) - (((5*a)/(c*d) + (7*d)/e^2)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/60 + (a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2)/(6*c*d*e*(d + e*x)) + ((c*d^2 - a*e^2)^5*(7*c*d^2 + 5*a*e^2)
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(1024*c^(7/2)*d^(7/2)*e^(9/2))
```


Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
```

```
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6cde(d+ex)}+\frac{1}{12}\left(-\frac{7d}{e}-\frac{5ae}{cd}\right)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+\right.}{d+ex}$$

$$=-\frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{6cde(d+ex)}$$

$$=\frac{\left(cd^2-ae^2\right)\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192c^2d^2e^3}$$

$$=-\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^4}$$

Mathematica [A] time = 3.61385, size = 506, normalized size = 1.33

$$\frac{(ae+cdx)((d+ex)(ae+cdx))^{5/2}\left(7-\frac{7\sqrt{cd}\sqrt{cd^2-ae^2}\left(5ae^2+7cd^2\right)\left(\frac{cd(d+ex)}{cd^2-ae^2}\right)^{3/2}\left(15\sqrt{e}\sqrt{cd}\left(cd^2-ae^2\right)^{11/2}(ae+cdx)\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}-10e^{3/2}\sqrt{cd}\left(cd^2-ae^2\right)^{9/2}(ae\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*(7 - (7*Sqrt[c*d]*Sqrt[c*d^2
 - a*e^2]*(7*c*d^2 + 5*a*e^2)*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(3/2)*(15*S
```

```
qrt[c*d]*Sqrt[e]*(c*d^2 - a*e^2)^(11/2)*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/
(c*d^2 - a*e^2)] - 10*Sqrt[c*d]*e^(3/2)*(c*d^2 - a*e^2)^(9/2)*(a*e + c*d*x)
^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + 8*Sqrt[c*d]*e^(5/2)*(c*d^2 - a*e
^2)^(7/2)*(a*e + c*d*x)^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + 16*Sqrt[c
*d]*e^(7/2)*(c*d^2 - a*e^2)^(3/2)*(a*e + c*d*x)^4*Sqrt[(c*d*(d + e*x))/(c*d
^2 - a*e^2)]*(-3*a*e^2 + c*d*(11*d + 8*e*x)) - 15*Sqrt[c]*Sqrt[d]*(c*d^2 -
a*e^2)^6*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*
x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(1280*c^5*d^5*e^(7/2)*(a*e + c*d*x)^
4*(d + e*x)^4)))/(42*c*d*e)
```

Maple [B] time = 0.06, size = 2411, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

```
[Out] -1/8*d*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+1/16*d^4/e^3*c*(c*
d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-3/128*d^7/e^4*c^2*(c*d*e*(d/e+x)
^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/16*e*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)
*(d/e+x))^(3/2)+5/48*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a+1/12/d/c
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*a-75/1024*e^4*d/c*ln((1/2*a*e^2+1/
2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*
e*c)^(1/2)*a^4+9/64*d^4/e*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
*x-3/256/d*e^6*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)
+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-15/256*d^7/e^
2*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)
^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+15/256*d*e^4*a^4/c*ln((1/2*a
*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(1/2))/(d*e*c)^(1/2)-5/64/e*d^4*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)*x*a+15/512/e^2*d^7*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a+15/512*e^6/d/c^
2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^5-5/1024*e^8/d^3/c^3*ln((1/2*a*e^2+1/2*c*d^2
+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1
/2)*a^6-5/96*e^2/d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*a^2+5/256*e^
5/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4-15/512*e^4/d/c^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+5/256*e^2*d/c*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)*a^3-75/1024*d^5*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*
e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+5/256
/e^3*d^6*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+5/512*e^6/d^3/c^3*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+15/128*e*d^2*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)*x*a^2-3/64*d*e^2*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(
d/e+x))^(1/2)+1/8*d^3/e^2*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x
-15/128*d^3*e^2*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c
*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/128/d*e^4*a^4/
c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+3/64*d^5/e^2*a*c*(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-9/64*d^2*e*a^2*(c*d*e*(d/e+x)^2+(a*e^
2-c*d^2)*(d/e+x))^(1/2)*x+3/64*e^3*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/
e+x))^(1/2)*x+15/128*d^5*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*
c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/256
*d^9/e^4*c^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d
/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+25/256*e^2*d^3*ln((1/2*
a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/(d*e*c)^(1/2)*a^3-5/64*e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x
*a^3-3/64*d^6/e^3*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-5/96/
```

```
e^2*d^3*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+5/192*e/c*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(3/2)*a^2+5/192/e*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)*a-5/192/e^3*d^4*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+5/512/e^4
*d^7*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/6/e*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(5/2)*x+1/12/e^2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-1/
5*d/e^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)+5/256*d^3*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-5/1024/e^4*d^9*c^3*ln((1/2*a*e^2+1/2*c*d^2
+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1
/2)-5/192*e^3/d^2/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a^3-15/512/e^
2*d^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="m
axima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.96248, size = 2264, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="f
ricas")

[Out] [-1/30720*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3
*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e)
 + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*e
+ 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245*a^4*c
^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^7)*x^4
+ 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3 - 8*(7*c
^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^3*e^9)*x^
2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6 + 80*a^3*c
^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(c^4*d^4*e^5), -1/15360*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2
*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10
- 5*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*
e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11*
e + 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245*a^4
*c^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^7)*x^
4 + 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3 - 8*(7
*c^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^3*e^9)*
x^2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6 + 80*a^3
*c^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c^4*d^4*e^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.460 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=274

$$\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^2d^2e^3}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}+$$

```
[Out] (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 +
2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/16 + (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - (3*(c*d^2 - a*e^2)^5*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(256*c^(5/2)*d^(5/2)*e^(7/2))
```

Rubi [A] time = 0.185465, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {664, 612, 621, 206}

$$\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^2d^2e^3}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```


```
[Out] (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 +
2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/16 + (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - (3*(c*d^2 - a*e^2)^5*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(256*c^(5/2)*d^(5/2)*e^(7/2))
```


Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e} - \frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int\left(ade+\left(cd^2+ae\right.\right.}{2e^2}$$

$$= \frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{\left(ade+\left(\right.\right.}{}$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{16}\left(\frac{a}{cd}-\right.$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{16}\left(\frac{a}{cd}-\right.$$

$$= \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3} + \frac{1}{16}\left(\frac{a}{cd}-\right.$$

Mathematica [A] time = 1.2586, size = 384, normalized size = 1.4

$$\frac{\sqrt{cd}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{cd}(d+ex)\left(2a^3c^2d^2e^5\left(64d^2+268dex+129e^2x^2\right)+2a^2c^3d^3e^3\left(87d^2ex-35d^3+489de^2x^2+292e^3x^3\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[c*d]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[e]*(d + e*x)*(-15*a^5*e^9 + 5*a^
4*c*d*e^7*(14*d - e*x) + 2*a^3*c^2*d^2*e^5*(64*d^2 + 268*d*e*x + 129*e^2*x^
2) + 2*a^2*c^3*d^3*e^3*(-35*d^3 + 87*d^2*e*x + 489*d*e^2*x^2 + 292*e^3*x^3)
 + c^5*d^5*x*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*e^4
*x^4) + a*c^4*d^4*e*(15*d^4 - 80*d^3*e*x + 54*d^2*e^2*x^2 + 688*d*e^3*x^3 +
 464*e^4*x^4)) - 15*(c*d^2 - a*e^2)^(11/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d +
 e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])
/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(640*c^(7/2)*d^(7/2)*e^(7/2)*Sqrt[(a*e
+ c*d*x)*(d + e*x)])
```

Maple [B] time = 0.056, size = 1123, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

```
[Out] 1/5/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)+3/256*e^7*a^5/d^2/c^2*l
n((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2
-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/128/e^3*c^2*d^6*(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2)+3/64*e^3*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2)-1/8/e*c*d^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-1/
16/e^2*c*d^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+1/8*e*a*(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+15/128*e^3*a^3*d^2*ln((1/2*a*e^2-1/
2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x)
)^(1/2))/(d*e*c)^(1/2)-3/64/e*a*d^4*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x
))^(1/2)-15/256*e^5*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1
/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/256/e^3*
c^3*d^8*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)
^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/64/e^2*c^2*d^5*(c*d*e*(d/e
+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+1/16*e^2*a^2/d/c*(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(3/2)+9/64*e^2*a^2*d*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e
+x))^(1/2)*x-3/128*e^5*a^4/d^2/c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^
(1/2)-3/64*e^4*a^3/d/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-9/64
*a*d^3*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-15/128*e*a^2*d^4*c
*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+15/256/e*a*d^6*c^2*ln((1/2*a*e^2-1/
2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x)
)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47216, size = 1789, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x)
 + 4*(128*c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d
^5*e^5 + 70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^
4*d^4*e^6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^
2 - 2*(5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d
^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/12
80*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^
6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
```

```
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(128*c^5*d^5*e^5*x^
4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 + 70*a^3*c^2*d^3*
e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^4*d^4*e^6)*x^3 + 8*(c^5*
d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^2 - 2*(5*c^5*d^8*e^2 - 2
3*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError
```

3.461 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=394

$$-\frac{\left(-83a^2cd^2e^4-5a^3e^6-11ac^2d^4e^2+2cdex\left(cd^2-5ae^2\right)\left(ae^2+3cd^2\right)+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64cde^2}+\frac{\left(90a^2c^2d^4\right.}{}$$

```
[Out] -((3*c^3*d^6 - 11*a*c^2*d^4*e^2 - 83*a^2*c*d^2*e^4 - 5*a^3*e^6 + 2*c*d*e*(c
*d^2 - 5*a*e^2)*(3*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(64*c*d*e^2) + ((3*c*d^2 + 11*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))/(24*e) + ((3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^
2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(128*c^(3/2)*d^(3/2)*e^(5/2)) - a^(5/2)*d^(3/2)*e^(5/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rubi [A] time = 0.449671, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 814, 843, 621, 206, 724}

$$-\frac{\left(-83a^2cd^2e^4-5a^3e^6-11ac^2d^4e^2+2cdex\left(cd^2-5ae^2\right)\left(ae^2+3cd^2\right)+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64cde^2}+\frac{\left(90a^2c^2d^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```


```
[Out] -((3*c^3*d^6 - 11*a*c^2*d^4*e^2 - 83*a^2*c*d^2*e^4 - 5*a^3*e^6 + 2*c*d*e*(c
*d^2 - 5*a*e^2)*(3*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(64*c*d*e^2) + ((3*c*d^2 + 11*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))/(24*e) + ((3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^
2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(128*c^(3/2)*d^(3/2)*e^(5/2)) - a^(5/2)*d^(3/2)*e^(5/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x}\,dx$$

$$=\frac{\left(3cd^2+11ae^2+6cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24e}-\int \frac{\left(-8a^2cd^2e^3+\frac{1}{2}cd\left(cd^2-5a\right.\right.}{}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{\cdot}}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{\cdot}}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{\cdot}}{64cde^2}$$

$$=-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{\cdot}}{64cde^2}$$

Mathematica [A] time = 2.03991, size = 390, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{e}\sqrt{ae+cdx}\left(a^2cde^4(337d+118ex)+15a^3e^6+ac^2d^2e^2\left(57d^2+244dex+136e^2x^2\right)+c^3\left(72d^4e^2x^2\right.\right.\right.}{192cde^{5/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e]*Sqrt[a*e + c*d*x]*(15*a^3*e^6 + a^2
*c*d*e^4*(337*d + 118*e*x) + a*c^2*d^2*e^2*(57*d^2 + 244*d*e*x + 136*e^2*x^
2) + c^3*(-9*d^6 + 6*d^5*e*x + 72*d^4*e^2*x^2 + 48*d^3*e^3*x^3)) + (3*Sqrt[
c]*Sqrt[d]*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^
2*e^6 - 5*a^4*e^8)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqr
t[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d +
 e*x))/(c*d^2 - a*e^2)]) - (384*a^(5/2)*c*d^(5/2)*e^5*ArcTanh[(Sqrt[d]*Sqrt
[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d + e*x]))/(192*c*d*e
^(5/2)*Sqrt[a*e + c*d*x])
```

Maple [B] time = 0.063, size = 2180, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x)
```

```
[Out] 1/8*d^3*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/128*d^5*c^2/e^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+83/64*d*a^2*e^2*(a*d*e+(a*e^2+c*d^2)*x+c
```

```
*d*e*x^2)^(1/2)+1/8*d*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-9/64*a^2*
e^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+3/64*d^3*a*c*(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/8*d*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(3/2)*x-1/5/d*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)+1/5/d*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+15/128*d^3*a^2*e^2*c*ln((1/2*a*e^2-
1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+
x))^(1/2))/(d*e*c)^(1/2)-25/256/d*a^4*e^6/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x
)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-3/64
/d^2*a^3*e^5/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+15/256/d*a^4*e^6/c
*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/64/d^2*a^3*e^5/c*(c*d*e*(d/e+x)^2
+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+9/64*d^2*a*e*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^
2)*(d/e+x))^(1/2)*x-3/256/d^3*a^5*e^8/c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c
*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^
(1/2)+11/24*a*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/8/d*a^3*e^4/c*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-d^2*a^3*e^3/(a*d*e)^(1/2)*ln((2*a*d*e+
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)
+75/128*d^3*a^2*e^2*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+19/64*d^2*a*e*c*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/16*d^2*c/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)
*(d/e+x))^(3/2)-3/128*d^5*c^2/e^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
1/2)+1/16*d^2*c/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+19/64*a^2*e^3*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-15/128*d*a^3*e^4*ln((1/2*a*e^2-1/2*c
*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
1/2))/(d*e*c)^(1/2)-3/64*d^4*c^2/e*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^
(1/2)*x+3/256*d^7*c^3/e^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1
/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1/8/d*a*e^
2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-3/64*d^4*c^2/e*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+3/128/d^3*a^4*e^6/c^2*(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2)-3/64/d*a^3*e^4/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2)-15/256*d^5*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c
)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1/16/d
^2*a^2*e^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+1/8/d*a*e^2*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/16/d^2*a^2*e^3/c*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)-3/128/d^3*a^4*e^6/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)-25/256*d^5*a*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3/256*d^7*c^3/e^2*ln((
1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/(d*e*c)^(1/2)+75/128*d*a^3*e^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d
*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3/256/d^
3*a^5*e^8/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="m
axima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] sage0*x
```

3.462 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=352

$$\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e}-\frac{\left(-45a^2cd^2e^4-5a^3e^6-15ac^2d^4e^2+\right.}{16}$$

```
[Out] ((c^2*d^4 + 28*a*c*d^2*e^2 + 19*a^2*e^4 + 2*c*d*e*(c*d^2 + 7*a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e) - ((3*a*e - c*d*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*x) - ((c^3*d^6 - 15*a*c^2*d^4*e^2
- 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt
[c]*Sqrt[d]*e^(3/2)) - (a^(3/2)*Sqrt[d]*e^(3/2)*(5*c*d^2 + 3*a*e^2)*ArcTanh
[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])])/2
```

Rubi [A] time = 0.432167, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {849, 812, 814, 843, 621, 206, 724}

$$\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e}-\frac{\left(-45a^2cd^2e^4-5a^3e^6-15ac^2d^4e^2+\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```


```
[Out] ((c^2*d^4 + 28*a*c*d^2*e^2 + 19*a^2*e^4 + 2*c*d*e*(c*d^2 + 7*a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e) - ((3*a*e - c*d*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*x) - ((c^3*d^6 - 15*a*c^2*d^4*e^2
- 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt
[c]*Sqrt[d]*e^(3/2)) - (a^(3/2)*Sqrt[d]*e^(3/2)*(5*c*d^2 + 3*a*e^2)*ArcTanh
[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])])/2
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
```

```
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{(3ae-cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{\left(-ae\left(5cd^2+3ae^2\right)-c\ldots\right.}{\ldots}$$

$$= \frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd\ldots}}{8e}$$

$$= \frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd\ldots}}{8e}$$

$$= \frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd\ldots}}{8e}$$

$$= \frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd\ldots}}{8e}$$

Mathematica [A] time = 2.21351, size = 350, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{e}\sqrt{ae+cdx}\left(3a^2e^3(11ex-8d)+2acde^2x(34d+13ex)+c^2d^2x\left(3d^2+14dex+8e^2x^2\right)\right)}{x} - \frac{3\sqrt{c}\sqrt{d}\left(-45a^2cd^2e^4-5a^3e^6-15ac^2d^4e^2+c^3d^6\right)\ldots}{\sqrt{cd}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex\ldots}{cd^2-ae^2\ldots}}}\right.}{24e^{3/2}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*Sqrt[a*e + c*d*x]*(3*a^2*e^3*(-8*d
 + 11*e*x) + 2*a*c*d*e^2*x*(34*d + 13*e*x) + c^2*d^2*x*(3*d^2 + 14*d*e*x +
8*e^2*x^2)))/x - (3*Sqrt[c]*Sqrt[d]*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*
d^2*e^4 - 5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(S
qrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)]) - (24*a^(3/2)*Sqrt[d]*e^3*(5*c*d^2 + 3*a*e^2)*Ar
cTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d
+ e*x]))/(24*e^(3/2)*Sqrt[a*e + c*d*x])
```

Maple [B] time = 0.066, size = 2364, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x)
```

```
[Out] -3/256/e*d^6*c^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*
e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/16*e^4/d^3*a^2/c*
(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+9/64*e^4/d*a^2*(c*d*e*(d/e+x)
```

```
^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/128*e^7/d^4*a^4/c^2*(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2)-3/64*e*d^2*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2)+1/8*e^3/d^2*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+
3/64*e^5/d^2*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/8*e^3/d^
2*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-3/2*e^4*d*a^3/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/x)-13/256/e*d^6*c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/16*e^4/d^3*a^2/c*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-3/64*e^5/d^2*a^3/c*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)+227/64*e*d^2*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)-9/64*e^4/d*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+3/128*e^7/d^
4*a^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/d*c/a*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(5/2)*x-1/d^2/a/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2
)+1/5*e/d^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)-1/16*d*c*(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+4/5*e/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(5/2)+19/8*e^3*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+67/48*d*c
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+15/128*e^5*a^3*ln((1/2*a*e^2-1/2*c
*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
1/2))/(d*e*c)^(1/2)+3/64*d^3*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2)*x-1/8*e*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+3/128/e*d^4*c
^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/a/e*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(5/2)*c+25/128*e^5*a^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*
c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+13/64*d^3*c
^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+e^2/d*a*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)+9/8*e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+13/128/
e*d^4*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-15/256*e^7/d^2*a^4/c*ln((
1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/256*e^9/d^4*a^5/c^2*ln((1/2*a*e^2-1/2*
c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^
(1/2))/(d*e*c)^(1/2)+225/256*e*d^4*a*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(
d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+15/256*
e^7/d^2*a^4/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+121/64*e^2*d*a*c*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)*x-3/256*e^9/d^4*a^5/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*
e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3
75/128*e^3*d^2*a^2*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3/64*e^6/d^3*a^3/c*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-15/128*e^3*d^2*a^2*c*ln((1/2*a*e^2-1/2*c*
d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2))/(d*e*c)^(1/2)+15/256*e*d^4*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e
)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2
)-3/64*e^6/d^3*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-9/64*e
^2*d*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-5/2*d^3*a^2*e^2/(a
*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/x)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^2), x)
```

Fricas [A] time = 104.892, size = 3652, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(
c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*
sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2
- 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*c^3*d^3*e^3*x^3 -
 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e
 + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c*d*e^2*x), 1/48*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*
e^4 - 5*a^3*e^6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c
*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 12*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^
5)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x
^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 2*(8*c^3*d^3*e^3*x^
3 - 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^
5*e + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(c*d*e^2*x), 1/96*(48*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*sqrt(
-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*
e + a^2*d*e^3)*x)) - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a
^3*e^6)*sqrt(c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(8*c^3*d^3*e^3*x^3 - 2
4*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e +
 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(c*d*e^2*x), 1/48*(24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*sqrt(-a*d*
e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a
^2*d*e^3)*x)) + 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^
6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2
*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^
2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c^3*d^3*e^3*x^3 - 24*a^2*c*d^2*e^4 + 2*(7*c
^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c^3*d^5*e + 68*a*c^2*d^3*e^3 + 33*a
^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.463 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=339

$$\frac{3\sqrt{c}\sqrt{d}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{e}}-\frac{3\sqrt{a}\sqrt{e}\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\right.}{8\sqrt{d}}$$

```
[Out] (-3*(a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(4*x) - ((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(2*x^2) + (3*Sqrt[c]*Sqrt[d]*(c^2*d^4 + 10*a*c*d^2*e^2 +
 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*Sqrt[e]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e
^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*Sqrt[d])
```

Rubi [A] time = 0.38817, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 812, 843, 621, 206, 724}

$$\frac{3\sqrt{c}\sqrt{d}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{e}}-\frac{3\sqrt{a}\sqrt{e}\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\right.}{8\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```


```
[Out] (-3*(a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(4*x) - ((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(2*x^2) + (3*Sqrt[c]*Sqrt[d]*(c^2*d^4 + 10*a*c*d^2*e^2 +
 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*Sqrt[e]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e
^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*Sqrt[d])
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3}\,dx$$

$$= -\frac{(ae-cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{\left(-2ae\left(3cd^2+ae^2\right)-2cd\left(c\right.\right.}{}$$

$$= -\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x} - \frac{(ae-c}{}$$

$$= -\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x} - \frac{(ae-c}{}$$

$$= -\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x} - \frac{(ae-c}{}$$

$$= -\frac{3\left(ae\left(3cd^2+ae^2\right)-cd\left(cd^2+3ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4x} - \frac{(ae-c}{}$$

Mathematica [A] time = 2.37238, size = 334, normalized size = 0.99

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{d}\sqrt{ae+cdx}\left(-a^2e^2(2d+5ex)-9acdex(d-ex)+c^2d^2x^2(5d+2ex)\right)}{x^2}+\frac{3\sqrt{cd}\sqrt{cd}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)}{\sqrt{e}\sqrt{cd^2-ae^2}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}}-\right.}{4\sqrt{d}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[d]*Sqrt[a*e + c*d*x]*(-9*a*c*d*e*x*(d
 - e*x) + c^2*d^2*x^2*(5*d + 2*e*x) - a^2*e^2*(2*d + 5*e*x)))/x^2 + (3*Sqrt
[c]*d*Sqrt[c*d]*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcSinh[(Sqrt[c]*Sqr
t[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[e]*
Sqrt[c*d^2 - a*e^2]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d
*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[d + e*x]))/(4*Sqrt[d]*Sqrt[a*e
+ c*d*x])
```

Maple [B] time = 0.075, size = 2688, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x)
```

```
[Out] 1/8*e^4/d^3*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/8*e^2/d*c*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/d*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(5/2)*c+1/4/d^3/a/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+3/256*d^5*c^3*l
n((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2
-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/4*e^3/d^2*a*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(3/2)+3/4*e^4/d*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/8*
e^5*a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+93/256*d^5*c^3*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2
)+15/128*e^6/d*a^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+147/64*e*d^2*c^2*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+3/64*e^6/d^3*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)+333/64*e^2*d*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/16
*e^5/d^4*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+9/64*e^5/d^2*a^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/2/d^2/a/e/x^2*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(7/2)+3/4*d/a^2/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c
^2+5/4*d^2/a/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+5/4*d/a*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*c^2+3/4/a^2/e*c^2*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(5/2)*x-3/128*e^8/d^5*a^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+39/64*e^3*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/16*e^5/d^4*
a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-9/64*e^5/d^2*a^2*(c*d*e
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+3/128*e^8/d^5*a^4/c^2*(c*d*e*(d/e
+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+3/64*e^2*d*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c
*d^2)*(d/e+x))^(1/2)-15/128*e^6/d*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e
)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2
)-3/64*e*d^2*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/8*e^4/d^
3*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-3/64*e^6/d^3*a^3/c*(c*d
*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/8*e^2/d*c*(c*d*e*(d/e+x)^2+(a*e
```

```
^2-c*d^2)*(d/e+x))^(3/2)*x+9/64*e^3*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e
+x))^(1/2)*x+387/128*d^3*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/5*e^
2/d^3*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)+1/16*e*c*(c*d*e*(d/e+x)
^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)-3/128*d^3*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2
)*(d/e+x))^(1/2)-1/20*e^2/d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+31/16
*e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-15/4*d^2*a^2*e^3/(a*d*e)^(1/2)
*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/x)*c-3/4/d/a^2/e^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c-1
/4/d^2/a*e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x+15/256*e^8/d^3*a^4/c
*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/64*e^7/d^4*a^3/c*(c*d*e*(d/e+x)^2
+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/256*e^10/d^5*a^5/c^2*ln((1/2*a*e^2-1/2*c*
d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2))/(d*e*c)^(1/2)-3/64*e^7/d^4*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)*x+3/256*e^10/d^5*a^5/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+225/128*e^4*d*a^2*c*
ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/(d*e*c)^(1/2)+975/256*e^2*d^3*a*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*
d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)
-15/256*e^8/d^3*a^4/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+15/128*e^4*d*a^2*c*ln((1/2
*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2
)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-15/256*e^2*d^3*a*c^2*ln((1/2*a*e^2-1/2*c*d^
2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2
))/(d*e*c)^(1/2)-15/8*d^4*a*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^3), x)
```

Fricas [A] time = 46.7206, size = 3301, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*d
^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e +
 a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^
3*e + a*c*d*e^3)*x) + 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*e/d)*
x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqr
```

```
t(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*c^2*d^2*e*x^3 - 2*a^2*d
*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/16*(6*(c^2*d^4 + 10*a*c*d^2*e
^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*
d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4
)*sqrt(a*e/d)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a
*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*c^2*d^2*e
*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e
^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, 1/16*(6*(5*c^2*d^4
 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d
*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 3*(c^2*d^4 + 10*a*c*d^2*
e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*
d^2*e*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*
a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/8*(3*(c^2*
d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^
2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c
*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2
+ a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*(2*c^2*d^2*e*x^3 - 2*a^2*d*e^2
+ (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.464 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=371

$$-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8dx}-\frac{\left(15a^2cd^2e^4-a^3e^6+45ac^2d^4e^2+5c^3d\right.}{16\sqrt{a}}$$

```
[Out] -((5*c^2*d^4 + 12*a*c*d^2*e^2 - a^2*e^4 - 2*c*d*e*(7*c*d^2 + a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*d*x) - ((4*a*d*e + 3*(3*c*d^2 +
 a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*d*x^3) + (c^(
3/2)*d^(3/2)*Sqrt[e]*(3*c*d^2 + 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x
)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])
/2 - ((5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh[(
2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.470245, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {849, 810, 812, 843, 621, 206, 724}

$$-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8dx}-\frac{\left(15a^2cd^2e^4-a^3e^6+45ac^2d^4e^2+5c^3d\right.}{16\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```


```
[Out] -((5*c^2*d^4 + 12*a*c*d^2*e^2 - a^2*e^4 - 2*c*d*e*(7*c*d^2 + a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*d*x) - ((4*a*d*e + 3*(3*c*d^2 +
 a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*d*x^3) + (c^(
3/2)*d^(3/2)*Sqrt[e]*(3*c*d^2 + 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x
)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])
/2 - ((5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh[(
2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt[a]*d^(3/2)*Sqrt[e])
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
```

```
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}dx = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4}dx$$

$$= -\frac{\left(4ade+3\left(3cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12dx^3} - \int \frac{\left(-\frac{1}{2}ae\left(5c^2d^4+12acd\right.\right.}{}$$

$$= -\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8dx}$$

$$= -\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8dx}$$

$$= -\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8dx}$$

$$= -\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8dx}$$

Mathematica [A] time = 3.32594, size = 357, normalized size = 0.96

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d}\sqrt{e}(d+ex)\sqrt{ae+cdx}\left(a^2e^2\left(8d^2+14dex+3e^2x^2\right)+2acd^2ex(13d+34ex)+3c^2d^3x^2(11d-8ex)\right)}{x^3}-\frac{3\sqrt{d+ex}\left(15a^2cd^2e^4-a^3e^6+45ac^2d^4e^2+5c^3d^6\right)\tanh}{\sqrt{a}}\right.}{24d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(3*c^2*d
^3*x^2*(11*d - 8*e*x) + 2*a*c*d^2*e*x*(13*d + 34*e*x) + a^2*e^2*(8*d^2 + 14
*d*e*x + 3*e^2*x^2)))/x^3) + (24*(c*d)^(5/2)*e*Sqrt[c*d^2 - a*e^2]*(3*c*d^2
 + 5*a*e^2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*
Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]])/c^(3/2) - (3*(
5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[d + e*x]*Ar
cTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]])/Sqrt[a]
))/(24*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.082, size = 3144, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x)
```

```
[Out] 3/128*e^9/d^6*a^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+93/64*e^2*d*c
^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+387/256*e*d^4*c^3*ln((1/2*a*e^
```

2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(d*e*c)^(1/2)-3/64*e^7/d^4*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1
5/128*e^5*a^2*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/3/d^2/a/e/x^3*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(7/2)+19/24/d^2/a*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2)*c+1/8*d^2/a^3/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^3-3/8/d^4/a*
e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+5/24*d^3/a^2/e^2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)*c^3+5/8*d^4/a/e*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+5/8*d^3/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3+5/6/d/a^
2*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x-1/8/a^3/e^3/x*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(7/2)*c^2+5/6/a*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)*x*c^2+1/16*e^6/d^5*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+
9/64*e^6/d^3*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/128*e^9/
d^6*a^4/c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+15/128*e^7/d^2*a^
3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*
e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/64*e^2*d*c^2*(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/256*e*d^4*c^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x
)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*
c)^(1/2)+1/8*e^5/d^4*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+3/64
*e^7/d^4*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/8*e^3/d^2*c*
(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-15/128*e^5*a^2*c*ln((1/2*a*
e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(
d/e+x))^(1/2))/(d*e*c)^(1/2)+1/5*e^3/d^4*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/
e+x))^(5/2)+7/40*e^3/d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/12*e^3/d
^2*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-15/128*e^7/d^2*a^3*ln((1/2*a
*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(d*e*c)^(1/2)-1/16*e^6/d^5*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)-9/64*e^6/d^3*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-1/8*e^5/d^4*a*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/16*e^6/d*a^3/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/x)-3/64*e^3*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/16*e^
2/d*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+3/128*e*d^2*c^2*(c*d*e*
(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/24*e^4/d^3*a*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)-1/8*e^5/d^2*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
37/48*e^2/d*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+493/128*e*d^2*c^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+107/64*e^3*a*c*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+35/24*d/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+5/12
/d^3/a/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)-5/16*d^5/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/x)*c^3+25/24/a^2/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^2-15/
256*e^9/d^4*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d
e(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/64*e^8/d^5*a^3/c
*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-9/64*e^4/d*a*c*(c*d*e*(d/e
+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+3/256*e^11/d^6*a^5/c^2*ln((1/2*a*e^2-1
/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x
))^(1/2))/(d*e*c)^(1/2)+15/256*e^3*d^2*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x
)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*
c)^(1/2)+15/256*e^9/d^4*a^4/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+625/256*e^3*d^2*a*
c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3/64*e^8/d^5*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)*x+1/64*e^4/d*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3
/256*e^11/d^6*a^5/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+1/8*d/a^3/e^2*c^3*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x-45/16*d^3*a*e^2/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x
)*c^2-1/12/d/a^2/e^2/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c-15/16*d*
a^2*e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c-5/6/d^2/a^2/e/x*(a*d*e+(a*e^2+c*d^2)*x

```
+c*d*e*x^2)^(7/2)*c+5/24*d^2/a^2/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*
x*c^3+3/8/d^3/a*e^2*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^4), x)
```

Fricas [A] time = 59.4654, size = 3684, normalized size = 9.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/96*(24*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*e
^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*
c*d*e^3)*x) - 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)
*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x
^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(24*a*c^2*d^4*e^2
*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^
2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x))/(a*d^2*e*x^3), -1/96*(48*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sq
rt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d
^3*e + a*c*d*e^3)*x)) + 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4
- a^3*e^6)*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(24*a*c
^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3
*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^6 + 45*a*c^2*d^4*e^
2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*
c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 12*(3*a*c^2*d^5
*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 2*(24*a*
c^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^
3*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^6 + 45*a*c^2*d^4*e
^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a
```

```
*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 24*(3*a*c^2*d^
5*e + 5*a^2*c*d^3*e^3)*sqrt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(24*a*c^2*d^4*e^2*x^3 -
8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d*e^5)*x^2 - 2*(
13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(a*d^2*e*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.465 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=404

$$-\frac{\left(x\left(11a^2cd^2e^4-3a^3e^6+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64ad^2ex^2}+\frac{\left(-90a^2c\right.}{}$$

```
[Out] -((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (5*c^3*d^6 + 83*a*c^2*d^4*
e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(64*a*d^2*e*x^2) - ((6*a*d*e + (11*c*d^2 + 3*a*e^2)*x)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*d*x^4) + c^(5/2)*d^(5/2)*e^(3/2)*Ar
cTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])] + ((5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c
^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^
2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(128*a^(3/2)*d^(5/2)*e^(3/2))
```

Rubi [A] time = 0.45556, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 810, 843, 621, 206, 724}

$$-\frac{\left(x\left(11a^2cd^2e^4-3a^3e^6+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64ad^2ex^2}+\frac{\left(-90a^2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```


```
[Out] -((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (5*c^3*d^6 + 83*a*c^2*d^4*
e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(64*a*d^2*e*x^2) - ((6*a*d*e + (11*c*d^2 + 3*a*e^2)*x)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*d*x^4) + c^(5/2)*d^(5/2)*e^(3/2)*Ar
cTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])] + ((5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c
^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^
2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(128*a^(3/2)*d^(5/2)*e^(3/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
```

```
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5}\,dx$$

$$=-\frac{\left(6ade+\left(11cd^2+3ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24dx^4}-\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-ae^2\right)(c\right.}{}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6\right)x\right)}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6\right)x\right)}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6\right)x\right)}{64ad^2ex^2}$$

$$=-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6\right)x\right)}{64ad^2ex^2}$$

Mathematica [A] time = 3.72771, size = 404, normalized size = 1.

$$\frac{\sqrt{ae+cdx}\left(-\frac{\sqrt{d}\sqrt{e}(d+ex)\sqrt{ae+cdx}\left(a^2cd^2e^2x\left(136d^2+244dex+57e^2x^2\right)+3a^3e^3\left(24d^2ex+16d^3+2de^2x^2-3e^3x^3\right)+ac^2d^4ex^2(118d+337ex)+15c^3d^6x^3\right)}{ax^4}+\frac{3\sqrt{d+ex}}{}\right.}{192d^{5/2}e^{3/2}\sqrt{(d+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*(-((Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(d + e*x)*(15*c^3*
d^6*x^3 + a*c^2*d^4*e*x^2*(118*d + 337*e*x) + a^2*c*d^2*e^2*x*(136*d^2 + 24
4*d*e*x + 57*e^2*x^2) + 3*a^3*e^3*(16*d^3 + 24*d^2*e*x + 2*d*e^2*x^2 - 3*e^
3*x^3)))/(a*x^4)) + 384*c^(3/2)*d^4*Sqrt[c*d]*e^3*Sqrt[c*d^2 - a*e^2]*Sqrt[
(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e
+ c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])] + (3*(5*c^4*d^8 - 60*a*c^3*d^6*e
^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*Sqrt[d + e*x]*ArcTa
nh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/a^(3/2)))/
(192*d^(5/2)*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.087, size = 3646, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x)
```

```
[Out] 3/64*e^8/d^5*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/16*e^7/d^6*a^2
/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+1/8*e^6/d^5*a*(a*d*e+(a*e^
```

2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-7/64*e^4/d^3*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)*x-9/64*e^7/d^4*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*
x+3/128*e^10/d^7*a^4/c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+3/64
*e^4/d*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-13/32*e/d^4/a/x^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+19/96/e^2*d/a^3*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)*c^3+25/64*e^2/d^5/a/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(7/2)-15/32*e^2/d^3/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c-15/32*e*d
^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^3+35/96/e*d^2/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(3/2)*c^3+85/192*d/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*c
^3+43/192/e/a^3*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x+5/32*e^5*a^2/
(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/x)*c+31/64/d^3/a^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(7/2)*c-17/64*e^4/d*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/128*e
^7/d^2*a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+9/64*e^7/d^4*a^2*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)*x-3/128*e^10/d^7*a^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+15/128*e^8/d^3*a^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+253/256*e^2*d^3*c^3
*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2))/(d*e*c)^(1/2)+1/16*e^7/d^6*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
x^2)^(3/2)-1/4/d^2/a/e/x^4(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)-5/192*d
^4/a^3/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^4-1/64*d^3/a^4/e^4*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^4-5/64*d^5/a^2/e^2*c^4*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+1/96/a^3/e^3/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(7/2)*c^2-15/128*e^8/d^3*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e
*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/25
6*e^2*d^3*c^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1/8*e^6/d^5*a*(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x-3/64*e^8/d^5*a^3/c*(c*d*e*(d/e+x)^2
+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/8*e^4/d^3*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*
(d/e+x))^(3/2)*x-1/5*e^4/d^5*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)-
61/320*e^4/d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/8/d^3/a/x^3*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+25/32*d^3/a*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)+1/96/d/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^2+35/96
e/a(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2+1/64*e^5/d^4*a*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/64*e^6/d^3*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-19/96*e^3/d^2*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+127/128*
e^2*d*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/8*e^3*c^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3/64*e^3*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d
/e+x))^(1/2)*x+1/16*e^3/d^2*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)
-3/128*e^2*d*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-15/256*e^10/
d^5*a^4/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+15/256*e^4*d*a*c^2*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c
)^(1/2)-3/64*e^9/d^6*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3/32*e
^5/d^2*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+3/256*e^12/d^7*a^5/c^2
*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2))/(d*e*c)^(1/2)-15/128*e^6/d*a^2*c*ln((1/2*a*e^2+1/2*c*d^2+c*d*
e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-4
3/192/e^2/d/a^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^2-25/64*e^3/d^4
/a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x-31/64*e/d^2/a^2*c^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x-13/48/e/d^2/a^2/x^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(7/2)*c-45/64*e^3*d^2*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2
)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2-35/192*
e^2/d/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*c^2+15/256*e^10/d^5*a^4/c
*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+3/64*e^9/d^6*a^3/c*(c*d*e*(d/e+x)^2
+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+9/64*e^5/d^2*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*

```
d^2)*(d/e+x))^(1/2)*x-3/256*e^12/d^7*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x
)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*
c)^(1/2)+15/128*e^6/d*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^
(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-15/256*e
^4*d*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e
+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+45/64*e*d^2/a*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3-1/64*d^2/a^4/e^3*c^4*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)*x+1/24/d/a^2/e^2/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(7/2)*c-5/192*d^3/a^3/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*c^4-5/
64*d^4/a^2/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^4+5/128*d^6/a/e/(a
*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/x)*c^4+1/64*d/a^4/e^4/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(7/2)*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^5), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.466 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=289

$$\frac{3\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^2d^3e^2x^2}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}$$

```
[Out] (3*(c*d^2 - a*e^2)^3*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(128*a^2*d^3*e^2*x^2) - ((c/(a*e) - e/d^2)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(16*x^4)
- (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*d*x^5) - (3*(c*d^2 - a*e
^2)^5*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(5/2)*d^(7/2)*e^(5/2))
```

Rubi [A] time = 0.327656, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 806, 720, 724, 206}

$$\frac{3\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^2d^3e^2x^2}-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(128*a^2*d^3*e^2*x^2) - ((c/(a*e) - e/d^2)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(16*x^4)
- (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*d*x^5) - (3*(c*d^2 - a*e
^2)^5*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5dx^5}-\frac{\left(-2acd^2e+ae\left(cd^2+ae^2\right)\right)\int\frac{\left(ade+\left(cd^2+ae\right.\right.}{x}}{2ade}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{16x^4}-\frac{\left(ade+\left(\right.\right.}{}$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-\frac{\left(\frac{c}{ae}-\frac{e}{d}\right.}{}$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-\frac{\left(\frac{c}{ae}-\frac{e}{d}\right.}{}$$

$$=\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-\frac{\left(\frac{c}{ae}-\frac{e}{d}\right.}{}$$

Mathematica [A] time = 0.960561, size = 295, normalized size = 1.02

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(\frac{5\left(cd^2-ae^2\right)\left(\frac{x\left(cd^2-ae^2\right)\left(\frac{x\left(ae^2-cd^2\right)\left(3x^2\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(ae(2d+5ex)-3cd^2x\right)\right)}{a^{5/2}\sqrt{d}e^{5/2}}-8(d+ex)^{5/2}\sqrt{ae+cdx}\right)}{d}\right)}{64dx^4(d+ex)^{3/2}(ae+cdx)^{3/2}}\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*((-2*(a*e + c*d*x)*(d + e*x))/x^5 + (5*(c*
d^2 - a*e^2)*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a*e^2)*x*
(-8*Sqrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt[a]*Sqrt
[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)
) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sq
rt[e]*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d))/(64*d*x^4*(a*e + c*
d*x)^(3/2)*(d + e*x)^(3/2))))/(10*d)
```

Maple [B] time = 0.102, size = 3991, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x)
```

```
[Out] -17/640*d^2/a^4/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^4+3/640*d^4/a
^5/e^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^5+25/128*e^5/d^6*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-15/128*e^3*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+1/5*e^5/d^6*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)+3/128*
e^3*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/16*e^4/d^3*c*(c*d*e
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+3/32*e*d^2/a*c^3*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)-15/128*e^2*d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2
)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^3-31/80*e
/d^4/a/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+1/5/d/a^3*c^3*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x+109/320/d^3/a^2/x^2*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(7/2)*c-1/5/d^2/a/e/x^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+
1/128*d^5/a^4/e^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^5+3/128*d^6/a^3
/e^3*c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-9/128*d^4/a^2/e*c^4*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/80/a^3/e^3/x^3*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(7/2)*c^2+11/320/a^4/e^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2
)*c^3+1/16*e^8/d^7*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)+9/64
*e^8/d^5*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/128*e^11/d^8
*a^4/c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-3/64*e^5/d^2*a*c*(c*
d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+15/128*e^9/d^4*a^3*ln((1/2*a*e^2
-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e
+x))^(1/2))/(d*e*c)^(1/2)+3/64*e^4/d*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/
e+x))^(1/2)*x-3/256*e^3*d^2*c^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e
*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+1/8*
e^7/d^6*a*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+3/64*e^9/d^6*a^3/
c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/8*e^5/d^4*c*(c*d*e*(d/e+x
)^2+(a*e^2-c*d^2)*(d/e+x))^(3/2)*x+15/256*e^5*a*c^2*ln((1/2*a*e^2-1/2*c*d^2
+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
)/(d*e*c)^(1/2)-3/64*e^9/d^6*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
15/128*e^5/d^2*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-15/128*e^9/d^4*a
^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/(d*e*c)^(1/2)-1/16*e^8/d^7*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)-9/64*e^8/d^5*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+3/
128*e^11/d^8*a^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/256*e^3*d^2*
c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-15/256*e^5*a*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+
3/256*e^8/d^3*a^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)
```

```
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-1/8*e^7/d^6*a*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)*x+15/128*e^5/d^4*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)*x-7/128*d^3/a^3/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^4+15/
256*d^5/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^4-3/64*d^3/a^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)*x*c^4+273/640*e^3/d^4/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)*c+129/320*e^2/d^5/a/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+47/
160*e/d^2/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^2-253/640*e^3/d^6/a
/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)-15/256*e^11/d^6*a^4/c*ln((1/2*a*
e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(
d/e+x))^(1/2))/(d*e*c)^(1/2)-3/64*e^10/d^7*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2)*x-9/64*e^6/d^3*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+
x))^(1/2)*x+3/256*e^13/d^8*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(
d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-1
5/128*e^7/d^2*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c
*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+15/256*e^11/d^6*
a^4/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+3/64*e^10/d^7*a^3/c*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)*x+15/128*e^6/d^3*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)*x-3/256*e^13/d^8*a^5/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+15/128*e^7/d^2*a^2*
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/(d*e*c)^(1/2)+139/320*e^2/d^3/a^2*c^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)*x+15/128*e^4*d*a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*
x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2-1/5/e/d^2
/a^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^2-139/320*e/d^4/a^2/x*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c-5/64*d^2/a^3/e*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)*x*c^4-1/5/d^2/a^2/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(7/2)*c+3/40/d/a^2/e^2/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c-3/256
*d^7/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^5+19/320/d/a^3/e^2/x^2*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^2-11/320*d/a^4/e^2*c^4*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(5/2)*x+3/128*d^5/a^3/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)*x*c^5-1/320*d/a^4/e^4/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^3+
3/640*d^3/a^5/e^4*c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x-3/640*d^2/a
^5/e^5/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^4+1/128*d^4/a^4/e^3*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*c^5-15/256*e^6/d*a^2/(a*d*e)^(1/2)*ln
((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/x)*c+5/64*e^3/d^2/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x*c^2+2
53/640*e^4/d^5/a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x+17/160/e/a^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^3-1/128*e^6/d^5*a*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)-3/128*e^7/d^4*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)+15/128*e^4/d^3*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+13/40/d^3/a
/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.467 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=386

$$-\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512a^3d^4e^3x^2}+\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+\right.\right.}{}$$

```
[Out] -((c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^3*d^4*e^3*x^2) + ((c*d^2 - a
*e^2)*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))/(192*a^2*d^3*e^2*x^4) - (a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2)/(6*d*x^6) - (((5*c)/(a*e) - (7*e)/d^2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(60*x^5) + ((c*d^2 - a*e^2)^5*(5*c*d^2 +
7*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*a^(7/2)*d^(9/2)*e^(7/2)
)
```

Rubi [A] time = 0.49468, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512a^3d^4e^3x^2}+\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] -((c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^3*d^4*e^3*x^2) + ((c*d^2 - a
*e^2)*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))/(192*a^2*d^3*e^2*x^4) - (a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(5/2)/(6*d*x^6) - (((5*c)/(a*e) - (7*e)/d^2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(60*x^5) + ((c*d^2 - a*e^2)^5*(5*c*d^2 +
7*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*a^(7/2)*d^(9/2)*e^(7/2)
)
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
```

```
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{x^7(d + ex)}\,dx = \int \frac{(ae + cdx)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{x^7}\,dx$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{6dx^6} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-7ae^2\right)+acde^2x\right)\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{x^6}}{6ade}$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae} - \frac{7e}{d^2}\right)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5}}{60x^5}$$

$$= \frac{\left(cd^2 - ae^2\right)\left(5cd^2 + 7ae^2\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right.}{192a^2d^3e^2x^4}$$

$$= -\frac{\left(cd^2 - ae^2\right)^3\left(5cd^2 + 7ae^2\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + c}}{512a^3d^4e^3x^2}$$

$$= -\frac{\left(cd^2 - ae^2\right)^3\left(5cd^2 + 7ae^2\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + c}}{512a^3d^4e^3x^2}$$

$$= -\frac{\left(cd^2 - ae^2\right)^3\left(5cd^2 + 7ae^2\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{ade + \left(cd^2 + ae^2\right)x + c}}{512a^3d^4e^3x^2}$$

Mathematica [A] time = 1.02725, size = 344, normalized size = 0.89

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(-\frac{\left(7ae^2+5cd^2\right)\left(5x\left(cd^2-ae^2\right)\left(\frac{x\left(cd^2-ae^2\right)\left(\frac{x\left(ae^2-cd^2\right)\left(3x^2\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(ae(2d+5ex)-3cd^2x\right)\right)}{a^{5/2}\sqrt{d}e^{5/2}}-8\right.}{d}\right.}{1280d^2x^5(d+ex)^{3/2}(ae+cdx)^{3/2}}\right.}{6ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^6) - ((5*
c*d^2 + 7*a*e^2)*(-128*d*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) + 5*(c*d^2 - a
*e^2)*x*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a*e^2)*x*(-8*S
qrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)) + 3
*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]
*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d)))/(1280*d^2*x^5*(a*e + c*
d*x)^(3/2)*(d + e*x)^(3/2))))/(6*a*d*e)
```

Maple [B] time = 0.121, size = 4735, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x)
```

```
[Out] -1/5*e^6/d^7*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(5/2)-101/512*e^6/d^7*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-59/320/d/a^3*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)*c^3-35/384*e^5/d^4*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)+25/512*e^4/d*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+7/1536*e^7/d^6
*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+7/512*e^8/d^5*a^2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+1/16*e^5/d^4*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/
e+x))^(3/2)-3/128*e^4/d*c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1
7/60/d^3/a/x^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+1/512*d^3/a^2*c^4*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-2681/7680*e^2/d^3/a^2*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(5/2)*c^2-1543/3840*e^3/d^6/a/x^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(7/2)-35/1536*e^3/d^2/a*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)+1017/2560*e^4/d^7/a/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)-1/64*e^2
*d/a*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-397/960*e^4/d^5/a*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c+377/960*e^2/d^5/a/x^3*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(7/2)+5/256*e^3*d^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*
x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^3-5/256*e^3
/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3-45/1024*e^5*a/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/x)*c^2+381/1280/d^3/a^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/
2)*c^2+9/64*e^9/d^6*a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3/128*e^1
2/d^9*a^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/128*e^10/d^5*a^3*l
n((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/(d*e*c)^(1/2)+15/512*e^5/d^2*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)*x-3/256*e^4*d*c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-185/1536*e^6/d^5*c*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-7/1024*e^9/d^4*a^3/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/x)+3/64*e^10/d^7*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/6
4*e^6/d^3*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*e^8/d^7*a*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/16*e^9/d^8*a^2/c*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)-57/160*e/d^4/a/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/
2)-221/7680/e^2*d/a^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^4+1/120/e^4
*d^3/a^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c^5+49/1536/e*d^2/a^3*c^4*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+7/384/e^3*d^4/a^4*c^5*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(3/2)+1/64/e^2*d^5/a^3*c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)+89/320/d^3/a^2/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c+3
5/768*d/a^3*c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-81/1280/e/a^4*c^4
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x-11/480/e^3/a^4/x^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(7/2)*c^3-1/16*e^9/d^8*a^2/c*(c*d*e*(d/e+x)^2+(a*e^2-c
*d^2)*(d/e+x))^(3/2)-9/64*e^9/d^6*a^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x
))^(1/2)*x+3/128*e^12/d^9*a^4/c^2*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
1/2)+3/64*e^6/d^3*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-15/128*
e^10/d^5*a^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d
/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-3/64*e^5/d^2*c^2*(c*d*e
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-1/8*e^8/d^7*a*(c*d*e*(d/e+x)^2+(a
*e^2-c*d^2)*(d/e+x))^(3/2)*x-3/64*e^10/d^7*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*
d^2)*(d/e+x))^(1/2)+1/8*e^6/d^5*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
3/2)*x+3/256*e^4*d*c^3*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)
+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-5/1536*d^6/a^
5/e^5*c^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-5/512*d^7/a^4/e^4*c^6*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/6/d^2/a/e/x^6*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(7/2)-1/512*d^5/a^6/e^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
*c^6-1/32/a^3/e^3/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^2-65/512*e^
7/d^4*a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+3/256*e^14/d^9*a^5/c^2*
ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e
```

```
*x^2)^(1/2))/(d*e*c)^(1/2)-15/128*e^8/d^3*a^2*c*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-
89/7680/e^4*d/a^5/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^4+43/1536/e^2
*d^3/a^4*c^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-113/640/e/d^2/a^3/x^
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^2+7/256*e*d^2/a^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^4+3/512/e*d^4/a^3*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*x*c^5+15/1024*e*d^4/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)
*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^4-9/512/e*
d^6/a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^5+29/320/e^2/d/a^3/x^3*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(7/2)*c^2+89/7680/e^3*d^2/a^5*c^5*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)*x+81/1280/e^2/d/a^4/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
7/2)*c^3-43/240/e/d^2/a^2/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c+15/
512*e^7/d^2*a^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c-11/30*e/d^4/a^2/x^2*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c-65/1536*e^4/d^3/a*c^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)*x-1017/2560*e^5/d^6/a*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)*x-3211/7680*e^3/d^4/a^2*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
*x-381/1280*e/d^2/a^3*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x+3211/76
80*e^2/d^5/a^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c-25/768*e^2/d/a^2
*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/12/d/a^2/e^2/x^5*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c+1/512*d^3/a^6/e^6/x*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(7/2)*c^5-5/1536*d^5/a^5/e^4*c^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)*x-5/512*d^6/a^4/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^6
+5/1024*d^8/a^3/e^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^6+1/192*d/a^4/e^4/x^3*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)*c^3+1/768*d^2/a^5/e^5/x^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(7/2)*c^4-1/512*d^4/a^6/e^5*c^6*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)*x+15/256*e^12/d^7*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c
*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^
(1/2)+3/64*e^11/d^8*a^3/c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+9
/64*e^7/d^4*a*c*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-3/256*e^14/
d^9*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/
e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)+15/128*e^8/d^3*a^2*c*ln(
(1/2*a*e^2-1/2*c*d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c
*d^2)*(d/e+x))^(1/2))/(d*e*c)^(1/2)-15/256*e^6/d*a*c^2*ln((1/2*a*e^2-1/2*c*
d^2+(d/e+x)*c*d*e)/(d*e*c)^(1/2)+(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1
/2))/(d*e*c)^(1/2)+15/256*e^6/d*a*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e
*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-15/256*e^1
2/d^7*a^4/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-3/64*e^11/d^8*a^3/c*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^7), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.468 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx$

Optimal. Leaf size=500

$$\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024a^4d^5e^4x^2}-\frac{\left(9a^2e^4+10acd^2e^2+5\right.}{}$$

```
[Out] ((c*d^2 - a*e^2)^3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*a^4*d^5*e
^4*x^2) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*
a^3*d^4*e^3*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(7*d*x^7)
- (((5*c)/(a*e) - (9*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))
/(84*x^6) + ((35*c^2*d^4 + 20*a*c*d^2*e^2 - 63*a^2*e^4)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(5/2))/(840*a^2*d^3*e^2*x^5) - ((c*d^2 - a*e^2)^5*(5*c
^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/
(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
2048*a^(9/2)*d^(11/2)*e^(9/2))
```

Rubi [A] time = 0.639465, antiderivative size = 500, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 834, 806, 720, 724, 206}

$$\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024a^4d^5e^4x^2}-\frac{\left(9a^2e^4+10acd^2e^2+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*a^4*d^5*e
^4*x^2) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*
a^3*d^4*e^3*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(7*d*x^7)
- (((5*c)/(a*e) - (9*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))
/(84*x^6) + ((35*c^2*d^4 + 20*a*c*d^2*e^2 - 63*a^2*e^4)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(5/2))/(840*a^2*d^3*e^2*x^5) - ((c*d^2 - a*e^2)^5*(5*c
^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/
(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
2048*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
```

```
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^8}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-9ae^2\right)+2acde^2x\right)\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{x^7}}{7ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5}}{84x^6}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5}}{84x^6}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+\right.\right.}{384a^3d^4e^3x^4}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2\right.}}{1024a^4d^5e^4x^2}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2\right.}}{1024a^4d^5e^4x^2}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2\right.}}{1024a^4d^5e^4x^2}$$

Mathematica [A] time = 0.780827, size = 408, normalized size = 0.82

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(\frac{7\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(5x\left(cd^2-ae^2\right)\left(\frac{x\left(cd^2-ae^2\right)\left(\frac{x\left(ae^2-cd^2\right)\left(3x^2\left(cd^2-ae^2\right)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)+\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(ae(2d+5e\right.\right.}{a^{5/2}\sqrt{d}e^{5/2}}\right.}{d}\right.\right.}{15360ad^3ex^5(d+ex)^{3/2}(ae+cdx)^{3/2}}\right.}{7ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^7) + ((7*
c*d^2 + 9*a*e^2)*(a*e + c*d*x)^2*(d + e*x))/(12*a*d*e*x^6) + (7*(5*c^2*d^4
+ 10*a*c*d^2*e^2 + 9*a^2*e^4)*(-128*d*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) +
 5*(c*d^2 - a*e^2)*x*(-16*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + ((c*d^2 - a
*e^2)*x*(-8*Sqrt[a*e + c*d*x]*(d + e*x)^(5/2) + ((-(c*d^2) + a*e^2)*x*(Sqrt
[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-3*c*d^2*x + a*e*(2*d
+ 5*e*x)) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sq
rt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(a^(5/2)*Sqrt[d]*e^(5/2))))/d)))/(15360*a*d
^3*e*x^5*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(7*a*d*e)
```

Maple [B] time = 0.138, size = 5353, normalized size = 10.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.469 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

Optimal. Leaf size=628

$$-\frac{3\left(45a^2cd^2e^4+33a^3e^6+35ac^2d^4e^2+15c^3d^6\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384a^5d^6e^5x^2}+\frac{\left(45a^2\right.}{}$$

```
[Out] (-3*(c*d^2 - a*e^2)^3*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 3
3*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(16384*a^5*d^6*e^5*x^2) + ((c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^
2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*a^4*d^5*e^4*x^4) - (a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(8*d*x^8) - (((5*c)/(a*e) - (11*e)
/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*x^7) + ((15*c^2*d
^4 + 10*a*c*d^2*e^2 - 33*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(448*a^2*d^3*e^2*x^6) - ((105*c^3*d^6 + 95*a*c^2*d^4*e^2 + 15*a^2*c*d
^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4480*
a^3*d^4*e^3*x^5) + (3*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45
*a^2*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*a^
(11/2)*d^(13/2)*e^(11/2))
```

Rubi [A] time = 0.885273, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {849, 834, 806, 720, 724, 206}

$$-\frac{3\left(45a^2cd^2e^4+33a^3e^6+35ac^2d^4e^2+15c^3d^6\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384a^5d^6e^5x^2}+\frac{\left(45a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```


```
[Out] (-3*(c*d^2 - a*e^2)^3*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 3
3*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(16384*a^5*d^6*e^5*x^2) + ((c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^
2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*a^4*d^5*e^4*x^4) - (a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(8*d*x^8) - (((5*c)/(a*e) - (11*e)
/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*x^7) + ((15*c^2*d
^4 + 10*a*c*d^2*e^2 - 33*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(448*a^2*d^3*e^2*x^6) - ((105*c^3*d^6 + 95*a*c^2*d^4*e^2 + 15*a^2*c*d
^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4480*
a^3*d^4*e^3*x^5) + (3*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45
*a^2*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*a^
(11/2)*d^(13/2)*e^(11/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^9}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-11ae^2\right)+3acde^2x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{x^8}}{8ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7}$$

$$= \frac{\left(cd^2-ae^2\right)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)x\right)(}{2048a^4d^5e^4x^4}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)\right.}{16384a^5d^6e^5x^2}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)\right.}{16384a^5d^6e^5x^2}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)\right.}{16384a^5d^6e^5x^2}$$

Mathematica [A] time = 1.50025, size = 512, normalized size = 0.82

$$\frac{((d+ex)(ae+cdx))^{3/2}\left(-\frac{(d+ex)\left(33a^2e^4+34acd^2e^2+21c^2d^4\right)(ae+cdx)^2}{56a^2d^2e^2x^6}+\frac{\left(45a^2cd^2e^4+33a^3e^6+35ac^2d^4e^2+15c^3d^6\right)\left(5x\left(cd^2-ae^2\right)\left(x\left(cd^2-ae^2\right)\left(8a^{5/2}\sqrt{d}e^{5/2}\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(3/2)*(-(((a*e + c*d*x)^2*(d + e*x))/x^8) + ((9*
c*d^2 + 11*a*e^2)*(a*e + c*d*x)^2*(d + e*x))/(14*a*d*e*x^7) - ((21*c^2*d^4
+ 34*a*c*d^2*e^2 + 33*a^2*e^4)*(a*e + c*d*x)^2*(d + e*x))/(56*a^2*d^2*e^2*x
^6) + ((15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(128
*a^(5/2)*d^(5/2)*e^(5/2)*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2) + 5*(c*d^2 - a
*e^2)*x*(16*a^(5/2)*d^(3/2)*e^(5/2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(5/2) + (
c*d^2 - a*e^2)*x*(8*a^(5/2)*Sqrt[d]*e^(5/2)*Sqrt[a*e + c*d*x]*(d + e*x)^(5/
2) + (c*d^2 - a*e^2)*x*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*(-3*c*d^2*x + a*e*(2*d + 5*e*x)) + 3*(c*d^2 - a*e^2)^2*x^2*ArcTanh[(Sq
rt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))))/(10240*a^(9/
2)*d^(11/2)*e^(9/2)*x^5*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(8*a*d*e)
```

Maple [B] time = 0.201, size = 6030, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.470 $\int \frac{x^3}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=271

$$\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}-\frac{3\left(ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^2d^2e^3}-\frac{2d^3\sqrt{x}}{e}$$

```
[Out] (-3*(3*c*d^2 + a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^2*d
^2*e^3) - (2*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(c*d^2 -
 a*e^2)*(d + e*x)) + ((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(2*c*d*e^3) + (3*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a
*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(7/2))
```

Rubi [A] time = 0.341119, antiderivative size = 298, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 779, 621, 206}

$$\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)-2cdex\left(5cd^2-ae^2\right)\right)\sqrt{x}}{4c^2d^2e^3\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(e*(c*d^2 - a*e^2)
^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (((5*c*d^2 - 3*a*e^2)*(3*
c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(4*c^2*d^2*e^3*(c*d^2 - a*e^2)) + (3*(5*c^2*d^4 + 2*a*c*d^2
*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(7/
2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
```

```
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\int \frac{x^3(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$=-\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\int \frac{x\left(2acd^2e\left(cd^2-ae^2\right)+\frac{1}{2}cd}{\sqrt{ade+\left(cd^2+\right.}}}{cde\left(cd^2\right.}$$

$$=-\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+\right.\right.}{}$$

$$=-\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+\right.\right.}{}$$

$$=-\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{e\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+\right.\right.}{}$$

Mathematica [A] time = 0.536852, size = 331, normalized size = 1.22

$$\frac{c^{3/2}d^{3/2}\sqrt{e}\left(a^2cde^3\left(4d^2+5dex+e^2x^2\right)+3a^3e^5(d+ex)-ac^2d^2e\left(d^2ex+15d^3-4de^2x^2+2e^3x^3\right)+c^3d^4x\left(-15d^2-5de\right.\right.}{4c^{7/2}d^{7/2}e^{7/2}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (c^(3/2)*d^(3/2)*Sqrt[e]*(3*a^3*e^5*(d + e*x) + a^2*c*d*e^3*(4*d^2 + 5*d*e*
x + e^2*x^2) + c^3*d^4*x*(-15*d^2 - 5*d*e*x + 2*e^2*x^2) - a*c^2*d^2*e*(15*
d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)) + 3*Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]
*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[a*e + c*d*x]*
Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(4*c^(7/2)*d^(7/2)*e^(7/2)
*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.06, size = 391, normalized size = 1.4

$$\frac{x}{2\,de^2c}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}-\frac{3\,a}{4\,c^2d^2e}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}-\frac{7}{4\,e^3c}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}+\frac{3\,a^2e}{8\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/2/e^2*x/d/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4/e/d^2/c^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-7/4/e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+3/8*e/d^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+3/4/e/c*ln((1/2*a*e^
2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(d*e*c)^(1/2)*a+15/8*d^2/e^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+2*d^3/e^4/(a*e^2-c
*d^2)/(d/e+x)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.75561, size = 1530, normalized size = 5.65

$$\left[\frac{3\left(5\,c^3d^7-3\,ac^2d^5e^2-a^2cd^3e^4-a^3de^6+\left(5\,c^3d^6e-3\,ac^2d^4e^3-a^2cd^2e^5-a^3e^7\right)x\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(5*c^3*d^7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*
d^6*e - 3*a*c^2*d^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d*e)*log(8*c^2
*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*
e + a*c*d*e^3)*x) - 4*(15*c^3*d^6*e - 4*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2
*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a
^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^6*e^4 -
a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)*x), -1/8*(3*(5*c^3*d^7 - 3*a*
c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*d^6*e - 3*a*c^2*d^4*e^3 -
a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*
x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*c^3*d^6*e - 4*a*c^2
*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d
^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c^4*d^6*e^4 - a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)
*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.471 $\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{(ae^2+3cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}+\frac{2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cde^2}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d*e^2) + (2*d^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(c*d^2 - a*e^2)*(d + e*x)) - ((3*c*d^
2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*e^(5/2))
```

Rubi [A] time = 0.346458, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1638, 792, 621, 206}

$$-\frac{(ae^2+3cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}+\frac{2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cde^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d*e^2) + (2*d^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(c*d^2 - a*e^2)*(d + e*x)) - ((3*c*d^
2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2}+\frac{\int \frac{-\frac{1}{2}de\left(cd^2+ae^2\right)-\frac{1}{2}e^2\left(3cd^2+ae^2\right)x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cde^3}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2}+\frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}-\frac{1}{2}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2}+\frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}-\Big($$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2}+\frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2\left(cd^2-ae^2\right)(d+ex)}-$$

Mathematica [A] time = 0.377895, size = 255, normalized size = 1.31

$$\frac{c^{3/2}d^{3/2}\sqrt{e}\left(-a^2e^3(d+ex)+acde\left(3d^2-e^2x^2\right)+c^2d^3x(3d+ex)\right)-\sqrt{cd}\sqrt{cd^2-ae^2}\left(-a^2e^4-2acd^2e^2+3c^2d^4\right)\sqrt{ae+cd}}{c^{5/2}d^{5/2}e^{5/2}\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (c^(3/2)*d^(3/2)*Sqrt[e]*(-(a^2*e^3*(d + e*x)) + c^2*d^3*x*(3*d + e*x) + a*
c*d*e*(3*d^2 - e^2*x^2)) - Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*(3*c^2*d^4 - 2*a*c
*d^2*e^2 - a^2*e^4)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]
*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2
- a*e^2])])/(c^(5/2)*d^(5/2)*e^(5/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

Maple [A] time = 0.059, size = 241, normalized size = 1.2

$$\frac{1}{de^2c}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}-\frac{a}{2\,cd}\ln\left(\left(\frac{ae^2}{2}+\frac{cd^2}{2}+cdex\right)\frac{1}{\sqrt{dec}}+\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}\right)\frac{1}{\sqrt{dec}}-\frac{3\,d}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] (a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/e^2/c-1/2/d/c*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c
)^(1/2)*a-3/2/e^2*d*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-2*d^2/e^3/(a*e^2-c*d^2)/(d/e
+x)*(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.6423, size = 1195, normalized size = 6.13

$$\left[\frac{\left(3\,c^2d^5-2\,acd^3e^2-a^2de^4+\left(3\,c^2d^4e-2\,acd^2e^3-a^2e^5\right)x\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+ad\cdots}\right.}{4\left(c^3d^5e^3-ac\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/4*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3
 - a^2*e^5)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(3*c^2*d^4*e - a*
c*d^2*e^3 + (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x),
 1/2*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3
 - a^2*e^5)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2
*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*c^2*d^4*e - a*c*d^2*e^3 + (c^2*d^
3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^5
*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.472 $\int \frac{x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

```
[Out] (-2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d +
e*x)) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rubi [A] time = 0.109496, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {792, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}-\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```


```
[Out] (-2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d +
e*x)) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*e^(3/2))
```


Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)} + \frac{\int \frac{1}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{e}$$

$$= -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)} + \frac{2\,\text{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae}{\sqrt{ade+\left(cd^2\right.}}\right.}{e}$$

$$= -\frac{2d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e\left(cd^2-ae^2\right)(d+ex)} + \frac{\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}\right.}{\sqrt{c}\sqrt{d}e^{3/2}}$$

Mathematica [A] time = 0.418805, size = 189, normalized size = 1.36

$$\frac{2\sqrt{cd}\left(cd^2-ae^2\right)^{3/2}\sqrt{ae+cdx}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cd^2-ae^2}}\right)-2c^{3/2}d^{5/2}\sqrt{e}(ae+cdx)}{c^{3/2}d^{3/2}e^{3/2}\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*c^(3/2)*d^(5/2)*Sqrt[e]*(a*e + c*d*x) + 2*Sqrt[c*d]*(c*d^2 - a*e^2)^(3/
2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(c^(3
/2)*d^(3/2)*e^(3/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.055, size = 131, normalized size = 0.9

$$\frac{1}{e}\ln\left(\left(\frac{ae^2}{2}+\frac{cd^2}{2}+cdex\right)\frac{1}{\sqrt{dec}}+\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}\right)\frac{1}{\sqrt{dec}}+2\frac{d}{e^2\left(ae^2-cd^2\right)}\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2-cd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+2*d/e^2/(a*e^2-c*d^2)/(d/e+x)*(c*d*e*(d/e+x)
^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.51495, size = 922, normalized size = 6.63

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}cd^2e-\left(cd^3-ade^2+\left(cd^2e-ae^3\right)x\right)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{\cdots}\right.}{2\left(c^2d^4e^2-acd^2e^4+\left(c^2d^3e^3-acde^5\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d^2*e - (c*d^3 - a*d
*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6
*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c
*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^2*d^
4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)*x), -(2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*c*d^2*e + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*
sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)))/(c^2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)
*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.473 $\int \frac{1}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

Rubi [A] time = 0.0235734, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {650}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\left(cd^2-ae^2\right)(d+ex)}$$

Mathematica [A] time = 0.0156808, size = 42, normalized size = 0.81

$$\frac{2(ae+cdx)}{\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.048, size = 51, normalized size = 1.

$$-2\frac{cdx+ae}{\left(ae^{2}-cd^{2}\right)\sqrt{cde\,x^{2}+ae^{2}x+cd^{2}x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2*(c*d*x+a*e)/(a*e^2-c*d^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48177, size = 117, normalized size = 2.25

$$\frac{2\sqrt{cde\,x^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{cd^{3}-ade^{2}+\left(cd^{2}e-ae^{3}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e -
 a*e^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError
```

3.474 $\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}$$

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.174722, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 12, 724, 206}

$$-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\int -\frac{ae\left(cd^2-ae^2\right)^2}{2x\sqrt{ade+\left(cd^2+ae^2\right)x+}}}{ade\left(cd^2-ae^2\right)^{}}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\int \frac{1}{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}}{d}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{4ade-x^2}\,dx,\right.}{}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\tanh^{-1}\left(\frac{2ade+(c}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+}}\right.}{\sqrt{a}d^{3/2}\sqrt{}}$$

Mathematica [A] time = 0.139461, size = 131, normalized size = 0.92

$$\frac{2\left(-\frac{\sqrt{d}e^{3/2}(ae+cdx)}{cd^2-ae^2}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}\right)}{d^{3/2}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(-((Sqrt[d]*e^(3/2)*(a*e + c*d*x))/(c*d^2 - a*e^2)) - (Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d
+ e*x])])/Sqrt[a]))/(d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.061, size = 136, normalized size = 1.

$$-\frac{1}{d}\ln\left(\frac{1}{x}\left(2\,ade+\left(ae^{2}+cd^{2}\right)x+2\,\sqrt{ade}\sqrt{ade+\left(ae^{2}+cd^{2}\right)x+cdex^{2}}\right)\right)\frac{1}{\sqrt{ade}}+2\,\frac{1}{d\left(ae^{2}-cd^{2}\right)}\sqrt{cde\left(\frac{d}{e}+x\right)^{2}+\left(ae^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -1/d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+2/d/(a*e^2-c*d^2)/(d/e+x)*(c*d*e*(d/e+x)^2+
(a*e^2-c*d^2)*(d/e+x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x), x)
```

Fricas [A] time = 4.64008, size = 938, normalized size = 6.56

$$\left[-\frac{4\,\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}ade^{2}-\left(cd^{3}-ade^{2}+\left(cd^{2}e-ae^{3}\right)x\right)\sqrt{ade}\log\left(\frac{8\,a^{2}d^{2}e^{2}+\left(c^{2}d^{4}+6\,acd^{2}e^{2}+a^{2}e^{4}\right)x^{2}-4\,\sqrt{cdex^{2}+ade+\left(cd^{2}\right.}}{x^{2}}\right.}{2\left(acd^{5}e-a^{2}d^{3}e^{3}+\left(acd^{4}e^{2}-a^{2}d^{2}e^{4}\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d
*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2
))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 - a^2*d^2*e^4)*x), -(2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d*e^2 + (c*d^2*e -
a*e^3)*x)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^
2 + (a*c*d^3*e + a^2*d*e^3)*x)))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 -
a^2*d^2*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.475 $\int \frac{1}{x^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=229

$$\frac{\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ad^2ex\left(cd^2-ae^2\right)}-\frac{2e(ae\ -}{dx\left(cd^2-ae^2\right)\sqrt{x\left(ae^2\right.}}$$

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]) - ((c*d^2 - 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(a*d^2*e*(c*d^2 - a*e^2)*x) + ((c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d
^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(2*a^(3/2)*d^(5/2)*e^(3/2))
```

Rubi [A] time = 0.292443, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 806, 724, 206}

$$\frac{\left(3ae^2+cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{ad^2ex\left(cd^2-ae^2\right)}-\frac{2e(ae\ -}{dx\left(cd^2-ae^2\right)\sqrt{x\left(ae^2\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```


```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]) - ((c*d^2 - 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(a*d^2*e*(c*d^2 - a*e^2)*x) + ((c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d
^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(2*a^(3/2)*d^(5/2)*e^(3/2))
```


Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\int \frac{ae+cdx}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$$

$$=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\int \frac{-\frac{1}{2}ae\left(cd^2-3ae^2\right)\left(cd^2\right.}{x^2\sqrt{ade+\left(cd\right.}}}{ade\left(cd^2\right.}$$

$$=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{ade+}}{ad^2e\left(c\right.}$$

$$=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{ade+}}{ad^2e\left(c\right.}$$

$$=-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(cd^2-3ae^2\right)\sqrt{ade+}}{ad^2e\left(c\right.}$$

Mathematica [A] time = 0.129399, size = 201, normalized size = 0.88

$$\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(a^2e^3(d+3ex)-acde\left(d^2-3e^2x^2\right)-c^2d^3x(d+ex)\right)+x\sqrt{d+ex}\left(-3a^2e^4+2acd^2e^2+c^2d^4\right)\sqrt{ae+cdx}\tanh^{-}}{a^{3/2}d^{5/2}e^{3/2}x\left(cd^2-ae^2\right)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c^2*d^3*x*(d + e*x)) + a^2*e^3*(d + 3*e*x) - a*
c*d*e*(d^2 - 3*e^2*x^2)) + (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*x*Sqrt[a*e
```

```
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/(a^(3/2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)*x*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Maple [A] time = 0.064, size = 253, normalized size = 1.1

$$\frac{3e}{2d^2}\ln\left(\frac{1}{x}\left(2ade+\left(ae^2+cd^2\right)x+2\sqrt{ade}\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}\right)\right)\frac{1}{\sqrt{ade}}-2\frac{e}{d^2\left(ae^2-cd^2\right)}\sqrt{cde\left(\frac{d}{e}+x\right)^2+\left(ae^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 3/2*e/d^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-2*e/d^2/(a*e^2-c*d^2)/(d/e+x)*(c*d*e*(
d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/d^2/a/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)+1/2/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^2), x)
```

Fricas [A] time = 11.0505, size = 1245, normalized size = 5.44

$$\left[\frac{\sqrt{ade}\left(\left(c^2d^4e+2\,acd^2e^3-3\,a^2e^5\right)x^2+\left(c^2d^5+2\,acd^3e^2-3\,a^2de^4\right)x\right)\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2+4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{x^2}\right.}{4\left(\left(a^2cd^5e^3-a^3d^3e^5\right)x^2+\left(a^2c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/4*(sqrt(a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + (c^2*d^5 +
 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(
a*c*d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*c*d^6*
e^2 - a^3*d^4*e^4)*x), -1/2*(sqrt(-a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a
```

```
^2*e^5)*x^2 + (c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*
d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(a*c*
d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*c*d^6*e^2
- a^3*d^4*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.476 $\int \frac{1}{x^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=329

$$-\frac{3\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}+\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{4a^2d^3e^2x\left(cd^2-ae^2\right)}$$

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) - ((c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(2*a*d^2*e*(c*d^2 - a*e^2)*x^2) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*a^2*d^3*e^2*(c*d^2 - a*e
^2)*x) - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(8*a^(5/2)*d^(7/2)*e^(5/2))
```

Rubi [A] time = 0.507791, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{3\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}+\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{4a^2d^3e^2x\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```


```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) - ((c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(2*a*d^2*e*(c*d^2 - a*e^2)*x^2) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*a^2*d^3*e^2*(c*d^2 - a*e
^2)*x) - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(8*a^(5/2)*d^(7/2)*e^(5/2))
```


Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
```

```
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}dx = \int \frac{ae+cdx}{x^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}dx$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{2\int \frac{-\frac{1}{2}ae(cd^2-5ae^2)(cd^2-ae}{x^3\sqrt{ade+(cd^2+}}}{ade(cd^2 -}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2-5ae^2)\sqrt{ade+(}}{2ad^2e(cd^2}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2-5ae^2)\sqrt{ade+(}}{2ad^2e(cd^2}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2-5ae^2)\sqrt{ade+(}}{2ad^2e(cd^2}$$

$$= -\frac{2e(ae+cdx)}{d(cd^2-ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2-5ae^2)\sqrt{ade+(}}{2ad^2e(cd^2}$$

Mathematica [A] time = 0.205207, size = 283, normalized size = 0.86

$$\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-a^2cde^2\left(-4d^2ex+2d^3+de^2x^2+15e^3x^3\right)+a^3e^4\left(2d^2-5dex-15e^2x^2\right)+ac^2d^3ex\left(d^2+5dex+4e^2x^2\right)+3c^3d^5}{4a^{5/2}d^{7/2}e^{5/2}x^2\left(cd^2-ae^2\right)\sqrt{(d+ex)(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^3*d^5*x^2*(d + e*x) + a^3*e^4*(2*d^2 - 5*d*e*
x - 15*e^2*x^2) + a*c^2*d^3*e*x*(d^2 + 5*d*e*x + 4*e^2*x^2) - a^2*c*d*e^2*(
2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 15*e^3*x^3)) - 3*(c^3*d^6 + a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*x^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(
Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(4*a^(5/2)*d^(
7/2)*e^(5/2)*(c*d^2 - a*e^2)*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.066, size = 414, normalized size = 1.3

$$-\frac{15e^2}{8d^3}\ln\left(\frac{1}{x}\left(2ade+(ae^2+cd^2)x+2\sqrt{ade}\sqrt{ade+(ae^2+cd^2)x+cdex^2}\right)\right)\frac{1}{\sqrt{ade}}+2\frac{e^2}{d^3(ae^2-cd^2)}\sqrt{cde\left(\frac{d}{e}+x\right)^2+(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -15/8*e^2/d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+2*e^2/d^3/(a*e^2-c*d^2)/(d/e+x)*(c
```

```
*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/2/d^2/a/e/x^2*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)+7/4/d^3/a/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)+3/4/d/a^2/e^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c-3/4/d/a/(a*d*e)
^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/x)*c-3/8*d/a^2/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)
*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(ex + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^3), x)
```

Fricas [A] time = 27.7265, size = 1593, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/16*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 + (
c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(a*d*e)*l
og((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e)
+ 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 -
(3*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2
*a^2*c*d^4*e^3 - 5*a^3*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x))/((a^3*c*d^6*e^4 - a^4*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2
), 1/8*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 +
(c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(-a*d*e)
*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d
*e^3)*x)) - 2*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 - (3*a*c^2*d^5*e^2 + 4*a^2*c
*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2*a^2*c*d^4*e^3 - 5*a^3*d^2
*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c*d^6*e^4 - a^4
*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.477 $\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=515

$$-\frac{2x^2\left(x\left(cd^2-ae^2\right)\left(-a^2cd^2e^4-3a^3e^6-11ac^2d^4e^2+7c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(-3a^2e^4-12acd^2e^2+7c^2d^4\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-2cde\right.}{}$$

```
[Out] (-2*d*x^4*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x^2*(a*d*e*(c*d^2
- a*e^2)*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(7*c^3*
d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 -
 a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((105*c^4*d^8 - 19
0*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^4*e^8 - 2*c*
d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*e^6)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^3*d^3*e^4*(c*d^2 - a*e^2)^3)
+ (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(8*c^(7/2)*d^(7/2)*e^(9/2))
```

Rubi [A] time = 0.618946, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 779, 621, 206}

$$-\frac{2x^2\left(x\left(cd^2-ae^2\right)\left(-a^2cd^2e^4-3a^3e^6-11ac^2d^4e^2+7c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(-3a^2e^4-12acd^2e^2+7c^2d^4\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-2cde\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] (-2*d*x^4*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x^2*(a*d*e*(c*d^2
- a*e^2)*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(7*c^3*
d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 -
 a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((105*c^4*d^8 - 19
0*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^4*e^8 - 2*c*
d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*e^6)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^3*d^3*e^4*(c*d^2 - a*e^2)^3)
+ (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(8*c^(7/2)*d^(7/2)*e^(9/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^5(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\int \frac{x^3\left(4acd^2e\left(cd^2-\ldots\right.\right.}{\left(ade+\ldots\right.}}{3c\ldots}$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x^2\left(ade\left(cd^2-\ldots\right.\right.}{\ldots}$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x^2\left(ade\left(cd^2-\ldots\right.\right.}{\ldots}$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x^2\left(ade\left(cd^2-\ldots\right.\right.}{\ldots}$$

$$=-\frac{2dx^4\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x^2\left(ade\left(cd^2-\ldots\right.\right.}{\ldots}$$

Mathematica [C] time = 8.66389, size = 2132, normalized size = 4.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (x^4*(a*e + c*d*x))/(2*c*d*e*((a*e + c*d*x)*(d + e*x))^(3/2)) + ((a*e + c*d
*x)^(3/2)*(d + e*x)^(3/2)*(((-7*c*d^2 - 5*a*e^2)*((x^2*Sqrt[a*e + c*d*x])/(
c*d*e*(d + e*x)^(3/2)) + ((4*Sqrt[a*e + c*d*x]*(2*a^2*d^2*e^4 - (3*c*d^4*(-
5*c*d^2 - a*e^2))/4 + (5*a*d^2*e^2*(-5*c*d^2 - a*e^2))/4 + e*(3*a^2*d*e^4 -
 c*d^3*(-5*c*d^2 - a*e^2) - d*e*(a*c*d^2*e - (3*a*e*(-5*c*d^2 - a*e^2))/2))
*x))/(3*e^2*(-(c*d^2) + a*e^2)^2*(d + e*x)^(3/2)) + ((-5*c*d^2 - a*e^2)*Sqr
t[c*d^2 - a*e^2]*Sqrt[(c*d)/((c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2
 - a*e^2))]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)]*S
qrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[
a*e + c*d*x])/(Sqrt[c*d^2 - a*e^2]*Sqrt[(c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*
e^2)/(c*d^2 - a*e^2)])])/(c^(3/2)*d^(3/2)*e^(5/2)*Sqrt[d + e*x]))/(c*d*e)))
/(2*c*d) + (a^3*e^3*(-4*a*c*d^2*e - (a*e*(-7*c*d^2 - 5*a*e^2))/2)*((c*d)/((
c^2*d^3)/(c*d^2 - a*e^2) - (a*c*d*e^2)/(c*d^2 - a*e^2)))^(5/2)*Sqrt[(c*d*(d
 + e*x))/(c*d^2 - a*e^2)]*((2520*c*d*(d + e*x))/(a*e^2*Sqrt[(c*d*(d + e*x))
/(c*d^2 - a*e^2)]) - (1330*c*d*(d + e*x))/(e*(a*e + c*d*x)*Sqrt[(c*d*(d + e
*x))/(c*d^2 - a*e^2)]) - (1050*c*d*(a*e + c*d*x)*(d + e*x))/(a^2*e^3*Sqrt[(
c*d*(d + e*x))/(c*d^2 - a*e^2)]) + (196*c*d*(a*e + c*d*x)^2*(d + e*x))/(a^3
*e^4*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) + 1568*Sqrt[(c*d*(d + e*x))/(c*
d^2 - a*e^2)] + (1575*(c*d^2 - a*e^2)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2
)])/(a^2*e^4) + (1995*(c*d^2 - a*e^2)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2
)])/(e^2*(a*e + c*d*x)^2) - (3780*(c*d^2 - a*e^2)^2*Sqrt[(c*d*(d + e*x))/(c
*d^2 - a*e^2)])/(a*e^3*(a*e + c*d*x)) - (294*(c*d^2 - a*e^2)^2*(a*e + c*d*x
)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(a^3*e^5) - 504*(1 + (c*d*x)/(a*e)
```

```
)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + 336*(1 + (c*d*x)/(a*e))^2*Sqrt[(c
*d*(d + e*x))/(c*d^2 - a*e^2)] - 56*(1 + (c*d*x)/(a*e))^3*Sqrt[(c*d*(d + e*
x))/(c*d^2 - a*e^2)] - (1995*ArcSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^
2)]])/((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^(5/2) + (3780*(a*e + c*d*x)*Ar
cSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/(a*e*((e*(a*e + c*d*x))/(
-(c*d^2) + a*e^2))^(5/2)) - (1575*(a*e + c*d*x)^2*ArcSin[Sqrt[(e*(a*e + c*d
*x))/(-(c*d^2) + a*e^2)]])/(a^2*e^2*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^
(5/2)) + (294*(a*e + c*d*x)^3*ArcSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e
^2)]])/(a^3*e^3*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^(5/2)) - (168*e^2*(a
*e + c*d*x)^2*Hypergeometric2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2)
 + a*e^2)])/(c*d^2 - a*e^2)^2 + (392*e*(a*e + c*d*x)^3*Hypergeometric2F1[3/
2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a*(c*d^2 - a*e^2)^2)
- (280*(a*e + c*d*x)^4*Hypergeometric2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/
(-(c*d^2) + a*e^2)])/(a^2*(c*d^2 - a*e^2)^2) + (56*(a*e + c*d*x)^5*Hypergeo
metric2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^3*e*(c*
d^2 - a*e^2)^2) - (96*e*(a*e + c*d*x)*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {
1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(-(c*d^2) + a*e^2) + (28
8*(a*e + c*d*x)^2*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e
+ c*d*x))/(-(c*d^2) + a*e^2)])/(a*(-(c*d^2) + a*e^2)) - (288*(a*e + c*d*x)^
3*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d
^2) + a*e^2)])/(a^2*(-(c*d^2*e) + a*e^3)) + (96*(a*e + c*d*x)^4*Hypergeomet
ricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]
)/(a^3*e^2*(-(c*d^2) + a*e^2))))/(252*c^3*d^3*(c*d^2 - a*e^2)^2*Sqrt[a*e +
c*d*x]*Sqrt[d + e*x]))/(2*c*d*e*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] time = 0.069, size = 1680, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 15/8/d^3/c^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a^2+2*d^4/e^5*(2*c*d*e*x+a*e^2+c*d^
2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
2/3*d^5/e^6/(a*e^2-c*d^2)/(d/e+x)/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(
1/2)-8/3*d^8/e^5*c^2/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x)
)^(1/2)-15/8/d^3/c^3*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+1/2/e^2*
x^3/d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+35/8/e^4*d/c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(
d*e*c)^(1/2)+9/8*e/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)*a^3+95/16/e^3*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+51/16/e^5*d^6*c/(-a^2*e^4+2*a*c*d^2*
e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-35/8/e^4*d/c*x/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/16*e/d^4/c^4/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)*a^3+5/16/e/d^2/c^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^
2-9/4/e^3/c*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-7/16/e^3/c^2/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+51/16/e^5*d^2/c/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+15/8*e^4/d^3/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4+5/2*e^2/d/c^2/(-a^2*e^4+2*a*c*d^2*e^2
-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3+51/8/e^4*d^5*c/(-a^
2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+15/1
6*e^5/d^4/c^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)*a^5+35/16*e^3/d^2/c^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4-16/3*d^7/e^4*c^2/(a*e^2-c*d^2)^3/(c*d*e
*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-8/3*d^6/e^3*c/(a*e^2-c*d^2)^3/(c*
```

```
d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*a-1/4*d/c/(-a^2*e^4+2*a*c*d^2*e^
2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+21/8/e*d^2/c/(-a^2
*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-5/4
/e/d^2/c^2*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-15/4/e^2/d/c^2*x/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+11/2/e^2*d^3/(-a^2*e^4+2*a*c*d^2*e
^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+15/4/e^2/d/c^2*ln((
1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/(d*e*c)^(1/2)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 50.9616, size = 4361, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/48*(15*(7*a*c^5*d^12*e - 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4
*c^2*d^6*e^7 + 3*a^5*c*d^4*e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*c^
5*d^9*e^4 + 6*a^2*c^4*d^7*e^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 - 3*
a^5*c*d*e^12)*x^3 + (14*c^6*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*e^5
+ 10*a^3*c^3*d^6*e^7 + 8*a^4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^13)*x
^2 + (7*c^6*d^13 - a*c^5*d^11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d^7*e^6
 + 7*a^4*c^2*d^5*e^8 + 3*a^5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(c*d*e)*log(
8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x) - 4*(105*a*c^5*d^11*e^2 - 190*a^2*c^4*d^9*e^4 + 36*a
^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c*d^3*e^10 - 6*(c^6*d^9*e^4 -
3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*d^3*e^10)*x^4 + 3*(7*c^6*d^10
*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 + 8*a^3*c^3*d^4*e^9 - 5*a^4*c^2
*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5*d^9*e^4 + 12*a^2*c^4*d^7*e^6
 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 + (105*c^6*d^12*e - 50*a*c^5*d
^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6*e^7 + 45*a^4*c^2*d^4*e^9 - 9
0*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^7*d^
12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^10 - a^4*c^4*d^6*e^12 + (c^8*
d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^11 - a^3*c^5*d^5*e^13)*x^3 + (
2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*d^8*e^10 + a^3*c^5*d^6*e^12 -
 a^4*c^4*d^4*e^14)*x^2 + (c^8*d^13*e^5 - a*c^7*d^11*e^7 - 3*a^2*c^6*d^9*e^9
 + 5*a^3*c^5*d^7*e^11 - 2*a^4*c^4*d^5*e^13)*x), -1/24*(15*(7*a*c^5*d^12*e -
 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4*c^2*d^6*e^7 + 3*a^5*c*d^4*
e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*c^5*d^9*e^4 + 6*a^2*c^4*d^7*e
^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 - 3*a^5*c*d*e^12)*x^3 + (14*c^6
*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*e^5 + 10*a^3*c^3*d^6*e^7 + 8*a^
```

```
4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^13)*x^2 + (7*c^6*d^13 - a*c^5*d^
11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d^7*e^6 + 7*a^4*c^2*d^5*e^8 + 3*a^
5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*
e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(105*a*c^5*d^11*e^2
 - 190*a^2*c^4*d^9*e^4 + 36*a^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c
*d^3*e^10 - 6*(c^6*d^9*e^4 - 3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*
d^3*e^10)*x^4 + 3*(7*c^6*d^10*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 +
8*a^3*c^3*d^4*e^9 - 5*a^4*c^2*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5
*d^9*e^4 + 12*a^2*c^4*d^7*e^6 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 +
 (105*c^6*d^12*e - 50*a*c^5*d^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6
*e^7 + 45*a^4*c^2*d^4*e^9 - 90*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a*c^7*d^12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^
10 - a^4*c^4*d^6*e^12 + (c^8*d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^1
1 - a^3*c^5*d^5*e^13)*x^3 + (2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*
d^8*e^10 + a^3*c^5*d^6*e^12 - a^4*c^4*d^4*e^14)*x^2 + (c^8*d^13*e^5 - a*c^7
*d^11*e^7 - 3*a^2*c^6*d^9*e^9 + 5*a^3*c^5*d^7*e^11 - 2*a^4*c^4*d^5*e^13)*x)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

3.478 $\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=438

$$-\frac{2x\left(x\left(cd^2-ae^2\right)\left(-a^2cd^2e^4-3a^3e^6-9ac^2d^4e^2+5c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(-3a^2e^4-10acd^2e^2+5c^2d^4\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\left(9a^2cd^2e\cdots\right.}{}$$

```
[Out] (-2*d*x^3*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x*(a*d*e*(c*d^2 -
a*e^2)*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(5*c^3*d^
6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*
e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^3*d^6 - 31*a*c
^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(3*c^2*d^2*e^3*(c*d^2 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*ArcTan
h[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(5/2)*d^(5/2)*e^(7/2))
```

Rubi [A] time = 0.541497, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 640, 621, 206}

$$-\frac{2x\left(x\left(cd^2-ae^2\right)\left(-a^2cd^2e^4-3a^3e^6-9ac^2d^4e^2+5c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(-3a^2e^4-10acd^2e^2+5c^2d^4\right)\right)}{3cde^2\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\left(9a^2cd^2e\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] (-2*d*x^3*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x*(a*d*e*(c*d^2 -
a*e^2)*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(5*c^3*d^
6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*
e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^3*d^6 - 31*a*c
^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(3*c^2*d^2*e^3*(c*d^2 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*ArcTan
h[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(5/2)*d^(5/2)*e^(7/2))
```


Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
```

```
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^4(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$=-\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\int \frac{x^2\left(3acd^2e\left(cd^2-ae^2\right)\cdots\right.}{\left(ade+\left(cd^2\cdots\right.\right.}}{3cde\left(\cdots\right.}$$

$$=-\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x\left(ade\left(cd^2-ae^2\right)\cdots\right.}{\cdots}$$

$$=-\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x\left(ade\left(cd^2-ae^2\right)\cdots\right.}{\cdots}$$

$$=-\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x\left(ade\left(cd^2-ae^2\right)\cdots\right.}{\cdots}$$

$$=-\frac{2dx^3\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2x\left(ade\left(cd^2-ae^2\right)\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.43002, size = 387, normalized size = 0.88

$$\frac{(ae+cdx)\left(-\frac{ae\left(3ae^2-cd^2\right)\left(a^2e^2\left(8d^2+12dex+3e^2x^2\right)+2acd^2ex(2d+3ex)-c^2d^4x^2\right)}{cd\left(cd^2-ae^2\right)^3}-\frac{\left(3ae^2+5cd^2\right)\sqrt{ae+cdx}\left(c^{3/2}d^{7/2}\sqrt{e}\left(cd^2-ae^2\right)\sqrt{ae+cdx}-(d+ex)\left(2c^{3/2}d^{5/}\cdots\right)\right)}{c^{5/2}\cdots}\right)}{3cde((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] ((a*e + c*d*x)*(3*x^3 - (a*e*(-(c*d^2) + 3*a*e^2)*(-(c^2*d^4*x^2) + 2*a*c*d
^2*e*x*(2*d + 3*e*x) + a^2*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2)))/(c*d*(c*d^2
 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*Sqrt[a*e + c*d*x]*(c^(3/2)*d^(7/2)*Sqrt
[e]*(c*d^2 - a*e^2)*Sqrt[a*e + c*d*x] - (d + e*x)*(2*c^(3/2)*d^(5/2)*Sqrt[e
]*(2*c*d^2 - 3*a*e^2)*Sqrt[a*e + c*d*x] - 3*Sqrt[c*d]*(c*d^2 - a*e^2)^(5/2)
*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])))/(c^(5/2)*d^(5/2)*e^(5/2
)*(c*d^2 - a*e^2)^2)))/(3*c*d*e*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] time = 0.063, size = 1266, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 5/2/e^3/c*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4/d^3/c^3/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-9/4/e^4*d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)-5/2/e^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)-9/2/e^3*d^4*c/(-a^2*e^4+2*a*
c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3/4*e^4/d^3/c^
3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
a^4-2*d^3/e^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/2/e/d^2/c^2*x/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)*a-3/2*e/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)*x*a^2-9/2/e*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+8/3*d^7/e^4*c^2/(a*e^2-c*d^2)^3/(
c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+1/e^2*x^2/d/c/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)-3*d/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-9/2/e^2*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-9/4/e^4*d^5*c/(-a^2*e^4+2*a*c*d
^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*e^2/d/c^2/(-a^2
*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-3/2
/e/d^2/c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)*a-2/3*d^4/e^5/(a*e^2-c*d^2)/(d/e+x)/(
c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+16/3*d^6/e^3*c^2/(a*e^2-c*d^2)
^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+8/3*d^5/e^2*c/(a*e^2-c*d
^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*a-3/2*e^3/d^2/c^2/(-a^2
*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 21.3408, size = 3615, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*c
*d^4*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*d^
5*e^6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*c^4
*d^8*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*c^5*
d^11 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a^4*c*
d^3*e^8 - 6*a^5*d*e^10)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(15*a
*c^4*d^9*e^2 - 31*a^2*c^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^3*e^8 + 3
*(c^5*d^8*e^3 - 3*a*c^4*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2*e^9)*x^3
+ (20*c^5*d^9*e^2 - 39*a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3*c^2*d^3*e^
8 - 9*a^4*c*d*e^10)*x^2 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 33*a^2*c^3*d^
6*e^5 + 15*a^3*c^2*d^4*e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a*c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9
 - a^4*c^3*d^5*e^11 + (c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^10
- a^3*c^4*d^4*e^12)*x^3 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^5*d^7
*e^9 + a^3*c^4*d^5*e^11 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^12*e^4 - a*c^6*d^1
0*e^6 - 3*a^2*c^5*d^8*e^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^3*d^4*e^12)*x), 1/
6*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*c*d^4
*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*d^5*e^
6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*c^4*d^8
*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*c^5*d^11
 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a^4*c*d^3*
e^8 - 6*a^5*d*e^10)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 +
 a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*a*c^4*d^9*e^2 - 31*a^2*c
^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^3*e^8 + 3*(c^5*d^8*e^3 - 3*a*c^4
*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2*e^9)*x^3 + (20*c^5*d^9*e^2 - 39*
a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3*c^2*d^3*e^8 - 9*a^4*c*d*e^10)*x^2
 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 33*a^2*c^3*d^6*e^5 + 15*a^3*c^2*d^4*
e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*
c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9 - a^4*c^3*d^5*e^11 + (
c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^10 - a^3*c^4*d^4*e^12)*x^3
 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^5*d^7*e^9 + a^3*c^4*d^5*e^11
 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^12*e^4 - a*c^6*d^10*e^6 - 3*a^2*c^5*d^8*e
^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^3*d^4*e^12)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, undef, 1]
```

3.479 $\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=297

$$-\frac{2\left(x\left(-a^2cd^2e^4-3a^3e^6-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\right)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}-$$

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*(a*d*e*(c*d^2 - 3*
a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3
*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^(3/2)*d^(3/2)*e^(5/2))
```

Rubi [A] time = 0.292511, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {849, 818, 777, 621, 206}

$$-\frac{2\left(x\left(-a^2cd^2e^4-3a^3e^6-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\right)}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*(a*d*e*(c*d^2 - 3*
a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3
*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x\left(2acd^2e\left(cd^2-a\right.\right.}{\left(ade+\left(cd^2+\right.\right.}}{3cde\left(c\right.}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(cd^2-3ae\right.\right.}{3cd}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(cd^2-3ae\right.\right.}{3cd}$$

$$= -\frac{2dx^2\left(ae\left(cd^2-ae^2\right)+cd\left(cd^2-ae^2\right)x\right)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(ade\left(cd^2-3ae\right.\right.}{3cd}$$

Mathematica [C] time = 4.99437, size = 1443, normalized size = 4.86

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (a^3*e^3*(a*e + c*d*x)^2*((c*d*(d + e*x))/(c*d^2 - a*e^2))^(5/2)*((2520*c*d
*(d + e*x))/(a*e^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (1330*c*d*(d +
e*x))/(e*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) - (1050*c*d*(
a*e + c*d*x)*(d + e*x))/(a^2*e^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]) + (
196*c*d*(a*e + c*d*x)^2*(d + e*x))/(a^3*e^4*Sqrt[(c*d*(d + e*x))/(c*d^2 - a
*e^2)]) + 1568*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] + (1575*(c*d^2 - a*e^2
)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(a^2*e^4) + (1995*(c*d^2 - a*e^2
)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(e^2*(a*e + c*d*x)^2) - (3780*(c
*d^2 - a*e^2)^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])/(a*e^3*(a*e + c*d*x)
) - (294*(c*d^2 - a*e^2)^2*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)])/(a^3*e^5) - 504*(1 + (c*d*x)/(a*e))*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)] + 336*(1 + (c*d*x)/(a*e))^2*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - 56*
(1 + (c*d*x)/(a*e))^3*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)] - (1995*ArcSin[
Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/((e*(a*e + c*d*x))/(-(c*d^2) +
 a*e^2))^(5/2) + (3780*(a*e + c*d*x)*ArcSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2
) + a*e^2)]])/(a*e*((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^(5/2)) - (1575*(a
*e + c*d*x)^2*ArcSin[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/(a^2*e^2*
((e*(a*e + c*d*x))/(-(c*d^2) + a*e^2))^(5/2)) + (294*(a*e + c*d*x)^3*ArcSin
[Sqrt[(e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)]])/(a^3*e^3*((e*(a*e + c*d*x))/(
-(c*d^2) + a*e^2))^(5/2)) - (168*e^2*(a*e + c*d*x)^2*Hypergeometric2F1[3/2,
 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(c*d^2 - a*e^2)^2 + (392
*e*(a*e + c*d*x)^3*Hypergeometric2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c
*d^2) + a*e^2)])/(a*(c*d^2 - a*e^2)^2) - (280*(a*e + c*d*x)^4*Hypergeometri
c2F1[3/2, 9/2, 11/2, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^2*(c*d^2 - a
*e^2)^2) + (56*(a*e + c*d*x)^5*Hypergeometric2F1[3/2, 9/2, 11/2, (e*(a*e +
c*d*x))/(-(c*d^2) + a*e^2)])/(a^3*e*(c*d^2 - a*e^2)^2) - (96*e*(a*e + c*d*x
)*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d
^2) + a*e^2)])/(-(c*d^2) + a*e^2) + (288*(a*e + c*d*x)^2*HypergeometricPFQ[
{1/2, 2, 2, 7/2}, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a*(-
(c*d^2) + a*e^2)) - (288*(a*e + c*d*x)^3*HypergeometricPFQ[{1/2, 2, 2, 7/2}
, {1, 1, 9/2}, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^2*(-(c*d^2*e) + a*
e^3)) + (96*(a*e + c*d*x)^4*HypergeometricPFQ[{1/2, 2, 2, 7/2}, {1, 1, 9/2}
, (e*(a*e + c*d*x))/(-(c*d^2) + a*e^2)])/(a^3*e^2*(-(c*d^2) + a*e^2))))/(25
2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2))
```

Maple [B] time = 0.06, size = 977, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -1/e^2*x/d/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2/e/d^2/c^2/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+3/2/e^3/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+e^2/d/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)*x*a^2+4*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)*x*a+3/e^2*d^3*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/2*e^3/d^2/c^2/(-a^2*e^4+2*a*c*d^2*
e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+5/2*e/c/(-a^2*e^4+
2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+7/2/e*d^
2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
a+3/2/e^3*d^4*c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)+1/e^2/d/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(d*e*c)^(1/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(d*e*c)^(1/2)+2*d^2/e^3*(2*c*d*e*x+a*e
^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)+2/3*d^3/e^4/(a*e^2-c*d^2)/(d/e+x)/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/
```

```
e+x))^(1/2)-16/3*d^5/e^2*c^2/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)
*(d/e+x))^(1/2)*x-8/3*d^4/e*c/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2
)*(d/e+x))^(1/2)*a-8/3*d^6/e^3*c^2/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 23.41, size = 2908, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*
c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*
x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^
4*d*e^8)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 +
 a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3*a*c^3*d^7*e^2 - 8
*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c^3*d^5*e^4 - 3*a
^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^2*d^4*e^5 - 6*a^
3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^5*d^10*e^
4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^10 + (c^6*d^9*e^5
 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*x^3 + (2*c^6*d^1
0*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e^10 - a^4*c^2*d^
2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7*e^7 + 5*a^3*c^3
*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x), -1/3*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3
 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c
^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*
d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c
^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*a
*c^3*d^7*e^2 - 8*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c
^3*d^5*e^4 - 3*a^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^
2*d^4*e^5 - 6*a^3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(a*c^5*d^10*e^4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^1
0 + (c^6*d^9*e^5 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*
x^3 + (2*c^6*d^10*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e
^10 - a^4*c^2*d^2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7
*e^7 + 5*a^3*c^3*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.480 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.107981, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {854, 12, 636}

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\int -\frac{2ade^2\left(cd^2\right.}{\left(ade+\left(cd^2+ae^2\right)\right.}}{3de\left(cd^2-\right.}$$
$$= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(4ae)\int \frac{}{\left(ade+\left(cd^2+a\right.\right.}}{3\left(cd^2-\right.}$$
$$= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8ae\left(2ad\right.}{3\left(cd^2-ae^2\right)^3\sqrt{a}}$$

Mathematica [A] time = 0.0768139, size = 99, normalized size = 0.79

$$\frac{-2a^2e^2\left(8d^2+12dex+3e^2x^2\right)-4acd^2ex(2d+3ex)+2c^2d^4x^2}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*c^2*d^4*x^2 - 4*a*c*d^2*e*x*(2*d + 3*e*x) - 2*a^2*e^2*(8*d^2 + 12*d*e*x
+ 3*e^2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.072, size = 145, normalized size = 1.2

$$\frac{(2\,cdx+2\,ae)\left(3\,a^2e^4x^2+6\,acd^2e^2x^2-c^2d^4x^2+12\,a^2de^3x+4\,acd^3ex+8\,a^2d^2e^2\right)}{3\,a^3e^6-9\,a^2cd^2e^4+9\,ac^2d^4e^2-3\,c^3d^6}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(3*a^2*e^4*x^2+6*a*c*d^2*e^2*x^2-c^2*d^4*x^2+12*a^2*d*e^3*x
+4*a*c*d^3*e*x+8*a^2*d^2*e^2)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*
d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.2837, size = 602, normalized size = 4.78

$$-\frac{2\left(8\,a^2d^2e^2-\left(c^2d^4-6\,acd^2e^2-3\,a^2e^4\right)x^2+4\left(acd^3e+3\,a^2de^3\right)\right.}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3d^6e^3+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2/3*(8*a^2*d^2*e^2 - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*x^2 + 4*(a*c*d^
3*e + 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^
8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*
a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c
^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 -
a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef},\mathrm{undef},\mathrm{undef},1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.481 $\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 - a
*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0937172, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {792, 613}

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] (-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 - a
*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\left(cd^2+3ae^2\right)\int}{3e}$$

$$=-\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\left(cd^2+3\right.}{3e\left(cd^2-ae^2\right)^3}$$

Mathematica [A] time = 0.0373915, size = 100, normalized size = 0.72

$$\frac{2\left(a^2e^3(2d+3ex)+2acde\left(3d^2+5dex+3e^2x^2\right)+c^2d^3x(3d+2ex)\right)}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(c^2*d^3*x*(3*d + 2*e*x) + a^2*e^3*(2*d + 3*e*x) + 2*a*c*d*e*(3*d^2 + 5*
d*e*x + 3*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d +
 e*x)])
```

Maple [A] time = 0.052, size = 149, normalized size = 1.1

$$-\frac{(2\,cdx+2\,ae)\left(6\,acde^3x^2+2\,c^2d^3ex^2+3\,a^2e^4x+10\,acd^2e^2x+3\,c^2d^4x+2\,a^2de^3+6\,acd^3e\right)}{3\,a^3e^6-9\,a^2cd^2e^4+9\,ac^2d^4e^2-3\,c^3d^6}\left(cdex^2+ae^2x+cd^2x+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(6*a*c*d*e^3*x^2+2*c^2*d^3*e*x^2+3*a^2*e^4*x+10*a*c*d^2*e^
2*x+3*c^2*d^4*x+2*a^2*d*e^3+6*a*c*d^3*e)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d
^4*e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 20.4064, size = 621, normalized size = 4.5

$$\frac{2\left(6\,acd^3e+2\,a^2de^3+2\left(c^2d^3e+3\,acde^3\right)x^2+\left(3\,c^2d^4+10\,acd^2e^2+3\,a\right.\right.}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3d^6e^3+3\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(6*a*c*d^3*e + 2*a^2*d*e^3 + 2*(c^2*d^3*e + 3*a*c*d*e^3)*x^2 + (3*c^2*d
^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^
```

```
4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4
*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2
 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d
*e^8)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] [undef, undef, undef, 1]
```

3.482 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0415744, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {658, 613}

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```


```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 658

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((m + p + 1)*(2*c
*d - b*e)), x] + Dist[(c*Simplify[m + 2*p + 2])/((m + p + 1)*(2*c*d - b*e))
, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !In
tegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(4cd)\int \frac{}{(ade+(c}}{3\,(c}$$

$$=\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8cd\,(}{3\left(cd^2-ae^2\right)^3}$$

Mathematica [A] time = 0.0337375, size = 95, normalized size = 0.79

$$\frac{2a^2e^4-4acde^2(3d+2ex)-2c^2d^2\left(3d^2+12dex+8e^2x^2\right)}{3(d+ex)\left(cd^2-ae^2\right)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*e^4 - 4*a*c*d*e^2*(3*d + 2*e*x) - 2*c^2*d^2*(3*d^2 + 12*d*e*x + 8*e^
2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.053, size = 138, normalized size = 1.1

$$-\frac{(2\,cdx+2\,ae)\left(-8\,x^2c^2d^2e^2-4\,xacde^3-12\,c^2d^3ex+a^2e^4-6\,acd^2e^2-3\,c^2d^4\right)}{3\,a^3e^6-9\,a^2cd^2e^4+9\,ac^2d^4e^2-3\,c^3d^6}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4-6
*a*c*d^2*e^2-3*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)/(
c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 14.1597, size = 599, normalized size = 4.95

$$-\frac{2\left(8\,c^2d^2e^2x^2+3\,c^2d^4+6\,acd^2e^2-a^2e^4+4\left(3\,c^2d^3e+acde^3\right)x\right)\sqrt{}}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+\left(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8\right)x^3+\left(2\,c^4d^8e-5\,ac^3d^6e^3+3\,a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(8*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 4*(3*c^2*d^
3*e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^8*
e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*
c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3
```

```
*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*
c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] [undef, undef, undef, 1]
```

3.483 $\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{2\left(7a^2cd^2e^4-3a^3e^6+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}-\frac{}{3d\left(c\right.}$$

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6
 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)
^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])]/(a^(3/2)*d^(5/2)*e^(3/2))
```

Rubi [A] time = 0.337703, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 12, 724, 206}

$$\frac{2\left(7a^2cd^2e^4-3a^3e^6+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}-\frac{}{3d\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6
 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)
^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])]/(a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{3}{2}ae\left(cd^2-ae^2\right)^{2}\cdots}{x\left(ade+\left(cd^2+\cdots\right.\right.}}{3ade\left(c\cdots\right.}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^{\cdots}\right.}{3ad\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^{\cdots}\right.}{3ad\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^{\cdots}\right.}{3ad\cdots}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac^2d^{\cdots}\right.}{3ad\cdots}$$

Mathematica [A] time = 0.444473, size = 262, normalized size = 0.97

$$\frac{2\left(-\frac{(d+ex)(ae+cdx)^{3/2}\left(\sqrt{a}\sqrt{d}\sqrt{e}\left(3a^2e^5-8acd^2e^3-3c^2d^4e\right)\sqrt{ae+cdx}+3\sqrt{d+ex}\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)\right)}{3\sqrt{a}d^{5/2}\sqrt{e}\left(cd^2-ae^2\right)^2}+\frac{\left(ae^3+3cd^2e\right)(ae+cdx)^2}{3cd^3-3ade^2}+cd(ae+cdx\cdots\right.}{ae\left(cd^2-ae^2\right)((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(c*d*(a*e + c*d*x) + ((3*c*d^2*e + a*e^3)*(a*e + c*d*x)^2)/(3*c*d^3 - 3*
a*d*e^2) - ((a*e + c*d*x)^(3/2)*(d + e*x)*(Sqrt[a]*Sqrt[d]*Sqrt[e]*(-3*c^2*
d^4*e - 8*a*c*d^2*e^3 + 3*a^2*e^5)*Sqrt[a*e + c*d*x] + 3*(c*d^2 - a*e^2)^3*
Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d +
 e*x])]))/(3*Sqrt[a]*d^(5/2)*Sqrt[e]*(c*d^2 - a*e^2)^2)))/(a*e*(c*d^2 - a*e
^2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] time = 0.059, size = 682, normalized size = 2.5

$$\frac{1}{ad^2e}\frac{1}{\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}}-2\frac{e^2xc}{d\left(-a^2e^4+2\,acd^2e^2-c^2d^4\right)\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}}-2\frac{}{a\left(-a^2e^4+2\,acd^2e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 1/d^2/a/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-2/d*e^2/(-a^2*e^4+2*a*c*d
^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c-2*d/a/(-a^2*e^4
+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^2-1/d^2
*a*e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)-2*e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)*c-d^2/a/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)*c^2-1/d^2/a/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a
*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+2/3/d/(a*e^2-c*d^2)
/(d/e+x)/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-16/3*d*e^2*c^2/(a*e^
2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x-8/3*e^3*c/(a*e^2
-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*a-8/3*d^2*e*c^2/(a*
e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x)
```

Fricas [B] time = 39.1122, size = 2923, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*
c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*
x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^
4*d*e^8)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2
*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(3*a*c^3*d
^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3 + 8*a^2*c^2*d^4*e
^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d^5*e^4 + 4*a^3*c*
d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3
*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*d^5*e^9 + (a^2*c^
4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c*d^4*e^10)*x^3 +
(2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^7 + a^5*c*d^5*e^9
 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^4 - 3*a^4*c^2*d^8
*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x), 1/3*(3*(a*c^3*d^8*e - 3*a^2*c^
2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4
+ 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3
*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2
- 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-a*d*e)*arctan
(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x
)) + 2*(3*a*c^3*d^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3
+ 8*a^2*c^2*d^4*e^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d
^5*e^4 + 4*a^3*c*d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(a^3*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*
d^5*e^9 + (a^2*c^4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c
*d^4*e^10)*x^3 + (2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^
7 + a^5*c*d^5*e^9 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^
4 - 3*a^4*c^2*d^8*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.484 $\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=394

$$-\frac{\left(31a^2cd^2e^4-15a^3e^6-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x\left(cd^2-ae^2\right)^3}+\frac{2\left(cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+9a^2cd^2 \ldots\right.}{3ad^2ex\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+c \ldots\right.}}$$

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^3*e
^6 + c*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 -
 a*e^2)^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((9*c^3*d^6 - 9*
a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x) + ((3*c*d^2 + 5*a*e^2)
*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(5/2)*d^(7/2)*e^(5/2))
```

Rubi [A] time = 0.590436, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {851, 822, 806, 724, 206}

$$-\frac{\left(31a^2cd^2e^4-15a^3e^6-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x\left(cd^2-ae^2\right)^3}+\frac{2\left(cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+9a^2cd^2 \ldots\right.}{3ad^2ex\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+c \ldots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^3*e
^6 + c*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 -
 a*e^2)^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((9*c^3*d^6 - 9*
a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x) + ((3*c*d^2 + 5*a*e^2)
*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}ae\left(3cd^2-\right.}{x^2(}}{3}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac\right.}{3}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac\right.}{3}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac\right.}{3}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+ac\right.}{3}$$

Mathematica [A] time = 0.589288, size = 370, normalized size = 0.94

$$\frac{(ae+cdx)\left(\sqrt{a}d^{3/2}\sqrt{e}x\left(ae^2-cd^2\right)\left(5a^2e^5-6acd^2e^3+9c^2d^4e\right)(ae+cdx)+x(d+ex)\sqrt{ae+cdx}\left(\sqrt{a}\sqrt{d}\sqrt{e}\left(-31a^2cd^2e^5+\cdots\right.\right.\right.}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(3*a^(3/2)*d^(5/2)*e^(3/2)*(-(c*d^2) + a*e^2)^3 + 3*Sqrt[a]*
c*d^(7/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-3*c*d^2 + a*e^2)*x + Sqrt[a]*d^(3/2)*
Sqrt[e]*(-(c*d^2) + a*e^2)*(9*c^2*d^4*e - 6*a*c*d^2*e^3 + 5*a^2*e^5)*x*(a*e
 + c*d*x) + x*Sqrt[a*e + c*d*x]*(d + e*x)*(Sqrt[a]*Sqrt[d]*Sqrt[e]*(-9*c^3*
d^6*e + 9*a*c^2*d^4*e^3 - 31*a^2*c*d^2*e^5 + 15*a^3*e^7)*Sqrt[a*e + c*d*x]
+ 3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sq
rt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])))/(3*a^(5/2)*d^(7/2)*e^(
5/2)*(c*d^2 - a*e^2)^3*x*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] time = 0.065, size = 912, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -5/2/d^3/a/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*e^3/d^2/(-a^2*e^4+2*a*
c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c+5/2*e^4/d^3*
a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
5/2*e^2/d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)*c+3/2*d/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)*c^2+5/2/d^3/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-2/3*e/d^2/(a*e^2-c
*d^2)/(d/e+x)/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+16/3*e^3*c^2/(a
*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*x+8/3*e^4/d*c/(
a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)*a+8/3*e^2*d*c^
2/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)-1/d^2/a/e/x
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2/d/a^2/e^2/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)*c+3*d^2/a^2/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3+3/2*d^3/a^2/e^2/(-a^2*e^4+2*a*c*d^2*e
^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3+3/2/d/a^2/e^2/(a*d*
e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/x)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
x)
```

Fricas [B] time = 128.576, size = 3671, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/12*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3*d^5*e^6 + 12*a^3*c^2
*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 16*a^2*c
^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*a^5*e^11)*x^3 + (3*c^
5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4*c*d^3*e^8 - 10*a^5*d
*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a^3*c^2*d^6*e^5 + 12*a
^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4
+ 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x
)/x^2) - 4*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^6 - 3*a^5
*d^3*e^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*e^7 - 15*a
^4*c*d^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*a^3*c^2*d^5
*e^6 + 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e - 3*a^2*c^3*
d^8*e^3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^9)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*a^4*c^3*d^9*e^
7 + 3*a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^12*e^4 - 5*a^4*c
^3*d^10*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*e^12)*x^3 + (a^3
*c^4*d^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*
a^7*d^5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^6 + 3*a^6*c*d^8*e^
8 - a^7*d^6*e^10)*x), -1/6*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3
*d^5*e^6 + 12*a^3*c^2*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c
^4*d^8*e^3 - 16*a^2*c^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*
a^5*e^11)*x^3 + (3*c^5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4
*c*d^3*e^8 - 10*a^5*d*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a
^3*c^2*d^6*e^5 + 12*a^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(-a*d*e)*arctan(1
/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 + 2*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^6 - 3*a^5*d^3*e
^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*e^7 - 15*a^4*c*d
^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*a^3*c^2*d^5*e^6 +
 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e - 3*a^2*c^3*d^8*e^
3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^9)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*a^4*c^3*d^9*e^7 + 3*
a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^12*e^4 - 5*a^4*c^3*d^1
0*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*e^12)*x^3 + (a^3*c^4*d
^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*a^7*d^
5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^6 + 3*a^6*c*d^8*e^8 - a^
7*d^6*e^10)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.485 $\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=522

$$-\frac{\left(61a^2cd^2e^4-35a^3e^6-9ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6a^2d^3e^2x^2\left(cd^2-ae^2\right)^3}+\frac{\left(-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e^8-\cdots\right.}{12a^3d^4\cdots}$$

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((15*c^3*d^
6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(6*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^2) + ((45*c^4*d^8
 - 30*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^
2)^3*x) - (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])])/(8*a^(7/2)*d^(9/2)*e^(7/2))
```

Rubi [A] time = 0.800339, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{\left(61a^2cd^2e^4-35a^3e^6-9ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6a^2d^3e^2x^2\left(cd^2-ae^2\right)^3}+\frac{\left(-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e^8-\cdots\right.}{12a^3d^4\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((15*c^3*d^
6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(6*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^2) + ((45*c^4*d^8
 - 30*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^
2)^3*x) - (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])])/(8*a^(7/2)*d^(9/2)*e^(7/2))
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}ae\left(3cd^2\right.}{x^3}}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+a\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+a\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+a\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+a\right.}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(3c^3d^6+a\right.}{}$$

Mathematica [A] time = 0.972066, size = 467, normalized size = 0.89

$$\frac{(ae+cdx)\left(x\left(-3\sqrt{a}d^{5/2}\sqrt{e}x\left(7a^2cde^4-15c^3d^5\right)\left(cd^2-ae^2\right)^2-\sqrt{a}d^{3/2}\sqrt{e}x\left(ae^2-cd^2\right)\left(-33a^2cd^2e^5+35a^3e^7-15ac^2d^4\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(6*a^(5/2)*d^(7/2)*e^(5/2)*(-(c*d^2) + a*e^2)^3 + x*(3*a^(3/
2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2) - 3*Sqrt[a]*d^(5/2
)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-15*c^3*d^5 + 7*a^2*c*d*e^4)*x - Sqrt[a]*d^(3/
2)*Sqrt[e]*(-(c*d^2) + a*e^2)*(45*c^3*d^6*e - 15*a*c^2*d^4*e^3 - 33*a^2*c*d
^2*e^5 + 35*a^3*e^7)*x*(a*e + c*d*x) - x*Sqrt[a*e + c*d*x]*(d + e*x)*(Sqrt[
a]*Sqrt[d]*Sqrt[e]*(-45*c^4*d^8*e + 30*a*c^3*d^6*e^3 + 36*a^2*c^2*d^4*e^5 -
 190*a^3*c*d^2*e^7 + 105*a^4*e^9)*Sqrt[a*e + c*d*x] + 15*(c*d^2 - a*e^2)^3*
(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt
[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))))/(12*a^(7/2)*d^(9/2)*e^(
7/2)*(c*d^2 - a*e^2)^3*x^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] time = 0.067, size = 1319, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 9/4/d^3/a/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/8/a^3/e^3/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2+1/4*e/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2-35/8*e/d^4/a/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/x)+2/3*e^2/d^3/(a*e^2-c*d^2)/(d/e+x)/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(
d/e+x))^(1/2)-35/8*e^5/d^4*a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)-7/2*e^3/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c-15/4*d^3/a^3/e^2/(-a^2*e^4+2*a*c*d^2
*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^4-35/4*e^4/d^3/(-
a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c+
7/4*e^2/d/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)*x*c^2-16/3*e^4/d*c^2/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^
2)*(d/e+x))^(1/2)*x-8/3*e^5/d^2*c/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c
*d^2)*(d/e+x))^(1/2)*a+35/8*e/d^4/a/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-8/3*e^3*c^2/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-c*d^2)*(d/e+x))^(1/2)+
15/4/d^2/a^2/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c-1/2/d^2/a/e/x^2/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-15/8/a^3/e^3/(a*d*e)^(1/2)*ln((2*a*d*
e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
x)*c^2-5/2*d^2/a^2/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)*c^3-15/8*d^4/a^3/e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^4-15/4/d^2/a^2/e/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/x)*c+5/4/d/a^2/e^2/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c-5/4*d/
a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)*x*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}(ex + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.486 $\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=664

$$-\frac{\left(33a^2cd^2e^4-21a^3e^6-3ac^2d^4e^2+7c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x^3\left(cd^2-ae^2\right)^3}-\frac{\left(-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-31\right.}{24a}$$

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^4 - 9*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((7*c^3*d^6
 - 3*a*c^2*d^4*e^2 + 33*a^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^3) + ((35*c^4*d^8
- 16*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*e^8)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^2
)^3*x^2) - ((105*c^5*d^10 - 55*a*c^4*d^8*e^2 - 54*a^2*c^3*d^6*e^4 - 78*a^3*
c^2*d^4*e^6 + 525*a^4*c*d^2*e^8 - 315*a^5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(24*a^4*d^5*e^4*(c*d^2 - a*e^2)^3*x) + (5*(7*c^3*d^6 + 15
*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 +
 a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*a^(9/2)*d^(11/2)*e^(9/2))
```

Rubi [A] time = 1.16988, antiderivative size = 664, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {851, 822, 834, 806, 724, 206}

$$-\frac{\left(33a^2cd^2e^4-21a^3e^6-3ac^2d^4e^2+7c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x^3\left(cd^2-ae^2\right)^3}-\frac{\left(-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-31\right.}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^4 - 9*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((7*c^3*d^6
 - 3*a*c^2*d^4*e^2 + 33*a^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^3) + ((35*c^4*d^8
- 16*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*e^8)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^2
)^3*x^2) - ((105*c^5*d^10 - 55*a*c^4*d^8*e^2 - 54*a^2*c^3*d^6*e^4 - 78*a^3*
c^2*d^4*e^6 + 525*a^4*c*d^2*e^8 - 315*a^5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(24*a^4*d^5*e^4*(c*d^2 - a*e^2)^3*x) + (5*(7*c^3*d^6 + 15
*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 +
 a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 851

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
```

```
p))/(a/d + (c*x)/e)^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{ae+cdx}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2\int \frac{-\frac{3}{2}ae\left(cd^2-3ae^2\right.}{x^4\left(ade+\right.}}{3a}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6+ac^2d\right.}{3a}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6+ac^2d\right.}{3a}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6+ac^2d\right.}{3a}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6+ac^2d\right.}{3a}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6+ac^2d\right.}{3a}$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6+ac^2d\right.}{3a}$$

Mathematica [A] time = 1.44048, size = 593, normalized size = 0.89

$$(ae+cdx)\left(x\left(x^2\left(3\sqrt{a}d^{3/2}\sqrt{e}\left(ae^2-cd^2\right)\left(-42a^2c^2d^4e^5-84a^3cd^2e^7+105a^4e^9-20ac^3d^6e^3+105c^4d^8e\right)(ae+cdx)+(d+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((a*e + c*d*x)*(24*a^(7/2)*d^(9/2)*e^(7/2)*(-(c*d^2) + a*e^2)^3 + x*(6*a^(5
/2)*d^(7/2)*e^(5/2)*(c*d^2 - a*e^2)^3*(7*c*d^2 + 9*a*e^2) + 3*a^(3/2)*d^(5/
2)*e^(3/2)*(-(c*d^2) + a*e^2)^3*(35*c^2*d^4 + 54*a*c*d^2*e^2 + 63*a^2*e^4)*
x + x^2*(9*Sqrt[a]*c*d^(7/2)*Sqrt[e]*(c*d^2 - a*e^2)^2*(-35*c^3*d^6 - 5*a*c
^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 21*a^3*e^6) + 3*Sqrt[a]*d^(3/2)*Sqrt[e]*(-(c
*d^2) + a*e^2)*(105*c^4*d^8*e - 20*a*c^3*d^6*e^3 - 42*a^2*c^2*d^4*e^5 - 84*
a^3*c*d^2*e^7 + 105*a^4*e^9)*(a*e + c*d*x) + Sqrt[a*e + c*d*x]*(d + e*x)*(3
*Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^5*d^10*e + 55*a*c^4*d^8*e^3 + 54*a^2*c^3*d
^6*e^5 + 78*a^3*c^2*d^4*e^7 - 525*a^4*c*d^2*e^9 + 315*a^5*e^11)*Sqrt[a*e +
c*d*x] + 45*(c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*
e^4 + 21*a^3*e^6)*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a
]*Sqrt[e]*Sqrt[d + e*x])])))))/(72*a^(9/2)*d^(11/2)*e^(9/2)*(c*d^2 - a*e^2)
```

```
^3*x^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] time = 0.072, size = 1705, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 105/16*e^2/d^5/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-35/16*d/a^4/e^4/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)*c^3-89/24/d^4/a*e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)-75/16/d/a^3/e^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2-1/3/d
^2/a/e/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+105/16/d^3/a^2/(a*d*e)^(
1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/x)*c+23/24*d/a^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3-35/24/a^3/e^3/x/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)*c^2-2/3*e^3/d^4/(a*e^2-c*d^2)/(d/e+x)/(c*d*e*(d/e+x)^2+(a*e^
2-c*d^2)*(d/e+x))^(1/2)+8/3*e^4/d*c^2/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2)-41/12*e^3/d^2/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^2+35/8*d^4/a^4/e^3/(-a^2*e^4+2*a
*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^5+25/12*d^2
/a^3/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)*x*c^4-105/16*e^2/d^5/a/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-105/16/
d^3/a^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c+13/12/d^3/a/x^2/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+105/16*e^6/d^5*a/(-a^2*e^4+2*a*c*d^2*e^2-c^2
*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+233/48*e^4/d^3/(-a^2*e^4+2*a*
c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c+7/12/d/a^2/e^2
/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c+105/8*e^5/d^4/(-a^2*e^4+2*a*
c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c-43/24*e^2/d/
a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
c^2-1/6/a^2*e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)*x*c^3+16/3*e^5/d^2*c^2/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e^2-
c*d^2)*(d/e+x))^(1/2)*x+8/3*e^6/d^3*c/(a*e^2-c*d^2)^3/(c*d*e*(d/e+x)^2+(a*e
^2-c*d^2)*(d/e+x))^(1/2)*a+75/16/d/a^3/e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2
+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2+3
5/16*d/a^4/e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^3+155/48*d^3/a^3/e^2/(-a^2*e^4+
2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^4+35/16*d^
5/a^4/e^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)*c^5-17/6/d^2/a^2/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}(ex + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4),
x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] [undef, undef, undef, 1]
```

3.487 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=259

$$\frac{8\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8\left(x\left(a^2cd^2e^4-2a^3e^6+c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)}{15e\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{5(}$$

```
[Out] (2*x^2)/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) - (8*(a*d*e*(c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (c^3*d^6 + a^2*c*d
^2*e^4 - 2*a^3*e^6)*x))/(15*e*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*
e^2 + 2*c*d*e*x))/(15*e*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rubi [A] time = 0.235826, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {854, 777, 613}

$$\frac{8\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8\left(x\left(a^2cd^2e^4-2a^3e^6+c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)}{15e\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{5(}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*x^2)/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) - (8*(a*d*e*(c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (c^3*d^6 + a^2*c*d
^2*e^4 - 2*a^3*e^6)*x))/(15*e*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*
e^2 + 2*c*d*e*x))/(15*e*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int \frac{x\left(-2ade^2\left(cd^2\right.\right.}{\left(ade+\left(cd\right.\right.}}{5de\left(\right.}$$

$$= \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8\left(ade\left(cd^2-a\right.\right.}{15e\left(cd^2\right.}$$

$$= \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{8\left(ade\left(cd^2-a\right.\right.}{15e\left(cd^2\right.}$$

Mathematica [A] time = 0.142492, size = 235, normalized size = 0.91

$$\frac{2\left(2a^2c^2d^2e^2\left(189d^2e^2x^2+110d^3ex+20d^4+110de^3x^3+20e^4x^4\right)+4a^3cde^4\left(53d^2ex+20d^3+45de^2x^2+15e^3x^3\right)+a^4e^6\left(\right.\right.}{15(d+ex)\left(cd^2-ae^2\right)^5\left((d+e\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(c^4*d^6*x^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2) + a^4*e^6*(8*d^2 + 20*d*e*x
 + 15*e^2*x^2) + 4*a^3*c*d*e^4*(20*d^3 + 53*d^2*e*x + 45*d*e^2*x^2 + 15*e^3
*x^3) + 4*a*c^3*d^4*e*x*(15*d^3 + 45*d^2*e*x + 53*d*e^2*x^2 + 20*e^3*x^3) +
 2*a^2*c^2*d^2*e^2*(20*d^4 + 110*d^3*e*x + 189*d^2*e^2*x^2 + 110*d*e^3*x^3
+ 20*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(
3/2))
```

Maple [A] time = 0.054, size = 366, normalized size = 1.4

$$-\frac{(2\,cdx+2\,ae)\left(40\,a^2c^2d^2e^6x^4+80\,ac^3d^4e^4x^4+8\,c^4d^6e^2x^4+60\,a^3cde^7x^3+220\,a^2c^2d^3e^5x^3+212\,ac^3d^5e^3x^3+20\,c^4d^7ex^3\right.}{15\,a^5e^{10}-7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(40*a^2*c^2*d^2*e^6*x^4+80*a*c^3*d^4*e^4*x^4+8*c^4*d^6*e^
2*x^4+60*a^3*c*d*e^7*x^3+220*a^2*c^2*d^3*e^5*x^3+212*a*c^3*d^5*e^3*x^3+20*c
^4*d^7*e*x^3+15*a^4*e^8*x^2+180*a^3*c*d^2*e^6*x^2+378*a^2*c^2*d^4*e^4*x^2+1
80*a*c^3*d^6*e^2*x^2+15*c^4*d^8*x^2+20*a^4*d*e^7*x+212*a^3*c*d^3*e^5*x+220*
a^2*c^2*d^5*e^3*x+60*a*c^3*d^7*e*x+8*a^4*d^2*e^6+80*a^3*c*d^4*e^4+40*a^2*c^
2*d^6*e^2)/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+
5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[undef, undef, undef, undef, undef, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"giac")

[Out] [undef, undef, undef, undef, undef, 1]
```

3.488 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{128cd\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{8\left(x\left(3a^2\right.\right.}{35e\left(cd^2\right.}$$

```
[Out] (2*x^2)/(7*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)) - (8*(2*a*d*e*(c*d^2 + 2*a*e^2) + (2*c^2*d^4 + a*c*d^2*e^2 + 3*a^2
*e^4)*x))/(35*e*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)) + (16*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d
*e*x))/(105*e*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)) - (128*c*d*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*
c*d*e*x))/(105*(c*d^2 - a*e^2)^7*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])
```

Rubi [A] time = 0.288833, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {854, 777, 614, 613}

$$-\frac{128cd\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{8\left(x\left(3a^2\right.\right.}{35e\left(cd^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```


```
[Out] (2*x^2)/(7*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)) - (8*(2*a*d*e*(c*d^2 + 2*a*e^2) + (2*c^2*d^4 + a*c*d^2*e^2 + 3*a^2
*e^4)*x))/(35*e*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)) + (16*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d
*e*x))/(105*e*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)) - (128*c*d*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*
c*d*e*x))/(105*(c*d^2 - a*e^2)^7*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])
```


Rule 854

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> -Simp[((2*c*d - b*e)*(f + g*x)^n*(a + b*x +
 c*x^2)^(p + 1))/(e*p*(b^2 - 4*a*c)*(d + e*x)), x] - Dist[1/(d*e*p*(b^2 - 4
*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f*(
2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p + 1)
*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && IGt
Q[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
```

```
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}dx = \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} + \frac{2\int \frac{x(-2ade^2(}{(ade}}{7}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8\left(2ade\left(cd^2\right.\right.}{35e\left(cd^2-\right.}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8\left(2ade\left(cd^2\right.\right.}{35e\left(cd^2-\right.}$$

$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} - \frac{8\left(2ade\left(cd^2\right.\right.}{35e\left(cd^2-\right.}$$

Mathematica [A] time = 0.213206, size = 433, normalized size = 1.27

$$-\frac{2\sqrt{(d+ex)(ae+cdx)}\left(5a^4c^2d^2e^6\left(1859d^2e^2x^2+1288d^3ex+336d^4+1288de^3x^3+336e^4x^4\right)+20a^3c^3d^3e^4\left(1001d^3e^2x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (-2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a^6*e^10*(8*d^2 + 28*d*e*x + 35*e^2*x^
2)) + 2*a^5*c*d*e^8*(112*d^3 + 382*d^2*e*x + 455*d*e^2*x^2 + 140*e^3*x^3) +
 3*c^6*d^8*x^2*(35*d^4 + 280*d^3*e*x + 560*d^2*e^2*x^2 + 448*d*e^3*x^3 + 12
8*e^4*x^4) + 5*a^4*c^2*d^2*e^6*(336*d^4 + 1288*d^3*e*x + 1859*d^2*e^2*x^2 +
 1288*d*e^3*x^3 + 336*e^4*x^4) + 20*a^3*c^3*d^3*e^4*(56*d^5 + 406*d^4*e*x +
 1001*d^3*e^2*x^2 + 1084*d^2*e^3*x^3 + 560*d*e^4*x^4 + 112*e^5*x^5) + 2*a*c
^5*d^6*e*x*(70*d^5 + 1295*d^4*e*x + 4060*d^3*e^2*x^2 + 5600*d^2*e^3*x^3 + 3
616*d*e^4*x^4 + 896*e^5*x^5) + a^2*c^4*d^4*e^2*(56*d^6 + 2996*d^5*e*x + 131
95*d^4*e^2*x^2 + 24080*d^3*e^3*x^3 + 20320*d^2*e^4*x^4 + 7616*d*e^5*x^5 + 8
96*e^6*x^6)))/(105*(c*d^2 - a*e^2)^7*(a*e + c*d*x)^3*(d + e*x)^4)
```

Maple [B] time = 0.06, size = 663, normalized size = 1.9

$$-\frac{(2cdx+2ae)\left(-896a^2c^4d^4e^8x^6-1792ac^5d^6e^6x^6-384c^6d^8e^4x^6-2240a^3c^3d^3e^9x^5-7616a^2c^4d^5e^7x^5-7232ac^5d^7e^5x^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x)
```

```
[Out] -2/105*(c*d*x+a*e)*(-896*a^2*c^4*d^4*e^8*x^6-1792*a*c^5*d^6*e^6*x^6-384*c^6
*d^8*e^4*x^6-2240*a^3*c^3*d^3*e^9*x^5-7616*a^2*c^4*d^5*e^7*x^5-7232*a*c^5*d
^7*e^5*x^5-1344*c^6*d^9*e^3*x^5-1680*a^4*c^2*d^2*e^10*x^4-11200*a^3*c^3*d^4
*e^8*x^4-20320*a^2*c^4*d^6*e^6*x^4-11200*a*c^5*d^8*e^4*x^4-1680*c^6*d^10*e^
2*x^4-280*a^5*c*d*e^11*x^3-6440*a^4*c^2*d^3*e^9*x^3-21680*a^3*c^3*d^5*e^7*x
^3-24080*a^2*c^4*d^7*e^5*x^3-8120*a*c^5*d^9*e^3*x^3-840*c^6*d^11*e*x^3+35*a
^6*e^12*x^2-910*a^5*c*d^2*e^10*x^2-9295*a^4*c^2*d^4*e^8*x^2-20020*a^3*c^3*d
^6*e^6*x^2-13195*a^2*c^4*d^8*e^4*x^2-2590*a*c^5*d^10*e^2*x^2-105*c^6*d^12*x
^2+28*a^6*d*e^11*x-764*a^5*c*d^3*e^9*x-6440*a^4*c^2*d^5*e^7*x-8120*a^3*c^3*
d^7*e^5*x-2996*a^2*c^4*d^9*e^3*x-140*a*c^5*d^11*e*x+8*a^6*d^2*e^10-224*a^5*
c*d^4*e^8-1680*a^4*c^2*d^6*e^6-1120*a^3*c^3*d^8*e^4-56*a^2*c^4*d^10*e^2)/(a
^7*e^14-7*a^6*c*d^2*e^12+21*a^5*c^2*d^4*e^10-35*a^4*c^3*d^6*e^8+35*a^3*c^4*
d^8*e^6-21*a^2*c^5*d^10*e^4+7*a*c^6*d^12*e^2-c^7*d^14)/(c*d*e*x^2+a*e^2*x+c
*d^2*x+a*d*e)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 1]
```

3.489 $\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=170

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}+\frac{2}{11}\sqrt{x+1}\sqrt{x^2-x+1}x^4+\frac{6}{55}$$

```
[Out] (6*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/11 - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[
(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 +
x^3))
```

Rubi [A] time = 0.0656172, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 279, 321, 218}

$$\frac{2}{11}\sqrt{x+1}\sqrt{x^2-x+1}x^4+\frac{6}{55}\sqrt{x+1}\sqrt{x^2-x+1}x-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-\right.}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (6*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/11 - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[
(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 +
x^3))
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int x^3\sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x^3\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{x^3}{\sqrt{1+x^3}}\,dx}{11\sqrt{1+x^3}}$$

$$= \frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\left(6\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{55\sqrt{1+x^3}}$$

$$= \frac{6}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x^4\sqrt{1+x}\sqrt{1-x+x^2} - \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x}}{55\sqrt{\frac{}{(1}}}$$

Mathematica [C] time = 0.926628, size = 221, normalized size = 1.3

$$\frac{2\left(x\sqrt{x+1}\left(5x^5-5x^4+5x^3+3x^2-3x+3\right)+\sqrt{-\frac{6i}{\sqrt{3}+3i}}\left(\sqrt{3}+3i\right)(x+1)\sqrt{\frac{\left(\sqrt{3}-3i\right)x+\sqrt{3}+3i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{\frac{\left(\sqrt{3}+3i\right)x+\sqrt{3}-3i}{\left(\sqrt{3}+3i\right)(x+1)}}\text{EllipticF}\left(\right.\right.}{55\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*(x*Sqrt[1 + x]*(3 - 3*x + 3*x^2 + 5*x^3 - 5*x^4 + 5*x^5) + Sqrt[(-6*I)/(
3*I + Sqrt[3])]*(3*I + Sqrt[3])*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I + Sqrt[
3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt[3])*x
)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3]
)]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(55*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.589, size = 257, normalized size = 1.5

$$\frac{2}{55x^3+55}\sqrt{1+x}\sqrt{x^2-x+1}\left(5x^7+3i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i}{i}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

```
[Out] 2/55*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(5*x^7+3*I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*
((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3)
)^(1/2))*3^(1/2)-9*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(
1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/
(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))+8*x^4+3*x)/(x^3+
1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2-x+1}\sqrt{x+1}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^2-x+1}\sqrt{x+1}x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2-x+1}\sqrt{x+1}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

3.490 $\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=23

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

```
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Rubi [A] time = 0.0178384, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int x^2\sqrt{1+x}\sqrt{1-x+x^2}\,dx=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

Mathematica [A] time = 0.0344631, size = 23, normalized size = 1.

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Maple [A] time = 0.043, size = 18, normalized size = 0.8

$$\frac{2}{9}\,(1+x)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

```
[Out] 2/9*(1+x)^(3/2)*(x^2-x+1)^(3/2)
```

Maxima [A] time = 1.47234, size = 30, normalized size = 1.3

$$\frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Fricas [A] time = 2.19205, size = 61, normalized size = 2.65

$$\frac{2}{9}\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Giac [A] time = 1.13955, size = 36, normalized size = 1.57

$$\frac{2}{9}\sqrt{(x+1)^2-3\,x((x+1)(x-2)+3)(x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt((x + 1)^2 - 3*x)*((x + 1)*(x - 2) + 3)*(x + 1)^(3/2)
```

3.491 $\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=294

$$\frac{2\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}+\frac{2}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2+\frac{6\sqrt{x+1}\ldots}{7\left(x+\ldots\right.}$$

```
[Out] (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (6*Sqrt[1 + x]*Sqrt[1 - x + x^2])
/(7*(1 + Sqrt[3] + x)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1
- x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3
] + x)^2]*(1 + x^3)) + (2*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1
 + x^3))
```

Rubi [A] time = 0.0945421, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {809, 279, 303, 218, 1877}

$$\frac{2}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2+\frac{6\sqrt{x+1}\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\ldots\right.}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (6*Sqrt[1 + x]*Sqrt[1 - x + x^2])
/(7*(1 + Sqrt[3] + x)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1
- x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3
] + x)^2]*(1 + x^3)) + (2*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1
 + x^3))
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}\\ &= \frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{x}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}}\\ &= \frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int\frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}}+\frac{\left(3\sqrt{2\left(2-\sqrt{3}\right)}\sqrt{1+x}\sqrt{1-\right.}{7\sqrt{1+x^3}}\\ &= \frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{6\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+}{(1+\sqrt{3}}}}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(}\end{aligned}$$

Mathematica [C] time = 0.514735, size = 347, normalized size = 1.18

$$\frac{\sqrt{x+1}\left(3\sqrt{2}\left(\sqrt{3}-i\right)\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)+4\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\left(x^2-x+1\right)x^2-3\right.}{14\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (Sqrt[1 + x]*(4*x^2*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]*(1 - x + x^2) - 3*
Sqrt[2]*(-3*I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt
[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])] + 3*Sq
rt[2]*(-I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I
 + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((
-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(14*Sqrt
[((-I)*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.569, size = 361, normalized size = 1.2

$$\frac{1}{7x^3+7}\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\mathrm{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

```
[Out] 1/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(
1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*El
lipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2)
)*3^(1/2)+2*x^5+9*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1
/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(
I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-18*(-2*(1+x)/(I*3
^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2)*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)
-3)/(I*3^(1/2)+3))^(1/2))+2*x^2)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2-x+1}\sqrt{x+1}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{x^2-x+1}\sqrt{x+1}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x+1}\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2-x+1}\sqrt{x+1}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

3.492 $\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=144

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}+\frac{2}{5}x\sqrt{x^2-x+1}\sqrt{x+1}$$

```
[Out] (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x
)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rubi [A] time = 0.0324381, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {713, 195, 218}

$$\frac{2}{5}x\sqrt{x^2-x+1}\sqrt{x+1}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x
)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

& PosQ[a]

Rubi steps

$$\int \sqrt{1+x}\sqrt{1-x+x^2}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$
$$= \frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{5\sqrt{1+x^3}}$$
$$= \frac{2}{5}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-\right.}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Mathematica [C] time = 0.506257, size = 169, normalized size = 1.17

$$\frac{2x\sqrt{x+1}\left(x^2-x+1\right)+\frac{i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{5\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2) + (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3]
)*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[
Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]
)/Sqrt[(-I)/(3*I + Sqrt[3])])/(5*Sqrt[1 - x + x^2])
```

Maple [B] time = 0.565, size = 252, normalized size = 1.8

$$-\frac{1}{5x^3+5}\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

```
[Out] -1/5*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^
(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*E
llipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2
))*3^(1/2)-9*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3
))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(
1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-2*x^4-2*x)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2-x+1}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^2 - x + 1}\sqrt{x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x + 1}\sqrt{x^2 - x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 - x + 1}\sqrt{x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

3.493 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} dx$

Optimal. Leaf size=66

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcT
anh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x^3])
```

Rubi [A] time = 0.032602, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 50, 63, 207}

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcT
anh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x^3])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x}\,dx &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x}\,dx}{\sqrt{1+x^3}}\\
&= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{\left(2\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.411064, size = 197, normalized size = 2.98

$$\frac{\sqrt{x+1}\left(2\left(x^2-x+1\right)+\frac{3i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\Pi\left(\frac{3}{2}-\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{3\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (Sqrt[1 + x]*(2*(1 - x + x^2) + ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] - (2*I)*x)
/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticP
i[3/2 - (I/2)*Sqrt[3], I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3]
)]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]
))/(3*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.091, size = 43, normalized size = 0.7

$$-\frac{2}{3}\sqrt{1+x}\sqrt{x^2-x+1}\left(-\sqrt{x^3+1}+\text{Artanh}\left(\sqrt{x^3+1}\right)\right)\frac{1}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x)
```

```
[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-(x^3+1)^(1/2)+arctanh((x^3+1)^(1/2)))/(x
^3+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

Fricas [A] time = 1.70636, size = 169, normalized size = 2.56

$$\frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) +
 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

3.494 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx$

Optimal. Leaf size=287

$$\frac{\sqrt{2}3^{3/4}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}-\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}+\frac{3\sqrt{x^2-x+1}\sqrt{}}{x+\sqrt{3}+}$$

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + x]*Sqrt[1 - x + x^2])/(1
 + Sqrt[3] + x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*(1 + x^3)) + (Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rubi [A] time = 0.0974466, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 277, 303, 218, 1877}

$$-\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x}+\frac{3\sqrt{x^2-x+1}\sqrt{x+1}}{x+\sqrt{3}+1}+\frac{\sqrt{2}3^{3/4}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```


```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + x]*Sqrt[1 - x + x^2])/(1
 + Sqrt[3] + x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*(1 + x^3)) + (Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```


Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x^2}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}} + \frac{\left(3\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\sqrt{1+x}\sqrt{1-x- \cdots\right.}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+x}\sqrt{1-x+x^2}}{1+\sqrt{3}+x} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x) \cdots}}}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+ \cdots\right.}$$

Mathematica [C] time = 0.402792, size = 349, normalized size = 1.22

$$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x} + \frac{3\sqrt{1+\frac{2i(x+1)}{\sqrt{3}-3i}}\sqrt{1-\frac{2i(x+1)}{\sqrt{3}+3i}}\left(\frac{\left(\sqrt{3}-i\right)\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}} - \frac{\left(\sqrt{3}-3i\right)\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x \cdots}}{\cdots}\right.}{2\sqrt{2}\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{(x+1)^2-3(x+1)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + ((2*I)*(1 + x))/(-3*I +
Sqrt[3])]*Sqrt[1 - ((2*I)*(1 + x))/(3*I + Sqrt[3])]*(-(((-3*I + Sqrt[3])*Sq
rt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 + x]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((-I)
*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(
1 + x))/(3*I + Sqrt[3])]) + ((-I + Sqrt[3])*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt
[1 + x]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]],
(3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(2
*Sqrt[2]*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

Maple [A] time = 1.33, size = 363, normalized size = 1.3

$$\frac{1}{2x\left(x^{3}+1\right)}\sqrt{1+x}\sqrt{x^{2}-x+1}\left(3i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-}{i\sqrt{3}+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x)
```

```
[Out] 1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(
1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*El
lipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2)
)*3^(1/2)*x+9*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+
3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^
(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x-18*(-2*(1+x)/(I*3^(
1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2)*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3
)/(I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/x/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

3.495 $\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx$

Optimal. Leaf size=146

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{2x^2}$$

```
[Out] -(Sqrt[1 + x]*Sqrt[1 - x + x^2])/(2*x^2) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 +
x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elliptic
F[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rubi [A] time = 0.0443619, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {915, 277, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

```
[Out] -(Sqrt[1 + x]*Sqrt[1 - x + x^2])/(2*x^2) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 +
x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elliptic
F[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x^3}\,dx}{\sqrt{1+x^3}}$$
$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2} + \frac{\left(3\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{4\sqrt{1+x^3}}$$
$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\left(1+x^3\right)}$$

Mathematica [C] time = 0.410136, size = 185, normalized size = 1.27

$$\frac{\sqrt{x+1}\left(-\frac{2\left(x^2-x+1\right)}{x^2}-\frac{3i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{4\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

```
[Out] (Sqrt[1 + x]*((-2*(1 - x + x^2))/x^2 - ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] - (
2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqr
t[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(4*Sqrt[1 -
x + x^2])
```

Maple [B] time = 1.089, size = 259, normalized size = 1.8

$$-\frac{1}{\left(4x^3+4\right)x^2}\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{-2\frac{1+x}{i\sqrt{3}-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x)
```

```
[Out] -1/4*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/
2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3
))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*(-2*(1+x)/(I*3^(1/2)-3))^(1/
2)*3^(1/2)*x^2-9*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/
2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I
*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^2+2*x^3+2)/(x^3+
1)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

3.496 $\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=201

$$-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{935\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}+\frac{2}{17}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+\right.$$

```
[Out] (54*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/935 + (18*x^4*Sqrt[1 + x]*Sqrt[1 - x +
 x^2])/187 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/17 - (36*3^(3/
4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(935*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rubi [A] time = 0.0761565, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 279, 321, 218}

$$\frac{2}{17}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)x^4+\frac{18}{187}\sqrt{x+1}\sqrt{x^2-x+1}x^4+\frac{54}{935}\sqrt{x+1}\sqrt{x^2-x+1}x-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2}}{93}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```


```
[Out] (54*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/935 + (18*x^4*Sqrt[1 + x]*Sqrt[1 - x +
 x^2])/187 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/17 - (36*3^(3/
4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(935*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```


Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x^3\left(1+x^3\right)^{3/2}dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x^3\sqrt{1+x^3}\,dx}{17\sqrt{1+x^3}}$$

$$= \frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(27\sqrt{1+x}\sqrt{1-x\right.}{187\sqrt{1}}$$

$$= \frac{54}{935}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}\left(1\right.$$

$$= \frac{54}{935}x\sqrt{1+x}\sqrt{1-x+x^2}+\frac{18}{187}x^4\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{17}x^4\sqrt{1+x}\sqrt{1-x+x^2}\left(1\right.$$

Mathematica [C] time = 0.890009, size = 235, normalized size = 1.17

$$\frac{2\left(x\sqrt{x+1}\left(55x^8-55x^7+55x^6+100x^5-100x^4+100x^3+27x^2-27x+27\right)-\frac{9i\sqrt{6}(x+1)\sqrt{\frac{\left(\sqrt{3}-3i\right)x+\sqrt{3}+3i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{\frac{\left(\sqrt{3}+3i\right)x+\sqrt{3}-3i}{\left(\sqrt{3}+3i\right)(x+1)}}\text{Ell}}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}\right.}{935\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (2*(x*Sqrt[1 + x]*(27 - 27*x + 27*x^2 + 100*x^3 - 100*x^4 + 100*x^5 + 55*x^
6 - 55*x^7 + 55*x^8) - ((9*I)*Sqrt[6]*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I +
 Sqrt[3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt
[3])*x)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + S
qrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + S
qrt[3])]))/(935*Sqrt[1 - x + x^2])
```

Maple [A] time = 1.13, size = 262, normalized size = 1.3

$$\frac{2}{935x^3+935}\sqrt{1+x}\sqrt{x^2-x+1}\left(55x^{10}+155x^7+27i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\mathrm{EllipticF}\left(\sqrt{-2\frac{1}{i\sqrt{}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

```
[Out] 2/935*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(55*x^10+155*x^7+27*I*(-2*(1+x)/(I*3^(1/2
)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(
I*3^(1/2)+3))^(1/2))*3^(1/2)-81*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-
2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*Ellipti
cF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))+127
*x^4+27*x)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(x^6+x^3\right)\sqrt{x^2-x+1}\sqrt{x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^6 + x^3)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 - x + 1\right)^{\frac{3}{2}} (x + 1)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

3.497 $\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

```
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Rubi [A] time = 0.0208217, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]

[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

Mathematica [A] time = 0.0430488, size = 23, normalized size = 1.

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]

[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Maple [A] time = 0.042, size = 18, normalized size = 0.8

$$\frac{2}{15}\,(1+x)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

```
[Out] 2/15*(1+x)^(5/2)*(x^2-x+1)^(5/2)
```

Maxima [A] time = 1.44561, size = 36, normalized size = 1.57

$$\frac{2}{15}\left(x^6+2x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Fricas [A] time = 1.7053, size = 73, normalized size = 3.17

$$\frac{2}{15}\left(x^6+2x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Giac [B] time = 1.19915, size = 100, normalized size = 4.35

$$\frac{2}{45}\left(((3((x+1)(x-5)+15)(x+1)-59)(x+1)+42)(x+1)-15\right)\sqrt{(x+1)^2-3x}(x+1)^{\frac{3}{2}}+\frac{2}{9}\sqrt{(x+1)^2-3x}((x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45*(((3*((x + 1)*(x - 5) + 15)*(x + 1) - 59)*(x + 1) + 42)*(x + 1) - 15)*
sqrt((x + 1)^2 - 3*x)*(x + 1)^(3/2) + 2/9*sqrt((x + 1)^2 - 3*x)*((x + 1)*(x
 - 2) + 3)*(x + 1)^(3/2)
```

3.498 $\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=325

$$\frac{18\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{91\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}+\frac{2}{13}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)x^2+\frac{1}{9}$$

```
[Out] (18*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/91 + (54*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/(91*(1 + Sqrt[3] + x)) + (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3)
)/13 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + S
qrt[3] + x)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x
^3)) + (18*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rubi [A] time = 0.106791, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {809, 279, 303, 218, 1877}

$$\frac{2}{13}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)x^2+\frac{18}{91}\sqrt{x+1}\sqrt{x^2-x+1}x^2+\frac{54\sqrt{x+1}\sqrt{x^2-x+1}}{91\left(x+\sqrt{3}+1\right)}+\frac{18\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\cdots}}{91\sqrt{\frac{\cdots}{(x+\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (18*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/91 + (54*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/(91*(1 + Sqrt[3] + x)) + (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3)
)/13 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + S
qrt[3] + x)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x
^3)) + (18*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx &= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x\left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}\\
&= \frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{13\sqrt{1+x^3}}\\
&= \frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(27\sqrt{1+x}\sqrt{1-x+\cdots}\right.}{91\sqrt{1+\cdots}}\\
&= \frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(27\sqrt{1+x}\sqrt{1-x+\cdots}\right.}{91\sqrt{1+\cdots}}\\
&= \frac{18}{91}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{54\sqrt{1+x}\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)}+\frac{2}{13}x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\cdots\right.
\end{aligned}
$$

Mathematica [C] time = 0.488259, size = 244, normalized size = 0.75

$$
\frac{\sqrt{x+1}\left(4x^2\left(x^2-x+1\right)\left(7x^3+16\right)-\frac{27\sqrt{2}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\left(\left(\sqrt{3}-3i\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)-\left(\sqrt{3}-i\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\right),\cdots\right.\right.}{\sqrt{-\frac{i(x+1)}{-2ix+\sqrt{3}+i}}}\right.}{182\sqrt{x^2-x+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + x]*(4*x^2*(1 - x + x^2)*(16 + 7*x^3) - (27*Sqrt[2]*Sqrt[(-I + Sqr
t[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + S
qrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I + Sqrt
[3] - (2*I)*x)]))/(182*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.577, size = 366, normalized size = 1.1

$$\frac{1}{91\,x^3+91}\sqrt{1+x}\sqrt{x^2-x+1}\left(14\,x^8+27\,i\sqrt{-2\,\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2\,x+1}{i\sqrt{3}+3}}\sqrt{\frac{2\,x-1+i\sqrt{3}}{i\sqrt{3}-3}}\mathrm{EllipticF}\left(\sqrt{-2\,\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{}}{i\sqrt{}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

```
[Out] 1/91*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(14*x^8+27*I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2
)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+
3))^(1/2))*3^(1/2)+46*x^5+81*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x
+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF(
(-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-162*(-
2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),
(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))+32*x^2)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(x^4+x\right)\sqrt{x^2-x+1}\sqrt{x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + x)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

3.499 $\int (1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=173

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}+\frac{2}{11}x\sqrt{x^2-x+1}\left(x^3+1\right)\sqrt{x+}$$

```
[Out] (18*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2
]*(1 + x^3))/11 + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x +
x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*(1 + x^3))
```

Rubi [A] time = 0.0457231, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {713, 195, 218}

$$\frac{2}{11}x\sqrt{x^2-x+1}\left(x^3+1\right)\sqrt{x+1}+\frac{18}{55}x\sqrt{x^2-x+1}\sqrt{x+1}+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x}{x}\right.\right.}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (18*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2
]*(1 + x^3))/11 + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x +
x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*(1 + x^3))
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int (1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}$$
$$= \frac{2}{11}x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right) + \frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}\,dx}{11\sqrt{1+x^3}}$$
$$= \frac{18}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right) + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)}{55\sqrt{1+x^3}}$$
$$= \frac{18}{55}x\sqrt{1+x}\sqrt{1-x+x^2} + \frac{2}{11}x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right) + \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/}}{}$$

Mathematica [C] time = 0.645399, size = 176, normalized size = 1.02

$$\frac{2x\sqrt{x+1}\left(x^2-x+1\right)\left(5x^3+14\right)+\frac{9i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{55\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2)*(14 + 5*x^3) + ((9*I)*(1 + x)*Sqrt[1 + (6*I)
/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*Ell
ipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(55*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.575, size = 257, normalized size = 1.5

$$-\frac{1}{55x^3+55}\sqrt{1+x}\sqrt{x^2-x+1}\left(-10x^7+27i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2),x)
```

```
[Out] -1/55*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-10*x^7+27*I*(-2*(1+x)/(I*3^(1/2)-3))^(1
/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2
)+3))^(1/2))*3^(1/2)-81*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(
I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(
1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-38*x^4-28*x
```

```
)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

3.500 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2}{9}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)+\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 + (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 +
 x^3))/9 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt
[1 + x^3])
```

Rubi [A] time = 0.0406819, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 50, 63, 207}

$$\frac{2}{9}\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)+\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 + (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 +
 x^3))/9 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt
[1 + x^3])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x}\,dx}{\sqrt{1+x^3}}$$
$$= \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}$$
$$= \frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx,x,x^3\right)}{3\sqrt{1+x^3}}$$
$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int\right.}{3\sqrt{1+x^3}}$$
$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(2\sqrt{1+x}\sqrt{1-x+x^2}\right)\text{Subst}\left(\int\right.}{3\sqrt{1+x^3}}$$
$$= \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{2\sqrt{1+x}\sqrt{1-x+x^2}\tanh^{-1}\left(\sqrt{\right.}{3\sqrt{1+x^3}}$$

Mathematica [C] time = 0.298377, size = 201, normalized size = 2.14

$$\frac{\sqrt{x+1}\left(\frac{2}{9}\left(x^2-x+1\right)\left(x^3+4\right)+\frac{i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\Pi\left(\frac{3}{2}-\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[1 + x]*((2*(1 - x + x^2)*(4 + x^3))/9 + (I*Sqrt[2]*Sqrt[(I + Sqrt[3]
- (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]
*EllipticPi[3/2 - (I/2)*Sqrt[3], I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I
 + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I +
 Sqrt[3])]))/Sqrt[1 - x + x^2]
```

Maple [A] time = 0.09, size = 57, normalized size = 0.6

$$-\frac{2}{9}\sqrt{1+x}\sqrt{x^2-x+1}\left(-x^3\sqrt{x^3+1}+3\,\text{Artanh}\left(\sqrt{x^3+1}\right)-4\sqrt{x^3+1}\right)\frac{1}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x)
```

```
[Out] -2/9*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-x^3*(x^3+1)^(1/2)+3*arctanh((x^3+1)^(1/2
))-4*(x^3+1)^(1/2))/(x^3+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

Fricas [A] time = 1.74663, size = 182, normalized size = 1.94

$$\frac{2}{9}\left(x^3+4\right)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

3.501 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=323

$$\frac{9\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}+\frac{9}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2+\frac{27\sqrt{x+1}\sqrt{x}}{7\left(x+\sqrt{}\right.}$$

```
[Out] (9*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (27*Sqrt[1 + x]*Sqrt[1 - x + x^2]
)/(7*(1 + Sqrt[3] + x)) - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/x - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(14*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3)) + (9*
Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rubi [A] time = 0.111433, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {915, 277, 279, 303, 218, 1877}

$$\frac{9}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2+\frac{27\sqrt{x+1}\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}-\frac{\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}{x}+\frac{9\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)}}}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```


```
[Out] (9*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (27*Sqrt[1 + x]*Sqrt[1 - x + x^2]
)/(7*(1 + Sqrt[3] + x)) - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/x - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(14*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3)) + (9*
Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```


Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x^2}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{x} + \frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{2\sqrt{1+x^3}}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{x} + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int}{14\sqrt{1+x^3}}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{x} + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int}{14\sqrt{1+x^3}}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2} + \frac{27\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{x} - \frac{27}{}$$

Mathematica [C] time = 0.506683, size = 244, normalized size = 0.76

$$\frac{\sqrt{x+1}\left(\frac{4(x^2-x+1)(2x^3-7)}{x}-\frac{27\sqrt{2}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\left((\sqrt{3}-3i)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)-(\sqrt{3}-i)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)\right)}{\sqrt{-\frac{i(x+1)}{-2ix+\sqrt{3}+i}}}\right)}{28\sqrt{x^2-x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[1 + x]*((4*(1 - x + x^2)*(-7 + 2*x^3))/x - (27*Sqrt[2]*Sqrt[(-I + Sqr
t[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + S
qrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I + Sqrt
[3] - (2*I)*x)]))/(28*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.6, size = 368, normalized size = 1.1

$$\frac{1}{14x(x^3+1)}\sqrt{1+x}\sqrt{x^2-x+1}\left(27i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x)
```

```
[Out] 1/14*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(27*I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3
^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*
EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/
2))*3^(1/2)*x+4*x^6+81*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I
*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1
+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x-162*(-2*(1
+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I
*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x-10*x^3-14)/x/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^3+1)\sqrt{x^2-x+1}\sqrt{x+1}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**2,x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

3.502 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=175

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{10\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}-\frac{\sqrt{x^2-x+1}\left(x^3+1\right)\sqrt{x+1}}{2x^2}+$$

```
[Out] (9*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/10 - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
+ x^3))/(2*x^2) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(10*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*(1 + x^3))
```

Rubi [A] time = 0.0584062, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 277, 195, 218}

$$-\frac{\sqrt{x^2-x+1}\left(x^3+1\right)\sqrt{x+1}}{2x^2}+\frac{9}{10}x\sqrt{x^2-x+1}\sqrt{x+1}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right.\right.}{10\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

```
[Out] (9*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/10 - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
+ x^3))/(2*x^2) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(10*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*(1 + x^3))
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \frac{\left(\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x^3}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{2x^2} + \frac{\left(9\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}\,dx}{4\sqrt{1+x^3}}$$

$$= \frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{2x^2} + \frac{\left(27\sqrt{1+x}\sqrt{1-x+x^2}\right)\int \frac{}{\sqrt{}}}{20\sqrt{1+x^3}}$$

$$= \frac{9}{10}x\sqrt{1+x}\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)}{2x^2} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}(1+x)^{3/2}\sqrt{1}}{}$$

Mathematica [C] time = 0.354324, size = 192, normalized size = 1.1

$$\frac{\sqrt{x+1}\left(\frac{2\left(x^2-x+1\right)\left(4x^3-5\right)}{x^2} - \frac{27i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{20\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[1 + x]*((2*(1 - x + x^2)*(-5 + 4*x^3))/x^2 - ((27*I)*Sqrt[2]*Sqrt[(I
+ Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I +
 Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]
], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))
/(20*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.592, size = 264, normalized size = 1.5

$$-\frac{1}{\left(20x^3+20\right)x^2}\sqrt{1+x}\sqrt{x^2-x+1}\left(27i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\,\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x)
```

```
[Out] -1/20*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(27*I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*
3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1
/2))*3^(1/2)*x^2-81*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^
(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)
/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^2-8*x^6+2*x^3
+10)/(x^3+1)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**3,x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

3.503 $\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=142

$$\frac{2x(x^3+1)}{5\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}}$$

```
[Out] (2*x*(1 + x^3))/(5*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (4*Sqrt[2 + Sqrt[3]]*Sq
rt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 +
 Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0472602, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {915, 321, 218}

$$\frac{2x(x^3+1)}{5\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*x*(1 + x^3))/(5*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (4*Sqrt[2 + Sqrt[3]]*Sq
rt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 +
 Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{x^3}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\ &= \frac{2x\left(1+x^3\right)}{5\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{5\sqrt{1+x}\sqrt{1-x+x^2}}\\ &= \frac{2x\left(1+x^3\right)}{5\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}\end{aligned}$$

Mathematica [C] time = 0.617742, size = 169, normalized size = 1.19

$$\frac{6x\sqrt{x+1}\left(x^2-x+1\right)-\frac{2i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{15\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (6*x*Sqrt[1 + x]*(1 - x + x^2) - ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqr
t[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcS
inh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[
3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(15*Sqrt[1 - x + x^2])
```

Maple [B] time = 1.324, size = 248, normalized size = 1.8

$$\frac{2}{5x^3+5}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 2/5*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/
2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*Elli
pticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*
3^(1/2)-3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^
(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2
)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))+x^4+x)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}x^3}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3/(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.504 $\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Rubi [A] time = 0.0187211, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{1+x}\sqrt{1-x+x^2}$$

Mathematica [A] time = 0.0255489, size = 23, normalized size = 1.

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Maple [A] time = 0.043, size = 18, normalized size = 0.8

$$\frac{2}{3}\sqrt{1+x}\sqrt{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

Maxima [A] time = 1.47163, size = 30, normalized size = 1.3

$$\frac{2\left(x^3+1\right)}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^3 + 1)/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Fricas [A] time = 1.68422, size = 47, normalized size = 2.04

$$\frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [A] time = 1.17004, size = 24, normalized size = 1.04

$$\frac{2}{3}\sqrt{(x+1)^2-3x}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

3.505 $\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=253

$$\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{2\left(x^3+1\right)}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x\ \cdots}}{\cdots}$$

```
[Out] (2*(1 + x^3))/(Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (2*Sqrt[2]*Sqrt[1 + x]*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0628543, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {809, 303, 218, 1877}

$$\frac{2\left(x^3+1\right)}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\cdots}}{\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*(1 + x^3))/(Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (2*Sqrt[2]*Sqrt[1 + x]*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 - x + x^2])
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2\left(1+x^3\right)}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 1.01365, size = 375, normalized size = 1.48

$$\frac{(x+1)^{3/2}\left(\frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{x+1}}+\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}}{}}}{\sqrt{}}\right)}{6\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (
3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[
3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*Ar
cSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqr
t[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I
)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + S
qrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*
I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(6*Sqrt[(-I)/(3*I + Sqrt[3])]*
Sqrt[1 - x + x^2])
```

Maple [A] time = 1.325, size = 275, normalized size = 1.1

$$\frac{i\sqrt{3}-3}{2x^3+2}\sqrt{1+x}\sqrt{x^2-x+1}\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\left(i\mathrm{EllipticE}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*3^(1/2)-3)*(-2*(1+x)/(I*3^(1/2)-3))^(1/2
)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2)*(I*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/
2)+3))^(1/2))*3^(1/2)-I*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/
2)-3)/(I*3^(1/2)+3))^(1/2))*3^(1/2)+3*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1
/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-EllipticF((-2*(1+x)/(I*3^(1/2)-3)
)^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2)))/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}x}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x/(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2 - x + 1}\sqrt{x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.506 $\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=110

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

```
[Out] (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0219877, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {713, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.144856, size = 148, normalized size = 1.35

$$\frac{i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{\frac{2}{3}-\frac{4i}{\left(\sqrt{3}+3i\right)(x+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[2/3 - (4*I)/((3*
I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqr
t[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/(Sqrt[(-I)/(3*I + Sqrt[3])]*Sq
rt[1 - x + x^2])
```

Maple [A] time = 0.651, size = 137, normalized size = 1.3

$$\frac{-i\sqrt{3}+3}{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] (-I*3^(1/2)+3)*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*(
(I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))
^(1/2))/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

3.507 $\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] (-2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0263797, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 266, 63, 207}

$$-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (-2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= -\frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 12.7928, size = 2463, normalized size = 58.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] 2*(((-I)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*S
qrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]]
, (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sq
rt[3 - 3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sq
rt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x
])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1
/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(S
qrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sq
rt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqr
t[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6
/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[Arc
Sin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3
])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*
(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqr
t[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sq
rt[3]))/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/S
qrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-S
qrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[
3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 +
 x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])]
 - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3
 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1
/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt
[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) - (Sqrt[
3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt
[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*
(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] +
 Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*
Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x
]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (
I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sq
rt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] +
Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1 + Sqr
```

```
t[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[
3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqr
t[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]],
(Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3
 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt[1/2 -
 (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/(
(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I
/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(S
qrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I
*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3
]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2])
)/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/
2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3
 - 3*(1 + x) + (1 + x)^2]))
```

Maple [A] time = 0.872, size = 33, normalized size = 0.8

$$-\frac{2}{3}\mathrm{Artanh}\left(\sqrt{x^3+1}\right)\sqrt{1+x}\sqrt{x^2-x+1}\frac{1}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

Fricas [A] time = 1.76303, size = 122, normalized size = 2.9

$$-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

3.508 $\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{x^3+1}{x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{x^3+1}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

```
[Out] -((1 + x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2])) + (1 + x^3)/(Sqrt[1 + x]*(1
+ Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 - x + x^2]) + (Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0966822, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 325, 303, 218, 1877}

$$-\frac{x^3+1}{x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{x^3+1}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -((1 + x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2])) + (1 + x^3)/(Sqrt[1 + x]*(1
+ Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 - x + x^2]) + (Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}} \\
&= -\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{1+x^3}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-\cdots}{(1+\cdots}}\cdots}{2\sqrt{\frac{1\cdots}{(1+\cdots}}\cdots}
\end{aligned}$$

Mathematica [C] time = 0.871238, size = 400, normalized size = 1.42

$$-\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}+\frac{(x+1)^{3/2}\left(\frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{x+1}}+\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2}+\cdots\right)}{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I +
 Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I
+ Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1
 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/S
qrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*
I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3
*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-
6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[
1 + x]))/(12*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.349, size = 363, normalized size = 1.3

$$\frac{1}{2x(x^3+1)}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*3^(1/2)*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*(
(I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))
^(1/2))*x-6*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3)
)^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticE((-2*(1+x)/(I*3^(1
/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x+3*(-2*(1+x)/(I*3^(1/2
)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(
I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/x/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^5+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^5 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x+1}\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

3.509 $\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{x^3+1}{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] -(1 + x^3)/(2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (Sqrt[2 + Sqrt[3]]*Sqrt[
1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0450628, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {915, 325, 218}

$$-\frac{x^3+1}{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -(1 + x^3)/(2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (Sqrt[2 + Sqrt[3]]*Sqrt[
1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= -\frac{1+x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{4\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= -\frac{1+x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.667534, size = 171, normalized size = 1.19

$$\frac{-\frac{6\sqrt{x+1}\left(x^2-x+1\right)}{x^2}-\frac{i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{12\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] ((-6*Sqrt[1 + x]*(1 - x + x^2))/x^2 - (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sq
rt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*Arc
Sinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt
[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(12*Sqrt[1 - x + x^2])
```

Maple [B] time = 0.347, size = 259, normalized size = 1.8

$$\frac{1}{\left(4x^3+4\right)x^2}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)
```

```
[Out] 1/4*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/
2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*Elli
pticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*
3^(1/2)*x^2-3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+
3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^
(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^2-2*x^3-2)/(x^3+1)/
x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - x + 1}\sqrt{x + 1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^6 + x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x + 1}\sqrt{x^2 - x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - x + 1}\sqrt{x + 1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

3.510 $\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] (-2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0483607, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {915, 288, 218}

$$\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (-2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 - x + x^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x^3}{\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.580536, size = 161, normalized size = 1.18

$$\frac{-\frac{6x}{\sqrt{x+1}} + \frac{2i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{9\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] ((-6*x)/Sqrt[1 + x] + ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 +
x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-
6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[
(-I)/(3*I + Sqrt[3])])/(9*Sqrt[1 - x + x^2])
```

Maple [B] time = 1.175, size = 245, normalized size = 1.8

$$-\frac{2}{3x^3+3}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1
/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*Ell
ipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))
*3^(1/2)-3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))
^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/
2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))+x)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}x^3}{x^6+2x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3/(x^6 + 2*x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**3/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.511 $\int \frac{x^2}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0211387, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = -\frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [A] time = 0.0304251, size = 23, normalized size = 1.

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Maple [A] time = 0.044, size = 18, normalized size = 0.8

$$-\frac{2}{3}\frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Maxima [A] time = 1.47613, size = 23, normalized size = 1.

$$-\frac{2}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Fricas [A] time = 1.74493, size = 62, normalized size = 2.7

$$-\frac{2\sqrt{x^2-x+1}\sqrt{x+1}}{3\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**2/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.512 $\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=282

$$-\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\left(x^3+1\right)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x}}$$

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*(1 + x^3))/(3*Sqrt[1 + x]*(1
 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) + (Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1
 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sq
rt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 - x + x^2]) - (2*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3
]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0839747, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {809, 290, 303, 218, 1877}

$$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\left(x^3+1\right)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}-\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*(1 + x^3))/(3*Sqrt[1 + x]*(1
 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) + (Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1
 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sq
rt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 - x + x^2]) - (2*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3
]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{x}{\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2x^2}{3\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2\left(1+x^3\right)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} + \frac{\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x\cdots}{(1+\sqrt{\cdots}}}\cdots}{3^{3/4}\sqrt{\cdots}}
\end{aligned}$$

Mathematica [C] time = 0.75637, size = 402, normalized size = 1.43

$$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{(x+1)^{3/2}\left(\frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{x+1}} + \frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2-x+1\right)}{(x+1)^2} + \cdots\right)}{18\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - ((1 + x)^(3/2)*((12*Sqrt[(-I)/(
3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[
(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*
I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3
])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2
]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqr
t[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sq
rt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/
Sqrt[1 + x]))/(18*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A] time = 1.463, size = 356, normalized size = 1.3

$$-\frac{1}{3x^3+3}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -1/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1
/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*Ell
ipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))
*3^(1/2)+3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))
^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/
2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-6*(-2*(1+x)/(I*3^(1/2)-3
))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2)*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3
^(1/2)+3))^(1/2))-2*x^2)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}x}{x^6+2x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x/(x^6 + 2*x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2-x+1)^{\frac{3}{2}}\,(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.513 $\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] (2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0330799, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {713, 199, 218}

$$\frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```


```
[Out] (2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 - x + x^2])
```


Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.372938, size = 216, normalized size = 1.58

$$\sqrt{(x+1)^2-3(x+1)+3}\left(\frac{2(x+1)^{3/2}}{9\left((x+1)^2-3(x+1)+3\right)}-\frac{2}{9\sqrt{x+1}}\right)+\frac{i\sqrt{\frac{2}{3}}(x+1)\sqrt{1-\frac{6}{\left(3-i\sqrt{3}\right)(x+1)}}\sqrt{1-\frac{6}{\left(3+i\sqrt{3}\right)(x+1)}}\text{Elli}}{3\sqrt{-\frac{1}{3-i\sqrt{3}}}\sqrt{(x+1)^2-3(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Sqrt[3 - 3*(1 + x) + (1 + x)^2]*(-2/(9*Sqrt[1 + x]) + (2*(1 + x)^(3/2))/(9*
(3 - 3*(1 + x) + (1 + x)^2))) + ((I/3)*Sqrt[2/3]*(1 + x)*Sqrt[1 - 6/((3 - I
*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])*(1 + x))]*EllipticF[I*ArcSi
nh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])
/(Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

Maple [B] time = 1.325, size = 247, normalized size = 1.8

$$-\frac{1}{3x^3+3}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -1/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1
/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*Ell
ipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))
*3^(1/2)-3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))
^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/
2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-2*x)/(x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^6+2\,x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

3.514 $\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3
]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0338986, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 51, 63, 207}

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3
]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{1}{x\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx,x,x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}
$$

Mathematica [C] time = 6.08176, size = 2511, normalized size = 38.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2/(9*(1 + x)) - (2*(-2 + x))/(9*(1 - x + x^2
))) + 2*(((-I)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3
 + I*Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1
 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-
1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]
] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] +
1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-
Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))
/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]
] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])
)]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3]
)] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(
3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*Ellipti
cF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(S
qrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 +
 I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3
- I*Sqrt[3]))/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (
I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]
])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[(
(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sq
```

```
rt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqr
t[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(
Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 -
Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2
)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) -
(Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*
(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]
))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt
[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 +
x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt
[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1
/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3
+ I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[
3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1
 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] -
 Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 -
 I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]
))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] -
Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt
[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3
]]))/((1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/
2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3
]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt
[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*
Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]
])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2
 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*
Sqrt[3 - 3*(1 + x) + (1 + x)^2]))
```

Maple [A] time = 0.937, size = 43, normalized size = 0.7

$$-\frac{2}{3\,x^{3}+3}\sqrt{1+x}\sqrt{x^{2}-x+1}\left(\operatorname{Artanh}\left(\sqrt{x^{3}+1}\right)\sqrt{x^{3}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)-1)/(
x^3+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{2}-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

Fricas [A] time = 1.74596, size = 205, normalized size = 3.11

$$-\frac{\left(x^3+1\right)\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)-\left(x^3+1\right)\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)-2\sqrt{x^2-x+1}\sqrt{x+1}}{3\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*((x^3 + 1)*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) - (x^3 + 1)*log(sqrt
(x^2 - x + 1)*sqrt(x + 1) - 1) - 2*sqrt(x^2 - x + 1)*sqrt(x + 1))/(x^3 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

3.515 $\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{5\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{5\left(x^3+1\right)}{3x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{5\left(x^3+1\right)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-}}$$

```
[Out] 2/(3*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (5*(1 + x^3))/(3*x*Sqrt[1 + x]*Sqrt
[1 - x + x^2]) + (5*(1 + x^3))/(3*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x
+ x^2]) - (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]])/(2*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (5*S
qrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.109126, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {915, 290, 325, 303, 218, 1877}

$$-\frac{5\left(x^3+1\right)}{3x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{5\left(x^3+1\right)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{2}{3x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{5\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin\right.}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```


```
[Out] 2/(3*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (5*(1 + x^3))/(3*x*Sqrt[1 + x]*Sqrt
[1 - x + x^2]) + (5*(1 + x^3))/(3*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x
+ x^2]) - (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]])/(2*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (5*S
qrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```


Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^2\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5\left(1+x^3\right)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{6\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5\left(1+x^3\right)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{6\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{\frac{1}{2}\left(2\right.}\right.}{3}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5\left(1+x^3\right)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{5\left(1+x^3\right)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} -$$

Mathematica [C] time = 0.878586, size = 409, normalized size = 1.29

$$-\frac{5x^3+3}{3x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{5(x+1)^{3/2}\left(\frac{i\sqrt{2}(\sqrt{3}+3i)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{x+1}}+\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}(x^2-x+}{(x+1)^2}\right.}{36\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -(3 + 5*x^3)/(3*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (5*(1 + x)^(3/2)*((12*Sq
rt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[
3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt
[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I
 + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] +
(I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt
[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*A
rcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sq
rt[3])])/Sqrt[1 + x]))/(36*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A] time = 1.331, size = 363, normalized size = 1.2

$$\frac{1}{(6x^3+6)x}\sqrt{1+x}\sqrt{x^2-x+1}\left(5i\sqrt{3}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-}{i\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] 1/6*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(5*I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)
-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x*(-2*(1+x)/(I*3^(1/2)-3))
^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2)+15*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)
+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3
^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x-30*(-2*(1+x)/(I*3^
(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2)*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-
3)/(I*3^(1/2)+3))^(1/2))*x-10*x^3-6)/(x^3+1)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^8 + 2x^5 + x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^8 + 2*x^5 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (x + 1)^{\frac{3}{2}} \left(x^2 - x + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2 - x + 1\right)^{\frac{3}{2}} (x + 1)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

3.516 $\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{7\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{7\left(x^3+1\right)}{6x^2\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{3x^2\sqrt{x+1}\sqrt{x^2-x}}$$

```
[Out] 2/(3*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (7*(1 + x^3))/(6*x^2*Sqrt[1 + x]*
Sqrt[1 - x + x^2]) - (7*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -
7 - 4*Sqrt[3]])/(6*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rubi [A] time = 0.0652856, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 290, 325, 218}

$$-\frac{7\left(x^3+1\right)}{6x^2\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{3x^2\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{7\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```


```
[Out] 2/(3*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (7*(1 + x^3))/(6*x^2*Sqrt[1 + x]*
Sqrt[1 - x + x^2]) - (7*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -
7 - 4*Sqrt[3]])/(6*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```


Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{x^3\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\left(1+x^3\right)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{12\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\left(1+x^3\right)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\mathrm{si}\right.}{6\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{}}$$

Mathematica [C] time = 0.402872, size = 170, normalized size = 1.

$$\frac{-\frac{6\left(7x^3+3\right)}{x^2\sqrt{x+1}} - \frac{7i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{36\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] ((-6*(3 + 7*x^3))/(x^2*Sqrt[1 + x]) - ((7*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I
+ Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(36*Sqrt[1 - x + x^2])
```

Maple [A] time = 1.35, size = 259, normalized size = 1.5

$$\frac{1}{\left(12x^3+12\right)x^2}\sqrt{1+x}\sqrt{x^2-x+1}\left(7i\sqrt{3}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^2\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-}{i\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)
```

```
[Out] 1/12*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(7*I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2
)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^2*(-2*(1+x)/(I*3^(1/2)-
3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2)-21*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1
/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(
I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^2-14*x^3-6)/(x^
3+1)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^9+2\,x^6+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^9 + 2*x^6 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\,(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

3.517 $\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{4x}{27\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}\,(\ldots}$$

```
[Out] (4*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
+ x^2]*(1 + x^3)) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rubi [A] time = 0.0573901, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 288, 199, 218}

$$\frac{4x}{27\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}+\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (4*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
+ x^2]*(1 + x^3)) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x^3}{\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right.}}}$$

Mathematica [C] time = 0.537443, size = 178, normalized size = 1.06

$$\frac{\frac{6x\left(2x^3-1\right)}{(x+1)^{3/2}\left(x^2-x+1\right)} + \frac{2i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{81\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((6*x*(-1 + 2*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((2*I)*(1 + x)*Sqrt[1 +
 (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x)
)]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sq
rt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x^2])
```

Maple [B] time = 1.419, size = 467, normalized size = 2.8

$$-\frac{2}{27}\left(i\sqrt{3}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}} - 3\,\text{EllipticF}\left(\sqrt{-2\frac{}{i}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -2/27*(I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(
I*3^(1/2)+3))^(1/2))*x^3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/
(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)-3*EllipticF((-
2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^3*(-2*
(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2)+I*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2
)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*Ellip
ticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*3
^(1/2)-3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(
1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)
-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-2*x^4+x)/(x^2-x+1)^(3/2)/(
1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}x^3}{x^9+3x^6+3x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3/(x^9 + 3*x^6 + 3*x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(x**3/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.518 $\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

```
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Rubi [A] time = 0.0205386, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {913}

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Rule 913

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(c*e*
(m + 2*p + 3)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx=-\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0409086, size = 23, normalized size = 1.

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Maple [A] time = 0.042, size = 18, normalized size = 0.8

$$-\frac{2}{9}(1+x)^{-\frac{3}{2}}\left(x^2-x+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -2/9/(1+x)^(3/2)/(x^2-x+1)^(3/2)
```

Maxima [A] time = 1.49092, size = 32, normalized size = 1.39

$$-\frac{2}{9\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/9/((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Fricas [A] time = 1.71457, size = 73, normalized size = 3.17

$$-\frac{2\sqrt{x^2-x+1}\sqrt{x+1}}{9\left(x^6+2x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(x**2/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.519 $\int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal. Leaf size=318

$$-\frac{10\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{10x^2}{27\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2x^2}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+\right.}$$

```
[Out] (10*x^2)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x^2)/(9*Sqrt[1 + x]*Sqrt[1
 - x + x^2]*(1 + x^3)) - (10*(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*S
qrt[1 - x + x^2]) + (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(9*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^
2]) - (10*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.104718, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {809, 290, 303, 218, 1877}

$$\frac{10x^2}{27\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2x^2}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}-\frac{10\left(x^3+1\right)}{27\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}-\frac{10\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right.}}}{27\sqrt[4]{3}\sqrt{\frac{}{(}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (10*x^2)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x^2)/(9*Sqrt[1 + x]*Sqrt[1
 - x + x^2]*(1 + x^3)) - (10*(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*S
qrt[1 - x + x^2]) + (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(9*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^
2]) - (10*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 809

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)
^FracPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x}{\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(5\sqrt{\cdots\right.}}{\cdots}$$

$$= \frac{10x^2}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{10\left(1+x^3\right)}{27\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x\cdots}}$$

Mathematica [C] time = 0.88498, size = 409, normalized size = 1.29

$$\frac{2x^2\left(5x^3+8\right)}{27(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}-\frac{5(x+1)^{3/2}\left(\frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{x+1}}+\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\left(x^2\right.}{(x+1)^2}\right.}{162\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (2*x^2*(8 + 5*x^3))/(27*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) - (5*(1 + x)^(3/
2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1
 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-
3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(
-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt
[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(
3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*Ell
ipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(162*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x
 + x^2])
```

Maple [B] time = 1.191, size = 688, normalized size = 2.2

$$-\frac{1}{27}\left(5i\sqrt{3}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}}+15\,\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -1/27*(5*I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)
/(I*3^(1/2)+3))^(1/2))*x^3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1
)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)+15*EllipticF
((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^3*(
-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x
-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)-30*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1
/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2
)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2)+5*I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)
/(I*3^(1/2)+3))^(1/2))*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I
*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)-10*x^5+15*(-2*(1
+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I
*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-30*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(
1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*El
lipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2)
)-16*x^2)/(x^2-x+1)^(3/2)/(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2 - x + 1\right)^{\frac{5}{2}} (x + 1)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}x}{x^9 + 3\,x^6 + 3\,x^3 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x/(x^9 + 3*x^6 + 3*x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x + 1)^{\frac{5}{2}} \left(x^2 - x + 1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(x/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2 - x + 1\right)^{\frac{5}{2}} (x + 1)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.520 $\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$\frac{14\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{14x}{27\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] (14*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) + (14*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
-7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x +
 x^2])
```

Rubi [A] time = 0.0438769, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {713, 199, 218}

$$\frac{14x}{27\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}+\frac{14\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (14*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) + (14*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
-7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x +
 x^2])
```

Rule 713

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^FracPart[p])/(a*d +
c*e*x^3)^FracPart[p], Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{27\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{14x}{27\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{14\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+\right.}}}$$

Mathematica [C] time = 0.492418, size = 178, normalized size = 1.06

$$\frac{\frac{6x\left(7x^3+10\right)}{(x+1)^{3/2}\left(x^2-x+1\right)} + \frac{7i(x+1)\sqrt{1+\frac{6i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{6-\frac{36i}{\left(\sqrt{3}+3i\right)(x+1)}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{81\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((6*x*(10 + 7*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((7*I)*(1 + x)*Sqrt[1 +
 (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x)
)]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sq
rt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x^2])
```

Maple [B] time = 1.175, size = 469, normalized size = 2.8

$$-\frac{1}{27}\left(7i\sqrt{3}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^3\sqrt{-2\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{2x-1+i\sqrt{3}}{i\sqrt{3}-3}} - 21\,\text{EllipticF}\left(\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -1/27*(7*I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)
/(I*3^(1/2)+3))^(1/2))*x^3*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1
)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)-21*EllipticF
```

```
((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^3*(
-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x
-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)+7*I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/
2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*(-2*(1+x)/(I*3^(1/2)-3))
^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2)-21*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)
+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3
^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))-14*x^4-20*x)/(x^2-x+
1)^(3/2)/(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^9+3x^6+3x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^9 + 3*x^6 + 3*x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)^{\frac{5}{2}}\left(x^2-x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

3.521 $\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
 + x^3)) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 -
 x + x^2])
```

Rubi [A] time = 0.0368954, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {915, 266, 51, 63, 207}

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
 + x^3)) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 -
 x + x^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{\sqrt{1+x^3}\,\mathrm{Subst}\left(\int \frac{1}{x(1+x)^{5/2}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\sqrt{1+x^3}\,\mathrm{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\sqrt{1+x^3}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x,\right.}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx\right.}{3\sqrt{1+x}\sqrt{1-x+x}}$$

$$= \frac{2}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 6.08822, size = 2539, normalized size = 26.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] Sqrt[1 + x]*Sqrt[1 - x + x^2]*(2/(81*(1 + x)^2) + 22/(81*(1 + x)) - (2*(-1
+ x))/(27*(1 - x + x^2)^2) - (2*(-21 + 11*x))/(81*(1 - x + x^2))) + 2*(((-I
)*(1 + x)*Sqrt[1 - 6/((3 - I*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])
*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 -
I*Sqrt[3])/(3 + I*Sqrt[3])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 -
3*(1 + x) + (1 + x)^2]) + (Sqrt[3/2]*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2
+ (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sq
rt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]
))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I
/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2
 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2
+ (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqr
t[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1
 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3]
)] - 6/Sqrt[1 + x]))]*((1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqr
t[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6
```

```
/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*
Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^
2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))
/3]*EllipticPi[((-1 + Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]]
 + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2
 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])), ArcSin[Sqrt[((Sqrt[3 - I*S
qrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/(
(Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sq
rt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sq
rt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I
/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] -
Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1 + x) + (1 + x)^2]) - (Sqrt[3/2]*(S
qrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]])*(1 + x)*(-Sqrt[1/2 -
(I/2)/Sqrt[3]] + 1/Sqrt[1 + x])^2*Sqrt[(Sqrt[(2*(3 - I*Sqrt[3]))/3]*(-Sqrt[
1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1
/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))]*Sqrt[(S
qrt[(2*(3 - I*Sqrt[3]))/3]*(Sqrt[1/2 + (I/2)/Sqrt[3]] + 1/Sqrt[1 + x]))/((S
qrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sq
rt[3]] + 1/Sqrt[1 + x]))]*Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])
*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3
+ I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]*((-1 + Sqrt[1/2 -
 (I/2)/Sqrt[3]])*EllipticF[ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*S
qrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] +
 Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3
 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sq
rt[3]])^2] - Sqrt[(2*(3 - I*Sqrt[3]))/3]*EllipticPi[((1 + Sqrt[1/2 - (I/2)/
Sqrt[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqrt[3]]))/((1 - Sq
rt[1/2 - (I/2)/Sqrt[3]])*(-Sqrt[1/2 - (I/2)/Sqrt[3]] + Sqrt[1/2 + (I/2)/Sqr
t[3]])), ArcSin[Sqrt[((Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])*(Sqrt[6*(
3 - I*Sqrt[3])] + 6/Sqrt[1 + x]))/((Sqrt[3 - I*Sqrt[3]] + Sqrt[3 + I*Sqrt[3
]])*(Sqrt[6*(3 - I*Sqrt[3])] - 6/Sqrt[1 + x]))]], (Sqrt[3 - I*Sqrt[3]] + Sq
rt[3 + I*Sqrt[3]])^2/(Sqrt[3 - I*Sqrt[3]] - Sqrt[3 + I*Sqrt[3]])^2]))/(Sqrt
[3 - I*Sqrt[3]]*(-1 - Sqrt[1/2 - (I/2)/Sqrt[3]])*(1 - Sqrt[1/2 - (I/2)/Sqrt
[3]])*(Sqrt[1/2 - (I/2)/Sqrt[3]] - Sqrt[1/2 + (I/2)/Sqrt[3]])*Sqrt[3 - 3*(1
 + x) + (1 + x)^2]))
```

Maple [A] time = 1.008, size = 69, normalized size = 0.7

$$-\frac{2}{9\,x^3+9}\left(3\,\mathrm{Artanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}x^3-3\,x^3+3\,\mathrm{Artanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}-4\right)\frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] -2/9*(3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)*x^3-3*x^3+3*arctanh((x^3+1)^(1
/2))*(x^3+1)^(1/2)-4)/(x^3+1)/(x^2-x+1)^(1/2)/(1+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

Fricas [A] time = 1.73839, size = 258, normalized size = 2.69

$$\frac{2\left(3\,x^{3}+4\right)\sqrt{x^{2}-x+1}\sqrt{x+1}-3\left(x^{6}+2\,x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\sqrt{x+1}+1\right)+3\left(x^{6}+2\,x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\sqrt{x+\right.}{9\left(x^{6}+2\,x^{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(3*x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 3*(x^6 + 2*x^3 + 1)*log(
sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 3*(x^6 + 2*x^3 + 1)*log(sqrt(x^2 - x +
 1)*sqrt(x + 1) - 1))/(x^6 + 2*x^3 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{2}-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

3.522 $\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=349

$$\frac{55\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{55\left(x^3+1\right)}{27x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{55\left(x^3+1\right)}{27\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{\cdots}}$$

```
[Out] 22/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x*Sqrt[1 + x]*Sqrt[1 - x + x
^2]*(1 + x^3)) - (55*(1 + x^3))/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (55*
(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (55*Sqrt[
2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (55*Sqrt[2]*Sqrt[1 +
x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.139555, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {915, 290, 325, 303, 218, 1877}

$$-\frac{55\left(x^3+1\right)}{27x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{55\left(x^3+1\right)}{27\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{22}{27x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9x\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```


```
[Out] 22/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x*Sqrt[1 + x]*Sqrt[1 - x + x
^2]*(1 + x^3)) - (55*(1 + x^3))/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (55*
(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (55*Sqrt[
2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (55*Sqrt[2]*Sqrt[1 +
x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 - x + x^2])
```


Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^2\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(11\sqrt{1+x^3}\right)\int \frac{1}{x^2\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(55\sqrt{1+x^3}\right)\int \frac{1}{x^2\sqrt{1+x^3}}}{27\sqrt{1+x}\sqrt{1-x+x}}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{55\left(1+x^3\right)}{27x\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{55\left(1+x^3\right)}{27x\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{55\left(1+x^3\right)}{27x\sqrt{1+x}\sqrt{1-x+x^2}}$$

Mathematica [C] time = 0.965548, size = 414, normalized size = 1.19

$$-\frac{55x^6+88x^3+27}{27x(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}} + \frac{55(x+1)^{3/2}\left(\frac{i\sqrt{2}\left(\sqrt{3}+3i\right)\sqrt{\frac{-\frac{6i}{x+1}+\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt{\frac{\frac{6i}{x+1}+\sqrt{3}-3i}{\sqrt{3}-3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{x+1}} + \frac{12\sqrt{-\frac{i}{\sqrt{3+}}}}{(x\cdot}\right.}{324\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] -(27 + 88*x^3 + 55*x^6)/(27*x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) + (55*(1 +
 x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqr
t[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*
Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh
[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1
+ x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3
])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + S
qrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(324*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqr
t[1 - x + x^2])
```

Maple [B] time = 1.474, size = 695, normalized size = 2.

$$\frac{1}{54x}\left(55\,i\text{EllipticF}\left(\sqrt{-2\,\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}x^4\sqrt{-2\,\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2\,x+1}{i\sqrt{3}+3}}\sqrt{\frac{2\,x-1+i\sqrt{3}}{i\sqrt{3}-3}}+165\,\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] 1/54*(55*I*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1
/2)+3))^(1/2))*3^(1/2)*x^4*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1
)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)+165*Elliptic
F((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^4*
(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*
x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)-330*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^
(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^4*(-2*(1+x)/(I*3^(1/2)-3))^(1
/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2)+55*I*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3
^(1/2)+3))^(1/2))*3^(1/2)*x*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+
1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)-110*x^6+165
*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2
*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/
2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x-330*(-2*(1+x)/(I*3^(1/2)-3))^(1/
2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2)*EllipticE((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)
+3))^(1/2))*x-176*x^3-54)/x/(x^2-x+1)^(3/2)/(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^{11}+3x^8+3x^5+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^11 + 3*x^8 + 3*x^5 + x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2 - x + 1\right)^{\frac{5}{2}} (x + 1)^{\frac{5}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

3.523 $\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{91\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{54\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{91\left(x^3+1\right)}{54x^2\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{26}{27x^2\sqrt{x+1}\sqrt{x^2-x}}$$

```
[Out] 26/(27*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x^2*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) - (91*(1 + x^3))/(54*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])
- (91*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]
*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5
4*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rubi [A] time = 0.0724538, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {915, 290, 325, 218}

$$-\frac{91\left(x^3+1\right)}{54x^2\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{26}{27x^2\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{2}{9x^2\sqrt{x+1}\sqrt{x^2-x+1}\left(x^3+1\right)}-\frac{91\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}{54\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 26/(27*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x^2*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) - (91*(1 + x^3))/(54*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])
- (91*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]
*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5
4*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 915

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[p]*(a + b*x + c*x^2)^Fr
acPart[p])/(a*d + c*e*x^3)^FracPart[p], Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(13\sqrt{1+x^3}\right)\int \frac{1}{x^3\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(91\sqrt{1+x^3}\right)\int \frac{1}{x^3\sqrt{1+}}}{27\sqrt{1+x}\sqrt{1-x+}}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{91\left(1+x^3\right)}{54x^2\sqrt{1+x}\sqrt{1-x+}}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{91\left(1+x^3\right)}{54x^2\sqrt{1+x}\sqrt{1-x+}}$$

Mathematica [C] time = 0.649276, size = 183, normalized size = 0.9

$$\frac{-\frac{6\left(91x^6+130x^3+27\right)}{x^2(x+1)^{3/2}} - \frac{91i(x+1)\left(x^2-x+1\right)\sqrt{6+\frac{36i}{\left(\sqrt{3}-3i\right)(x+1)}}\sqrt{1-\frac{6i}{\left(\sqrt{3}+3i\right)(x+1)}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{\sqrt{3}+3i}}}{\sqrt{x+1}}\right),\frac{\sqrt{3}+3i}{-\sqrt{3}+3i}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{324\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((-6*(27 + 130*x^3 + 91*x^6))/(x^2*(1 + x)^(3/2)) - ((91*I)*(1 + x)*(1 - x
+ x^2)*Sqrt[6 + (36*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[1 - (6*I)/((3*I + S
qrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 +
 x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(324*(1
 - x + x^2)^(3/2))
```

Maple [B] time = 1.411, size = 481, normalized size = 2.4

$$\frac{1}{108\,x^2}\left(91\,i\sqrt{3}\text{EllipticF}\left(\sqrt{-2\,\frac{1+x}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)x^5\sqrt{-2\,\frac{1+x}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2\,x+1}{i\sqrt{3}+3}}\sqrt{\frac{2\,x-1+i\sqrt{3}}{i\sqrt{3}-3}}-273\,\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)
```

```
[Out] 1/108*(91*I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3
)/(I*3^(1/2)+3))^(1/2))*x^5*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+
1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)-273*Ellipti
cF((-2*(1+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^5
*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2
*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)+91*I*3^(1/2)*EllipticF((-2*(1+x)/(I*3^
(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^2*(-2*(1+x)/(I*3^(1
/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2)-273*(-2*(1+x)/(I*3^(1/2)-3))^(1/2)*((I*3^(1/2)-2*x+1)/(I
*3^(1/2)+3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)*EllipticF((-2*(1
+x)/(I*3^(1/2)-3))^(1/2),(-(I*3^(1/2)-3)/(I*3^(1/2)+3))^(1/2))*x^2-182*x^6-
260*x^3-54)/x^2/(x^2-x+1)^(3/2)/(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^{12}+3\,x^9+3\,x^6+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^12 + 3*x^9 + 3*x^6 + x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2 - x + 1\right)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

3.524 $\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2} dx$

Optimal. Leaf size=97

$$\frac{44x+39}{276(1-x)^2(4x^2+5x+3)} - \frac{11\log(4x^2+5x+3)}{4608} - \frac{97}{4416(1-x)} - \frac{21}{736(1-x)^2} + \frac{11\log(1-x)}{2304} + \frac{6023\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

```
[Out] -21/(736*(1 - x)^2) - 97/(4416*(1 - x)) + (39 + 44*x)/(276*(1 - x)^2*(3 + 5
*x + 4*x^2)) + (6023*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23]) + (11*Log
[1 - x])/2304 - (11*Log[3 + 5*x + 4*x^2])/4608
```

Rubi [A] time = 0.0713876, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {822, 800, 634, 618, 204, 628}

$$\frac{44x+39}{276(1-x)^2(4x^2+5x+3)} - \frac{11\log(4x^2+5x+3)}{4608} - \frac{97}{4416(1-x)} - \frac{21}{736(1-x)^2} + \frac{11\log(1-x)}{2304} + \frac{6023\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -21/(736*(1 - x)^2) - 97/(4416*(1 - x)) + (39 + 44*x)/(276*(1 - x)^2*(3 + 5
*x + 4*x^2)) + (6023*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23]) + (11*Log
[1 - x])/2304 - (11*Log[3 + 5*x + 4*x^2])/4608
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx = \frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{1}{276}\int \frac{57+132x}{(-1+x)^3\left(3+5x+4x^2\right)}\,dx$$

$$= \frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{1}{276}\int \left(\frac{63}{4(-1+x)^3}-\frac{97}{16(-1+x)^2}+\frac{253}{192(-1+x)}+\right.$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}+$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}-$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{11\log(1-x)}{2304}-$$

$$= -\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2\left(3+5x+4x^2\right)}+\frac{6023\tan^{-1}\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}}+$$

Mathematica [A] time = 0.0471454, size = 78, normalized size = 0.8

$$\frac{\frac{184(2204x+975)}{4x^2+5x+3}-17457\log\left(4x^2+5x+3\right)+\frac{59248}{x-1}-\frac{25392}{(x-1)^2}+34914\log(1-x)+36138\sqrt{23}\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{7312896}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] (-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4*x^2
) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 17457*Lo
g[3 + 5*x + 4*x^2])/7312896
```

Maple [A] time = 0.052, size = 68, normalized size = 0.7

$$-\frac{1}{288\,(-1+x)^2}+\frac{7}{-864+864\,x}+\frac{11\,\ln(-1+x)}{2304}-\frac{1}{6912}\left(-\frac{2204\,x}{23}-\frac{975}{23}\right)\left(x^2+\frac{5\,x}{4}+\frac{3}{4}\right)^{-1}-\frac{11\,\ln\left(4\,x^2+5\,x+3\right)}{4608}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-1+x)^3/(4*x^2+5*x+3)^2,x)
```

```
[Out] -1/288/(-1+x)^2+7/864/(-1+x)+11/2304*ln(-1+x)-1/6912*(-2204/23*x-975/23)/(x
^2+5/4*x+3/4)-11/4608*ln(4*x^2+5*x+3)+6023/1218816*arctan(1/23*(5+8*x)*23^(
1/2))*23^(1/2)
```

Maxima [A] time = 1.46368, size = 101, normalized size = 1.04

$$\frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right)+\frac{388x^3-407x^2-120x-45}{4416\left(4x^4-3x^3-3x^2-x+3\right)}-\frac{11}{4608}\log\left(4x^2+5x+3\right)+\frac{11}{2304}\log\left(x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 11/4608*log(4*x^2 +
5*x + 3) + 11/2304*log(x - 1)
```

Fricas [A] time = 1.75029, size = 377, normalized size = 3.89

$$\frac{214176x^3+12046\sqrt{23}\left(4x^4-3x^3-3x^2-x+3\right)\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right)-224664x^2-5819\left(4x^4-3x^3-3x^2-x+\right.}{2437632\left(4x^4-3x^3-3x^2-x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arct
an(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 - x
+ 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x - 1
) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

Sympy [A] time = 0.211788, size = 88, normalized size = 0.91

$$\frac{388x^3-407x^2-120x-45}{17664x^4-13248x^3-13248x^2-4416x+13248}+\frac{11\log(x-1)}{2304}-\frac{11\log\left(x^2+\frac{5x}{4}+\frac{3}{4}\right)}{4608}+\frac{6023\sqrt{23}\operatorname{atan}\left(\frac{8\sqrt{23}x}{23}+\frac{5\sqrt{23}}{23}\right)}{1218816}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] (388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2 -
4416*x + 13248) + 11*log(x - 1)/2304 - 11*log(x**2 + 5*x/4 + 3/4)/4608 + 60
23*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

Giac [A] time = 1.15479, size = 96, normalized size = 0.99

$$\frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right)+\frac{388x^3-407x^2-120x-45}{4416(4x^2+5x+3)(x-1)^2}-\frac{11}{4608}\log\left(4x^2+5x+3\right)+\frac{11}{2304}\log\left(|x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 11/4608*log(4*x^2 + 5*
x + 3) + 11/2304*log(abs(x - 1))
```

3.525 $\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt{2}\left(-\frac{-5a^2bc^2e+2a^2c^3d-4ab^2c^2d+5ab^3ce+b^4cd+b^5(-e)}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\sqrt{2}$$

```
[Out] (-2*b*(b^2 - 2*a*c)*Sqrt[d + e*x])/c^4 + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d -
 a*e))*(d + e*x)^(3/2))/(3*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^(5/2))/(5*
c^2*e^3) + (2*(d + e*x)^(7/2))/(7*c*e^3) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d
- b^4*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d
- b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d -
 b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d -
 b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 14.8468, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{-5a^2bc^2e+2a^2c^3d-4ab^2c^2d+5ab^3ce+b^4cd+b^5(-e)}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```


```
[Out] (-2*b*(b^2 - 2*a*c)*Sqrt[d + e*x])/c^4 + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d -
 a*e))*(d + e*x)^(3/2))/(3*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^(5/2))/(5*
c^2*e^3) + (2*(d + e*x)^(7/2))/(7*c*e^3) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d
- b^4*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d
- b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d -
 b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d -
 b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{\left(b^3-2abc\right)e}{c^4}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)x^2}{c^3e^2}-\frac{(2cd+be)x^4}{c^2e^2}+\frac{x^6}{ce^2}+\frac{b\left(b^2-2ac\right)\left(cd^2-bde+ae^2\right)-\left(b^3cd-2abc^2d\right.}{c^4e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x}{e^2}\right.}\right.\right.}{e}$$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3}+2($$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3}+2($$

$$= -\frac{2b\left(b^2-2ac\right)\sqrt{d+ex}}{c^4}+\frac{2\left(c^2d^2+b^2e^2+ce(bd-ae)\right)(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3}+2($$

Mathematica [A] time = 0.76614, size = 568, normalized size = 1.16

$$-\frac{\sqrt{2}\left(a^2c^2\left(e\sqrt{b^2-4ac}+2cd\right)+abc^2\left(2d\sqrt{b^2-4ac}-5ae\right)+b^4\left(e\sqrt{b^2-4ac}+cd\right)+b^3c\left(5ae-d\sqrt{b^2-4ac}\right)-ab^2c\left(3\right.\right.}{c^{9/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-105*b^3*e^3 - 7*c^2*e*(d + e*x)*(-2*b*d + 5*a*e + 3*b*e*
x) + c^3*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) + 35*b*c*e^2*(6*a*e
 + b*(d + e*x))))/(105*c^4*e^3) - (Sqrt[2]*(-(b^5*e) + a*b*c^2*(2*Sqrt[b^2
- 4*a*c]*d - 5*a*e) + b^3*c*(-(Sqrt[b^2 - 4*a*c]*d) + 5*a*e) + b^4*(c*d + S
qrt[b^2 - 4*a*c]*e) + a^2*c^2*(2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*c*(4*c*
d + 3*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^5*e - b^3*c*(Sqrt[b^2 - 4*a*c]*d
 + 5*a*e) + a*b*c^2*(2*Sqrt[b^2 - 4*a*c]*d + 5*a*e) + a*b^2*c*(4*c*d - 3*Sq
rt[b^2 - 4*a*c]*e) + a^2*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e) + b^4*(-(c*d) +
 Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e])
```

Maple [B] time = 0.301, size = 2218, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 4*e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))*a*b^2*d+4*e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b
*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d+2/3/e^2/c^2*(e*x+d)^(3
/2)*b*d-2/c^4*b^3*(e*x+d)^(1/2)+4/c^3*a*b*(e*x+d)^(1/2)-2/5/e^2/c^2*(e*x+d)
^(5/2)*b-4/5/e^3/c*(e*x+d)^(5/2)*d+2/3/e^3/c*(e*x+d)^(3/2)*d^2-2/3/e/c^2*(e
*x+d)^(3/2)*a+2/3/e/c^3*(e*x+d)^(3/2)*b^2+e^2/c^4/(-e^2*(4*a*c-b^2))^(1/2)*
2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)
*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5-3*e/c^3*2^(1
/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2
^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2+e^2/c^4/(-e^2*
(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*b^5+3*e/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*a*b^2-2/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*a*b*d+2/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
a*b*d-5*e^2/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2))*a*b^3-e/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4*d-2*e/c/(-e^2*(4*a*
c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^
2*d-2*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2))*a^2*d+5*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*
e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b-5*e^2/c^3/(-e^2*(4*a*c
-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(
```

```
(e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b
^3-e/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))*b^4*d+5*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-
b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b+2/7*(e*x+d)^(7/2)/c/e^3
-e/c^4*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x
+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4+e/
c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(
1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2+e/c^4*2^
(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c
*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4-1/c^3*2^(1/2)/
((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d+1/c^3*2^(1/2)/((-b
*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d-e/c^2*2^(1/2)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}x^4}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^4/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 6.28687, size = 11800, normalized size = 24.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16
*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^
3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c
^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 +
 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^
3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^
8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4
*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^
2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^12*c - 12*
a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^
2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4
*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e - (b^6*c^9 - 8*a*b^4*c^10 + 18*a
^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4
 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2
 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*
b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
```

^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b^12*c - 12*a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e)*sqrt(e*x + d)) + 105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^12*c - 12*a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e + (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9

```
*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*
a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5
 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*
a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*
b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*
b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b
^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 -
4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*
d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e)*
sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^3*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^
4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 -
 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 1
2*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5
*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^
3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 -
4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a
^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b
^12*c - 12*a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5
- 32*a^5*b^2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^
3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e + (b^6*c^9 - 8*a*b^4*
c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a
^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b
^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4
 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e +
(b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c
^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*
c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9
*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a
^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b
^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4
 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e +
(b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c
^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 1
0*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 -
10*a^7*b^2*c^3 + a^8*c^4)*e)*sqrt(e*x + d)) + 4*(15*c^3*e^3*x^3 + 8*c^3*d^3
 + 14*b*c^2*d^2*e + 35*(b^2*c - a*c^2)*d*e^2 - 105*(b^3 - 2*a*b*c)*e^3 + 3*
(c^3*d*e^2 - 7*b*c^2*e^3)*x^2 - (4*c^3*d^2*e + 7*b*c^2*d*e^2 - 35*(b^2*c -
a*c^2)*e^3)*x)*sqrt(e*x + d))/(c^4*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.526 $\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=326

$$\frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}+\frac{\left(-\sqrt{b^2-4ac}\left(b^2-ac\right)-3abc+b^3\right)\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}}-\cdots$$

```
[Out] (2*(b^2 - a*c)*Sqrt[d + e*x])/c^3 - (2*(c*d + b*e)*(d + e*x)^(3/2))/(3*c^2*
e^2) + (2*(d + e*x)^(5/2))/(5*c*e^2) + ((b^3 - 3*a*b*c - Sqrt[b^2 - 4*a*c]*
(b^2 - a*c))*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2
)*Sqrt[b^2 - 4*a*c]) - ((b^3 - 3*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 - a*c))*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a
*c])
```

Rubi [A] time = 7.46584, antiderivative size = 397, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {897, 1287, 1166, 208}

$$-\frac{\sqrt{2}\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2 - a*c)*Sqrt[d + e*x])/c^3 - (2*(c*d + b*e)*(d + e*x)^(3/2))/(3*c^2*
e^2) + (2*(d + e*x)^(5/2))/(5*c*e^2) - (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e
+ 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/S
qrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d
- b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
```

```
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}-\frac{2\,\mathrm{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+ae^2)+(-b^2cd+ac\cdots}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}\cdots}\right.}{c^3e^2}$$

$$= \frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}+\frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{2\left(b^2-ac\right)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}-\frac{\sqrt{2}\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-\cdots}{\cdots}\right.}{c^{7/2}\sqrt{2cd\cdots}}$$

Mathematica [A] time = 0.553366, size = 466, normalized size = 1.43

$$\frac{2\sqrt{d+ex}\left(-5ce(3ae+b(d+ex))+15b^2e^2+c^2\left(-2d^2+dex+3e^2x^2\right)\right)}{15c^3e^2}+\frac{\sqrt{2}\left(ac^2\left(d\sqrt{b^2-4ac}-2ae\right)+b^3\left(e\sqrt{b^2-4ac}+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x + 3*e^2*x^2) - 5*c*e*(3*
a*e + b*(d + e*x))))/(15*c^3*e^2) + (Sqrt[2]*(-(b^4*e) + a*c^2*(Sqrt[b^2 -
4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(c*d + Sqr
```

```
t[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(7/2)*
Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^4
*e + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a
*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*
c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e])
```

Maple [B] time = 0.283, size = 1764, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/5*(e*x+d)^(5/2)/c/e^2-2/3/e/c^2*(e*x+d)^(3/2)*b-2/3/e^2/c*(e*x+d)^(3/2)*d
-2/c^2*a*(e*x+d)^(1/2)+2/c^3*b^2*(e*x+d)^(1/2)-2*e^2/c/(-e^2*(4*a*c-b^2))^(
1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^
(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2+4*e^2/c
^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2))*a*b^2-3*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e
^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b*d-e^2/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(
1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*
2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4+e/c^2/(-e^2*(4*
a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
b^3*d+2*e/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*
b-1/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d-e/c^3*
2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)
*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3+1/c^2*2^(1/2
)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(
1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d-2*e^2/c/(-e^2*(4
*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*a^2+4*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2-3*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b*d-e^2/c^3/(-e^2*(
4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*b^4+e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*b^3*d-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2))*a*b+1/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))*a*d+e/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*b^3-1/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
```

```
(1/2))*c)^(1/2))*b^2*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}x^3}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^3/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 4.15671, size = 8779, normalized size = 26.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3
*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4
*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9
*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c
^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*
c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 +
 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e - (b^5*c^7 -
 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b
^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*
b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e
 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*
a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3
 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c
 + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*
a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6
*c^3)*e)*sqrt(e*x + d)) - 15*sqrt(2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9
*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
- 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^
7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((
b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (
b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^
5*c^5)*e - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8
*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9
*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6
)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
```

```
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c -
 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 +
 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9
*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e
+ (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))
 + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c
+ 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(e*x + d)) + 15*sqrt(2)*c^3*e^2*sqrt(((b^
6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^
2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*
c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*
a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)
*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c
^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*
c^8))*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 +
 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 2
9*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(
((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)
*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b
^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^
15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*
a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10
*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 -
 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5
 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/
(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^
3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(e*x + d)) - 15*sqrt(
2)*c^3*e^2*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7
 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt((
(b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*
d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^
3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^1
5)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c
^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8
+ 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4
*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a
^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^
2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a
^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 -
 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 +
 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5
*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^
2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 +
 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt
(e*x + d)) + 4*(3*c^2*e^2*x^2 - 2*c^2*d^2 - 5*b*c*d*e + 15*(b^2 - a*c)*e^2
+ (c^2*d*e - 5*b*c*e^2)*x)*sqrt(e*x + d))/(c^3*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.527 $\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=316

$$\frac{\sqrt{2}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-\right.}{c^{5/2}\sqrt{2cd-e\left(\right.}}$$

```
[Out] (-2*b*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c*e) + (Sqrt[2]*(b*c*d -
b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b*c*
d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.15817, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-\right.}{c^{5/2}\sqrt{2cd-e\left(\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c*e) + (Sqrt[2]*(b*c*d -
b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b*c*
d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
```

```
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b\left(cd^2-bde+ae^2\right)-\left(bcd-b^2e+ace\right)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx,x,\sqrt{d+ex}\right)}{e}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{2\,\text{Subst}\left(\int \frac{b\left(cd^2-bde+ae^2\right)+\left(-bcd+b^2e-ace\right)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx,x,\sqrt{d+ex}\right)}{c^2e^2}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx^2}{e^2}}\,dx\right.}{c^2e^2}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{\sqrt{2}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.467621, size = 375, normalized size = 1.19

$$\frac{\sqrt{2}\left(-b^2\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(d\sqrt{b^2-4ac}-3ae\right)+ac\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\left(b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-3*b*e + c*(d + e*x)))/(3*c^2*e) + (Sqrt[2]*(b^3*e + b*c*
(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*
d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-
```

```
b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*c]*d + 3
*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a*c]
*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e])
```

Maple [B] time = 0.295, size = 1329, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/3*(e*x+d)^(3/2)/c/e-2*b*(e*x+d)^(1/2)/c^2-3*e^2/c/(-e^2*(4*a*c-b^2))^(1/2
)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b+2*e/(-e^2*
(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))*a*d+e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2))*b^3-e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d-e/c*2^(1/2)/((b*e-2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a+e/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*b^2-1/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))*b*d-3*e^2/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e
+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b+2*e/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d+e^2/c^2/
(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2))*b^3-e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d+e/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2))*a-e/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2))*b^2+1/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x^2/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 2.68347, size = 5998, normalized size = 18.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a
*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3
+ 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4
)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^6*c - 6*a*b^4*c
^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e
- (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^
2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a
^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*sqrt(2)*c^2*e*sqrt(((
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b
^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^
7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*
c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e - (b^4*c^5 - 6*a*b^2*c^6 +
8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5
*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c
 - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^
5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c -
 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 -
 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*e)*sqrt(e*x + d)) + 3*sqrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2
*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^
6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)
*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c
^2 - 4*a^3*b*c^3)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 -
4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*
d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2
- 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c -
 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) - 3*s
qrt(2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
```

```
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*
a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e + (b^4*
c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4
)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*
c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^
2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*
a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4
- 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(e*x + d)) + 4*(c*e*x + c*d - 3*b*e)*sqrt(e
*x + d))/(c^2*e)
```

Sympy [B] time = 120.946, size = 1630, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] 2*a*b*e**2*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a
**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c
*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4)
 + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2
) + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*
_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 6
4*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3
+ 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e -
 4*_t*c*d**2 + sqrt(d + e*x))))/c**2 - 2*a*e*RootSum(_t**4*(256*a**2*c**3*e
**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a
*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, La
mbda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*
_t*c*d + sqrt(d + e*x))))/c - 2*b**2*d*e*RootSum(_t**4*(256*a**3*c**2*e**6
- 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 +
 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5
*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 +
 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5
 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4
+ 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**
4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2
*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(d + e*x))))/c**2 + 2*b**2
*e*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**
4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e*
*2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16*
_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x))))/c**2 + 2*b*d**2*
RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*
d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 1
28*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-
16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lamb
da(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**
3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c
**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*
b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**
2 + sqrt(d + e*x))))/c - 2*b*d*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b*
*2*c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 +
```

```
 4*b**3*e**3 - 8*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*lo
g(64*_t**3*a*c**2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(
d + e*x))))/c - 2*b*sqrt(d + e*x)/c**2 + 2*(d + e*x)**(3/2)/(3*c*e)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.528 $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=287

$$\frac{\sqrt{2}\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-}}$$

```
[Out] (2*Sqrt[d + e*x])/c + (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]
*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d
- b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rubi [A] time = 3.19951, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {824, 826, 1166, 208}

$$\frac{\sqrt{2}\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x])/c + (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]
*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d
- b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx &= \frac{2\sqrt{d+ex}}{c}+\frac{\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{c}\\ &= \frac{2\sqrt{d+ex}}{c}+\frac{2\,\mathrm{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)}{c}\\ &= \frac{2\sqrt{d+ex}}{c}-\frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx,x,\sqrt{d+\right.}{c\sqrt{b^2-4ac}}\\ &= \frac{2\sqrt{d+ex}}{c}+\frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\sqrt{2}\left(bcd-b^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.427114, size = 301, normalized size = 1.05

$$\frac{\frac{\sqrt{2}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+2}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x] + (Sqrt[2]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e
+ 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d +
b^2*e - 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c^(3/2)
```

Maple [B] time = 0.278, size = 926, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2*(e*x+d)^(1/2)/c+2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2))*a*e^2-1/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b
*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/
((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^2+1/(-e^2*(4*a*c-b^2)
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+
d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d*e-1/
c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*e+2^(1/2)/((
b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)
/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d+2/(-e^2*(4*a*c-b^2))^(1/
2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^
(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*e^2-1/c/
(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2))*b^2*e^2+1/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d*e+1/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*e-2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}x}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*x/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 2.13617, size = 3465, normalized size = 12.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4
*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c
- 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sq
rt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2
)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^
2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b
^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*
```

```
d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d
 - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a
*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d
*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c -
2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*
(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7
)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) +
 sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c
^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c - 4*
a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((
b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^
3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c
 + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d -
(a*b^2 - a^2*c)*e)*sqrt(e*x + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (
b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c
^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*
c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3
*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/
(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(e*x + d)) + 4*s
qrt(e*x + d))/c
```

Sympy [B] time = 74.7427, size = 1435, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] -2*a*e**2*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a*
*2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*
d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4)
+ _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2)
 + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_
t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64
*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 +
 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e -
4*_t*c*d**2 + sqrt(d + e*x))))/c + 2*b*d*e*RootSum(_t**4*(256*a**3*c**2*e**
6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4
 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b*
*5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2
 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e*
*5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**
4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e
**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 -
 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(d + e*x))))/c - 2*b*e*R
ootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4) +
 _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2)
+ a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16*_t**
3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x))))/c - 2*d**2*RootSum(_
```

```
t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 +
256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2
*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*
e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t
*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c
- 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*
e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d*
*3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(
d + e*x)))) + 2*d*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4
+ 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3
- 8*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*
c**2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x)))) +
 2*sqrt(d + e*x)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.529 $\int \frac{\sqrt{d+ex}}{a+bx+cx^2} dx$

Optimal. Leaf size=198

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2
 - 4*a*c])) + (Sqrt[2]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.271493, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {699, 1130, 208}

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2
 - 4*a*c])) + (Sqrt[2]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = (2e)\,\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$
$$= -\left(\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex}\right)\right)+\left(e-\frac{2c}{\sqrt{b}}\right.$$
$$= -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\tanh}{\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.407417, size = 175, normalized size = 0.88

$$\frac{\sqrt{2}\left(\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)-\sqrt{e\sqrt{b^2-4ac}-be+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Sqrt[2]*(-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.273, size = 545, normalized size = 2.8

$$e^2\sqrt{2}b\arctan\left(c\sqrt{2}\sqrt{ex+d}\frac{1}{\sqrt{\left(be-2\,cd+\sqrt{-e^2\left(4\,ac-b^2\right)}\right)c}}\right)\frac{1}{\sqrt{-e^2\left(4\,ac-b^2\right)}}\frac{1}{\sqrt{\left(be-2\,cd+\sqrt{-e^2\left(4\,ac-b^2\right)}\right)c}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a),x)
```

```
[Out] e^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*b-2*c*e/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*d+e*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))+e^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b-2*c*e/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-e*2^(1/2)/((-b*e+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b
*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 2.00312, size = 1466, normalized size = 7.4

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\sqrt{2}\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}\sqrt{\frac{2\,cd-be+\left(b^2c-4\,ac^2\right)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^
3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*
e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log
(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e
 + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*
sqrt(e*x + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e
^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*s
qrt(e^2/(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2
/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + 2*sqrt(e*x + d)*e) - 1/2*sqrt(2
)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2
*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3))
*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*
c - 4*a*c^2)) + 2*sqrt(e*x + d)*e)
```

Sympy [A] time = 10.8374, size = 155, normalized size = 0.78

$$2e\,\mathrm{RootSum}\left(t^4\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^3+32ac^2de^2+4b^3e^3-8b^2cde^2\right)+ae^2-bde+cd^2,\left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] 2*e*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e*
*4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e
**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16
*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x))))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.530 $\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx$

Optimal. Leaf size=275

$$\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a + (Sqrt[2]*Sqrt[c]*(b*d + Sqr
t[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.12901, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {897, 1287, 206, 1166, 208}

$$\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a + (Sqrt[2]*Sqrt[c]*(b*d + Sqr
t[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{de}{a\left(d-x^2\right)}+\frac{e\left(cd^2-bde+ae^2-cdx^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a}$$

$$= -\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x\right.}{a\sqrt{b^2-4ac}}$$

$$= -\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{2}\sqrt{}}{}$$

Mathematica [A] time = 0.923132, size = 267, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b
^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^
```

```
2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a
```

Maple [B] time = 0.283, size = 581, normalized size = 2.1

$$-2\,\frac{\sqrt{d}}{a}\mathrm{Artanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)-2\,\frac{ce^{2}\sqrt{2}}{\sqrt{-e^{2}\left(4\,ac-b^{2}\right)}\sqrt{\left(be-2\,cd+\sqrt{-e^{2}\left(4\,ac-b^{2}\right)}\right)c}}\arctan\left(\frac{\sqrt{ex+d}c\sqrt{2}}{\sqrt{\left(be-2\,cd+\sqrt{-e^{2}\left(4\,ac-b\right.\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x)
```

```
[Out] -2*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a-2*e^2*c/(-e^2*(4*a*c-b^2))^(1/2
)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+e/a*c/(-e^2*(4
*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arc
tan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
*b*d-1/a*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e
*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-2*e
^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))+e/a*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d+1/a*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.13064, size = 5018, normalized size = 18.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*l
og(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
```

```
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d
)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d
)) + sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)
) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^
2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log
(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)
) + 2*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/a, 1/2*(sqrt(2)
*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a
*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*a
*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + 4*sqrt(-d
)*arctan(sqrt(e*x + d)*sqrt(-d)/d))/a]
```

Sympy [B] time = 60.5149, size = 1295, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x/(c*x**2+b*x+a),x)
```

```
[Out] 2*e**2*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*
b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e
```

```
**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _
t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) +
c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**
3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t
**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16
*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_
t*c*d**2 + sqrt(d + e*x)))) - 2*b*d*e*RootSum(_t**4*(256*a**3*c**2*e**6 - 1
28*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16
*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*
e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*
b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 -
64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 9
6*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c
 - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t
*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(d + e*x))))/a + 2*c*d**2*Roo
tSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e
**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*
a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*
a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(
_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e
**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2
*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**
2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 +
 sqrt(d + e*x))))/a - 2*c*d*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*
c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*
b**3*e**3 - 8*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(6
4*_t**3*a*c**2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d +
 e*x))))/a + 2*d*atan(sqrt(d + e*x)/sqrt(-d))/(a*sqrt(-d))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.531 $\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=368

$$-\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\left(-b\left(d\sqrt{b^2-4ac}+ae\right)-a\left(2cd\right.\right.}{a^2\sqrt{b^2-4ac}}$$

```
[Out] -(Sqrt[d + e*x]/(a*x)) + (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a*Sqrt[d]) + (
2*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt[2]*Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c
]*(b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e))
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.66411, antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2\right.}{a^2\sqrt{b^2-4ac}\sqrt{2cd-}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -(Sqrt[d + e*x]/(a*x)) + (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a*Sqrt[d]) + (
2*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt[2]*Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c
]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqr
t[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
```

```
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{de^2}{a\left(d-x^2\right)^2}-\frac{e(-bd+ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2} + \frac{(2de)\,\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex}\right)}{a} -$$

$$= -\frac{\sqrt{d+ex}}{ax} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{\left(c\left(b^2d-2acd-\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex}}{ax} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2}\right.}{a^2\sqrt{b^2-4ac}}$$

Mathematica [A] time = 1.59864, size = 364, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}}{\sqrt{2cd-e\left(\sqrt{b^2}\right.}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*Sqrt[d + e*x])/x) + (a*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (2
*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (Sqrt[2]*Sqrt[c]*(-(
b^2*d) + 2*a*c*d - b*Sqrt[b^2 - 4*a*c]*d + a*b*e + a*Sqrt[b^2 - 4*a*c]*e)*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]
*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[
2]*Sqrt[c]*(-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2
 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]))/a^2
```

Maple [B] time = 0.29, size = 999, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x)
```

```
[Out] -(e*x+d)^(1/2)/a/x-e*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+2/a^2*d^(1/2)
*arctanh((e*x+d)^(1/2)/d^(1/2))*b+e^2/a*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/
((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b+2*e/a*c^2/(-e^2*(4*a*c
-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(
(e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-e
/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2))*b^2*d-e/a*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2))+1/a^2*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2))*b*d+e^2/a*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2))*b+2*e/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-e/a^2*c/(-e^2*(4*a*c-
b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh
((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b
^2*d+e/a*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(
(e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/
a^2*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+
d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 72.0699, size = 9783, normalized size = 26.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^
2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2
- 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*
c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*
a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b
^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 -
 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c
^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b
^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2
 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x
*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 -
 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*
c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8
*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e +
(a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5
- 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(
a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a
*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) + sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^
2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6
 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d
*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2
- 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a
^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6
- 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*
e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4
 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*s
qrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3
*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/
(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e
)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d -
(a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt
```

```
(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c
^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2
*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*
c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b
^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))
 + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - (
2*b*d - a*e)*sqrt(d)*x*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*sqr
t(e*x + d)*a*d)/(a^2*d*x), 1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b
^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^
2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*
c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c
 + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^
4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b
^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^
6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*
d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2
 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e
*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 -
 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^
6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e -
(a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d
- (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^
2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a
^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b
^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) + sqrt(2)*a
^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4
*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*
a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*
b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2
*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*
c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a
^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b
^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)
*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a
*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e
^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d
 - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(e*x + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4
*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(
((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c
^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4
*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5
 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(
((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c
^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt
(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^
5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c +
 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9
*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*
c^3)*e)*sqrt(e*x + d)) - 2*(2*b*d - a*e)*sqrt(-d)*x*arctan(sqrt(e*x + d)*sq
rt(-d)/d) - 2*sqrt(e*x + d)*a*d)/(a^2*d*x)]
```

Sympy [B] time = 91.2937, size = 1588, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] -2*b*e**2*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a*
*2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*
d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4)
+ _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2)
 + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_
t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64
*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 +
 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e -
4*_t*c*d**2 + sqrt(d + e*x))))/a - 2*c*e*RootSum(_t**4*(256*a**2*c**3*e**4
- 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**
2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, Lambda
(_t, _t*log(64*_t**3*a*c**2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c
*d + sqrt(d + e*x))))/a - d*e*sqrt(d**(-3))*log(-d**2*sqrt(d**(-3)) + sqrt(
d + e*x))/(2*a) + d*e*sqrt(d**(-3))*log(d**2*sqrt(d**(-3)) + sqrt(d + e*x))
/(2*a) + 2*e*atan(sqrt(d + e*x)/sqrt(-d))/(a*sqrt(-d)) - sqrt(d + e*x)/(a*x
) + 2*b**2*d*e*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 2
56*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b*
*3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e
**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*
e**2) + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4
- 8*_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3
 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e
**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d
*e - 4*_t*c*d**2 + sqrt(d + e*x))))/a**2 - 2*b*c*d**2*RootSum(_t**4*(256*a*
*3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**
3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e
**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*
c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t*log(32*_t**
3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c - 16*_t**3*a
*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t*
*3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*
_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(d + e*x))))/
a**2 + 2*b*c*d*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 1
6*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8
*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**
2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x))))/a**2
 - 2*b*d*atan(sqrt(d + e*x)/sqrt(-d))/(a**2*sqrt(-d))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.532 $\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=531

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2a\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] -Sqrt[d + e*x]/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*d*x) + ((b*d - a*e)*Sqr
t[d + e*x])/(a^2*d*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*d^(3/2)
) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*d^(3/2)) - (2*(b^2*
d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[2]
*Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c
]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d - b^2
*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c
*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.569, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2a\right.\right.}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -Sqrt[d + e*x]/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*d*x) + ((b*d - a*e)*Sqr
t[d + e*x])/(a^2*d*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*d^(3/2)
) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*d^(3/2)) - (2*(b^2*
d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[2]
*Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c
]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d - b^2
*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c
*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{de^3}{a\left(d-x^2\right)^3}+\frac{e^2(-bd+ae)}{a^2\left(d-x^2\right)^2}+\frac{e\left(-b^2d+acd+abe\right)}{a^3\left(d-x^2\right)}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2\right)}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx\right.}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}dx, x, \sqrt{d+ex}\right)}{a^3} - \frac{\left(2de^2\right)\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^3}dx, x\right.}{a}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{2\left(b^2d-acd-abe\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-\frac{\left(3e^2\right)\text{Subst}\left(\int \frac{1}{\left(d-x\right.}\right.}{2a}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}-\frac{2\left(b^2d-acd-\right.}{}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{}}\right.}{a^2d^{3/2}}$$

Mathematica [A] time = 2.38842, size = 516, normalized size = 0.97

$$-\frac{\frac{3a^2e\left(ex\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)-\sqrt{d}\sqrt{d+ex}\right)}{d^{3/2}x}+\frac{2a^2\sqrt{d+ex}}{x^2}+\frac{8\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{\sqrt{d}}+\frac{4\sqrt{2}\sqrt{c}\left(b^2\left(ae-d\sqrt{b^2-4ac}\right)+ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(d\sqrt{b^2-4}\right.\right.}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -((2*a^2*Sqrt[d + e*x])/x^2 - (4*a*(b*d - a*e)*Sqrt[d + e*x])/(d*x) - (4*a*
e*(-(b*d) + a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(3/2) + (8*(b^2*d - a*c*
d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (3*a^2*e*(-(Sqrt[d]*Sq
rt[d + e*x]) + e*x*ArcTanh[Sqrt[d + e*x]/Sqrt[d]]))/(d^(3/2)*x) + (4*Sqrt[2
]*Sqrt[c]*(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 -
 4*a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b^3*d
 - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*
b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e]))/(4*a^3)
```

Maple [B] time = 0.294, size = 1486, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x)
```

```
[Out] -1/4/a/x^2/d*(e*x+d)^(3/2)+1/e/a^2/x^2*(e*x+d)^(3/2)*b-1/e/a^2/x^2*(e*x+d)^
(1/2)*b*d-1/4*(e*x+d)^(1/2)/a/x^2+1/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/
d^(3/2)+e/a^2/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b+2/a^2*d^(1/2)*arctan
h((e*x+d)^(1/2)/d^(1/2))*c-2/a^3*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b^2
+2*e^2/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2))-e^2/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*
d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2-3*e/a^2*c^2/(-e^2*(4*a*c-b^2))
^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d
)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d+e/a^3
*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2))*b^3*d+e/a^2*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2))*b+1/a^2*c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*d-1/a^3*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
b^2*d+2*e^2/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))-e^2/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((
-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2-3*e/a^2*c^2/(-e^2*(
4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*b*d+e/a^3*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d-e/a^2*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*b-1/a^2*c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2))*d+1/a^3*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2))*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.533 $\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=650

$$\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)+ab^2c^2\left(4cd^2-9ae^2\right)-10ab^3c^2de-b^4c\left(cd^2-6ae^2\right)+2b^5cde+b^6\left(-e^2\right)}{\sqrt{b^2-4ac}}+\left(ace+b^2(-e)+bcd\right)\left(3abce-2ac^2d+b^2cd\right.\right.}{c^{11/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] (-2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Sqrt[d + e*x]
)/c^5 - (2*b*(b^2 - 2*a*c)*(d + e*x)^(3/2))/(3*c^4) + (2*(c^2*d^2 + b^2*e^2
 + c*e*(b*d - a*e))*(d + e*x)^(5/2))/(5*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*
x)^(7/2))/(7*c^2*e^3) + (2*(d + e*x)^(9/2))/(9*c*e^3) + (Sqrt[2]*((b*c*d -
b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 1
0*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2
) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]])/(c^(11/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((b*c
*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*c*d*
e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*
a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(11/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 2.67918, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3\left(cd^2-ae^2\right)+ab^2c^2\left(4cd^2-9ae^2\right)-10ab^3c^2de-b^4c\left(cd^2-6ae^2\right)+2b^5cde+b^6\left(-e^2\right)}{\sqrt{b^2-4ac}}+\left(ace+b^2(-e)+bcd\right)\left(3abce-2ac^2d+b^2cd\right.\right.}{c^{11/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Sqrt[d + e*x]
)/c^5 - (2*b*(b^2 - 2*a*c)*(d + e*x)^(3/2))/(3*c^4) + (2*(c^2*d^2 + b^2*e^2
 + c*e*(b*d - a*e))*(d + e*x)^(5/2))/(5*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*
x)^(7/2))/(7*c^2*e^3) + (2*(d + e*x)^(9/2))/(9*c*e^3) + (Sqrt[2]*((b*c*d -
b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 1
0*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2
) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]])/(c^(11/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((b*c
*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*c*d*
e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*
a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(11/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{e\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)}{c^5}-\frac{b\left(b^2-2ac\right)ex^2}{c^4}+\frac{\left(c^2d^2+b^2e^2+ce(bd-ae)\right)x^4}{c^3e^2}-\frac{(2cd+be)x^6}{c^2e^2}+\frac{x^8}{ce^2}+\right.\right.}{e}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2+b^2e^2\right.}{}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2+b^2e^2\right.}{}$$

$$= -\frac{2\left(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e\right)\sqrt{d+ex}}{c^5}-\frac{2b\left(b^2-2ac\right)(d+ex)^{3/2}}{3c^4}+\frac{2\left(c^2d^2+b^2e^2\right.}{}$$

Mathematica [A] time = 1.16443, size = 808, normalized size = 1.24

$$\frac{2\sqrt{d+ex}\left((d+ex)^2\left(8d^2-20exd+35e^2x^2\right)c^4-9e(d+ex)^2(-2bd+7ae+5bex)c^3+21e^2\left(15a^2e^2+10ab(4d+ex)e+3b^2\right.\right.}{315c^5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(315*b^4*e^4 - 105*b^2*c*e^3*(4*b*d + 9*a*e + b*e*x) - 9*c
^3*e*(d + e*x)^2*(-2*b*d + 7*a*e + 5*b*e*x) + c^4*(d + e*x)^2*(8*d^2 - 20*d
*e*x + 35*e^2*x^2) + 21*c^2*e^2*(15*a^2*e^2 + 3*b^2*(d + e*x)^2 + 10*a*b*e*
(4*d + e*x))))/(315*c^5*e^3) + (Sqrt[2]*(-(b^6*e^2) + b^5*e*(2*c*d + Sqrt[b
^2 - 4*a*c]*e) + a*b^2*c^2*(4*c*d^2 + 6*Sqrt[b^2 - 4*a*c]*d*e - 9*a*e^2) +
b^3*c*(-4*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 10*a*e)) + a
*b*c^2*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - 2*c*d*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) -
b^4*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + 2*a^2*c^3*(-(c*d^2) + e
*(-(Sqrt[b^2 - 4*a*c]*d) + a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(11/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^6*e^2 + b^5*e*(-2*c*d + S
qrt[b^2 - 4*a*c]*e) + a*b^2*c^2*(-4*c*d^2 + 6*Sqrt[b^2 - 4*a*c]*d*e + 9*a*e
^2) - 2*a^2*c^3*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b^4*c*(c*d^2 -
 2*e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + a*b*c^2*(3*a*Sqrt[b^2 - 4*a*c]*e^2 -
2*c*d*(Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + b^3*c*(-4*a*Sqrt[b^2 - 4*a*c]*e^2 +
c*d*(Sqrt[b^2 - 4*a*c]*d + 10*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(11/2)*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Maple [B] time = 0.318, size = 3685, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b*d^2-e^
3/c^5/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2))*b^6+2*e/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2))*a^2*d+4*e^2/c^4*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2))*a*b^3+2*e/c^4*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b^4*d+3*e^2/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2))*a^2*b-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*a^2*d-4*e^2/c^4*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2))*a*b^3+2*e^3/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*
```

d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^3+2*e^3/c^2/(-e^2*(4*a*c-b^2))^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d
)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^3-e^3/
c^5/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2))*b^6-3*e^2/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2))*a^2*b+10*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d
+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b*d+4/3/c^3*(e*x+d)^(3/2)*a*b-2
/c^4*b^3*d*(e*x+d)^(1/2)+6*e/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*a*b^2*d-9*e^3/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b
*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b^2-9*e^3/c^3/(-e^2*(
4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
)*a^2*b^2+2*e^2/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2))*b^5*d-6*e/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d-e/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2
)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4*d^2-2*e/c/(-e^2*(4*a*
c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^
2*d^2+6*e^3/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2))*a*b^4+2*e^2/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2
^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5*d-e/c^3/(-e^2*(
4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*b^4*d^2-2*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*d^2+6*e^3/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2
^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*
c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^4+2/5/e/c^3*(
e*x+d)^(5/2)*b^2+2*e/c^3*a^2*(e*x+d)^(1/2)+2*e/c^5*b^4*(e*x+d)^(1/2)-4/7/e^
3/c*(e*x+d)^(7/2)*d+2/5/e^3/c*(e*x+d)^(5/2)*d^2-2/7/e^2/c^2*(e*x+d)^(7/2)*b
-2/5/e/c^2*(e*x+d)^(5/2)*a-2/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*a*b*d^2-2*e/c^4*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))*b^4*d-2/3/c^4*(e*x+d)^(3/2)*b^3+2/5/e^2/c^2*(e*x+d)^
(5/2)*b*d-6*e/c^4*a*b^2*(e*x+d)^(1/2)+4/c^3*a*b*d*(e*x+d)^(1/2)-10*e^2/c^3/
(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*a*b^3*d+4*e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d^2+10*e^2/c^2/(-e^2*(4*a*c-b^2))^(
1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d
)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b*d-
10*e^2/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2))*a*b^3*d+4*e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-
b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d^2+2/9*(e*x+d)^(9

```
/2)/c/e^3-1/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
an((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
b^3*d^2+1/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
nh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
*b^3*d^2-e^2/c^5*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arc
tan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
*b^5+e^2/c^5*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
h((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*
b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}x^4}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^4/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 90.312, size = 31622, normalized size = 48.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/630*(315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 -
 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^
4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2
*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*
a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*
e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c
^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12
)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 2
39*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e +
 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 183
5*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*
a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 29
70*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 -
700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 +
 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^
7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^
2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1
068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 +
95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c
^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^
8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^
6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12))*log(sqrt(2)*((b^12*c^4 -
12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5
*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7
*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*
c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6
```

```
- 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*
b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^
5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 1
04*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 34
2*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 - ((b^6*c^12 - 8*a*b^4*c^13
 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*
c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8
- 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d
^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*
a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*
(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a
^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8
*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*
a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700
*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 53
0*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 +
 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e
^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068
*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*
a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2
- 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c
^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/
(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*
a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 -
30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6
*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^
9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3
+ (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 -
 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^
6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a
^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(
5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^
4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*
c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a
^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*
a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530
*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 +
3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^
4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*
a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a
^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 -
 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^
6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(
b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5
*c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4
+ 55*a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 2
2*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*
(2*a^4*b^10*c - 15*a^5*b^8*c^2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)
*d^2*e^3 + (a^4*b^11 - 6*a^5*b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^
8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2
- 35*a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) - 315*sqr
t(2)*c^5*e^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6
 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*
c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a
^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^
2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11
 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^
8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15
*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9
```

- 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4
- 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 -
 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e
^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6
 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^1
0 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c
^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^
6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19
*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^
5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9
- 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^
14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^
7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 -
4*a*c^23)))/(b^2*c^11 - 4*a*c^12))*log(-sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5
+ 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4
- (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4
*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^1
2*c^3 + 154*a^2*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*
c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360
*a^2*b^11*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 56
1*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^
2 - 352*a^3*b^10*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6
- 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 - ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*
c^14 - 8*a^3*c^15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*
b*c^14)*e)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*
c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c
^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 -
166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 -
70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1
570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2
 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 +
 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10
+ 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4
 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*
c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c
 - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5
- 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 -
5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14
*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*
b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*
a*c^23)))*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 +
2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5
 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*
b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b
^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11 -
4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c
^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^
5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 1
66*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 7
0*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 15
70*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2
- 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 +
5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 +
 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4
- 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c
^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c
- 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 -
 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5
*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*

c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5*b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) + 315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12))*log(sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 + ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3

- 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6) *d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^ 4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3 *b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^ 6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c ^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^ 3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^ 7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11) *d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12 *c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b ^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c ^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d* e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4* b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^ 8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c ^11 - 4*a*c^12)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7 *b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2 *c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^ 4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^ 2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5* b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e ^4 - (a^5*b^10 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2 *c^4 - a^10*c^5)*e^5)*sqrt(e*x + d)) - 315*sqrt(2)*c^5*e^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^ 2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3* (b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80 *a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2 *b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b ^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^1 2*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b ^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b ^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^ 5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855 *a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 12 5*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2* b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a ^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c ^12))*log(-sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^ 6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 42 0*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7* c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9* c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8) *d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^ 4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8) *e^4 + ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c

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^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 -
12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a
^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b
^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3
*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*
c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4
*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^1
5*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*
b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*
(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a
^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*
a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^
15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6
*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 -
 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 22
32*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 3
0*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*
a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 -
 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^
10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*
a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*
a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 1
2*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^
5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^
11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*
c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c
^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*
c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15
*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b
^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(
5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^
4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a
^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^1
5*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*
b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 -
18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 223
2*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30
*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12
)) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 -
(4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c
^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3
*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^10*c - 15*a^5*b^8*c^2 + 35*a^6*b^6
*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^11 - 6*a^5*b^9*c + 4*a^6*
b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 - (a^5*b^10
 - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^10*c^
5)*e^5)*sqrt(e*x + d)) - 4*(35*c^4*e^4*x^4 + 8*c^4*d^4 + 18*b*c^3*d^3*e + 6
3*(b^2*c^2 - a*c^3)*d^2*e^2 - 420*(b^3*c - 2*a*b*c^2)*d*e^3 + 315*(b^4 - 3*
a*b^2*c + a^2*c^2)*e^4 + 5*(10*c^4*d*e^3 - 9*b*c^3*e^4)*x^3 + 3*(c^4*d^2*e^
2 - 24*b*c^3*d*e^3 + 21*(b^2*c^2 - a*c^3)*e^4)*x^2 - (4*c^4*d^3*e + 9*b*c^3
*d^2*e^2 - 126*(b^2*c^2 - a*c^3)*d*e^3 + 105*(b^3*c - 2*a*b*c^2)*e^4)*x)*sq
rt(e*x + d))/(c^5*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.534 $\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=581

$$\frac{\sqrt{2}\left(-\frac{4a^2c^3de-8ab^2c^2de-b^3c\left(cd^2-5ae^2\right)+abc^2\left(3cd^2-5ae^2\right)+2b^4cde+b^5\left(-e^2\right)}{\sqrt{b^2-4ac}}-b^2c\left(cd^2-3ae^2\right)-4abc^2de+ac^2\left(cd^2-ae^2\right)+2b^3cde+\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] (2*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Sqrt[d + e*x])/c^4 + (2*(b^2 - a
*c)*(d + e*x)^(3/2))/(3*c^3) - (2*(c*d + b*e)*(d + e*x)^(5/2))/(5*c^2*e^2)
+ (2*(d + e*x)^(7/2))/(7*c*e^2) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b
^4*e^2 - b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) - (2*b^4*c*d*e - 8
*a*b^2*c^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^
2*(3*c*d^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 -
 b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) + (2*b^4*c*d*e - 8*a*b^2*c
^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*d
^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e])
```

Rubi [A] time = 15.2472, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(-\frac{4a^2c^3de-8ab^2c^2de-b^3c\left(cd^2-5ae^2\right)+abc^2\left(3cd^2-5ae^2\right)+2b^4cde+b^5\left(-e^2\right)}{\sqrt{b^2-4ac}}-b^2c\left(cd^2-3ae^2\right)-4abc^2de+ac^2\left(cd^2-ae^2\right)+2b^3cde+\right.}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Sqrt[d + e*x])/c^4 + (2*(b^2 - a
*c)*(d + e*x)^(3/2))/(3*c^3) - (2*(c*d + b*e)*(d + e*x)^(5/2))/(5*c^2*e^2)
+ (2*(d + e*x)^(7/2))/(7*c*e^2) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b
^4*e^2 - b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) - (2*b^4*c*d*e - 8
*a*b^2*c^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^
2*(3*c*d^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 -
 b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) + (2*b^4*c*d*e - 8*a*b^2*c
^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*d
^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{e\left(b^2cd-ac^2d-b^3e+2abce\right)}{c^4}+\frac{\left(b^2-ac\right)ex^2}{c^3}-\frac{(cd+be)x^4}{c^2e}+\frac{x^6}{ce}-\frac{\left(b^2cd-ac^2d-b^3e+2abce\right)\left(cd^2-bde+ae^2\right)+\left(2b^3cde-\right.}{c^4e\left(\frac{cd^2-bde+ae^2}{e^2}-\right.}\right.\right.}{e}$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\frac{2(d+ex)}{7ce^2}$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\frac{2(d+ex)}{7ce^2}$$

$$= \frac{2\left(b^2cd-ac^2d-b^3e+2abce\right)\sqrt{d+ex}}{c^4}+\frac{2\left(b^2-ac\right)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\frac{2(d+ex)}{7ce^2}$$

Mathematica [A] time = 0.949256, size = 680, normalized size = 1.17

$$-\frac{\sqrt{2}\left(abc^2\left(e\left(4d\sqrt{b^2-4ac}-5ae\right)+3cd^2\right)+ac^2\left(cd\left(4ae-d\sqrt{b^2-4ac}\right)+ae^2\sqrt{b^2-4ac}\right)-b^3c\left(e\left(2d\sqrt{b^2-4ac}-5ae\right)+\right.\right.}{c^{9/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b\right.}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(105*b^3*e^3 + 3*c^3*(2*d - 5*e*x)*(d + e*x)^2 - 35*b*c*e
^2*(4*b*d + 6*a*e + b*e*x) + 7*c^2*e*(3*b*(d + e*x)^2 + 5*a*e*(4*d + e*x)))
)/(105*c^4*e^2) - (Sqrt[2]*(-(b^5*e^2) + b^4*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e
) + b^2*c*(-3*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e))
- b^3*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 5*a*e)) + a*b*c^2*(3*c*d^2 + e*
(4*Sqrt[b^2 - 4*a*c]*d - 5*a*e)) + a*c^2*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(-(
Sqrt[b^2 - 4*a*c]*d) + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqr
t[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(9/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b^5*e^2 + b^4*e*(-2*c*d + Sqrt
[b^2 - 4*a*c]*e) + a*c^2*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*
d + 4*a*e)) + b^3*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + a*b*c^2*(
-3*c*d^2 + e*(4*Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + b^2*c*(-3*a*Sqrt[b^2 - 4*a*
c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Maple [B] time = 0.317, size = 2988, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] -2/5/e^2/c*(e*x+d)^(5/2)*d-2/c^2*a*d*(e*x+d)^(1/2)+2/c^3*b^2*d*(e*x+d)^(1/2
)-2/5/e/c^2*(e*x+d)^(5/2)*b-2*e/c^4*b^3*(e*x+d)^(1/2)+8*e^2/c^2/(-e^2*(4*a*
c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan
((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*
b^2*d-3*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))*a*b*d^2+8*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-
b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d-3*e/c/(-e^2*(4*a
*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
anh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
)*a*b*d^2-2/3/c^2*(e*x+d)^(3/2)*a+2/3/c^3*(e*x+d)^(3/2)*b^2-4*e^2/c/(-e^2*(
4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
)*a^2*d+e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2))*b^3*d^2+4*e/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2))*a*b*d-4*e/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*a*b*d-5*e^3/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^3-2*e^2/c^3/(-e^
2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2))*b^4*d+5*e^3/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b-5*e^3/c^3/(-e^2*(4*a*c-b^2))^(1/2
)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/
```

```
2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^3-2*e^2/c^
3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2))*b^4*d+e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d^2-4*e^2/c/(-e^2*(4*a*c-b^2))^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)
*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*d+5*e^3/c^2
/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2))*a^2*b+3*e^2/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*a*b^2+2*e/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2))*b^3*d+e^3/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d
+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5+e^3/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2
^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2
)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5-3*e^2/c^3*
2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)
*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2-2*e/c^3*2^
(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c
*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d+2/7*(e*x+d)^
(7/2)/c/e^2+4*e/c^3*a*b*(e*x+d)^(1/2)-1/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2))*a*d^2+1/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2))*b^2*d^2+1/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))*a*d^2-1/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*b^2*d^2+e^2/c^4*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*b^4-e^2/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))*a^2-e^2/c^4*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))*b^4+e^2/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2))*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}x^3}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^3/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 43.7331, size = 24287, normalized size = 41.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/210*(105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 -
2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d
^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5
)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)
*e^3 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8
- 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2
*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^1
2*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8
 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 +
 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39
*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 64
0*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*
c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^
4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5
 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c
^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^
2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(sqrt(2)*((b^9*c^4 - 9*a*b^7*
c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 4
0*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*
c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^
5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 2
16*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^
6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181
*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 - ((b^5*c^10 - 7*a*b^3*c^1
1 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^
12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*
a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5
*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5
 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3
*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 91
0*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 +
 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a
^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^1
5*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 -
 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*
c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5
+ 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*
sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2
- 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^
2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c +
 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*
sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2
*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 +
22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*
a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^
10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b
^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b
^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*
c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 1
3*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^
5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*
a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a
^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^
9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^
3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b
```

^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*
b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3
+ (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d
*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*
e^5)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*
a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 -
7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c
^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3
+ 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 2
2*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^
9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^
5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 2
30*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 11
0*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^
5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2
*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^
2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 -
174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^
7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 +
367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8
*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b^9
*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 -
(4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^
2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 -
155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48
*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5
*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^
3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 - ((b^5*c^1
0 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*
c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a
^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c
^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4
- 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*
a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a
^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b
*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*
b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d
^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 23
9*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^
16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 3
14*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18
- 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^
3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8
*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^
9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + (b^2*c
^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*
c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40
*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*
b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2
*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*
c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d
^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5
+ 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4
- 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b
^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14
*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*
b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*
c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c
^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4

e + 2(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*
e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^
7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 -
11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*
c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) + 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6
*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14
*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3
 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 -
30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 -
8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^
11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3
*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*
a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(1
0*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b
^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^1
2*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*
c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*
a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6
*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230
*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^
7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log
(sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4
*b*c^8)*d^4 - (4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^
6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88
*a^2*b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 -
(4*b^12*c - 48*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4
*c^5 - 141*a^5*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^
9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e
^4 + ((b^5*c^10 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10
 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*
b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6
+ 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e +
3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4
*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^
11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*
c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*
c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8
- 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^
3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^
8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a
^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*
e^6)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 -
 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*
d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^
5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4
)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8
 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^
2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^
12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^
8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4
+ 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 3
9*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 6
40*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7
*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c
^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^
5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*
c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b
^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c
^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 -

3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) - 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*log(-sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 + ((b^5*c^10 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21

```
*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^
5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5
*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^
5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c +
 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) - 4*(15*c^3
*e^3*x^3 - 6*c^3*d^3 - 21*b*c^2*d^2*e + 140*(b^2*c - a*c^2)*d*e^2 - 105*(b^
3 - 2*a*b*c)*e^3 + 3*(8*c^3*d*e^2 - 7*b*c^2*e^3)*x^2 + (3*c^3*d^2*e - 42*b*
c^2*d*e^2 + 35*(b^2*c - a*c^2)*e^3)*x)*sqrt(e*x + d))/(c^4*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.535 $\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=441

$$\frac{\sqrt{2}\left(\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+2b^3cde+b^4\left(-e^2\right)}{\sqrt{b^2-4ac}}+(cd-be)\left(2ace+b^2(-e)+bcd\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left((c\right.}{}$$

```
[Out] (-2*(b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x])/c^3 - (2*b*(d + e*x)^(3/2))/(3*c
^2) + (2*(d + e*x)^(5/2))/(5*c*e) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e +
2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2)
 + 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c
*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*
a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 2.15049, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {897, 1287, 1166, 208}

$$\frac{\sqrt{2}\left(\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+2b^3cde+b^4\left(-e^2\right)}{\sqrt{b^2-4ac}}+(cd-be)\left(2ace+b^2(-e)+bcd\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left((c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x])/c^3 - (2*b*(d + e*x)^(3/2))/(3*c
^2) + (2*(d + e*x)^(5/2))/(5*c*e) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e +
2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2)
 + 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c
*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*
a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(-\frac{e\left(bcd-b^2e+ace\right)}{c^3}-\frac{bex^2}{c^2}+\frac{x^4}{c}+\frac{\left(bcd-b^2e+ace\right)\left(cd^2-bde+ae^2\right)-(cd-be)\left(bcd-b^2e+2ace\right)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right) dx, x, \sqrt{d\cdots}\right.}{e}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{2\,\mathrm{Subst}\left(\int \frac{\left(bcd-b^2e+ace\right)\left(cd^2-bde+ae\cdots\right.}{\frac{cd^2-bde+ae^2}{e^2}-\cdots}\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}-\frac{\left((cd-be)\left(bcd-b^2e+2ace\right)-\cdots\right.}{\cdots}$$

$$= -\frac{2\left(bcd-b^2e+ace\right)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{\sqrt{2}\left((cd-be)\left(bcd-b^2e+2ace\right)\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.759562, size = 538, normalized size = 1.22

$$\frac{\sqrt{2}\left(2ac^2\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)-b^2c\left(2e\left(d\sqrt{b^2-4ac}-2ae\right)+cd^2\right)+bc\left(cd\left(d\sqrt{b^2-4ac}-6ae\right)-2ae^2\sqrt{b^2-4\cdots}\right.\right.}{c^{7/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(15*b^2*e^2 + 3*c^2*(d + e*x)^2 - 5*c*e*(4*b*d + 3*a*e + b
*e*x)))/(15*c^3*e) + (Sqrt[2]*(-(b^4*e^2) + b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c
]*e) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 6*a*e))
 - b^2*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + 2*a*c^2*(c*d^2 + e*(
Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2
*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^4*e^2 + b^3*e*(-2*c*d + Sqrt[b^
2 - 4*a*c]*e) + 2*a*c^2*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b^2*c*
(c*d^2 - 2*e*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e
^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 6*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Maple [B] time = 0.308, size = 2358, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] -6*e^2/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))*a*b*d-6*e^2/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*
e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b*d-e^3/c^3/(-e^2*(4*a*c-b^2)
)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+
d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4+2*e^
2/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b-2*e/c*
2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)
*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d+2*e/c^2*2^(1
/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2
^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d-2*e^3/c/(-e^2*
(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*a^2-e^3/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4-2*e^2/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b+2*e/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2))*a*d-2*e/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*b^2*d+2*e/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d^2+2*e/(-e^2*(4*a*c-b^2
))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x
+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d^2-2
*e^3/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(
1/2))*c)^(1/2))*a^2+2*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d-e/c/(-e^2*(4*a*c-b^2))^(1/2)*2
^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*
c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d^2+4*e^3/c^2
/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
```

```
^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))*c)^(1/2))*a*b^2+2*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d-e/c/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d^2+4*e^
3/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2))*a*b^2-2/3*b*(e*x+d)^(3/2)/c^2+2/5*(e*x+d)^(5/2)/c/e-2/c^2*b*
d*(e*x+d)^(1/2)-2*e/c^2*a*(e*x+d)^(1/2)+2*e/c^3*b^2*(e*x+d)^(1/2)-1/c*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^
(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2+1/c*2^(1/2)/((-
b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2
)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2-e^2/c^3*2^(1/2)/((
b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)
/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3+e^2/c^3*2^(1/2)/((-b*e
+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/(
(-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*x^2/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 17.2234, size = 17508, normalized size = 39.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*c^3*e*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(
b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2
*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)
*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5
*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*
e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29
*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*
a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2
 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 +
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*l
og(sqrt(2)*((b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a
*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c
^3 + 39*a^2*b^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a
*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10
- 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5
```

)*e^4 - ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 +
12*a^2*b*c^9)*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7
*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30
*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b
^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d
^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 +
 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7
*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^
2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a
*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7
 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c
^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 +
 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*
a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^
2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11
*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5
*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c
^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a
^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b
*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d
^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3
*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) - 15*sqrt
(2)*c^3*e*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*
b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a
^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 + (b^2
*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*
c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*
a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^
9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^
3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 +
45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*
c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((
b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53
*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b
^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 10
7*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 - ((b^4
*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)
*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^
5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 +
55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a
*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5
*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c
^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3
*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^
6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 -
3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*
a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*
b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c
^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3
*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d
^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 +

```
29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 1
20*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*
c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5
 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)
) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 +
3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2
 - 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2
*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*
b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) + 15*sqrt(2)*c^3*e*sqr
t(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a
^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2
 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^
8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5
*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 5
5*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*
b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*
b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^
6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*
b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6
)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(sqrt(2)*((b^6*c^4 - 6*a*
b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53*a^2*b^3*c^5 -
 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b^4*c^4 - 29*a^
3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 107*a^2*b^5*c^3
- 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c
^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 + ((b^4*c^8 - 6*a*b^2
*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*e)*sqrt(((b^6
*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*
b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6
 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160
*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40
*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7
)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*
a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 -
 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5
*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 -
2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (
b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b
^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 -
30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10
*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)
*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5
 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b
^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*
a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^2*b^
3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 + 3*a^4*c^5)*d^4
*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2 - 2*(2*a^2*b^
6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2*b^7 - 2*a^3*b
^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*
b^2*c^2 - a^6*c^3)*e^5)*sqrt(e*x + d)) - 15*sqrt(2)*c^3*e*sqrt(((b^4*c^3 -
4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*
e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*
b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*
c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b
^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6
- 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*
```

```
a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*
a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)
*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a
^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 -
4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-sqrt(2)*((b^6*c^4 - 6*a*b^4*c^5 + 8*a
^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6
)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b^4*c^4 - 29*a^3*b^2*c^5 + 4
*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3
*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b
^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 + ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*
c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*e)*sqrt(((b^6*c^6 - 4*a*b^
4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a
^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2
*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5
- 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 +
110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6
*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 -
 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*
sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 +
5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*
e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 - (b^2*c^7 - 4*a
*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*
b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5
+ 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70
*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*
(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2
*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a
^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37
*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*
e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3
*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2
*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b
^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*
b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6
*c^3)*e^5)*sqrt(e*x + d)) - 4*(3*c^2*e^2*x^2 + 3*c^2*d^2 - 20*b*c*d*e + 15*
(b^2 - a*c)*e^2 + (6*c^2*d*e - 5*b*c*e^2)*x)*sqrt(e*x + d))/(c^3*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.536 $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=453

$$\frac{\sqrt{2}\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] (2*(c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c) + (Sqrt[2]*(b
^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d
 - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - (Sqrt[2]*(b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 4.52867, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {824, 826, 1166, 208}

$$\frac{\sqrt{2}\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```


```
[Out] (2*(c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c) + (Sqrt[2]*(b
^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d
 - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - (Sqrt[2]*(b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{\sqrt{d+ex}(-ae+(cd-be)x)}{a+bx+cx^2}\,dx}{c}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{-ae(2cd-be)+\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{2\,\mathrm{Subst}\left(\int \frac{-ae^2(2cd-be)-d\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)+\left(c^2d^2+b^2e^2-ce(2bd+\cdots\right.}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\cdots\right.}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} - \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}e\right)+c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2\cdots}\right.\right.\right.}{\cdots}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\sqrt{2}\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}e\right)+c\left(a\sqrt{b^2-4ac}e^2-cd\left(\cdots\right.\right.\right.}{c^{5/2}\sqrt{b^2\cdots}}$$

Mathematica [A] time = 1.71299, size = 779, normalized size = 1.72

$$\frac{2\left(\frac{3\sqrt{c}d\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{3\left(3c^2de\left(d\sqrt{b^2-4ac}-2ae-bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+\cdots\right.\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\cdots}\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(-3*c*d*e*Sqrt[d + e*x] - 3*e*(-2*c*d + b*e)*Sqrt[d + e*x] + c*e*(d + e*
x)^(3/2) + (3*Sqrt[c]*d*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*
(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2
```

```
*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (3*(2*c^3*d^3 + b^2*(-b + Sqrt[b^2 -
4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 2*a*e) + c*e^2*(3*b
^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])
 - (3*Sqrt[c]*d*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + S
qrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]) + (3*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^
3 - 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt
[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]
*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/(3*c^2*e)
```

Maple [B] time = 0.286, size = 1714, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2/3*(e*x+d)^(3/2)/c-2/c^2*(e*x+d)^(1/2)*b*e+2/c*(e*x+d)^(1/2)*d-3/c/(-e^2*(
4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
)*a*b*e^3+4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2))*a*d*e^2+1/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*
e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*e^3-2/c/(-e^2*(4*a*c-b^2))^
(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)
^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d*e^2+
1/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2))*b*d^2*e-1/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2))*a*e^2+1/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))*b^2*e^2-2/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
n((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b
*d*e+2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^
(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-3/c/(-e
^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*a*b*e^3+4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*d*e^2+1/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/
2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*
2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*e^3-2/c/(-e^2*
(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*b^2*d*e^2+1/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2*e+1/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(
4*a*c-b^2))^(1/2))*c)^(1/2))*a*e^2-1/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4
```

```
*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^2+2/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))*b*d*e-2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}x}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")

[Out] integrate((e*x + d)^(3/2)*x/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 7.54289, size = 11256, normalized size = 24.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")

[Out] -1/6*(3*sqrt(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)
*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^
6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3
 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*
a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^3
*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(
2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2
+ 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4
*a^3*b*c^3)*e^4 - ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2
*c^7)*e)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 1
0*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*
c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^
2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*
a*c^11)))*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3
*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*
e^3 + (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e +
 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^
3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^
4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c
^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^
6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2
*c^3 - 3*a^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^
4*c - 3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2
*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 3*sqrt(2)*c^2*sqrt(((b^
2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*
```

```
a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*
a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^
5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*
e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b
^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 1
2*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4
)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 - ((b^3*c^6
 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*sqrt((b^2*c^6*d^6 -
 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4
*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2
 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c
^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^2*c^3 - 2
*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^
2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*sq
rt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5
 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^
3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(
b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(
b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2*
(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 - a^3*c^
3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2
)*e^5)*sqrt(e*x + d)) + 3*sqrt(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^
3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5
 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6
*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e
^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 -
20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2
 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
*log(sqrt(2)*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^
2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6
*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c +
13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 + ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6
*a*b^2*c^6 + 8*a^2*c^7)*e)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e
+ 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b
^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c
^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*
c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e
^6)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*
a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3
*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5
 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10
*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c
^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*
c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d
^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b
*c^2)*d*e^4 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 3*sqr
t(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*
(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e
^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e +
3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3
*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4
 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^
```

```
4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^3*c^4 - 4*a
*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(2*b^5*c^2
- 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b
^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3
)*e^4 + ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*s
qrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^
5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e
^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*
(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))*
sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c -
4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2
*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*
c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19
*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*
c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a
^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^
2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - 3*a
^3*b^2*c + a^4*c^2)*e^5)*sqrt(e*x + d)) - 4*(c*e*x + 4*c*d - 3*b*e)*sqrt(e*
x + d))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.537 $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2} \, dx$

Optimal. Leaf size=322

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}\right.\right.\ldots}{\ldots}$$

```
[Out] (2*e*Sqrt[d + e*x])/c - (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c
^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.24375, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {703, 826, 1166, 208}

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}\right.\right.\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```


```
[Out] (2*e*Sqrt[d + e*x])/c - (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c
^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c} + \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{2\,\text{Subst}\left(\int \frac{-de(2cd-be)+e\left(cd^2-ae^2\right)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}\left(\int \frac{}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\cdots}\right.}{c\sqrt{b^2-4ac}}$$

$$= \frac{2e\sqrt{d+ex}}{c} - \frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{2cd-\left(b-\cdots\right.}}\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.735757, size = 317, normalized size = 0.98

$$\frac{\frac{\sqrt{2}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2c\cdots}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\cdots}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x] + (Sqrt[2]*(-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*
c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c^(3/2)
```

Maple [B] time = 0.28, size = 1138, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] 2*e*(e*x+d)^(1/2)/c+2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4
*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*e^3-1/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2
)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^3+2/(-e^2*(4*a*c-b^
2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*
x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d*e^
2-2*e*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2))*d^2-1/c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b*e^2+2*e*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))*d+2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2))*a*e^3-1/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*e^3+2/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d*e^2-2*e*
c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/
2))*c)^(1/2))*d^2+1/c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b*e^2-2*e*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*
arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(c*x^2 + b*x + a), x)
```

Fricas [B] time = 3.39844, size = 5581, normalized size = 17.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2
 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3
*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b
```

```
^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (
b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*
a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3
)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b
^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3
+ 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - sqrt(2)*c*sqrt((2*
c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 +
 (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2
- 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2
)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-sqrt(2)*(3*(b^2*c^2
- 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4
*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c -
a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqr
t((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*
e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2
*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6
*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b
^2 - a^2*c)*e^5)*sqrt(e*x + d)) + sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e
 + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sq
rt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*
(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(
b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4
- 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*
e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 -
2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d
^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4
)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4
- 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^
2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*
x + d)) - sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(-sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^
3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3
 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*
a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c
^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 -
 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d
*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*
a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 -
(b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(e*x + d)) - 4*sqrt(e*x +
d)*e)/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.538 $\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=340

$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}+4\right.\right.}{a}$$

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*S
qrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2
]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 +
 a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rubi [A] time = 1.57768, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {897, 1287, 206, 1166, 208}

$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}+4\right.\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*S
qrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2
]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 +
 a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
```

```
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{d^2e}{a\left(d-x^2\right)}+\frac{e\left(d\left(cd^2-bde+ae^2\right)-\left(cd^2-ae^2\right)x^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{d\left(cd^2-bde+ae^2\right)+\left(-cd^2+ae^2\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)-b\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\ldots}\right.}{a\sqrt{b^2-4ac}}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)-b\left(cd^2+ae^2\right)\right)\tanh^{-1}\left(\ldots\right.}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 1.14756, size = 331, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\sqrt{2}\left(cd\left(d\sqrt{b^2-4ac}+4ae\right)-ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right.}}\ldots\right.}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*(-(a*Sqrt[b^2 - 4*a*c
]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqr
t[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]
*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) - b*(c*d^2
 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]))/a
```

Maple [B] time = 0.291, size = 944, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x)
```

```
[Out] -2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a+e^3/(-e^2*(4*a*c-b^2))^(1/2)*2^
(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c
*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b-4*e^2*c/(-e^2*(4
*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arc
tan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
*d+e/a*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))*b*d^2+e^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2)
)*c)^(1/2))-1/a*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
)*d^2+e^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2))*b-4*e^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*
e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d+e/a*c/(-e^2*(4*a*c-b^2))^(1/2
)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(
1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2-e^2*2
^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2
)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+1/a*c*2^(1/2)/
((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 54.4261, size = 10272, normalized size = 30.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c
- 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*
e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4
*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*
b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((
a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6
 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^
4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e
^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*
c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c
*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) +
4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 -
(b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e -
6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)
*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^
3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a
^3*c^2))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^
3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2
*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2
+ 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3))
)*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3
+ (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d
^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^
5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*
c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d))
 + sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e +
9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2
*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^
2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*
b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6
*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 +
a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 +
 a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*
d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2
*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b
*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2
*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^
2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqr
t((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 -
6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c
^2))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e
+ (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c -
 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*
a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sq
rt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a
^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e
^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^
3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^
2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - 2
*d^(3/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/a, -1/2*(sqrt(2)*a*s
qrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (
a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*
e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c
^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*
```

```
b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*
b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e
 + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*
b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (
b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^
2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6
)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a
*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*
d^4*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^
2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6
 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^
4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(-sqrt
(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4
*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)
*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^
3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d
^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a
^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c
*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*
c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*
d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) + sqrt(2)*a*sqrt(
-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*
b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2
+ 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3))
)/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*
c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^
2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9
*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*
c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*
c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^
5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a
^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c
*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*
e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*
e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6
*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 +
a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(-sqrt(2)*
((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3
*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqr
t((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 -
6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e
 - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c
^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3
*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c -
4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d*e^
4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*sqrt(e*x + d)) - 4*sqrt(-d)*d*arctan
(sqrt(e*x + d)*sqrt(-d)/d))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x/(c*x**2+b*x+a),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.539 $\int \frac{(d+ex)^{3/2}}{x^2\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=403

$$-\frac{\sqrt{2}\sqrt{c}\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\sqrt{2}\sqrt{c}\left(-2a\right.$$

```
[Out] -((d*Sqrt[d + e*x])/(a*x)) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a +
 (2*Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2 - (Sqrt[2]*Sq
rt[c]*(b^2*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*(c*d^2 + e*(Sqrt[b
^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(b^2*d^2 - b*d*(Sqrt[b^2 - 4*a*c]*d +
 2*a*e) - 2*a*(c*d^2 - e*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt[b^2
 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.07458, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{2}\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\sqrt{2}\sqrt{c}\left(-bd\left(d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((d*Sqrt[d + e*x])/(a*x)) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a +
 (2*Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2 - (Sqrt[2]*Sq
rt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*(Sqr
t[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(Sqrt[
b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt
[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{d^2e^2}{a\left(d-x^2\right)^2}-\frac{de(-bd+2ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-(bd-ae)\left(cd^2-bde+ae^2\right)+cd(bd-2ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{-(bd-ae)\left(cd^2-bde+ae^2\right)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2} + \frac{\left(2d^2e\right)\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d-\right.}{a}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{2\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right)}{a} + \frac{\left(c\left(b^2d^2\right.\right.}{}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{2\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} - \frac{\sqrt{2}\sqrt{c}\left(b^2d^2-2acd^2+\right.}{}$$

Mathematica [A] time = 1.58731, size = 393, normalized size = 0.98

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(2ae-d\sqrt{b^2-4ac}\right)-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}+2ae\right)-2ae\left(d\sqrt{b^2-4ac}+ae\right)+2acd^2-b^2d^2\right)\text{ t}}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x])/x) + a*Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + 2*
Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*Sqrt[c]*(-(
b^2*d^2) + b*d*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) + 2*a*(c*d^2 + e*(Sqrt[b^2
- 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*
e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 -
 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(-(b^2*d^2) + 2*a*c*d^2 - 2*a*e*(Sqrt[b^2 -
 4*a*c]*d + a*e) + b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]*Sqrt
[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a^2
```

Maple [B] time = 0.293, size = 1215, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x)
```

```
[Out] -d*(e*x+d)^(1/2)/a/x-3*e*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+2*d^(3/2)
/a^2*arctanh((e*x+d)^(1/2)/d^(1/2))*b-2*e^3*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1
/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2
^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+2*e^2/a*c/(-e^2*(4*a
*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
n((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b
*d+2*e/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b
^2))^(1/2))*c)^(1/2))*d^2-e/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e
-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d^2-2*e/a*c*2^(1/2)/((b*e-2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e
-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d+1/a^2*c*2^(1/2)/((b*e-2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d
+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2-2*e^3*c/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+2*e^2/a*c/(-
e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
c)^(1/2))*b*d+2*e/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-e/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)
*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d^2+2*e/a*c
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1
/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-1/a^2*c*2^
```

```
(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)
*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**2/(c*x**2+b*x+a),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")

[Out] Timed out
```

3.540 $\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx$

Optimal. Leaf size=607

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2-4ac}-\right.\right.}{a^3\sqrt{b^2-}}$$

```
[Out] -(d*Sqrt[d + e*x])/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*x) + ((b*d - 2*a*e)
*Sqrt[d + e*x])/(a^2*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*Sqrt[
d]) - (e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (2*(
b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a
^3*Sqrt[d]) + (Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*
e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - a*b*
(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*
d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d +
 a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.93064, antiderivative size = 607, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {897, 1287, 199, 206, 1166, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2-4ac}-\right.\right.}{a^3\sqrt{b^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```


```
[Out] -(d*Sqrt[d + e*x])/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*x) + ((b*d - 2*a*e)
*Sqrt[d + e*x])/(a^2*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*Sqrt[
d]) - (e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (2*(
b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a
^3*Sqrt[d]) + (Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*
e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - a*b*
(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*
d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d +
 a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(-\frac{d^2e^3}{a\left(d-x^2\right)^3}+\frac{de^2(-bd+2ae)}{a^2\left(d-x^2\right)^2}+\frac{e\left(-b^2d^2+2abde+a\left(cd^2-ae^2\right)\right)}{a^3\left(d-x^2\right)}+\frac{e\left(\left(b^2d-acd-abe\right)\left(cd^2-bde+ae^2\right)-c\left(b^2\right.\right.}{a^3\left(cd^2-bde+ae^2-(2cd\right.}\right.\right.}{e}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{\left(b^2d-acd-abe\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^3} - \frac{\left(2d^2e^2\right)\mathrm{Subst}}{}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{2\left(b^2d^2-2abde-a\left(cd^2-ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-\frac{(3de}{}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{e(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{2\left(b^2d^2-2a\right.}{}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}-\frac{e(bd-2ae)\tanh^{-1}}{a^2\sqrt{d}}$$

Mathematica [A] time = 2.87339, size = 587, normalized size = 0.97

$$\frac{-\frac{2a^2d\sqrt{d+ex}}{x^2}+3a^2e\left(\frac{\sqrt{d+ex}}{x}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}\right)-\frac{8\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde+a\left(ae^2-cd^2\right)+b^2d^2\right)}{\sqrt{d}}+\frac{4\sqrt{2}\sqrt{c}\left(ab\left(e\left(ae-2d\sqrt{b^2-4ac}\right)-3cd^2\right)+a\left(cd\left(4ac\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*a^2*d*Sqrt[d + e*x])/x^2 + (4*a*(b*d - 2*a*e)*Sqrt[d + e*x])/x + (4*a*
e*(-(b*d) + 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] - (8*(b^2*d^2 -
2*a*b*d*e + a*(-(c*d^2) + a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] +
 3*a^2*e*(Sqrt[d + e*x]/x - (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d]) + (
4*Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*b*(-3*
c*d^2 + e*(-2*Sqrt[b^2 - 4*a*c]*d + a*e)) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*
d*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*d*(Sqrt[b
^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d + a*e)) +
 a*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(4*a^3)
```

Maple [B] time = 0.304, size = 1880, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x)
```

```
[Out] 3*e/a^2*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b-3*e/a^2*c^2/(-e^2*(4*a*c-b
^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e
*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d^2
+e/a^3*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))
^(1/2))*c)^(1/2))*b^3*d^2-2*e^2/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b
*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d-3*e/a^2*c^2/(-e^2*(
4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2))*b*d^2+e/a^3*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d^2-2*e^2/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2
)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^2*d+2/a^2*d^(
3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c-2/a^3*d^(3/2)*arctanh((e*x+d)^(1/2)/d
^(1/2))*b^2+1/a^3*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)
*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2))*b^2*d^2-1/a^3*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^
(1/2))*b^2*d^2+2*e/a^2*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c
)^(1/2))*b*d+4*e^2/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^
2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d
+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d+e^3/a*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*
c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b+e^3/a*c/(-e^2*
(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*a
rctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
))*b+4*e^2/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-
b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a
*c-b^2))^(1/2))*c)^(1/2))*d-2*e/a^2*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2))*b*d-5/4/a/x^2*(e*x+d)^(3/2)+1/a^2*c^2*2^(1/2)/((b*
e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan((e*x+d)^(1/2)*c*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2-1/a^2*c^2*2^(1/2)/((-b*e
+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x+d)^(1/2)*c*2^(1/2)/(
(-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d^2+1/e/a^2/x^2*(e*x+d)^(3/
2)*b*d-e^2/a*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
n((e*x+d)^(1/2)*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+e
^2/a*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh((e*x
+d)^(1/2)*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/e/a^
2/x^2*(e*x+d)^(1/2)*b*d^2-3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+
3/4*d*(e*x+d)^(1/2)/a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.541 $\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=201

$$\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+\right.}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+\right.}$$

```
[Out] (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), (-2*c*
x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1
+ m)*(1 + (f*x)/e)^n) - (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
 + m, -((f*x)/e), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b
+ Sqrt[b^2 - 4*a*c])*(1 + m)*(1 + (f*x)/e)^n)
```

Rubi [A] time = 0.363151, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {911, 135, 133}

$$\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+\right.}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), (-2*c*
x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1
+ m)*(1 + (f*x)/e)^n) - (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
 + m, -((f*x)/e), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b
+ Sqrt[b^2 - 4*a*c])*(1 + m)*(1 + (f*x)/e)^n)
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) dx$$

$$= \frac{(2c)\int \frac{x^m(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x^m(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n} F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right.}{\sqrt{b^2-\ldots}}$$

Mathematica [F] time = 0.309209, size = 0, normalized size = 0.

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2), x]
```

Maple [F] time = 1.344, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^m}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^m}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^m}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^m}{cx^2+bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

3.542 $\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=290

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n\right.}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}\right.\right.}$$

```
[Out] -(((c*e + b*f)*(e + f*x)^(1 + n))/(c^2*f^2*(1 + n))) + (e + f*x)^(2 + n)/(c
*f^2*(2 + n)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (
b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))
 + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e + f*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2
- 4*a*c])*f)])/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rubi [A] time = 0.769534, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1628, 68}

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n\right.}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] -(((c*e + b*f)*(e + f*x)^(1 + n))/(c^2*f^2*(1 + n))) + (e + f*x)^(2 + n)/(c
*f^2*(2 + n)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (
b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))
 + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e + f*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2
- 4*a*c])*f)])/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}dx = \int \left(\frac{(-ce-bf)(e+fx)^n}{c^2 f} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}-\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx} + \frac{(e+f}{cf} \right.$$

$$= -\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}dx - \frac{\left(a-\right.}{}$$

$$= -\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+}{2ce-(b-\sqrt{}}\right.}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

Mathematica [A] time = 0.772312, size = 261, normalized size = 0.9

$$\frac{(e+fx)^{n+1}\left(\frac{c\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{(n+1)\left(f\left(\sqrt{b^2-4ac}-b\right)+2ce\right)}+\frac{c\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{bf+ce}{f^2(n+1)}+\frac{c(e+fx)}{f^2(n+2)}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((c*e + b*f)/(f^2*(1 + n))) + (c*(e + f*x))/(f^2*(2 +
n)) + (c*(a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)
])/((2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (c*(a - b^2/c - (b*(b^2
 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(
e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/((2*c*e - (b + Sqrt[b^2 - 4
*a*c])*f)*(1 + n))))/c^2
```

Maple [F] time = 1.278, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^3}{cx^2+bx+a}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^3}{cx^2+bx+a}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x^{3}}{cx^{2}+bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}x^{3}}{cx^{2}+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

3.543 $\int \frac{x^2(e+fx)^n}{a+bx+cx^2} \, dx$

Optimal. Leaf size=237

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] (e + f*x)^(1 + n)/(c*f*(1 + n)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e -
 (b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n
)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])
/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rubi [A] time = 0.382054, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1628, 68}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (e + f*x)^(1 + n)/(c*f*(1 + n)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e -
 (b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n
)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])
/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{(e+fx)^n}{c} + \frac{\left(-\frac{b}{c}+\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right)dx$$

$$= \frac{(e+fx)^{1+n}}{cf(1+n)} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{c}$$

$$= \frac{(e+fx)^{1+n}}{cf(1+n)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e\cdots}{c(\cdots}$$

Mathematica [A] time = 0.36375, size = 202, normalized size = 0.85

$$\frac{(e+fx)^{n+1}\left(\frac{\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)} + \frac{1}{f}\right)}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] ((e + f*x)^(1 + n)*(f^(-1) + ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Hyperg
eometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*
c])*f)])/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f) + ((b + (b^2 - 2*a*c)/Sqrt[b^
2 - 4*a*c])*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b
+ Sqrt[b^2 - 4*a*c])*f)])/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)))/(c*(1 + n))
```

Maple [F] time = 1.309, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^2}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^2}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

3.544 $\int \frac{x(e+fx)^n}{a+bx+cx^2} \, dx$

Optimal. Leaf size=198

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(}{2ce-\left(b+\right.}\right.}{(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] -(((1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/((2*c*e - (b -
 Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - ((1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1
+ n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)])/((2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rubi [A] time = 0.189035, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {830, 68}

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(}{2ce-\left(b+\right.}\right.}{(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] -(((1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/((2*c*e - (b -
 Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - ((1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1
+ n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)])/((2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right)dx$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1\right.}{\left(2ce-\left(b+\sqrt{b^2-\right.\right.}$$

Mathematica [A] time = 0.275087, size = 183, normalized size = 0.92

$$\frac{(e+fx)^{n+1}\left(-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce} - \frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(((1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n
, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-
b + Sqrt[b^2 - 4*a*c])*f)) - ((1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1
, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*
e - (b + Sqrt[b^2 - 4*a*c])*f)))/(1 + n)
```

Maple [F] time = 1.308, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n x}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x}{cx^{2}+bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}x}{cx^{2}+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

3.545 $\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=191

$$\frac{2c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[Out] (-2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/
(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2 - 4*a*c]*(2*c*e - (b - Sqrt
[b^2 - 4*a*c])*f)*(1 + n)) + (2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1
+ n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2
 - 4*a*c]*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rubi [A] time = 0.259567, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {711, 68}

$$\frac{2c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(a + b*x + c*x^2),x]
```

```
[Out] (-2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/
(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2 - 4*a*c]*(2*c*e - (b - Sqrt
[b^2 - 4*a*c])*f)*(1 + n)) + (2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1
+ n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2
 - 4*a*c]*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 711

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) dx$$

$$= \frac{(2c)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{2c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{2c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e}{2ce-(b+}\right.}{\sqrt{b^2-4ac}\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(}$$

Mathematica [A] time = 0.246821, size = 163, normalized size = 0.85

$$\frac{2c(e+fx)^{n+1}\left(\frac{{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce}\right)}{(n+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x)
)/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)]/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f
)) + Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)]/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)))/(Sqrt[b^2 - 4*a*c]*
(1 + n))
```

Maple [F] time = 1.303, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n}{cx^2+bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

3.546 $\int \frac{(e+fx)^n}{x\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=242

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\ldots\right.}{a(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b
 - Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^
(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sq
rt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 +
n))
```

Rubi [A] time = 0.385396, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {960, 65, 830, 68}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\ldots\right.}{a(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b
 - Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^
(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sq
rt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 +
n))
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^n}{x\left(a+bx+cx^2\right)}\,dx &= \int \left(\frac{(e+fx)^n}{ax}+\frac{(-b-cx)(e+fx)^n}{a\left(a+bx+cx^2\right)}\right)dx\\ &= \frac{\int \frac{(e+fx)^n}{x}\,dx}{a}+\frac{\int \frac{(-b-cx)(e+fx)^n}{a+bx+cx^2}\,dx}{a}\\ &= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)}+\frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{a}\\ &= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4a}}\right.\right.}{}\\ &= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F}{a\left(2ce-\left(b+\sqrt{}\right.\right.}\end{aligned}$$

Mathematica [A] time = 0.41423, size = 207, normalized size = 0.86

$$\frac{(e+fx)^{n+1}\left(\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{e}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n
, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-
b + Sqrt[b^2 - 4*a*c])*f) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[
1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(2*c
*e - (b + Sqrt[b^2 - 4*a*c])*f) - Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f
*x)/e]/e))/(a*(1 + n))
```

Maple [F] time = 1.257, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^n}{x\left(cx^2+bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n}{cx^3+bx^2+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^3 + b*x^2 + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(cx^2+bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

3.547 $\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=296

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(}{2ce-(b+}\right.}{a^2(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] -((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/
(a^2*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - (c*(b - (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*
c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(a^2*(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)*(1 + n)) + (b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, 1 + (f*x)/e])/(a^2*e*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometri
c2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rubi [A] time = 0.477245, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {960, 65, 830, 68}

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(}{2ce-(b+}\right.}{a^2(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/
(a^2*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - (c*(b - (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*
c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(a^2*(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)*(1 + n)) + (b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, 1 + (f*x)/e])/(a^2*e*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometri
c2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{(e+fx)^n}{ax^2} - \frac{b(e+fx)^n}{a^2x} + \frac{\left(b^2-ac+bcx\right)(e+fx)^n}{a^2\left(a+bx+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(b^2-ac+bcx\right)(e+fx)^n}{a+bx+cx^2}\,dx}{a^2} + \frac{\int \frac{(e+fx)^n}{x^2}\,dx}{a} - \frac{b\int \frac{(e+fx)^n}{x}\,dx}{a^2}$$

$$= \frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)} + \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)} + \frac{\int\Big(}{}$$

$$= \frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)} + \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)} + \frac{\Big(c\Big(}{}$$

$$= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} - \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{}}{a^2\Big(2ce-\Big(}$$

Mathematica [A] time = 0.438454, size = 246, normalized size = 0.83

$$\frac{(e+fx)^{n+1}\left(-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce+\left(\sqrt{b^2-4ac}-b\right)f}\right)}{f\left(\sqrt{b^2-4ac}-b\right)+2ce} - \frac{c\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-f\left(\sqrt{b^2-4ac}+b\right)} + \frac{af\,{}_2F_1\left(2,n+1;n+2;\frac{fx}{e}+1\right)}{e^2} + \frac{b}{}\right.}{a^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)
])/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[b^2
- 4*a*c])*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b +
Sqrt[b^2 - 4*a*c])*f)])/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f) + (b*Hypergeome
tric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/e + (a*f*Hypergeometric2F1[2, 1 + n,
 2 + n, 1 + (f*x)/e])/e^2))/(a^2*(1 + n))
```

Maple [F] time = 1.307, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}}{x^{2}\left(c\,x^{2}+b x+a\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}}{\left(c x^{2}+b x+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(fx+e\right)^{n}}{cx^{4}+bx^{3}+ax^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^4 + b*x^3 + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{n}}{\left(c x^{2}+b x+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

3.548 $\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2} dx$

Optimal. Leaf size=141

$$-\frac{dx^2\left(4d^2g^2+7defg+2e^2f^2\right)}{e}-\frac{d^2x\left(8d^2g^2+16defg+7e^2f^2\right)}{e^2}-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{1}{2}egx^4(2dg+ef)-\frac{1}{3}x^3(d$$

```
[Out] -((d^2*(7*e^2*f^2 + 16*d*e*f*g + 8*d^2*g^2)*x)/e^2) - (d*(2*e^2*f^2 + 7*d*e
*f*g + 4*d^2*g^2)*x^2)/e - ((e*f + d*g)*(e*f + 7*d*g)*x^3)/3 - (e*g*(e*f +
2*d*g)*x^4)/2 - (e^2*g^2*x^5)/5 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Rubi [A] time = 0.182809, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{dx^2\left(4d^2g^2+7defg+2e^2f^2\right)}{e}-\frac{d^2x\left(8d^2g^2+16defg+7e^2f^2\right)}{e^2}-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{1}{2}egx^4(2dg+ef)-\frac{1}{3}x^3(d$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d^2*(7*e^2*f^2 + 16*d*e*f*g + 8*d^2*g^2)*x)/e^2) - (d*(2*e^2*f^2 + 7*d*e
*f*g + 4*d^2*g^2)*x^2)/e - ((e*f + d*g)*(e*f + 7*d*g)*x^3)/3 - (e*g*(e*f +
2*d*g)*x^4)/2 - (e^2*g^2*x^5)/5 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}dx &= \int \frac{(d+ex)^3(f+gx)^2}{d-ex}dx\\ &= \int \left(-\frac{d^2\left(7e^2f^2+16defg+8d^2g^2\right)}{e^2}-\frac{2d\left(2e^2f^2+7defg+4d^2g^2\right)x}{e}+(-ef-7dg)(ef+dg)x^2\right.\\ &= -\frac{d^2\left(7e^2f^2+16defg+8d^2g^2\right)x}{e^2}-\frac{d\left(2e^2f^2+7defg+4d^2g^2\right)x^2}{e}-\frac{1}{3}(ef+dg)(ef+7dg)x^3-\end{aligned}$$

Mathematica [A] time = 0.0765786, size = 134, normalized size = 0.95

$$-\frac{x\left(70d^2e^2\left(3f^2+3fgx+g^2x^2\right)+120d^3eg(4f+gx)+240d^4g^2+10de^3x\left(6f^2+8fgx+3g^2x^2\right)+e^4x^2\left(10f^2+15fgx+\right.\right.}{30e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -(x*(240*d^4*g^2 + 120*d^3*e*g*(4*f + g*x) + 70*d^2*e^2*(3*f^2 + 3*f*g*x +
g^2*x^2) + 10*d*e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2) + e^4*x^2*(10*f^2 + 15*
f*g*x + 6*g^2*x^2)))/(30*e^2) - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Maple [A] time = 0.043, size = 186, normalized size = 1.3

$$-\frac{e^2g^2x^5}{5}-ex^4dg^2-\frac{e^2x^4fg}{2}-\frac{7x^3d^2g^2}{3}-\frac{8ex^3dfg}{3}-\frac{e^2x^3f^2}{3}-4\frac{x^2d^3g^2}{e}-7x^2d^2fg-2ex^2df^2-8\frac{d^4g^2x}{e^2}-16\frac{d^3fgx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/5*e^2*g^2*x^5-e*x^4*d*g^2-1/2*e^2*x^4*f*g-7/3*x^3*d^2*g^2-8/3*e*x^3*d*f*
g-1/3*e^2*x^3*f^2-4/e*x^2*d^3*g^2-7*x^2*d^2*f*g-2*e*x^2*d*f^2-8/e^2*d^4*g^2
*x-16/e*d^3*f*g*x-7*d^2*f^2*x-8*d^5/e^3*ln(e*x-d)*g^2-16*d^4/e^2*ln(e*x-d)*
f*g-8*d^3/e*ln(e*x-d)*f^2
```

Maxima [A] time = 0.984163, size = 236, normalized size = 1.67

$$-\frac{6e^4g^2x^5+15\left(e^4fg+2de^3g^2\right)x^4+10\left(e^4f^2+8de^3fg+7d^2e^2g^2\right)x^3+30\left(2de^3f^2+7d^2e^2fg+4d^3eg^2\right)x^2+30\left(7d\right.}{30e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*e^4*g^2*x^5 + 15*(e^4*f*g + 2*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 8*d*e
^3*f*g + 7*d^2*e^2*g^2)*x^3 + 30*(2*d*e^3*f^2 + 7*d^2*e^2*f*g + 4*d^3*e*g^2
)*x^2 + 30*(7*d^2*e^2*f^2 + 16*d^3*e*f*g + 8*d^4*g^2)*x)/e^2 - 8*(d^3*e^2*f
^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.69555, size = 371, normalized size = 2.63

$$-\frac{6e^5g^2x^5+15\left(e^5fg+2de^4g^2\right)x^4+10\left(e^5f^2+8de^4fg+7d^2e^3g^2\right)x^3+30\left(2de^4f^2+7d^2e^3fg+4d^3e^2g^2\right)x^2+30\left(7\right.}{30e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*e^5*g^2*x^5 + 15*(e^5*f*g + 2*d*e^4*g^2)*x^4 + 10*(e^5*f^2 + 8*d*e
^4*f*g + 7*d^2*e^3*g^2)*x^3 + 30*(2*d*e^4*f^2 + 7*d^2*e^3*f*g + 4*d^3*e^2*g
^2)*x^2 + 30*(7*d^2*e^3*f^2 + 16*d^3*e^2*f*g + 8*d^4*e*g^2)*x + 240*(d^3*e^
2*f^2 + 2*d^4*e*f*g + d^5*g^2)*log(e*x - d))/e^3
```

Sympy [A] time = 0.698339, size = 156, normalized size = 1.11

$$-\frac{8d^3\left(dg+ef\right)^2\log\left(-d+ex\right)}{e^3}-\frac{e^2g^2x^5}{5}-x^4\left(deg^2+\frac{e^2fg}{2}\right)-x^3\left(\frac{7d^2g^2}{3}+\frac{8defg}{3}+\frac{e^2f^2}{3}\right)-\frac{x^2\left(4d^3g^2+7d^2efg+2de^2f\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -8*d**3*(d*g + e*f)**2*log(-d + e*x)/e**3 - e**2*g**2*x**5/5 - x**4*(d*e*g*
*2 + e**2*f*g/2) - x**3*(7*d**2*g**2/3 + 8*d*e*f*g/3 + e**2*f**2/3) - x**2*
(4*d**3*g**2 + 7*d**2*e*f*g + 2*d*e**2*f**2)/e - x*(8*d**4*g**2 + 16*d**3*e
*f*g + 7*d**2*e**2*f**2)/e**2
```

Giac [A] time = 1.17453, size = 336, normalized size = 2.38

$$-4\left(d^5g^2e^3+2\,d^4fge^4+d^3f^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{30}\left(6\,g^2x^5e^{12}+30\,dg^2x^4e^{11}+70\,d^2g^2x^3e^{10}+120\,d^3g^2x^2e^9+240\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -4*(d^5*g^2*e^3 + 2*d^4*f*g*e^4 + d^3*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2
)) - 1/30*(6*g^2*x^5*e^12 + 30*d*g^2*x^4*e^11 + 70*d^2*g^2*x^3*e^10 + 120*d
^3*g^2*x^2*e^9 + 240*d^4*g^2*x*e^8 + 15*f*g*x^4*e^12 + 80*d*f*g*x^3*e^11 +
210*d^2*f*g*x^2*e^10 + 480*d^3*f*g*x*e^9 + 10*f^2*x^3*e^12 + 60*d*f^2*x^2*e
^11 + 210*d^2*f^2*x*e^10)*e^(-10) - 4*(d^6*g^2*e^4 + 2*d^5*f*g*e^5 + d^4*f^
2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(
d)
```

3.549 $\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2} dx$

Optimal. Leaf size=109

$$-\frac{x^2\left(4d^2g^2+6defg+e^2f^2\right)}{2e}-\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3}-\frac{dx(2dg+ef)(2dg+3ef)}{e^2}-\frac{1}{3}gx^3(3dg+2ef)-\frac{1}{4}eg^2x^4$$

```
[Out] -((d*(e*f + 2*d*g)*(3*e*f + 2*d*g)*x)/e^2) - ((e^2*f^2 + 6*d*e*f*g + 4*d^2*
g^2)*x^2)/(2*e) - (g*(2*e*f + 3*d*g)*x^3)/3 - (e*g^2*x^4)/4 - (4*d^2*(e*f +
 d*g)^2*Log[d - e*x])/e^3
```

Rubi [A] time = 0.135804, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{x^2\left(4d^2g^2+6defg+e^2f^2\right)}{2e}-\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3}-\frac{dx(2dg+ef)(2dg+3ef)}{e^2}-\frac{1}{3}gx^3(3dg+2ef)-\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d*(e*f + 2*d*g)*(3*e*f + 2*d*g)*x)/e^2) - ((e^2*f^2 + 6*d*e*f*g + 4*d^2*
g^2)*x^2)/(2*e) - (g*(2*e*f + 3*d*g)*x^3)/3 - (e*g^2*x^4)/4 - (4*d^2*(e*f +
 d*g)^2*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)^2(f+gx)^2}{d-ex}\,dx \\
&= \int \left(\frac{d(-3ef-2dg)(ef+2dg)}{e^2}-\frac{\left(e^2f^2+6defg+4d^2g^2\right)x}{e}-g(2ef+3dg)x^2-eg^2x^3-\frac{4d^2}{e^2}\right. \\
&= -\frac{d(ef+2dg)(3ef+2dg)x}{e^2}-\frac{\left(e^2f^2+6defg+4d^2g^2\right)x^2}{2e}-\frac{1}{3}g(2ef+3dg)x^3-\frac{1}{4}eg^2x^4-
\end{aligned}$$

Mathematica [A] time = 0.0500337, size = 103, normalized size = 0.94

$$-\frac{ex\left(24d^2eg(4f+gx)+48d^3g^2+12de^2\left(3f^2+3fgx+g^2x^2\right)+e^3x\left(6f^2+8fgx+3g^2x^2\right)\right)+48d^2(dg+ef)^2\log(d-}{12e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -(e*x*(48*d^3*g^2 + 24*d^2*e*g*(4*f + g*x) + 12*d*e^2*(3*f^2 + 3*f*g*x + g^
2*x^2) + e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2)) + 48*d^2*(e*f + d*g)^2*Log[d
- e*x])/(12*e^3)
```

Maple [A] time = 0.046, size = 145, normalized size = 1.3

$$-\frac{eg^2x^4}{4} - x^3dg^2 - \frac{2\,ex^3fg}{3} - 2\,\frac{x^2d^2g^2}{e} - 3\,x^2dfg - \frac{ex^2f^2}{2} - 4\,\frac{d^3g^2x}{e^2} - 8\,\frac{d^2fgx}{e} - 3\,df^2x - 4\,\frac{d^4\ln(ex-d)\,g^2}{e^3} - 8\,\frac{d^3\ln(e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/4*e*g^2*x^4-x^3*d*g^2-2/3*e*x^3*f*g-2/e*x^2*d^2*g^2-3*x^2*d*f*g-1/2*e*x^
2*f^2-4/e^2*d^3*g^2*x-8/e*d^2*f*g*x-3*d*f^2*x-4*d^4/e^3*ln(e*x-d)*g^2-8*d^3
/e^2*ln(e*x-d)*f*g-4*d^2/e*ln(e*x-d)*f^2
```

Maxima [A] time = 0.96125, size = 186, normalized size = 1.71

$$-\frac{3\,e^3g^2x^4 + 4\left(2\,e^3fg + 3\,de^2g^2\right)x^3 + 6\left(e^3f^2 + 6\,de^2fg + 4\,d^2eg^2\right)x^2 + 12\left(3\,de^2f^2 + 8\,d^2efg + 4\,d^3g^2\right)x}{12\,e^2} - \frac{4\left(d^2e^2f^2 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*e^3*g^2*x^4 + 4*(2*e^3*f*g + 3*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 6*d*e
^2*f*g + 4*d^2*e*g^2)*x^2 + 12*(3*d*e^2*f^2 + 8*d^2*e*f*g + 4*d^3*g^2)*x)/e
^2 - 4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.70013, size = 292, normalized size = 2.68

$$-\frac{3\,e^4g^2x^4 + 4\left(2\,e^4fg + 3\,de^3g^2\right)x^3 + 6\left(e^4f^2 + 6\,de^3fg + 4\,d^2e^2g^2\right)x^2 + 12\left(3\,de^3f^2 + 8\,d^2e^2fg + 4\,d^3eg^2\right)x + 48\left(d^2e^2f^2\right.}{12\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*e^4*g^2*x^4 + 4*(2*e^4*f*g + 3*d*e^3*g^2)*x^3 + 6*(e^4*f^2 + 6*d*e
^3*f*g + 4*d^2*e^2*g^2)*x^2 + 12*(3*d*e^3*f^2 + 8*d^2*e^2*f*g + 4*d^3*e*g^2
)*x + 48*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)*log(e*x - d))/e^3
```

Sympy [A] time = 0.706913, size = 116, normalized size = 1.06

$$-\frac{4d^2\left(dg + ef\right)^2\log\left(-d + ex\right)}{e^3} - \frac{eg^2x^4}{4} - x^3\left(dg^2 + \frac{2efg}{3}\right) - \frac{x^2\left(4d^2g^2 + 6defg + e^2f^2\right)}{2e} - \frac{x\left(4d^3g^2 + 8d^2efg + 3de^2f^2\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -4*d**2*(d*g + e*f)**2*log(-d + e*x)/e**3 - e*g**2*x**4/4 - x**3*(d*g**2 +
2*e*f*g/3) - x**2*(4*d**2*g**2 + 6*d*e*f*g + e**2*f**2)/(2*e) - x*(4*d**3*g
**2 + 8*d**2*e*f*g + 3*d*e**2*f**2)/e**2
```

Giac [B] time = 1.16436, size = 285, normalized size = 2.61

$$-2\left(d^4g^2e^3+2\,d^3fge^4+d^2f^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{12}\left(3\,g^2x^4e^9+12\,dg^2x^3e^8+24\,d^2g^2x^2e^7+48\,d^3g^2xe^6+8\,fgx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -2*(d^4*g^2*e^3 + 2*d^3*f*g*e^4 + d^2*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2
)) - 1/12*(3*g^2*x^4*e^9 + 12*d*g^2*x^3*e^8 + 24*d^2*g^2*x^2*e^7 + 48*d^3*g
^2*x*e^6 + 8*f*g*x^3*e^9 + 36*d*f*g*x^2*e^8 + 96*d^2*f*g*x*e^7 + 6*f^2*x^2*
e^9 + 36*d*f^2*x*e^8)*e^(-8) - 2*(d^5*g^2*e^2 + 2*d^4*f*g*e^3 + d^3*f^2*e^4
)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

3.550 $\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{2dgx(dg+ef)}{e^2}-\frac{2d(dg+ef)^2\log(d-ex)}{e^3}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}$$

```
[Out] (-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*(
e*f + d*g)^2*Log[d - e*x])/e^3
```

Rubi [A] time = 0.0600605, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 77}

$$-\frac{2dgx(dg+ef)}{e^2}-\frac{2d(dg+ef)^2\log(d-ex)}{e^3}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] (-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*(
e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{2dg(ef+dg)}{e^2}-\frac{2d(ef+dg)^2}{e^2(-d+ex)}-\frac{2dg(f+gx)}{e}-(f+gx)^2\right)dx\\ &= -\frac{2dg(ef+dg)x}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}-\frac{2d(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0348435, size = 73, normalized size = 1.12

$$-\frac{ex\left(6d^2g^2+3deg(4f+gx)+e^2\left(3f^2+3fgx+g^2x^2\right)\right)+6d(dg+ef)^2\log(d-ex)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -(e*x*(6*d^2*g^2 + 3*d*e*g*(4*f + g*x) + e^2*(3*f^2 + 3*f*g*x + g^2*x^2)) +
 6*d*(e*f + d*g)^2*Log[d - e*x])/(3*e^3)
```

Maple [A] time = 0.045, size = 110, normalized size = 1.7

$$-\frac{g^2x^3}{3}-\frac{dx^2g^2}{e}-x^2fg-2\frac{d^2g^2x}{e^2}-4\frac{dfgx}{e}-xf^2-2\frac{d^3\ln(ex-d)g^2}{e^3}-4\frac{d^2\ln(ex-d)fg}{e^2}-2\frac{d\ln(ex-d)f^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/3*g^2*x^3-1/e*x^2*d*g^2-x^2*f*g-2/e^2*d^2*g^2*x-4/e*d*f*g*x-x*f^2-2*d^3/
e^3*ln(e*x-d)*g^2-4*d^2/e^2*ln(e*x-d)*f*g-2*d/e*ln(e*x-d)*f^2
```

Maxima [A] time = 0.954936, size = 131, normalized size = 2.02

$$-\frac{e^2g^2x^3+3\left(e^2fg+deg^2\right)x^2+3\left(e^2f^2+4defg+2d^2g^2\right)x}{3e^2}-\frac{2\left(de^2f^2+2d^2efg+d^3g^2\right)\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/3*(e^2*g^2*x^3 + 3*(e^2*f*g + d*e*g^2)*x^2 + 3*(e^2*f^2 + 4*d*e*f*g + 2*
d^2*g^2)*x)/e^2 - 2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.80452, size = 204, normalized size = 3.14

$$-\frac{e^3g^2x^3+3\left(e^3fg+de^2g^2\right)x^2+3\left(e^3f^2+4de^2fg+2d^2eg^2\right)x+6\left(de^2f^2+2d^2efg+d^3g^2\right)\log(ex-d)}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/3*(e^3*g^2*x^3 + 3*(e^3*f*g + d*e^2*g^2)*x^2 + 3*(e^3*f^2 + 4*d*e^2*f*g
+ 2*d^2*e*g^2)*x + 6*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*log(e*x - d))/e^3
```

Sympy [A] time = 0.540506, size = 75, normalized size = 1.15

$$-\frac{2d\left(dg+ef\right)^2\log(-d+ex)}{e^3}-\frac{g^2x^3}{3}-\frac{x^2\left(dg^2+efg\right)}{e}-\frac{x\left(2d^2g^2+4defg+e^2f^2\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -2*d*(d*g + e*f)**2*log(-d + e*x)/e**3 - g**2*x**3/3 - x**2*(d*g**2 + e*f*g
)/e - x*(2*d**2*g**2 + 4*d*e*f*g + e**2*f**2)/e**2
```

Giac [B] time = 1.15648, size = 232, normalized size = 3.57

$$-\left(d^3g^2e + 2\,d^2fge^2 + df^2e^3\right)e^{(-4)}\log\left(\left|x^2e^2 - d^2\right|\right) - \frac{1}{3}\left(g^2x^3e^6 + 3\,dg^2x^2e^5 + 6\,d^2g^2xe^4 + 3\,fgx^2e^6 + 12\,dfgxe^5 + 3\,f^2xe^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(d^3*g^2*e + 2*d^2*f*g*e^2 + d*f^2*e^3)*e^(-4)*log(abs(x^2*e^2 - d^2)) - 1
/3*(g^2*x^3*e^6 + 3*d*g^2*x^2*e^5 + 6*d^2*g^2*x*e^4 + 3*f*g*x^2*e^6 + 12*d*
f*g*x*e^5 + 3*f^2*x*e^6)*e^(-6) - (d^4*g^2*e^2 + 2*d^3*f*g*e^3 + d^2*f^2*e^
4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

3.551 $\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=50

$$-\frac{gx(dg+ef)}{e^2}-\frac{(dg+ef)^2\log(d-ex)}{e^3}-\frac{(f+gx)^2}{2e}$$

```
[Out] -((g*(e*f + d*g)*x)/e^2) - (f + g*x)^2/(2*e) - ((e*f + d*g)^2*Log[d - e*x])
/e^3
```

Rubi [A] time = 0.0340709, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {799, 43}

$$-\frac{gx(dg+ef)}{e^2}-\frac{(dg+ef)^2\log(d-ex)}{e^3}-\frac{(f+gx)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((g*(e*f + d*g)*x)/e^2) - (f + g*x)^2/(2*e) - ((e*f + d*g)^2*Log[d - e*x])
/e^3
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{g(ef+dg)}{e^2}+\frac{(ef+dg)^2}{e^2(d-ex)}-\frac{g(f+gx)}{e}\right)dx\\ &= -\frac{g(ef+dg)x}{e^2}-\frac{(f+gx)^2}{2e}-\frac{(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0192171, size = 43, normalized size = 0.86

$$-\frac{egx(2dg+4ef+egx)+2(dg+ef)^2\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -(e*g*x*(4*e*f + 2*d*g + e*g*x) + 2*(e*f + d*g)^2*Log[d - e*x])/(2*e^3)
```

Maple [A] time = 0.044, size = 82, normalized size = 1.6

$$-\frac{g^2x^2}{2e}-\frac{g^2dx}{e^2}-2\frac{fgx}{e}-\frac{\ln(ex-d)\,d^2g^2}{e^3}-2\frac{\ln(ex-d)\,dfg}{e^2}-\frac{\ln(ex-d)\,f^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/2*g^2*x^2/e-g^2/e^2*d*x-2*g/e*f*x-1/e^3*ln(e*x-d)*d^2*g^2-2/e^2*ln(e*x-d
)*d*f*g-1/e*ln(e*x-d)*f^2
```

Maxima [A] time = 0.947378, size = 85, normalized size = 1.7

$$-\frac{eg^2x^2+2\left(2\,efg+dg^2\right)x}{2\,e^2}-\frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/2*(e*g^2*x^2 + 2*(2*e*f*g + d*g^2)*x)/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g
^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.78179, size = 136, normalized size = 2.72

$$-\frac{e^2g^2x^2+2\left(2\,e^2fg+deg^2\right)x+2\left(e^2f^2+2\,defg+d^2g^2\right)\log(ex-d)}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/2*(e^2*g^2*x^2 + 2*(2*e^2*f*g + d*e*g^2)*x + 2*(e^2*f^2 + 2*d*e*f*g + d^
2*g^2)*log(e*x - d))/e^3
```

Sympy [A] time = 0.402451, size = 46, normalized size = 0.92

$$-\frac{g^2x^2}{2e}-\frac{x\left(dg^2+2efg\right)}{e^2}-\frac{\left(dg+ef\right)^2\log(-d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -g**2*x**2/(2*e) - x*(d*g**2 + 2*e*f*g)/e**2 - (d*g + e*f)**2*log(-d + e*x)
/e**3
```

Giac [B] time = 1.19032, size = 181, normalized size = 3.62

$$-\frac{1}{2}\left(d^2g^2e+2\,dfge^2+f^2e^3\right)e^{(-4)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{2}\left(g^2x^2e^3+2\,dg^2xe^2+4\,fgxe^3\right)e^{(-4)}-\frac{\left(d^3g^2+2\,d^2fge+df^2e^2\right)e}{2\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/2*(d^2*g^2*e + 2*d*f*g*e^2 + f^2*e^3)*e^(-4)*log(abs(x^2*e^2 - d^2)) - 1
/2*(g^2*x^2*e^3 + 2*d*g^2*x*e^2 + 4*f*g*x*e^3)*e^(-4) - 1/2*(d^3*g^2 + 2*d^
2*f*g*e + d*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*a
bs(d)*e))/abs(d)
```

3.552 $\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=62

$$-\frac{(dg+ef)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}-\frac{g^2x}{e^2}$$

```
[Out] -((g^2*x)/e^2) - ((e*f + d*g)^2*Log[d - e*x])/(2*d*e^3) + ((e*f - d*g)^2*Lo
g[d + e*x])/(2*d*e^3)
```

Rubi [A] time = 0.0800188, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {702, 633, 31}

$$-\frac{(dg+ef)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}-\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] -((g^2*x)/e^2) - ((e*f + d*g)^2*Log[d - e*x])/(2*d*e^3) + ((e*f - d*g)^2*Lo
g[d + e*x])/(2*d*e^3)
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \left(-\frac{g^2}{e^2}+\frac{e^2f^2+d^2g^2+2e^2fgx}{e^2\left(d^2-e^2x^2\right)}\right)dx\\
&= -\frac{g^2x}{e^2}+\frac{\int \frac{e^2f^2+d^2g^2+2e^2fgx}{d^2-e^2x^2}\,dx}{e^2}\\
&= -\frac{g^2x}{e^2}-\frac{(ef-dg)^2\int \frac{1}{-de-e^2x}\,dx}{2de}+\frac{(ef+dg)^2\int \frac{1}{de-e^2x}\,dx}{2de}\\
&= -\frac{g^2x}{e^2}-\frac{(ef+dg)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}
\end{aligned}$$

Mathematica [A] time = 0.0221898, size = 55, normalized size = 0.89

$$\frac{\left(d^2g^2+e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)-deg\left(f\log\left(d^2-e^2x^2\right)+gx\right)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] ((e^2*f^2 + d^2*g^2)*ArcTanh[(e*x)/d] - d*e*g*(g*x + f*Log[d^2 - e^2*x^2]))
/(d*e^3)
```

Maple [A] time = 0.049, size = 107, normalized size = 1.7

$$-\frac{g^2x}{e^2}-\frac{d\ln(ex-d)g^2}{2e^3}-\frac{\ln(ex-d)fg}{e^2}-\frac{\ln(ex-d)f^2}{2de}+\frac{d\ln(ex+d)g^2}{2e^3}-\frac{\ln(ex+d)fg}{e^2}+\frac{\ln(ex+d)f^2}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -g^2*x/e^2-1/2*d/e^3*ln(e*x-d)*g^2-1/e^2*ln(e*x-d)*f*g-1/2/d/e*ln(e*x-d)*f^
2+1/2/e^3*d*ln(e*x+d)*g^2-1/e^2*ln(e*x+d)*f*g+1/2/e/d*ln(e*x+d)*f^2
```

Maxima [A] time = 0.992196, size = 111, normalized size = 1.79

$$-\frac{g^2x}{e^2}+\frac{\left(e^2f^2-2defg+d^2g^2\right)\log(ex+d)}{2de^3}-\frac{\left(e^2f^2+2defg+d^2g^2\right)\log(ex-d)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -g^2*x/e^2 + 1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d*e^3) - 1/2
*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d*e^3)
```

Fricas [A] time = 1.74218, size = 165, normalized size = 2.66

$$-\frac{2deg^2x-\left(e^2f^2-2defg+d^2g^2\right)\log(ex+d)+\left(e^2f^2+2defg+d^2g^2\right)\log(ex-d)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*e*g^2*x - (e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d) + (e^2*f^2
 + 2*d*e*f*g + d^2*g^2)*log(e*x - d))/(d*e^3)
```

Sympy [B] time = 0.733054, size = 112, normalized size = 1.81

$$-\frac{g^2x}{e^2}+\frac{\left(dg-ef\right)^2\log\left(x+\frac{2d^2fg+\frac{d\left(dg-ef\right)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}-\frac{\left(dg+ef\right)^2\log\left(x+\frac{2d^2fg-\frac{d\left(dg+ef\right)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -g**2*x/e**2 + (d*g - e*f)**2*log(x + (2*d**2*f*g + d*(d*g - e*f)**2/e)/(d*
*2*g**2 + e**2*f**2))/(2*d*e**3) - (d*g + e*f)**2*log(x + (2*d**2*f*g - d*(
d*g + e*f)**2/e)/(d**2*g**2 + e**2*f**2))/(2*d*e**3)
```

Giac [A] time = 1.12473, size = 109, normalized size = 1.76

$$-g^2xe^{(-2)}-fge^{(-2)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{\left(d^2g^2+f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d||e|\right|}{\left|2\,xe^2+2\,|d||e|\right|}\right)}{2\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -g^2*x*e^(-2) - f*g*e^(-2)*log(abs(x^2*e^2 - d^2)) - 1/2*(d^2*g^2 + f^2*e^2
)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

3.553 $\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)}\,dx$

Optimal. Leaf size=86

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

```
[Out] -(e*f - d*g)^2/(2*d*e^3*(d + e*x)) - ((e*f + d*g)^2*Log[d - e*x])/(4*d^2*e^
3) + ((e*f - d*g)*(e*f + 3*d*g)*Log[d + e*x])/(4*d^2*e^3)
```

Rubi [A] time = 0.0855682, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(2*d*e^3*(d + e*x)) - ((e*f + d*g)^2*Log[d - e*x])/(4*d^2*e^
3) + ((e*f - d*g)*(e*f + 3*d*g)*Log[d + e*x])/(4*d^2*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^2}\,dx\\ &= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{2de^2(d+ex)^2}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)}\right)dx\\ &= -\frac{(ef-dg)^2}{2de^3(d+ex)}-\frac{(ef+dg)^2\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)(ef+3dg)\log(d+ex)}{4d^2e^3}\end{aligned}$$

Mathematica [A] time = 0.0486066, size = 82, normalized size = 0.95

$$\frac{(ef-dg)((d+ex)(3dg+ef)\log(d+ex)+2d(dg-ef))-(d+ex)(dg+ef)^2\log(d-ex)}{4d^2e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] (-((e*f + d*g)^2*(d + e*x)*Log[d - e*x]) + (e*f - d*g)*(2*d*(-(e*f) + d*g)
+ (e*f + 3*d*g)*(d + e*x)*Log[d + e*x]))/(4*d^2*e^3*(d + e*x))
```

Maple [A] time = 0.054, size = 149, normalized size = 1.7

$$-\frac{\ln(ex-d)g^2}{4e^3}-\frac{\ln(ex-d)fg}{2de^2}-\frac{\ln(ex-d)f^2}{4d^2e}-\frac{3\ln(ex+d)g^2}{4e^3}+\frac{\ln(ex+d)fg}{2de^2}+\frac{\ln(ex+d)f^2}{4d^2e}-\frac{g^2d}{2e^3(ex+d)}+\frac{}{e^2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x)
```

```
[Out] -1/4/e^3*ln(e*x-d)*g^2-1/2/d/e^2*ln(e*x-d)*f*g-1/4/d^2/e*ln(e*x-d)*f^2-3/4/
e^3*ln(e*x+d)*g^2+1/2/d/e^2*ln(e*x+d)*f*g+1/4/d^2/e*ln(e*x+d)*f^2-1/2*d/e^3
/(e*x+d)*g^2+1/e^2/(e*x+d)*f*g-1/2/d/e/(e*x+d)*f^2
```

Maxima [A] time = 0.985877, size = 153, normalized size = 1.78

$$-\frac{e^2f^2-2\,defg+d^2g^2}{2\left(de^4x+d^2e^3\right)}+\frac{\left(e^2f^2+2\,defg-3\,d^2g^2\right)\log(ex+d)}{4\,d^2e^3}-\frac{\left(e^2f^2+2\,defg+d^2g^2\right)\log(ex-d)}{4\,d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(d*e^4*x + d^2*e^3) + 1/4*(e^2*f^2 + 2
*d*e*f*g - 3*d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 + 2*d*e*f*g + d
^2*g^2)*log(e*x - d)/(d^2*e^3)
```

Fricas [B] time = 1.83723, size = 343, normalized size = 3.99

$$-\frac{2\,de^2f^2-4\,d^2efg+2\,d^3g^2-\left(de^2f^2+2\,d^2efg-3\,d^3g^2+\left(e^3f^2+2\,de^2fg-3\,d^2eg^2\right)x\right)\log(ex+d)+\left(de^2f^2+2\,d^2efg\right.}{4\left(d^2e^4x+d^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*e^2*f^2 - 4*d^2*e*f*g + 2*d^3*g^2 - (d*e^2*f^2 + 2*d^2*e*f*g - 3*
d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x + d) + (d*e^2*f^
2 + 2*d^2*e*f*g + d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x
- d))/(d^2*e^4*x + d^3*e^3)
```

Sympy [B] time = 1.14778, size = 182, normalized size = 2.12

$$-\frac{d^2g^2-2defg+e^2f^2}{2d^2e^3+2de^4x}-\frac{\left(dg-ef\right)\left(3dg+ef\right)\log\left(x+\frac{-2d^3g^2+d\left(dg-ef\right)\left(3dg+ef\right)}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3}-\frac{\left(dg+ef\right)^2\log\left(x+\frac{-2d^3g^2+d\left(dg+ef\right)^2}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2),x)
```

```
[Out] -(d**2*g**2 - 2*d*e*f*g + e**2*f**2)/(2*d**2*e**3 + 2*d*e**4*x) - (d*g - e*
f)*(3*d*g + e*f)*log(x + (-2*d**3*g**2 + d*(d*g - e*f)*(3*d*g + e*f))/(d**2
*e*g**2 - 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3) - (d*g + e*f)**2*log(x +
 (-2*d**3*g**2 + d*(d*g + e*f)**2)/(d**2*e*g**2 - 2*d*e**2*f*g - e**3*f**2)
)/(4*d**2*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.554 $\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)}\,dx$

Optimal. Leaf size=87

$$-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

```
[Out] -(e*f - d*g)^2/(4*d*e^3*(d + e*x)^2) - ((e*f - d*g)*(e*f + 3*d*g))/(4*d^2*e
^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rubi [A] time = 0.0966412, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(4*d*e^3*(d + e*x)^2) - ((e*f - d*g)*(e*f + 3*d*g))/(4*d^2*e
^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)}\,dx = \int \frac{(f+gx)^2}{(d-ex)(d+ex)^3}\,dx$$
$$= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^3} + \frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^2} + \frac{(ef+dg)^2}{4d^2e^2\left(d^2-e^2x^2\right)}\right)dx$$
$$= -\frac{(ef-dg)^2}{4de^3(d+ex)^2} - \frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)} + \frac{(ef+dg)^2\int\frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2}$$
$$= -\frac{(ef-dg)^2}{4de^3(d+ex)^2} - \frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)} + \frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

Mathematica [A] time = 0.07658, size = 87, normalized size = 1.

$$\frac{\frac{2d(dg-ef)\left(2d^2g+de(2f+3gx)+e^2fx\right)}{(d+ex)^2} + (dg+ef)^2(-\log(d-ex)) + (dg+ef)^2\log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] ((2*d*(-(e*f) + d*g)*(2*d^2*g + e^2*f*x + d*e*(2*f + 3*g*x)))/(d + e*x)^2 -
 (e*f + d*g)^2*Log[d - e*x] + (e*f + d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

Maple [B] time = 0.055, size = 206, normalized size = 2.4

$$-\frac{\ln(ex-d)\,g^2}{8\,de^3} - \frac{\ln(ex-d)\,fg}{4\,d^2e^2} - \frac{\ln(ex-d)\,f^2}{8\,d^3e} + \frac{3\,g^2}{4\,e^3\,(ex+d)} - \frac{fg}{2\,de^2\,(ex+d)} - \frac{f^2}{4\,d^2e\,(ex+d)} - \frac{g^2d}{4\,e^3\,(ex+d)^2} + \frac{}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x)
```

```
[Out] -1/8/e^3/d*ln(e*x-d)*g^2-1/4/e^2/d^2*ln(e*x-d)*f*g-1/8/e/d^3*ln(e*x-d)*f^2+
3/4/e^3/(e*x+d)*g^2-1/2/d/e^2/(e*x+d)*f*g-1/4/d^2/e/(e*x+d)*f^2-1/4*d/e^3/(
e*x+d)^2*g^2+1/2/e^2/(e*x+d)^2*f*g-1/4/d/e/(e*x+d)^2*f^2+1/8/e^3/d*ln(e*x+d
)*g^2+1/4/e^2/d^2*ln(e*x+d)*f*g+1/8/e/d^3*ln(e*x+d)*f^2
```

Maxima [A] time = 0.972392, size = 201, normalized size = 2.31

$$-\frac{2\,de^2f^2 - 2\,d^3g^2 + \left(e^3f^2 + 2\,de^2fg - 3\,d^2eg^2\right)x}{4\left(d^2e^5x^2 + 2\,d^3e^4x + d^4e^3\right)} + \frac{\left(e^2f^2 + 2\,defg + d^2g^2\right)\log(ex+d)}{8\,d^3e^3} - \frac{\left(e^2f^2 + 2\,defg + d^2g^2\right)\log(}{8\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*e^2*f^2 - 2*d^3*g^2 + (e^3*f^2 + 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d
^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*l
og(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d
```

```
^3*e^3)
```

Fricas [B] time = 1.82193, size = 548, normalized size = 6.3

$$-\frac{4\,d^2e^2f^2-4\,d^4g^2+2\left(de^3f^2+2\,d^2e^2fg-3\,d^3eg^2\right)x-\left(d^2e^2f^2+2\,d^3efg+d^4g^2+\left(e^4f^2+2\,de^3fg+d^2e^2g^2\right)x^2+2\left(de^3\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g - 3*d^3*e*g^
2)*x - (d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*
e^2*g^2)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d) +
(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2
)*x^2 + 2*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3*e^5
*x^2 + 2*d^4*e^4*x + d^5*e^3)
```

Sympy [B] time = 1.11523, size = 185, normalized size = 2.13

$$\frac{2d^3g^2-2de^2f^2+x\left(3d^2eg^2-2de^2fg-e^3f^2\right)}{4d^4e^3+8d^3e^4x+4d^2e^5x^2}-\frac{\left(dg+ef\right)^2\log\left(-\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right)}+x\right)}{8d^3e^3}+\frac{\left(dg+ef\right)^2\log\left(\frac{d\left(dg+ef\right)^2}{e\left(d^2g^2+2defg+e^2f^2\right.}\right.}{8d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2),x)
```

```
[Out] (2*d**3*g**2 - 2*d*e**2*f**2 + x*(3*d**2*e*g**2 - 2*d*e**2*f*g - e**3*f**2)
)/(4*d**4*e**3 + 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g + e*f)**2*log(-d*
(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3) +
 (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)
) + x)/(8*d**3*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.555 $\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx$

Optimal. Leaf size=113

$$-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

```
[Out] -(e*f - d*g)^2/(6*d*e^3*(d + e*x)^3) - ((e*f - d*g)*(e*f + 3*d*g))/(8*d^2*e
^3*(d + e*x)^2) - (e*f + d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcT
anh[(e*x)/d])/(8*d^4*e^3)
```

Rubi [A] time = 0.115919, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(6*d*e^3*(d + e*x)^3) - ((e*f - d*g)*(e*f + 3*d*g))/(8*d^2*e
^3*(d + e*x)^2) - (e*f + d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcT
anh[(e*x)/d])/(8*d^4*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)}\,dx = \int \frac{(f+gx)^2}{(d-ex)(d+ex)^4}\,dx$$
$$= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^4} + \frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^3} + \frac{(ef+dg)^2}{8d^3e^2(d+ex)^2} + \frac{(ef+dg)^2}{8d^3e^2\left(d^2-e^2x^2\right)}\right)dx$$
$$= -\frac{(ef-dg)^2}{6de^3(d+ex)^3} - \frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2} - \frac{(ef+dg)^2}{8d^3e^3(d+ex)} + \frac{(ef+dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2}$$
$$= -\frac{(ef-dg)^2}{6de^3(d+ex)^3} - \frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2} - \frac{(ef+dg)^2}{8d^3e^3(d+ex)} + \frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Mathematica [A] time = 0.0583331, size = 122, normalized size = 1.08

$$\frac{\frac{6d^2\left(3d^2g^2-2defg-e^2f^2\right)}{(d+ex)^2} - \frac{8d^3(ef-dg)^2}{(d+ex)^3} - \frac{6d(dg+ef)^2}{d+ex} - 3(dg+ef)^2\log(d-ex) + 3(dg+ef)^2\log(d+ex)}{48d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] ((-8*d^3*(e*f - d*g)^2)/(d + e*x)^3 + (6*d^2*(-(e^2*f^2) - 2*d*e*f*g + 3*d^
2*g^2))/(d + e*x)^2 - (6*d*(e*f + d*g)^2)/(d + e*x) - 3*(e*f + d*g)^2*Log[d
 - e*x] + 3*(e*f + d*g)^2*Log[d + e*x])/(48*d^4*e^3)
```

Maple [B] time = 0.054, size = 259, normalized size = 2.3

$$-\frac{\ln(ex-d)\,g^2}{16\,d^2e^3} - \frac{\ln(ex-d)\,fg}{8\,d^3e^2} - \frac{\ln(ex-d)\,f^2}{16\,d^4e} + \frac{3\,g^2}{8\,e^3\,(ex+d)^2} - \frac{fg}{4\,de^2\,(ex+d)^2} - \frac{f^2}{8\,d^2e\,(ex+d)^2} - \frac{g^2d}{6\,e^3\,(ex+d)^3} + \frac{}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x)
```

```
[Out] -1/16/e^3/d^2*ln(e*x-d)*g^2-1/8/e^2/d^3*ln(e*x-d)*f*g-1/16/e/d^4*ln(e*x-d)*
f^2+3/8/e^3/(e*x+d)^2*g^2-1/4/d/e^2/(e*x+d)^2*f*g-1/8/d^2/e/(e*x+d)^2*f^2-1
/6*d/e^3/(e*x+d)^3*g^2+1/3/e^2/(e*x+d)^3*f*g-1/6/d/e/(e*x+d)^3*f^2+1/16/e^3
/d^2*ln(e*x+d)*g^2+1/8/e^2/d^3*ln(e*x+d)*f*g+1/16/e/d^4*ln(e*x+d)*f^2-1/8/d
/e^3/(e*x+d)*g^2-1/4/d^2/e^2/(e*x+d)*f*g-1/8/d^3/e/(e*x+d)*f^2
```

Maxima [A] time = 0.992291, size = 278, normalized size = 2.46

$$-\frac{10\,d^2e^2f^2 + 4\,d^3efg - 2\,d^4g^2 + 3\left(e^4f^2 + 2\,de^3fg + d^2e^2g^2\right)x^2 + 3\left(3\,de^3f^2 + 6\,d^2e^2fg - d^3eg^2\right)x}{24\left(d^3e^6x^3 + 3\,d^4e^5x^2 + 3\,d^5e^4x + d^6e^3\right)} + \frac{\left(e^2f^2 + 2\,defg + d^2\right.}{16\,d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/24*(10*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 + 3*(e^4*f^2 + 2*d*e^3*f*g
+ d^2*e^2*g^2)*x^2 + 3*(3*d*e^3*f^2 + 6*d^2*e^2*f*g - d^3*e*g^2)*x)/(d^3*e^
```

```
6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3) + 1/16*(e^2*f^2 + 2*d*e*f*g
+ d^2*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*lo
g(e*x - d)/(d^4*e^3)
```

Fricas [B] time = 1.80371, size = 807, normalized size = 7.14

$$-\frac{20\,d^3e^2f^2+8\,d^4efg-4\,d^5g^2+6\left(de^4f^2+2\,d^2e^3fg+d^3e^2g^2\right)x^2+6\left(3\,d^2e^3f^2+6\,d^3e^2fg-d^4eg^2\right)x-3\left(d^3e^2f^2+2\,\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/48*(20*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 + 6*(d*e^4*f^2 + 2*d^2*e^3*
f*g + d^3*e^2*g^2)*x^2 + 6*(3*d^2*e^3*f^2 + 6*d^3*e^2*f*g - d^4*e*g^2)*x -
3*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g
^2)*x^3 + 3*(d*e^4*f^2 + 2*d^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2
+ 2*d^3*e^2*f*g + d^4*e*g^2)*x)*log(e*x + d) + 3*(d^3*e^2*f^2 + 2*d^4*e*f*g
 + d^5*g^2 + (e^5*f^2 + 2*d*e^4*f*g + d^2*e^3*g^2)*x^3 + 3*(d*e^4*f^2 + 2*d
^2*e^3*f*g + d^3*e^2*g^2)*x^2 + 3*(d^2*e^3*f^2 + 2*d^3*e^2*f*g + d^4*e*g^2)
*x)*log(e*x - d))/(d^4*e^6*x^3 + 3*d^5*e^5*x^2 + 3*d^6*e^4*x + d^7*e^3)
```

Sympy [B] time = 1.40659, size = 248, normalized size = 2.19

$$-\frac{-2d^4g^2+4d^3efg+10d^2e^2f^2+x^2\left(3d^2e^2g^2+6de^3fg+3e^4f^2\right)+x\left(-3d^3eg^2+18d^2e^2fg+9de^3f^2\right)}{24d^6e^3+72d^5e^4x+72d^4e^5x^2+24d^3e^6x^3}-\frac{\left(dg+ef\right)^2\log\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**4*g**2 + 4*d**3*e*f*g + 10*d**2*e**2*f**2 + x**2*(3*d**2*e**2*g**2
+ 6*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 18*d**2*e**2*f*g + 9*d*
e**3*f**2))/(24*d**6*e**3 + 72*d**5*e**4*x + 72*d**4*e**5*x**2 + 24*d**3*e*
*6*x**3) - (d*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g +
 e**2*f**2)) + x)/(16*d**4*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(
d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(16*d**4*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.556 $\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx$

Optimal. Leaf size=139

$$-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

```
[Out] -(e*f - d*g)^2/(8*d*e^3*(d + e*x)^4) - ((e*f - d*g)*(e*f + 3*d*g))/(12*d^2*
e^3*(d + e*x)^3) - (e*f + d*g)^2/(16*d^3*e^3*(d + e*x)^2) - (e*f + d*g)^2/(
16*d^4*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(16*d^5*e^3)
```

Rubi [A] time = 0.133073, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}+\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -(e*f - d*g)^2/(8*d*e^3*(d + e*x)^4) - ((e*f - d*g)*(e*f + 3*d*g))/(12*d^2*
e^3*(d + e*x)^3) - (e*f + d*g)^2/(16*d^3*e^3*(d + e*x)^2) - (e*f + d*g)^2/(
16*d^4*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(16*d^5*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)}\,dx=\int \frac{(f+gx)^2}{(d-ex)(d+ex)^5}\,dx$$
$$=\int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^5}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^4}+\frac{(ef+dg)^2}{8d^3e^2(d+ex)^3}+\frac{(ef+dg)^2}{16d^4e^2(d+ex)^2}+\frac{(ef}{16d^4e^2(}\right.$$
$$=-\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+dg)^2}{16}$$
$$=-\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+dg)^2}{16}$$

Mathematica [A] time = 0.0940853, size = 142, normalized size = 1.02

$$-\frac{\frac{8d^3\left(-3d^2g^2+2defg+e^2f^2\right)}{(d+ex)^3}+\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{6d^2(dg+ef)^2}{(d+ex)^2}+\frac{6d(dg+ef)^2}{d+ex}+3(dg+ef)^2\log(d-ex)-3(dg+ef)^2\log(d+ex)}{96d^5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -((12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(e^2*f^2 + 2*d*e*f*g - 3*d^2*
g^2))/(d + e*x)^3 + (6*d^2*(e*f + d*g)^2)/(d + e*x)^2 + (6*d*(e*f + d*g)^2)
/(d + e*x) + 3*(e*f + d*g)^2*Log[d - e*x] - 3*(e*f + d*g)^2*Log[d + e*x])/(
96*d^5*e^3)
```

Maple [B] time = 0.056, size = 312, normalized size = 2.2

$$-\frac{\ln(ex-d)\,g^2}{32\,d^3e^3}-\frac{\ln(ex-d)\,fg}{16\,d^4e^2}-\frac{\ln(ex-d)\,f^2}{32\,d^5e}+\frac{g^2}{4\,e^3\,(ex+d)^3}-\frac{fg}{6\,de^2\,(ex+d)^3}-\frac{f^2}{12\,d^2e\,(ex+d)^3}-\frac{g^2d}{8\,e^3\,(ex+d)^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x)
```

```
[Out] -1/32/e^3/d^3*ln(e*x-d)*g^2-1/16/e^2/d^4*ln(e*x-d)*f*g-1/32/e/d^5*ln(e*x-d)
*f^2+1/4/e^3/(e*x+d)^3*g^2-1/6/d/e^2/(e*x+d)^3*f*g-1/12/d^2/e/(e*x+d)^3*f^2
-1/8*d/e^3/(e*x+d)^4*g^2+1/4/e^2/(e*x+d)^4*f*g-1/8/d/e/(e*x+d)^4*f^2+1/32/e
^3/d^3*ln(e*x+d)*g^2+1/16/e^2/d^4*ln(e*x+d)*f*g+1/32/e/d^5*ln(e*x+d)*f^2-1/
16/d^2/e^3/(e*x+d)*g^2-1/8/d^3/e^2/(e*x+d)*f*g-1/16/d^4/e/(e*x+d)*f^2-1/16/
d/e^3/(e*x+d)^2*g^2-1/8/d^2/e^2/(e*x+d)^2*f*g-1/16/d^3/e/(e*x+d)^2*f^2
```

Maxima [A] time = 1.04339, size = 319, normalized size = 2.29

$$-\frac{16\,d^3ef^2+8\,d^4fg+3\left(e^4f^2+2\,de^3fg+d^2e^2g^2\right)x^3+12\left(de^3f^2+2\,d^2e^2fg+d^3eg^2\right)x^2+\left(19\,d^2e^2f^2+38\,d^3efg+3\,d\right.}{48\left(d^4e^6x^4+4\,d^5e^5x^3+6\,d^6e^4x^2+4\,d^7e^3x+d^8e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/48*(16*d^3*e*f^2 + 8*d^4*f*g + 3*(e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2)*x
^3 + 12*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x^2 + (19*d^2*e^2*f^2 + 38*
d^3*e*f*g + 3*d^4*g^2)*x)/(d^4*e^6*x^4 + 4*d^5*e^5*x^3 + 6*d^6*e^4*x^2 + 4*
d^7*e^3*x + d^8*e^2) + 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d
^5*e^3) - 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

Fricas [B] time = 1.75234, size = 1031, normalized size = 7.42

$$-\frac{32\,d^4e^2f^2+16\,d^5efg+6\left(de^5f^2+2\,d^2e^4fg+d^3e^3g^2\right)x^3+24\left(d^2e^4f^2+2\,d^3e^3fg+d^4e^2g^2\right)x^2+2\left(19\,d^3e^3f^2+38\,d^4e^2f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/96*(32*d^4*e^2*f^2 + 16*d^5*e*f*g + 6*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e
^3*g^2)*x^3 + 24*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 2*(19*d^
3*e^3*f^2 + 38*d^4*e^2*f*g + 3*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + 2*d^5*e*f*g
+ d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x^4 + 4*(d*e^5*f^2 + 2*d^
2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2
)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)*x)*log(e*x + d) + 3*(d^
4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x
^4 + 4*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d
^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)
*x)*log(e*x - d))/(d^5*e^7*x^4 + 4*d^6*e^6*x^3 + 6*d^7*e^5*x^2 + 4*d^8*e^4*
x + d^9*e^3)
```

Sympy [B] time = 1.69364, size = 282, normalized size = 2.03

$$-\frac{8d^4fg+16d^3ef^2+x^3\left(3d^2e^2g^2+6de^3fg+3e^4f^2\right)+x^2\left(12d^3eg^2+24d^2e^2fg+12de^3f^2\right)+x\left(3d^4g^2+38d^3efg+19d^2e\right.}{48d^8e^2+192d^7e^3x+288d^6e^4x^2+192d^5e^5x^3+48d^4e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2),x)
```

```
[Out] -(8*d**4*f*g + 16*d**3*e*f**2 + x**3*(3*d**2*e**2*g**2 + 6*d*e**3*f*g + 3*e
**4*f**2) + x**2*(12*d**3*e*g**2 + 24*d**2*e**2*f*g + 12*d*e**3*f**2) + x*(
3*d**4*g**2 + 38*d**3*e*f*g + 19*d**2*e**2*f**2))/(48*d**8*e**2 + 192*d**7*
e**3*x + 288*d**6*e**4*x**2 + 192*d**5*e**5*x**3 + 48*d**4*e**6*x**4) - (d*
g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) +
 x)/(32*d**5*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*
d*e*f*g + e**2*f**2)) + x)/(32*d**5*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.557 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right)+\frac{d^2x^2\left(80d^2g^2+98defg+23e^2f^2\right)}{2e}+\frac{d^3x\left(112d^2g^2\right.}{}$$

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x)) +
 (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*Log[d - e*x])/e^3
```

Rubi [A] time = 0.28372, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right)+\frac{d^2x^2\left(80d^2g^2+98defg+23e^2f^2\right)}{2e}+\frac{d^3x\left(112d^2g^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x)) +
 (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(d+ex)^5(f+gx)^2}{(d-ex)^2}\,dx$$

$$=\int \left(\frac{d^3\left(49e^2f^2+160defg+112d^2g^2\right)}{e^2}+\frac{d^2\left(23e^2f^2+98defg+80d^2g^2\right)x}{e}+d\left(7e^2f^2+46de\right.\right.$$

$$=\frac{d^3\left(49e^2f^2+160defg+112d^2g^2\right)x}{e^2}+\frac{d^2\left(23e^2f^2+98defg+80d^2g^2\right)x^2}{2e}+\frac{1}{3}d\left(7e^2f^2+46de\right.$$

Mathematica [A] time = 0.126223, size = 226, normalized size = 1.04

$$\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right)+\frac{d^2x^2\left(80d^2g^2+98defg+23e^2f^2\right)}{2e}+\frac{d^3x\left(112d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 - (32*d^5*(e*f + d*g)^2)/(e^3*(-d + e*x))
+ (16*d^4*(5*e^2*f^2 + 14*d*e*f*g + 9*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A] time = 0.053, size = 286, normalized size = 1.3

$$\frac{e^3g^2x^6}{6}+\frac{7e^2x^5dg^2}{5}+\frac{2e^3x^5fg}{5}+\frac{23ex^4d^2g^2}{4}+\frac{7e^2x^4dfg}{2}+\frac{e^3x^4f^2}{4}+\frac{49x^3d^3g^2}{3}+\frac{46ex^3d^2fg}{3}+\frac{7e^2x^3df^2}{3}+40\frac{x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/6*e^3*g^2*x^6+7/5*e^2*x^5*d*g^2+2/5*e^3*x^5*f*g+23/4*e*x^4*d^2*g^2+7/2*e^
2*x^4*d*f*g+1/4*e^3*x^4*f^2+49/3*x^3*d^3*g^2+46/3*e*x^3*d^2*f*g+7/3*e^2*x^3
*d*f^2+40/e*x^2*d^4*g^2+49*x^2*d^3*f*g+23/2*e*x^2*d^2*f^2+112/e^2*d^5*g^2*x
+160/e*d^4*f*g*x+49*d^3*f^2*x+144*d^6/e^3*ln(e*x-d)*g^2+224*d^5/e^2*ln(e*x-
d)*f*g+80*d^4/e*ln(e*x-d)*f^2-32*d^7/e^3/(e*x-d)*g^2-64*d^6/e^2/(e*x-d)*f*g
-32*d^5/e/(e*x-d)*f^2
```

Maxima [A] time = 0.990392, size = 348, normalized size = 1.6

$$-\frac{32\left(d^5e^2f^2+2d^6efg+d^7g^2\right)}{e^4x-de^3}+\frac{10e^5g^2x^6+12\left(2e^5fg+7de^4g^2\right)x^5+15\left(e^5f^2+14de^4fg+23d^2e^3g^2\right)x^4+20\left(7de\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -32*(d^5*e^2*f^2 + 2*d^6*e*f*g + d^7*g^2)/(e^4*x - d*e^3) + 1/60*(10*e^5*g^
2*x^6 + 12*(2*e^5*f*g + 7*d*e^4*g^2)*x^5 + 15*(e^5*f^2 + 14*d*e^4*f*g + 23*
d^2*e^3*g^2)*x^4 + 20*(7*d*e^4*f^2 + 46*d^2*e^3*f*g + 49*d^3*e^2*g^2)*x^3 +
 30*(23*d^2*e^3*f^2 + 98*d^3*e^2*f*g + 80*d^4*e*g^2)*x^2 + 60*(49*d^3*e^2*f
^2 + 160*d^4*e*f*g + 112*d^5*g^2)*x)/e^2 + 16*(5*d^4*e^2*f^2 + 14*d^5*e*f*g
 + 9*d^6*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.70984, size = 716, normalized size = 3.28

$$\frac{10e^7g^2x^7-1920d^5e^2f^2-3840d^6efg-1920d^7g^2+2\left(12e^7fg+37de^6g^2\right)x^6+3\left(5e^7f^2+62de^6fg+87d^2e^5g^2\right)x^5+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*e^7*g^2*x^7 - 1920*d^5*e^2*f^2 - 3840*d^6*e*f*g - 1920*d^7*g^2 + 2
*(12*e^7*f*g + 37*d*e^6*g^2)*x^6 + 3*(5*e^7*f^2 + 62*d*e^6*f*g + 87*d^2*e^5
*g^2)*x^5 + 5*(25*d*e^6*f^2 + 142*d^2*e^5*f*g + 127*d^3*e^4*g^2)*x^4 + 10*(
55*d^2*e^5*f^2 + 202*d^3*e^4*f*g + 142*d^4*e^3*g^2)*x^3 + 90*(25*d^3*e^4*f^
2 + 74*d^4*e^3*f*g + 48*d^5*e^2*g^2)*x^2 - 60*(49*d^4*e^3*f^2 + 160*d^5*e^2
*f*g + 112*d^6*e*g^2)*x - 960*(5*d^5*e^2*f^2 + 14*d^6*e*f*g + 9*d^7*g^2 - (
5*d^4*e^3*f^2 + 14*d^5*e^2*f*g + 9*d^6*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e
^3)
```

Sympy [A] time = 1.20428, size = 255, normalized size = 1.17

$$\frac{16d^4\left(dg+ef\right)\left(9dg+5ef\right)\log\left(-d+ex\right)}{e^3}+\frac{e^3g^2x^6}{6}+x^5\left(\frac{7de^2g^2}{5}+\frac{2e^3fg}{5}\right)+x^4\left(\frac{23d^2eg^2}{4}+\frac{7de^2fg}{2}+\frac{e^3f^2}{4}\right)+x^3\left(\frac{49d^3}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 16*d**4*(d*g + e*f)*(9*d*g + 5*e*f)*log(-d + e*x)/e**3 + e**3*g**2*x**6/6 +
 x**5*(7*d*e**2*g**2/5 + 2*e**3*f*g/5) + x**4*(23*d**2*e*g**2/4 + 7*d*e**2*
f*g/2 + e**3*f**2/4) + x**3*(49*d**3*g**2/3 + 46*d**2*e*f*g/3 + 7*d*e**2*f*
*2/3) - (32*d**7*g**2 + 64*d**6*e*f*g + 32*d**5*e**2*f**2)/(-d*e**3 + e**4*
x) + x**2*(80*d**4*g**2 + 98*d**3*e*f*g + 23*d**2*e**2*f**2)/(2*e) + x*(112
*d**5*g**2 + 160*d**4*e*f*g + 49*d**3*e**2*f**2)/e**2
```

Giac [A] time = 1.16513, size = 495, normalized size = 2.27

$$8\left(9\,d^6g^2e^7+14\,d^5fge^8+5\,d^4f^2e^9\right)e^{(-10)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{60}\left(10\,g^2x^6e^{27}+84\,dg^2x^5e^{26}+345\,d^2g^2x^4e^{25}+980\,d^3g^2x^3e^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 8*(9*d^6*g^2*e^7 + 14*d^5*f*g*e^8 + 5*d^4*f^2*e^9)*e^(-10)*log(abs(x^2*e^2
- d^2)) + 1/60*(10*g^2*x^6*e^27 + 84*d*g^2*x^5*e^26 + 345*d^2*g^2*x^4*e^25
+ 980*d^3*g^2*x^3*e^24 + 2400*d^4*g^2*x^2*e^23 + 6720*d^5*g^2*x*e^22 + 24*f
*g*x^5*e^27 + 210*d*f*g*x^4*e^26 + 920*d^2*f*g*x^3*e^25 + 2940*d^3*f*g*x^2*
e^24 + 9600*d^4*f*g*x*e^23 + 15*f^2*x^4*e^27 + 140*d*f^2*x^3*e^26 + 690*d^2
*f^2*x^2*e^25 + 2940*d^3*f^2*x*e^24)*e^(-24) + 8*(9*d^7*g^2*e^6 + 14*d^6*f*
g*e^7 + 5*d^5*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2
*abs(d)*e))/abs(d) - 32*(d^8*g^2*e^7 + 2*d^7*f*g*e^8 + d^6*f^2*e^9 + (d^7*g
^2*e^8 + 2*d^6*f*g*e^9 + d^5*f^2*e^10)*x)*e^(-10)/(x^2*e^2 - d^2)
```

3.558 $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=177

$$\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3e^2f^2\right)}{e}+\frac{d^2x\left(48d^2g^2+64defg+17e^2f^2\right)}{e^2}+\frac{16d^4(dg+ef)}{e^3(d-ex)}$$

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3*(d
 - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rubi [A] time = 0.232196, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3e^2f^2\right)}{e}+\frac{d^2x\left(48d^2g^2+64defg+17e^2f^2\right)}{e^2}+\frac{16d^4(dg+ef)}{e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3*(d
 - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^2}\,dx$$

$$=\int \left(\frac{d^2\left(17e^2f^2+64defg+48d^2g^2\right)}{e^2}+\frac{2d\left(3e^2f^2+17defg+16d^2g^2\right)x}{e}+\left(e^2f^2+12defg\right.\right.$$

$$=\frac{d^2\left(17e^2f^2+64defg+48d^2g^2\right)x}{e^2}+\frac{d\left(3e^2f^2+17defg+16d^2g^2\right)x^2}{e}+\frac{1}{3}\left(e^2f^2+12defg\right.$$

Mathematica [A] time = 0.120351, size = 185, normalized size = 1.05

$$\frac{1}{3}x^3\left(17d^2g^2+12defg+e^2f^2\right)+\frac{dx^2\left(16d^2g^2+17defg+3e^2f^2\right)}{e}+\frac{d^2x\left(48d^2g^2+64defg+17e^2f^2\right)}{e^2}+\frac{32d^3\left(2d^2g^2+3d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 - (16*d^4*(e*f + d*g)^2)/(e^3*(-
d + e*x)) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A] time = 0.049, size = 245, normalized size = 1.4

$$\frac{e^2g^2x^5}{5}+\frac{3\,ex^4dg^2}{2}+\frac{e^2x^4fg}{2}+\frac{17\,x^3d^2g^2}{3}+4\,ex^3dfg+\frac{e^2x^3f^2}{3}+16\,\frac{x^2d^3g^2}{e}+17\,x^2d^2fg+3\,ex^2df^2+48\,\frac{d^4g^2x}{e^2}+64\,\frac{d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/5*e^2*g^2*x^5+3/2*e*x^4*d*g^2+1/2*e^2*x^4*f*g+17/3*x^3*d^2*g^2+4*e*x^3*d*
f*g+1/3*e^2*x^3*f^2+16/e*x^2*d^3*g^2+17*x^2*d^2*f*g+3*e*x^2*d*f^2+48/e^2*d^
4*g^2*x+64/e*d^3*f*g*x+17*d^2*f^2*x+64*d^5/e^3*ln(e*x-d)*g^2+96*d^4/e^2*ln(
e*x-d)*f*g+32*d^3/e*ln(e*x-d)*f^2-16*d^6/e^3/(e*x-d)*g^2-32*d^5/e^2/(e*x-d)
*f*g-16*d^4/e/(e*x-d)*f^2
```

Maxima [A] time = 0.98052, size = 294, normalized size = 1.66

$$-\frac{16\left(d^4e^2f^2+2\,d^5efg+d^6g^2\right)}{e^4x-de^3}+\frac{6\,e^4g^2x^5+15\left(e^4fg+3\,de^3g^2\right)x^4+10\left(e^4f^2+12\,de^3fg+17\,d^2e^2g^2\right)x^3+30\left(3\,de^3f^2+\right.}{30\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -16*(d^4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2)/(e^4*x - d*e^3) + 1/30*(6*e^4*g^2
*x^5 + 15*(e^4*f*g + 3*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 12*d*e^3*f*g + 17*d^2
*e^2*g^2)*x^3 + 30*(3*d*e^3*f^2 + 17*d^2*e^2*f*g + 16*d^3*e*g^2)*x^2 + 30*(
17*d^2*e^2*f^2 + 64*d^3*e*f*g + 48*d^4*g^2)*x)/e^2 + 32*(d^3*e^2*f^2 + 3*d^
4*e*f*g + 2*d^5*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.76268, size = 612, normalized size = 3.46

$$\frac{6\,e^6g^2x^6-480\,d^4e^2f^2-960\,d^5efg-480\,d^6g^2+3\left(5\,e^6fg+13\,de^5g^2\right)x^5+5\left(2\,e^6f^2+21\,de^5fg+25\,d^2e^4g^2\right)x^4+10\left(8\,de^{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/30*(6*e^6*g^2*x^6 - 480*d^4*e^2*f^2 - 960*d^5*e*f*g - 480*d^6*g^2 + 3*(5*
e^6*f*g + 13*d*e^5*g^2)*x^5 + 5*(2*e^6*f^2 + 21*d*e^5*f*g + 25*d^2*e^4*g^2)
*x^4 + 10*(8*d*e^5*f^2 + 39*d^2*e^4*f*g + 31*d^3*e^3*g^2)*x^3 + 30*(14*d^2*
e^4*f^2 + 47*d^3*e^3*f*g + 32*d^4*e^2*g^2)*x^2 - 30*(17*d^3*e^3*f^2 + 64*d^
4*e^2*f*g + 48*d^5*e*g^2)*x - 960*(d^4*e^2*f^2 + 3*d^5*e*f*g + 2*d^6*g^2 -
(d^3*e^3*f^2 + 3*d^4*e^2*f*g + 2*d^5*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3
)
```

Sympy [A] time = 1.08256, size = 204, normalized size = 1.15

$$\frac{32d^3\left(dg+ef\right)\left(2dg+ef\right)\log\left(-d+ex\right)}{e^3}+\frac{e^2g^2x^5}{5}+x^4\left(\frac{3deg^2}{2}+\frac{e^2fg}{2}\right)+x^3\left(\frac{17d^2g^2}{3}+4defg+\frac{e^2f^2}{3}\right)-\frac{16d^6g^2+32\ldots}{-\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 32*d**3*(d*g + e*f)*(2*d*g + e*f)*log(-d + e*x)/e**3 + e**2*g**2*x**5/5 + x
**4*(3*d*e*g**2/2 + e**2*f*g/2) + x**3*(17*d**2*g**2/3 + 4*d*e*f*g + e**2*f
**2/3) - (16*d**6*g**2 + 32*d**5*e*f*g + 16*d**4*e**2*f**2)/(-d*e**3 + e**4
*x) + x**2*(16*d**3*g**2 + 17*d**2*e*f*g + 3*d*e**2*f**2)/e + x*(48*d**4*g*
*2 + 64*d**3*e*f*g + 17*d**2*e**2*f**2)/e**2
```

Giac [A] time = 1.18565, size = 441, normalized size = 2.49

$$16\left(2\,d^5g^2e^5+3\,d^4fge^6+d^3f^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{30}\left(6\,g^2x^5e^{22}+45\,dg^2x^4e^{21}+170\,d^2g^2x^3e^{20}+480\,d^3g^2x^2e^{19}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 16*(2*d^5*g^2*e^5 + 3*d^4*f*g*e^6 + d^3*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d
^2)) + 1/30*(6*g^2*x^5*e^22 + 45*d*g^2*x^4*e^21 + 170*d^2*g^2*x^3*e^20 + 48
0*d^3*g^2*x^2*e^19 + 1440*d^4*g^2*x*e^18 + 15*f*g*x^4*e^22 + 120*d*f*g*x^3*
e^21 + 510*d^2*f*g*x^2*e^20 + 1920*d^3*f*g*x*e^19 + 10*f^2*x^3*e^22 + 90*d*
f^2*x^2*e^21 + 510*d^2*f^2*x*e^20)*e^(-20) + 16*(2*d^6*g^2*e^6 + 3*d^5*f*g*
e^7 + d^4*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs
(d)*e))/abs(d) - 16*(d^7*g^2*e^5 + 2*d^6*f*g*e^6 + d^5*f^2*e^7 + (d^6*g^2*e
^6 + 2*d^5*f*g*e^7 + d^4*f^2*e^8)*x)*e^(-8)/(x^2*e^2 - d^2)
```

3.559 $\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3}$$

```
[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d^3
*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*Log[d
- e*x])/e^3
```

Rubi [A] time = 0.181323, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d^3
*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*Log[d
- e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^2}\,dx$$

$$=\int \left(\frac{d\left(5e^2f^2+24defg+20d^2g^2\right)}{e^2}+\frac{\left(e^2f^2+10defg+12d^2g^2\right)x}{e}+g(2ef+5dg)x^2+eg^2x^3+\right.$$

$$=\frac{d\left(5e^2f^2+24defg+20d^2g^2\right)x}{e^2}+\frac{\left(e^2f^2+10defg+12d^2g^2\right)x^2}{2e}+\frac{1}{3}g(2ef+5dg)x^3+\frac{1}{4}eg^2x^4$$

Mathematica [A] time = 0.0918917, size = 154, normalized size = 1.05

$$\frac{x^2\left(12d^2g^2+10defg+e^2f^2\right)}{2e}+\frac{dx\left(20d^2g^2+24defg+5e^2f^2\right)}{e^2}+\frac{4d^2\left(7d^2g^2+10defg+3e^2f^2\right)\log(d-ex)}{e^3}-\frac{8d^3(dg}{e^3(ex}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 - (8*d^3
*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d^2*(3*e^2*f^2 + 10*d*e*f*g + 7*d^2*g
^2)*Log[d - e*x])/e^3
```

Maple [A] time = 0.052, size = 204, normalized size = 1.4

$$\frac{eg^2x^4}{4}+\frac{5\,x^3dg^2}{3}+\frac{2\,ex^3fg}{3}+6\,\frac{x^2d^2g^2}{e}+5\,x^2dfg+\frac{ex^2f^2}{2}+20\,\frac{d^3g^2x}{e^2}+24\,\frac{d^2fgx}{e}+5\,df^2x+28\,\frac{d^4\ln(ex-d)\,g^2}{e^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/4*e*g^2*x^4+5/3*x^3*d*g^2+2/3*e*x^3*f*g+6/e*x^2*d^2*g^2+5*x^2*d*f*g+1/2*e
*x^2*f^2+20/e^2*d^3*g^2*x+24/e*d^2*f*g*x+5*d*f^2*x+28*d^4/e^3*ln(e*x-d)*g^2
+40*d^3/e^2*ln(e*x-d)*f*g+12*d^2/e*ln(e*x-d)*f^2-8*d^5/e^3/(e*x-d)*g^2-16*d
^4/e^2/(e*x-d)*f*g-8*d^3/e/(e*x-d)*f^2
```

Maxima [A] time = 0.969659, size = 246, normalized size = 1.68

$$-\frac{8\left(d^3e^2f^2+2\,d^4efg+d^5g^2\right)}{e^4x-de^3}+\frac{3\,e^3g^2x^4+4\left(2\,e^3fg+5\,de^2g^2\right)x^3+6\left(e^3f^2+10\,de^2fg+12\,d^2eg^2\right)x^2+12\left(5\,de^2f^2+\right.}{12\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -8*(d^3*e^2*f^2 + 2*d^4*e*f*g + d^5*g^2)/(e^4*x - d*e^3) + 1/12*(3*e^3*g^2*
x^4 + 4*(2*e^3*f*g + 5*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 10*d*e^2*f*g + 12*d^2*
e*g^2)*x^2 + 12*(5*d*e^2*f^2 + 24*d^2*e*f*g + 20*d^3*g^2)*x)/e^2 + 4*(3*d^2
*e^2*f^2 + 10*d^3*e*f*g + 7*d^4*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.72244, size = 529, normalized size = 3.62

$$\frac{3\,e^5g^2x^5-96\,d^3e^2f^2-192\,d^4efg-96\,d^5g^2+\left(8\,e^5fg+17\,de^4g^2\right)x^4+2\left(3\,e^5f^2+26\,de^4fg+26\,d^2e^3g^2\right)x^3+6\left(9\,de^4f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*e^5*g^2*x^5 - 96*d^3*e^2*f^2 - 192*d^4*e*f*g - 96*d^5*g^2 + (8*e^5*
f*g + 17*d*e^4*g^2)*x^4 + 2*(3*e^5*f^2 + 26*d*e^4*f*g + 26*d^2*e^3*g^2)*x^3
 + 6*(9*d*e^4*f^2 + 38*d^2*e^3*f*g + 28*d^3*e^2*g^2)*x^2 - 12*(5*d^2*e^3*f^
2 + 24*d^3*e^2*f*g + 20*d^4*e*g^2)*x - 48*(3*d^3*e^2*f^2 + 10*d^4*e*f*g + 7
*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f*g + 7*d^4*e*g^2)*x)*log(e*x - d))/
(e^4*x - d*e^3)
```

Sympy [A] time = 0.952896, size = 167, normalized size = 1.14

$$\frac{4d^2\left(dg+ef\right)\left(7dg+3ef\right)\log\left(-d+ex\right)}{e^3}+\frac{eg^2x^4}{4}+x^3\left(\frac{5dg^2}{3}+\frac{2efg}{3}\right)-\frac{8d^5g^2+16d^4efg+8d^3e^2f^2}{-de^3+e^4x}+\frac{x^2\left(12d^2g^2+10d\right.}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d**2*(d*g + e*f)*(7*d*g + 3*e*f)*log(-d + e*x)/e**3 + e*g**2*x**4/4 + x**
3*(5*d*g**2/3 + 2*e*f*g/3) - (8*d**5*g**2 + 16*d**4*e*f*g + 8*d**3*e**2*f**
2)/(-d*e**3 + e**4*x) + x**2*(12*d**2*g**2 + 10*d*e*f*g + e**2*f**2)/(2*e)
+ x*(20*d**3*g**2 + 24*d**2*e*f*g + 5*d*e**2*f**2)/e**2
```

Giac [B] time = 1.23424, size = 393, normalized size = 2.69

$$2\left(7\,d^4g^2e^5+10\,d^3fge^6+3\,d^2f^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{12}\left(3\,g^2x^4e^{17}+20\,dg^2x^3e^{16}+72\,d^2g^2x^2e^{15}+240\,d^3g^2xe^{14}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 2*(7*d^4*g^2*e^5 + 10*d^3*f*g*e^6 + 3*d^2*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 -
 d^2)) + 1/12*(3*g^2*x^4*e^17 + 20*d*g^2*x^3*e^16 + 72*d^2*g^2*x^2*e^15 + 2
40*d^3*g^2*x*e^14 + 8*f*g*x^3*e^17 + 60*d*f*g*x^2*e^16 + 288*d^2*f*g*x*e^15
 + 6*f^2*x^2*e^17 + 60*d*f^2*x*e^16)*e^(-16) + 2*(7*d^5*g^2*e^4 + 10*d^4*f*
g*e^5 + 3*d^3*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2
*abs(d)*e))/abs(d) - 8*(d^6*g^2*e^5 + 2*d^5*f*g*e^6 + d^4*f^2*e^7 + (d^5*g^
2*e^6 + 2*d^4*f*g*e^7 + d^3*f^2*e^8)*x)*e^(-8)/(x^2*e^2 - d^2)
```

3.560 $\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=107

$$\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d^2(dg+ef)^2}{e^3(d-ex)}+\frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 + (4*d^2*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d*(e*f + d*g)*(e*f + 3*
d*g)*Log[d - e*x])/e^3
```

Rubi [A] time = 0.136072, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d^2(dg+ef)^2}{e^3(d-ex)}+\frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 + (4*d^2*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d*(e*f + d*g)*(e*f + 3*
d*g)*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^2}\,dx$$
$$=\int \left(\frac{e^2f^2+8defg+8d^2g^2}{e^2}+\frac{2g(ef+2dg)x}{e}+g^2x^2+\frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)}+\frac{4d^2(ef+}{e^2(-d+}\right.$$
$$=\frac{\left(e^2f^2+8defg+8d^2g^2\right)x}{e^2}+\frac{g(ef+2dg)x^2}{e}+\frac{g^2x^3}{3}+\frac{4d^2(ef+dg)^2}{e^3(d-ex)}+\frac{4d(ef+dg)(ef+3}{e^3}$$

Mathematica [A] time = 0.0884237, size = 115, normalized size = 1.07

$$\frac{x\left(8d^2g^2+8defg+e^2f^2\right)}{e^2}+\frac{4d\left(3d^2g^2+4defg+e^2f^2\right)\log(d-ex)}{e^3}-\frac{4d^2(dg+ef)^2}{e^3(ex-d)}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 - (4*d^2*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d*(e^2*f^2 + 4*d*e*f*g
 + 3*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A] time = 0.05, size = 167, normalized size = 1.6

$$\frac{g^2x^3}{3}+2\,\frac{dx^2g^2}{e}+x^2fg+8\,\frac{d^2g^2x}{e^2}+8\,\frac{dfgx}{e}+xf^2+12\,\frac{d^3\ln(ex-d)\,g^2}{e^3}+16\,\frac{d^2\ln(ex-d)\,fg}{e^2}+4\,\frac{d\ln(ex-d)\,f^2}{e}-4\,\frac{}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/3*g^2*x^3+2/e*x^2*d*g^2+x^2*f*g+8/e^2*d^2*g^2*x+8/e*d*f*g*x+x*f^2+12*d^3/
e^3*ln(e*x-d)*g^2+16*d^2/e^2*ln(e*x-d)*f*g+4*d/e*ln(e*x-d)*f^2-4*d^4/e^3/(e
*x-d)*g^2-8*d^3/e^2/(e*x-d)*f*g-4*d^2/e/(e*x-d)*f^2
```

Maxima [A] time = 0.97571, size = 190, normalized size = 1.78

$$-\frac{4\left(d^2e^2f^2+2\,d^3efg+d^4g^2\right)}{e^4x-de^3}+\frac{e^2g^2x^3+3\left(e^2fg+2\,deg^2\right)x^2+3\left(e^2f^2+8\,defg+8\,d^2g^2\right)x}{3\,e^2}+\frac{4\left(de^2f^2+4\,d^2efg+3\,d^3\right.}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -4*(d^2*e^2*f^2 + 2*d^3*e*f*g + d^4*g^2)/(e^4*x - d*e^3) + 1/3*(e^2*g^2*x^3
 + 3*(e^2*f*g + 2*d*e*g^2)*x^2 + 3*(e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2
 + 4*(d*e^2*f^2 + 4*d^2*e*f*g + 3*d^3*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.75058, size = 421, normalized size = 3.93

$$\frac{e^4g^2x^4-12\,d^2e^2f^2-24\,d^3efg-12\,d^4g^2+\left(3\,e^4fg+5\,de^3g^2\right)x^3+3\left(e^4f^2+7\,de^3fg+6\,d^2e^2g^2\right)x^2-3\left(de^3f^2+8\,d^2e^2fg\right.}{3\left(e^4x-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(e^4*g^2*x^4 - 12*d^2*e^2*f^2 - 24*d^3*e*f*g - 12*d^4*g^2 + (3*e^4*f*g
+ 5*d*e^3*g^2)*x^3 + 3*(e^4*f^2 + 7*d*e^3*f*g + 6*d^2*e^2*g^2)*x^2 - 3*(d*e
^3*f^2 + 8*d^2*e^2*f*g + 8*d^3*e*g^2)*x - 12*(d^2*e^2*f^2 + 4*d^3*e*f*g + 3
*d^4*g^2 - (d*e^3*f^2 + 4*d^2*e^2*f*g + 3*d^3*e*g^2)*x)*log(e*x - d))/(e^4*
x - d*e^3)
```

Sympy [A] time = 0.829104, size = 122, normalized size = 1.14

$$\frac{4d\left(dg+ef\right)\left(3dg+ef\right)\log\left(-d+ex\right)}{e^3}+\frac{g^2x^3}{3}-\frac{4d^4g^2+8d^3efg+4d^2e^2f^2}{-de^3+e^4x}+\frac{x^2\left(2dg^2+efg\right)}{e}+\frac{x\left(8d^2g^2+8defg+\right.}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d*(d*g + e*f)*(3*d*g + e*f)*log(-d + e*x)/e**3 + g**2*x**3/3 - (4*d**4*g*
*2 + 8*d**3*e*f*g + 4*d**2*e**2*f**2)/(-d*e**3 + e**4*x) + x**2*(2*d*g**2 +
 e*f*g)/e + x*(8*d**2*g**2 + 8*d*e*f*g + e**2*f**2)/e**2
```

Giac [B] time = 1.18845, size = 338, normalized size = 3.16

$$2\left(3\,d^3g^2e^3+4\,d^2fge^4+df^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{3}\left(g^2x^3e^{12}+6\,dg^2x^2e^{11}+24\,d^2g^2xe^{10}+3\,fgx^2e^{12}+24\,dfgxe\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 2*(3*d^3*g^2*e^3 + 4*d^2*f*g*e^4 + d*f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2)
) + 1/3*(g^2*x^3*e^12 + 6*d*g^2*x^2*e^11 + 24*d^2*g^2*x*e^10 + 3*f*g*x^2*e^
12 + 24*d*f*g*x*e^11 + 3*f^2*x*e^12)*e^(-12) + 2*(3*d^4*g^2*e^4 + 4*d^3*f*g
*e^5 + d^2*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*ab
s(d)*e))/abs(d) - 4*(d^5*g^2*e^3 + 2*d^4*f*g*e^4 + d^3*f^2*e^5 + (d^4*g^2*e
^4 + 2*d^3*f*g*e^5 + d^2*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2)
```

3.561 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{2d(dg+ef)^2}{e^3(d-ex)}+\frac{gx(3dg+2ef)}{e^2}+\frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x^2}{2e}$$

```
[Out] (g*(2*e*f + 3*d*g)*x)/e^2 + (g^2*x^2)/(2*e) + (2*d*(e*f + d*g)^2)/(e^3*(d -
 e*x)) + ((e*f + d*g)*(e*f + 5*d*g)*Log[d - e*x])/e^3
```

Rubi [A] time = 0.0961631, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 77}

$$\frac{2d(dg+ef)^2}{e^3(d-ex)}+\frac{gx(3dg+2ef)}{e^2}+\frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g*(2*e*f + 3*d*g)*x)/e^2 + (g^2*x^2)/(2*e) + (2*d*(e*f + d*g)^2)/(e^3*(d -
 e*x)) + ((e*f + d*g)*(e*f + 5*d*g)*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(d+ex)(f+gx)^2}{(d-ex)^2}\,dx\\ &= \int \left(\frac{g(2ef+3dg)}{e^2}+\frac{g^2x}{e}+\frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)}+\frac{2d(ef+dg)^2}{e^2(-d+ex)^2}\right)dx\\ &= \frac{g(2ef+3dg)x}{e^2}+\frac{g^2x^2}{2e}+\frac{2d(ef+dg)^2}{e^3(d-ex)}+\frac{(ef+dg)(ef+5dg)\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0633919, size = 83, normalized size = 1.06

$$\frac{2\left(5d^2g^2+6defg+e^2f^2\right)\log(d-ex)+\frac{4d(dg+ef)^2}{d-ex}+2egx(3dg+2ef)+e^2g^2x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (2*e*g*(2*e*f + 3*d*g)*x + e^2*g^2*x^2 + (4*d*(e*f + d*g)^2)/(d - e*x) + 2*
(e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*Log[d - e*x])/(2*e^3)
```

Maple [A] time = 0.05, size = 138, normalized size = 1.8

$$\frac{g^2x^2}{2e}+3\frac{dg^2x}{e^2}+2\frac{fgx}{e}+5\frac{\ln(ex-d)d^2g^2}{e^3}+6\frac{\ln(ex-d)dfg}{e^2}+\frac{\ln(ex-d)f^2}{e}-2\frac{d^3g^2}{e^3(ex-d)}-4\frac{d^2fg}{e^2(ex-d)}-2\frac{d}{e(e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/2*g^2*x^2/e+3*g^2/e^2*d*x+2*g/e*f*x+5/e^3*ln(e*x-d)*d^2*g^2+6/e^2*ln(e*x-
d)*d*f*g+1/e*ln(e*x-d)*f^2-2*d^3/e^3/(e*x-d)*g^2-4*d^2/e^2/(e*x-d)*f*g-2*d/
e/(e*x-d)*f^2
```

Maxima [A] time = 1.00677, size = 140, normalized size = 1.79

$$-\frac{2\left(de^2f^2+2d^2efg+d^3g^2\right)}{e^4x-de^3}+\frac{eg^2x^2+2\left(2efg+3dg^2\right)x}{2e^2}+\frac{\left(e^2f^2+6defg+5d^2g^2\right)\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)/(e^4*x - d*e^3) + 1/2*(e*g^2*x^2 + 2
*(2*e*f*g + 3*d*g^2)*x)/e^2 + (e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*log(e*x - d
)/e^3
```

Fricas [B] time = 1.71681, size = 321, normalized size = 4.12

$$\frac{e^3g^2x^3-4de^2f^2-8d^2efg-4d^3g^2+\left(4e^3fg+5de^2g^2\right)x^2-2\left(2de^2fg+3d^2eg^2\right)x-2\left(de^2f^2+6d^2efg+5d^3g^2-\left(\right.\right.}{2\left(e^4x-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(e^3*g^2*x^3 - 4*d*e^2*f^2 - 8*d^2*e*f*g - 4*d^3*g^2 + (4*e^3*f*g + 5*d
*e^2*g^2)*x^2 - 2*(2*d*e^2*f*g + 3*d^2*e*g^2)*x - 2*(d*e^2*f^2 + 6*d^2*e*f*
g + 5*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)*log(e*x - d))/(e^4
*x - d*e^3)
```

Sympy [A] time = 0.723033, size = 92, normalized size = 1.18

$$-\frac{2d^3g^2+4d^2efg+2de^2f^2}{-de^3+e^4x}+\frac{g^2x^2}{2e}+\frac{x\left(3dg^2+2efg\right)}{e^2}+\frac{\left(dg+ef\right)\left(5dg+ef\right)\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] -(2*d**3*g**2 + 4*d**2*e*f*g + 2*d*e**2*f**2)/(-d*e**3 + e**4*x) + g**2*x**
2/(2*e) + x*(3*d*g**2 + 2*e*f*g)/e**2 + (d*g + e*f)*(5*d*g + e*f)*log(-d +
e*x)/e**3
```

Giac [B] time = 1.24317, size = 286, normalized size = 3.67

$$\frac{1}{2}\left(5\,d^2g^2e^3+6\,dfge^4+f^2e^5\right)e^{(-6)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{1}{2}\left(g^2x^2e^7+6\,dg^2xe^6+4\,fgxe^7\right)e^{(-8)}+\frac{\left(5\,d^3g^2e^2+6\,d^2fge^3+df^2e\right.}{2\left|d\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*d^2*g^2*e^3 + 6*d*f*g*e^4 + f^2*e^5)*e^(-6)*log(abs(x^2*e^2 - d^2))
+ 1/2*(g^2*x^2*e^7 + 6*d*g^2*x*e^6 + 4*f*g*x*e^7)*e^(-8) + 1/2*(5*d^3*g^2*e
^2 + 6*d^2*f*g*e^3 + d*f^2*e^4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*
x*e^2 + 2*abs(d)*e))/abs(d) - 2*(d^4*g^2*e^3 + 2*d^3*f*g*e^4 + d^2*f^2*e^5
+ (d^3*g^2*e^4 + 2*d^2*f*g*e^5 + d*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2)
```

3.562 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=50

$$\frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

```
[Out] (g^2*x)/e^2 + (e*f + d*g)^2/(e^3*(d - e*x)) + (2*g*(e*f + d*g)*Log[d - e*x]
)/e^3
```

Rubi [A] time = 0.0570058, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 43}

$$\frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g^2*x)/e^2 + (e*f + d*g)^2/(e^3*(d - e*x)) + (2*g*(e*f + d*g)*Log[d - e*x]
)/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx &= \int \frac{(f+gx)^2}{(d-ex)^2} dx\\
&= \int \left(\frac{g^2}{e^2}+\frac{(ef+dg)^2}{e^2(-d+ex)^2}+\frac{2g(ef+dg)}{e^2(-d+ex)}\right) dx\\
&= \frac{g^2x}{e^2}+\frac{(ef+dg)^2}{e^3(d-ex)}+\frac{2g(ef+dg)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.04354, size = 46, normalized size = 0.92

$$\frac{\frac{(dg+ef)^2}{d-ex}+2g(dg+ef)\log(d-ex)+eg^2x}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (e*g^2*x + (e*f + d*g)^2/(d - e*x) + 2*g*(e*f + d*g)*Log[d - e*x])/e^3
```

Maple [A] time = 0.048, size = 96, normalized size = 1.9

$$\frac{g^2x}{e^2}+2\,\frac{d\ln(ex-d)\,g^2}{e^3}+2\,\frac{\ln(ex-d)\,fg}{e^2}-\frac{d^2g^2}{e^3\,(ex-d)}-2\,\frac{dfg}{e^2\,(ex-d)}-\frac{f^2}{e\,(ex-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] g^2*x/e^2+2*d/e^3*ln(e*x-d)*g^2+2/e^2*ln(e*x-d)*f*g-1/e^3/(e*x-d)*d^2*g^2-2
/e^2/(e*x-d)*d*f*g-1/e/(e*x-d)*f^2
```

Maxima [A] time = 0.984881, size = 93, normalized size = 1.86

$$\frac{g^2x}{e^2}-\frac{e^2f^2+2\,defg+d^2g^2}{e^4x-de^3}+\frac{2\left(efg+dg^2\right)\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] g^2*x/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(e^4*x - d*e^3) + 2*(e*f*g + d*
g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.70168, size = 184, normalized size = 3.68

$$\frac{e^2g^2x^2-deg^2x-e^2f^2-2\,defg-d^2g^2-2\left(defg+d^2g^2-\left(e^2fg+deg^2\right)x\right)\log(ex-d)}{e^4x-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] (e^2*g^2*x^2 - d*e*g^2*x - e^2*f^2 - 2*d*e*f*g - d^2*g^2 - 2*(d*e*f*g + d^2
*g^2 - (e^2*f*g + d*e*g^2)*x)*log(e*x - d))/(e^4*x - d*e^3)
```

Sympy [A] time = 0.545168, size = 60, normalized size = 1.2

$$-\frac{d^2g^2+2defg+e^2f^2}{-de^3+e^4x}+\frac{g^2x}{e^2}+\frac{2g\left(dg+ef\right)\log(-d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] -(d**2*g**2 + 2*d*e*f*g + e**2*f**2)/(-d*e**3 + e**4*x) + g**2*x/e**2 + 2*g
*(d*g + e*f)*log(-d + e*x)/e**3
```

Giac [B] time = 1.19154, size = 216, normalized size = 4.32

$$g^2xe^{(-2)} + \left(dg^2e + fge^2\right)e^{(-4)}\log\left(\left|x^2e^2 - d^2\right|\right) + \frac{\left(d^2g^2e^2 + dfge^3\right)e^{(-5)}\log\left(\frac{\left|2\,xe^2 - 2\,|d||e|\right|}{\left|2\,xe^2 + 2\,|d||e|\right|}\right)}{|d|} - \frac{\left(d^3g^2e + 2\,d^2fge^2 + df^2e^3 + \right.}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] g^2*x*e^(-2) + (d*g^2*e + f*g*e^2)*e^(-4)*log(abs(x^2*e^2 - d^2)) + (d^2*g^
2*e^2 + d*f*g*e^3)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs
(d)*e))/abs(d) - (d^3*g^2*e + 2*d^2*f*g*e^2 + d*f^2*e^3 + (d^2*g^2*e^2 + 2*
d*f*g*e^3 + f^2*e^4)*x)*e^(-4)/(x^2*e^2 - d^2)
```

3.563 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=86

$$\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}-\frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3}+\frac{(dg+ef)^2}{2de^3(d-ex)}$$

```
[Out] (e*f + d*g)^2/(2*d*e^3*(d - e*x)) - ((e*f - 3*d*g)*(e*f + d*g)*Log[d - e*x]
)/(4*d^2*e^3) + ((e*f - d*g)^2*Log[d + e*x])/(4*d^2*e^3)
```

Rubi [A] time = 0.0815934, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {799, 88}

$$\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}-\frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3}+\frac{(dg+ef)^2}{2de^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (e*f + d*g)^2/(2*d*e^3*(d - e*x)) - ((e*f - 3*d*g)*(e*f + d*g)*Log[d - e*x]
)/(4*d^2*e^3) + ((e*f - d*g)^2*Log[d + e*x])/(4*d^2*e^3)
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)}\,dx\\ &= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)}\right)dx\\ &= \frac{(ef+dg)^2}{2de^3(d-ex)}-\frac{(ef-3dg)(ef+dg)\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}\end{aligned}$$

Mathematica [A] time = 0.0483859, size = 91, normalized size = 1.06

$$\frac{(d-ex)\left(3d^2g^2+2defg-e^2f^2\right)\log(d-ex)+(d-ex)(ef-dg)^2\log(d+ex)+2d(dg+ef)^2}{4d^2e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (2*d*(e*f + d*g)^2 + (-(e^2*f^2) + 2*d*e*f*g + 3*d^2*g^2)*(d - e*x)*Log[d -
 e*x] + (e*f - d*g)^2*(d - e*x)*Log[d + e*x])/(4*d^2*e^3*(d - e*x))
```

Maple [A] time = 0.09, size = 156, normalized size = 1.8

$$-\frac{g^2 d}{2 e^3 (ex-d)}-\frac{fg}{e^2 (ex-d)}-\frac{f^2}{2de (ex-d)}+\frac{3\ln(ex-d) g^2}{4e^3}+\frac{\ln(ex-d) fg}{2de^2}-\frac{\ln(ex-d) f^2}{4d^2 e}+\frac{\ln(ex+d) g^2}{4e^3}-\frac{\ln(e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/2*d/e^3/(e*x-d)*g^2-1/e^2/(e*x-d)*f*g-1/2/d/e/(e*x-d)*f^2+3/4/e^3*ln(e*x
-d)*g^2+1/2/d/e^2*ln(e*x-d)*f*g-1/4/d^2/e*ln(e*x-d)*f^2+1/4/e^3*ln(e*x+d)*g
^2-1/2/d/e^2*ln(e*x+d)*f*g+1/4/d^2/e*ln(e*x+d)*f^2
```

Maxima [A] time = 0.975871, size = 154, normalized size = 1.79

$$-\frac{e^2 f^2+2 defg+d^2 g^2}{2 \left(de^4 x-d^2 e^3\right)}+\frac{\left(e^2 f^2-2 defg+d^2 g^2\right) \log(ex+d)}{4 d^2 e^3}-\frac{\left(e^2 f^2-2 defg-3 d^2 g^2\right) \log(ex-d)}{4 d^2 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(d*e^4*x - d^2*e^3) + 1/4*(e^2*f^2 - 2
*d*e*f*g + d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 - 2*d*e*f*g - 3*d
^2*g^2)*log(e*x - d)/(d^2*e^3)
```

Fricas [B] time = 1.71683, size = 343, normalized size = 3.99

$$-\frac{2 de^2 f^2+4 d^2 efg+2 d^3 g^2+\left(de^2 f^2-2 d^2 efg+d^3 g^2-\left(e^3 f^2-2 de^2 fg+d^2 eg^2\right)x\right)\log(ex+d)-\left(de^2 f^2-2 d^2 efg\right.}{4\left(d^2 e^4 x-d^3 e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*e^2*f^2 + 4*d^2*e*f*g + 2*d^3*g^2 + (d*e^2*f^2 - 2*d^2*e*f*g + d^
3*g^2 - (e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x + d) - (d*e^2*f^2 -
2*d^2*e*f*g - 3*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x
- d))/(d^2*e^4*x - d^3*e^3)
```

Sympy [B] time = 1.14259, size = 180, normalized size = 2.09

$$-\frac{d^2g^2+2defg+e^2f^2}{-2d^2e^3+2de^4x}+\frac{\left(dg-ef\right)^2\log\left(x+\frac{2d^3g^2-d\left(dg-ef\right)^2}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}+\frac{\left(dg+ef\right)\left(3dg-ef\right)\log\left(x+\frac{2d^3g^2-d\left(dg+ef\right)\left(3dg-ef\right)}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] -(d**2*g**2 + 2*d*e*f*g + e**2*f**2)/(-2*d**2*e**3 + 2*d*e**4*x) + (d*g - e
*f)**2*log(x + (2*d**3*g**2 - d*(d*g - e*f)**2)/(d**2*e*g**2 + 2*d*e**2*f*g
 - e**3*f**2))/(4*d**2*e**3) + (d*g + e*f)*(3*d*g - e*f)*log(x + (2*d**3*g*
*2 - d*(d*g + e*f)*(3*d*g - e*f))/(d**2*e*g**2 + 2*d*e**2*f*g - e**3*f**2))
/(4*d**2*e**3)
```

Giac [A] time = 1.14363, size = 215, normalized size = 2.5

$$\frac{1}{2}\,g^2e^{(-3)}\log\left(\left|x^2e^2-d^2\right|\right)+\frac{\left(d^2g^2+2\,dfge-f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{4\,d|d|}-\frac{\left(\left(d^2g^2+2\,dfge+f^2e^2\right)x+\left(d^3g^2e+2\,d^2fge^2\right.\right.}{2\left(x^2e^2-d^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*g^2*e^(-3)*log(abs(x^2*e^2 - d^2)) + 1/4*(d^2*g^2 + 2*d*f*g*e - f^2*e^2
)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/(d*abs(d)
) - 1/2*((d^2*g^2 + 2*d*f*g*e + f^2*e^2)*x + (d^3*g^2*e + 2*d^2*f*g*e^2 + d
*f^2*e^3)*e^(-2))*e^(-2)/((x^2*e^2 - d^2)*d)
```

3.564 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{(f+gx)\left(d^2g+e^2fx\right)}{2d^2e^2\left(d^2-e^2x^2\right)}+\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(2*d^2*e^2*(d^2 - e^2*x^2)) + ((e*f - d*g)*(e
*f + d*g)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Rubi [A] time = 0.0291274, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {723, 208}

$$\frac{(f+gx)\left(d^2g+e^2fx\right)}{2d^2e^2\left(d^2-e^2x^2\right)}+\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(2*d^2*e^2*(d^2 - e^2*x^2)) + ((e*f - d*g)*(e
*f + d*g)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx &= \frac{\left(d^2g+e^2fx\right)(f+gx)}{2d^2e^2\left(d^2-e^2x^2\right)}-\frac{1}{2}\left(-\frac{f^2}{d^2}+\frac{g^2}{e^2}\right)\int \frac{1}{d^2-e^2x^2}\,dx\\ &= \frac{\left(d^2g+e^2fx\right)(f+gx)}{2d^2e^2\left(d^2-e^2x^2\right)}+\frac{(ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}\end{aligned}$$

Mathematica [A] time = 0.038201, size = 85, normalized size = 1.15

$$\frac{-2d^2fg-d^2g^2x-e^2f^2x}{2d^2e^2\left(e^2x^2-d^2\right)}-\frac{\left(d^2g^2-e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (-2*d^2*f*g - e^2*f^2*x - d^2*g^2*x)/(2*d^2*e^2*(-d^2 + e^2*x^2)) - ((-(e^2
*f^2) + d^2*g^2)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Maple [B] time = 0.055, size = 180, normalized size = 2.4

$$\frac{\ln (ex-d) g^2}{4 e^3 d} - \frac{\ln (ex-d) f^2}{4 e d^3} - \frac{g^2}{4 e^3 (ex-d)} - \frac{fg}{2 d e^2 (ex-d)} - \frac{f^2}{4 e d^2 (ex-d)} - \frac{\ln (ex+d) g^2}{4 e^3 d} + \frac{\ln (ex+d) f^2}{4 e d^3} - \frac{g^2}{4 e^3 (ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/4/e^3/d*ln(e*x-d)*g^2-1/4/e/d^3*ln(e*x-d)*f^2-1/4/e^3/(e*x-d)*g^2-1/2/d/e
^2/(e*x-d)*f*g-1/4/d^2/e/(e*x-d)*f^2-1/4/e^3/d*ln(e*x+d)*g^2+1/4/e/d^3*ln(e
*x+d)*f^2-1/4/e^3/(e*x+d)*g^2+1/2/d/e^2/(e*x+d)*f*g-1/4/d^2/e/(e*x+d)*f^2
```

Maxima [A] time = 0.98369, size = 150, normalized size = 2.03

$$-\frac{2 d^2 fg + \left(e^2 f^2 + d^2 g^2\right)x}{2 \left(d^2 e^4 x^2 - d^4 e^2\right)} + \frac{\left(e^2 f^2 - d^2 g^2\right) \log (ex+d)}{4 d^3 e^3} - \frac{\left(e^2 f^2 - d^2 g^2\right) \log (ex-d)}{4 d^3 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*d^2*f*g + (e^2*f^2 + d^2*g^2)*x)/(d^2*e^4*x^2 - d^4*e^2) + 1/4*(e^2
*f^2 - d^2*g^2)*log(e*x + d)/(d^3*e^3) - 1/4*(e^2*f^2 - d^2*g^2)*log(e*x -
d)/(d^3*e^3)
```

Fricas [B] time = 1.72916, size = 286, normalized size = 3.86

$$-\frac{4 d^3 efg + 2 \left(de^3 f^2 + d^3 eg^2\right)x + \left(d^2 e^2 f^2 - d^4 g^2 - \left(e^4 f^2 - d^2 e^2 g^2\right)x^2\right) \log (ex+d) - \left(d^2 e^2 f^2 - d^4 g^2 - \left(e^4 f^2 - d^2 e^2 g^2\right)x^2\right)}{4 \left(d^3 e^5 x^2 - d^5 e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d^3*e*f*g + 2*(d*e^3*f^2 + d^3*e*g^2)*x + (d^2*e^2*f^2 - d^4*g^2 -
(e^4*f^2 - d^2*e^2*g^2)*x^2)*log(e*x + d) - (d^2*e^2*f^2 - d^4*g^2 - (e^4*f
^2 - d^2*e^2*g^2)*x^2)*log(e*x - d))/(d^3*e^5*x^2 - d^5*e^3)
```

Sympy [B] time = 0.814184, size = 155, normalized size = 2.09

$$-\frac{2d^2 fg + x \left(d^2 g^2 + e^2 f^2\right)}{-2d^4 e^2 + 2d^2 e^4 x^2} + \frac{\left(dg - ef\right)\left(dg + ef\right) \log \left(-\frac{d\left(dg-ef\right)\left(dg+ef\right)}{e\left(d^2 g^2 - e^2 f^2\right)} + x\right)}{4d^3 e^3} - \frac{\left(dg - ef\right)\left(dg + ef\right) \log \left(\frac{d\left(dg-ef\right)\left(dg+ef\right)}{e\left(d^2 g^2 - e^2 f^2\right)} + x\right)}{4d^3 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] -(2*d**2*f*g + x*(d**2*g**2 + e**2*f**2))/(-2*d**4*e**2 + 2*d**2*e**4*x**2)
 + (d*g - e*f)*(d*g + e*f)*log(-d*(d*g - e*f)*(d*g + e*f)/(e*(d**2*g**2 - e
**2*f**2)) + x)/(4*d**3*e**3) - (d*g - e*f)*(d*g + e*f)*log(d*(d*g - e*f)*(
d*g + e*f)/(e*(d**2*g**2 - e**2*f**2)) + x)/(4*d**3*e**3)
```

Giac [A] time = 1.15209, size = 136, normalized size = 1.84

$$\frac{\left(d^2g^2 - f^2e^2\right)e^{(-3)}\log\left(\frac{|2\,xe^2 - 2\,|d|e|}{|2\,xe^2 + 2\,|d|e|}\right)}{4\,d^2|d|} - \frac{\left(d^2g^2x + 2\,d^2fg + f^2xe^2\right)e^{(-2)}}{2\left(x^2e^2 - d^2\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^2*g^2 - f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 +
2*abs(d)*e))/(d^2*abs(d)) - 1/2*(d^2*g^2*x + 2*d^2*f*g + f^2*x*e^2)*e^(-2)/
((x^2*e^2 - d^2)*d^2)
```

3.565 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=121

$$-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}+\frac{(dg+ef)^2}{8d^3e^3(d-ex)}+\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

```
[Out] (e*f + d*g)^2/(8*d^3*e^3*(d - e*x)) - (e*f - d*g)^2/(8*d^2*e^3*(d + e*x)^2)
 - (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d + e*x)) + ((3*e*f - d*g)*(e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rubi [A] time = 0.135801, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}+\frac{(dg+ef)^2}{8d^3e^3(d-ex)}+\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(8*d^3*e^3*(d - e*x)) - (e*f - d*g)^2/(8*d^2*e^3*(d + e*x)^2)
 - (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d + e*x)) + ((3*e*f - d*g)*(e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^3}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{8d^3e^2(d-ex)^2} + \frac{(-ef+dg)^2}{4d^2e^2(d+ex)^3} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^2} + \frac{(3ef-dg)(ef+dg)}{8d^3e^2\left(d^2-e^2x^2\right)}\right) dx$$
$$= \frac{(ef+dg)^2}{8d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^2e^3(d+ex)^2} - \frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)} + \frac{((3ef-dg)(ef+dg))\int \frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2}$$
$$= \frac{(ef+dg)^2}{8d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^2e^3(d+ex)^2} - \frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)} + \frac{(3ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Mathematica [A] time = 0.104534, size = 139, normalized size = 1.15

$$\frac{\frac{4d\left(d^2g^2-e^2f^2\right)}{d+ex} + \left(d^2g^2-2defg-3e^2f^2\right)\log(d-ex) + \left(-d^2g^2+2defg+3e^2f^2\right)\log(d+ex) - \frac{2d^2(ef-dg)^2}{(d+ex)^2} + \frac{2d(dg+ef)^2}{d-ex}}{16d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((2*d*(e*f + d*g)^2)/(d - e*x) - (2*d^2*(e*f - d*g)^2)/(d + e*x)^2 + (4*d*(
-(e^2*f^2) + d^2*g^2))/(d + e*x) + (-3*e^2*f^2 - 2*d*e*f*g + d^2*g^2)*Log[d
 - e*x] + (3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

Maple [B] time = 0.055, size = 253, normalized size = 2.1

$$\frac{\ln(ex-d)g^2}{16e^3d^2} - \frac{\ln(ex-d)fg}{8e^2d^3} - \frac{3\ln(ex-d)f^2}{16ed^4} - \frac{g^2}{8de^3(ex-d)} - \frac{fg}{4d^2e^2(ex-d)} - \frac{f^2}{8ed^3(ex-d)} + \frac{g^2}{4de^3(ex+d)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x)
```

```
[Out] 1/16/e^3/d^2*ln(e*x-d)*g^2-1/8/e^2/d^3*ln(e*x-d)*f*g-3/16/e/d^4*ln(e*x-d)*f
^2-1/8/e^3/d/(e*x-d)*g^2-1/4/e^2/d^2/(e*x-d)*f*g-1/8/e/d^3/(e*x-d)*f^2+1/4/
d/e^3/(e*x+d)*g^2-1/4/d^3/e/(e*x+d)*f^2-1/16/e^3/d^2*ln(e*x+d)*g^2+1/8/e^2/
d^3*ln(e*x+d)*f*g+3/16/e/d^4*ln(e*x+d)*f^2-1/8/e^3/(e*x+d)^2*g^2+1/4/d/e^2/
(e*x+d)^2*f*g-1/8/d^2/e/(e*x+d)^2*f^2
```

Maxima [A] time = 0.982601, size = 286, normalized size = 2.36

$$\frac{2d^2e^2f^2-4d^3efg-2d^4g^2-\left(3e^4f^2+2de^3fg-d^2e^2g^2\right)x^2-\left(3de^3f^2+2d^2e^2fg+3d^3eg^2\right)x}{8\left(d^3e^6x^3+d^4e^5x^2-d^5e^4x-d^6e^3\right)} + \frac{\left(3e^2f^2+2defg-d\right.}{16d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^2*e^2*f^2 - 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 + 2*d*e^3*f*g - d
^2*e^2*g^2)*x^2 - (3*d*e^3*f^2 + 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x
```

```
^3 + d^4*e^5*x^2 - d^5*e^4*x - d^6*e^3) + 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2
*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(e
*x - d)/(d^4*e^3)
```

Fricas [B] time = 1.80002, size = 810, normalized size = 6.69

$$\frac{4\,d^3e^2f^2 - 8\,d^4efg - 4\,d^5g^2 - 2\left(3\,de^4f^2 + 2\,d^2e^3fg - d^3e^2g^2\right)x^2 - 2\left(3\,d^2e^3f^2 + 2\,d^3e^2fg + 3\,d^4eg^2\right)x - \left(3\,d^3e^2f^2 + 2\,d^4e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d^3*e^2*f^2 - 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 + 2*d^2*e^3*
f*g - d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x
- (3*d^3*e^2*f^2 + 2*d^4*e*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e
^3*g^2)*x^3 - (3*d*e^4*f^2 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*
f^2 + 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) + (3*d^3*e^2*f^2 + 2*d^4*e
*f*g - d^5*g^2 - (3*e^5*f^2 + 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e^4*f^2
 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 + (3*d^2*e^3*f^2 + 2*d^3*e^2*f*g - d^4*
e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 + d^5*e^5*x^2 - d^6*e^4*x - d^7*e^3)
```

Sympy [B] time = 1.50707, size = 279, normalized size = 2.31

$$\frac{-2d^4g^2 - 4d^3efg + 2d^2e^2f^2 + x^2\left(d^2e^2g^2 - 2de^3fg - 3e^4f^2\right) + x\left(-3d^3eg^2 - 2d^2e^2fg - 3de^3f^2\right)}{-8d^6e^3 - 8d^5e^4x + 8d^4e^5x^2 + 8d^3e^6x^3} + \frac{\left(dg - 3ef\right)\left(dg + ef\right)\text{lo}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**4*g**2 - 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 - 2*
d*e**3*f*g - 3*e**4*f**2) + x*(-3*d**3*e*g**2 - 2*d**2*e**2*f*g - 3*d*e**3*
f**2))/(-8*d**6*e**3 - 8*d**5*e**4*x + 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
 + (d*g - 3*e*f)*(d*g + e*f)*log(-d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2
 - 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - 3*e*f)*(d*g + e*f
)*log(d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2 - 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.566 $\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}+\frac{(dg+ef)^2}{16d^4e^3(d-ex)}-\frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)}+\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}$$

```
[Out] (e*f + d*g)^2/(16*d^4*e^3*(d - e*x)) - (e*f - d*g)^2/(12*d^2*e^3*(d + e*x)^
3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e*x)^2) - ((3*e*f - d*g)*(e*f + d*
g))/(16*d^4*e^3*(d + e*x)) + (f*(e*f + d*g)*ArcTanh[(e*x)/d])/(4*d^5*e^2)
```

Rubi [A] time = 0.155293, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}+\frac{(dg+ef)^2}{16d^4e^3(d-ex)}-\frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)}+\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(16*d^4*e^3*(d - e*x)) - (e*f - d*g)^2/(12*d^2*e^3*(d + e*x)^
3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e*x)^2) - ((3*e*f - d*g)*(e*f + d*
g))/(16*d^4*e^3*(d + e*x)) + (f*(e*f + d*g)*ArcTanh[(e*x)/d])/(4*d^5*e^2)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}\,dx = \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^4}\,dx$$

$$= \int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^2} + \frac{(-ef+dg)^2}{4d^2e^2(d+ex)^4} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^3} + \frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^2} + \frac{f(ef+}{4d^4e\left(d^2\right.}\right.$$

$$= \frac{(ef+dg)^2}{16d^4e^3(d-ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2} - \frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)} + \frac{(f(ef+dg))}{4d^4}$$

$$= \frac{(ef+dg)^2}{16d^4e^3(d-ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2} - \frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)} + \frac{f(ef+dg)\,\mathrm{ta}}{4d^5e}$$

Mathematica [A] time = 0.0990623, size = 171, normalized size = 1.17

$$\frac{2d\left(d^3e^2f(gx-4f)+d^2e^3fx(f+6gx)+2d^4eg(f+2gx)+2d^5g^2+3de^4fx^2(2f+gx)+3e^5f^2x^3\right)+3ef(ex-d)(d+ex)^3}{24d^5e^3(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (2*d*(2*d^5*g^2 + 3*e^5*f^2*x^3 + d^3*e^2*f*(-4*f + g*x) + 3*d*e^4*f*x^2*(2
*f + g*x) + 2*d^4*e*g*(f + 2*g*x) + d^2*e^3*f*x*(f + 6*g*x)) + 3*e*f*(e*f +
 d*g)*(-d + e*x)*(d + e*x)^3*Log[d - e*x] + 3*e*f*(e*f + d*g)*(d - e*x)*(d
+ e*x)^3*Log[d + e*x])/(24*d^5*e^3*(d - e*x)*(d + e*x)^3)
```

Maple [A] time = 0.056, size = 270, normalized size = 1.9

$$-\frac{g^2}{16\,d^2e^3\,(ex-d)} - \frac{fg}{8\,e^2d^3\,(ex-d)} - \frac{f^2}{16\,ed^4\,(ex-d)} - \frac{\ln(ex-d)\,fg}{8\,e^2d^4} - \frac{\ln(ex-d)\,f^2}{8\,d^5e} + \frac{g^2}{8\,de^3\,(ex+d)^2} - \frac{f^2}{8\,ed^3\,(ex+d)^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/16/e^3/d^2/(e*x-d)*g^2-1/8/e^2/d^3/(e*x-d)*f*g-1/16/e/d^4/(e*x-d)*f^2-1/
8/e^2/d^4*ln(e*x-d)*f*g-1/8/e/d^5*ln(e*x-d)*f^2+1/8/d/e^3/(e*x+d)^2*g^2-1/8
/d^3/e/(e*x+d)^2*f^2+1/16/d^2/e^3/(e*x+d)*g^2-1/8/d^3/e^2/(e*x+d)*f*g-3/16/
d^4/e/(e*x+d)*f^2-1/12/e^3/(e*x+d)^3*g^2+1/6/d/e^2/(e*x+d)^3*f*g-1/12/d^2/e
/(e*x+d)^3*f^2+1/8/e^2/d^4*ln(e*x+d)*f*g+1/8/e/d^5*ln(e*x+d)*f^2
```

Maxima [A] time = 1.01457, size = 266, normalized size = 1.82

$$\frac{4\,d^3e^2f^2-2\,d^4efg-2\,d^5g^2-3\left(e^5f^2+de^4fg\right)x^3-6\left(de^4f^2+d^2e^3fg\right)x^2-\left(d^2e^3f^2+d^3e^2fg+4\,d^4eg^2\right)x}{12\left(d^4e^7x^4+2\,d^5e^6x^3-2\,d^7e^4x-d^8e^3\right)} + \frac{\left(ef^2+dfg\right)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*d^3*e^2*f^2 - 2*d^4*e*f*g - 2*d^5*g^2 - 3*(e^5*f^2 + d*e^4*f*g)*x^3
 - 6*(d*e^4*f^2 + d^2*e^3*f*g)*x^2 - (d^2*e^3*f^2 + d^3*e^2*f*g + 4*d^4*e*g
```

```
^2)*x)/(d^4*e^7*x^4 + 2*d^5*e^6*x^3 - 2*d^7*e^4*x - d^8*e^3) + 1/8*(e*f^2 +
 d*f*g)*log(e*x + d)/(d^5*e^2) - 1/8*(e*f^2 + d*f*g)*log(e*x - d)/(d^5*e^2)
```

Fricas [B] time = 1.74351, size = 674, normalized size = 4.62

$$\frac{8\,d^4e^2f^2 - 4\,d^5efg - 4\,d^6g^2 - 6\left(de^5f^2 + d^2e^4fg\right)x^3 - 12\left(d^2e^4f^2 + d^3e^3fg\right)x^2 - 2\left(d^3e^3f^2 + d^4e^2fg + 4\,d^5eg^2\right)x - 3\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(8*d^4*e^2*f^2 - 4*d^5*e*f*g - 4*d^6*g^2 - 6*(d*e^5*f^2 + d^2*e^4*f*g)
*x^3 - 12*(d^2*e^4*f^2 + d^3*e^3*f*g)*x^2 - 2*(d^3*e^3*f^2 + d^4*e^2*f*g +
4*d^5*e*g^2)*x - 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2
*(d*e^5*f^2 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x +
 d) + 3*(d^4*e^2*f^2 + d^5*e*f*g - (e^6*f^2 + d*e^5*f*g)*x^4 - 2*(d*e^5*f^2
 + d^2*e^4*f*g)*x^3 + 2*(d^3*e^3*f^2 + d^4*e^2*f*g)*x)*log(e*x - d))/(d^5*e
^7*x^4 + 2*d^6*e^6*x^3 - 2*d^8*e^4*x - d^9*e^3)
```

Sympy [A] time = 1.62728, size = 236, normalized size = 1.62

$$-\frac{2d^5g^2 + 2d^4efg - 4d^3e^2f^2 + x^3\left(3de^4fg + 3e^5f^2\right) + x^2\left(6d^2e^3fg + 6de^4f^2\right) + x\left(4d^4eg^2 + d^3e^2fg + d^2e^3f^2\right)}{-12d^8e^3 - 24d^7e^4x + 24d^5e^6x^3 + 12d^4e^7x^4} - \frac{f\left(dg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] -(2*d**5*g**2 + 2*d**4*e*f*g - 4*d**3*e**2*f**2 + x**3*(3*d*e**4*f*g + 3*e*
*5*f**2) + x**2*(6*d**2*e**3*f*g + 6*d*e**4*f**2) + x*(4*d**4*e*g**2 + d**3
*e**2*f*g + d**2*e**3*f**2))/(-12*d**8*e**3 - 24*d**7*e**4*x + 24*d**5*e**6
*x**3 + 12*d**4*e**7*x**4) - f*(d*g + e*f)*log(-d*f*(d*g + e*f)/(e*(d*f*g +
 e*f**2)) + x)/(8*d**5*e**2) + f*(d*g + e*f)*log(d*f*(d*g + e*f)/(e*(d*f*g
+ e*f**2)) + x)/(8*d**5*e**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.567 $\int \frac{(f+gx)^2}{(d+ex)^3 \left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=178

$$-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}+\frac{(dg+ef)^2}{32d^5e^3(d-ex)}-\frac{f(dg+ef)}{8d^5e^2(d+ex)}-\frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2}+\frac{(dg+ef)(dg+5ef)\tanh}{32d^6e^3}$$

```
[Out] (e*f + d*g)^2/(32*d^5*e^3*(d - e*x)) - (e*f - d*g)^2/(16*d^2*e^3*(d + e*x)^
4) - (e^2*f^2 - d^2*g^2)/(12*d^3*e^3*(d + e*x)^3) - ((3*e*f - d*g)*(e*f + d
*g))/(32*d^4*e^3*(d + e*x)^2) - (f*(e*f + d*g))/(8*d^5*e^2*(d + e*x)) + ((e
*f + d*g)*(5*e*f + d*g)*ArcTanh[(e*x)/d])/(32*d^6*e^3)
```

Rubi [A] time = 0.199561, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}+\frac{(dg+ef)^2}{32d^5e^3(d-ex)}-\frac{f(dg+ef)}{8d^5e^2(d+ex)}-\frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2}+\frac{(dg+ef)(dg+5ef)\tanh}{32d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(32*d^5*e^3*(d - e*x)) - (e*f - d*g)^2/(16*d^2*e^3*(d + e*x)^
4) - (e^2*f^2 - d^2*g^2)/(12*d^3*e^3*(d + e*x)^3) - ((3*e*f - d*g)*(e*f + d
*g))/(32*d^4*e^3*(d + e*x)^2) - (f*(e*f + d*g))/(8*d^5*e^2*(d + e*x)) + ((e
*f + d*g)*(5*e*f + d*g)*ArcTanh[(e*x)/d])/(32*d^6*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx = \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^5}\,dx$$

$$= \int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^2} + \frac{(-ef+dg)^2}{4d^2e^2(d+ex)^5} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^4} + \frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^3} + \frac{f(ef}{8d^5e(}\right.$$

$$= \frac{(ef+dg)^2}{32d^5e^3(d-ex)} - \frac{(ef-dg)^2}{16d^2e^3(d+ex)^4} - \frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3} - \frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2} - \frac{f(ef+}{8d^5e^2(d-}$$

$$= \frac{(ef+dg)^2}{32d^5e^3(d-ex)} - \frac{(ef-dg)^2}{16d^2e^3(d+ex)^4} - \frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3} - \frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2} - \frac{f(ef+}{8d^5e^2(d-}$$

Mathematica [A] time = 0.149004, size = 195, normalized size = 1.1

$$\frac{\frac{16d^3\left(d^2g^2-e^2f^2\right)}{(d+ex)^3} + \frac{6d^2\left(d^2g^2-2defg-3e^2f^2\right)}{(d+ex)^2} - 3\left(d^2g^2+6defg+5e^2f^2\right)\log(d-ex) + 3\left(d^2g^2+6defg+5e^2f^2\right)\log(d+ex) -}{192d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((6*d*(e*f + d*g)^2)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (16*d
^3*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^3 + (6*d^2*(-3*e^2*f^2 - 2*d*e*f*g + d
^2*g^2))/(d + e*x)^2 - (24*d*e*f*(e*f + d*g))/(d + e*x) - 3*(5*e^2*f^2 + 6*
d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*Log[d
 + e*x])/(192*d^6*e^3)
```

Maple [B] time = 0.059, size = 341, normalized size = 1.9

$$-\frac{\ln(ex-d)\,g^2}{64\,e^3d^4} - \frac{3\,\ln(ex-d)\,fg}{32\,e^2d^5} - \frac{5\,\ln(ex-d)\,f^2}{64\,ed^6} - \frac{g^2}{32\,d^3e^3\,(ex-d)} - \frac{fg}{16\,e^2d^4\,(ex-d)} - \frac{f^2}{32\,ed^5\,(ex-d)} + \frac{\ln(ex+}{64\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/64/e^3/d^4*ln(e*x-d)*g^2-3/32/e^2/d^5*ln(e*x-d)*f*g-5/64/e/d^6*ln(e*x-d)
*f^2-1/32/e^3/d^3/(e*x-d)*g^2-1/16/e^2/d^4/(e*x-d)*f*g-1/32/e/d^5/(e*x-d)*f
^2+1/64/e^3/d^4*ln(e*x+d)*g^2+3/32/e^2/d^5*ln(e*x+d)*f*g+5/64/e/d^6*ln(e*x+
d)*f^2+1/12/e^3/d/(e*x+d)^3*g^2-1/12/e/d^3/(e*x+d)^3*f^2+1/32/e^3/d^2/(e*x+
d)^2*g^2-1/16/e^2/d^3/(e*x+d)^2*f*g-3/32/e/d^4/(e*x+d)^2*f^2-1/16/e^3/(e*x+
d)^4*g^2+1/8/d/e^2/(e*x+d)^4*f*g-1/16/d^2/e/(e*x+d)^4*f^2-1/8/e^2*f/d^4/(e*
x+d)*g-1/8/e*f^2/d^5/(e*x+d)
```

Maxima [A] time = 1.03643, size = 402, normalized size = 2.26

$$\frac{32\,d^4e^2f^2 - 8\,d^6g^2 - 3\left(5\,e^6f^2 + 6\,de^5fg + d^2e^4g^2\right)x^4 - 9\left(5\,de^5f^2 + 6\,d^2e^4fg + d^3e^3g^2\right)x^3 - 7\left(5\,d^2e^4f^2 + 6\,d^3e^3fg + \right.}{96\left(d^5e^8x^5 + 3\,d^6e^7x^4 + 2\,d^7e^6x^3 - 2\,d^8e^5x^2 - 3\,d^9e^4x - d^{10}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(32*d^4*e^2*f^2 - 8*d^6*g^2 - 3*(5*e^6*f^2 + 6*d*e^5*f*g + d^2*e^4*g^2
)*x^4 - 9*(5*d*e^5*f^2 + 6*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 - 7*(5*d^2*e^4*f^
2 + 6*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 3*(5*d^3*e^3*f^2 + 6*d^4*e^2*f*g - 7
*d^5*e*g^2)*x)/(d^5*e^8*x^5 + 3*d^6*e^7*x^4 + 2*d^7*e^6*x^3 - 2*d^8*e^5*x^2
 - 3*d^9*e^4*x - d^10*e^3) + 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x
 + d)/(d^6*e^3) - 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^6*
e^3)
```

Fricas [B] time = 2.02957, size = 1303, normalized size = 7.32

$$\frac{64\,d^5e^2f^2-16\,d^7g^2-6\left(5\,de^6f^2+6\,d^2e^5fg+d^3e^4g^2\right)x^4-18\left(5\,d^2e^5f^2+6\,d^3e^4fg+d^4e^3g^2\right)x^3-14\left(5\,d^3e^4f^2+6\,d^4e^3fg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(64*d^5*e^2*f^2 - 16*d^7*g^2 - 6*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^3*e
^4*g^2)*x^4 - 18*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 14*(5*
d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 6*(5*d^4*e^3*f^2 + 6*d^5*e
^2*f*g - 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7*g^2 - (5*e^7
*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2*e^5*f*g + d^
3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 + 2*(5
*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3*f^2 + 6*d^5*
e^2*f*g + d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 6*d^6*e*f*g + d^7
*g^2 - (5*e^7*f^2 + 6*d*e^6*f*g + d^2*e^5*g^2)*x^5 - 3*(5*d*e^6*f^2 + 6*d^2
*e^5*f*g + d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^4*e^3*g^
2)*x^3 + 2*(5*d^3*e^4*f^2 + 6*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + 3*(5*d^4*e^3
*f^2 + 6*d^5*e^2*f*g + d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5 + 3*d^7*e^7
*x^4 + 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 - 3*d^10*e^4*x - d^11*e^3)
```

Sympy [B] time = 2.15653, size = 371, normalized size = 2.08

$$-\frac{8d^6g^2-32d^4e^2f^2+x^4\left(3d^2e^4g^2+18de^5fg+15e^6f^2\right)+x^3\left(9d^3e^3g^2+54d^2e^4fg+45de^5f^2\right)+x^2\left(7d^4e^2g^2+42d^3e^3fg\right.}{-96d^{10}e^3-288d^9e^4x-192d^8e^5x^2+192d^7e^6x^3+288d^6e^7x^4+96d^5e^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2)**2,x)
```

```
[Out] -(8*d**6*g**2 - 32*d**4*e**2*f**2 + x**4*(3*d**2*e**4*g**2 + 18*d*e**5*f*g
+ 15*e**6*f**2) + x**3*(9*d**3*e**3*g**2 + 54*d**2*e**4*f*g + 45*d*e**5*f**
2) + x**2*(7*d**4*e**2*g**2 + 42*d**3*e**3*f*g + 35*d**2*e**4*f**2) + x*(21
*d**5*e*g**2 - 18*d**4*e**2*f*g - 15*d**3*e**3*f**2))/(-96*d**10*e**3 - 288
*d**9*e**4*x - 192*d**8*e**5*x**2 + 192*d**7*e**6*x**3 + 288*d**6*e**7*x**4
 + 96*d**5*e**8*x**5) - (d*g + e*f)*(d*g + 5*e*f)*log(-d*(d*g + e*f)*(d*g +
 5*e*f)/(e*(d**2*g**2 + 6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3) + (d*
g + e*f)*(d*g + 5*e*f)*log(d*(d*g + e*f)*(d*g + 5*e*f)/(e*(d**2*g**2 + 6*d*
e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.568 $\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=210

$$-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}+$$

```
[Out] (e*f + d*g)^2/(64*d^6*e^3*(d - e*x)) - (e*f - d*g)^2/(20*d^2*e^3*(d + e*x)^
5) - (e^2*f^2 - d^2*g^2)/(16*d^3*e^3*(d + e*x)^4) - ((3*e*f - d*g)*(e*f + d
*g))/(48*d^4*e^3*(d + e*x)^3) - (f*(e*f + d*g))/(16*d^5*e^2*(d + e*x)^2) -
((e*f + d*g)*(5*e*f + d*g))/(64*d^6*e^3*(d + e*x)) + ((e*f + d*g)*(3*e*f +
d*g)*ArcTanh[(e*x)/d])/(32*d^7*e^3)
```

Rubi [A] time = 0.241934, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```


```
[Out] (e*f + d*g)^2/(64*d^6*e^3*(d - e*x)) - (e*f - d*g)^2/(20*d^2*e^3*(d + e*x)^
5) - (e^2*f^2 - d^2*g^2)/(16*d^3*e^3*(d + e*x)^4) - ((3*e*f - d*g)*(e*f + d
*g))/(48*d^4*e^3*(d + e*x)^3) - (f*(e*f + d*g))/(16*d^5*e^2*(d + e*x)^2) -
((e*f + d*g)*(5*e*f + d*g))/(64*d^6*e^3*(d + e*x)) + ((e*f + d*g)*(3*e*f +
d*g)*ArcTanh[(e*x)/d])/(32*d^7*e^3)
```


Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx = \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^6}\,dx$$

$$= \int \left(\frac{(ef+dg)^2}{64d^6e^2(d-ex)^2} + \frac{(-ef+dg)^2}{4d^2e^2(d+ex)^6} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^5} + \frac{(3ef-dg)(ef+dg)}{16d^4e^2(d+ex)^4} + \frac{f(ef}{8d^5e(}\right.$$

$$= \frac{(ef+dg)^2}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{20d^2e^3(d+ex)^5} - \frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4} - \frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3} - \frac{f(ef+}{16d^5e^2(d}$$

$$= \frac{(ef+dg)^2}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{20d^2e^3(d+ex)^5} - \frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4} - \frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3} - \frac{f(ef+}{16d^5e^2(d}$$

Mathematica [A] time = 0.18319, size = 229, normalized size = 1.09

$$\frac{\frac{60d^4\left(d^2g^2-e^2f^2\right)}{(d+ex)^4} + \frac{20d^3\left(d^2g^2-2defg-3e^2f^2\right)}{(d+ex)^3} - \frac{15d\left(d^2g^2+6defg+5e^2f^2\right)}{d+ex} - 15\left(d^2g^2+4defg+3e^2f^2\right)\log(d-ex) + 15\left(d^2g^2+4def\right.}{960d^7e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((15*d*(e*f + d*g)^2)/(d - e*x) - (48*d^5*(e*f - d*g)^2)/(d + e*x)^5 + (60*
d^4*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^4 + (20*d^3*(-3*e^2*f^2 - 2*d*e*f*g +
 d^2*g^2))/(d + e*x)^3 - (60*d^2*e*f*(e*f + d*g))/(d + e*x)^2 - (15*d*(5*e^
2*f^2 + 6*d*e*f*g + d^2*g^2))/(d + e*x) - 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g
^2)*Log[d - e*x] + 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*Log[d + e*x])/(960*
d^7*e^3)
```

Maple [B] time = 0.062, size = 394, normalized size = 1.9

$$-\frac{\ln(ex-d)\,g^2}{64\,e^3d^5} - \frac{\ln(ex-d)\,fg}{16\,e^2d^6} - \frac{3\,\ln(ex-d)\,f^2}{64\,ed^7} - \frac{g^2}{64\,d^4e^3\,(ex-d)} - \frac{fg}{32\,e^2d^5\,(ex-d)} - \frac{f^2}{64\,ed^6\,(ex-d)} + \frac{\ln(ex+d)}{64\,e^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x)
```

```
[Out] -1/64/e^3/d^5*ln(e*x-d)*g^2-1/16/e^2/d^6*ln(e*x-d)*f*g-3/64/e/d^7*ln(e*x-d)
*f^2-1/64/e^3/d^4/(e*x-d)*g^2-1/32/e^2/d^5/(e*x-d)*f*g-1/64/e/d^6/(e*x-d)*f
^2+1/64/e^3/d^5*ln(e*x+d)*g^2+1/16/e^2/d^6*ln(e*x+d)*f*g+3/64/e/d^7*ln(e*x+
d)*f^2-1/64/e^3/d^4/(e*x+d)*g^2-3/32/e^2/d^5/(e*x+d)*f*g-5/64/e/d^6/(e*x+d)
*f^2+1/16/e^3/d/(e*x+d)^4*g^2-1/16/e/d^3/(e*x+d)^4*f^2+1/48/e^3/d^2/(e*x+d)
^3*g^2-1/24/e^2/d^3/(e*x+d)^3*f*g-1/16/e/d^4/(e*x+d)^3*f^2-1/20/e^3/(e*x+d)
^5*g^2+1/10/d/e^2/(e*x+d)^5*f*g-1/20/d^2/e/(e*x+d)^5*f^2-1/16/e^2*f/d^4/(e*
x+d)^2*g-1/16/e*f^2/d^5/(e*x+d)^2
```

Maxima [A] time = 1.08751, size = 462, normalized size = 2.2

$$\frac{144\,d^5e^2f^2 + 32\,d^6efg - 16\,d^7g^2 - 15\left(3\,e^7f^2 + 4\,de^6fg + d^2e^5g^2\right)x^5 - 60\left(3\,de^6f^2 + 4\,d^2e^5fg + d^3e^4g^2\right)x^4 - 80\left(3\,d^2e^5\right.}{480\left(d^6e^9x^6 + 4\,d^7e^8x^5 + 5\,d^8e^7x^4 - 5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/480*(144*d^5*e^2*f^2 + 32*d^6*e*f*g - 16*d^7*g^2 - 15*(3*e^7*f^2 + 4*d*e^
6*f*g + d^2*e^5*g^2)*x^5 - 60*(3*d*e^6*f^2 + 4*d^2*e^5*f*g + d^3*e^4*g^2)*x
^4 - 80*(3*d^2*e^5*f^2 + 4*d^3*e^4*f*g + d^4*e^3*g^2)*x^3 - 20*(3*d^3*e^4*f
^2 + 4*d^4*e^3*f*g + d^5*e^2*g^2)*x^2 + (141*d^4*e^3*f^2 + 188*d^5*e^2*f*g
- 49*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 4*d^7*e^8*x^5 + 5*d^8*e^7*x^4 - 5*d^10*e^
5*x^2 - 4*d^11*e^4*x - d^12*e^3) + 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*l
og(e*x + d)/(d^7*e^3) - 1/64*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*log(e*x - d)
/(d^7*e^3)
```

Fricas [B] time = 2.13187, size = 1413, normalized size = 6.73

$$\frac{288\,d^6e^2f^2 + 64\,d^7efg - 32\,d^8g^2 - 30\left(3\,de^7f^2 + 4\,d^2e^6fg + d^3e^5g^2\right)x^5 - 120\left(3\,d^2e^6f^2 + 4\,d^3e^5fg + d^4e^4g^2\right)x^4 - 160\left(3\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] 1/960*(288*d^6*e^2*f^2 + 64*d^7*e*f*g - 32*d^8*g^2 - 30*(3*d*e^7*f^2 + 4*d^
2*e^6*f*g + d^3*e^5*g^2)*x^5 - 120*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4
*g^2)*x^4 - 160*(3*d^3*e^5*f^2 + 4*d^4*e^4*f*g + d^5*e^3*g^2)*x^3 - 40*(3*d
^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 2*(141*d^5*e^3*f^2 + 188*d^
6*e^2*f*g - 49*d^7*e*g^2)*x - 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g + d^8*g^2 - (
3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 + 4*d^2*e^6*f*g
 + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*e^4*g^2)*x^4 +
 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d^5*e^3*f^2 + 4
*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x + d) + 15*(3*d^6*e^2*f^2 + 4*d^7*e*f*g
 + d^8*g^2 - (3*e^8*f^2 + 4*d*e^7*f*g + d^2*e^6*g^2)*x^6 - 4*(3*d*e^7*f^2 +
 4*d^2*e^6*f*g + d^3*e^5*g^2)*x^5 - 5*(3*d^2*e^6*f^2 + 4*d^3*e^5*f*g + d^4*
e^4*g^2)*x^4 + 5*(3*d^4*e^4*f^2 + 4*d^5*e^3*f*g + d^6*e^2*g^2)*x^2 + 4*(3*d
^5*e^3*f^2 + 4*d^6*e^2*f*g + d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 4*d
^8*e^8*x^5 + 5*d^9*e^7*x^4 - 5*d^11*e^5*x^2 - 4*d^12*e^4*x - d^13*e^3)
```

Sympy [B] time = 2.58207, size = 420, normalized size = 2.

$$-\frac{16d^7g^2 - 32d^6efg - 144d^5e^2f^2 + x^5\left(15d^2e^5g^2 + 60de^6fg + 45e^7f^2\right) + x^4\left(60d^3e^4g^2 + 240d^2e^5fg + 180de^6f^2\right) + x^3\left(80\right.}{-480d^{12}e^3 - 1920d^{11}e^4x - 2400d^{10}e^5x^2 + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2)**2,x)
```

```
[Out] -(16*d**7*g**2 - 32*d**6*e*f*g - 144*d**5*e**2*f**2 + x**5*(15*d**2*e**5*g*
*2 + 60*d*e**6*f*g + 45*e**7*f**2) + x**4*(60*d**3*e**4*g**2 + 240*d**2*e**
5*f*g + 180*d*e**6*f**2) + x**3*(80*d**4*e**3*g**2 + 320*d**3*e**4*f*g + 24
0*d**2*e**5*f**2) + x**2*(20*d**5*e**2*g**2 + 80*d**4*e**3*f*g + 60*d**3*e*
*4*f**2) + x*(49*d**6*e*g**2 - 188*d**5*e**2*f*g - 141*d**4*e**3*f**2))/(-4
80*d**12*e**3 - 1920*d**11*e**4*x - 2400*d**10*e**5*x**2 + 2400*d**8*e**7*x
**4 + 1920*d**7*e**8*x**5 + 480*d**6*e**9*x**6) - (d*g + e*f)*(d*g + 3*e*f)
*log(-d*(d*g + e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f**2))
 + x)/(64*d**7*e**3) + (d*g + e*f)*(d*g + 3*e*f)*log(d*(d*g + e*f)*(d*g + 3
```

```
*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f**2)) + x)/(64*d**7*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.569 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=179

$$-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{32d^3(dg+ef)(2dg+e}{e^3(d-ex)}$$

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e*f + 2*d*g)*(e*f +
12*d*g)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*d^4*(e*
f + d*g)^2)/(e^3*(d - e*x)^2) - (32*d^3*(e*f + d*g)*(e*f + 2*d*g))/(e^3*(d
- e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rubi [A] time = 0.241137, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{32d^3(dg+ef)(2dg+e}{e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e*f + 2*d*g)*(e*f +
12*d*g)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*d^4*(e*
f + d*g)^2)/(e^3*(d - e*x)^2) - (32*d^3*(e*f + d*g)*(e*f + 2*d*g))/(e^3*(d
- e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^3}\,dx\\
&= \int \left(-\frac{d\left(7e^2f^2+48defg+56d^2g^2\right)}{e^2}+\frac{(-ef-12dg)(ef+2dg)x}{e}-g(2ef+7dg)x^2-eg^2x^3+\right.\\
&= -\frac{d\left(7e^2f^2+48defg+56d^2g^2\right)x}{e^2}-\frac{(ef+2dg)(ef+12dg)x^2}{2e}-\frac{1}{3}g(2ef+7dg)x^3-\frac{1}{4}eg^2x^4+
\end{aligned}$$

Mathematica [A] time = 0.0978451, size = 193, normalized size = 1.08

$$-\frac{x^2\left(24d^2g^2+14defg+e^2f^2\right)}{2e}+\frac{32d^3\left(2d^2g^2+3defg+e^2f^2\right)}{e^3(ex-d)}-\frac{dx\left(56d^2g^2+48defg+7e^2f^2\right)}{e^2}-\frac{8d^2\left(13d^2g^2+14d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e^2*f^2 + 14*d*e*f*g
 + 24*d^2*g^2)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*
d^4*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2
*g^2))/(e^3*(-d + e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[
d - e*x])/e^3
```

Maple [A] time = 0.052, size = 263, normalized size = 1.5

$$-\frac{eg^2x^4}{4}-\frac{7x^3dg^2}{3}-\frac{2ex^3fg}{3}-12\frac{x^2d^2g^2}{e}-7x^2dfg-\frac{ex^2f^2}{2}-56\frac{d^3g^2x}{e^2}-48\frac{d^2fgx}{e}-7df^2x-104\frac{d^4\ln(ex-d)g^2}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/4*e*g^2*x^4-7/3*x^3*d*g^2-2/3*e*x^3*f*g-12/e*x^2*d^2*g^2-7*x^2*d*f*g-1/2
*e*x^2*f^2-56/e^2*d^3*g^2*x-48/e*d^2*f*g*x-7*d*f^2*x-104*d^4/e^3*ln(e*x-d)*
g^2-112*d^3/e^2*ln(e*x-d)*f*g-24*d^2/e*ln(e*x-d)*f^2+8*d^6/e^3/(e*x-d)^2*g^
2+16*d^5/e^2/(e*x-d)^2*f*g+8*d^4/e/(e*x-d)^2*f^2+64*d^5/e^3/(e*x-d)*g^2+96*
d^4/e^2/(e*x-d)*f*g+32*d^3/e/(e*x-d)*f^2
```

Maxima [A] time = 0.991082, size = 306, normalized size = 1.71

$$-\frac{8\left(3d^4e^2f^2+10d^5efg+7d^6g^2-4\left(d^3e^3f^2+3d^4e^2fg+2d^5eg^2\right)x\right)}{e^5x^2-2de^4x+d^2e^3}-\frac{3e^3g^2x^4+4\left(2e^3fg+7de^2g^2\right)x^3+6\left(e^3f^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -8*(3*d^4*e^2*f^2 + 10*d^5*e*f*g + 7*d^6*g^2 - 4*(d^3*e^3*f^2 + 3*d^4*e^2*f
*g + 2*d^5*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/12*(3*e^3*g^2*x^4
+ 4*(2*e^3*f*g + 7*d*e^2*g^2)*x^3 + 6*(e^3*f^2 + 14*d*e^2*f*g + 24*d^2*e*g^
2)*x^2 + 12*(7*d*e^2*f^2 + 48*d^2*e*f*g + 56*d^3*g^2)*x)/e^2 - 8*(3*d^2*e^2
*f^2 + 14*d^3*e*f*g + 13*d^4*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.77242, size = 722, normalized size = 4.03

$$-\frac{3e^6g^2x^6+288d^4e^2f^2+960d^5efg+672d^6g^2+2\left(4e^6fg+11de^5g^2\right)x^5+\left(6e^6f^2+68de^5fg+91d^2e^4g^2\right)x^4+4\left(18\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*e^6*g^2*x^6 + 288*d^4*e^2*f^2 + 960*d^5*e*f*g + 672*d^6*g^2 + 2*(4
*e^6*f*g + 11*d*e^5*g^2)*x^5 + (6*e^6*f^2 + 68*d*e^5*f*g + 91*d^2*e^4*g^2)*
x^4 + 4*(18*d*e^5*f^2 + 104*d^2*e^4*f*g + 103*d^3*e^3*g^2)*x^3 - 6*(27*d^2*
e^4*f^2 + 178*d^3*e^3*f*g + 200*d^4*e^2*g^2)*x^2 - 12*(25*d^3*e^3*f^2 + 48*
d^4*e^2*f*g + 8*d^5*e*g^2)*x + 96*(3*d^4*e^2*f^2 + 14*d^5*e*f*g + 13*d^6*g^
2 + (3*d^2*e^4*f^2 + 14*d^3*e^3*f*g + 13*d^4*e^2*g^2)*x^2 - 2*(3*d^3*e^3*f^
2 + 14*d^4*e^2*f*g + 13*d^5*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x +
d^2*e^3)
```

Sympy [A] time = 1.73791, size = 223, normalized size = 1.25

$$-\frac{8d^2\left(13d^2g^2+14defg+3e^2f^2\right)\log\left(-d+ex\right)}{e^3}-\frac{eg^2x^4}{4}-x^3\left(\frac{7dg^2}{3}+\frac{2efg}{3}\right)+\frac{-56d^6g^2-80d^5efg-24d^4e^2f^2+x\left(64d^5\right.}{d^2e^3-2de^4x+e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -8*d**2*(13*d**2*g**2 + 14*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - e*g*
*2*x**4/4 - x**3*(7*d*g**2/3 + 2*e*f*g/3) + (-56*d**6*g**2 - 80*d**5*e*f*g
- 24*d**4*e**2*f**2 + x*(64*d**5*e*g**2 + 96*d**4*e**2*f*g + 32*d**3*e**3*f
**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - x**2*(24*d**2*g**2 + 14*d*e*f*
g + e**2*f**2)/(2*e) - x*(56*d**3*g**2 + 48*d**2*e*f*g + 7*d*e**2*f**2)/e**
2
```

Giac [B] time = 1.16236, size = 491, normalized size = 2.74

$$-4\left(13\,d^4g^2e^7+14\,d^3fge^8+3\,d^2f^2e^9\right)e^{(-10)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{12}\left(3\,g^2x^4e^{25}+28\,dg^2x^3e^{24}+144\,d^2g^2x^2e^{23}+672\,d^3g^2xe^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -4*(13*d^4*g^2*e^7 + 14*d^3*f*g*e^8 + 3*d^2*f^2*e^9)*e^(-10)*log(abs(x^2*e^
2 - d^2)) - 1/12*(3*g^2*x^4*e^25 + 28*d*g^2*x^3*e^24 + 144*d^2*g^2*x^2*e^23
 + 672*d^3*g^2*x*e^22 + 8*f*g*x^3*e^25 + 84*d*f*g*x^2*e^24 + 576*d^2*f*g*x*
e^23 + 6*f^2*x^2*e^25 + 84*d*f^2*x*e^24)*e^(-24) - 4*(13*d^5*g^2*e^6 + 14*d
^4*f*g*e^7 + 3*d^3*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^
2 + 2*abs(d)*e))/abs(d) - 8*(7*d^8*g^2*e^7 + 10*d^7*f*g*e^8 + 3*d^6*f^2*e^9
 - 4*(2*d^5*g^2*e^10 + 3*d^4*f*g*e^11 + d^3*f^2*e^12)*x^3 - (9*d^6*g^2*e^9
+ 14*d^5*f*g*e^10 + 5*d^4*f^2*e^11)*x^2 + 2*(3*d^7*g^2*e^8 + 4*d^6*f*g*e^9
+ d^5*f^2*e^10)*x)*e^(-10)/(x^2*e^2 - d^2)^2
```

3.570 $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg+3e}{e^3(d-ex)}$$

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - (4*d^2*(e*f + d*g)*
(3*e*f + 7*d*g))/(e^3*(d - e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g + 19*d^2*g^
2)*Log[d - e*x])/e^3
```

Rubi [A] time = 0.197255, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg+3e}{e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - (4*d^2*(e*f + d*g)*
(3*e*f + 7*d*g))/(e^3*(d - e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g + 19*d^2*g^
2)*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^3}\,dx$$

$$=\int\left(\frac{-e^2f^2-12defg-18d^2g^2}{e^2}-\frac{2g(ef+3dg)x}{e}-g^2x^2+\frac{4d^2(-3ef-7dg)(ef+dg)}{e^2(d-ex)^2}-\frac{8d^3}{e^2(}\right.$$

$$=-\frac{\left(e^2f^2+12defg+18d^2g^2\right)x}{e^2}-\frac{g(ef+3dg)x^2}{e}-\frac{g^2x^3}{3}+\frac{4d^3(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d^2(ef+dg)(3e}{e^3(d-e}$$

Mathematica [A] time = 0.0875661, size = 157, normalized size = 1.05

$$\frac{4d^2\left(7d^2g^2+10defg+3e^2f^2\right)}{e^3(ex-d)}-\frac{x\left(18d^2g^2+12defg+e^2f^2\right)}{e^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log(d-ex)}{e^3}+\frac{4d^3(dg+ef}{e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (4*d^2*(3*e^2*f^2 +
 10*d*e*f*g + 7*d^2*g^2))/(e^3*(-d + e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g +
 19*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A] time = 0.052, size = 228, normalized size = 1.5

$$-\frac{g^2x^3}{3}-3\frac{dx^2g^2}{e}-x^2fg-18\frac{d^2g^2x}{e^2}-12\frac{dfgx}{e}-xf^2-38\frac{d^3\ln(ex-d)g^2}{e^3}-36\frac{d^2\ln(ex-d)fg}{e^2}-6\frac{d\ln(ex-d)f^2}{e}+4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/3*g^2*x^3-3/e*x^2*d*g^2-x^2*f*g-18/e^2*d^2*g^2*x-12/e*d*f*g*x-x*f^2-38*d
^3/e^3*ln(e*x-d)*g^2-36*d^2/e^2*ln(e*x-d)*f*g-6*d/e*ln(e*x-d)*f^2+4*d^5/e^3
/(e*x-d)^2*g^2+8*d^4/e^2/(e*x-d)^2*f*g+4*d^3/e/(e*x-d)^2*f^2+28*d^4/e^3/(e*
x-d)*g^2+40*d^3/e^2/(e*x-d)*f*g+12*d^2/e/(e*x-d)*f^2
```

Maxima [A] time = 0.974452, size = 254, normalized size = 1.7

$$-\frac{4\left(2\,d^3e^2f^2+8\,d^4efg+6\,d^5g^2-\left(3\,d^2e^3f^2+10\,d^3e^2fg+7\,d^4eg^2\right)x\right)}{e^5x^2-2\,de^4x+d^2e^3}-\frac{e^2g^2x^3+3\left(e^2fg+3\,deg^2\right)x^2+3\left(e^2f^2+12\,def\right.}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -4*(2*d^3*e^2*f^2 + 8*d^4*e*f*g + 6*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f
*g + 7*d^4*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/3*(e^2*g^2*x^3 + 3
*(e^2*f*g + 3*d*e*g^2)*x^2 + 3*(e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2 -
 2*(3*d*e^2*f^2 + 18*d^2*e*f*g + 19*d^3*g^2)*log(e*x - d)/e^3
```

Fricas [A] time = 1.73908, size = 620, normalized size = 4.16

$$-\frac{e^5g^2x^5+24\,d^3e^2f^2+96\,d^4efg+72\,d^5g^2+\left(3\,e^5fg+7\,de^4g^2\right)x^4+\left(3\,e^5f^2+30\,de^4fg+37\,d^2e^3g^2\right)x^3-3\left(2\,de^4f^2+23\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(e^5*g^2*x^5 + 24*d^3*e^2*f^2 + 96*d^4*e*f*g + 72*d^5*g^2 + (3*e^5*f*g
 + 7*d*e^4*g^2)*x^4 + (3*e^5*f^2 + 30*d*e^4*f*g + 37*d^2*e^3*g^2)*x^3 - 3*(
2*d*e^4*f^2 + 23*d^2*e^3*f*g + 33*d^3*e^2*g^2)*x^2 - 3*(11*d^2*e^3*f^2 + 28
*d^3*e^2*f*g + 10*d^4*e*g^2)*x + 6*(3*d^3*e^2*f^2 + 18*d^4*e*f*g + 19*d^5*g
^2 + (3*d*e^4*f^2 + 18*d^2*e^3*f*g + 19*d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2
 + 18*d^3*e^2*f*g + 19*d^4*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d
^2*e^3)
```

Sympy [A] time = 1.50012, size = 180, normalized size = 1.21

$$-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\log\left(-d+ex\right)}{e^3}-\frac{g^2x^3}{3}+\frac{-24d^5g^2-32d^4efg-8d^3e^2f^2+x\left(28d^4eg^2+40d^3e^2fg+12\right.}{d^2e^3-2de^4x+e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -2*d*(19*d**2*g**2 + 18*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - g**2*x*
*3/3 + (-24*d**5*g**2 - 32*d**4*e*f*g - 8*d**3*e**2*f**2 + x*(28*d**4*e*g**
2 + 40*d**3*e**2*f*g + 12*d**2*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x
**2) - x**2*(3*d*g**2 + e*f*g)/e - x*(18*d**2*g**2 + 12*d*e*f*g + e**2*f**2
)/e**2
```

Giac [B] time = 1.16219, size = 437, normalized size = 2.93

$$-\left(19\,d^3g^2e^5+18\,d^2fge^6+3\,df^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{3}\left(g^2x^3e^{18}+9\,dg^2x^2e^{17}+54\,d^2g^2xe^{16}+3\,fgx^2e^{18}+36\,df\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -(19*d^3*g^2*e^5 + 18*d^2*f*g*e^6 + 3*d*f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d
^2)) - 1/3*(g^2*x^3*e^18 + 9*d*g^2*x^2*e^17 + 54*d^2*g^2*x*e^16 + 3*f*g*x^2
*e^18 + 36*d*f*g*x*e^17 + 3*f^2*x*e^18)*e^(-18) - (19*d^4*g^2*e^6 + 18*d^3*
f*g*e^7 + 3*d^2*f^2*e^8)*e^(-9)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 +
 2*abs(d)*e))/abs(d) - 4*(6*d^7*g^2*e^5 + 8*d^6*f*g*e^6 + 2*d^5*f^2*e^7 - (
7*d^4*g^2*e^8 + 10*d^3*f*g*e^9 + 3*d^2*f^2*e^10)*x^3 - 4*(2*d^5*g^2*e^7 + 3
*d^4*f*g*e^8 + d^3*f^2*e^9)*x^2 + (5*d^6*g^2*e^6 + 6*d^5*f*g*e^7 + d^4*f^2*
e^8)*x)*e^(-8)/(x^2*e^2 - d^2)^2
```

3.571 $\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)}{e^3}+\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

```
[Out] -((g*(2*e*f + 5*d*g)*x)/e^2) - (g^2*x^2)/(2*e) + (2*d^2*(e*f + d*g)^2)/(e^3
*(d - e*x)^2) - (4*d*(e*f + d*g)*(e*f + 3*d*g))/(e^3*(d - e*x)) - ((e^2*f^2
 + 10*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rubi [A] time = 0.143006, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 88}

$$-\frac{\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)}{e^3}+\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g*(2*e*f + 5*d*g)*x)/e^2) - (g^2*x^2)/(2*e) + (2*d^2*(e*f + d*g)^2)/(e^3
*(d - e*x)^2) - (4*d*(e*f + d*g)*(e*f + 3*d*g))/(e^3*(d - e*x)) - ((e^2*f^2
 + 10*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^3}\,dx\\ &= \int \left(-\frac{g(2ef+5dg)}{e^2}-\frac{g^2x}{e}+\frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)^2}-\frac{4d^2(ef+dg)^2}{e^2(-d+ex)^3}+\frac{-e^2f^2-10defg-13}{e^2(-d+ex)}\right.\\ &= -\frac{g(2ef+5dg)x}{e^2}-\frac{g^2x^2}{2e}+\frac{2d^2(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d(ef+dg)(ef+3dg)}{e^3(d-ex)}-\frac{\left(e^2f^2+10defg+13d^2g\right.}{e^3}\end{aligned}$$

Mathematica [A] time = 0.0928828, size = 118, normalized size = 1.

$$-\frac{\frac{8d\left(3d^2g^2+4defg+e^2f^2\right)}{d-ex}+2\left(13d^2g^2+10defg+e^2f^2\right)\log(d-ex)-\frac{4d^2(dg+ef)^2}{(d-ex)^2}+2egx(5dg+2ef)+e^2g^2x^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(2*e*g*(2*e*f + 5*d*g)*x + e^2*g^2*x^2 - (4*d^2*(e*f + d*g)^2)/(d - e*x)^2
 + (8*d*(e^2*f^2 + 4*d*e*f*g + 3*d^2*g^2))/(d - e*x) + 2*(e^2*f^2 + 10*d*e*
f*g + 13*d^2*g^2)*Log[d - e*x])/(2*e^3)
```

Maple [A] time = 0.051, size = 198, normalized size = 1.7

$$-\frac{g^2x^2}{2e}-5\frac{dg^2x}{e^2}-2\frac{fgx}{e}-13\frac{\ln(ex-d)d^2g^2}{e^3}-10\frac{\ln(ex-d)dfg}{e^2}-\frac{\ln(ex-d)f^2}{e}+2\frac{d^4g^2}{e^3(ex-d)^2}+4\frac{d^3fg}{e^2(ex-d)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/2*g^2*x^2/e-5*g^2/e^2*d*x-2*g/e*f*x-13/e^3*ln(e*x-d)*d^2*g^2-10/e^2*ln(e
*x-d)*d*f*g-1/e*ln(e*x-d)*f^2+2*d^4/e^3/(e*x-d)^2*g^2+4*d^3/e^2/(e*x-d)^2*f
*g+2*d^2/e/(e*x-d)^2*f^2+12*d^3/e^3/(e*x-d)*g^2+16*d^2/e^2/(e*x-d)*f*g+4*d/
e/(e*x-d)*f^2
```

Maxima [A] time = 0.975768, size = 201, normalized size = 1.7

$$-\frac{2\left(d^2e^2f^2+6d^3efg+5d^4g^2-2\left(de^3f^2+4d^2e^2fg+3d^3eg^2\right)x\right)}{e^5x^2-2de^4x+d^2e^3}-\frac{eg^2x^2+2\left(2efg+5dg^2\right)x}{2e^2}-\frac{\left(e^2f^2+10defg+1\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -2*(d^2*e^2*f^2 + 6*d^3*e*f*g + 5*d^4*g^2 - 2*(d*e^3*f^2 + 4*d^2*e^2*f*g +
3*d^3*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 1/2*(e*g^2*x^2 + 2*(2*e*f
*g + 5*d*g^2)*x)/e^2 - (e^2*f^2 + 10*d*e*f*g + 13*d^2*g^2)*log(e*x - d)/e^3
```

Fricas [B] time = 1.73834, size = 504, normalized size = 4.27

$$-\frac{e^4g^2x^4+4d^2e^2f^2+24d^3efg+20d^4g^2+4\left(e^4fg+2de^3g^2\right)x^3-\left(8de^3fg+19d^2e^2g^2\right)x^2-2\left(4de^3f^2+14d^2e^2fg+\right.}{2\left(e^5x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(e^4*g^2*x^4 + 4*d^2*e^2*f^2 + 24*d^3*e*f*g + 20*d^4*g^2 + 4*(e^4*f*g
+ 2*d*e^3*g^2)*x^3 - (8*d*e^3*f*g + 19*d^2*e^2*g^2)*x^2 - 2*(4*d*e^3*f^2 +
14*d^2*e^2*f*g + 7*d^3*e*g^2)*x + 2*(d^2*e^2*f^2 + 10*d^3*e*f*g + 13*d^4*g^
2 + (e^4*f^2 + 10*d*e^3*f*g + 13*d^2*e^2*g^2)*x^2 - 2*(d*e^3*f^2 + 10*d^2*e
^2*f*g + 13*d^3*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 1.34291, size = 148, normalized size = 1.25

$$\frac{-10d^4g^2-12d^3efg-2d^2e^2f^2+x\left(12d^3eg^2+16d^2e^2fg+4de^3f^2\right)}{d^2e^3-2de^4x+e^5x^2}-\frac{g^2x^2}{2e}-\frac{x\left(5dg^2+2efg\right)}{e^2}-\frac{\left(13d^2g^2+10defg+e^2f^2\right.}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (-10*d**4*g**2 - 12*d**3*e*f*g - 2*d**2*e**2*f**2 + x*(12*d**3*e*g**2 + 16*
d**2*e**2*f*g + 4*d*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - g**2
*x**2/(2*e) - x*(5*d*g**2 + 2*e*f*g)/e**2 - (13*d**2*g**2 + 10*d*e*f*g + e*
*2*f**2)*log(-d + e*x)/e**3
```

Giac [B] time = 1.14559, size = 369, normalized size = 3.13

$$-\frac{1}{2}\left(13\,d^2g^2e^5+10\,dfge^6+f^2e^7\right)e^{(-8)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{1}{2}\left(g^2x^2e^{11}+10\,dg^2xe^{10}+4\,fgxe^{11}\right)e^{(-12)}-\frac{\left(13\,d^3g^2e^4+10\,d^2f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*(13*d^2*g^2*e^5 + 10*d*f*g*e^6 + f^2*e^7)*e^(-8)*log(abs(x^2*e^2 - d^2
)) - 1/2*(g^2*x^2*e^11 + 10*d*g^2*x*e^10 + 4*f*g*x*e^11)*e^(-12) - 1/2*(13*
d^3*g^2*e^4 + 10*d^2*f*g*e^5 + d*f^2*e^6)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)
*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d) - 2*(5*d^6*g^2*e^5 + 6*d^5*f*g*e^6 +
d^4*f^2*e^7 - 2*(3*d^3*g^2*e^8 + 4*d^2*f*g*e^9 + d*f^2*e^10)*x^3 - (7*d^4*g
^2*e^7 + 10*d^3*f*g*e^8 + 3*d^2*f^2*e^9)*x^2 + 4*(d^5*g^2*e^6 + d^4*f*g*e^7
)*x)*e^(-8)/(x^2*e^2 - d^2)^2
```

3.572 $\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2}-\frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

```
[Out] -((g^2*x)/e^2) + (d*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - ((e*f + d*g)*(e*f +
5*d*g))/(e^3*(d - e*x)) - (2*g*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rubi [A] time = 0.100875, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 77}

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2}-\frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g^2*x)/e^2) + (d*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - ((e*f + d*g)*(e*f +
5*d*g))/(e^3*(d - e*x)) - (2*g*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^4(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(d+ex)(f+gx)^2}{(d-ex)^3}\,dx\\
&= \int \left(-\frac{g^2}{e^2}+\frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)^2}-\frac{2d(ef+dg)^2}{e^2(-d+ex)^3}-\frac{2g(ef+2dg)}{e^2(-d+ex)}\right)dx\\
&= -\frac{g^2x}{e^2}+\frac{d(ef+dg)^2}{e^3(d-ex)^2}-\frac{(ef+dg)(ef+5dg)}{e^3(d-ex)}-\frac{2g(ef+2dg)\log(d-ex)}{e^3}
\end{aligned}$$

Mathematica [A] time = 0.0402026, size = 93, normalized size = 1.15

$$\frac{4d^2eg(gx-f)-4d^3g^2+2de^2gx(3f+gx)-2g(d-ex)^2(2dg+ef)\log(d-ex)+e^3x\left(f^2-g^2x^2\right)}{e^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (-4*d^3*g^2 + 4*d^2*e*g*(-f + g*x) + 2*d*e^2*g*x*(3*f + g*x) + e^3*x*(f^2 -
 g^2*x^2) - 2*g*(e*f + 2*d*g)*(d - e*x)^2*Log[d - e*x])/(e^3*(d - e*x)^2)
```

Maple [A] time = 0.05, size = 151, normalized size = 1.9

$$-\frac{g^2x}{e^2}-4\frac{d\ln(ex-d)g^2}{e^3}-2\frac{\ln(ex-d)fg}{e^2}+\frac{d^3g^2}{e^3(ex-d)^2}+2\frac{d^2fg}{e^2(ex-d)^2}+\frac{df^2}{e(ex-d)^2}+5\frac{d^2g^2}{e^3(ex-d)}+6\frac{dfg}{e^2(ex-d)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -g^2*x/e^2-4*d/e^3*ln(e*x-d)*g^2-2/e^2*ln(e*x-d)*f*g+d^3/e^3/(e*x-d)^2*g^2+
2*d^2/e^2/(e*x-d)^2*f*g+d/e/(e*x-d)^2*f^2+5/e^3/(e*x-d)*d^2*g^2+6/e^2/(e*x-
d)*d*f*g+1/e/(e*x-d)*f^2
```

Maxima [A] time = 0.984322, size = 142, normalized size = 1.75

$$-\frac{g^2x}{e^2}-\frac{4\,d^2efg+4\,d^3g^2-\left(e^3f^2+6\,de^2fg+5\,d^2eg^2\right)x}{e^5x^2-2\,de^4x+d^2e^3}-\frac{2\left(efg+2\,dg^2\right)\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -g^2*x/e^2 - (4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^
2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 2*(e*f*g + 2*d*g^2)*log(e*x - d)/e^
3
```

Fricas [A] time = 1.77384, size = 320, normalized size = 3.95

$$-\frac{e^3g^2x^3-2\,de^2g^2x^2+4\,d^2efg+4\,d^3g^2-\left(e^3f^2+6\,de^2fg+4\,d^2eg^2\right)x+2\left(d^2efg+2\,d^3g^2+\left(e^3fg+2\,de^2g^2\right)x^2-2\left(de^2\right.\right.}{e^5x^2-2\,de^4x+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -(e^3*g^2*x^3 - 2*d*e^2*g^2*x^2 + 4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*
e^2*f*g + 4*d^2*e*g^2)*x + 2*(d^2*e*f*g + 2*d^3*g^2 + (e^3*f*g + 2*d*e^2*g^
2)*x^2 - 2*(d*e^2*f*g + 2*d^2*e*g^2)*x)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x
+ d^2*e^3)
```

Sympy [A] time = 1.04675, size = 99, normalized size = 1.22

$$\frac{-4d^3g^2-4d^2efg+x\left(5d^2eg^2+6de^2fg+e^3f^2\right)}{d^2e^3-2de^4x+e^5x^2}-\frac{g^2x}{e^2}-\frac{2g\left(2dg+ef\right)\log(-d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (-4*d**3*g**2 - 4*d**2*e*f*g + x*(5*d**2*e*g**2 + 6*d*e**2*f*g + e**3*f**2)
)/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - g**2*x/e**2 - 2*g*(2*d*g + e*f)*lo
g(-d + e*x)/e**3
```

Giac [B] time = 1.16408, size = 306, normalized size = 3.78

$$-g^2xe^{(-2)} - \left(2\,dg^2e^3 + fge^4\right)e^{(-6)}\log\left(\left|x^2e^2 - d^2\right|\right) - \frac{\left(2\,d^2g^2e^4 + dfge^5\right)e^{(-7)}\log\left(\frac{\left|2\,xe^2 - 2\,|d||e|\right|}{\left|2\,xe^2 + 2\,|d||e|\right|}\right)}{|d|} - \frac{\left(4\,d^5g^2e^3 + 4\,d^4fge^4 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -g^2*x*e^(-2) - (2*d*g^2*e^3 + f*g*e^4)*e^(-6)*log(abs(x^2*e^2 - d^2)) - (2
*d^2*g^2*e^4 + d*f*g*e^5)*e^(-7)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2
+ 2*abs(d)*e))/abs(d) - (4*d^5*g^2*e^3 + 4*d^4*f*g*e^4 - (5*d^2*g^2*e^6 + 6
*d*f*g*e^7 + f^2*e^8)*x^3 - 2*(3*d^3*g^2*e^5 + 4*d^2*f*g*e^6 + d*f^2*e^7)*x
^2 + (3*d^4*g^2*e^4 + 2*d^3*f*g*e^5 - d^2*f^2*e^6)*x)*e^(-6)/(x^2*e^2 - d^2
)^2
```

3.573 $\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal. Leaf size=61

$$-\frac{2g(dg+ef)}{e^3(d-ex)}+\frac{(dg+ef)^2}{2e^3(d-ex)^2}-\frac{g^2\log(d-ex)}{e^3}$$

```
[Out] (e*f + d*g)^2/(2*e^3*(d - e*x)^2) - (2*g*(e*f + d*g))/(e^3*(d - e*x)) - (g^
2*Log[d - e*x])/e^3
```

Rubi [A] time = 0.0597248, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {848, 43}

$$-\frac{2g(dg+ef)}{e^3(d-ex)}+\frac{(dg+ef)^2}{2e^3(d-ex)^2}-\frac{g^2\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(2*e^3*(d - e*x)^2) - (2*g*(e*f + d*g))/(e^3*(d - e*x)) - (g^
2*Log[d - e*x])/e^3
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3}\,dx\\ &= \int \left(\frac{(ef+dg)^2}{e^2(d-ex)^3}-\frac{2g(ef+dg)}{e^2(d-ex)^2}+\frac{g^2}{e^2(d-ex)}\right)dx\\ &= \frac{(ef+dg)^2}{2e^3(d-ex)^2}-\frac{2g(ef+dg)}{e^3(d-ex)}-\frac{g^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A] time = 0.025836, size = 49, normalized size = 0.8

$$\frac{\frac{(dg+ef)(e(f+4gx)-3dg)}{(d-ex)^2}-2g^2\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (((e*f + d*g)*(-3*d*g + e*(f + 4*g*x)))/(d - e*x)^2 - 2*g^2*Log[d - e*x])/(
2*e^3)
```

Maple [A] time = 0.05, size = 105, normalized size = 1.7

$$-\frac{\ln(ex-d)\,g^2}{e^3}+\frac{d^2g^2}{2\,e^3\,(ex-d)^2}+\frac{dfg}{e^2\,(ex-d)^2}+\frac{f^2}{2\,e\,(ex-d)^2}+2\,\frac{g^2d}{e^3\,(ex-d)}+2\,\frac{fg}{e^2\,(ex-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/e^3*ln(e*x-d)*g^2+1/2/e^3/(e*x-d)^2*d^2*g^2+1/e^2/(e*x-d)^2*d*f*g+1/2/e/
(e*x-d)^2*f^2+2*d/e^3/(e*x-d)*g^2+2/e^2/(e*x-d)*f*g
```

Maxima [A] time = 1.00053, size = 109, normalized size = 1.79

$$\frac{e^2f^2-2\,defg-3\,d^2g^2+4\left(e^2fg+deg^2\right)x}{2\left(e^5x^2-2\,de^4x+d^2e^3\right)}-\frac{g^2\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x)/(e^5*x^2 -
2*d*e^4*x + d^2*e^3) - g^2*log(e*x - d)/e^3
```

Fricas [A] time = 1.72297, size = 205, normalized size = 3.36

$$\frac{e^2f^2-2\,defg-3\,d^2g^2+4\left(e^2fg+deg^2\right)x-2\left(e^2g^2x^2-2\,deg^2x+d^2g^2\right)\log(ex-d)}{2\left(e^5x^2-2\,de^4x+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x - 2*(e^2*g^2
*x^2 - 2*d*e*g^2*x + d^2*g^2)*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)
```

Sympy [A] time = 0.708844, size = 80, normalized size = 1.31

$$\frac{-3d^2g^2-2defg+e^2f^2+x\left(4deg^2+4e^2fg\right)}{2d^2e^3-4de^4x+2e^5x^2}-\frac{g^2\log(-d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (-3*d**2*g**2 - 2*d*e*f*g + e**2*f**2 + x*(4*d*e*g**2 + 4*e**2*f*g))/(2*d**
2*e**3 - 4*d*e**4*x + 2*e**5*x**2) - g**2*log(-d + e*x)/e**3
```

Giac [B] time = 1.17624, size = 263, normalized size = 4.31

$$-\frac{dg^2e^{(-3)}\log\left(\frac{|2\,xe^2-2\,|d||e||}{|2\,xe^2+2\,|d||e||}\right)}{2\,|d|} - \frac{1}{2}\,g^2e^{(-3)}\log\left(\left|x^2e^2-d^2\right|\right) + \frac{\left(4\left(d^2g^2e^4+dfge^5\right)x^3+\left(5\,d^3g^2e^3+6\,d^2fge^4+df^2e^5\right)x^2-2\left(d^4\right.\right.}{2\left(x^2e^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*d*g^2*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/
abs(d) - 1/2*g^2*e^(-3)*log(abs(x^2*e^2 - d^2)) + 1/2*(4*(d^2*g^2*e^4 + d*f
*g*e^5)*x^3 + (5*d^3*g^2*e^3 + 6*d^2*f*g*e^4 + d*f^2*e^5)*x^2 - 2*(d^4*g^2*
e^2 - d^2*f^2*e^4)*x - (3*d^5*g^2*e^3 + 2*d^4*f*g*e^4 - d^3*f^2*e^5)*e^(-2)
)*e^(-4)/((x^2*e^2 - d^2)^2*d)
```

3.574 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=88

$$\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

```
[Out] (e*f + d*g)^2/(4*d*e^3*(d - e*x)^2) + ((e*f - 3*d*g)*(e*f + d*g))/(4*d^2*e^
3*(d - e*x)) + ((e*f - d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rubi [A] time = 0.0985019, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(4*d*e^3*(d - e*x)^2) + ((e*f - 3*d*g)*(e*f + d*g))/(4*d^2*e^
3*(d - e*x)) + ((e*f - d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx=\int \frac{(f+gx)^2}{(d-ex)^3(d+ex)}\,dx$$
$$=\int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^3}+\frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2\left(d^2-e^2x^2\right)}\right)dx$$
$$=\frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2}$$
$$=\frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

Mathematica [A] time = 0.0810217, size = 90, normalized size = 1.02

$$\frac{-\frac{2d(dg+ef)\left(2d^2g-de(2f+3gx)+e^2fx\right)}{(d-ex)^2}+(ef-dg)^2(-\log(d-ex))+(ef-dg)^2\log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((-2*d*(e*f + d*g)*(2*d^2*g + e^2*f*x - d*e*(2*f + 3*g*x)))/(d - e*x)^2 - (
e*f - d*g)^2*Log[d - e*x] + (e*f - d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

Maple [B] time = 0.051, size = 218, normalized size = 2.5

$$\frac{3g^2}{4e^3(ex-d)}+\frac{fg}{2de^2(ex-d)}-\frac{f^2}{4d^2e(ex-d)}+\frac{g^2d}{4e^3(ex-d)^2}+\frac{fg}{2e^2(ex-d)^2}+\frac{f^2}{4de(ex-d)^2}-\frac{\ln(ex-d)g^2}{8de^3}+\frac{\ln(ex-}{4d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 3/4/e^3/(e*x-d)*g^2+1/2/d/e^2/(e*x-d)*f*g-1/4/d^2/e/(e*x-d)*f^2+1/4*d/e^3/(
e*x-d)^2*g^2+1/2/e^2/(e*x-d)^2*f*g+1/4/d/e/(e*x-d)^2*f^2-1/8/e^3/d*ln(e*x-d
)*g^2+1/4/e^2/d^2*ln(e*x-d)*f*g-1/8/e/d^3*ln(e*x-d)*f^2+1/8/e^3/d*ln(e*x+d)
*g^2-1/4/e^2/d^2*ln(e*x+d)*f*g+1/8/e/d^3*ln(e*x+d)*f^2
```

Maxima [A] time = 1.06183, size = 203, normalized size = 2.31

$$\frac{2de^2f^2-2d^3g^2-\left(e^3f^2-2de^2fg-3d^2eg^2\right)x}{4\left(d^2e^5x^2-2d^3e^4x+d^4e^3\right)}+\frac{\left(e^2f^2-2defg+d^2g^2\right)\log(ex+d)}{8d^3e^3}-\frac{\left(e^2f^2-2defg+d^2g^2\right)\log(ex-}{8d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*e^2*f^2 - 2*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)/(d^
2*e^5*x^2 - 2*d^3*e^4*x + d^4*e^3) + 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*lo
g(e*x + d)/(d^3*e^3) - 1/8*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x - d)/(d^
```

```
3*e^3)
```

Fricas [B] time = 1.80241, size = 547, normalized size = 6.22

$$\frac{4\,d^{2}e^{2}f^{2}-4\,d^{4}g^{2}-2\left(de^{3}f^{2}-2\,d^{2}e^{2}fg-3\,d^{3}eg^{2}\right)x+\left(d^{2}e^{2}f^{2}-2\,d^{3}efg+d^{4}g^{2}+\left(e^{4}f^{2}-2\,de^{3}fg+d^{2}e^{2}g^{2}\right)x^{2}-2\left(de\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*d^2*e^2*f^2 - 4*d^4*g^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g - 3*d^3*e*g^2
)*x + (d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e
^2*g^2)*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x + d) - (
d^2*e^2*f^2 - 2*d^3*e*f*g + d^4*g^2 + (e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)
*x^2 - 2*(d*e^3*f^2 - 2*d^2*e^2*f*g + d^3*e*g^2)*x)*log(e*x - d))/(d^3*e^5*
x^2 - 2*d^4*e^4*x + d^5*e^3)
```

Sympy [B] time = 1.21287, size = 185, normalized size = 2.1

$$\frac{-2d^{3}g^{2}+2de^{2}f^{2}+x\left(3d^{2}eg^{2}+2de^{2}fg-e^{3}f^{2}\right)}{4d^{4}e^{3}-8d^{3}e^{4}x+4d^{2}e^{5}x^{2}}-\frac{\left(dg-ef\right)^{2}\log\left(-\frac{d\left(dg-ef\right)^{2}}{e\left(d^{2}g^{2}-2defg+e^{2}f^{2}\right)}+x\right)}{8d^{3}e^{3}}+\frac{\left(dg-ef\right)^{2}\log\left(\frac{d\left(dg-e\right.}{e\left(d^{2}g^{2}-2def\right.}\right.}{8d^{3}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (-2*d**3*g**2 + 2*d*e**2*f**2 + x*(3*d**2*e*g**2 + 2*d*e**2*f*g - e**3*f**2
))/(4*d**4*e**3 - 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g - e*f)**2*log(-d
*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3)
+ (d*g - e*f)**2*log(d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2
)) + x)/(8*d**3*e**3)
```

Giac [B] time = 1.15882, size = 266, normalized size = 3.02

$$-\frac{\left(d^{2}g^{2}e^{2}-2\,dfge^{3}+f^{2}e^{4}\right)e^{(-5)}\log\left(\frac{|2\,xe^{2}-2\,|d|e|}{|2\,xe^{2}+2\,|d|e|}\right)}{8\,d^{2}|d|}+\frac{\left(3\,d^{2}g^{2}x^{3}e^{4}+4\,d^{3}g^{2}x^{2}e^{3}-d^{4}g^{2}xe^{2}-2\,d^{5}g^{2}e+2\,dfgx^{3}e^{5}+4\,d^{2}fg\right.}{4\left(x^{2}e^{2}-d^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*e^(-5)*log(abs(2*x*e^2 - 2*abs(d
)*e)/abs(2*x*e^2 + 2*abs(d)*e))/(d^2*abs(d)) + 1/4*(3*d^2*g^2*x^3*e^4 + 4*d
^3*g^2*x^2*e^3 - d^4*g^2*x*e^2 - 2*d^5*g^2*e + 2*d*f*g*x^3*e^5 + 4*d^2*f*g*
x^2*e^4 + 2*d^3*f*g*x*e^3 - f^2*x^3*e^6 + 3*d^2*f^2*x*e^4 + 2*d^3*f^2*e^3)*
e^(-4)/((x^2*e^2 - d^2)^2*d^2)
```

3.575 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3} \, dx$

Optimal. Leaf size=122

$$\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2}+\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

```
[Out] (e*f + d*g)^2/(8*d^2*e^3*(d - e*x)^2) + (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d -
 e*x)) - (e*f - d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f - d*g)*(3*e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rubi [A] time = 0.11539, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {799, 88, 208}

$$\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2}+\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(8*d^2*e^3*(d - e*x)^2) + (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d -
 e*x)) - (e*f - d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f - d*g)*(3*e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 799

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c*x)/g)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^2}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)^3}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d-ex)^2}+\frac{(-ef+dg)^2}{8d^3e^2(d+ex)^2}+\frac{(ef-dg)(3ef+dg)}{8d^3e^2\left(d^2-e^2x^2\right)}\right)dx \\
&= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{((ef-dg)(3ef+dg))\int \frac{1}{d^2-e^2x^2}\,dx}{8d^3e^2} \\
&= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(ef-dg)(3ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}
\end{aligned}
$$

Mathematica [A] time = 0.104088, size = 140, normalized size = 1.15

$$
\frac{\frac{4de^2f^2-4d^3g^2}{d-ex}+\left(d^2g^2+2defg-3e^2f^2\right)\log(d-ex)+\left(-d^2g^2-2defg+3e^2f^2\right)\log(d+ex)+\frac{2d^2(dg+ef)^2}{(d-ex)^2}-\frac{2d(ef-dg)^2}{d+ex}}{16d^4e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((2*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (4*d*e^2*f^2 - 4*d^3*g^2)/(d - e*x) -
(2*d*(e*f - d*g)^2)/(d + e*x) + (-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2)*Log[d -
e*x] + (3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

Maple [B] time = 0.055, size = 257, normalized size = 2.1

$$
\frac{g^2}{4\,de^3\,(ex-d)}-\frac{f^2}{4\,ed^3\,(ex-d)}+\frac{g^2}{8\,e^3\,(ex-d)^2}+\frac{fg}{4\,de^2\,(ex-d)^2}+\frac{f^2}{8\,ed^2\,(ex-d)^2}+\frac{\ln(ex-d)\,g^2}{16\,e^3d^2}+\frac{\ln(ex-d)\,fg}{8\,e^2d^3}-3
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/4/e^3/d/(e*x-d)*g^2-1/4/e/d^3/(e*x-d)*f^2+1/8/e^3/(e*x-d)^2*g^2+1/4/d/e^2
/(e*x-d)^2*f*g+1/8/d^2/e/(e*x-d)^2*f^2+1/16/e^3/d^2*ln(e*x-d)*g^2+1/8/e^2/d
^3*ln(e*x-d)*f*g-3/16/e/d^4*ln(e*x-d)*f^2-1/16/e^3/d^2*ln(e*x+d)*g^2-1/8/e^
2/d^3*ln(e*x+d)*f*g+3/16/e/d^4*ln(e*x+d)*f^2-1/8/d/e^3/(e*x+d)*g^2+1/4/d^2/
e^2/(e*x+d)*f*g-1/8/d^3/e/(e*x+d)*f^2
```

Maxima [A] time = 1.03317, size = 285, normalized size = 2.34

$$
\frac{2\,d^2e^2f^2+4\,d^3efg-2\,d^4g^2-\left(3\,e^4f^2-2\,de^3fg-d^2e^2g^2\right)x^2+\left(3\,de^3f^2-2\,d^2e^2fg+3\,d^3eg^2\right)x}{8\left(d^3e^6x^3-d^4e^5x^2-d^5e^4x+d^6e^3\right)}+\frac{\left(3\,e^2f^2-2\,defg-d\right.}{16\,d^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*d^2*e^2*f^2 + 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 - 2*d*e^3*f*g - d
^2*e^2*g^2)*x^2 + (3*d*e^3*f^2 - 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x
```

```
^3 - d^4*e^5*x^2 - d^5*e^4*x + d^6*e^3) + 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2
*g^2)*log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*log(e
*x - d)/(d^4*e^3)
```

Fricas [B] time = 1.71767, size = 810, normalized size = 6.64

$$\frac{4\,d^3e^2f^2+8\,d^4efg-4\,d^5g^2-2\left(3\,de^4f^2-2\,d^2e^3fg-d^3e^2g^2\right)x^2+2\left(3\,d^2e^3f^2-2\,d^3e^2fg+3\,d^4eg^2\right)x+\left(3\,d^3e^2f^2-2\,d^4e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d^3*e^2*f^2 + 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 - 2*d^2*e^3*
f*g - d^3*e^2*g^2)*x^2 + 2*(3*d^2*e^3*f^2 - 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x
+ (3*d^3*e^2*f^2 - 2*d^4*e*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e
^3*g^2)*x^3 - (3*d*e^4*f^2 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*
f^2 - 2*d^3*e^2*f*g - d^4*e*g^2)*x)*log(e*x + d) - (3*d^3*e^2*f^2 - 2*d^4*e
*f*g - d^5*g^2 + (3*e^5*f^2 - 2*d*e^4*f*g - d^2*e^3*g^2)*x^3 - (3*d*e^4*f^2
 - 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - (3*d^2*e^3*f^2 - 2*d^3*e^2*f*g - d^4*
e*g^2)*x)*log(e*x - d))/(d^4*e^6*x^3 - d^5*e^5*x^2 - d^6*e^4*x + d^7*e^3)
```

Sympy [B] time = 1.45898, size = 277, normalized size = 2.27

$$\frac{-2d^4g^2+4d^3efg+2d^2e^2f^2+x^2\left(d^2e^2g^2+2de^3fg-3e^4f^2\right)+x\left(3d^3eg^2-2d^2e^2fg+3de^3f^2\right)}{8d^6e^3-8d^5e^4x-8d^4e^5x^2+8d^3e^6x^3}+\frac{\left(dg-ef\right)\left(dg+3ef\right)\log}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (-2*d**4*g**2 + 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 + 2*
d*e**3*f*g - 3*e**4*f**2) + x*(3*d**3*e*g**2 - 2*d**2*e**2*f*g + 3*d*e**3*f
**2))/(8*d**6*e**3 - 8*d**5*e**4*x - 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3) +
 (d*g - e*f)*(d*g + 3*e*f)*log(-d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 +
 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - e*f)*(d*g + 3*e*f)*
log(d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 2*d*e*f*g - 3*e**2*f**2)) +
 x)/(16*d**4*e**3)
```

Giac [A] time = 1.14288, size = 258, normalized size = 2.11

$$\frac{\left(d^2g^2+2\,dfge-3\,f^2e^2\right)e^{(-3)}\log\left(\frac{|2\,xe^2-2\,|d|e|}{|2\,xe^2+2\,|d|e|}\right)}{16\,d^3|d|}+\frac{\left(d^2g^2x^3e^4+4\,d^3g^2x^2e^3+d^4g^2xe^2-2\,d^5g^2e+2\,dfgx^3e^5+2\,d^3fgxe^3+4\right.}{8\left(x^2e^2-d^2\right)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(d^2*g^2 + 2*d*f*g*e - 3*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)
/abs(2*x*e^2 + 2*abs(d)*e))/(d^3*abs(d)) + 1/8*(d^2*g^2*x^3*e^4 + 4*d^3*g^2
```

```
*x^2*e^3 + d^4*g^2*x*e^2 - 2*d^5*g^2*e + 2*d*f*g*x^3*e^5 + 2*d^3*f*g*x*e^3
+ 4*d^4*f*g*e^2 - 3*f^2*x^3*e^6 + 5*d^2*f^2*x*e^4 + 2*d^3*f^2*e^3)*e^(-4)/(
(x^2*e^2 - d^2)^2*d^3)
```

3.576 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=127

$$\frac{x\left(3e^2f^2-d^2g^2\right)+2d^2fg}{8d^4e^2\left(d^2-e^2x^2\right)}+\frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}+\frac{(f+gx)\left(d^2g+e^2fx\right)}{4d^2e^2\left(d^2-e^2x^2\right)^2}$$

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(4*d^2*e^2*(d^2 - e^2*x^2)^2) + (2*d^2*f*g +
(3*e^2*f^2 - d^2*g^2)*x)/(8*d^4*e^2*(d^2 - e^2*x^2)) + ((3*e^2*f^2 - d^2*g^
2)*ArcTanh[(e*x)/d])/(8*d^5*e^3)
```

Rubi [A] time = 0.0614481, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {739, 639, 208}

$$\frac{x\left(3e^2f^2-d^2g^2\right)+2d^2fg}{8d^4e^2\left(d^2-e^2x^2\right)}+\frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}+\frac{(f+gx)\left(d^2g+e^2fx\right)}{4d^2e^2\left(d^2-e^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(4*d^2*e^2*(d^2 - e^2*x^2)^2) + (2*d^2*f*g +
(3*e^2*f^2 - d^2*g^2)*x)/(8*d^4*e^2*(d^2 - e^2*x^2)) + ((3*e^2*f^2 - d^2*g^
2)*ArcTanh[(e*x)/d])/(8*d^5*e^3)
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx = \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2} - \frac{\int \frac{-3e^2f^2+d^2g^2-2e^2fgx}{\left(d^2-e^2x^2\right)^2}\,dx}{4d^2e^2}$$

$$= \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2} + \frac{2d^2fg+\left(3e^2f^2-d^2g^2\right)x}{8d^4e^2\left(d^2-e^2x^2\right)} - \frac{\left(-\frac{3e^2f^2}{d^2}+g^2\right)\int \frac{1}{d^2-e^2x^2}\,dx}{8d^2e^2}$$

$$= \frac{\left(d^2g+e^2fx\right)(f+gx)}{4d^2e^2\left(d^2-e^2x^2\right)^2} + \frac{2d^2fg+\left(3e^2f^2-d^2g^2\right)x}{8d^4e^2\left(d^2-e^2x^2\right)} + \frac{\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}$$

Mathematica [A] time = 0.044333, size = 110, normalized size = 0.87

$$\frac{d^3e^3x\left(5f^2+g^2x^2\right)+\left(d^2-e^2x^2\right)^2\left(3e^2f^2-d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)+d^5eg(4f+gx)-3de^5f^2x^3}{8d^5e^3\left(d^2-e^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (-3*d*e^5*f^2*x^3 + d^5*e*g*(4*f + g*x) + d^3*e^3*x*(5*f^2 + g^2*x^2) + (3*
e^2*f^2 - d^2*g^2)*(d^2 - e^2*x^2)^2*ArcTanh[(e*x)/d])/(8*d^5*e^3*(d^2 - e^
2*x^2)^2)
```

Maple [B] time = 0.057, size = 298, normalized size = 2.4

$$\frac{\ln(ex-d)\,g^2}{16\,e^3d^3} - \frac{3\,\ln(ex-d)\,f^2}{16\,ed^5} + \frac{g^2}{16\,e^3d\,(ex-d)^2} + \frac{fg}{8\,d^2e^2\,(ex-d)^2} + \frac{f^2}{16\,ed^3\,(ex-d)^2} + \frac{g^2}{16\,e^3d^2\,(ex-d)} - \frac{fg}{8\,e^2d^3\,(ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/16/e^3/d^3*ln(e*x-d)*g^2-3/16/e/d^5*ln(e*x-d)*f^2+1/16/e^3/d/(e*x-d)^2*g^
2+1/8/e^2/d^2/(e*x-d)^2*f*g+1/16/e/d^3/(e*x-d)^2*f^2+1/16/e^3/d^2/(e*x-d)*g
^2-1/8/e^2/d^3/(e*x-d)*f*g-3/16/e/d^4/(e*x-d)*f^2-1/16/e^3/d^3*ln(e*x+d)*g^
2+3/16/e/d^5*ln(e*x+d)*f^2+1/16/d^2/e^3/(e*x+d)*g^2+1/8/d^3/e^2/(e*x+d)*f*g
-3/16/d^4/e/(e*x+d)*f^2-1/16/d/e^3/(e*x+d)^2*g^2+1/8/d^2/e^2/(e*x+d)^2*f*g-
1/16/d^3/e/(e*x+d)^2*f^2
```

Maxima [A] time = 0.987638, size = 205, normalized size = 1.61

$$\frac{4\,d^4fg-\left(3\,e^4f^2-d^2e^2g^2\right)x^3+\left(5\,d^2e^2f^2+d^4g^2\right)x}{8\left(d^4e^6x^4-2\,d^6e^4x^2+d^8e^2\right)} + \frac{\left(3\,e^2f^2-d^2g^2\right)\log(ex+d)}{16\,d^5e^3} - \frac{\left(3\,e^2f^2-d^2g^2\right)\log(ex-d)}{16\,d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*d^4*f*g - (3*e^4*f^2 - d^2*e^2*g^2)*x^3 + (5*d^2*e^2*f^2 + d^4*g^2)*
x)/(d^4*e^6*x^4 - 2*d^6*e^4*x^2 + d^8*e^2) + 1/16*(3*e^2*f^2 - d^2*g^2)*log
(e*x + d)/(d^5*e^3) - 1/16*(3*e^2*f^2 - d^2*g^2)*log(e*x - d)/(d^5*e^3)
```

Fricas [B] time = 1.66353, size = 475, normalized size = 3.74

$$\frac{8\,d^5efg - 2\left(3\,de^5f^2 - d^3e^3g^2\right)x^3 + 2\left(5\,d^3e^3f^2 + d^5eg^2\right)x + \left(3\,d^4e^2f^2 - d^6g^2 + \left(3\,e^6f^2 - d^2e^4g^2\right)x^4 - 2\left(3\,d^2e^4f^2 - d^4e^2g^2\right.\right.}{16\left(d^5e^7x^4 - 2\,d^7e^5x^2 + d^9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(8*d^5*e*f*g - 2*(3*d*e^5*f^2 - d^3*e^3*g^2)*x^3 + 2*(5*d^3*e^3*f^2 +
d^5*e*g^2)*x + (3*d^4*e^2*f^2 - d^6*g^2 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2
*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log(e*x + d) - (3*d^4*e^2*f^2 - d^6*g^2
 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)*log
(e*x - d))/(d^5*e^7*x^4 - 2*d^7*e^5*x^2 + d^9*e^3)
```

Sympy [A] time = 1.13298, size = 143, normalized size = 1.13

$$\frac{4d^4fg + x^3\left(d^2e^2g^2 - 3e^4f^2\right) + x\left(d^4g^2 + 5d^2e^2f^2\right)}{8d^8e^2 - 16d^6e^4x^2 + 8d^4e^6x^4} + \frac{\left(d^2g^2 - 3e^2f^2\right)\log\left(-\frac{d}{e} + x\right)}{16d^5e^3} - \frac{\left(d^2g^2 - 3e^2f^2\right)\log\left(\frac{d}{e} + x\right)}{16d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (4*d**4*f*g + x**3*(d**2*e**2*g**2 - 3*e**4*f**2) + x*(d**4*g**2 + 5*d**2*e
**2*f**2))/(8*d**8*e**2 - 16*d**6*e**4*x**2 + 8*d**4*e**6*x**4) + (d**2*g**
2 - 3*e**2*f**2)*log(-d/e + x)/(16*d**5*e**3) - (d**2*g**2 - 3*e**2*f**2)*l
og(d/e + x)/(16*d**5*e**3)
```

Giac [A] time = 1.13916, size = 171, normalized size = 1.35

$$\frac{\left(d^2g^2 - 3\,f^2e^2\right)e^{(-3)}\log\left(\frac{|2\,xe^2 - 2\,|d|e|}{|2\,xe^2 + 2\,|d|e|}\right)}{16\,d^4|d|} + \frac{\left(d^2g^2x^3e^2 + d^4g^2x + 4\,d^4fg - 3\,f^2x^3e^4 + 5\,d^2f^2xe^2\right)e^{(-2)}}{8\left(x^2e^2 - d^2\right)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(d^2*g^2 - 3*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2
 + 2*abs(d)*e))/(d^4*abs(d)) + 1/8*(d^2*g^2*x^3*e^2 + d^4*g^2*x + 4*d^4*f*g
 - 3*f^2*x^3*e^4 + 5*d^2*f^2*x*e^2)*e^(-2)/((x^2*e^2 - d^2)^2*d^4)
```

3.577 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=188

$$-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}+\frac{\left(-d^2g^2+2defg+5e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2}+\frac{f(dg+ef)}{8d^5e^2(d-ex)}+$$

```
[Out] (e*f + d*g)^2/(32*d^4*e^3*(d - e*x)^2) + (f*(e*f + d*g))/(8*d^5*e^2*(d - e*
x)) - (e*f - d*g)^2/(24*d^3*e^3*(d + e*x)^3) - ((e*f - d*g)*(3*e*f + d*g))/
(32*d^4*e^3*(d + e*x)^2) - (3*e^2*f^2 - d^2*g^2)/(16*d^5*e^3*(d + e*x)) + (
(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(16*d^6*e^3)
```

Rubi [A] time = 0.209689, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}+\frac{\left(-d^2g^2+2defg+5e^2f^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2}+\frac{f(dg+ef)}{8d^5e^2(d-ex)}+$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(32*d^4*e^3*(d - e*x)^2) + (f*(e*f + d*g))/(8*d^5*e^2*(d - e*
x)) - (e*f - d*g)^2/(24*d^3*e^3*(d + e*x)^3) - ((e*f - d*g)*(3*e*f + d*g))/
(32*d^4*e^3*(d + e*x)^2) - (3*e^2*f^2 - d^2*g^2)/(16*d^5*e^3*(d + e*x)) + (
(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(16*d^6*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx = \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^4}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^3} + \frac{f(ef+dg)}{8d^5e(d-ex)^2} + \frac{(-ef+dg)^2}{8d^3e^2(d+ex)^4} + \frac{(ef-dg)(3ef+dg)}{16d^4e^2(d+ex)^3} + \frac{3e^2f^2-d\ldots}{16d^5e^2(d+\ldots}\right.$$
$$= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2} + \frac{f(ef+dg)}{8d^5e^2(d-ex)} - \frac{(ef-dg)^2}{24d^3e^3(d+ex)^3} - \frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2} - \frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex\ldots}$$
$$= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2} + \frac{f(ef+dg)}{8d^5e^2(d-ex)} - \frac{(ef-dg)^2}{24d^3e^3(d+ex)^3} - \frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2} - \frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex\ldots}$$

Mathematica [A] time = 0.157574, size = 197, normalized size = 1.05

$$\frac{\frac{3d^2\left(d^2g^2+2defg-3e^2f^2\right)}{(d+ex)^2} + \frac{6d\left(d^2g^2-3e^2f^2\right)}{d+ex} + 3\left(d^2g^2-2defg-5e^2f^2\right)\log(d-ex) + 3\left(-d^2g^2+2defg+5e^2f^2\right)\log(d+ex) - \frac{4d\ldots}{\ldots}}{96d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((3*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (12*d*e*f*(e*f + d*g))/(d - e*x) - (4*
d^3*(e*f - d*g)^2)/(d + e*x)^3 + (3*d^2*(-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))
/(d + e*x)^2 + (6*d*(-3*e^2*f^2 + d^2*g^2))/(d + e*x) + 3*(-5*e^2*f^2 - 2*d
*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d
+ e*x])/(96*d^6*e^3)
```

Maple [A] time = 0.058, size = 348, normalized size = 1.9

$$\frac{g^2}{32\,d^2e^3\,(ex-d)^2} + \frac{fg}{16\,e^2d^3\,(ex-d)^2} + \frac{f^2}{32\,ed^4\,(ex-d)^2} + \frac{\ln(ex-d)\,g^2}{32\,e^3d^4} - \frac{\ln(ex-d)\,fg}{16\,e^2d^5} - \frac{5\,\ln(ex-d)\,f^2}{32\,ed^6} - \frac{fg}{8\,e^2d^4\,(ex-\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x)
```

```
[Out] 1/32/e^3/d^2/(e*x-d)^2*g^2+1/16/e^2/d^3/(e*x-d)^2*f*g+1/32/e/d^4/(e*x-d)^2*
f^2+1/32/e^3/d^4*ln(e*x-d)*g^2-1/16/e^2/d^5*ln(e*x-d)*f*g-5/32/e/d^6*ln(e*x
-d)*f^2-1/8/e^2/d^4/(e*x-d)*f*g-1/8/e/d^5/(e*x-d)*f^2+1/16/e^3/d^3/(e*x+d)*
g^2-3/16/e*f^2/d^5/(e*x+d)+1/32/e^3/d^2/(e*x+d)^2*g^2+1/16/e^2/d^3/(e*x+d)^
2*f*g-3/32/e/d^4/(e*x+d)^2*f^2-1/32/e^3/d^4*ln(e*x+d)*g^2+1/16/e^2/d^5*ln(e
*x+d)*f*g+5/32/e/d^6*ln(e*x+d)*f^2-1/24/e^3/d/(e*x+d)^3*g^2+1/12/e^2/d^2/(e
*x+d)^3*f*g-1/24/e/d^3/(e*x+d)^3*f^2
```

Maxima [A] time = 1.07283, size = 416, normalized size = 2.21

$$-\frac{8\,d^4e^2f^2 - 16\,d^5efg - 4\,d^6g^2 + 3\left(5\,e^6f^2 + 2\,de^5fg - d^2e^4g^2\right)x^4 + 3\left(5\,de^5f^2 + 2\,d^2e^4fg - d^3e^3g^2\right)x^3 - 5\left(5\,d^2e^4f^2 + 2\,d^3\ldots\right.}{48\left(d^5e^8x^5 + d^6e^7x^4 - 2\,d^7e^6x^3 - 2\,d^8e^5x^2 + d^9e^4x + d^{10}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(8*d^4*e^2*f^2 - 16*d^5*e*f*g - 4*d^6*g^2 + 3*(5*e^6*f^2 + 2*d*e^5*f*
g - d^2*e^4*g^2)*x^4 + 3*(5*d*e^5*f^2 + 2*d^2*e^4*f*g - d^3*e^3*g^2)*x^3 -
5*(5*d^2*e^4*f^2 + 2*d^3*e^3*f*g - d^4*e^2*g^2)*x^2 - (25*d^3*e^3*f^2 + 10*
d^4*e^2*f*g + 7*d^5*e*g^2)*x)/(d^5*e^8*x^5 + d^6*e^7*x^4 - 2*d^7*e^6*x^3 -
2*d^8*e^5*x^2 + d^9*e^4*x + d^10*e^3) + 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g
^2)*log(e*x + d)/(d^6*e^3) - 1/32*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*log(e*x
 - d)/(d^6*e^3)
```

Fricas [B] time = 1.79197, size = 1307, normalized size = 6.95

$$-\frac{16\,d^5e^2f^2-32\,d^6efg-8\,d^7g^2+6\left(5\,de^6f^2+2\,d^2e^5fg-d^3e^4g^2\right)x^4+6\left(5\,d^2e^5f^2+2\,d^3e^4fg-d^4e^3g^2\right)x^3-10\left(5\,d^3e^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(16*d^5*e^2*f^2 - 32*d^6*e*f*g - 8*d^7*g^2 + 6*(5*d*e^6*f^2 + 2*d^2*e
^5*f*g - d^3*e^4*g^2)*x^4 + 6*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)
*x^3 - 10*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - 2*(25*d^4*e^3
*f^2 + 10*d^5*e^2*f*g + 7*d^6*e*g^2)*x - 3*(5*d^5*e^2*f^2 + 2*d^6*e*f*g - d
^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^2 + 2*d^2
*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^
2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*d^4*e^3*f
^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x + d) + 3*(5*d^5*e^2*f^2 + 2*d^6*
e*f*g - d^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^
2 + 2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d
^4*e^3*g^2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*
d^4*e^3*f^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x)*log(e*x - d))/(d^6*e^8*x^5 + d^
7*e^7*x^4 - 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 + d^10*e^4*x + d^11*e^3)
```

Sympy [A] time = 2.1155, size = 320, normalized size = 1.7

$$\frac{4d^6g^2+16d^5efg-8d^4e^2f^2+x^4\left(3d^2e^4g^2-6de^5fg-15e^6f^2\right)+x^3\left(3d^3e^3g^2-6d^2e^4fg-15de^5f^2\right)+x^2\left(-5d^4e^2g^2+1\right.}{48d^{10}e^3+48d^9e^4x-96d^8e^5x^2-96d^7e^6x^3+48d^6e^7x^4+48d^5e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (4*d**6*g**2 + 16*d**5*e*f*g - 8*d**4*e**2*f**2 + x**4*(3*d**2*e**4*g**2 -
6*d*e**5*f*g - 15*e**6*f**2) + x**3*(3*d**3*e**3*g**2 - 6*d**2*e**4*f*g - 1
5*d*e**5*f**2) + x**2*(-5*d**4*e**2*g**2 + 10*d**3*e**3*f*g + 25*d**2*e**4*
f**2) + x*(7*d**5*e*g**2 + 10*d**4*e**2*f*g + 25*d**3*e**3*f**2))/(48*d**10
*e**3 + 48*d**9*e**4*x - 96*d**8*e**5*x**2 - 96*d**7*e**6*x**3 + 48*d**6*e*
*7*x**4 + 48*d**5*e**8*x**5) + (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*log(-d
/e + x)/(32*d**6*e**3) - (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*log(d/e + x)
/(32*d**6*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.578 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3} dx$

Optimal. Leaf size=235

$$-\frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)}-\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}+\frac{(-d^2g^2+10defg+15e^2f^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{64d^7e^3}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(dg+3e}{48d^4e^3}$$

```
[Out] (e*f + d*g)^2/(64*d^5*e^3*(d - e*x)^2) + ((e*f + d*g)*(5*e*f + d*g))/(64*d^
6*e^3*(d - e*x)) - (e*f - d*g)^2/(32*d^3*e^3*(d + e*x)^4) - ((e*f - d*g)*(3
*e*f + d*g))/(48*d^4*e^3*(d + e*x)^3) - (3*e^2*f^2 - d^2*g^2)/(32*d^5*e^3*(
d + e*x)^2) - (5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)/(32*d^6*e^3*(d + e*x)) + ((
15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(64*d^7*e^3)
```

Rubi [A] time = 0.273199, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {848, 88, 208}

$$-\frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)}-\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}+\frac{(-d^2g^2+10defg+15e^2f^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{64d^7e^3}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(dg+3e}{48d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(64*d^5*e^3*(d - e*x)^2) + ((e*f + d*g)*(5*e*f + d*g))/(64*d^
6*e^3*(d - e*x)) - (e*f - d*g)^2/(32*d^3*e^3*(d + e*x)^4) - ((e*f - d*g)*(3
*e*f + d*g))/(48*d^4*e^3*(d + e*x)^3) - (3*e^2*f^2 - d^2*g^2)/(32*d^5*e^3*(
d + e*x)^2) - (5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)/(32*d^6*e^3*(d + e*x)) + ((
15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(64*d^7*e^3)
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx = \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^5}\,dx$$

$$= \int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^3} + \frac{(ef+dg)(5ef+dg)}{64d^6e^2(d-ex)^2} + \frac{(-ef+dg)^2}{8d^3e^2(d+ex)^5} + \frac{(ef-dg)(3ef+dg)}{16d^4e^2(d+ex)^4} + \frac{3e}{16c}\right.$$

$$= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2} + \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3} - \frac{3e^2f}{32d^5e^3}$$

$$= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2} + \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3} - \frac{3e^2f}{32d^5e^3}$$

Mathematica [A] time = 0.173548, size = 244, normalized size = 1.04

$$\frac{\frac{8d^3\left(d^2g^2+2defg-3e^2f^2\right)}{(d+ex)^3} + \frac{12d^2\left(d^2g^2-3e^2f^2\right)}{(d+ex)^2} + \frac{6d\left(d^2g^2+6defg+5e^2f^2\right)}{d-ex} + \frac{12d\left(d^2g^2-2defg-5e^2f^2\right)}{d+ex} + 3\left(d^2g^2-10defg-15e^2f^2\right)\log(d-ex}{384d^7e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((6*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (6*d*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)
)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(-3*e^2*f^2 + 2*d
*e*f*g + d^2*g^2))/(d + e*x)^3 + (12*d^2*(-3*e^2*f^2 + d^2*g^2))/(d + e*x)^
2 + (12*d*(-5*e^2*f^2 - 2*d*e*f*g + d^2*g^2))/(d + e*x) + 3*(-15*e^2*f^2 -
10*d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*
Log[d + e*x])/(384*d^7*e^3)
```

Maple [A] time = 0.059, size = 421, normalized size = 1.8

$$-\frac{g^2}{64\,e^3d^4\,(ex-d)} - \frac{3\,fg}{32\,e^2d^5\,(ex-d)} - \frac{5\,f^2}{64\,ed^6\,(ex-d)} + \frac{g^2}{64\,d^3e^3\,(ex-d)^2} + \frac{fg}{32\,e^2d^4\,(ex-d)^2} + \frac{f^2}{64\,ed^5\,(ex-d)^2} + \frac{\ln(ex}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x)
```

```
[Out] -1/64/e^3/d^4/(e*x-d)*g^2-3/32/e^2/d^5/(e*x-d)*f*g-5/64/e/d^6/(e*x-d)*f^2+1
/64/e^3/d^3/(e*x-d)^2*g^2+1/32/e^2/d^4/(e*x-d)^2*f*g+1/64/e/d^5/(e*x-d)^2*f
^2+1/128/e^3/d^5*ln(e*x-d)*g^2-5/64/e^2/d^6*ln(e*x-d)*f*g-15/128/e/d^7*ln(e
*x-d)*f^2+1/32/e^3/d^3/(e*x+d)^2*g^2-3/32/e*f^2/d^5/(e*x+d)^2+1/48/e^3/d^2/
(e*x+d)^3*g^2+1/24/e^2/d^3/(e*x+d)^3*f*g-1/16/e/d^4/(e*x+d)^3*f^2+1/32/e^3/
d^4/(e*x+d)*g^2-1/16/e^2/d^5/(e*x+d)*f*g-5/32/e/d^6/(e*x+d)*f^2-1/128/e^3/d
^5*ln(e*x+d)*g^2+5/64/e^2/d^6*ln(e*x+d)*f*g+15/128/e/d^7*ln(e*x+d)*f^2-1/32
/e^3/d/(e*x+d)^4*g^2+1/16/e^2/d^2/(e*x+d)^4*f*g-1/32/e/d^3/(e*x+d)^4*f^2
```

Maxima [A] time = 1.09176, size = 485, normalized size = 2.06

$$-\frac{48\,d^5e^2f^2 - 32\,d^6efg - 16\,d^7g^2 + 3\left(15\,e^7f^2 + 10\,de^6fg - d^2e^5g^2\right)x^5 + 6\left(15\,de^6f^2 + 10\,d^2e^5fg - d^3e^4g^2\right)x^4 - 2\left(15\,d^2e^5\right.}{192\left(d^6e^9x^6 + 2\,d^7e^8x^5 - d^8e^7x^4 - 4\,d^9e^6x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(48*d^5*e^2*f^2 - 32*d^6*e*f*g - 16*d^7*g^2 + 3*(15*e^7*f^2 + 10*d*e
^6*f*g - d^2*e^5*g^2)*x^5 + 6*(15*d*e^6*f^2 + 10*d^2*e^5*f*g - d^3*e^4*g^2)
*x^4 - 2*(15*d^2*e^5*f^2 + 10*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 10*(15*d^3*e
^4*f^2 + 10*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - (51*d^4*e^3*f^2 + 34*d^5*e^2*f
*g + 35*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 2*d^7*e^8*x^5 - d^8*e^7*x^4 - 4*d^9*e^
6*x^3 - d^10*e^5*x^2 + 2*d^11*e^4*x + d^12*e^3) + 1/128*(15*e^2*f^2 + 10*d*
e*f*g - d^2*g^2)*log(e*x + d)/(d^7*e^3) - 1/128*(15*e^2*f^2 + 10*d*e*f*g -
d^2*g^2)*log(e*x - d)/(d^7*e^3)
```

Fricas [B] time = 1.85028, size = 1604, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/384*(96*d^6*e^2*f^2 - 64*d^7*e*f*g - 32*d^8*g^2 + 6*(15*d*e^7*f^2 + 10*d
^2*e^6*f*g - d^3*e^5*g^2)*x^5 + 12*(15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e
^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - 20*(1
5*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 - 2*(51*d^5*e^3*f^2 + 34*
d^6*e^2*f*g + 35*d^7*e*g^2)*x - 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^2
+ (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^2*
e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g^2
)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e^4
*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2*f
*g - d^7*e*g^2)*x)*log(e*x + d) + 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^
2 + (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^
2*e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g
^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e
^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2
*f*g - d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 2*d^8*e^8*x^5 - d^9*e^7*x
^4 - 4*d^10*e^6*x^3 - d^11*e^5*x^2 + 2*d^12*e^4*x + d^13*e^3)
```

Sympy [A] time = 2.40839, size = 371, normalized size = 1.58

$$\frac{16d^7g^2 + 32d^6efg - 48d^5e^2f^2 + x^5\left(3d^2e^5g^2 - 30de^6fg - 45e^7f^2\right) + x^4\left(6d^3e^4g^2 - 60d^2e^5fg - 90de^6f^2\right) + x^3\left(-2d^4e^3g \cdots\right.}{192d^{12}e^3 + 384d^{11}e^4x - 192d^{10}e^5x^2 - 768d^9e^6x \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (16*d**7*g**2 + 32*d**6*e*f*g - 48*d**5*e**2*f**2 + x**5*(3*d**2*e**5*g**2
- 30*d*e**6*f*g - 45*e**7*f**2) + x**4*(6*d**3*e**4*g**2 - 60*d**2*e**5*f*g
 - 90*d*e**6*f**2) + x**3*(-2*d**4*e**3*g**2 + 20*d**3*e**4*f*g + 30*d**2*e
**5*f**2) + x**2*(-10*d**5*e**2*g**2 + 100*d**4*e**3*f*g + 150*d**3*e**4*f*
*2) + x*(35*d**6*e*g**2 + 34*d**5*e**2*f*g + 51*d**4*e**3*f**2))/(192*d**12
*e**3 + 384*d**11*e**4*x - 192*d**10*e**5*x**2 - 768*d**9*e**6*x**3 - 192*d
**8*e**7*x**4 + 384*d**7*e**8*x**5 + 192*d**6*e**9*x**6) + (d**2*g**2 - 10*
d*e*f*g - 15*e**2*f**2)*log(-d/e + x)/(128*d**7*e**3) - (d**2*g**2 - 10*d*e
```

```
*f*g - 15*e**2*f**2)*log(d/e + x)/(128*d**7*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.579 $\int \frac{(d+ex)^3(f+gx)^5}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=269

$$\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{15d^3e^6\sqrt{d^2-e^2x^2}}-\frac{g^3\left(13d^2g^2+30defg+20e^2f^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{g^4\sqrt{d^2-e^2x^2}(3\ldots}{e^6}$$

```
[Out] ((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 23*
d*g)*(e*f + d*g)^4*(d + e*x)^2)/(15*d^2*e^6*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(15*d^3*e^6*Sqrt
[d^2 - e^2*x^2]) + (g^4*(5*e*f + 3*d*g)*Sqrt[d^2 - e^2*x^2])/e^6 + (g^5*x*S
qrt[d^2 - e^2*x^2])/(2*e^5) - (g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rubi [A] time = 0.974598, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1635, 1815, 641, 217, 203}

$$\frac{(d+ex)(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{15d^3e^6\sqrt{d^2-e^2x^2}}-\frac{g^3\left(13d^2g^2+30defg+20e^2f^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{g^4\sqrt{d^2-e^2x^2}(3\ldots}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```


```
[Out] ((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 23*
d*g)*(e*f + d*g)^4*(d + e*x)^2)/(15*d^2*e^6*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(15*d^3*e^6*Sqrt
[d^2 - e^2*x^2]) + (g^4*(5*e*f + 3*d*g)*Sqrt[d^2 - e^2*x^2])/e^6 + (g^5*x*S
qrt[d^2 - e^2*x^2])/(2*e^5) - (g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```


Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^5}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^5f^5-15de^4f^4g-30d^2e^3f^3g^2-30d^3e^2f^2g^3-15d^4efg^4-3d^5g^5}{e^5}+\frac{5dg^2\left(10e^3f^3+10de^2f^2g+5d^2e\right.}{e^4}\right.}{\left(d^2-e^2x^2\right)^{5/2}}}{5d}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^5f^5-15de^4f^4g+70d^2e^3f^3g^2+170d^3e^2f\right.}{e^5}\right.}{}}{}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21defg+127d\right.}{15d^3e^6\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21defg+127d\right.}{15d^3e^6\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21defg+127d\right.}{15d^3e^6\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21defg+127d\right.}{15d^3e^6\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-21defg+127d\right.}{15d^3e^6\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.99772, size = 193, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2(dg+ef)^3\left(127d^2g^2-21defg+2e^2f^2\right)}{d^3(d-ex)}+\frac{2(2ef-23dg)(dg+ef)^4}{d^2(d-ex)^2}+30g^4(3dg+5ef)+\frac{6(dg+ef)^5}{d(d-ex)^3}+15eg^5x\right)-15g^3\left(13d^2g^2+3\right.}{30e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(30*g^4*(5*e*f + 3*d*g) + 15*e*g^5*x + (6*(e*f + d*g)^
5)/(d*(d - e*x)^3) + (2*(2*e*f - 23*d*g)*(e*f + d*g)^4)/(d^2*(d - e*x)^2) +
 (2*(e*f + d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2))/(d^3*(d - e*x)))
```

```
- 15*g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(30*e^6)
```

Maple [B] time = 0.18, size = 1308, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 4/5*x/(-e^2*x^2+d^2)^(5/2)*d*f^5+1/15/d*x/(-e^2*x^2+d^2)^(3/2)*f^5+2/15/d^3
*x/(-e^2*x^2+d^2)^(1/2)*f^5-3*x^6/(-e^2*x^2+d^2)^(5/2)*d*g^5+16/e^3*x/(-e^2
*x^2+d^2)^(1/2)*f^2*g^3-10/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d
^2)^(1/2))*f^2*g^3+30*x^4/(-e^2*x^2+d^2)^(5/2)*d*f^2*g^3+10*x^4*e/(-e^2*x^2
+d^2)^(5/2)*f^3*g^2+44/3*d^5/e^4/(-e^2*x^2+d^2)^(5/2)*f^2*g^3+4/3*d^4/e^3/(
-e^2*x^2+d^2)^(5/2)*f^3*g^2+15*x^3/(-e^2*x^2+d^2)^(5/2)*d*f^3*g^2+5/2*x^3*e
/(-e^2*x^2+d^2)^(5/2)*f^4*g-1/2/e*x/(-e^2*x^2+d^2)^(3/2)*f^4*g+5*x^2/(-e^2*
x^2+d^2)^(5/2)*d*f^4*g-d^3/e^2/(-e^2*x^2+d^2)^(5/2)*f^4*g+13/10/e*g^5*d^2*x
^5/(-e^2*x^2+d^2)^(5/2)-13/6/e^3*g^5*d^2*x^3/(-e^2*x^2+d^2)^(3/2)+13/2/e^5*
g^5*d^2*x/(-e^2*x^2+d^2)^(1/2)-13/2/e^5*g^5*d^2/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-e^2*x^2+d^2)^(1/2))-5*x^6*e/(-e^2*x^2+d^2)^(5/2)*f*g^4+19*d^3/e^2*x
^4/(-e^2*x^2+d^2)^(5/2)*g^5-76/3*d^5/e^4*x^2/(-e^2*x^2+d^2)^(5/2)*g^5+24*d^
6/e^5/(-e^2*x^2+d^2)^(5/2)*f*g^4+3*x^5/(-e^2*x^2+d^2)^(5/2)*d*f*g^4+152/15*
d^7/e^6/(-e^2*x^2+d^2)^(5/2)*g^5-1/2*e*g^5*x^7/(-e^2*x^2+d^2)^(5/2)+1/3*x^2
*e/(-e^2*x^2+d^2)^(5/2)*f^5+7/15*d^2/e/(-e^2*x^2+d^2)^(5/2)*f^5+2*x^5*e/(-e
^2*x^2+d^2)^(5/2)*f^2*g^3-10/3/e*x^3/(-e^2*x^2+d^2)^(3/2)*f^2*g^3-110/3*d^3
/e^2*x^2/(-e^2*x^2+d^2)^(5/2)*f^2*g^3-10/3*d^2/e*x^2/(-e^2*x^2+d^2)^(5/2)*f
^3*g^2+5/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)*d^3*f*g^4+15*x^3/e/(-e^2*x^2+d^2)^(
5/2)*d^2*f^2*g^3-3/2*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)*f*g^4-9*d^4/e^3*x/(-e^2
*x^2+d^2)^(5/2)*f^2*g^3-7*d^3/e^2*x/(-e^2*x^2+d^2)^(5/2)*f^3*g^2+3/2*d^2/e*
x/(-e^2*x^2+d^2)^(5/2)*f^4*g+1/2/e^4*x/(-e^2*x^2+d^2)^(3/2)*d^3*f*g^4+3/e^3
*x/(-e^2*x^2+d^2)^(3/2)*d^2*f^2*g^3+7/3/e^2*x/(-e^2*x^2+d^2)^(3/2)*d*f^3*g^
2+14/3/d/e^2*x/(-e^2*x^2+d^2)^(1/2)*f^3*g^2-1/d^2/e*x/(-e^2*x^2+d^2)^(1/2)*
f^4*g+45*d^2/e*x^4/(-e^2*x^2+d^2)^(5/2)*f*g^4-60*d^4/e^3*x^2/(-e^2*x^2+d^2)
^(5/2)*f*g^4-5/e^2*x^3/(-e^2*x^2+d^2)^(3/2)*d*f*g^4+16/e^4*x/(-e^2*x^2+d^2)
^(1/2)*d*f*g^4-15/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)*d*f*g^4
```

Maxima [B] time = 1.58141, size = 2167, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*e*g^5*x^7/(-e^2*x^2 + d^2)^(5/2) + 7/30*d^2*e*g^5*x*(15*x^4/((-e^2*x^2
 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2
*x^2 + d^2)^(5/2)*e^6)) - 7/6*d^2*g^5*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2)
 - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e + 1/5*d*f^5*x/(-e^2*x^2 + d^2)^(5/
2) + 3/5*d^2*f^5/((-e^2*x^2 + d^2)^(5/2)*e) + d^3*f^4*g/((-e^2*x^2 + d^2)^(
5/2)*e^2) + 4/15*f^5*x/((-e^2*x^2 + d^2)^(3/2)*d) + 14/15*d^4*g^5*x/((-e^2*
x^2 + d^2)^(3/2)*e^5) + 1/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5
```

```
)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) + 8/15*f^5*x/(sqrt(-e^2*x^2 +
 d^2)*d^3) - 49/30*d^2*g^5*x/(sqrt(-e^2*x^2 + d^2)*e^5) - (5*e^3*f*g^4 + 3*
d*e^2*g^5)*x^6/((-e^2*x^2 + d^2)^(5/2)*e^2) - 7/2*d^2*g^5*arcsin(e^2*x/sqrt
(d^2*e^2))/(sqrt(e^2)*e^5) - 1/3*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e
*g^5)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)
*e^4))/e^2 + 6*(5*e^3*f*g^4 + 3*d*e^2*g^5)*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*
e^4) + (10*e^3*f^3*g^2 + 30*d*e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*x^4/(
(-e^2*x^2 + d^2)^(5/2)*e^2) + 5/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^
2*g^3 + d^3*f*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 8*(5*e^3*f*g^4 + 3*d*
e^2*g^5)*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^6) - 4/3*(10*e^3*f^3*g^2 + 30*d*
e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4
) + 1/3*(e^3*f^5 + 15*d*e^2*f^4*g + 30*d^2*e*f^3*g^2 + 10*d^3*f^2*g^3)*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^2) - 3/2*(e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f
^2*g^3 + d^3*f*g^4)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*(3*d*e^2*f^5 +
 15*d^2*e*f^4*g + 10*d^3*f^3*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 16/5*(5*
e^3*f*g^4 + 3*d*e^2*g^5)*d^6/((-e^2*x^2 + d^2)^(5/2)*e^8) + 8/15*(10*e^3*f^
3*g^2 + 30*d*e^2*f^2*g^3 + 15*d^2*e*f*g^4 + d^3*g^5)*d^4/((-e^2*x^2 + d^2)^
(5/2)*e^6) - 2/15*(e^3*f^5 + 15*d*e^2*f^4*g + 30*d^2*e*f^3*g^2 + 10*d^3*f^2
*g^3)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 4/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*
g^4 + 3*d^2*e*g^5)*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^6) + 1/2*(e^3*f^4*g + 6*
d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 + d^3*f*g^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4)
 - 1/15*(3*d*e^2*f^5 + 15*d^2*e*f^4*g + 10*d^3*f^3*g^2)*x/((-e^2*x^2 + d^2)
^(3/2)*d^2*e^2) - 7/15*(10*e^3*f^2*g^3 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*x/(s
qrt(-e^2*x^2 + d^2)*e^6) + (e^3*f^4*g + 6*d*e^2*f^3*g^2 + 6*d^2*e*f^2*g^3 +
 d^3*f*g^4)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/15*(3*d*e^2*f^5 + 15*d^2*e
*f^4*g + 10*d^3*f^3*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (10*e^3*f^2*g^3
 + 15*d*e^2*f*g^4 + 3*d^2*e*g^5)*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^6
)
```

Fricas [B] time = 2.51113, size = 1693, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(14*d^3*e^5*f^5 - 30*d^4*e^4*f^4*g + 40*d^5*e^3*f^3*g^2 + 440*d^6*e^2
*f^2*g^3 + 720*d^7*e*f*g^4 + 304*d^8*g^5 - 2*(7*e^8*f^5 - 15*d*e^7*f^4*g +
20*d^2*e^6*f^3*g^2 + 220*d^3*e^5*f^2*g^3 + 360*d^4*e^4*f*g^4 + 152*d^5*e^3*
g^5)*x^3 + 6*(7*d*e^7*f^5 - 15*d^2*e^6*f^4*g + 20*d^3*e^5*f^3*g^2 + 220*d^4
*e^4*f^2*g^3 + 360*d^5*e^3*f*g^4 + 152*d^6*e^2*g^5)*x^2 - 6*(7*d^2*e^6*f^5
- 15*d^3*e^5*f^4*g + 20*d^4*e^4*f^3*g^2 + 220*d^5*e^3*f^2*g^3 + 360*d^6*e^2
*f*g^4 + 152*d^7*e*g^5)*x + 30*(20*d^6*e^2*f^2*g^3 + 30*d^7*e*f*g^4 + 13*d^
8*g^5 - (20*d^3*e^5*f^2*g^3 + 30*d^4*e^4*f*g^4 + 13*d^5*e^3*g^5)*x^3 + 3*(2
0*d^4*e^4*f^2*g^3 + 30*d^5*e^3*f*g^4 + 13*d^6*e^2*g^5)*x^2 - 3*(20*d^5*e^3*
f^2*g^3 + 30*d^6*e^2*f*g^4 + 13*d^7*e*g^5)*x)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) - (15*d^3*e^4*g^5*x^4 - 14*d^2*e^5*f^5 + 30*d^3*e^4*f^4*g - 40
*d^4*e^3*f^3*g^2 - 440*d^5*e^2*f^2*g^3 - 720*d^6*e*f*g^4 - 304*d^7*g^5 + 15
*(10*d^3*e^4*f*g^4 + 3*d^4*e^3*g^5)*x^3 - (4*e^7*f^5 - 30*d*e^6*f^4*g + 140
*d^2*e^5*f^3*g^2 + 640*d^3*e^4*f^2*g^3 + 1170*d^4*e^3*f*g^4 + 479*d^5*e^2*g
^5)*x^2 + 3*(4*d*e^6*f^5 - 30*d^2*e^5*f^4*g + 40*d^3*e^4*f^3*g^2 + 340*d^4*
e^3*f^2*g^3 + 570*d^5*e^2*f*g^4 + 239*d^6*e*g^5)*x)*sqrt(-e^2*x^2 + d^2))/(
d^3*e^9*x^3 - 3*d^4*e^8*x^2 + 3*d^5*e^7*x - d^6*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**5/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25827, size = 725, normalized size = 2.7

$$-\frac{1}{2}\left(13\,d^2g^5+30\,dfg^4e+20\,f^2g^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}\left(d\right)+\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(\left(\left(15\left(g^5xe+\frac{2\left(3\,d^5g^5e^{12}+5\,d^4fg^4e^{13}\right)e^{(-12)}}{d^4}\right.\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/2*(13*d^2*g^5 + 30*d*f*g^4*e + 20*f^2*g^3*e^2)*arcsin(x*e/d)*e^(-6)*sgn(
d) + 1/30*sqrt(-x^2*e^2 + d^2)*((((((15*(g^5*x*e + 2*(3*d^5*g^5*e^12 + 5*d^
4*f*g^4*e^13)*e^(-12)/d^4)*x - (299*d^6*g^5*e^11 + 720*d^5*f*g^4*e^12 + 640
*d^4*f^2*g^3*e^13 + 140*d^3*f^3*g^2*e^14 - 30*d^2*f^4*g*e^15 + 4*d*f^5*e^16
)*e^(-12)/d^4)*x - 30*(19*d^7*g^5*e^10 + 45*d^6*f*g^4*e^11 + 30*d^5*f^2*g^3
*e^12 + 10*d^4*f^3*g^2*e^13)*e^(-12)/d^4)*x + 5*(91*d^8*g^5*e^9 + 210*d^7*f
*g^4*e^10 + 140*d^6*f^2*g^3*e^11 - 20*d^5*f^3*g^2*e^12 - 30*d^4*f^4*g*e^13
+ 2*d^3*f^5*e^14)*e^(-12)/d^4)*x + 10*(76*d^9*g^5*e^8 + 180*d^8*f*g^4*e^9 +
 110*d^7*f^2*g^3*e^10 + 10*d^6*f^3*g^2*e^11 - 15*d^5*f^4*g*e^12 - d^4*f^5*e
^13)*e^(-12)/d^4)*x - 15*(13*d^10*g^5*e^7 + 30*d^9*f*g^4*e^8 + 20*d^8*f^2*g
^3*e^9 + 2*d^5*f^5*e^12)*e^(-12)/d^4)*x - 2*(152*d^11*g^5*e^6 + 360*d^10*f*
g^4*e^7 + 220*d^9*f^2*g^3*e^8 + 20*d^8*f^3*g^2*e^9 - 15*d^7*f^4*g*e^10 + 7*
d^6*f^5*e^11)*e^(-12)/d^4)/(x^2*e^2 - d^2)^3
```

3.580 $\int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=215

$$\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{15d^3e^5\sqrt{d^2-e^2x^2}}-\frac{g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+\ldots}{5de^{\ldots}}$$

```
[Out] ((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 9*d
*g)*(e*f + d*g)^3*(d + e*x)^2)/(15*d^2*e^5*(d^2 - e^2*x^2)^(3/2)) + (2*(e*f
 + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(15*d^3*e^5*Sqrt[d^
2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/e^5 - (g^3*(4*e*f + 3*d*g)*ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rubi [A] time = 0.66619, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1635, 641, 217, 203}

$$\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{15d^3e^5\sqrt{d^2-e^2x^2}}-\frac{g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+\ldots}{5de^{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```


```
[Out] ((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 9*d
*g)*(e*f + d*g)^3*(d + e*x)^2)/(15*d^2*e^5*(d^2 - e^2*x^2)^(3/2)) + (2*(e*f
 + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(15*d^3*e^5*Sqrt[d^
2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/e^5 - (g^3*(4*e*f + 3*d*g)*ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```


Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^4f^4-12de^3f^3g-18d^2e^2f^2g^2-12d^3efg^3-3d^4g^4}{e^4}+\frac{5dg^2\left(6e^2f^2+4defg+d^2g^2\right)x}{e^3}+\frac{5dg^3(4e}{e}\right.}{\left(d^2-e^2x^2\right)^{5/2}}}{5d}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^4f^4-12de^3f^3g+42d^2e^2f^2g^2+68d^3e}{e^4}\right.}{\left(d^2-e^2\right.}}{15}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+36d\right.}{15d^3e^5\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+36d\right.}{15d^3e^5\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+36d\right.}{15d^3e^5\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-8defg+36d\right.}{15d^3e^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.759525, size = 168, normalized size = 0.78

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(2(d-ex)^2(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)+3d^2(dg+ef)^4+15d^3g^4(d-ex)^3+2d(d-ex)(ef-9dg)(dg+ef)^3\right)}{d^3(d-ex)^3}-15g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2}}\right.}{15e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^2*(e*f + d*g)^4 + 2*d*(e*f - 9*d*g)*(e*f + d*g)^
3*(d - e*x) + 2*(e*f + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d - e*x)^
2 + 15*d^3*g^4*(d - e*x)^3))/(d^3*(d - e*x)^3) - 15*g^3*(4*e*f + 3*d*g)*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(15*e^5)
```

Maple [B] time = 0.121, size = 1030, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 9/e*g^4*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-12/e^3*g^4*d^4*x^2/(-e^2*x^2+d^2)^(5/2
)+4/5*x^5*e/(-e^2*x^2+d^2)^(5/2)*f*g^3-1/e^2*x^3/(-e^2*x^2+d^2)^(3/2)*d*g^4
-4/3/e*x^3/(-e^2*x^2+d^2)^(3/2)*f*g^3+16/5/e^4*x/(-e^2*x^2+d^2)^(1/2)*d*g^4
+32/5/e^3*x/(-e^2*x^2+d^2)^(1/2)*f*g^3-3/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))*d*g^4-4/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2
*x^2+d^2)^(1/2))*f*g^3+12*x^4/(-e^2*x^2+d^2)^(5/2)*d*f*g^3+6*x^4*e/(-e^2*x^
2+d^2)^(5/2)*f^2*g^2+88/15*d^5/e^4/(-e^2*x^2+d^2)^(5/2)*f*g^3+4/5*d^4/e^3/(
-e^2*x^2+d^2)^(5/2)*f^2*g^2+1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)*d^3*g^4+9*x^3/
(-e^2*x^2+d^2)^(5/2)*d*f^2*g^2+2*x^3*e/(-e^2*x^2+d^2)^(5/2)*f^3*g-3/10*d^5/
e^4*x/(-e^2*x^2+d^2)^(5/2)*g^4+1/10/e^4*x/(-e^2*x^2+d^2)^(3/2)*d^3*g^4-2/5/
e*x/(-e^2*x^2+d^2)^(3/2)*f^3*g+4*x^2/(-e^2*x^2+d^2)^(5/2)*d*f^3*g-4/5*d^3/e
^2/(-e^2*x^2+d^2)^(5/2)*f^3*g-e*g^4*x^6/(-e^2*x^2+d^2)^(5/2)+24/5/e^5*g^4*d
^6/(-e^2*x^2+d^2)^(5/2)+3/5*x^5/(-e^2*x^2+d^2)^(5/2)*d*g^4+4/5*x/(-e^2*x^2+
d^2)^(5/2)*d*f^4+1/15/d*x/(-e^2*x^2+d^2)^(3/2)*f^4+2/15/d^3*x/(-e^2*x^2+d^2
)^(1/2)*f^4+1/3*x^2*e/(-e^2*x^2+d^2)^(5/2)*f^4+7/15*d^2/e/(-e^2*x^2+d^2)^(5
/2)*f^4-44/3*d^3/e^2*x^2/(-e^2*x^2+d^2)^(5/2)*f*g^3-2*d^2/e*x^2/(-e^2*x^2+d
^2)^(5/2)*f^2*g^2+6*x^3/e/(-e^2*x^2+d^2)^(5/2)*d^2*f*g^3-18/5*d^4/e^3*x/(-e
^2*x^2+d^2)^(5/2)*f*g^3-21/5*d^3/e^2*x/(-e^2*x^2+d^2)^(5/2)*f^2*g^2+6/5*d^2
/e*x/(-e^2*x^2+d^2)^(5/2)*f^3*g+6/5/e^3*x/(-e^2*x^2+d^2)^(3/2)*d^2*f*g^3+7/
5/e^2*x/(-e^2*x^2+d^2)^(3/2)*d*f^2*g^2+14/5/d/e^2*x/(-e^2*x^2+d^2)^(1/2)*f^
2*g^2-4/5/d^2/e*x/(-e^2*x^2+d^2)^(1/2)*f^3*g
```

Maxima [B] time = 1.56566, size = 1608, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -e*g^4*x^6/(-e^2*x^2 + d^2)^(5/2) + 6*d^2*g^4*x^4/((-e^2*x^2 + d^2)^(5/2)*e
) - 8*d^4*g^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) + 1/5*d*f^4*x/(-e^2*x^2 + d^
2)^(5/2) + 1/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/
2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)
^(5/2)*e^6)) + 3/5*d^2*f^4/((-e^2*x^2 + d^2)^(5/2)*e) + 4/5*d^3*f^3*g/((-e^
2*x^2 + d^2)^(5/2)*e^2) + 16/5*d^6*g^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 4/15*
f^4*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^4*x/(sqrt(-e^2*x^2 + d^2)*d^3) -
1/3*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d
^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^2 + 3*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d
^2*e*g^4)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) + 1/2*(4*e^3*f^3*g + 18*d*e^2*f^
2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4*(2*e
^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4
) + 1/3*(e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*x^2/((-
e^2*x^2 + d^2)^(5/2)*e^2) - 3/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e
*f*g^3 + d^3*g^4)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^4 + 4*d
^2*e*f^3*g + 2*d^3*f^2*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(2*e^3*f^2
*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(
e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*d^2/((-e^2*x^2 +
 d^2)^(5/2)*e^4) + 4/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*d^2*x/((-e^2*x^2 + d^2)
^(3/2)*e^6) + 1/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g
^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^4 + 4*d^2*e*f^3*g + 2*d^3
*f^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 7/15*(4*e^3*f*g^3 + 3*d*e^2*
g^4)*x/(sqrt(-e^2*x^2 + d^2)*e^6) + 1/5*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 1
2*d^2*e*f*g^3 + d^3*g^4)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^4
+ 4*d^2*e*f^3*g + 2*d^3*f^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (4*e^3*
f*g^3 + 3*d*e^2*g^4)*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^6)
```

Fricas [B] time = 3.22675, size = 1277, normalized size = 5.94

$$-\frac{7\,d^3e^4f^4-12\,d^4e^3f^3g+12\,d^5e^2f^2g^2+88\,d^6efg^3+72\,d^7g^4-\left(7\,e^7f^4-12\,de^6f^3g+12\,d^2e^5f^2g^2+88\,d^3e^4fg^3+72\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^4*f^4 - 12*d^4*e^3*f^3*g + 12*d^5*e^2*f^2*g^2 + 88*d^6*e*f*g
^3 + 72*d^7*g^4 - (7*e^7*f^4 - 12*d*e^6*f^3*g + 12*d^2*e^5*f^2*g^2 + 88*d^3
*e^4*f*g^3 + 72*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 - 12*d^2*e^5*f^3*g + 12*d
^3*e^4*f^2*g^2 + 88*d^4*e^3*f*g^3 + 72*d^5*e^2*g^4)*x^2 - 3*(7*d^2*e^5*f^4
- 12*d^3*e^4*f^3*g + 12*d^4*e^3*f^2*g^2 + 88*d^5*e^2*f*g^3 + 72*d^6*e*g^4)*
x + 30*(4*d^6*e*f*g^3 + 3*d^7*g^4 - (4*d^3*e^4*f*g^3 + 3*d^4*e^3*g^4)*x^3 +
 3*(4*d^4*e^3*f*g^3 + 3*d^5*e^2*g^4)*x^2 - 3*(4*d^5*e^2*f*g^3 + 3*d^6*e*g^4
)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (15*d^3*e^3*g^4*x^3 - 7*d^
2*e^4*f^4 + 12*d^3*e^3*f^3*g - 12*d^4*e^2*f^2*g^2 - 88*d^5*e*f*g^3 - 72*d^6
*g^4 - (2*e^6*f^4 - 12*d*e^5*f^3*g + 42*d^2*e^4*f^2*g^2 + 128*d^3*e^3*f*g^3
 + 117*d^4*e^2*g^4)*x^2 + 3*(2*d*e^5*f^4 - 12*d^2*e^4*f^3*g + 12*d^3*e^3*f^
2*g^2 + 68*d^4*e^2*f*g^3 + 57*d^5*e*g^4)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^8*
x^3 - 3*d^4*e^7*x^2 + 3*d^5*e^6*x - d^6*e^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3\left(f+gx\right)^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**4/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [B] time = 1.21178, size = 555, normalized size = 2.58

$$-\left(3\,dg^4+4\,fg^3e\right)\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}\left(d\right)+\frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(\left(\left(15\,g^4xe-\frac{2\left(36\,d^5g^4e^{10}+64\,d^4fg^3e^{11}+21\,d^3f^2g^2e^{12}-6\,d^2f^3ge^{13}+df\right.}{d^4}\right.\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -(3*d*g^4 + 4*f*g^3*e)*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/15*sqrt(-x^2*e^2 + d
^2)*((((((15*g^4*x*e - 2*(36*d^5*g^4*e^10 + 64*d^4*f*g^3*e^11 + 21*d^3*f^2*
g^2*e^12 - 6*d^2*f^3*g*e^13 + d*f^4*e^14)*e^(-10)/d^4)*x - 45*(3*d^6*g^4*e^
9 + 4*d^5*f*g^3*e^10 + 2*d^4*f^2*g^2*e^11)*e^(-10)/d^4)*x + 5*(21*d^7*g^4*e
^8 + 28*d^6*f*g^3*e^9 - 6*d^5*f^2*g^2*e^10 - 12*d^4*f^3*g*e^11 + d^3*f^4*e^
12)*e^(-10)/d^4)*x + 5*(36*d^8*g^4*e^7 + 44*d^7*f*g^3*e^8 + 6*d^6*f^2*g^2*e
```

```
^9 - 12*d^5*f^3*g*e^10 - d^4*f^4*e^11)*e^(-10)/d^4)*x - 15*(3*d^9*g^4*e^6 +
 4*d^8*f*g^3*e^7 + d^5*f^4*e^10)*e^(-10)/d^4)*x - (72*d^10*g^4*e^5 + 88*d^9
*f*g^3*e^6 + 12*d^8*f^2*g^2*e^7 - 12*d^7*f^3*g*e^8 + 7*d^6*f^4*e^9)*e^(-10)
/d^4)/(x^2*e^2 - d^2)^3
```

3.581 $\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=183

$$\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{15d^3e^4\sqrt{d^2-e^2x^2}}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{g^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] ((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 13*
d*g)*(e*f + d*g)^2*(d + e*x)^2)/(15*d^2*e^4*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(15*d^3*e^4*Sqrt[d^
2 - e^2*x^2]) - (g^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4
```

Rubi [A] time = 0.40349, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1635, 778, 217, 203}

$$\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{15d^3e^4\sqrt{d^2-e^2x^2}}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{g^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 13*
d*g)*(e*f + d*g)^2*(d + e*x)^2)/(15*d^2*e^4*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(15*d^3*e^4*Sqrt[d^
2 - e^2*x^2]) - (g^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^3f^3-9de^2f^2g-9d^2efg^2-3d^3g^3}{e^3}+\frac{5dg^2(3ef+dg)x}{e^2}+\frac{5dg^3x^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^3f^3-9de^2f^2g+21d^2efg^2+17d^3g^3}{e^3}+ \frac{15d}{\ldots}\right.}{\left(d^2-e^2x^2\right)^{3/2}}}{15d^2}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg+32d^2g\right.}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg+32d^2g\right.}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-11defg+32d^2g\right.}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.809311, size = 182, normalized size = 0.99

$$\frac{(d+ex)\left(\sqrt{1-\frac{e^2x^2}{d^2}}(dg+ef)\left(d^2e^2\left(7f^2+33fgx+32g^2x^2\right)-d^3eg(16f+51gx)+22d^4g^2-de^3fx(6f+11gx)+2e^4f^2x^2\right)\right.}{15d^3e^4(d-ex)^2\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*((e*f + d*g)*Sqrt[1 - (e^2*x^2)/d^2]*(22*d^4*g^2 + 2*e^4*f^2*x^2
 - d*e^3*f*x*(6*f + 11*g*x) - d^3*e*g*(16*f + 51*g*x) + d^2*e^2*(7*f^2 + 33
*f*g*x + 32*g^2*x^2)) - 15*d^2*g^3*(d - e*x)^3*ArcSin[(e*x)/d]))/(15*d^3*e^
4*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [B] time = 0.093, size = 713, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15/d*x/(-e^2*x^2+d^2)^(3/2)*f^3+2/15/d^3*x/(-e^2*x^2+d^2)^(1/2)*f^3+1/3*x
^2*e/(-e^2*x^2+d^2)^(5/2)*f^3+1/5*e*g^3*x^5/(-e^2*x^2+d^2)^(5/2)-1/3/e*g^3*
x^3/(-e^2*x^2+d^2)^(3/2)+8/5/e^3*g^3*x/(-e^2*x^2+d^2)^(1/2)-1/e^3*g^3/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+3*x^4/(-e^2*x^2+d^2)^(5/2
)*d*g^3+22/15*d^5/e^4/(-e^2*x^2+d^2)^(5/2)*g^3+7/15*d^2/e/(-e^2*x^2+d^2)^(5
/2)*f^3+4/5*x/(-e^2*x^2+d^2)^(5/2)*d*f^3+3*x^4*e/(-e^2*x^2+d^2)^(5/2)*f*g^2
```

```
-11/3*d^3/e^2*x^2/(-e^2*x^2+d^2)^(5/2)*g^3+2/5*d^4/e^3/(-e^2*x^2+d^2)^(5/2)
*f*g^2+3/2*x^3/e/(-e^2*x^2+d^2)^(5/2)*d^2*g^3+9/2*x^3/(-e^2*x^2+d^2)^(5/2)*
d*f*g^2+3/2*x^3*e/(-e^2*x^2+d^2)^(5/2)*f^2*g-9/10*d^4/e^3*x/(-e^2*x^2+d^2)^
(5/2)*g^3+3/10/e^3*x/(-e^2*x^2+d^2)^(3/2)*d^2*g^3-3/10/e*x/(-e^2*x^2+d^2)^(
3/2)*f^2*g+3*x^2/(-e^2*x^2+d^2)^(5/2)*d*f^2*g-3/5*d^3/e^2/(-e^2*x^2+d^2)^(5
/2)*f^2*g-21/10*d^3/e^2*x/(-e^2*x^2+d^2)^(5/2)*f*g^2+9/10*d^2/e*x/(-e^2*x^2
+d^2)^(5/2)*f^2*g+7/10/e^2*x/(-e^2*x^2+d^2)^(3/2)*d*f*g^2+7/5/d/e^2*x/(-e^2
*x^2+d^2)^(1/2)*f*g^2-3/5/d^2/e*x/(-e^2*x^2+d^2)^(1/2)*f^2*g-d^2/e*x^2/(-e^
2*x^2+d^2)^(5/2)*f*g^2
```

Maxima [B] time = 1.57726, size = 1220, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*e^3*g^3*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2
 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e*g^3*x*(3*x
^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 1/5
*d*f^3*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^3/((-e^2*x^2 + d^2)^(5/2)*e) +
3/5*d^3*f^2*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f^3*x/((-e^2*x^2 + d^2)^(
3/2)*d) + 4/15*d^2*g^3*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 8/15*f^3*x/(sqrt(-e
^2*x^2 + d^2)*d^3) - 7/15*g^3*x/(sqrt(-e^2*x^2 + d^2)*e^3) + 3*(e^3*f*g^2 +
 d*e^2*g^3)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - g^3*arcsin(e^2*x/sqrt(d^2*e^
2))/(sqrt(e^2)*e^3) + 3/2*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x^3/((-e^
2*x^2 + d^2)^(5/2)*e^2) - 4*(e^3*f*g^2 + d*e^2*g^3)*d^2*x^2/((-e^2*x^2 + d^
2)^(5/2)*e^4) + 1/3*(e^3*f^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2 + d^3*g^3)*x^2
/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3
)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3
*f*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(e^3*f*g^2 + d*e^2*g^3)*d^4/((
-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(e^3*f^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2
+ d^3*g^3)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/10*(e^3*f^2*g + 3*d*e^2*f*g
^2 + d^2*e*g^3)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^3 + 3*d^2*e*f
^2*g + d^3*f*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) + 3/5*(e^3*f^2*g + 3*d
*e^2*f*g^2 + d^2*e*g^3)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^3 +
 3*d^2*e*f^2*g + d^3*f*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

Fricas [B] time = 2.21653, size = 917, normalized size = 5.01

$$-\frac{7\,d^3e^3f^3 - 9\,d^4e^2f^2g + 6\,d^5efg^2 + 22\,d^6g^3 - \left(7\,e^6f^3 - 9\,de^5f^2g + 6\,d^2e^4fg^2 + 22\,d^3e^3g^3\right)x^3 + 3\left(7\,de^5f^3 - 9\,d^2e^4f^2g\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^3*f^3 - 9*d^4*e^2*f^2*g + 6*d^5*e*f*g^2 + 22*d^6*g^3 - (7*e^
6*f^3 - 9*d*e^5*f^2*g + 6*d^2*e^4*f*g^2 + 22*d^3*e^3*g^3)*x^3 + 3*(7*d*e^5*
f^3 - 9*d^2*e^4*f^2*g + 6*d^3*e^3*f*g^2 + 22*d^4*e^2*g^3)*x^2 - 3*(7*d^2*e^
4*f^3 - 9*d^3*e^3*f^2*g + 6*d^4*e^2*f*g^2 + 22*d^5*e*g^3)*x - 30*(d^3*e^3*g
^3*x^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*arctan(-(d - sqrt(-e^
```

```
2*x^2 + d^2))/(e*x)) + (7*d^2*e^3*f^3 - 9*d^3*e^2*f^2*g + 6*d^4*e*f*g^2 + 2
2*d^5*g^3 + (2*e^5*f^3 - 9*d*e^4*f^2*g + 21*d^2*e^3*f*g^2 + 32*d^3*e^2*g^3)
*x^2 - 3*(2*d*e^4*f^3 - 9*d^2*e^3*f^2*g + 6*d^3*e^2*f*g^2 + 17*d^4*e*g^3)*x
)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^3 - 3*d^4*e^6*x^2 + 3*d^5*e^5*x - d^6*e^
4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3 \left(f+gx\right)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.20298, size = 417, normalized size = 2.28

$$-g^3 \arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}\left(d\right) - \frac{\sqrt{-x^2e^2+d^2}\left(\left(\left(\left(x\left(\frac{\left(32\,d^4g^3e^8+21\,d^3fg^2e^9-9\,d^2f^2ge^{10}+2\,df^3e^{11}\right)xe^{(-7)}}{d^4}+\frac{45\left(d^5g^3e^7+d^4fg^2e^8\right)e^{(-7)}}{d^4}\right)-\frac{5\left(7\,d^6\right.}{}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -g^3*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/15*sqrt(-x^2*e^2 + d^2)*((((x*((32*d^4
*g^3*e^8 + 21*d^3*f*g^2*e^9 - 9*d^2*f^2*g*e^10 + 2*d*f^3*e^11)*x*e^(-7)/d^4
 + 45*(d^5*g^3*e^7 + d^4*f*g^2*e^8)*e^(-7)/d^4) - 5*(7*d^6*g^3*e^6 - 3*d^5*
f*g^2*e^7 - 9*d^4*f^2*g*e^8 + d^3*f^3*e^9)*e^(-7)/d^4)*x - 5*(11*d^7*g^3*e^
5 + 3*d^6*f*g^2*e^6 - 9*d^5*f^2*g*e^7 - d^4*f^3*e^8)*e^(-7)/d^4)*x + 15*(d^
8*g^3*e^4 + d^5*f^3*e^7)*e^(-7)/d^4)*x + (22*d^9*g^3*e^3 + 6*d^8*f*g^2*e^4
- 9*d^7*f^2*g*e^5 + 7*d^6*f^3*e^6)*e^(-7)/d^4)/(x^2*e^2 - d^2)^3
```

3.582 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(d+ex)\left(7d^2g^2-6defg+2e^2f^2\right)}{15d^3e^3\sqrt{d^2-e^2x^2}}+\frac{(d+ex)^3(dg+ef)^2}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}$$

```
[Out] ((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 4*d
*g)*(e*f + d*g)*(d + e*x)^2)/(15*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*f
^2 - 6*d*e*f*g + 7*d^2*g^2)*(d + e*x))/(15*d^3*e^3*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.223916, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1635, 789, 637}

$$\frac{(d+ex)\left(7d^2g^2-6defg+2e^2f^2\right)}{15d^3e^3\sqrt{d^2-e^2x^2}}+\frac{(d+ex)^3(dg+ef)^2}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```


```
[Out] ((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 4*d
*g)*(e*f + d*g)*(d + e*x)^2)/(15*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*f
^2 - 6*d*e*f*g + 7*d^2*g^2)*(d + e*x))/(15*d^3*e^3*Sqrt[d^2 - e^2*x^2])
```


Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-2f^2+\frac{6dfg}{e}+\frac{3d^2g^2}{e^2}+\frac{5dg^2x}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}$$

$$= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\left(2e^2f^2-6defg+7d^2g^2\right)\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}}{15d^2e^2}$$

$$= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\left(2e^2f^2-6defg+7d^2g^2\right)(d+ex)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.419794, size = 110, normalized size = 0.76

$$\frac{(d+ex)\left(d^2e^2\left(7f^2+18fgx+7g^2x^2\right)-6d^3eg(f+gx)+2d^4g^2-6de^3fx(f+gx)+2e^4f^2x^2\right)}{15d^3e^3(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(2*d^4*g^2 + 2*e^4*f^2*x^2 - 6*d^3*e*g*(f + g*x) - 6*d*e^3*f*x*(
f + g*x) + d^2*e^2*(7*f^2 + 18*f*g*x + 7*g^2*x^2)))/(15*d^3*e^3*(d - e*x)^2
*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.048, size = 131, normalized size = 0.9

$$\frac{(-ex+d)(ex+d)^4\left(7d^2e^2g^2x^2-6de^3fgx^2+2e^4f^2x^2-6d^3eg^2x+18d^2e^2fgx-6de^3f^2x+2d^4g^2-6d^3efg+7d^2e^2f^2\right)}{15d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(7*d^2*e^2*g^2*x^2-6*d*e^3*f*g*x^2+2*e^4*f^2*x^2-6*
d^3*e*g^2*x+18*d^2*e^2*f*g*x-6*d*e^3*f^2*x+2*d^4*g^2-6*d^3*e*f*g+7*d^2*e^2*
f^2)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)
```

Maxima [B] time = 1.0094, size = 787, normalized size = 5.43

$$\frac{eg^2x^4}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{4\,d^2g^2x^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{df^2x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{3\,d^2f^2}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{2\,d^3fg}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} + \frac{8\,d^4g^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^3} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] e*g^2*x^4/(-e^2*x^2 + d^2)^(5/2) - 4/3*d^2*g^2*x^2/((-e^2*x^2 + d^2)^(5/2)*
e) + 1/5*d*f^2*x/(-e^2*x^2 + d^2)^(5/2) + 3/5*d^2*f^2/((-e^2*x^2 + d^2)^(5/
```

```
2)*e) + 2/5*d^3*f*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/15*d^4*g^2/((-e^2*x^2
+ d^2)^(5/2)*e^3) + 4/15*f^2*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^2*x/(sqr
t(-e^2*x^2 + d^2)*d^3) + 1/2*(2*e^3*f*g + 3*d*e^2*g^2)*x^3/((-e^2*x^2 + d^2
)^(5/2)*e^2) + 1/3*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*x^2/((-e^2*x^2 + d
^2)^(5/2)*e^2) - 3/10*(2*e^3*f*g + 3*d*e^2*g^2)*d^2*x/((-e^2*x^2 + d^2)^(5/
2)*e^4) + 1/5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)^(5/
2)*e^2) - 2/15*(e^3*f^2 + 6*d*e^2*f*g + 3*d^2*e*g^2)*d^2/((-e^2*x^2 + d^2)^
(5/2)*e^4) + 1/10*(2*e^3*f*g + 3*d*e^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*e^4)
- 1/15*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*
e^2) + 1/5*(2*e^3*f*g + 3*d*e^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/1
5*(3*d*e^2*f^2 + 6*d^2*e*f*g + d^3*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

Fricas [B] time = 1.87037, size = 562, normalized size = 3.88

$$-\frac{7\,d^3e^2f^2-6\,d^4efg+2\,d^5g^2-\left(7\,e^5f^2-6\,de^4fg+2\,d^2e^3g^2\right)x^3+3\left(7\,de^4f^2-6\,d^2e^3fg+2\,d^3e^2g^2\right)x^2-3\left(7\,d^2e^3f^2-\right.}{15\left(d^3e^6x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e^2*f^2 - 6*d^4*e*f*g + 2*d^5*g^2 - (7*e^5*f^2 - 6*d*e^4*f*g +
 2*d^2*e^3*g^2)*x^3 + 3*(7*d*e^4*f^2 - 6*d^2*e^3*f*g + 2*d^3*e^2*g^2)*x^2 -
 3*(7*d^2*e^3*f^2 - 6*d^3*e^2*f*g + 2*d^4*e*g^2)*x + (7*d^2*e^2*f^2 - 6*d^3
*e*f*g + 2*d^4*g^2 + (2*e^4*f^2 - 6*d*e^3*f*g + 7*d^2*e^2*g^2)*x^2 - 6*(d*e
^3*f^2 - 3*d^2*e^2*f*g + d^3*e*g^2)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 -
 3*d^4*e^5*x^2 + 3*d^5*e^4*x - d^6*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3\left(f+gx\right)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.1802, size = 267, normalized size = 1.84

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(15\,df^2+\left(\left(\left(15\,g^2e+\frac{\left(7\,d^3g^2e^6-6\,d^2fge^7+2\,df^2e^8\right)xe^{(-4)}}{d^4}\right)x+\frac{5\left(d^5g^2e^4+6\,d^4fge^5-d^3f^2e^6\right)e^{(-4)}}{d^4}\right)x-\frac{5\left(d^6g^2e^3-6\,d^5fge^4-d^4\right.}{d^4}\right.\right.\right.}{15\left(x^2e^2-d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((15*d*f^2 + (((15*g^2*e + (7*d^3*g^2*e^6 - 6*d^
2*f*g*e^7 + 2*d*f^2*e^8)*x*e^(-4)/d^4)*x + 5*(d^5*g^2*e^4 + 6*d^4*f*g*e^5 -
```

```
d^3*f^2*e^6)*e^(-4)/d^4)*x - 5*(d^6*g^2*e^3 - 6*d^5*f*g*e^4 - d^4*f^2*e^5)
*e^(-4)/d^4)*x)*x + (2*d^8*g^2*e - 6*d^7*f*g*e^2 + 7*d^6*f^2*e^3)*e^(-4)/d^
4)/(x^2*e^2 - d^2)^3
```

3.583 $\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{(d+ex)^3(dg+ef)}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)(2ef-3dg)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] ((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^(5/2)) + (2*(2*e*f - 3*d
*g)*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^(3/2)) + ((2*e*f - 3*d*g)*x)/(15*d
^3*e*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] time = 0.0593807, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {789, 653, 191}

$$\frac{(d+ex)^3(dg+ef)}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)(2ef-3dg)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^(5/2)) + (2*(2*e*f - 3*d
*g)*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^(3/2)) + ((2*e*f - 3*d*g)*x)/(15*d
^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{(-5ef+3(ef+dg))\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de}$$

$$= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{(-5ef+3(ef+dg))\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}$$

$$= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{(2ef-3dg)x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Mathematica [A] time = 0.234674, size = 83, normalized size = 0.71

$$-\frac{(d+ex)\left(-d^2e(7f+9gx)+3d^3g+3de^2x(2f+gx)-2e^3fx^2\right)}{15d^3e^2(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] -((d + e*x)*(3*d^3*g - 2*e^3*f*x^2 + 3*d*e^2*x*(2*f + g*x) - d^2*e*(7*f + 9
*g*x)))/(15*d^3*e^2*(d - e*x)^2*Sqrt[d^2 - e^2*x^2])
```

Maple [A] time = 0.05, size = 85, normalized size = 0.7

$$-\frac{(-ex+d)\,(ex+d)^4\left(3\,de^2gx^2-2\,e^3fx^2-9\,d^2egx+6\,de^2fx+3\,d^3g-7\,d^2ef\right)}{15\,d^3e^2}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] -1/15*(-e*x+d)*(e*x+d)^4*(3*d*e^2*g*x^2-2*e^3*f*x^2-9*d^2*e*g*x+6*d*e^2*f*x
+3*d^3*g-7*d^2*e*f)/d^3/e^2/(-e^2*x^2+d^2)^(7/2)
```

Maxima [B] time = 1.02423, size = 504, normalized size = 4.31

$$\frac{egx^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{dfx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{3\,d^2gx}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{3\,d^2f}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{d^3g}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e^2} + \frac{4\,fx}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*g*x^3/(-e^2*x^2 + d^2)^(5/2) + 1/5*d*f*x/(-e^2*x^2 + d^2)^(5/2) - 3/1
0*d^2*g*x/((-e^2*x^2 + d^2)^(5/2)*e) + 3/5*d^2*f/((-e^2*x^2 + d^2)^(5/2)*e)
 + 1/5*d^3*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 4/15*f*x/((-e^2*x^2 + d^2)^(3/2
)*d) + 1/10*g*x/((-e^2*x^2 + d^2)^(3/2)*e) + 8/15*f*x/(sqrt(-e^2*x^2 + d^2)
*d^3) + 1/5*g*x/(sqrt(-e^2*x^2 + d^2)*d^2*e) + 1/3*(e^3*f + 3*d*e^2*g)*x^2/
((-e^2*x^2 + d^2)^(5/2)*e^2) + 3/5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^
```

```
(5/2)*e^2) - 2/15*(e^3*f + 3*d*e^2*g)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) - 1/
5*(d*e^2*f + d^2*e*g)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 2/5*(d*e^2*f + d
^2*e*g)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)
```

Fricas [A] time = 1.89551, size = 378, normalized size = 3.23

$$-\frac{7 d^3 e f - 3 d^4 g - \left(7 e^4 f - 3 d e^3 g\right) x^3 + 3 \left(7 d e^3 f - 3 d^2 e^2 g\right) x^2 - 3 \left(7 d^2 e^2 f - 3 d^3 e g\right) x + \left(7 d^2 e f - 3 d^3 g + \left(2 e^3 f - 3 d e\right.\right.}{15 \left(d^3 e^5 x^3 - 3 d^4 e^4 x^2 + 3 d^5 e^3 x - d^6 e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*d^3*e*f - 3*d^4*g - (7*e^4*f - 3*d*e^3*g)*x^3 + 3*(7*d*e^3*f - 3*d
^2*e^2*g)*x^2 - 3*(7*d^2*e^2*f - 3*d^3*e*g)*x + (7*d^2*e*f - 3*d^3*g + (2*e
^3*f - 3*d*e^2*g)*x^2 - 3*(2*d*e^2*f - 3*d^2*e*g)*x)*sqrt(-e^2*x^2 + d^2))/
(d^3*e^5*x^3 - 3*d^4*e^4*x^2 + 3*d^5*e^3*x - d^6*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3 \left(f + gx\right)}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.23002, size = 188, normalized size = 1.61

$$-\frac{\sqrt{-x^2 e^2 + d^2}\left(\left(15 d f - \left(x\left(\frac{\left(3 d^2 g e^7 - 2 d f e^8\right) x^2 e^{(-4)}}{d^4} - \frac{5\left(3 d^4 g e^5 - d^3 f e^6\right) e^{(-4)}}{d^4}\right) - \frac{5\left(3 d^5 g e^4 + d^4 f e^5\right) e^{(-4)}}{d^4}\right) x\right) x - \frac{\left(3 d^7 g e^2 - 7 d^6 f e^3\right) e^{(-4)}}{d^4}\right)}{15\left(x^2 e^2 - d^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*((15*d*f - (x*((3*d^2*g*e^7 - 2*d*f*e^8)*x^2*e^(
-4)/d^4 - 5*(3*d^4*g*e^5 - d^3*f*e^6)*e^(-4)/d^4) - 5*(3*d^5*g*e^4 + d^4*f*
e^5)*e^(-4)/d^4)*x)*x - (3*d^7*g*e^2 - 7*d^6*f*e^3)*e^(-4)/d^4)/(x^2*e^2 -
d^2)^3
```

3.584 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rubi [A] time = 0.0493467, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {655, 659, 651}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}dx = \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}dx$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}dx}{15d^2}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Mathematica [A] time = 0.0618823, size = 58, normalized size = 0.56

$$\frac{(d+ex)\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((d + e*x)*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^2*Sqrt[d^2 -
e^2*x^2])
```

Maple [A] time = 0.047, size = 55, normalized size = 0.5

$$\frac{(-ex+d)(ex+d)^4\left(2x^2e^2-6dex+7d^2\right)}{15d^3e}\left(-x^2e^2+d^2\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^4*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2
)
```

Maxima [A] time = 1.00769, size = 136, normalized size = 1.32

$$\frac{ex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{4dx}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{7d^2}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d} + \frac{2x}{15\sqrt{-e^2x^2+d^2}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^
2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

Fricas [A] time = 1.77293, size = 215, normalized size = 2.09

$$\frac{7\,e^{3}x^{3}-21\,de^{2}x^{2}+21\,d^{2}ex-7\,d^{3}-\left(2\,e^{2}x^{2}-6\,dex+7\,d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15\,\left(d^{3}e^{4}x^{3}-3\,d^{4}e^{3}x^{2}+3\,d^{5}e^{2}x-d^{6}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
+ 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x -
 d^6*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{3}}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A] time = 1.19323, size = 95, normalized size = 0.92

$$-\frac{\sqrt{-x^{2}e^{2}+d^{2}}\left(7\,d^{2}e^{(-1)}+\left(\left(x\left(\frac{2\,x^{2}e^{4}}{d^{3}}-\frac{5\,e^{2}}{d}\right)+5\,e\right)x+15\,d\right)x\right)}{15\,\left(x^{2}e^{2}-d^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(7*d^2*e^(-1) + ((x*(2*x^2*e^4/d^3 - 5*e^2/d) +
5*e)*x + 15*d)*x)/(x^2*e^2 - d^2)^3
```

3.585 $\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=242

$$\frac{ex\left(22d^2g^2+9defg+2e^2f^2\right)+15d^3g^2}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^3}+\frac{g^3\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}}-\frac{5d(ef-dg)-ex(11dg+ef)}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^2}+\frac{4d(d}{5\left(d^2-e^2x^2\right)}$$

```
[Out] (4*d*(d + e*x))/(5*(e*f + d*g)*(d^2 - e^2*x^2)^(5/2)) - (5*d*(e*f - d*g) -
e*(e*f + 11*d*g)*x)/(15*d*(e*f + d*g)^2*(d^2 - e^2*x^2)^(3/2)) + (15*d^3*g^
2 + e*(2*e^2*f^2 + 9*d*e*f*g + 22*d^2*g^2)*x)/(15*d^3*(e*f + d*g)^3*Sqrt[d^
2 - e^2*x^2]) + (g^3*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/((e*f + d*g)^3*Sqrt[e^2*f^2 - d^2*g^2])
```

Rubi [A] time = 0.617526, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1647, 823, 12, 725, 204}

$$\frac{ex\left(22d^2g^2+9defg+2e^2f^2\right)+15d^3g^2}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^3}+\frac{g^3\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}}-\frac{5d(ef-dg)-ex(11dg+ef)}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^2}+\frac{4d(d}{5\left(d^2-e^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (4*d*(d + e*x))/(5*(e*f + d*g)*(d^2 - e^2*x^2)^(5/2)) - (5*d*(e*f - d*g) -
e*(e*f + 11*d*g)*x)/(15*d*(e*f + d*g)^2*(d^2 - e^2*x^2)^(3/2)) + (15*d^3*g^
2 + e*(2*e^2*f^2 + 9*d*e*f*g + 22*d^2*g^2)*x)/(15*d^3*(e*f + d*g)^3*Sqrt[d^
2 - e^2*x^2]) + (g^3*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/((e*f + d*g)^3*Sqrt[e^2*f^2 - d^2*g^2])
```


Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2(ef+5dg)}{ef+dg}-\frac{d^2e^3(5ef-11dg)x}{ef+dg}}{(f+gx)\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-\frac{d^3e^4(ef-dg)\left(2e^2f^2+7defg+15d^2g^2\right)}{ef+dg}-\frac{2d}{}}{(f+gx)\left(d^2-e^2x^2\right)^{3}}}{15d^4e^4\left(e^2f^2-d\right.}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9defg+\right.}{15d^3(ef+dg)^3\sqrt{d^2-e^2}}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9defg+\right.}{15d^3(ef+dg)^3\sqrt{d^2-e^2}}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9defg+\right.}{15d^3(ef+dg)^3\sqrt{d^2-e^2}}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e\left(2e^2f^2+9defg+\right.}{15d^3(ef+dg)^3\sqrt{d^2-e^2}}$$

Mathematica [A] time = 0.410837, size = 225, normalized size = 0.93

$$\frac{\frac{(d+ex)\left(d^2g^2-e^2f^2\right)\left(d^2e^2\left(7f^2-27fgx+22g^2x^2\right)+3d^3eg(8f-17gx)+32d^4g^2+3de^3fx(3gx-2f)+2e^4f^2x^2\right)}{d^3(d-ex)^2\sqrt{d^2-e^2x^2}} - 15g^3\sqrt{e^2f^2-d^2g^2}\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2}}\right.}{15(dg-ef)(dg+ef)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((-(e^2*f^2) + d^2*g^2)*(d + e*x)*(32*d^4*g^2 + 2*e^4*f^2*x^2 + 3*d^3*e*g*
(8*f - 17*g*x) + 3*d*e^3*f*x*(-2*f + 3*g*x) + d^2*e^2*(7*f^2 - 27*f*g*x + 2
2*g^2*x^2)))/(d^3*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]) - 15*g^3*Sqrt[e^2*f^2 -
d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x
```

```
^2])])/(15*(-(e*f) + d*g)*(e*f + d*g)^4)
```

Maple [B] time = 0.259, size = 3961, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 22/15*e/g/d^4*x/(-e^2*x^2+d^2)^(1/2)-g^4/(d^2*g^2-e^2*f^2)^3*f*d/(-(x+f/g)^
2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*e^2*x-3*g^2/(d^2*g^2-e
^2*f^2)^3*f^3/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1
/2)*e^4*x+g/(d^2*g^2-e^2*f^2)^3*f^4/d^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(
d^2*g^2-e^2*f^2)/g^2)^(1/2)*e^5*x-3/5/g^2*e^4*f^3/(d^2*g^2-e^2*f^2)/d/(-(x+
f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*x+8/15/g^3*e^5*f^
4/(d^2*g^2-e^2*f^2)/d^6/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)
/g^2)^(1/2)*x-1/3*g^2/(d^2*g^2-e^2*f^2)^2*e^2*f*d/(-(x+f/g)^2*e^2+2*e^2*f/g
*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+4/15/g^3*e^5*f^4/(d^2*g^2-e^2*f^2)/
d^4/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x-8/5/g^
2*e^4*f^3/(d^2*g^2-e^2*f^2)/d^5/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-
e^2*f^2)/g^2)^(1/2)*x-1/5/g^2/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(
x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*e^3*f^3-g^2/(d^2*g^2-e^2*f^2)^3/(-(x+f/
g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*e^3*f^3-g^5/(d^2*g^
2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^
2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2*e^2+2*e^2*f/g*(x+
f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))*d^3-3/5/(d^2*g^2-e^2*f^2)/(-(x+
f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)*d^2*e*f-2/(d^2*g^
2-e^2*f^2)^2*e^4*f^3/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2
)/g^2)^(1/2)*x+3/5/g/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d
^2*g^2-e^2*f^2)/g^2)^(5/2)*d*e^2*f^2-g^2/(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*e^
2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*d^2*e*f-2/3*g^2/(d^2*g^2-e
^2*f^2)^2*e^2*f/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^
(1/2)*x+3/5/g/(-e^2*x^2+d^2)^(5/2)*d+4/5/g*e^3*f^2/(d^2*g^2-e^2*f^2)/d^2/(-
(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x-4/5/g^2*e^4*
f^3/(d^2*g^2-e^2*f^2)/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^
2)/g^2)^(3/2)*x+3*g^4/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln(
(2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*
(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))*d^
2*e*f-3*g^3/(d^2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*g^
2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^
2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))*d*e^2*f^2+1/
5/g^3*e^5*f^4/(d^2*g^2-e^2*f^2)/d^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*
g^2-e^2*f^2)/g^2)^(5/2)*x+8/5/g*e^3*f^2/(d^2*g^2-e^2*f^2)/d^4/(-(x+f/g)^2*e
^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x+2*g/(d^2*g^2-e^2*f^2)^2
*e^3*f^2/d^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)
*x+2/3/g/(d^2*g^2-e^2*f^2)^2*e^5*f^4/d^4/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+
(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x+11/15*e/g/d^2*x/(-e^2*x^2+d^2)^(3/2)+g^2/(d^
2*g^2-e^2*f^2)^3/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*g^2-e^2*f^2)/g^2+
2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2*e^2+2*e^2*f/g
*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g))*e^3*f^3+3/5/g*e^3*f^2/(d^2*
g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)
*x+g/(d^2*g^2-e^2*f^2)^2*e^3*f^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2
-e^2*f^2)/g^2)^(3/2)*x+3*g^3/(d^2*g^2-e^2*f^2)^3*f^2/(-(x+f/g)^2*e^2+2*e^2*
f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*e^3*x-1/(d^2*g^2-e^2*f^2)^2*e^4*f^
3/d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x+4/15*e
^3/g^3*f^2/d^4*x/(-e^2*x^2+d^2)^(3/2)+8/15*e^3/g^3*f^2/d^6*x/(-e^2*x^2+d^2)
```

```
^(1/2)-1/5*e^2*f/(d^2*g^2-e^2*f^2)*d/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2
*g^2-e^2*f^2)/g^2)^(5/2)*x-4/15*e^2*f/(d^2*g^2-e^2*f^2)/d/(-(x+f/g)^2*e^2+2
*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*x-8/15*e^2*f/(d^2*g^2-e^2*f^2
)/d^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*x-1/3/
(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2
)^(3/2)*e^3*f^3+1/5*g/(d^2*g^2-e^2*f^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(
d^2*g^2-e^2*f^2)/g^2)^(5/2)*d^3+1/3*g^3/(d^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*e^2
+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)*d^3+g^5/(d^2*g^2-e^2*f^2)^3
/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*d^3+4/5*e/g
*x/(-e^2*x^2+d^2)^(5/2)-1/5*e/g^2/(-e^2*x^2+d^2)^(5/2)*f-3/5*e^2/g^2/d*f*x/
(-e^2*x^2+d^2)^(5/2)-4/5*e^2/g^2/d^3*f*x/(-e^2*x^2+d^2)^(3/2)-8/5*e^2/g^2/d
^5*f*x/(-e^2*x^2+d^2)^(1/2)+1/5*e^3/g^3*f^2*x/d^2/(-e^2*x^2+d^2)^(5/2)+g/(d
^2*g^2-e^2*f^2)^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^
(3/2)*d*e^2*f^2-3*g^4/(d^2*g^2-e^2*f^2)^3/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)
+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*d^2*e*f+3*g^3/(d^2*g^2-e^2*f^2)^3/(-(x+f/g)^2
*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)*d*e^2*f^2+1/3/g/(d^2*g^
2-e^2*f^2)^2*e^5*f^4/d^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2
)/g^2)^(3/2)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.26396, size = 3537, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*g
^3 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d^3
*e^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 25*d
^3*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 + 15*(d^3*e^3*g^3*x
^3 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(-e^2*f^2 + d^2*g^2)*
log((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sq
rt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x +
f)) - 3*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24*d^5*e^2
*f*g^3 - 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*d^4*e^2*f
^2*g^2 - 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g + 20*d^2*
e^4*f^2*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5*f^4 + 9*d^
2*e^4*f^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g^4)*x)*sqrt(
-e^2*x^2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^3*g^2 - 2*d^9
*e^2*f^2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d^4*e^7*f^4*g +
 2*d^5*e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d^8*e^3*g^5)*x^3
 + 3*(d^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2*d^7*e^4*f^2*g^3
 - 3*d^8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3*d^6*e^5*f^4*g +
2*d^7*e^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 - d^10*e*g^5)*x), 1
```

```
/15*(7*d^3*e^4*f^4 + 24*d^4*e^3*f^3*g + 25*d^5*e^2*f^2*g^2 - 24*d^6*e*f*g^3
 - 32*d^7*g^4 - (7*e^7*f^4 + 24*d*e^6*f^3*g + 25*d^2*e^5*f^2*g^2 - 24*d^3*e
^4*f*g^3 - 32*d^4*e^3*g^4)*x^3 + 3*(7*d*e^6*f^4 + 24*d^2*e^5*f^3*g + 25*d^3
*e^4*f^2*g^2 - 24*d^4*e^3*f*g^3 - 32*d^5*e^2*g^4)*x^2 - 30*(d^3*e^3*g^3*x^3
 - 3*d^4*e^2*g^3*x^2 + 3*d^5*e*g^3*x - d^6*g^3)*sqrt(e^2*f^2 - d^2*g^2)*arc
tan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e^2*f^2 - d^2*g^2)*x)) - 3
*(7*d^2*e^5*f^4 + 24*d^3*e^4*f^3*g + 25*d^4*e^3*f^2*g^2 - 24*d^5*e^2*f*g^3
- 32*d^6*e*g^4)*x + (7*d^2*e^4*f^4 + 24*d^3*e^3*f^3*g + 25*d^4*e^2*f^2*g^2
- 24*d^5*e*f*g^3 - 32*d^6*g^4 + (2*e^6*f^4 + 9*d*e^5*f^3*g + 20*d^2*e^4*f^2
*g^2 - 9*d^3*e^3*f*g^3 - 22*d^4*e^2*g^4)*x^2 - 3*(2*d*e^5*f^4 + 9*d^2*e^4*f
^3*g + 15*d^3*e^3*f^2*g^2 - 9*d^4*e^2*f*g^3 - 17*d^5*e*g^4)*x)*sqrt(-e^2*x^
2 + d^2))/(d^6*e^5*f^5 + 3*d^7*e^4*f^4*g + 2*d^8*e^3*f^3*g^2 - 2*d^9*e^2*f^
2*g^3 - 3*d^10*e*f*g^4 - d^11*g^5 - (d^3*e^8*f^5 + 3*d^4*e^7*f^4*g + 2*d^5*
e^6*f^3*g^2 - 2*d^6*e^5*f^2*g^3 - 3*d^7*e^4*f*g^4 - d^8*e^3*g^5)*x^3 + 3*(d
^4*e^7*f^5 + 3*d^5*e^6*f^4*g + 2*d^6*e^5*f^3*g^2 - 2*d^7*e^4*f^2*g^3 - 3*d^
8*e^3*f*g^4 - d^9*e^2*g^5)*x^2 - 3*(d^5*e^6*f^5 + 3*d^6*e^5*f^4*g + 2*d^7*e
^4*f^3*g^2 - 2*d^8*e^3*f^2*g^3 - 3*d^9*e^2*f*g^4 - d^10*e*g^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)), x)
```

Giac [B] time = 1.28356, size = 4004, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2*(d^3*g^6*e^2 - 3*d^2*f*g^5*e^3 + 3*d*f^2*g^4*e^4 - f^3*g^3*e^5)*arctan((
d*g*e + (d*e + sqrt(-x^2*e^2 + d^2)*e)*f/x)/sqrt(-d^2*g^2*e^2 + f^2*e^4))/(
(d^6*g^6*e - 3*d^4*f^2*g^4*e^3 + 3*d^2*f^4*g^2*e^5 - f^6*e^7)*sqrt(-d^2*g^2
*e^2 + f^2*e^4)) - 1/15*sqrt(-x^2*e^2 + d^2)*((((((22*d^18*g^17*e^9 + 339*d
^17*f*g^16*e^10 + 2447*d^16*f^2*g^15*e^11 + 10985*d^15*f^3*g^14*e^12 + 3433
5*d^14*f^4*g^13*e^13 + 79261*d^13*f^5*g^12*e^14 + 139867*d^12*f^6*g^11*e^15
 + 192621*d^11*f^7*g^10*e^16 + 209495*d^10*f^8*g^9*e^17 + 180895*d^9*f^9*g^
8*e^18 + 123981*d^8*f^10*g^7*e^19 + 67067*d^7*f^11*g^6*e^20 + 28301*d^6*f^1
2*g^5*e^21 + 9135*d^5*f^13*g^4*e^22 + 2185*d^4*f^14*g^3*e^23 + 367*d^3*f^15
*g^2*e^24 + 39*d^2*f^16*g*e^25 + 2*d*f^17*e^26)*x/(d^22*g^18*e^4 + 18*d^21*
f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f^3*g^15*e^7 + 3060*d^18*f^4*
g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*f^6*g^12*e^10 + 31824*d^15*f
^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620*d^13*f^9*g^9*e^13 + 43758*d
^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 + 18564*d^10*f^12*g^6*e^16 + 8
568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18 + 816*d^7*f^15*g^3*e^19 + 15
3*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*f^18*e^22) + 15*(d^19*g^17*e
^8 + 15*d^18*f*g^16*e^9 + 105*d^17*f^2*g^15*e^10 + 455*d^16*f^3*g^14*e^11 +
```

1365*d^15*f^4*g^13*e^12 + 3003*d^14*f^5*g^12*e^13 + 5005*d^13*f^6*g^11*e^1
4 + 6435*d^12*f^7*g^10*e^15 + 6435*d^11*f^8*g^9*e^16 + 5005*d^10*f^9*g^8*e^
17 + 3003*d^9*f^10*g^7*e^18 + 1365*d^8*f^11*g^6*e^19 + 455*d^7*f^12*g^5*e^2
0 + 105*d^6*f^13*g^4*e^21 + 15*d^5*f^14*g^3*e^22 + d^4*f^15*g^2*e^23)/(d^22
*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f^3*g^15*
e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*f^6*g^12
*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620*d^13*f^
9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 + 18564*d^
10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18 + 816*d^
7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*f^18*e^2
2))*x - 5*(11*d^20*g^17*e^7 + 171*d^19*f*g^16*e^8 + 1246*d^18*f^2*g^15*e^9
+ 5650*d^17*f^3*g^14*e^10 + 17850*d^16*f^4*g^13*e^11 + 41678*d^15*f^5*g^12*
e^12 + 74438*d^14*f^6*g^11*e^13 + 103818*d^13*f^7*g^10*e^14 + 114400*d^12*f
^8*g^9*e^15 + 100100*d^11*f^9*g^8*e^16 + 69498*d^10*f^10*g^7*e^17 + 38038*d
^9*f^11*g^6*e^18 + 16198*d^8*f^12*g^5*e^19 + 5250*d^7*f^13*g^4*e^20 + 1250*
d^6*f^14*g^3*e^21 + 206*d^5*f^15*g^2*e^22 + 21*d^4*f^16*g*e^23 + d^3*f^17*e
^24)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19
*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^1
6*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 486
20*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15
+ 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^1
8 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^
4*f^18*e^22))*x - 5*(7*d^21*g^17*e^6 + 105*d^20*f*g^16*e^7 + 734*d^19*f^2*g
^15*e^8 + 3170*d^18*f^3*g^14*e^9 + 9450*d^17*f^4*g^13*e^10 + 20566*d^16*f^5
*g^12*e^11 + 33670*d^15*f^6*g^11*e^12 + 42042*d^14*f^7*g^10*e^13 + 40040*d^
13*f^8*g^9*e^14 + 28600*d^12*f^9*g^8*e^15 + 14586*d^11*f^10*g^7*e^16 + 4550
*d^10*f^11*g^6*e^17 + 182*d^9*f^12*g^5*e^18 - 630*d^8*f^13*g^4*e^19 - 350*d
^7*f^14*g^3*e^20 - 98*d^6*f^15*g^2*e^21 - 15*d^5*f^16*g*e^22 - d^4*f^17*e^2
3)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19*f
^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^16*
f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 48620
*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15 +
18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^18
+ 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^4*
f^18*e^22))*x + 15*(3*d^22*g^17*e^5 + 48*d^21*f*g^16*e^6 + 361*d^20*f^2*g^1
5*e^7 + 1695*d^19*f^3*g^14*e^8 + 5565*d^18*f^4*g^13*e^9 + 13559*d^17*f^5*g^
12*e^10 + 25389*d^16*f^6*g^11*e^11 + 37323*d^15*f^7*g^10*e^12 + 43615*d^14*
f^8*g^9*e^13 + 40755*d^13*f^9*g^8*e^14 + 30459*d^12*f^10*g^7*e^15 + 18109*d
^11*f^11*g^6*e^16 + 8463*d^10*f^12*g^5*e^17 + 3045*d^9*f^13*g^4*e^18 + 815*
d^8*f^14*g^3*e^19 + 153*d^7*f^15*g^2*e^20 + 18*d^6*f^16*g*e^21 + d^5*f^17*e
^22)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g^16*e^6 + 816*d^19
*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^13*e^9 + 18564*d^1
6*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f^8*g^10*e^12 + 486
20*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d^11*f^11*g^7*e^15
+ 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 3060*d^8*f^14*g^4*e^1
8 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^5*f^17*g*e^21 + d^
4*f^18*e^22))*x + (32*d^23*g^17*e^4 + 504*d^22*f*g^16*e^5 + 3727*d^21*f^2*g
^15*e^6 + 17185*d^20*f^3*g^14*e^7 + 55335*d^19*f^4*g^13*e^8 + 132041*d^18*f
^5*g^12*e^9 + 241787*d^17*f^6*g^11*e^10 + 347061*d^16*f^7*g^10*e^11 + 39539
5*d^15*f^8*g^9*e^12 + 359645*d^14*f^9*g^8*e^13 + 261261*d^13*f^10*g^7*e^14
+ 150787*d^12*f^11*g^6*e^15 + 68341*d^11*f^12*g^5*e^16 + 23835*d^10*f^13*g^
4*e^17 + 6185*d^9*f^14*g^3*e^18 + 1127*d^8*f^15*g^2*e^19 + 129*d^7*f^16*g*e
^20 + 7*d^6*f^17*e^21)/(d^22*g^18*e^4 + 18*d^21*f*g^17*e^5 + 153*d^20*f^2*g
^16*e^6 + 816*d^19*f^3*g^15*e^7 + 3060*d^18*f^4*g^14*e^8 + 8568*d^17*f^5*g^
13*e^9 + 18564*d^16*f^6*g^12*e^10 + 31824*d^15*f^7*g^11*e^11 + 43758*d^14*f
^8*g^10*e^12 + 48620*d^13*f^9*g^9*e^13 + 43758*d^12*f^10*g^8*e^14 + 31824*d
^11*f^11*g^7*e^15 + 18564*d^10*f^12*g^6*e^16 + 8568*d^9*f^13*g^5*e^17 + 306
0*d^8*f^14*g^4*e^18 + 816*d^7*f^15*g^3*e^19 + 153*d^6*f^16*g^2*e^20 + 18*d^
5*f^17*g*e^21 + d^4*f^18*e^22))/(x^2*e^2 - d^2)^3

3.586 $\int \frac{(d+ex)^3}{(f+gx)^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=311

$$\frac{e\left(ex\left(57d^2g^2+14defg+2e^2f^2\right)+45d^3g^2\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^4}+\frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4}-$$

```
[Out] (4*d*e*(d + e*x))/(5*(e*f + d*g)^2*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d*(e*f -
3*d*g) - e*(e*f + 21*d*g)*x))/(15*d*(e*f + d*g)^3*(d^2 - e^2*x^2)^(3/2)) +
(e*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(15*d^3*(e*f +
 d*g)^4*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/((e*f - d*g)*(e*f
+ d*g)^4*(f + g*x)) + (e*g^3*(4*e*f - 3*d*g)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt
[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/((e*f - d*g)*(e*f + d*g)^4*Sqrt[
e^2*f^2 - d^2*g^2])
```

Rubi [A] time = 1.26448, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1647, 807, 725, 204}

$$\frac{e\left(ex\left(57d^2g^2+14defg+2e^2f^2\right)+45d^3g^2\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^4}+\frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (4*d*e*(d + e*x))/(5*(e*f + d*g)^2*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d*(e*f -
3*d*g) - e*(e*f + 21*d*g)*x))/(15*d*(e*f + d*g)^3*(d^2 - e^2*x^2)^(3/2)) +
(e*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(15*d^3*(e*f +
 d*g)^4*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/((e*f - d*g)*(e*f
+ d*g)^4*(f + g*x)) + (e*g^3*(4*e*f - 3*d*g)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt
[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/((e*f - d*g)*(e*f + d*g)^4*Sqrt[
e^2*f^2 - d^2*g^2])
```


Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^2f^2+10defg+5d^2g^2\right)}{(ef+dg)^2}-\frac{d^2e^3(ef-5dg)(5ef+3dg)x}{(ef+dg)^2}+\frac{16d^3e^4g^2x^2}{(ef+dg)^2}}{(f+gx)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{d^3e^4\left(2e^3f^3+12de^2f^2g+45d\ldots\right.}{(ef+dg)^3}\ldots}{\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2e^2f^2\ldots\right.\right.}{15d^3(ef+d\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2e^2f^2\ldots\right.\right.}{15d^3(ef+d\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2e^2f^2\ldots\right.\right.}{15d^3(ef+d\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e\left(45d^3g^2+e\left(2e^2f^2\ldots\right.\right.}{15d^3(ef+d\ldots}$$

Mathematica [A] time = 0.648739, size = 341, normalized size = 1.1

$$\frac{\frac{(d+ex)\left(e^2f^2-d^2g^2\right)\left(d^4e^2g^2\left(38f^2+164fgx+171g^2x^2\right)-3d^3e^3g\left(19f^2gx-9f^3+47fg^2x^2+24g^3x^3\right)+d^2e^4f\left(-29f^2gx+7f^3+7fg^2x^2+43g^3x^3\right)-9d^5eg^3(8f+13gx)+1\ldots\right.}{d^3(d-ex)^2\sqrt{d^2-e^2x^2}(f+gx)}\ldots}{15(ef-dg)^2(dg+e\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((e^2*f^2 - d^2*g^2)*(d + e*x)*(15*d^6*g^4 + 2*e^6*f^3*x^2*(f + g*x) - 9*d
^5*e*g^3*(8*f + 13*g*x) + 6*d*e^5*f^2*x*(-f^2 + f*g*x + 2*g^2*x^2) + d^4*e^
2*g^2*(38*f^2 + 164*f*g*x + 171*g^2*x^2) - 3*d^3*e^3*g*(-9*f^3 + 19*f^2*g*x
 + 47*f*g^2*x^2 + 24*g^3*x^3) + d^2*e^4*f*(7*f^3 - 29*f^2*g*x + 7*f*g^2*x^2
 + 43*g^3*x^3)))/(d^3*(d - e*x)^2*(f + g*x)*Sqrt[d^2 - e^2*x^2]) + 15*e*g^3
*(4*e*f - 3*d*g)*Sqrt[e^2*f^2 - d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2
*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/(15*(e*f - d*g)^2*(e*f + d*g)^5)
```

Maple [B] time = 0.211, size = 6760, normalized size = 21.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.69988, size = 6650, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*e^4*f^5*g^2 - 99*d^6*e^3*f
^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 - 15*d^9*f*g^6 - (7*e^9*f^6*
g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^3*e^6*f^3*g^4 - 23*d^4*e^5
*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4 - (7*e^9*f^7 + 6*d*e^8*f^
6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 + 274*d^4*e^5*f^3*g^4 + 141*
d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*g^7)*x^3 + 3*(7*d*e^8*f^7
+ 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*e^5*f^4*g^3 + 76*d^5*e^4*f
^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 + 15*d^8*e*g^7)*x^2 - 15*(4*
d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^2*g^4 - 3*d^4*e^4*f*g^5)*x
^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*d^5*e^3*f*g^5)*x^3 + 3*(4*
d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f*g^5)*x^2 - (12*d^5*e^3*f^
3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sqrt(-e^2*f^2 + d^2*g^2)*log
((d*e^2*f*g*x + d^3*g^2 - sqrt(-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sqrt(
-e^2*x^2 + d^2)*d*g) - (e^2*f^2 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x + f))
 - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5*g^2 - 328*d^5*e^4*f^
4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117*d^8*e*f*g^6 + 15*d^9
*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e^4*f^5*g^2 - 99*d^5*e
^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 15*d^8*f*g^6 + (2*e^8*
f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3*e^5*f^3*g^4 - 43*d^4
*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + 6*d*e^7*f^6*g + 5*d^2*e
^6*f^5*g^2 - 147*d^3*e^5*f^4*g^3 + 164*d^4*e^4*f^3*g^4 + 141*d^5*e^3*f^2*g^
5 - 171*d^6*e^2*f*g^6)*x^2 - (6*d*e^7*f^7 + 29*d^2*e^6*f^6*g + 51*d^3*e^5*f
^5*g^2 - 193*d^4*e^4*f^4*g^3 + 60*d^5*e^3*f^3*g^4 + 164*d^6*e^2*f^2*g^5 - 1
17*d^7*e*f*g^6)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*f^9 + 3*d^7*e^6*f^8*g + d
```

```
^8*e^5*f^7*g^2 - 5*d^9*e^4*f^6*g^3 - 5*d^10*e^3*f^5*g^4 + d^11*e^2*f^4*g^5
+ 3*d^12*e*f^3*g^6 + d^13*f^2*g^7 - (d^3*e^10*f^8*g + 3*d^4*e^9*f^7*g^2 + d
^5*e^8*f^6*g^3 - 5*d^6*e^7*f^5*g^4 - 5*d^7*e^6*f^4*g^5 + d^8*e^5*f^3*g^6 +
3*d^9*e^4*f^2*g^7 + d^10*e^3*f*g^8)*x^4 - (d^3*e^10*f^9 - 8*d^5*e^8*f^7*g^2
 - 8*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 + 16*d^8*e^5*f^4*g^5 - 8*d^10*e^3
*f^2*g^7 - 3*d^11*e^2*f*g^8)*x^3 + 3*(d^4*e^9*f^9 + 2*d^5*e^8*f^8*g - 2*d^6
*e^7*f^7*g^2 - 6*d^7*e^6*f^6*g^3 + 6*d^9*e^4*f^4*g^5 + 2*d^10*e^3*f^3*g^6 -
 2*d^11*e^2*f^2*g^7 - d^12*e*f*g^8)*x^2 - (3*d^5*e^8*f^9 + 8*d^6*e^7*f^8*g
- 16*d^8*e^5*f^6*g^3 - 10*d^9*e^4*f^5*g^4 + 8*d^10*e^3*f^4*g^5 + 8*d^11*e^2
*f^3*g^6 - d^13*f*g^8)*x), 1/15*(7*d^3*e^6*f^7 + 27*d^4*e^5*f^6*g + 31*d^5*
e^4*f^5*g^2 - 99*d^6*e^3*f^4*g^3 - 23*d^7*e^2*f^3*g^4 + 72*d^8*e*f^2*g^5 -
15*d^9*f*g^6 - (7*e^9*f^6*g + 27*d*e^8*f^5*g^2 + 31*d^2*e^7*f^4*g^3 - 99*d^
3*e^6*f^3*g^4 - 23*d^4*e^5*f^2*g^5 + 72*d^5*e^4*f*g^6 - 15*d^6*e^3*g^7)*x^4
 - (7*e^9*f^7 + 6*d*e^8*f^6*g - 50*d^2*e^7*f^5*g^2 - 192*d^3*e^6*f^4*g^3 +
274*d^4*e^5*f^3*g^4 + 141*d^5*e^4*f^2*g^5 - 231*d^6*e^3*f*g^6 + 45*d^7*e^2*
g^7)*x^3 + 3*(7*d*e^8*f^7 + 20*d^2*e^7*f^6*g + 4*d^3*e^6*f^5*g^2 - 130*d^4*
e^5*f^4*g^3 + 76*d^5*e^4*f^3*g^4 + 95*d^6*e^3*f^2*g^5 - 87*d^7*e^2*f*g^6 +
15*d^8*e*g^7)*x^2 + 30*(4*d^6*e^2*f^3*g^3 - 3*d^7*e*f^2*g^4 - (4*d^3*e^5*f^
2*g^4 - 3*d^4*e^4*f*g^5)*x^4 - (4*d^3*e^5*f^3*g^3 - 15*d^4*e^4*f^2*g^4 + 9*
d^5*e^3*f*g^5)*x^3 + 3*(4*d^4*e^4*f^3*g^3 - 7*d^5*e^3*f^2*g^4 + 3*d^6*e^2*f
*g^5)*x^2 - (12*d^5*e^3*f^3*g^3 - 13*d^6*e^2*f^2*g^4 + 3*d^7*e*f*g^5)*x)*sq
rt(e^2*f^2 - d^2*g^2)*arctan((d*g*x + d*f - sqrt(-e^2*x^2 + d^2)*f)/(sqrt(e
^2*f^2 - d^2*g^2)*x)) - (21*d^2*e^7*f^7 + 74*d^3*e^6*f^6*g + 66*d^4*e^5*f^5
*g^2 - 328*d^5*e^4*f^4*g^3 + 30*d^6*e^3*f^3*g^4 + 239*d^7*e^2*f^2*g^5 - 117
*d^8*e*f*g^6 + 15*d^9*g^7)*x + (7*d^2*e^6*f^7 + 27*d^3*e^5*f^6*g + 31*d^4*e
^4*f^5*g^2 - 99*d^5*e^3*f^4*g^3 - 23*d^6*e^2*f^3*g^4 + 72*d^7*e*f^2*g^5 - 1
5*d^8*f*g^6 + (2*e^8*f^6*g + 12*d*e^7*f^5*g^2 + 41*d^2*e^6*f^4*g^3 - 84*d^3
*e^5*f^3*g^4 - 43*d^4*e^4*f^2*g^5 + 72*d^5*e^3*f*g^6)*x^3 + (2*e^8*f^7 + 6*
d*e^7*f^6*g + 5*d^2*e^6*f^5*g^2 - 147*d^3*e^5*f^4*g^3 + 164*d^4*e^4*f^3*g^4
 + 141*d^5*e^3*f^2*g^5 - 171*d^6*e^2*f*g^6)*x^2 - (6*d*e^7*f^7 + 29*d^2*e^6
*f^6*g + 51*d^3*e^5*f^5*g^2 - 193*d^4*e^4*f^4*g^3 + 60*d^5*e^3*f^3*g^4 + 16
4*d^6*e^2*f^2*g^5 - 117*d^7*e*f*g^6)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*f^9
+ 3*d^7*e^6*f^8*g + d^8*e^5*f^7*g^2 - 5*d^9*e^4*f^6*g^3 - 5*d^10*e^3*f^5*g^
4 + d^11*e^2*f^4*g^5 + 3*d^12*e*f^3*g^6 + d^13*f^2*g^7 - (d^3*e^10*f^8*g +
3*d^4*e^9*f^7*g^2 + d^5*e^8*f^6*g^3 - 5*d^6*e^7*f^5*g^4 - 5*d^7*e^6*f^4*g^5
 + d^8*e^5*f^3*g^6 + 3*d^9*e^4*f^2*g^7 + d^10*e^3*f*g^8)*x^4 - (d^3*e^10*f^
9 - 8*d^5*e^8*f^7*g^2 - 8*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 + 16*d^8*e^5
*f^4*g^5 - 8*d^10*e^3*f^2*g^7 - 3*d^11*e^2*f*g^8)*x^3 + 3*(d^4*e^9*f^9 + 2*
d^5*e^8*f^8*g - 2*d^6*e^7*f^7*g^2 - 6*d^7*e^6*f^6*g^3 + 6*d^9*e^4*f^4*g^5 +
 2*d^10*e^3*f^3*g^6 - 2*d^11*e^2*f^2*g^7 - d^12*e*f*g^8)*x^2 - (3*d^5*e^8*f
^9 + 8*d^6*e^7*f^8*g - 16*d^8*e^5*f^6*g^3 - 10*d^9*e^4*f^5*g^4 + 8*d^10*e^3
*f^4*g^5 + 8*d^11*e^2*f^3*g^6 - d^13*f*g^8)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/((-e^2*x^2 + d^2)^(7/2)*(g*x + f)^2), x)
```

3.587 $\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=398

$$\frac{e^2\left(ex\left(107d^2g^2+19defg+2e^2f^2\right)+90d^3g^2\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^5}+\frac{e^2g^3\left(13d^2g^2-30defg+20e^2f^2\right)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}}+\frac{3eg^4}{2(f+}$$

```
[Out] (4*d*e^2*(d + e*x))/(5*(e*f + d*g)^3*(d^2 - e^2*x^2)^(5/2)) - (e^2*(5*d*(e*
f - 5*d*g) - e*(e*f + 31*d*g)*x))/(15*d*(e*f + d*g)^4*(d^2 - e^2*x^2)^(3/2)
) + (e^2*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x))/(15*d^3
*(e*f + d*g)^5*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/(2*(e*f - d
*g)*(e*f + d*g)^4*(f + g*x)^2) + (3*e*g^4*(3*e*f - 2*d*g)*Sqrt[d^2 - e^2*x^
2])/(2*(e*f - d*g)^2*(e*f + d*g)^5*(f + g*x)) + (e^2*g^3*(20*e^2*f^2 - 30*d
*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/(2*(e*f - d*g)^2*(e*f + d*g)^5*Sqrt[e^2*f^2 - d^2*g^2])
```

Rubi [A] time = 2.56842, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1647, 1651, 807, 725, 204}

$$\frac{e^2\left(ex\left(107d^2g^2+19defg+2e^2f^2\right)+90d^3g^2\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^5}+\frac{e^2g^3\left(13d^2g^2-30defg+20e^2f^2\right)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}}+\frac{3eg^4}{2(f+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```


```
[Out] (4*d*e^2*(d + e*x))/(5*(e*f + d*g)^3*(d^2 - e^2*x^2)^(5/2)) - (e^2*(5*d*(e*
f - 5*d*g) - e*(e*f + 31*d*g)*x))/(15*d*(e*f + d*g)^4*(d^2 - e^2*x^2)^(3/2)
) + (e^2*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x))/(15*d^3
*(e*f + d*g)^5*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/(2*(e*f - d
*g)*(e*f + d*g)^4*(f + g*x)^2) + (3*e*g^4*(3*e*f - 2*d*g)*Sqrt[d^2 - e^2*x^
2])/(2*(e*f - d*g)^2*(e*f + d*g)^5*(f + g*x)) + (e^2*g^3*(20*e^2*f^2 - 30*d
*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/(2*(e*f - d*g)^2*(e*f + d*g)^5*Sqrt[e^2*f^2 - d^2*g^2])
```


Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^3f^3+15de^2f^2g+15d^2efg^2+5d^3g^3\right)}{(ef+dg)^3} - \frac{d^2e^3\left(5e^3f^3-33de^2f^2g-45d^2efg^2-15d^3g^3\right)x}{(ef+dg)^3}}{(f+gx)^3\left(d^2-e^2x^2\right)^{5/2}}}{5d^2e^2}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{d^3e^4\left(2e^4f^4+17de^3f^3g+90d^2e^2\right.}{(ef+dg)}}{}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+\right.\right.}{15d^3(ef+dg}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+\right.\right.}{15d^3(ef+dg}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+\right.\right.}{15d^3(ef+dg}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+\right.\right.}{15d^3(ef+dg}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2\left(90d^3g^2+e\left(2e^2f^2+\right.\right.}{15d^3(ef+dg}$$

Mathematica [C] time = 1.26057, size = 387, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2e^2(107d^2g^2+19defg+2e^2f^2)}{d^3(d-ex)}+\frac{2e^2(dg+ef)(17dg+2ef)}{d^2(d-ex)^2}+\frac{6e^2(dg+ef)^2}{d(d-ex)^3}+\frac{45eg^4(3ef-2dg)}{(f+gx)(ef-dg)^2}+\frac{15g^4(dg+ef)}{(f+gx)^2(ef-dg)}\right)-\frac{15ie^2g^3(13d^2g^2-30}{}}{30(dg+ef)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((6*e^2*(e*f + d*g)^2)/(d*(d - e*x)^3) + (2*e^2*(e*f +
 d*g)*(2*e*f + 17*d*g))/(d^2*(d - e*x)^2) + (2*e^2*(2*e^2*f^2 + 19*d*e*f*g
+ 107*d^2*g^2))/(d^3*(d - e*x)) + (15*g^4*(e*f + d*g))/((e*f - d*g)*(f + g*
x)^2) + (45*e*g^4*(3*e*f - 2*d*g))/((e*f - d*g)^2*(f + g*x))) - ((15*I)*e^2
*g^3*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*Log[(4*(e*f - d*g)^2*(e*f + d*g
)^5*(I*d^2*g + I*e^2*f*x + Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2]))/(e
^2*g^2*Sqrt[e^2*f^2 - d^2*g^2]*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*(f +
g*x))])/((e*f - d*g)^2*Sqrt[e^2*f^2 - d^2*g^2]))/(30*(e*f + d*g)^5)
```

Maple [B] time = 0.258, size = 9593, normalized size = 24.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 25.1647, size = 11237, normalized size = 28.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/30*(14*d^3*e^8*f^10 + 60*d^4*e^7*f^9*g + 78*d^5*e^6*f^8*g^2 - 480*d^6*e^
5*f^7*g^3 + 312*d^7*e^4*f^6*g^4 + 330*d^8*e^3*f^5*g^5 - 419*d^9*e^2*f^4*g^6
 + 90*d^10*e*f^3*g^7 + 15*d^11*f^2*g^8 - (14*e^11*f^8*g^2 + 60*d*e^10*f^7*g
```

^3 + 78*d^2*e^9*f^6*g^4 - 480*d^3*e^8*f^5*g^5 + 312*d^4*e^7*f^4*g^6 + 330*d
^5*e^6*f^3*g^7 - 419*d^6*e^5*f^2*g^8 + 90*d^7*e^4*f*g^9 + 15*d^8*e^3*g^10)*
x^5 - (28*e^11*f^9*g + 78*d*e^10*f^8*g^2 - 24*d^2*e^9*f^7*g^3 - 1194*d^3*e^
8*f^6*g^4 + 2064*d^4*e^7*f^5*g^5 - 276*d^5*e^6*f^4*g^6 - 1828*d^6*e^5*f^3*g
^7 + 1437*d^7*e^4*f^2*g^8 - 240*d^8*e^3*f*g^9 - 45*d^9*e^2*g^10)*x^4 - (14*
e^11*f^10 - 24*d*e^10*f^9*g - 240*d^2*e^9*f^8*g^2 - 768*d^3*e^8*f^7*g^3 + 3
426*d^4*e^7*f^6*g^4 - 2982*d^5*e^6*f^5*g^5 - 1463*d^6*e^5*f^4*g^6 + 3594*d^
7*e^4*f^3*g^7 - 1782*d^8*e^3*f^2*g^8 + 180*d^9*e^2*f*g^9 + 45*d^10*e*g^10)*
x^3 + (42*d*e^10*f^10 + 96*d^2*e^9*f^9*g - 112*d^3*e^8*f^8*g^2 - 1848*d^4*e
^7*f^7*g^3 + 3894*d^5*e^6*f^6*g^4 - 1362*d^6*e^5*f^5*g^5 - 2925*d^7*e^4*f^4
*g^6 + 3114*d^8*e^3*f^3*g^7 - 914*d^9*e^2*f^2*g^8 + 15*d^11*g^10)*x^2 - 15*
(20*d^6*e^4*f^6*g^3 - 30*d^7*e^3*f^5*g^4 + 13*d^8*e^2*f^4*g^5 - (20*d^3*e^7
*f^4*g^5 - 30*d^4*e^6*f^3*g^6 + 13*d^5*e^5*f^2*g^7)*x^5 - (40*d^3*e^7*f^5*g
^4 - 120*d^4*e^6*f^4*g^5 + 116*d^5*e^5*f^3*g^6 - 39*d^6*e^4*f^2*g^7)*x^4 -
(20*d^3*e^7*f^6*g^3 - 150*d^4*e^6*f^5*g^4 + 253*d^5*e^5*f^4*g^5 - 168*d^6*e
^4*f^3*g^6 + 39*d^7*e^3*f^2*g^7)*x^3 + (60*d^4*e^6*f^6*g^3 - 210*d^5*e^5*f^
5*g^4 + 239*d^6*e^4*f^4*g^5 - 108*d^7*e^3*f^3*g^6 + 13*d^8*e^2*f^2*g^7)*x^2
 - (60*d^5*e^5*f^6*g^3 - 130*d^6*e^4*f^5*g^4 + 99*d^7*e^3*f^4*g^5 - 26*d^8*
e^2*f^3*g^6)*x)*sqrt(-e^2*f^2 + d^2*g^2)*log((d*e^2*f*g*x + d^3*g^2 - sqrt(
-e^2*f^2 + d^2*g^2)*(e^2*f*x + d^2*g + sqrt(-e^2*x^2 + d^2)*d*g) - (e^2*f^2
 - d^2*g^2)*sqrt(-e^2*x^2 + d^2))/(g*x + f)) - (42*d^2*e^9*f^10 + 152*d^3*e
^8*f^9*g + 114*d^4*e^7*f^8*g^2 - 1596*d^5*e^6*f^7*g^3 + 1896*d^6*e^5*f^6*g^
4 + 366*d^7*e^4*f^5*g^5 - 1917*d^8*e^3*f^4*g^6 + 1108*d^9*e^2*f^3*g^7 - 135
*d^10*e*f^2*g^8 - 30*d^11*f*g^9)*x + (14*d^2*e^8*f^10 + 60*d^3*e^7*f^9*g +
78*d^4*e^6*f^8*g^2 - 480*d^5*e^5*f^7*g^3 + 312*d^6*e^4*f^6*g^4 + 330*d^7*e^
3*f^5*g^5 - 419*d^8*e^2*f^4*g^6 + 90*d^9*e*f^3*g^7 + 15*d^10*f^2*g^8 + (4*e
^10*f^8*g^2 + 30*d*e^9*f^7*g^3 + 138*d^2*e^8*f^6*g^4 - 555*d^3*e^7*f^5*g^5
 + 162*d^4*e^6*f^4*g^6 + 525*d^5*e^5*f^3*g^7 - 304*d^6*e^4*f^2*g^8)*x^4 + (8
*e^10*f^9*g + 48*d*e^9*f^8*g^2 + 186*d^2*e^8*f^7*g^3 - 1224*d^3*e^7*f^6*g^4
 + 1539*d^4*e^6*f^5*g^5 + 459*d^5*e^5*f^4*g^6 - 1733*d^6*e^4*f^3*g^7 + 717*
d^7*e^3*f^2*g^8)*x^3 + (4*e^10*f^10 + 6*d*e^9*f^9*g - 28*d^2*e^8*f^8*g^2 -
828*d^3*e^7*f^7*g^3 + 2400*d^4*e^6*f^6*g^4 - 1197*d^5*e^5*f^5*g^5 - 1897*d^
6*e^4*f^4*g^6 + 2019*d^7*e^3*f^3*g^7 - 479*d^8*e^2*f^2*g^8)*x^2 - (12*d*e^9
*f^10 + 62*d^2*e^8*f^9*g + 114*d^3*e^7*f^8*g^2 - 1056*d^4*e^6*f^7*g^3 + 162
6*d^5*e^5*f^6*g^4 + 81*d^6*e^4*f^5*g^5 - 1707*d^7*e^3*f^4*g^6 + 913*d^8*e^2
*f^3*g^7 - 45*d^9*e*f^2*g^8)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^9*f^13 + 3*d^7
*e^8*f^12*g - 8*d^9*e^6*f^10*g^3 - 6*d^10*e^5*f^9*g^4 + 6*d^11*e^4*f^8*g^5
 + 8*d^12*e^3*f^7*g^6 - 3*d^14*e*f^5*g^8 - d^15*f^4*g^9 - (d^3*e^12*f^11*g^2
 + 3*d^4*e^11*f^10*g^3 - 8*d^6*e^9*f^8*g^5 - 6*d^7*e^8*f^7*g^6 + 6*d^8*e^7*
f^6*g^7 + 8*d^9*e^6*f^5*g^8 - 3*d^11*e^4*f^3*g^10 - d^12*e^3*f^2*g^11)*x^5
 - (2*d^3*e^12*f^12*g + 3*d^4*e^11*f^11*g^2 - 9*d^5*e^10*f^10*g^3 - 16*d^6*e
^9*f^9*g^4 + 12*d^7*e^8*f^8*g^5 + 30*d^8*e^7*f^7*g^6 - 2*d^9*e^6*f^6*g^7 -
24*d^10*e^5*f^5*g^8 - 6*d^11*e^4*f^4*g^9 + 7*d^12*e^3*f^3*g^10 + 3*d^13*e^2
*f^2*g^11)*x^4 - (d^3*e^12*f^13 - 3*d^4*e^11*f^12*g - 15*d^5*e^10*f^11*g^2
 + d^6*e^9*f^10*g^3 + 42*d^7*e^8*f^9*g^4 + 18*d^8*e^7*f^8*g^5 - 46*d^9*e^6*f
^7*g^6 - 30*d^10*e^5*f^6*g^7 + 21*d^11*e^4*f^5*g^8 + 17*d^12*e^3*f^4*g^9 -
3*d^13*e^2*f^3*g^10 - 3*d^14*e*f^2*g^11)*x^3 + (3*d^4*e^11*f^13 + 3*d^5*e^1
0*f^12*g - 17*d^6*e^9*f^11*g^2 - 21*d^7*e^8*f^10*g^3 + 30*d^8*e^7*f^9*g^4 +
 46*d^9*e^6*f^8*g^5 - 18*d^10*e^5*f^7*g^6 - 42*d^11*e^4*f^6*g^7 - d^12*e^3*
f^5*g^8 + 15*d^13*e^2*f^4*g^9 + 3*d^14*e*f^3*g^10 - d^15*f^2*g^11)*x^2 - (3
*d^5*e^10*f^13 + 7*d^6*e^9*f^12*g - 6*d^7*e^8*f^11*g^2 - 24*d^8*e^7*f^10*g^
3 - 2*d^9*e^6*f^9*g^4 + 30*d^10*e^5*f^8*g^5 + 12*d^11*e^4*f^7*g^6 - 16*d^12
*e^3*f^6*g^7 - 9*d^13*e^2*f^5*g^8 + 3*d^14*e*f^4*g^9 + 2*d^15*f^3*g^10)*x),
 1/30*(14*d^3*e^8*f^10 + 60*d^4*e^7*f^9*g + 78*d^5*e^6*f^8*g^2 - 480*d^6*e^
5*f^7*g^3 + 312*d^7*e^4*f^6*g^4 + 330*d^8*e^3*f^5*g^5 - 419*d^9*e^2*f^4*g^6
 + 90*d^10*e*f^3*g^7 + 15*d^11*f^2*g^8 - (14*e^11*f^8*g^2 + 60*d*e^10*f^7*g
^3 + 78*d^2*e^9*f^6*g^4 - 480*d^3*e^8*f^5*g^5 + 312*d^4*e^7*f^4*g^6 + 330*d
^5*e^6*f^3*g^7 - 419*d^6*e^5*f^2*g^8 + 90*d^7*e^4*f*g^9 + 15*d^8*e^3*g^10)*
x^5 - (28*e^11*f^9*g + 78*d*e^10*f^8*g^2 - 24*d^2*e^9*f^7*g^3 - 1194*d^3*e^

```
8*f^6*g^4 + 2064*d^4*e^7*f^5*g^5 - 276*d^5*e^6*f^4*g^6 - 1828*d^6*e^5*f^3*g
^7 + 1437*d^7*e^4*f^2*g^8 - 240*d^8*e^3*f*g^9 - 45*d^9*e^2*g^10)*x^4 - (14*
e^11*f^10 - 24*d*e^10*f^9*g - 240*d^2*e^9*f^8*g^2 - 768*d^3*e^8*f^7*g^3 + 3
426*d^4*e^7*f^6*g^4 - 2982*d^5*e^6*f^5*g^5 - 1463*d^6*e^5*f^4*g^6 + 3594*d^
7*e^4*f^3*g^7 - 1782*d^8*e^3*f^2*g^8 + 180*d^9*e^2*f*g^9 + 45*d^10*e*g^10)*
x^3 + (42*d*e^10*f^10 + 96*d^2*e^9*f^9*g - 112*d^3*e^8*f^8*g^2 - 1848*d^4*e
^7*f^7*g^3 + 3894*d^5*e^6*f^6*g^4 - 1362*d^6*e^5*f^5*g^5 - 2925*d^7*e^4*f^4
*g^6 + 3114*d^8*e^3*f^3*g^7 - 914*d^9*e^2*f^2*g^8 + 15*d^11*g^10)*x^2 + 30*
(20*d^6*e^4*f^6*g^3 - 30*d^7*e^3*f^5*g^4 + 13*d^8*e^2*f^4*g^5 - (20*d^3*e^7
*f^4*g^5 - 30*d^4*e^6*f^3*g^6 + 13*d^5*e^5*f^2*g^7)*x^5 - (40*d^3*e^7*f^5*g
^4 - 120*d^4*e^6*f^4*g^5 + 116*d^5*e^5*f^3*g^6 - 39*d^6*e^4*f^2*g^7)*x^4 -
(20*d^3*e^7*f^6*g^3 - 150*d^4*e^6*f^5*g^4 + 253*d^5*e^5*f^4*g^5 - 168*d^6*e
^4*f^3*g^6 + 39*d^7*e^3*f^2*g^7)*x^3 + (60*d^4*e^6*f^6*g^3 - 210*d^5*e^5*f^
5*g^4 + 239*d^6*e^4*f^4*g^5 - 108*d^7*e^3*f^3*g^6 + 13*d^8*e^2*f^2*g^7)*x^2
 - (60*d^5*e^5*f^6*g^3 - 130*d^6*e^4*f^5*g^4 + 99*d^7*e^3*f^4*g^5 - 26*d^8*
e^2*f^3*g^6)*x)*sqrt(e^2*f^2 - d^2*g^2)*arctan((d*g*x + d*f - sqrt(-e^2*x^2
 + d^2)*f)/(sqrt(e^2*f^2 - d^2*g^2)*x)) - (42*d^2*e^9*f^10 + 152*d^3*e^8*f^
9*g + 114*d^4*e^7*f^8*g^2 - 1596*d^5*e^6*f^7*g^3 + 1896*d^6*e^5*f^6*g^4 + 3
66*d^7*e^4*f^5*g^5 - 1917*d^8*e^3*f^4*g^6 + 1108*d^9*e^2*f^3*g^7 - 135*d^10
*e*f^2*g^8 - 30*d^11*f*g^9)*x + (14*d^2*e^8*f^10 + 60*d^3*e^7*f^9*g + 78*d^
4*e^6*f^8*g^2 - 480*d^5*e^5*f^7*g^3 + 312*d^6*e^4*f^6*g^4 + 330*d^7*e^3*f^5
*g^5 - 419*d^8*e^2*f^4*g^6 + 90*d^9*e*f^3*g^7 + 15*d^10*f^2*g^8 + (4*e^10*f
^8*g^2 + 30*d*e^9*f^7*g^3 + 138*d^2*e^8*f^6*g^4 - 555*d^3*e^7*f^5*g^5 + 162
*d^4*e^6*f^4*g^6 + 525*d^5*e^5*f^3*g^7 - 304*d^6*e^4*f^2*g^8)*x^4 + (8*e^10
*f^9*g + 48*d*e^9*f^8*g^2 + 186*d^2*e^8*f^7*g^3 - 1224*d^3*e^7*f^6*g^4 + 15
39*d^4*e^6*f^5*g^5 + 459*d^5*e^5*f^4*g^6 - 1733*d^6*e^4*f^3*g^7 + 717*d^7*e
^3*f^2*g^8)*x^3 + (4*e^10*f^10 + 6*d*e^9*f^9*g - 28*d^2*e^8*f^8*g^2 - 828*d
^3*e^7*f^7*g^3 + 2400*d^4*e^6*f^6*g^4 - 1197*d^5*e^5*f^5*g^5 - 1897*d^6*e^4
*f^4*g^6 + 2019*d^7*e^3*f^3*g^7 - 479*d^8*e^2*f^2*g^8)*x^2 - (12*d*e^9*f^10
 + 62*d^2*e^8*f^9*g + 114*d^3*e^7*f^8*g^2 - 1056*d^4*e^6*f^7*g^3 + 1626*d^5
*e^5*f^6*g^4 + 81*d^6*e^4*f^5*g^5 - 1707*d^7*e^3*f^4*g^6 + 913*d^8*e^2*f^3*
g^7 - 45*d^9*e*f^2*g^8)*x)*sqrt(-e^2*x^2 + d^2))/(d^6*e^9*f^13 + 3*d^7*e^8*
f^12*g - 8*d^9*e^6*f^10*g^3 - 6*d^10*e^5*f^9*g^4 + 6*d^11*e^4*f^8*g^5 + 8*d
^12*e^3*f^7*g^6 - 3*d^14*e*f^5*g^8 - d^15*f^4*g^9 - (d^3*e^12*f^11*g^2 + 3*
d^4*e^11*f^10*g^3 - 8*d^6*e^9*f^8*g^5 - 6*d^7*e^8*f^7*g^6 + 6*d^8*e^7*f^6*g
^7 + 8*d^9*e^6*f^5*g^8 - 3*d^11*e^4*f^3*g^10 - d^12*e^3*f^2*g^11)*x^5 - (2*
d^3*e^12*f^12*g + 3*d^4*e^11*f^11*g^2 - 9*d^5*e^10*f^10*g^3 - 16*d^6*e^9*f^
9*g^4 + 12*d^7*e^8*f^8*g^5 + 30*d^8*e^7*f^7*g^6 - 2*d^9*e^6*f^6*g^7 - 24*d^
10*e^5*f^5*g^8 - 6*d^11*e^4*f^4*g^9 + 7*d^12*e^3*f^3*g^10 + 3*d^13*e^2*f^2*
g^11)*x^4 - (d^3*e^12*f^13 - 3*d^4*e^11*f^12*g - 15*d^5*e^10*f^11*g^2 + d^6
*e^9*f^10*g^3 + 42*d^7*e^8*f^9*g^4 + 18*d^8*e^7*f^8*g^5 - 46*d^9*e^6*f^7*g^
6 - 30*d^10*e^5*f^6*g^7 + 21*d^11*e^4*f^5*g^8 + 17*d^12*e^3*f^4*g^9 - 3*d^1
3*e^2*f^3*g^10 - 3*d^14*e*f^2*g^11)*x^3 + (3*d^4*e^11*f^13 + 3*d^5*e^10*f^1
2*g - 17*d^6*e^9*f^11*g^2 - 21*d^7*e^8*f^10*g^3 + 30*d^8*e^7*f^9*g^4 + 46*d
^9*e^6*f^8*g^5 - 18*d^10*e^5*f^7*g^6 - 42*d^11*e^4*f^6*g^7 - d^12*e^3*f^5*g
^8 + 15*d^13*e^2*f^4*g^9 + 3*d^14*e*f^3*g^10 - d^15*f^2*g^11)*x^2 - (3*d^5*
e^10*f^13 + 7*d^6*e^9*f^12*g - 6*d^7*e^8*f^11*g^2 - 24*d^8*e^7*f^10*g^3 - 2
*d^9*e^6*f^9*g^4 + 30*d^10*e^5*f^8*g^5 + 12*d^11*e^4*f^7*g^6 - 16*d^12*e^3*
f^6*g^7 - 9*d^13*e^2*f^5*g^8 + 3*d^14*e*f^4*g^9 + 2*d^15*f^3*g^10)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

[Out] Timed out

Giac [B] time = 2.71614, size = 8123, normalized size = 20.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -(13*d^9*g^12*e^8 - 69*d^8*f*g^11*e^9 + 123*d^7*f^2*g^10*e^10 - 25*d^6*f^3*
g^9*e^11 - 195*d^5*f^4*g^8*e^12 + 237*d^4*f^5*g^7*e^13 - 31*d^3*f^6*g^6*e^1
4 - 123*d^2*f^7*g^5*e^15 + 90*d*f^8*g^4*e^16 - 20*f^9*g^3*e^17)*arctan((d*g
*e + (d*e + sqrt(-x^2*e^2 + d^2)*e)*f/x)/sqrt(-d^2*g^2*e^2 + f^2*e^4))/((d^
14*g^14*e^5 - 7*d^12*f^2*g^12*e^7 + 21*d^10*f^4*g^10*e^9 - 35*d^8*f^6*g^8*e
^11 + 35*d^6*f^8*g^6*e^13 - 21*d^4*f^10*g^4*e^15 + 7*d^2*f^12*g^2*e^17 - f^
14*e^19)*sqrt(-d^2*g^2*e^2 + f^2*e^4)) - 1/15*sqrt(-x^2*e^2 + d^2)*((((((10
7*d^28*g^27*e^11 + 2694*d^27*f*g^26*e^12 + 32577*d^26*f^2*g^25*e^13 + 25185
0*d^25*f^3*g^24*e^14 + 1397850*d^24*f^4*g^23*e^15 + 5929860*d^23*f^5*g^22*e
^16 + 19984470*d^22*f^6*g^21*e^17 + 54906060*d^21*f^7*g^20*e^18 + 125216025
*d^20*f^8*g^19*e^19 + 240109650*d^19*f^9*g^18*e^20 + 390736995*d^18*f^10*g^
17*e^21 + 543134190*d^17*f^11*g^16*e^22 + 647660220*d^16*f^12*g^15*e^23 + 6
64152600*d^15*f^13*g^14*e^24 + 586148100*d^14*f^14*g^13*e^25 + 444848520*d^
13*f^15*g^12*e^26 + 289619565*d^12*f^16*g^11*e^27 + 161082570*d^11*f^17*g^1
0*e^28 + 76070775*d^10*f^18*g^9*e^29 + 30246150*d^9*f^19*g^8*e^30 + 1001121
0*d^8*f^20*g^7*e^31 + 2717220*d^7*f^21*g^6*e^32 + 592710*d^6*f^22*g^5*e^33
+ 101100*d^5*f^23*g^4*e^34 + 12975*d^4*f^24*g^3*e^35 + 1182*d^3*f^25*g^2*e^
36 + 69*d^2*f^26*g*e^37 + 2*d*f^27*e^38)*x/(d^34*g^30*e^4 + 30*d^33*f*g^29*
e^5 + 435*d^32*f^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*
e^8 + 142506*d^29*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f
^7*g^23*e^11 + 5852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 3
0045015*d^24*f^10*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*
f^12*g^18*e^16 + 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e
^18 + 155117520*d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 11975
9850*d^17*f^17*g^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^1
9*g^11*e^23 + 30045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 +
5852925*d^12*f^22*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*
g^6*e^28 + 142506*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^
27*g^3*e^31 + 435*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34) +
 90*(d^29*g^27*e^10 + 25*d^28*f*g^26*e^11 + 300*d^27*f^2*g^25*e^12 + 2300*d
^26*f^3*g^24*e^13 + 12650*d^25*f^4*g^23*e^14 + 53130*d^24*f^5*g^22*e^15 + 1
77100*d^23*f^6*g^21*e^16 + 480700*d^22*f^7*g^20*e^17 + 1081575*d^21*f^8*g^1
9*e^18 + 2042975*d^20*f^9*g^18*e^19 + 3268760*d^19*f^10*g^17*e^20 + 4457400
*d^18*f^11*g^16*e^21 + 5200300*d^17*f^12*g^15*e^22 + 5200300*d^16*f^13*g^14
*e^23 + 4457400*d^15*f^14*g^13*e^24 + 3268760*d^14*f^15*g^12*e^25 + 2042975
*d^13*f^16*g^11*e^26 + 1081575*d^12*f^17*g^10*e^27 + 480700*d^11*f^18*g^9*e
^28 + 177100*d^10*f^19*g^8*e^29 + 53130*d^9*f^20*g^7*e^30 + 12650*d^8*f^21*
g^6*e^31 + 2300*d^7*f^22*g^5*e^32 + 300*d^6*f^23*g^4*e^33 + 25*d^5*f^24*g^3
*e^34 + d^4*f^25*g^2*e^35)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f
^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^2
9*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5
852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^1
0*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 +
 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*
d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g
^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30
045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^2
```

2*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 14250
6*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 43
5*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x - 5*(49*d^30*g
^27*e^9 + 1239*d^29*f*g^26*e^10 + 15051*d^28*f^2*g^25*e^11 + 116925*d^27*f^
3*g^24*e^12 + 652350*d^26*f^4*g^23*e^13 + 2782770*d^25*f^5*g^22*e^14 + 9434
370*d^24*f^6*g^21*e^15 + 26086830*d^23*f^7*g^20*e^16 + 59904075*d^22*f^8*g^
19*e^17 + 115728525*d^21*f^9*g^18*e^18 + 189852465*d^20*f^10*g^17*e^19 + 26
6218215*d^19*f^11*g^16*e^20 + 320487060*d^18*f^12*g^15*e^21 + 332076300*d^1
7*f^13*g^14*e^22 + 296417100*d^16*f^14*g^13*e^23 + 227773140*d^15*f^15*g^12
*e^24 + 150325815*d^14*f^16*g^11*e^25 + 84867585*d^13*f^17*g^10*e^26 + 4073
9325*d^12*f^18*g^9*e^27 + 16489275*d^11*f^19*g^8*e^28 + 5563470*d^10*f^20*g
^7*e^29 + 1540770*d^9*f^21*g^6*e^30 + 342930*d^8*f^22*g^5*e^31 + 59550*d^7*
f^23*g^4*e^32 + 7725*d^6*f^24*g^3*e^33 + 699*d^5*f^25*g^2*e^34 + 39*d^4*f^2
6*g*e^35 + d^3*f^27*e^36)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^
2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29
*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 58
52925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10
*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 +
119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d
^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^
13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 300
45015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22
*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506
*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435
*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x - 5*(41*d^31*g^
27*e^8 + 1029*d^30*f*g^26*e^9 + 12399*d^29*f^2*g^25*e^10 + 95475*d^28*f^3*g
^24*e^11 + 527550*d^27*f^4*g^23*e^12 + 2226630*d^26*f^5*g^22*e^13 + 7460970
*d^25*f^6*g^21*e^14 + 20363970*d^24*f^7*g^20*e^15 + 46090275*d^23*f^8*g^19*
e^16 + 87607575*d^22*f^9*g^18*e^17 + 141109485*d^21*f^10*g^17*e^18 + 193785
465*d^20*f^11*g^16*e^19 + 227773140*d^19*f^12*g^15*e^20 + 229556100*d^18*f^
13*g^14*e^21 + 198354300*d^17*f^14*g^13*e^22 + 146648460*d^16*f^15*g^12*e^2
3 + 92379615*d^15*f^16*g^11*e^24 + 49247715*d^14*f^17*g^10*e^25 + 21992025*
d^13*f^18*g^9*e^26 + 8102325*d^12*f^19*g^8*e^27 + 2406030*d^11*f^20*g^7*e^2
8 + 554070*d^10*f^21*g^6*e^29 + 91770*d^9*f^22*g^5*e^30 + 8850*d^8*f^23*g^4
*e^31 - 75*d^7*f^24*g^3*e^32 - 159*d^6*f^25*g^2*e^33 - 21*d^5*f^26*g*e^34 -
d^4*f^27*e^35)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6
 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*
e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26
*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14
 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*
d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g
^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 +
86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14
*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26
+ 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*
g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*
g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x + 15*(10*d^32*g^27*e^7 +
255*d^31*f*g^26*e^8 + 3126*d^30*f^2*g^25*e^9 + 24525*d^29*f^3*g^24*e^10 + 1
38300*d^28*f^4*g^23*e^11 + 596850*d^27*f^5*g^22*e^12 + 2049300*d^26*f^6*g^2
1*e^13 + 5745630*d^25*f^7*g^20*e^14 + 13396350*d^24*f^8*g^19*e^15 + 2631832
5*d^23*f^9*g^18*e^16 + 43984050*d^22*f^10*g^17*e^17 + 62960775*d^21*f^11*g^
16*e^18 + 77558760*d^20*f^12*g^15*e^19 + 82461900*d^19*f^13*g^14*e^20 + 757
75800*d^18*f^14*g^13*e^21 + 60174900*d^17*f^15*g^12*e^22 + 41230950*d^16*f^
16*g^11*e^23 + 24299385*d^15*f^17*g^10*e^24 + 12257850*d^14*f^18*g^9*e^25 +
 5256075*d^13*f^19*g^8*e^26 + 1897500*d^12*f^20*g^7*e^27 + 569250*d^11*f^21
*g^6*e^28 + 139380*d^10*f^22*g^5*e^29 + 27150*d^9*f^23*g^4*e^30 + 4050*d^8*
f^24*g^3*e^31 + 435*d^7*f^25*g^2*e^32 + 30*d^6*f^26*g*e^33 + d^5*f^27*e^34)
/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6 + 4060*d^31*f^
3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*e^9 + 593775*d^

28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))*x + (127*d^33*g^27*e^6 + 3219*d^32*f*g^26*e^7 + 39207*d^31*f^2*g^25*e^8 + 305475*d^30*f^3*g^24*e^9 + 1709850*d^29*f^4*g^23*e^10 + 7320210*d^28*f^5*g^22*e^11 + 24917970*d^27*f^6*g^21*e^12 + 69213210*d^26*f^7*g^20*e^13 + 159750525*d^25*f^8*g^19*e^14 + 310412025*d^24*f^9*g^18*e^15 + 512594445*d^23*f^10*g^17*e^16 + 724216065*d^22*f^11*g^16*e^17 + 879445020*d^21*f^12*g^15*e^18 + 920453100*d^20*f^13*g^14*e^19 + 831305100*d^19*f^14*g^13*e^20 + 647660220*d^18*f^15*g^12*e^21 + 434485065*d^17*f^16*g^11*e^22 + 250132245*d^16*f^17*g^10*e^23 + 122939025*d^15*f^18*g^9*e^24 + 51213525*d^14*f^19*g^8*e^25 + 17904810*d^13*f^20*g^7*e^26 + 5183970*d^12*f^21*g^6*e^27 + 1220610*d^11*f^22*g^5*e^28 + 227850*d^10*f^23*g^4*e^29 + 32475*d^9*f^24*g^3*e^30 + 3327*d^8*f^25*g^2*e^31 + 219*d^7*f^26*g*e^32 + 7*d^6*f^27*e^33)/(d^34*g^30*e^4 + 30*d^33*f*g^29*e^5 + 435*d^32*f^2*g^28*e^6 + 4060*d^31*f^3*g^27*e^7 + 27405*d^30*f^4*g^26*e^8 + 142506*d^29*f^5*g^25*e^9 + 593775*d^28*f^6*g^24*e^10 + 2035800*d^27*f^7*g^23*e^11 + 5852925*d^26*f^8*g^22*e^12 + 14307150*d^25*f^9*g^21*e^13 + 30045015*d^24*f^10*g^20*e^14 + 54627300*d^23*f^11*g^19*e^15 + 86493225*d^22*f^12*g^18*e^16 + 119759850*d^21*f^13*g^17*e^17 + 145422675*d^20*f^14*g^16*e^18 + 155117520*d^19*f^15*g^15*e^19 + 145422675*d^18*f^16*g^14*e^20 + 119759850*d^17*f^17*g^13*e^21 + 86493225*d^16*f^18*g^12*e^22 + 54627300*d^15*f^19*g^11*e^23 + 30045015*d^14*f^20*g^10*e^24 + 14307150*d^13*f^21*g^9*e^25 + 5852925*d^12*f^22*g^8*e^26 + 2035800*d^11*f^23*g^7*e^27 + 593775*d^10*f^24*g^6*e^28 + 142506*d^9*f^25*g^5*e^29 + 27405*d^8*f^26*g^4*e^30 + 4060*d^7*f^27*g^3*e^31 + 435*d^6*f^28*g^2*e^32 + 30*d^5*f^29*g*e^33 + d^4*f^30*e^34))/(x^2*e^2 - d^2)^3 + (2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^10*g^13*e^3/x^2 + 2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^9*f*g^12*e^6/x + 6*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^9*f*g^12*e^4/x^2 + 2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^9*f*g^12*e^2/x^3 + d^8*f^2*g^11*e^9 + 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*f^2*g^11*e^7/x - 51*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^8*f^2*g^11*e^5/x^2 + 3*d^7*f^3*g^10*e^10 - 79*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*f^3*g^10*e^8/x + 91*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^7*f^3*g^10*e^6/x^2 - 25*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^7*f^3*g^10*e^4/x^3 - 26*d^6*f^4*g^9*e^11 + 127*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6*f^4*g^9*e^9/x - 48*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*f^4*g^9*e^7/x^2 + 49*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^6*f^4*g^9*e^5/x^3 + 44*d^5*f^5*g^8*e^12 - 28*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^5*f^5*g^8*e^10/x - 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*f^5*g^8*e^8/x^2 - 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^5*f^5*g^8*e^6/x^3 - 11*d^4*f^6*g^7*e^13 - 110*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*f^6*g^7*e^11/x + 61*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*f^6*g^7*e^9/x^2 - 38*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*f^6*g^7*e^7/x^3 - 37*d^3*f^7*g^6*e^14 + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*f^7*g^6*e^12/x - 57*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*f^7*g^6*e^10/x^2 + 39*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*f^7*g^6*e^8/x^3 + 36*d^2*f^8*g^5*e^15 - 29*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*f^8*g^5*e^13/x + 36*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*f^8*g^5*e^11/x^2 - 11*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*f^8*g^5*e^9/x^3 - 10*d*f^9*g^4*e^16 - 10*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*f^9*g^4*e^12/x^2)/((d^12*f^2*g^12*e^5 - 6*d^10*f^4*g^10*e^7 + 15*d^8*f^6*g^8*e^9 - 20*d^6*f^8*g^6*e^11 + 15*d^4*f^10*g^4*e^13 - 6*d^2*f^12*g^2*e^15 + f^14*e^17)*(2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*g*e^(-1)/x + f*e^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^2*f*e^(-2)/x^2)^2)

3.588 $\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2\left(ae^2+cd^2\right)}{e^2\sqrt{d+ex}(ef-dg)}-\frac{2\left(ag^2+cf^2\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^2g}$$

```
[Out] (-2*(c*d^2 + a*e^2))/(e^2*(e*f - d*g)*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/
(e^2*g) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]
])/(g^(3/2)*(e*f - d*g)^(3/2))
```

Rubi [A] time = 0.215486, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {898, 1261, 205}

$$-\frac{2\left(ae^2+cd^2\right)}{e^2\sqrt{d+ex}(ef-dg)}-\frac{2\left(ag^2+cf^2\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*(c*d^2 + a*e^2))/(e^2*(e*f - d*g)*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/
(e^2*g) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]
])/(g^(3/2)*(e*f - d*g)^(3/2))
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cd^2+ae^2}{e^2}-\frac{2cdx^2}{e^2}+\frac{cx^4}{e^2}}{x^2\left(\frac{ef-dg}{e}+\frac{gx^2}{e}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cd^2+ae^2}{e(ef-dg)x^2}-\frac{e\left(cf^2+ag^2\right)}{g(-ef+dg)\left(-ef+dg-gx^2\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= -\frac{2\left(cd^2+ae^2\right)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}+\frac{\left(2\left(cf^2+ag^2\right)\right)\text{Subst}\left(\int \frac{1}{-ef+dg-gx^2}\,dx, x, \sqrt{d+ex}\right)}{g(ef-dg)}$$

$$= -\frac{2\left(cd^2+ae^2\right)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}-\frac{2\left(cf^2+ag^2\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

Mathematica [C] time = 0.0936026, size = 91, normalized size = 0.81

$$\frac{2c(ef-dg)(2dg+e(f+gx))-2e^2\left(ag^2+cf^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{g(d+ex)}{dg-ef}\right)}{e^2g^2\sqrt{d+ex}(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (2*c*(e*f - d*g)*(2*d*g + e*(f + g*x)) - 2*e^2*(c*f^2 + a*g^2)*Hypergeometr
ic2F1[-1/2, 1, 1/2, (g*(d + e*x))/(-(e*f) + d*g)])/(e^2*g^2*(e*f - d*g)*Sqr
t[d + e*x])
```

Maple [A] time = 0.239, size = 114, normalized size = 1.

$$2\frac{1}{e^2}\left(\frac{c\sqrt{ex+d}}{g}-\frac{e^2\left(ag^2+cf^2\right)}{\left(dg-ef\right)g\sqrt{\left(dg-ef\right)g}}\text{Artanh}\left(\frac{\sqrt{ex+d}g}{\sqrt{\left(dg-ef\right)g}}\right)-\frac{-ae^2-cd^2}{\left(dg-ef\right)\sqrt{ex+d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x)
```

```
[Out] 2/e^2*(c/g*(e*x+d)^(1/2)-e^2*(a*g^2+c*f^2)/(d*g-e*f)/g/((d*g-e*f)*g)^(1/2)*
arctanh((e*x+d)^(1/2)*g/((d*g-e*f)*g)^(1/2))-(-a*e^2-c*d^2)/(d*g-e*f)/(e*x+
d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89517, size = 1030, normalized size = 9.2

$$\left[\frac{\left(cde^2f^2 + ade^2g^2 + \left(ce^3f^2 + ae^3g^2\right)x\right)\sqrt{-efg + dg^2}\log\left(\frac{egx-ef+2\,dg-2\,\sqrt{-efg+dg^2}\sqrt{ex+d}}{gx+f}\right) + 2\left(cde^2f^2g - \left(3\,cd^2e + ae^3\right)fg\right.}{de^4f^2g^2 - 2\,d^2e^3fg^3 + d^3e^2g^4 + \left(e^5f^2g^2 - 2\,de^4fg^3 + d^2e^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="fricas")
```

```
[Out] [((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*sqrt(-e*f*g + d*g
^2)*log((e*g*x - e*f + 2*d*g - 2*sqrt(-e*f*g + d*g^2)*sqrt(e*x + d))/(g*x +
 f)) + 2*(c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)*g
^3 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*e^4
*f^2*g^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3 + d
^2*e^3*g^4)*x), 2*((c*d*e^2*f^2 + a*d*e^2*g^2 + (c*e^3*f^2 + a*e^3*g^2)*x)*
sqrt(e*f*g - d*g^2)*arctan(sqrt(e*f*g - d*g^2)*sqrt(e*x + d)/(e*g*x + d*g))
 + (c*d*e^2*f^2*g - (3*c*d^2*e + a*e^3)*f*g^2 + (2*c*d^3 + a*d*e^2)*g^3 + (
c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(e*x + d))/(d*e^4*f^2*g
^2 - 2*d^2*e^3*f*g^3 + d^3*e^2*g^4 + (e^5*f^2*g^2 - 2*d*e^4*f*g^3 + d^2*e^3
*g^4)*x)]
```

Sympy [A] time = 26.087, size = 107, normalized size = 0.96

$$\frac{2c\sqrt{d+ex}}{e^2g} + \frac{2\left(ag^2+cf^2\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{g^2\sqrt{-\frac{dg-ef}{g}}\left(dg-ef\right)} + \frac{2\left(ae^2+cd^2\right)}{e^2\sqrt{d+ex}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(3/2)/(g*x+f),x)
```

```
[Out] 2*c*sqrt(d + e*x)/(e**2*g) + 2*(a*g**2 + c*f**2)*atan(sqrt(d + e*x)/sqrt(-(
d*g - e*f)/g))/(g**2*sqrt(-(d*g - e*f)/g)*(d*g - e*f)) + 2*(a*e**2 + c*d**2
)/(e**2*sqrt(d + e*x)*(d*g - e*f))
```

Giac [A] time = 1.14253, size = 157, normalized size = 1.4

$$\frac{2\sqrt{xe+d}ce^{(-2)}}{g} + \frac{2\left(cf^2+ag^2\right)\arctan\left(\frac{\sqrt{xe+d}g}{\sqrt{-dg^2+fge}}\right)}{\left(dg^2-fge\right)\sqrt{-dg^2+fge}} + \frac{2\left(cd^2+ae^2\right)}{\left(dge^2-fe^3\right)\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*c*e^(-2)/g + 2*(c*f^2 + a*g^2)*arctan(sqrt(x*e + d)*g/sqrt(
-d*g^2 + f*g*e))/((d*g^2 - f*g*e)*sqrt(-d*g^2 + f*g*e)) + 2*(c*d^2 + a*e^2)
```

```
/((d*g*e^2 - f*e^3)*sqrt(x*e + d))
```

3.589 $\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=240

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}-$$

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3*
a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^6) - (2*(e*f - d*g)*(3
*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6)
 + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/
2))/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c*e^3*
(f + g*x)^(11/2))/(11*g^6)
```

Rubi [A] time = 0.344977, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1153}

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}-$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```


```
[Out] (-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3*
a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^6) - (2*(e*f - d*g)*(3
*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6)
 + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/
2))/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c*e^3*
(f + g*x)^(11/2))/(11*g^6)
```


Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int\left(\frac{(-ef+dg)^3(cf^2+ag^2)}{g^5}+\frac{(ef-dg)^2(3aeg^2+cf(5ef-2dg))x^2}{g^5}+\frac{(ef-dg)(-3ae^2g^2-c(10e^2f^2-8defg+d^2g^2))}{g^5}\right.\right.}{g}$$

$$= -\frac{2(ef-dg)^3(cf^2+ag^2)\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2(3aeg^2+cf(5ef-2dg))(f+gx)^{3/2}}{3g^6}-\frac{2(ef}{}$$

Mathematica [A] time = 0.254791, size = 207, normalized size = 0.86

$$2\sqrt{f+gx}\left(495e(f+gx)^3\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)-693(f+gx)^2(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3465*(e*f - d*g)^3*(c*f^2 + a*g^2) + 1155*(e*f - d*g)^2*
(3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x) - 693*(e*f - d*g)*(3*a*e^2*g^2
+ c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^2 + 495*e*(a*e^2*g^2 + c*
(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^3 - 385*c*e^2*(5*e*f - 3*d
*g)*(f + g*x)^4 + 315*c*e^3*(f + g*x)^5))/(3465*g^6)
```

Maple [A] time = 0.047, size = 365, normalized size = 1.5

$$630\,e^3cx^5g^5+2310\,cde^2g^5x^4-700\,ce^3fg^4x^4+990\,ae^3g^5x^3+2970\,cd^2eg^5x^3-2640\,cde^2fg^4x^3+800\,ce^3f^2g^3x^3+4158\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3465*(g*x+f)^(1/2)*(315*c*e^3*g^5*x^5+1155*c*d*e^2*g^5*x^4-350*c*e^3*f*g^
4*x^4+495*a*e^3*g^5*x^3+1485*c*d^2*e*g^5*x^3-1320*c*d*e^2*f*g^4*x^3+400*c*e
^3*f^2*g^3*x^3+2079*a*d*e^2*g^5*x^2-594*a*e^3*f*g^4*x^2+693*c*d^3*g^5*x^2-1
782*c*d^2*e*f*g^4*x^2+1584*c*d*e^2*f^2*g^3*x^2-480*c*e^3*f^3*g^2*x^2+3465*a
*d^2*e*g^5*x-2772*a*d*e^2*f*g^4*x+792*a*e^3*f^2*g^3*x-924*c*d^3*f*g^4*x+237
6*c*d^2*e*f^2*g^3*x-2112*c*d*e^2*f^3*g^2*x+640*c*e^3*f^4*g*x+3465*a*d^3*g^5
-6930*a*d^2*e*f*g^4+5544*a*d*e^2*f^2*g^3-1584*a*e^3*f^3*g^2+1848*c*d^3*f^2*
g^3-4752*c*d^2*e*f^3*g^2+4224*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

Maxima [A] time = 0.985909, size = 440, normalized size = 1.83

$$2\left(315\left(gx+f\right)^{\frac{11}{2}}ce^3-385\left(5\,ce^3f-3\,cde^2g\right)\left(gx+f\right)^{\frac{9}{2}}+495\left(10\,ce^3f^2-12\,cde^2fg+\left(3\,cd^2e+ae^3\right)g^2\right)\left(gx+f\right)^{\frac{7}{2}}-693\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f
)^(9/2) + 495*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*
x + f)^(7/2) - 693*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)
*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 12*
c*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*a*d*e^2)*f*g^3)*(g*x + f)^(3/2) - 3465*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a
*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d*e^2
)*f^2*g^3)*sqrt(g*x + f))/g^6
```

Fricas [A] time = 1.80019, size = 747, normalized size = 3.11

$$\frac{2\left(315\,ce^3g^5x^5 - 1280\,ce^3f^5 + 4224\,cde^2f^4g - 6930\,ad^2efg^4 + 3465\,ad^3g^5 - 1584\left(3\,cd^2e + ae^3\right)f^3g^2 + 1848\left(cd^3 + 3\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 4224*c*d*e^2*f^4*g - 6930*a*d^
2*e*f*g^4 + 3465*a*d^3*g^5 - 1584*(3*c*d^2*e + a*e^3)*f^3*g^2 + 1848*(c*d^3
 + 3*a*d*e^2)*f^2*g^3 - 35*(10*c*e^3*f*g^4 - 33*c*d*e^2*g^5)*x^4 + 5*(80*c*
e^3*f^2*g^3 - 264*c*d*e^2*f*g^4 + 99*(3*c*d^2*e + a*e^3)*g^5)*x^3 - 3*(160*
c*e^3*f^3*g^2 - 528*c*d*e^2*f^2*g^3 + 198*(3*c*d^2*e + a*e^3)*f*g^4 - 231*(
c*d^3 + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 2112*c*d*e^2*f^3*g^2 + 346
5*a*d^2*e*g^5 + 792*(3*c*d^2*e + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*a*d*e^2)*f
*g^4)*x)*sqrt(g*x + f)/g^6
```

Sympy [A] time = 97.7774, size = 1040, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d**3*f/sqrt(f + g*x) + 2*a*d**3*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) + 6*a*d**2*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 6*a*d**2*e*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 6*a*d*e**2
*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 6*a
*d*e**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
(f + g*x)**(5/2)/5)/g**2 + 2*a*e**3*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f
+ g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 + 2*a*e**3*(f**4/sqr
t(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)
**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 + 2*c*d**3*f*(f**2/sqrt(f + g*x) + 2*f
*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*d**3*(-f**3/sqrt(f + g*x) -
 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 6*c
*d**2*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2)
- (f + g*x)**(5/2)/5)/g**3 + 6*c*d**2*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f
 + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/
2)/7)/g**3 + 6*c*d*e**2*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f*
*2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 + 6
*c*d*e**2*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**
```

```
(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/
2)/9)/g**4 + 2*c*e**3*f*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f*
*3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 -
(f + g*x)**(9/2)/9)/g**5 + 2*c*e**3*(f**6/sqrt(f + g*x) + 6*f**5*sqrt(f + g
*x) - 5*f**4*(f + g*x)**(3/2) + 4*f**3*(f + g*x)**(5/2) - 15*f**2*(f + g*x)
**(7/2)/7 + 2*f*(f + g*x)**(9/2)/3 - (f + g*x)**(11/2)/11)/g**5)/g, Ne(g, 0
)), ((a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x*
*4*(a*e**3 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + c*d**3)/3)/sqrt(f), True))
```

Giac [A] time = 1.17097, size = 510, normalized size = 2.12

$$2\left(3465\sqrt{gx+f}ad^3 + \frac{3465\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)ad^2e}{g} + \frac{231\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)cd^3}{g^2} + \frac{693\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx\right.}}{g^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*a*d^2*e/g + 231*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*c*d^3/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sq
rt(g*x + f)*f^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*
f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 99*(5*(g*x
 + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x +
 f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(
g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*d*e
^2/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/
2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x
 + f)*f^5)*c*e^3/g^5)/g
```

3.590 $\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=175

$$\frac{2(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2+c\right.}{3g^5}$$

```
[Out] (2*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*g
^2 + c*f*(2*e*f - d*g))*(f + g*x)^(3/2))/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e^2
*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (4*c*e*(2*e*f - d*g
)*(f + g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rubi [A] time = 0.237012, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1153}

$$\frac{2(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2+c\right.}{3g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*g
^2 + c*f*(2*e*f - d*g))*(f + g*x)^(3/2))/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e^2
*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (4*c*e*(2*e*f - d*g
)*(f + g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{(-ef+dg)^2\left(cf^2+ag^2\right)}{g^4}+\frac{2(ef-dg)\left(-aeg^2-cf(2ef-dg)\right)x^2}{g^4}+\frac{\left(ae^2g^2+c\left(6e^2f^2-6defg+d^2g^2\right)\right)x^4}{g^4}-\frac{2ce(2e}{}\right.\right.}{g}$$

$$= \frac{2(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)(f+gx)^{3/2}}{3g^5}+\frac{2\left(ae^2g^2+\right.}{}$$

Mathematica [A] time = 0.154785, size = 149, normalized size = 0.85

$$\frac{2\sqrt{f+gx}\left(63(f+gx)^2\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)+315\left(ag^2+cf^2\right)(ef-dg)^2-210(f+gx)(ef-dg)\left(aeg^2+\right.\right.}{315g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(315*(e*f - d*g)^2*(c*f^2 + a*g^2) - 210*(e*f - d*g)*(a*e*
g^2 + c*f*(2*e*f - d*g))*(f + g*x) + 63*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f
*g + d^2*g^2))*(f + g*x)^2 - 90*c*e*(2*e*f - d*g)*(f + g*x)^3 + 35*c*e^2*(f
 + g*x)^4))/(315*g^5)
```

Maple [A] time = 0.048, size = 215, normalized size = 1.2

$$\frac{70\,ce^2x^4g^4+180\,cdeg^4x^3-80\,ce^2fg^3x^3+126\,ae^2g^4x^2+126\,cd^2g^4x^2-216\,cdefg^3x^2+96\,ce^2f^2g^2x^2+420\,adeg^4x-168}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/315*(g*x+f)^(1/2)*(35*c*e^2*g^4*x^4+90*c*d*e*g^4*x^3-40*c*e^2*f*g^3*x^3+6
3*a*e^2*g^4*x^2+63*c*d^2*g^4*x^2-108*c*d*e*f*g^3*x^2+48*c*e^2*f^2*g^2*x^2+2
10*a*d*e*g^4*x-84*a*e^2*f*g^3*x-84*c*d^2*f*g^3*x+144*c*d*e*f^2*g^2*x-64*c*e
^2*f^3*g*x+315*a*d^2*g^4-420*a*d*e*f*g^3+168*a*e^2*f^2*g^2+168*c*d^2*f^2*g^
2-288*c*d*e*f^3*g+128*c*e^2*f^4)/g^5
```

Maxima [A] time = 0.984542, size = 266, normalized size = 1.52

$$\frac{2\left(35\left(gx+f\right)^{\frac{9}{2}}ce^2-90\left(2\,ce^2f-cdeg\right)\left(gx+f\right)^{\frac{7}{2}}+63\left(6\,ce^2f^2-6\,cdefg+\left(cd^2+ae^2\right)g^2\right)\left(gx+f\right)^{\frac{5}{2}}-210\left(2\,ce^2f^3-3\,c\right.\right.}{315\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 90*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(7/2)
+ 63*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(5/2) - 21
```

```
0*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*(g*x +
f)^(3/2) + 315*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 + (c*
d^2 + a*e^2)*f^2*g^2)*sqrt(g*x + f))/g^5
```

Fricas [A] time = 1.7991, size = 452, normalized size = 2.58

$$\frac{2\left(35\,ce^2g^4x^4+128\,ce^2f^4-288\,cdef^3g-420\,adefg^3+315\,ad^2g^4+168\left(cd^2+ae^2\right)f^2g^2-10\left(4\,ce^2fg^3-9\,cdeg^4\right)x^3+\cdots\right.}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 - 288*c*d*e*f^3*g - 420*a*d*e*f*g^3
 + 315*a*d^2*g^4 + 168*(c*d^2 + a*e^2)*f^2*g^2 - 10*(4*c*e^2*f*g^3 - 9*c*d*
e*g^4)*x^3 + 3*(16*c*e^2*f^2*g^2 - 36*c*d*e*f*g^3 + 21*(c*d^2 + a*e^2)*g^4)
*x^2 - 2*(32*c*e^2*f^3*g - 72*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 42*(c*d^2 + a
*e^2)*f*g^3)*x)*sqrt(g*x + f)/g^5
```

Sympy [A] time = 61.3772, size = 673, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d**2*f/sqrt(f + g*x) + 2*a*d**2*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) + 4*a*d*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 4*a*d*e*(f**2/s
qrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 2*a*e**2*f*(f**2
/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*a*e**2*(-
f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)*
*(5/2)/5)/g**2 + 2*c*d**2*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f +
g*x)**(3/2)/3)/g**2 + 2*c*d**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x)
+ f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 4*c*d*e*f*(-f**3/sqrt(f +
 g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**
3 + 4*c*d*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**
(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 + 2*c*e**2*f*(f**
4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f +
 g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 + 2*c*e**2*(-f**5/sqrt(f + g*x) -
 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2
) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4)/g, Ne(g, 0)), ((a*d*
*2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3
)/sqrt(f), True))
```

Giac [A] time = 1.12678, size = 328, normalized size = 1.87

$$2\left(315\sqrt{gx+f}ad^2+\frac{210\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)ade}{g}+\frac{21\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)cd^2}{g^2}+\frac{21\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+\cdots}\right.}{g^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
a*d*e/g + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*c*d^2/g^2 + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*a*e^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*
x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + (35*(g*x + f)^(9/2) -
180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 +
 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

3.591 $\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=113

$$-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}\left(aeg^2+cf(3ef-2dg)\right)}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

```
[Out] (-2*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*e
*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^(5/2))
/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7*g^4)
```

Rubi [A] time = 0.0742448, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {772}

$$-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}\left(aeg^2+cf(3ef-2dg)\right)}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*e
*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^(5/2))
/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7*g^4)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3\sqrt{f+gx}}+\frac{\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^3}+\frac{c(-3ef+dg)(f+gx)^{3/2}}{g^3}\right.\\ &= -\frac{2(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{3/2}}{3g^4}-\frac{2c(3ef-dg)(f}{5g^4}\end{aligned}$$

Mathematica [A] time = 0.0912615, size = 94, normalized size = 0.83

$$\frac{2\sqrt{f+gx}\left(35ag^2(3dg-2ef+egx)+7cdg\left(8f^2-4fgx+3g^2x^2\right)-3ce\left(-8f^2gx+16f^3+6fg^2x^2-5g^3x^3\right)\right)}{105g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(35*a*g^2*(-2*e*f + 3*d*g + e*g*x) + 7*c*d*g*(8*f^2 - 4*f*
g*x + 3*g^2*x^2) - 3*c*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))/(
```

```
105*g^4)
```

Maple [A] time = 0.046, size = 101, normalized size = 0.9

$$\frac{30\,ce x^3 g^3 + 42\,cd g^3 x^2 - 36\,cef g^2 x^2 + 70\,ae g^3 x - 56\,cdf g^2 x + 48\,ce f^2 g x + 210\,ad g^3 - 140\,aef g^2 + 112\,cd f^2 g - 96\,ce f^3}{105\,g^4}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(15*c*e*g^3*x^3+21*c*d*g^3*x^2-18*c*e*f*g^2*x^2+35*a*e*
g^3*x-28*c*d*f*g^2*x+24*c*e*f^2*g*x+105*a*d*g^3-70*a*e*f*g^2+56*c*d*f^2*g-4
8*c*e*f^3)/g^4
```

Maxima [A] time = 1.00632, size = 140, normalized size = 1.24

$$\frac{2\left(15\left(gx+f\right)^{\frac{7}{2}}ce - 21\left(3\,cef - cdg\right)\left(gx+f\right)^{\frac{5}{2}} + 35\left(3\,cef^2 - 2\,cdfg + aeg^2\right)\left(gx+f\right)^{\frac{3}{2}} - 105\left(cef^3 - cdf^2g + aefg^2 - ad\right.\right.}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - c*d*g)*(g*x + f)^(5/2) + 35*(
3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*(g*x + f)^(3/2) - 105*(c*e*f^3 - c*d*f^2*g
 + a*e*f*g^2 - a*d*g^3)*sqrt(g*x + f))/g^4
```

Fricas [A] time = 1.65952, size = 243, normalized size = 2.15

$$\frac{2\left(15\,ceg^3x^3 - 48\,cef^3 + 56\,cdf^2g - 70\,aefg^2 + 105\,adg^3 - 3\left(6\,cefg^2 - 7\,cdg^3\right)x^2 + \left(24\,cef^2g - 28\,cdfg^2 + 35\,aeg^3\right)x\right)\sqrt{g}}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 56*c*d*f^2*g - 70*a*e*f*g^2 + 105*a*d*
g^3 - 3*(6*c*e*f*g^2 - 7*c*d*g^3)*x^2 + (24*c*e*f^2*g - 28*c*d*f*g^2 + 35*a
*e*g^3)*x)*sqrt(g*x + f)/g^4
```

Sympy [A] time = 35.1896, size = 374, normalized size = 3.31

$$\begin{cases} -\frac{\frac{2adf}{\sqrt{f+gx}} + 2ad\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) + \frac{2aef\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} + \frac{2ae\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} + \frac{2cdf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} + \frac{2cd\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \cdots\right.}{g^2}}{g} \\ \frac{adx + \frac{aex^2}{2} + \frac{cdx^3}{3} + \frac{cex^4}{4}}{\sqrt{f}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d*f/sqrt(f + g*x) + 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x
)) + 2*a*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 2*a*e*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 2*c*d*f*(f**2/sqrt(f + g*x
) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*d*(-f**3/sqrt(f + g*
x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 +
 2*c*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
 (f + g*x)**(5/2)/5)/g**3 + 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x
) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/
g**3)/g, Ne(g, 0)), ((a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/sqrt(f)
, True))
```

Giac [A] time = 1.1359, size = 181, normalized size = 1.6

$$\frac{2\left(105\sqrt{gx+f}ad+\frac{35\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)ae}{g}+\frac{7\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)cd}{g^2}+\frac{3\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-3\right.}{g^3}\right.}{105g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e
/g + 7*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*
d/g^2 + 3*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^
2 - 35*sqrt(g*x + f)*f^3)*c*e/g^3)/g
```

3.592 $\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

```
[Out] (2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^3 - (4*c*f*(f + g*x)^(3/2))/(3*g^3) + (
2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rubi [A] time = 0.0262261, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^3 - (4*c*f*(f + g*x)^(3/2))/(3*g^3) + (
2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx &= \int \left(\frac{cf^2+ag^2}{g^2\sqrt{f+gx}}-\frac{2cf\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx\\ &= \frac{2\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A] time = 0.0269882, size = 44, normalized size = 0.72

$$\frac{2\sqrt{f+gx}\left(15ag^2+c\left(8f^2-4fgx+3g^2x^2\right)\right)}{15g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(15*a*g^2 + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)
```

Maple [A] time = 0.045, size = 41, normalized size = 0.7

$$\frac{6\,c x^2 g^2 - 8\,c f x g + 30\,a g^2 + 16\,c f^2}{15\,g^3}\sqrt{g x + f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(3*c*g^2*x^2-4*c*f*g*x+15*a*g^2+8*c*f^2)/g^3
```

Maxima [A] time = 0.985573, size = 72, normalized size = 1.18

$$\frac{2\left(15\,\sqrt{g x + f}a + \frac{\left(3\,(g x+f)^{\frac{5}{2}} - 10\,(g x+f)^{\frac{3}{2}} f + 15\,\sqrt{g x+f} f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

Fricas [A] time = 1.70529, size = 96, normalized size = 1.57

$$\frac{2\left(3\,c g^2 x^2 - 4\,c f g x + 8\,c f^2 + 15\,a g^2\right)\sqrt{g x + f}}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*g^2*x^2 - 4*c*f*g*x + 8*c*f^2 + 15*a*g^2)*sqrt(g*x + f)/g^3
```

Sympy [A] time = 6.90782, size = 150, normalized size = 2.46

$$\begin{cases} -\frac{\frac{2af}{\sqrt{f+gx}} + 2a\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) + \frac{2cf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} + \frac{2c\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for}\; g \neq 0 \\ \frac{ax + \frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*f/sqrt(f + g*x) + 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) +
 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 +
```

```
2*c*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f
+ g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + c*x**3/3)/sqrt(f), True))
```

Giac [A] time = 1.15214, size = 72, normalized size = 1.18

$$\frac{2\left(15\sqrt{gx+f}a+\frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

3.593 $\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=104

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}(dg+ef)}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

```
[Out] (-2*c*(e*f + d*g)*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))/(3*e*g^2
) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e
^(5/2)*Sqrt[e*f - d*g])
```

Rubi [A] time = 0.123759, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {898, 1153, 208}

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}(dg+ef)}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*c*(e*f + d*g)*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))/(3*e*g^2
) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e
^(5/2)*Sqrt[e*f - d*g])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2\left(cd^2+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A] time = 0.169567, size = 92, normalized size = 0.88

$$\frac{2c\sqrt{f+gx}(-3dg-2ef+egx)}{3e^2g^2}-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x]*(-2*e*f - 3*d*g + e*g*x))/(3*e^2*g^2) - (2*(c*d^2 + a*e^
2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*
g])
```

Maple [A] time = 0.077, size = 132, normalized size = 1.3

$$\frac{2c}{3eg^2}(gx+f)^{\frac{3}{2}}-2\frac{cd\sqrt{gx+f}}{ge^2}-2\frac{cf\sqrt{gx+f}}{eg^2}+2\frac{a}{\sqrt{(dg-ef)e}}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+2\frac{cd^2}{e^2\sqrt{(dg-ef)e}}\arctan\left(\frac{e}{\sqrt{(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2/g*c/e^2*d*(g*x+f)^(1/2)-2/g^2*c/e*f*(g*x+f)^(1/
2)+2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*a+2/e^
2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83007, size = 628, normalized size = 6.04

$$\left[\frac{3\left(cd^2+ae^2\right)\sqrt{e^2f-deg}g^2\log\left(\frac{egx+2ef-dg-2\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2ce^3f^2+cde^2fg-3cd^2eg^2-\left(ce^3fg-cde^2g^2\right)x\right)\sqrt{g}}{3\left(e^4fg^2-de^3g^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c*d^2 + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f - d*g -
2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 + c*d*e^2*
f*g - 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^
2 - d*e^3*g^3), 2/3*(3*(c*d^2 + a*e^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt
(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*e^3*f^2 + c*d*e^2*f*g
- 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 -
d*e^3*g^3)]
```

Sympy [A] time = 20.3544, size = 100, normalized size = 0.96

$$\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3eg^2}-\frac{2c\sqrt{f+gx}\left(dg+ef\right)}{e^2g^2}-\frac{2\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{e^2\sqrt{\frac{e}{dg-ef}}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*e*g**2) - 2*c*sqrt(f + g*x)*(d*g + e*f)/(e**2*g**2)
 - 2*(a*e**2 + c*d**2)*atan(1/(sqrt(e/(d*g - e*f))*sqrt(f + g*x)))/(e**2*sq
rt(e/(d*g - e*f))*(d*g - e*f))
```

Giac [A] time = 1.18318, size = 144, normalized size = 1.38

$$\frac{2\left(cd^2+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)e^{(-2)}}{\sqrt{dge-fe^2}}-\frac{2\left(3\sqrt{gx+f}cdg^5e-\left(gx+f\right)^{\frac{3}{2}}cg^4e^2+3\sqrt{gx+f}cfg^4e^2\right)e^{(-3)}}{3g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))*e^(-2)/sqrt(d
*g*e - f*e^2) - 2/3*(3*sqrt(g*x + f)*c*d*g^5*e - (g*x + f)^(3/2)*c*g^4*e^2
+ 3*sqrt(g*x + f)*c*f*g^4*e^2)*e^(-3)/g^6
```

3.594 $\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((a + (c*d^2)/e^2)*Sqrt[f + g*x])/((e*f - d*g
)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g
*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/2))
```

Rubi [A] time = 0.203416, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1157, 388, 208}

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((a + (c*d^2)/e^2)*Sqrt[f + g*x])/((e*f - d*g
)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g
*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/2))
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)}+\frac{\text{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cf^2}{g^2}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g}-\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)}-\frac{\left(a+\frac{cd(4ef-3dg)}{e^2g}\right)\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g}-\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)}+\frac{\left(ae^2g+cd(4ef-3dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A] time = 0.32149, size = 171, normalized size = 1.4

$$\frac{\frac{\left(ae^2+cd^2\right)\left(\sqrt{e}\sqrt{f+gx}(dg-ef)+g(d+ex)\sqrt{dg-ef}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)\right)}{(d+ex)(ef-dg)^2}+\frac{4cd\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}+\frac{2c\sqrt{e}\sqrt{f+gx}}{g}}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] ((2*c*Sqrt[e]*Sqrt[f + g*x])/g + ((c*d^2 + a*e^2)*(Sqrt[e]*(-(e*f) + d*g)*S
qrt[f + g*x] + g*Sqrt[-(e*f) + d*g]*(d + e*x)*ArcTan[(Sqrt[e]*Sqrt[f + g*x]
)/Sqrt[-(e*f) + d*g]]))/((e*f - d*g)^2*(d + e*x)) + (4*c*d*ArcTanh[(Sqrt[e]
*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g])/e^(5/2)
```

Maple [B] time = 0.217, size = 237, normalized size = 1.9

$$2\,\frac{c\sqrt{gx+f}}{e^2g}+\frac{ag}{\left(dg-ef\right)\left(egx+dg\right)}\sqrt{gx+f}+\frac{cd^2g}{e^2\left(dg-ef\right)\left(egx+dg\right)}\sqrt{gx+f}+\frac{ag}{dg-ef}\arctan\left(e\sqrt{gx+f}\frac{1}{\sqrt{(dg-e}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e^2/g+g/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*a+g/e^2/(d*g-
e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*c*d^2+g/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan
(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*a-3*g/e^2/(d*g-e*f)/((d*g-e*f)*e)^(1/
2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d^2+4/e/(d*g-e*f)/((d*g-e*
f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86566, size = 1116, normalized size = 9.15

$$\left[-\frac{\left(4\,cd^2efg - \left(3\,cd^3 - ade^2\right)g^2 + \left(4\,cde^2fg - \left(3\,cd^2e - ae^3\right)g^2\right)x\right)\sqrt{e^2f - deg}\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right) - 2\left(2\,cde^3f \cdots\right.}{2\left(de^5f^2g - 2\,d^2e^4fg^2 + d^3e^3g^3 + \left(e^6f^2g - 2\,de^5 \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((4*c*d^2*e*f*g - (3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2
*e - a*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e
^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 +
 a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*
d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3
 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -((4*c*d^2*e*f*g - (3*c*d^
3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2*e - a*e^3)*g^2)*x)*sqrt(-e^2*f
 + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d
*e^3*f^2 - (5*c*d^2*e^2 + a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4
*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^
2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14689, size = 200, normalized size = 1.64

$$\frac{2\,\sqrt{gx+f}ce^{(-2)}}{g} - \frac{\left(3\,cd^2g - 4\,cdfe - age^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{\left(dge^2 - fe^3\right)\sqrt{dge - fe^2}} + \frac{\sqrt{gx+f}cd^2g + \sqrt{gx+f}age^2}{\left(dge^2 - fe^3\right)\left(dg + \left(gx+f\right)e - fe\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(g*x + f)*c*e^(-2)/g - (3*c*d^2*g - 4*c*d*f*e - a*g*e^2)*arctan(sqrt(
g*x + f)*e/sqrt(d*g*e - f*e^2))/((d*g*e^2 - f*e^3)*sqrt(d*g*e - f*e^2)) + (
sqrt(g*x + f)*c*d^2*g + sqrt(g*x + f)*a*g*e^2)/((d*g*e^2 - f*e^3)*(d*g + (g
*x + f)*e - f*e))
```

3.595 $\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=178

$$-\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}\left(3ae^2g+cd(8ef-5dg)\right)}{4e^2(d+ex)(ef-dg)^2}$$

```
[Out] -((a + (c*d^2)/e^2)*Sqrt[f + g*x])/(2*(e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*
g + c*d*(8*e*f - 5*d*g))*Sqrt[f + g*x])/(4*e^2*(e*f - d*g)^2*(d + e*x)) - (
(3*a*e^2*g^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt
[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rubi [A] time = 0.30305, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1157, 385, 208}

$$-\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}\left(3ae^2g+cd(8ef-5dg)\right)}{4e^2(d+ex)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```


```
[Out] -((a + (c*d^2)/e^2)*Sqrt[f + g*x])/(2*(e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*
g + c*d*(8*e*f - 5*d*g))*Sqrt[f + g*x])/(4*e^2*(e*f - d*g)^2*(d + e*x)) - (
(3*a*e^2*g^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt
[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(5/2)*(e*f - d*g)^(5/2))
```


Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{4cf^2}{g^2}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{2(ef-dg)}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(8ef-5dg)\right)\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{\left(3ae^2g^2+c\left(8e^2f^2-8defg+\right.\right.}{}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(8ef-5dg)\right)\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{\left(3ae^2g^2+c\left(8e^2f^2-8defg+\right.\right.}{4e^{5/2}(ef-}$$

Mathematica [C] time = 0.885958, size = 207, normalized size = 1.16

$$\frac{2\left(\frac{\sqrt{e}g^2\sqrt{f+gx}\left(ae^2+cd^2\right){}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{e(f+gx)}{ef-dg}\right)}{(dg-ef)^3}-\frac{cd\left(\sqrt{e}\sqrt{f+gx}(dg-ef)+g(d+ex)\sqrt{dg-ef}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)\right)}{(d+ex)(ef-dg)^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] (2*(-((c*d*(Sqrt[e]*(-(e*f) + d*g)*Sqrt[f + g*x] + g*Sqrt[-(e*f) + d*g]*(d
+ e*x)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]]))/((e*f - d*g)^2*
(d + e*x))) - (c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e*f
 - d*g] + (Sqrt[e]*(c*d^2 + a*e^2)*g^2*Sqrt[f + g*x]*Hypergeometric2F1[1/2,
 3, 3/2, (e*(f + g*x))/(e*f - d*g)])/(-(e*f) + d*g)^3))/e^(5/2)
```

Maple [B] time = 0.219, size = 384, normalized size = 2.2

$$2\frac{1}{\left(e\left(gx+f\right)+dg-ef\right)^2}\left(1/8\frac{g\left(3ae^2g-5cd^2g+8cdef\right)\left(gx+f\right)^{3/2}}{e\left(d^2g^2-2defg+e^2f^2\right)}+1/8\frac{\left(5ae^2g-3cd^2g+8cdef\right)g\sqrt{gx+f}}{e^2\left(dg-ef\right)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

```
[Out] 2*(1/8*g*(3*a*e^2*g-5*c*d^2*g+8*c*d*e*f)/e/(d^2*g^2-2*d*e*f*g+e^2*f^2)*(g*x
+f)^(3/2)+1/8*(5*a*e^2*g-3*c*d^2*g+8*c*d*e*f)/e^2*g/(d*g-e*f)*(g*x+f)^(1/2)
```

```
)/(e*(g*x+f)+d*g-e*f)^2+3/4/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)
*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*a*g^2+3/4/(d^2*g^2-2*d*e*f*g+e
^2*f^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
*c*d^2*g^2-2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*
x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d*f*g+2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((d*g
-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.00828, size = 1835, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*g^2 + (8*c*e
^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2
- 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((
e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(
2*(3*c*d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4*e -
5*a*d^2*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5*c*d^3
*e^2 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3
*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 -
 d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*
e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*
g^2 + (8*c*e^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*
c*d*e^3*f^2 - 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(-e^2*f +
 d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*(3*c*
d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4*e - 5*a*d^2
*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5*c*d^3*e^2 -
3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3*d^4*e^
4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 - d^3*e^
5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*e^4*g^3
)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14981, size = 375, normalized size = 2.11

$$\frac{\left(3\,cd^2g^2-8\,cdfge+8\,cf^2e^2+3\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{4\left(d^2g^2e^2-2\,dfge^3+f^2e^4\right)\sqrt{dge-fe^2}}-\frac{3\,\sqrt{gx+f}cd^3g^3+5\left(gx+f\right)^{\frac{3}{2}}cd^2g^2e-11\,\sqrt{gx+f}cd^2fg}{4\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*g^2 - 8*c*d*f*g*e + 8*c*f^2*e^2 + 3*a*g^2*e^2)*arctan(sqrt(g*x
 + f)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*sqrt(d*
g*e - f*e^2)) - 1/4*(3*sqrt(g*x + f)*c*d^3*g^3 + 5*(g*x + f)^(3/2)*c*d^2*g^
2*e - 11*sqrt(g*x + f)*c*d^2*f*g^2*e - 8*(g*x + f)^(3/2)*c*d*f*g*e^2 + 8*sq
rt(g*x + f)*c*d*f^2*g*e^2 - 5*sqrt(g*x + f)*a*d*g^3*e^2 - 3*(g*x + f)^(3/2)
*a*g^2*e^3 + 5*sqrt(g*x + f)*a*f*g^2*e^3)/((d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2
*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

3.596 $\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=238

$$\frac{2e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{3g^6}+\frac{2(}{}$$

```
[Out] (2*(e*f - d*g)^3*(c*f^2 + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d*g)^2*(3
*a*e*g^2 + c*f*(5*e*f - 2*d*g))*Sqrt[f + g*x])/g^6 - (2*(e*f - d*g)*(3*a*e^
2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) + (2
*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(
5*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*(f +
g*x)^(9/2))/(9*g^6)
```

Rubi [A] time = 0.268299, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1261}

$$\frac{2e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{3g^6}+\frac{2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```


```
[Out] (2*(e*f - d*g)^3*(c*f^2 + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d*g)^2*(3
*a*e*g^2 + c*f*(5*e*f - 2*d*g))*Sqrt[f + g*x])/g^6 - (2*(e*f - d*g)*(3*a*e^
2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) + (2
*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(
5*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*(f +
g*x)^(9/2))/(9*g^6)
```


Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\mathrm{Subst}\left(\int\left(\frac{(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{g^5}+\frac{(-ef+dg)^3\left(cf^2+ag^2\right)}{g^5x^2}+\frac{(ef-dg)\left(-3ae^2g^2-c\left(10e^2f^2-8defg+d^2g^2\right.\right.}{g^5}\right.\right.}{g}$$

$$= \frac{2(ef-dg)^3\left(cf^2+ag^2\right)}{g^6\sqrt{f+gx}} + \frac{2(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)\sqrt{f+gx}}{g^6} - \frac{2(ef-dg)\left(3a\right.}{}$$

Mathematica [A] time = 0.246201, size = 207, normalized size = 0.87

$$\frac{2\left(63e(f+gx)^3\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)-105(f+gx)^2(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)\right.}{31}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(315*(e*f - d*g)^3*(c*f^2 + a*g^2) + 315*(e*f - d*g)^2*(3*a*e*g^2 + c*f*
(5*e*f - 2*d*g))*(f + g*x) - 105*(e*f - d*g)*(3*a*e^2*g^2 + c*(10*e^2*f^2 -
 8*d*e*f*g + d^2*g^2))*(f + g*x)^2 + 63*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d
*e*f*g + 3*d^2*g^2))*(f + g*x)^3 - 45*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^4 + 3
5*c*e^3*(f + g*x)^5))/(315*g^6*Sqrt[f + g*x])
```

Maple [A] time = 0.049, size = 365, normalized size = 1.5

$$-\frac{-70\,e^3cx^5g^5-270\,cde^2g^5x^4+100\,ce^3fg^4x^4-126\,ae^3g^5x^3-378\,cd^2eg^5x^3+432\,cde^2fg^4x^3-160\,ce^3f^2g^3x^3-630\,ad}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/315/(g*x+f)^(1/2)*(-35*c*e^3*g^5*x^5-135*c*d*e^2*g^5*x^4+50*c*e^3*f*g^4*
x^4-63*a*e^3*g^5*x^3-189*c*d^2*e*g^5*x^3+216*c*d*e^2*f*g^4*x^3-80*c*e^3*f^2
*g^3*x^3-315*a*d*e^2*g^5*x^2+126*a*e^3*f*g^4*x^2-105*c*d^3*g^5*x^2+378*c*d^
2*e*f*g^4*x^2-432*c*d*e^2*f^2*g^3*x^2+160*c*e^3*f^3*g^2*x^2-945*a*d^2*e*g^5
*x+1260*a*d*e^2*f*g^4*x-504*a*e^3*f^2*g^3*x+420*c*d^3*f*g^4*x-1512*c*d^2*e*
f^2*g^3*x+1728*c*d*e^2*f^3*g^2*x-640*c*e^3*f^4*g*x+315*a*d^3*g^5-1890*a*d^2
*e*f*g^4+2520*a*d*e^2*f^2*g^3-1008*a*e^3*f^3*g^2+840*c*d^3*f^2*g^3-3024*c*d
^2*e*f^3*g^2+3456*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

Maxima [A] time = 1.03147, size = 451, normalized size = 1.89

$$2\left(\frac{35\left(gx+f\right)^{\frac{9}{2}}ce^3-45\left(5\,ce^3f-3\,cde^2g\right)\left(gx+f\right)^{\frac{7}{2}}+63\left(10\,ce^3f^2-12\,cde^2fg+\left(3\,cd^2e+ae^3\right)g^2\right)\left(gx+f\right)^{\frac{5}{2}}-105\left(10\,ce^3f^3-18\,cde^2f^2g+3\left(3\,cd^2e+ae^3\right)fg^2-\left(cd^3+3a\right.\right.}{g^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f)^(
7/2) + 63*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*x +
f)^(5/2) - 105*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)*f*g
^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 12*c*d*e
^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*a*d
*e^2)*f*g^3)*sqrt(g*x + f))/g^5 + 315*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a*d^
2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d*e^2)*f
^2*g^3)/(sqrt(g*x + f)*g^5))/g
```

Fricas [A] time = 1.75808, size = 749, normalized size = 3.15

$$\frac{2\left(35\,ce^3g^5x^5 + 1280\,ce^3f^5 - 3456\,cde^2f^4g + 1890\,ad^2efg^4 - 315\,ad^3g^5 + 1008\left(3\,cd^2e + ae^3\right)f^3g^2 - 840\left(cd^3 + 3\,ade^2\right)f^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 3456*c*d*e^2*f^4*g + 1890*a*d^2*
e*f*g^4 - 315*a*d^3*g^5 + 1008*(3*c*d^2*e + a*e^3)*f^3*g^2 - 840*(c*d^3 + 3
*a*d*e^2)*f^2*g^3 - 5*(10*c*e^3*f*g^4 - 27*c*d*e^2*g^5)*x^4 + (80*c*e^3*f^2
*g^3 - 216*c*d*e^2*f*g^4 + 63*(3*c*d^2*e + a*e^3)*g^5)*x^3 - (160*c*e^3*f^3
*g^2 - 432*c*d*e^2*f^2*g^3 + 126*(3*c*d^2*e + a*e^3)*f*g^4 - 105*(c*d^3 + 3
*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 1728*c*d*e^2*f^3*g^2 + 945*a*d^2*e*
g^5 + 504*(3*c*d^2*e + a*e^3)*f^2*g^3 - 420*(c*d^3 + 3*a*d*e^2)*f*g^4)*x)*s
qrt(g*x + f)/(g^7*x + f*g^6)
```

Sympy [A] time = 56.0701, size = 328, normalized size = 1.38

$$\frac{2ce^3\left(f+gx\right)^{\frac{9}{2}}}{9g^6} + \frac{\left(f+gx\right)^{\frac{7}{2}}\left(6cde^2g - 10ce^3f\right)}{7g^6} + \frac{\left(f+gx\right)^{\frac{5}{2}}\left(2ae^3g^2 + 6cd^2eg^2 - 24cde^2fg + 20ce^3f^2\right)}{5g^6} + \frac{\left(f+gx\right)^{\frac{3}{2}}\left(6ad \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**3*(f + g*x)**(9/2)/(9*g**6) + (f + g*x)**(7/2)*(6*c*d*e**2*g - 10*c*
e**3*f)/(7*g**6) + (f + g*x)**(5/2)*(2*a*e**3*g**2 + 6*c*d**2*e*g**2 - 24*c
*d*e**2*f*g + 20*c*e**3*f**2)/(5*g**6) + (f + g*x)**(3/2)*(6*a*d*e**2*g**3
- 6*a*e**3*f*g**2 + 2*c*d**3*g**3 - 18*c*d**2*e*f*g**2 + 36*c*d*e**2*f**2*g
 - 20*c*e**3*f**3)/(3*g**6) + sqrt(f + g*x)*(6*a*d**2*e*g**4 - 12*a*d*e**2*
f*g**3 + 6*a*e**3*f**2*g**2 - 4*c*d**3*f*g**3 + 18*c*d**2*e*f**2*g**2 - 24*
c*d*e**2*f**3*g + 10*c*e**3*f**4)/g**6 - 2*(a*g**2 + c*f**2)*(d*g - e*f)**3
/(g**6*sqrt(f + g*x))
```

Giac [B] time = 1.17236, size = 612, normalized size = 2.57

$$-\frac{2\left(cd^3f^2g^3+ad^3g^5-3\,cd^2f^3g^2e-3\,ad^2fg^4e+3\,cdf^4ge^2+3\,adf^2g^3e^2-cf^5e^3-af^3g^2e^3\right)}{\sqrt{gx+f}g^6}+\frac{2\left(105\left(gx+f\right)^{\frac{3}{2}}cd^3g^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^3*f^2*g^3 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e - 3*a*d^2*f*g^4*e + 3*c*d
*f^4*g*e^2 + 3*a*d*f^2*g^3*e^2 - c*f^5*e^3 - a*f^3*g^2*e^3)/(sqrt(g*x + f)*
g^6) + 2/315*(105*(g*x + f)^(3/2)*c*d^3*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^
51 + 189*(g*x + f)^(5/2)*c*d^2*g^50*e - 945*(g*x + f)^(3/2)*c*d^2*f*g^50*e
+ 2835*sqrt(g*x + f)*c*d^2*f^2*g^50*e + 945*sqrt(g*x + f)*a*d^2*g^52*e + 13
5*(g*x + f)^(7/2)*c*d*g^49*e^2 - 756*(g*x + f)^(5/2)*c*d*f*g^49*e^2 + 1890*
(g*x + f)^(3/2)*c*d*f^2*g^49*e^2 - 3780*sqrt(g*x + f)*c*d*f^3*g^49*e^2 + 31
5*(g*x + f)^(3/2)*a*d*g^51*e^2 - 1890*sqrt(g*x + f)*a*d*f*g^51*e^2 + 35*(g*
x + f)^(9/2)*c*g^48*e^3 - 225*(g*x + f)^(7/2)*c*f*g^48*e^3 + 630*(g*x + f)^
(5/2)*c*f^2*g^48*e^3 - 1050*(g*x + f)^(3/2)*c*f^3*g^48*e^3 + 1575*sqrt(g*x
+ f)*c*f^4*g^48*e^3 + 63*(g*x + f)^(5/2)*a*g^50*e^3 - 315*(g*x + f)^(3/2)*a
*f*g^50*e^3 + 945*sqrt(g*x + f)*a*f^2*g^50*e^3)/g^54
```

3.597 $\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{g^5}$$

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*Sqrt[f + g*x]) - (4*(e*f - d*g)*(a*
e*g^2 + c*f*(2*e*f - d*g))*Sqrt[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*f^
2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (4*c*e*(2*e*f - d*g)*(
f + g*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rubi [A] time = 0.203525, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {898, 1261}

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*Sqrt[f + g*x]) - (4*(e*f - d*g)*(a*
e*g^2 + c*f*(2*e*f - d*g))*Sqrt[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*f^
2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (4*c*e*(2*e*f - d*g)*(
f + g*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}dx,x,\sqrt{f+gx}\right)}{g}$$
$$= \frac{2\,\mathrm{Subst}\left(\int\left(\frac{2(ef-dg)(-aeg^2-cf(2ef-dg))}{g^4}+\frac{(-ef+dg)^2(cf^2+ag^2)}{g^4x^2}+\frac{(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))x^2}{g^4}-\frac{2ce(\ldots}{}\right.\right.}{g}$$
$$= -\frac{2(ef-dg)^2(cf^2+ag^2)}{g^5\sqrt{f+gx}}-\frac{4(ef-dg)(aeg^2+cf(2ef-dg))\sqrt{f+gx}}{g^5}+\frac{2(ae^2g^2+c(6e^2\ldots}{}$$

Mathematica [A] time = 0.160018, size = 149, normalized size = 0.86

$$\frac{2\left(35(f+gx)^2\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)-105\left(ag^2+cf^2\right)(ef-dg)^2-210(f+gx)(ef-dg)\left(aeg^2+cf(2e\ldots\right.\right.}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-105*(e*f - d*g)^2*(c*f^2 + a*g^2) - 210*(e*f - d*g)*(a*e*g^2 + c*f*(2*
e*f - d*g))*(f + g*x) + 35*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 42*c*e*(2*e*f - d*g)*(f + g*x)^3 + 15*c*e^2*(f + g*x)^4))/(
105*g^5*Sqrt[f + g*x])
```

Maple [A] time = 0.049, size = 215, normalized size = 1.2

$$-\frac{-30\,ce^2x^4g^4-84\,cdeg^4x^3+48\,ce^2fg^3x^3-70\,ae^2g^4x^2-70\,cd^2g^4x^2+168\,cdefg^3x^2-96\,ce^2f^2g^2x^2-420\,adeg^4x+2\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/105/(g*x+f)^(1/2)*(-15*c*e^2*g^4*x^4-42*c*d*e*g^4*x^3+24*c*e^2*f*g^3*x^3
-35*a*e^2*g^4*x^2-35*c*d^2*g^4*x^2+84*c*d*e*f*g^3*x^2-48*c*e^2*f^2*g^2*x^2-
210*a*d*e*g^4*x+140*a*e^2*f*g^3*x+140*c*d^2*f*g^3*x-336*c*d*e*f^2*g^2*x+192
*c*e^2*f^3*g*x+105*a*d^2*g^4-420*a*d*e*f*g^3+280*a*e^2*f^2*g^2+280*c*d^2*f^
2*g^2-672*c*d*e*f^3*g+384*c*e^2*f^4)/g^5
```

Maxima [A] time = 0.995829, size = 277, normalized size = 1.6

$$\frac{2\left(\frac{15\left(gx+f\right)^{\frac{7}{2}}ce^2-42\left(2\,ce^2f-cdeg\right)\left(gx+f\right)^{\frac{5}{2}}+35\left(6\,ce^2f^2-6\,cdefg+\left(cd^2+ae^2\right)g^2\right)\left(gx+f\right)^{\frac{3}{2}}-210\left(2\,ce^2f^3-3\,cdef^2g-adeg^3+\left(cd^2+ae^2\right)fg^2\right)\sqrt{gx+f}}{g^4}-\frac{105\left(c\ldots\right.}{}\right.}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 42*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(5/2)
 + 35*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(3/2) - 2
10*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*sqrt(g
*x + f))/g^4 - 105*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 +
 (c*d^2 + a*e^2)*f^2*g^2)/(sqrt(g*x + f)*g^4))/g
```

Fricas [A] time = 1.7252, size = 466, normalized size = 2.69

$$\frac{2\left(15\,ce^2g^4x^4 - 384\,ce^2f^4 + 672\,cdef^3g + 420\,adefg^3 - 105\,ad^2g^4 - 280\left(cd^2 + ae^2\right)f^2g^2 - 6\left(4\,ce^2fg^3 - 7\,cdeg^4\right)x^3 + (48\right.}{105\left(g^6x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 + 672*c*d*e*f^3*g + 420*a*d*e*f*g^3
 - 105*a*d^2*g^4 - 280*(c*d^2 + a*e^2)*f^2*g^2 - 6*(4*c*e^2*f*g^3 - 7*c*d*e
*g^4)*x^3 + (48*c*e^2*f^2*g^2 - 84*c*d*e*f*g^3 + 35*(c*d^2 + a*e^2)*g^4)*x^
2 - 2*(96*c*e^2*f^3*g - 168*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 70*(c*d^2 + a*e
^2)*f*g^3)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

Sympy [A] time = 30.8231, size = 204, normalized size = 1.18

$$\frac{2ce^2\left(f+gx\right)^{\frac{7}{2}}}{7g^5} + \frac{\left(f+gx\right)^{\frac{5}{2}}\left(4cdeg - 8ce^2f\right)}{5g^5} + \frac{\left(f+gx\right)^{\frac{3}{2}}\left(2ae^2g^2 + 2cd^2g^2 - 12cdefg + 12ce^2f^2\right)}{3g^5} + \frac{\sqrt{f+gx}\left(4adeg^3 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + (f + g*x)**(5/2)*(4*c*d*e*g - 8*c*e**2
*f)/(5*g**5) + (f + g*x)**(3/2)*(2*a*e**2*g**2 + 2*c*d**2*g**2 - 12*c*d*e*f
*g + 12*c*e**2*f**2)/(3*g**5) + sqrt(f + g*x)*(4*a*d*e*g**3 - 4*a*e**2*f*g*
*2 - 4*c*d**2*f*g**2 + 12*c*d*e*f**2*g - 8*c*e**2*f**3)/g**5 - 2*(a*g**2 +
c*f**2)*(d*g - e*f)**2/(g**5*sqrt(f + g*x))
```

Giac [A] time = 1.14735, size = 371, normalized size = 2.14

$$-\frac{2\left(cd^2f^2g^2 + ad^2g^4 - 2\,cdf^3ge - 2\,adfg^3e + cf^4e^2 + af^2g^2e^2\right)}{\sqrt{gx+f}g^5} + \frac{2\left(35\left(gx+f\right)^{\frac{3}{2}}cd^2g^{32} - 210\sqrt{gx+f}cd^2fg^{32} + 42\left(gx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*f^2*g^2 + a*d^2*g^4 - 2*c*d*f^3*g*e - 2*a*d*f*g^3*e + c*f^4*e^2 +
 a*f^2*g^2*e^2)/(sqrt(g*x + f)*g^5) + 2/105*(35*(g*x + f)^(3/2)*c*d^2*g^32
- 210*sqrt(g*x + f)*c*d^2*f*g^32 + 42*(g*x + f)^(5/2)*c*d*g^31*e - 210*(g*x
 + f)^(3/2)*c*d*f*g^31*e + 630*sqrt(g*x + f)*c*d*f^2*g^31*e + 210*sqrt(g*x
```

```
+ f)*a*d*g^33*e + 15*(g*x + f)^(7/2)*c*g^30*e^2 - 84*(g*x + f)^(5/2)*c*f*g^
30*e^2 + 210*(g*x + f)^(3/2)*c*f^2*g^30*e^2 - 420*sqrt(g*x + f)*c*f^3*g^30*
e^2 + 35*(g*x + f)^(3/2)*a*g^32*e^2 - 210*sqrt(g*x + f)*a*f*g^32*e^2)/g^35
```

3.598 $\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}} \, dx$

Optimal. Leaf size=111

$$\frac{2(ag^2+cf^2)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\left(aeg^2+cf(3ef-2dg)\right)}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

```
[Out] (2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(a*e*g^2 + c*f*(3*
e*f - 2*d*g))*Sqrt[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^(3/2))/(3*g
^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rubi [A] time = 0.0661105, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {772}

$$\frac{2(ag^2+cf^2)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\left(aeg^2+cf(3ef-2dg)\right)}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(a*e*g^2 + c*f*(3*
e*f - 2*d*g))*Sqrt[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^(3/2))/(3*g
^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{(-ef+dg)(cf^2+ag^2)}{g^3(f+gx)^{3/2}}+\frac{aeg^2+cf(3ef-2dg)}{g^3\sqrt{f+gx}}+\frac{c(-3ef+dg)\sqrt{f+gx}}{g^3}+\frac{ce(f+gx)^{3/2}}{g^3}\right)\\ &= \frac{2(ef-dg)(cf^2+ag^2)}{g^4\sqrt{f+gx}}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^4}-\frac{2c(3ef-dg)(f+gx)^{3/2}}{3g^4}+\frac{2ce(}{}\end{aligned}$$

Mathematica [A] time = 0.0860907, size = 92, normalized size = 0.83

$$\frac{30ag^2(-dg+2ef+egx)+10cdg\left(-8f^2-4fgx+g^2x^2\right)+6ce\left(8f^2gx+16f^3-2fg^2x^2+g^3x^3\right)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (30*a*g^2*(2*e*f - d*g + e*g*x) + 10*c*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 6
*c*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))/(15*g^4*Sqrt[f + g*x])
```

Maple [A] time = 0.046, size = 101, normalized size = 0.9

$$-\frac{-6\,ce x^3 g^3 - 10\,cd g^3 x^2 + 12\,cef g^2 x^2 - 30\,ae g^3 x + 40\,cdf g^2 x - 48\,ce f^2 g x + 30\,ad g^3 - 60\,aef g^2 + 80\,cd f^2 g - 96\,ce f^3}{15\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/15/(g*x+f)^(1/2)*(-3*c*e*g^3*x^3-5*c*d*g^3*x^2+6*c*e*f*g^2*x^2-15*a*e*g^
3*x+20*c*d*f*g^2*x-24*c*e*f^2*g*x+15*a*d*g^3-30*a*e*f*g^2+40*c*d*f^2*g-48*c
*e*f^3)/g^4
```

Maxima [A] time = 0.998517, size = 151, normalized size = 1.36

$$\frac{2\left(\frac{3\,(gx+f)^{\frac{5}{2}}ce - 5\,(3\,cef - cdg)(gx+f)^{\frac{3}{2}} + 15\,(3\,cef^2 - 2\,cdfg + aeg^2)\sqrt{gx+f}}{g^3} + \frac{15\,(cef^3 - cdf^2g + aefg^2 - adg^3)}{\sqrt{gx+f}\,g^3}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - c*d*g)*(g*x + f)^(3/2) + 15*(3*
c*e*f^2 - 2*c*d*f*g + a*e*g^2)*sqrt(g*x + f))/g^3 + 15*(c*e*f^3 - c*d*f^2*g
 + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^3))/g
```

Fricas [A] time = 1.72792, size = 252, normalized size = 2.27

$$\frac{2\left(3\,ceg^3x^3 + 48\,cef^3 - 40\,cdf^2g + 30\,aefg^2 - 15\,adg^3 - \left(6\,cefg^2 - 5\,cdg^3\right)x^2 + \left(24\,cef^2g - 20\,cdfg^2 + 15\,aeg^3\right)x\right)\sqrt{g}}{15\left(g^5x + fg^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 40*c*d*f^2*g + 30*a*e*f*g^2 - 15*a*d*g^3
 - (6*c*e*f*g^2 - 5*c*d*g^3)*x^2 + (24*c*e*f^2*g - 20*c*d*f*g^2 + 15*a*e*g^
3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

Sympy [A] time = 14.7237, size = 112, normalized size = 1.01

$$\frac{2ce\left(f+gx\right)^{\frac{5}{2}}}{5g^4} + \frac{\left(f+gx\right)^{\frac{3}{2}}\left(2cdg - 6cef\right)}{3g^4} + \frac{\sqrt{f+gx}\left(2aeg^2 - 4cdfg + 6cef^2\right)}{g^4} - \frac{2\left(ag^2 + cf^2\right)\left(dg - ef\right)}{g^4\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e*(f + g*x)**(5/2)/(5*g**4) + (f + g*x)**(3/2)*(2*c*d*g - 6*c*e*f)/(3*g
**4) + sqrt(f + g*x)*(2*a*e*g**2 - 4*c*d*f*g + 6*c*e*f**2)/g**4 - 2*(a*g**2
 + c*f**2)*(d*g - e*f)/(g**4*sqrt(f + g*x))
```

Giac [A] time = 1.12359, size = 193, normalized size = 1.74

$$-\frac{2\left(cdf^2g + adg^3 - cf^3e - afg^2e\right)}{\sqrt{gx+f}g^4} + \frac{2\left(5\left(gx+f\right)^{\frac{3}{2}}cdg^{17} - 30\sqrt{gx+f}cdfg^{17} + 3\left(gx+f\right)^{\frac{5}{2}}cg^{16}e - 15\left(gx+f\right)^{\frac{3}{2}}cfg^{16}e \cdots\right.}{15g^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d*f^2*g + a*d*g^3 - c*f^3*e - a*f*g^2*e)/(sqrt(g*x + f)*g^4) + 2/15*(
5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f*g^17 + 3*(g*x + f)^(5/2
)*c*g^16*e - 15*(g*x + f)^(3/2)*c*f*g^16*e + 45*sqrt(g*x + f)*c*f^2*g^16*e
+ 15*sqrt(g*x + f)*a*g^18*e)/g^20
```

3.599 $\int \frac{a+cx^2}{(f+gx)^{3/2}} \, dx$

Optimal. Leaf size=59

$$-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

```
[Out] (-2*(c*f^2 + a*g^2))/(g^3*Sqrt[f + g*x]) - (4*c*f*Sqrt[f + g*x])/g^3 + (2*c
*(f + g*x)^(3/2))/(3*g^3)
```

Rubi [A] time = 0.0257772, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {697}

$$-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g^3*Sqrt[f + g*x]) - (4*c*f*Sqrt[f + g*x])/g^3 + (2*c
*(f + g*x)^(3/2))/(3*g^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(f+gx)^{3/2}} \, dx &= \int \left(\frac{cf^2+ag^2}{g^2(f+gx)^{3/2}}-\frac{2cf}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right) \, dx\\ &= -\frac{2\left(cf^2+ag^2\right)}{g^3\sqrt{f+gx}}-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A] time = 0.0281284, size = 43, normalized size = 0.73

$$\frac{2\left(c\left(-8f^2-4fgx+g^2x^2\right)-3ag^2\right)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-3*a*g^2 + c*(-8*f^2 - 4*f*g*x + g^2*x^2)))/(3*g^3*Sqrt[f + g*x])
```

Maple [A] time = 0.043, size = 41, normalized size = 0.7

$$-\frac{-2\,c x^2 g^2+8\,c f x g+6\,a g^2+16\,c f^2}{3\,g^3}\frac{1}{\sqrt{g x+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/3/(g*x+f)^(1/2)*(-c*g^2*x^2+4*c*f*g*x+3*a*g^2+8*c*f^2)/g^3
```

Maxima [A] time = 1.01421, size = 73, normalized size = 1.24

$$\frac{2\left(\frac{(gx+f)^{\frac{3}{2}}c-6\,\sqrt{gx+f}cf}{g^2}-\frac{3\left(cf^2+ag^2\right)}{\sqrt{gx+f}g^2}\right)}{3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 6*sqrt(g*x + f)*c*f)/g^2 - 3*(c*f^2 + a*g^2)/(sqr
t(g*x + f)*g^2))/g
```

Fricas [A] time = 1.68355, size = 107, normalized size = 1.81

$$\frac{2\left(cg^2x^2-4\,cfgx-8\,cf^2-3\,ag^2\right)\sqrt{gx+f}}{3\left(g^4x+fg^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 4*c*f*g*x - 8*c*f^2 - 3*a*g^2)*sqrt(g*x + f)/(g^4*x + f*g^
3)
```

Sympy [A] time = 5.66873, size = 58, normalized size = 0.98

$$-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3g^3}-\frac{2\left(ag^2+cf^2\right)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] -4*c*f*sqrt(f + g*x)/g**3 + 2*c*(f + g*x)**(3/2)/(3*g**3) - 2*(a*g**2 + c*f
**2)/(g**3*sqrt(f + g*x))
```

<u>Giac</u> [A] time = 1.14379, size = 76, normalized size = 1.29

$$-\frac{2\left(cf^{2}+ag^{2}\right)}{\sqrt{gx+f}g^{3}}+\frac{2\left(\left(gx+f\right)^{\frac{3}{2}}cg^{6}-6\sqrt{gx+f}cfg^{6}\right)}{3g^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6 - 6*sqr
t(g*x + f)*c*f*g^6)/g^9
```

3.600 $\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2+cf^2\right)}{g^2\sqrt{f+gx}(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

```
[Out] (2*(c*f^2 + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f + g*x])/(
e*g^2) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rubi [A] time = 0.173682, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {898, 1261, 208}

$$-\frac{2\left(ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2+cf^2\right)}{g^2\sqrt{f+gx}(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f + g*x])/(
e*g^2) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, x, \sqrt{f+gx}\right)}{g}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$
$$= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{\left(2(cd^2+ae^2)\right)\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}\,dx, x, \sqrt{f+gx}\right)}{e(ef-dg)}$$
$$= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

Mathematica [C] time = 0.0714895, size = 90, normalized size = 0.8

$$-\frac{2\left(g^2(ae^2+cd^2)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)(dg+2ef+egx)\right)}{e^2g^2\sqrt{f+gx}(dg-ef)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*(e*f - d*g)*(2*e*f + d*g + e*g*x) + (c*d^2 + a*e^2)*g^2*Hypergeometr
ic2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*g)]))/(e^2*g^2*(-(e*f) + d*g)*Sq
rt[f + g*x])
```

Maple [A] time = 0.214, size = 165, normalized size = 1.5

$$2\,\frac{c\sqrt{gx+f}}{eg^2}-2\,\frac{ae}{(dg-ef)\sqrt{(dg-ef)e}}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)-2\,\frac{cd^2}{(dg-ef)e\sqrt{(dg-ef)e}}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e/g^2-2/(d*g-e*f)*e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^
(1/2)/((d*g-e*f)*e)^(1/2))*a-2/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*
x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d^2-2/(d*g-e*f)/(g*x+f)^(1/2)*a-2/g^2/(d*
g-e*f)/(g*x+f)^(1/2)*c*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 1.86707, size = 1015, normalized size = 9.06

$$\left[-\frac{\left(\left(cd^2+ae^2\right)g^3x+\left(cd^2+ae^2\right)fg^2\right)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2\,ce^3f^3-3\,cde^2f^2g-ade^2g^3+\left(cd^2\right.\right.}{e^4f^3g^2-2\,de^3f^2g^3+d^2e^2fg^4+\left(e^4f^2g^3-2\,de^3fg^4+d^2e^2g^5\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(e^2*f - d*e*g)*log(
(e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*
(2*c*e^3*f^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c
*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2
- 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*
g^5)*x), 2*(((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(-e^2*f + d
*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f
^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c*e^3*f^2*g
 - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*
f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x)]
```

Sympy [A] time = 20.3886, size = 104, normalized size = 0.93

$$\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2\left(ag^2+cf^2\right)}{g^2\sqrt{f+gx}\left(dg-ef\right)}-\frac{2\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*sqrt(f + g*x)/(e*g**2) - 2*(a*g**2 + c*f**2)/(g**2*sqrt(f + g*x)*(d*g -
 e*f)) - 2*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**2*
sqrt((d*g - e*f)/e)*(d*g - e*f))
```

Giac [A] time = 1.16887, size = 136, normalized size = 1.21

$$-\frac{2\left(cd^2+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{\left(dge-fe^2\right)^{\frac{3}{2}}}+\frac{2\sqrt{gx+f}ce^{(-1)}}{g^2}-\frac{2\left(cf^2+ag^2\right)}{\left(dg^3-fg^2e\right)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/(d*g*e - f*e
^2)^(3/2) + 2*sqrt(g*x + f)*c*e^(-1)/g^2 - 2*(c*f^2 + a*g^2)/((d*g^3 - f*g^
2*e)*sqrt(g*x + f))
```

3.601 $\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2+cf^2\right)}{g\sqrt{f+gx}(ef-dg)^2}$$

```
[Out] (-2*(c*f^2 + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqr
t[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((3*a*e^2*g + c*d*(4*e*f - d*g))*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2
))
```

Rubi [A] time = 0.271231, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {898, 1259, 453, 208}

$$-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2+cf^2\right)}{g\sqrt{f+gx}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqr
t[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((3*a*e^2*g + c*d*(4*e*f - d*g))*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2
))
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2+ag^2\right)}{g^5}+\frac{e\left(ae^2g^2-c\left(2e^2f^2-4defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, x, \sqrt{f+gx}\right)}{e^2(ef-dg)^2}$$

$$= -\frac{2\left(cf^2+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{\left(3ae^2g+cd(4ef-dg)\right)\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{eg(ef-dg)^2}$$

$$= -\frac{2\left(cf^2+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{\left(3ae^2g+cd(4ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

Mathematica [C] time = 0.0878971, size = 118, normalized size = 0.82

$$-\frac{2\left(g^2\left(ae^2+cd^2\right){}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+2cdg(ef-dg)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\right)}{e^2g\sqrt{f+gx}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*(e*f - d*g)^2 + 2*c*d*g*(e*f - d*g)*Hypergeometric2F1[-1/2, 1, 1/2,
(e*(f + g*x))/(e*f - d*g)] + (c*d^2 + a*e^2)*g^2*Hypergeometric2F1[-1/2, 2,
 1/2, (e*(f + g*x))/(e*f - d*g)]))/(e^2*g*(e*f - d*g)^2*Sqrt[f + g*x])
```

Maple [B] time = 0.245, size = 269, normalized size = 1.9

$$-2\,\frac{ag}{\left(dg-ef\right)^2\sqrt{gx+f}}-2\,\frac{cf^2}{g\left(dg-ef\right)^2\sqrt{gx+f}}-\frac{aeg}{\left(dg-ef\right)^2\left(egx+dg\right)}\sqrt{gx+f}-\frac{cd^2g}{\left(dg-ef\right)^2e\left(egx+dg\right)}\sqrt{gx+f}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x)
```

```
[Out] -2*g/(d*g-e*f)^2/(g*x+f)^(1/2)*a-2/g/(d*g-e*f)^2/(g*x+f)^(1/2)*c*f^2-g/(d*g
-e*f)^2*e*(g*x+f)^(1/2)/(e*g*x+d*g)*a-g/(d*g-e*f)^2/e*(g*x+f)^(1/2)/(e*g*x+
```

```
d*g)*c*d^2-3*g/(d*g-e*f)^2*e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d
*g-e*f)*e)^(1/2))*a+g/(d*g-e*f)^2/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1
/2)/((d*g-e*f)*e)^(1/2))*c*d^2-4/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*arctan(e*(
g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*d*c*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28051, size = 1817, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 - (c*
d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^2 -
(c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g +
2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*a*d^
2*e^2*g^3 - (c*d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^
4*f^3 - 2*c*d*e^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^
3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*
g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^
3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g
^5)*x), -((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 -
 (c*d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^
2 - (c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d
*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*
d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*c*d*e
^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^3)*g^3)*x)*sqrt
(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f
*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2
+ (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16049, size = 304, normalized size = 2.11

$$\frac{\left(cd^2g - 4\,cdfe - 3\,age^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{\left(d^2g^2e - 2\,dfge^2 + f^2e^3\right)\sqrt{dge - fe^2}} - \frac{\left(gx+f\right)cd^2g^2 + 2\,cdf^2ge + 2\,adg^3e + 2\left(gx+f\right)cf^2e^2 - 2\,cf^3e^2 + 3\left(gx + \right.}{\left(d^2g^3e - 2\,dfg^2e^2 + f^2ge^3\right)\left(\sqrt{gx+f}dg + \left(gx+f\right)^{\frac{3}{2}}e - \sqrt{gx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] (c*d^2*g - 4*c*d*f*e - 3*a*g*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2
))/((d^2*g^2*e - 2*d*f*g*e^2 + f^2*e^3)*sqrt(d*g*e - f*e^2)) - ((g*x + f)*c
*d^2*g^2 + 2*c*d*f^2*g*e + 2*a*d*g^3*e + 2*(g*x + f)*c*f^2*e^2 - 2*c*f^3*e^
2 + 3*(g*x + f)*a*g^2*e^2 - 2*a*f*g^2*e^2)/((d^2*g^3*e - 2*d*f*g^2*e^2 + f^
2*g*e^3)*(sqrt(g*x + f)*d*g + (g*x + f)^(3/2)*e - sqrt(g*x + f)*f*e))
```

3.602 $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(15ae^2g^2+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}+\frac{\sqrt{f+gx}\left(7ae^2g+cd(8ef-dg\right.}{4e(d+ex)(ef-dg)^3}$$

```
[Out] (2*(c*f^2 + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqrt[f
 + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((7*a*e^2*g + c*d*(8*e*f - d*g))
*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d + e*x)) - ((15*a*e^2*g^2 + c*(8*e^2*f
^2 + 8*d*e*f*g - d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(3/2)*(e*f - d*g)^(7/2))
```

Rubi [A] time = 0.504543, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {898, 1259, 456, 453, 208}

$$-\frac{\left(15ae^2g^2+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}\left(ae^2+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}+\frac{\sqrt{f+gx}\left(7ae^2g+cd(8ef-dg\right.}{4e(d+ex)(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqrt[f
 + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((7*a*e^2*g + c*d*(8*e*f - d*g))
*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d + e*x)) - ((15*a*e^2*g^2 + c*(8*e^2*f
^2 + 8*d*e*f*g - d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(3/2)*(e*f - d*g)^(7/2))
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)\left(cf^2+ag^2\right)}{g^5}+\frac{e\left(3ae^2g^2-c\left(4e^2f^2-8defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+g\ldots}\right.}{2e^2(ef-dg)^2}$$

$$= -\frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{g^3\,\text{Subst}\left(\int \frac{\frac{8e^2\left(cf^2+ag^2\right)}{g^4}+\frac{e\left(7ae^2g+\ldots\right.}{g^3\ldots}}{x^2\left(\frac{-ef+dg}{g}+\frac{e\ldots}{\ldots}\right.}\right.}{4e^2(ef-\ldots}$$

$$= \frac{2\left(cf^2+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{\left(15ae^2g^2\ldots\right.}{\ldots}$$

$$= \frac{2\left(cf^2+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{\left(7ae^2g+cd(8ef-dg)\right)\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} - \frac{\left(15ae^2g^2\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.107233, size = 140, normalized size = 0.65

$$\frac{2\left(g\left(g\left(ae^2+cd^2\right)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)+2cd(ef-dg)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)\right)+c(ef-dg)^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{e(f+gx)}{ef-dg}\right)\right)}{e^2\sqrt{f+gx}(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*(e*f - d*g)^2*Hypergeometric2F1[-1/2, 1, 1/2, (e*(f + g*x))/(e*f - d*
g)] + g*(2*c*d*(e*f - d*g)*Hypergeometric2F1[-1/2, 2, 1/2, (e*(f + g*x))/(e
```

```
*f - d*g)] + (c*d^2 + a*e^2)*g*Hypergeometric2F1[-1/2, 3, 1/2, (e*(f + g*x)
)/(e*f - d*g)])))/(e^2*(e*f - d*g)^3*Sqrt[f + g*x])
```

Maple [B] time = 0.224, size = 546, normalized size = 2.6

$$-2\frac{ag^2}{(dg-ef)^3\sqrt{gx+f}}-2\frac{cf^2}{(dg-ef)^3\sqrt{gx+f}}-\frac{7\,ae^2g^2}{4\,(dg-ef)^3\,(egx+dg)^2}(gx+f)^{\frac{3}{2}}+\frac{cd^2g^2}{4\,(dg-ef)^3\,(egx+dg)^2}(gx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x)
```

```
[Out] -2/(d*g-e*f)^3/(g*x+f)^(1/2)*a*g^2-2/(d*g-e*f)^3/(g*x+f)^(1/2)*c*f^2-7/4/(d
*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*a*e^2*g^2+1/4/(d*g-e*f)^3/(e*g*x+d*g)
^2*(g*x+f)^(3/2)*c*d^2*g^2-2/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*c*d*e*
f*g-9/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3*e*(g*x+f)^(1/2)*a*d+9/4/(d*g-e*f)^3/(
e*g*x+d*g)^2*g^2*e^2*(g*x+f)^(1/2)*a*f-1/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3/e*
(g*x+f)^(1/2)*c*d^3-7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*(g*x+f)^(1/2)*c*d^2*f
+2/(d*g-e*f)^3/(e*g*x+d*g)^2*g*e*(g*x+f)^(1/2)*c*d*f^2-15/4/(d*g-e*f)^3*e/(
(d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*a*g^2+1/4/(d
*g-e*f)^3/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
*c*d^2*g^2-2/(d*g-e*f)^3/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e
*f)*e)^(1/2))*c*d*f*g-2/(d*g-e*f)^3*e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^
(1/2)/((d*g-e*f)*e)^(1/2))*c*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14663, size = 3135, normalized size = 14.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((8*c*d^2*e^2*f^3 + 8*c*d^3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 +
(8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e
^4*f^3 + 24*c*d*e^3*f^2*g + 15*(c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*
a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a
*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g
*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a
*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*d^3*e^2 - 11*a*d*e^4)*f^
2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 3*(3*c*d^2*e^3 - 5*a*e^5
```

```
)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c*d*e^4*f^3 - (19*c*d^2*e
^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g^2 - (c*d^4*e + 25*a*d^2
*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3
*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6
*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*
f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3
*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*
f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 8*c*d^
3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2
 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 24*c*d*e^3*f^2*g + 15*(
c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*
f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^
2*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x +
f)/(e*g*x + e*f)) - (8*a*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*
d^3*e^2 - 11*a*d*e^4)*f^2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 -
3*(3*c*d^2*e^3 - 5*a*e^5)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c
*d*e^4*f^3 - (19*c*d^2*e^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g
^2 - (c*d^4*e + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e
^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4
*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x
^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d
^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*
e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18016, size = 487, normalized size = 2.28

$$\frac{\left(cd^2g^2-8\,cdfge-8\,cf^2e^2-15\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{4\left(d^3g^3e-3\,d^2fg^2e^2+3\,df^2ge^3-f^3e^4\right)\sqrt{dge-fe^2}}-\frac{2\left(cf^2+ag^2\right)}{\left(d^3g^3-3\,d^2fg^2e+3\,df^2ge^2-f^3e^3\right)\sqrt{gx+f}}-\frac{\sqrt{gx+f}cd^3g$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^2*g^2 - 8*c*d*f*g*e - 8*c*f^2*e^2 - 15*a*g^2*e^2)*arctan(sqrt(g*x
+ f)*e/sqrt(d*g*e - f*e^2))/((d^3*g^3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^3 -
 f^3*e^4)*sqrt(d*g*e - f*e^2)) - 2*(c*f^2 + a*g^2)/((d^3*g^3 - 3*d^2*f*g^2*
e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(g*x + f)) - 1/4*(sqrt(g*x + f)*c*d^3*g^3
- (g*x + f)^(3/2)*c*d^2*g^2*e + 7*sqrt(g*x + f)*c*d^2*f*g^2*e + 8*(g*x + f)
^(3/2)*c*d*f*g*e^2 - 8*sqrt(g*x + f)*c*d*f^2*g*e^2 + 9*sqrt(g*x + f)*a*d*g^
3*e^2 + 7*(g*x + f)^(3/2)*a*g^2*e^3 - 9*sqrt(g*x + f)*a*f*g^2*e^3)/((d^3*g^
3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^3 - f^3*e^4)*(d*g + (g*x + f)*e - f*e)^
2)
```

3.603 $\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=147

$$\frac{\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

```
[Out] -(c*(3*e*f + 5*d*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) + (c*(d + e*x)
^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g
+ 3*d^2*g^2))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(4*
e^(5/2)*g^(5/2))
```

Rubi [A] time = 0.141073, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {952, 80, 63, 217, 206}

$$\frac{\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -(c*(3*e*f + 5*d*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) + (c*(d + e*x)
^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g
+ 3*d^2*g^2))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(4*
e^(5/2)*g^(5/2))
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}ce(3ef+5dg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{1}{8}\left(8a+\frac{c\left(3e^2f^2+2defg+3d^2g^2\right)}{e^2g^2}\right)$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8a+\frac{c\left(3e^2f^2+2defg+3d^2g^2\right)}{e^2g^2}\right)\text{Subst}\left(\int \right.}{4e}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8a+\frac{c\left(3e^2f^2+2defg+3d^2g^2\right)}{e^2g^2}\right)\text{Subst}\left(\int \right.}{4e}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(8ae^2g^2+c\left(3e^2f^2+2defg+3d^2g^2\right)\right.}{4e^{5/2}g^{5/2}}$$

Mathematica [A] time = 0.576056, size = 155, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)+ce\sqrt{g}\sqrt{d+ex}(f+gx)(-3dg-3ef+2egx)}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (c*e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(-3*e*f - 3*d*g + 2*e*g*x) + Sqrt[e*f
- d*g]*(8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*Sqrt[(e*(f + g
*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(4*e^3*
g^(5/2)*Sqrt[f + g*x])
```

Maple [B] time = 0.331, size = 306, normalized size = 2.1

$$\frac{1}{8e^2g^2}\left(8\ln\left(1/2\,\frac{2\,egx+2\,\sqrt{(gx+f)(ex+d)}\sqrt{eg}+dg+ef}{\sqrt{eg}}\right)ae^2g^2+3\ln\left(1/2\,\frac{2\,egx+2\,\sqrt{(gx+f)(ex+d)}\sqrt{eg}+dg+}{\sqrt{eg}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)
^(1/2))*a*e^2*g^2+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e
*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2+4*(e*g)^(1/2)*((g*
x+f)*(e*x+d))^(1/2)*x*c*e*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d*g-6*(
e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)
^(1/2)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.35452, size = 787, normalized size = 5.35

$$\left[\frac{\left(3\,ce^2f^2+2\,cdefg+\left(3\,cd^2+8\,ae^2\right)g^2\right)\sqrt{eg}\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\left(2\,egx+ef+dg\right)\sqrt{eg}\sqrt{ex+d}\sqrt{\right.}{16\,e^3g^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(e*g)*log(
8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqr
t(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(2*c*e^2*
g^2*x - 3*c*e^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3),
-1/8*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(-e*g)*arct
an(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^
2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g
- 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [A] time = 1.20633, size = 209, normalized size = 1.42

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\sqrt{xe+d}\left(\frac{2\,(xe+d)ce^{(-3)}}{g}-\frac{\left(5\,cdg^2e^5+3\,cfge^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3\,cd^2g^2+2\,cdfge+3\,cf^2e^2+8\,ag^2e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2*g^2 + 2*c*d*f
*g*e + 3*c*f^2*e^2 + 8*a*g^2*e^2)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e
^(1/2) + sqrt((x*e + d)*g*e - d*g*e + f*e^2)))/g^(5/2)
```

3.604 $\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx$

Optimal. Leaf size=16

$$\sqrt{x-1}x\sqrt{x+1}$$

```
[Out] Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Rubi [A] time = 0.0091387, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {384}

$$\sqrt{x-1}x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Rule 384

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2), x] /; FreeQ[{a1, b1, a2, b2, c, d, n
, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d - b1*b2*
c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{-1+x}x\sqrt{1+x}$$

Mathematica [C] time = 0.0913363, size = 66, normalized size = 4.12

$$\frac{\sqrt{x-1}\left(x\sqrt{1-x^2}-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{\sqrt{1-x}}+2\tanh^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[-1 + x]*(x*Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/Sqrt[1 - x
] + 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]]
```

Maple [A] time = 0.042, size = 13, normalized size = 0.8

$$x\sqrt{-1+x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] x*(-1+x)^(1/2)*(1+x)^(1/2)
```

Maxima [A] time = 0.965673, size = 12, normalized size = 0.75

$$\sqrt{x^2-1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*x
```

Fricas [A] time = 1.67343, size = 36, normalized size = 2.25

$$\sqrt{x+1}\sqrt{x-1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

Sympy [C] time = 12.5668, size = 129, normalized size = 8.06

$$-\begin{cases} 2\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{|x+1|}{2}>1 \\ -2i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases} + \frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}} - \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1 \end{matrix}\right.}{2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -Piecewise((2*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1)/2 > 1), (-2*I*asin(s
qrt(2)*sqrt(x + 1)/2), True)) + meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)),
 ((-1, -3/4, -1/2, -1/4, 0, 0), ()), x**(-2))/(2*pi**(3/2)) - I*meijerg(((-
3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_
polar(2*I*pi)/x**2)/(2*pi**(3/2))
```

Giac [A] time = 1.16967, size = 16, normalized size = 1.

$$\sqrt{x+1}\sqrt{x-1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")

[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

3.605 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal. Leaf size=411

$$\frac{\left(\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}-\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\left(\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)+\frac{a\left(ae^2g-cd(dg+2\right.}{\sqrt{c}}\right.}{ac\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-}}}$$

```
[Out] (e*Sqrt[d + e*x]*Sqrt[f + g*x])/c + (Sqrt[e]*(e*f + 3*d*g)*ArcTanh[(Sqrt[g]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqrt[g]) + (((a*(a*e^2*g - c*d
*(2*e*f + d*g)))/Sqrt[c] - Sqrt[-a]*(c*d^2*f - a*e*(e*f + 2*d*g)))*ArcTanh[
(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*
Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a
]*g]) + (((a*(a*e^2*g - c*d*(2*e*f + d*g)))/Sqrt[c] + Sqrt[-a]*(c*d^2*f - a
*e*(e*f + 2*d*g)))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*
e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rubi [A] time = 2.51139, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {904, 80, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}-\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\left(\sqrt{-a}\left(cd^2f-ae(2dg+ef)\right)+\frac{a\left(ae^2g-cd(dg+2\right.}{\sqrt{c}}\right.}{ac\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (e*Sqrt[d + e*x]*Sqrt[f + g*x])/c + (Sqrt[e]*(e*f + 3*d*g)*ArcTanh[(Sqrt[g]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqrt[g]) + (((a*(a*e^2*g - c*d
*(2*e*f + d*g)))/Sqrt[c] - Sqrt[-a]*(c*d^2*f - a*e*(e*f + 2*d*g)))*ArcTanh[
(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*
Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a
]*g]) + (((a*(a*e^2*g - c*d*(2*e*f + d*g)))/Sqrt[c] + Sqrt[-a]*(c*d^2*f - a
*e*(e*f + 2*d*g)))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*
e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 904

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[g/c, Int[Simp[2*e*f + d*g + e*g*x, x]*(d + e*x)^(m
- 1)*(f + g*x)^(n - 2), x], x] + Dist[1/c, Int[(Simp[c*d*f^2 - 2*a*e*f*g -
a*d*g^2 + (c*e*f^2 + 2*c*d*f*g - a*e*g^2)*x, x]*(d + e*x)^(m - 1)*(f + g*x)
^(n - 2))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2
+ a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\int \frac{cd^2f-ae(ef+2dg)-\left(ae^2g-cd(2ef+dg)\right)x}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{c} + \frac{e\int \frac{ef+2dg+egx}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\int\left(\frac{-\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}}+\sqrt{-a}\left(cd^2f-ae(ef+2dg)\right)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}}+\sqrt{-a}\left(cd^2f-ae(ef+2\right.}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right.}{c}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{(ef+3dg)\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{c} + \frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}}+\right.}{}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{(ef+3dg)\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} + \frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}}+\sqrt{-a}\left(cd\right.\right.}{}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\sqrt{e}(ef+3dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(\frac{a\left(ae^2g-cd(2ef+dg)\right)}{\sqrt{c}}-\sqrt{-a}\left(cd^2f-ae(\right.\right.}{ac\sqrt{\sqrt{c}d-\sqrt{-}}}$$

Mathematica [B] time = 6.06434, size = 1587, normalized size = 3.86

$$\frac{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\frac{2\left(\sqrt{c}f+\sqrt{-a}g\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{-\sqrt{c}f-\sqrt{-a}g}} - \frac{2\sqrt{g}\sqrt{ef-dg}\sqrt{\frac{e}{\frac{e^2f}{ef-dg}-\frac{deg}{ef-dg}}}\sqrt{\frac{e^2f}{ef-dg}-\frac{deg}{ef-dg}}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}\sqrt{\frac{e^2f}{ef-dg}-\frac{de}{ef-}}}\right.}{\sqrt{c}e^{3/2}\sqrt{f+gx}}\right.}{\sqrt{c}}\right.}{2a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*((-2*Sqrt[d + e*x]*Sqrt[f + g*x]*(1 + (e
*g*(d + e*x))/((e*f - d*g)*((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g))))^(3
/2)*(1/(2*(1 + (e*g*(d + e*x))/((e*f - d*g)*((e^2*f)/(e*f - d*g) - (d*e*g)/
(e*f - d*g))))) + (Sqrt[e*f - d*g]*Sqrt[(e^2*f)/(e*f - d*g) - (d*e*g)/(e*f
- d*g)]*ArcSinh[(Sqrt[e]*Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e*f - d*g]*Sqrt[(e^2*
f)/(e*f - d*g) - (d*e*g)/(e*f - d*g)])])/(2*Sqrt[e]*Sqrt[g]*Sqrt[d + e*x]*(
1 + (e*g*(d + e*x))/((e*f - d*g)*((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g)
)))^(3/2))))/(Sqrt[c]*Sqrt[e/((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g))]*S
qrt[(e*(f + g*x))/(e*f - d*g)]) + ((Sqrt[c]*d + Sqrt[-a]*e)*((-2*Sqrt[g]*Sq
rt[e*f - d*g]*Sqrt[e/((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g))]*Sqrt[(e^2
*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g)]*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcS
inh[(Sqrt[e]*Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e*f - d*g]*Sqrt[(e^2*f)/(e*f - d*
g) - (d*e*g)/(e*f - d*g)])])/(Sqrt[c]*e^(3/2)*Sqrt[f + g*x]) + (2*(Sqrt[c]*
f + Sqrt[-a]*g)*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqr
t[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-
a]*e]*Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g])))/Sqrt[c]))/(2*a*Sqrt[c]) - (Sqrt[-a
```

```
]*(-(Sqrt[c]*d) + Sqrt[-a]*e)*((2*Sqrt[d + e*x]*Sqrt[f + g*x]*(1 + (e*g*(d
+ e*x))/((e*f - d*g)*((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g))))^(3/2)*(1
/(2*(1 + (e*g*(d + e*x))/((e*f - d*g)*((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f -
 d*g))))) + (Sqrt[e*f - d*g]*Sqrt[(e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g)
]*ArcSinh[(Sqrt[e]*Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e*f - d*g]*Sqrt[(e^2*f)/(e*
f - d*g) - (d*e*g)/(e*f - d*g)])])/(2*Sqrt[e]*Sqrt[g]*Sqrt[d + e*x]*(1 + (e
*g*(d + e*x))/((e*f - d*g)*((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g))))^(3
/2))))/(Sqrt[c]*Sqrt[e/((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g))]*Sqrt[(e
*(f + g*x))/(e*f - d*g)]) - ((-(Sqrt[c]*d) + Sqrt[-a]*e)*((2*Sqrt[g]*Sqrt[e
*f - d*g]*Sqrt[e/((e^2*f)/(e*f - d*g) - (d*e*g)/(e*f - d*g))]*Sqrt[(e^2*f)/
(e*f - d*g) - (d*e*g)/(e*f - d*g)]*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[
(Sqrt[e]*Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e*f - d*g]*Sqrt[(e^2*f)/(e*f - d*g) -
 (d*e*g)/(e*f - d*g)])])/(Sqrt[c]*e^(3/2)*Sqrt[f + g*x]) - (2*(-(Sqrt[c]*f)
 + Sqrt[-a]*g)*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(
Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[-(Sqrt[c]*d) + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])))/Sqrt[c]))/(2*a*Sqrt[c])
```

Maple [B] time = 0.493, size = 2497, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e
*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*a*c*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f
+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*
g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^
(1/2)))*d*e*g+(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e
*g)^(1/2)*a*c*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(e*g*x^2+d*g*x+e*f
*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*e^2*f+(-((-a*
c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*(-a*c)^(1/2
)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2
)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e^2*g-(-((-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*(-a*c)^(1/2)*ln((2*(-
a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d^2*g-2*(-((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*(-a*c)^(1/2)*ln((2*(-a*c)^(1/
2)*x*e*g+x*c*d*g+x*c*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*e*f-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*
f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e
*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)
^(1/2)))*c^2*d^2*f+3*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(
1/2)*(-a*c)^(1/2)*ln(1/2*(2*e*g*x+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(e*g)^(
1/2)+d*g+e*f)/(e*g)^(1/2))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)
/c)^(1/2)*c*d*e*g+(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2
)*(-a*c)^(1/2)*ln(1/2*(2*e*g*x+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(e*g)^(1/2
)+d*g+e*f)/(e*g)^(1/2))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)
^(1/2)*c*e^2*f-2*(e*g)^(1/2)*a*c*ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+
2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g*x^2+d*g*x
+e*f*x+d*f)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^
```

```
(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*d*e*g-(e*
g)^(1/2)*a*c*ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*c-
(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*e^2*f+(e*g)^(1/2)*(-a*c)^(1/2
)*ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*c-(-a*c)^(1/2
)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*a*e^2*g-(e*g)^(1/2)*(-a*c)^(1/2)*ln((-2*
(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a
*e*g-c*d*f)/c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a
*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f-a*e*g+c*d*f)/c)^(1/2)*c*d^2*g-2*(e*g)^(1/2)*(-a*c)^(1/2)*ln((-2*(-a*c)^
(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*
d*f)/c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/
2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e
*g+c*d*f)/c)^(1/2)*c*d*e*f+(e*g)^(1/2)*ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*
c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g*x^2
+d*g*x+e*f*x+d*f)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(
-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c^2
*d^2*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(
1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c*e)/(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)/(-a*c
)^(1/2)/c^2/(e*g)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)
^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
```

```
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/(a + c*x**2), x)
```

Giac [B] time = 48.7111, size = 4543, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 2*(c^2*d^7*g^(13/2)*e^(9/2) - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e
+ d)*g*e - d*g*e + f*e^2))^4*c^2*d^5*g^(9/2)*e^(5/2) + 4*(sqrt(x*e + d)*sqr
t(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^2*d^4*g^(7/2)*e^(3/
2) - (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
8*c^2*d^3*g^(5/2)*e^(1/2) + c^2*d^6*f*g^(11/2)*e^(11/2) - 8*(sqrt(x*e + d)*
sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d^4*f*g^(7/2)*
e^(7/2) + 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^6*c^2*d^3*f*g^(5/2)*e^(5/2) - (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt(
(x*e + d)*g*e - d*g*e + f*e^2))^8*c^2*d^2*f*g^(3/2)*e^(3/2) - 12*c^2*d^5*f^
2*g^(9/2)*e^(13/2) - 5*a*c*d^5*g^(13/2)*e^(13/2) - 28*(sqrt(x*e + d)*sqrt(g
)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d^3*f^2*g^(5/2)*e^(9
/2) - 36*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^
2))^4*a*c*d^3*g^(9/2)*e^(9/2) + 12*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x
*e + d)*g*e - d*g*e + f*e^2))^6*c^2*d^2*f^2*g^(3/2)*e^(7/2) + 12*(sqrt(x*e
+ d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c*d^2*g^(7/
2)*e^(7/2) - 2*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
+ f*e^2))^8*c^2*d*f^2*sqrt(g)*e^(5/2) - 3*(sqrt(x*e + d)*sqrt(g)*e^(1/2) -
sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*a*c*d*g^(5/2)*e^(5/2) + 18*c^2*d^4*f
^3*g^(7/2)*e^(15/2) + 17*a*c*d^4*f*g^(11/2)*e^(15/2) - 16*(sqrt(x*e + d)*sq
rt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d^2*f^3*g^(3/2)*
e^(11/2) - 32*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e +
 f*e^2))^4*a*c*d^2*f*g^(7/2)*e^(11/2) + 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) -
sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^2*d*f^3*sqrt(g)*e^(9/2) + 16*(sqrt
(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c*d*f*
g^(5/2)*e^(9/2) - (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g
*e + f*e^2))^8*a*c*f*g^(3/2)*e^(7/2) - 7*c^2*d^3*f^4*g^(5/2)*e^(17/2) - 18*
a*c*d^3*f^2*g^(9/2)*e^(17/2) - 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e
 + d)*g*e - d*g*e + f*e^2))^4*c^2*d*f^4*sqrt(g)*e^(13/2) - 52*(sqrt(x*e + d
)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c*d*f^2*g^(5/2
)*e^(13/2) - 48*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
 + f*e^2))^4*a^2*d*g^(9/2)*e^(13/2) + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sq
rt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c*f^2*g^(3/2)*e^(11/2) - 3*c^2*d^2*f
^5*g^(3/2)*e^(19/2) + 2*a*c*d^2*f^3*g^(7/2)*e^(19/2) - 8*(sqrt(x*e + d)*sqr
t(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c*f^3*g^(3/2)*e^(15
/2) - 16*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^
2))^4*a^2*f*g^(7/2)*e^(15/2) + 2*c^2*d*f^6*sqrt(g)*e^(21/2) + 7*a*c*d*f^4*g
^(5/2)*e^(21/2) - 3*a*c*f^5*g^(3/2)*e^(23/2))*log(abs(c*d^4*g^4*e^4 - 4*(sq
rt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^3*
g^3*e^3 + 6*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^4*c*d^2*g^2*e^2 - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*
g*e - d*g*e + f*e^2))^6*c*d*g*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*
e + d)*g*e - d*g*e + f*e^2))^8*c - 4*c*d^3*f*g^3*e^5 + 4*(sqrt(x*e + d)*sqr
t(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*f*g^2*e^4 + 4*(
```

sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d*
f*g*e^3 - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^6*c*f*e^2 + 6*c*d^2*f^2*g^2*e^6 + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) -
 sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d*f^2*g*e^5 + 6*(sqrt(x*e + d)*sq
rt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*f^2*e^4 + 16*(sqrt
(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*g^2*e^
4 - 4*c*d*f^3*g*e^7 - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e
 - d*g*e + f*e^2))^2*c*f^3*e^6 + c*f^4*e^8))/(c^3*d^6*g^6*e^4 - 6*(sqrt(x*e
 + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^3*d^4*g^4*
e^2 + 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2
))^6*c^3*d^3*g^3*e - 3*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e
- d*g*e + f*e^2))^8*c^3*d^2*g^2 + 2*c^3*d^5*f*g^5*e^5 - 24*(sqrt(x*e + d)*s
qrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^3*d^3*f*g^3*e^3 +
 24*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6
*c^3*d^2*f*g^2*e^2 - 2*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e
- d*g*e + f*e^2))^8*c^3*d*f*g*e - 17*c^3*d^4*f^2*g^4*e^6 - 8*a*c^2*d^4*g^6*
e^6 - 68*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^
2))^4*c^3*d^2*f^2*g^2*e^4 - 96*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e +
 d)*g*e - d*g*e + f*e^2))^4*a*c^2*d^2*g^4*e^4 + 24*(sqrt(x*e + d)*sqrt(g)*e
^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^3*d*f^2*g*e^3 + 32*(sqrt(
x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c^2*d*g
^3*e^3 - 3*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*
e^2))^8*c^3*f^2*e^2 - 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e
 - d*g*e + f*e^2))^8*a*c^2*g^2*e^2 + 28*c^3*d^3*f^3*g^3*e^7 + 32*a*c^2*d^3*
f*g^5*e^7 - 24*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
+ f*e^2))^4*c^3*d*f^3*g*e^5 - 64*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e
 + d)*g*e - d*g*e + f*e^2))^4*a*c^2*d*f*g^3*e^5 + 8*(sqrt(x*e + d)*sqrt(g)*
e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^3*f^3*e^4 + 32*(sqrt(x*e
 + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c^2*f*g^2*
e^4 - 17*c^3*d^2*f^4*g^2*e^8 - 48*a*c^2*d^2*f^2*g^4*e^8 - 6*(sqrt(x*e + d)*
sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^3*f^4*e^6 - 96*(
sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c^
2*f^2*g^2*e^6 - 128*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d
*g*e + f*e^2))^4*a^2*c*g^4*e^6 + 2*c^3*d*f^5*g*e^9 + 32*a*c^2*d*f^3*g^3*e^9
 + c^3*f^6*e^10 - 8*a*c^2*f^4*g^2*e^10) + sqrt((x*e + d)*g*e - d*g*e + f*e^
2)*sqrt(x*e + d)/c - 1/2*(3*d*g^(3/2)*e^(1/2) + f*sqrt(g)*e^(3/2))*log((sqr
t(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2)/(c*g)

3.606 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal. Leaf size=342

$$\frac{\left(-\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

```
[Out] (2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])]
)/c + ((c*d*f - a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x]
)])/(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])
- ((c*d*f - a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]*f +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rubi [A] time = 2.08574, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {906, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(-\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])]
)/c + ((c*d*f - a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x]
)])/(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])
- ((c*d*f - a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]*f +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 906

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(e*g)/c, Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1), x
], x] + Dist[1/c, Int[(Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x)
^(m - 1)*(f + g*x)^(n - 1))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0
] && GtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{c} + \frac{(eg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{-a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c} + \frac{(2g)\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+\cdots}\right.}{c}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}c} \cdots$$

$$= \frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\cdots\right.}{\sqrt{-a}c}$$

$$= \frac{2\sqrt{e}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} + \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{(cdf\cdots}{\cdots}$$

Mathematica [A] time = 1.34225, size = 381, normalized size = 1.11

$$\frac{-\frac{\left(a\sqrt{c}(dg+ef)-\sqrt{-a}cdf+\sqrt{-a}aeg\right)\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{a\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{-\sqrt{-a}g-\sqrt{c}f}} + \frac{\left(a\sqrt{c}(dg+ef)+\sqrt{-a}cdf+(-a)^{3/2}eg\right)\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{a\sqrt{\sqrt{-a}e-\sqrt{c}d}\sqrt{\sqrt{c}f-\sqrt{-a}g}} + \frac{2\sqrt{g}\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\cdots}{\sqrt{f+gx}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] ((2*Sqrt[g]*Sqrt[e*f - d*g]*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g
]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/Sqrt[f + g*x] - ((-(Sqrt[-a]*c*d*f) + Sq
rt[-a]*a*e*g + a*Sqrt[c]*(e*f + d*g))*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(a*Sqrt[Sq
rt[c]*d + Sqrt[-a]*e]*Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]) + ((Sqrt[-a]*c*d*f +
 (-a)^(3/2)*e*g + a*Sqrt[c]*(e*f + d*g))*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(a*Sqrt
[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]))/c
```

Maple [B] time = 0.378, size = 1569, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+2*(e*g*x^2+d*g*x+e*f*x+d
*f)^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f-a*e*g+c*d*f)/c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f
)/c)^(1/2)*(-a*c)^(1/2)*e*g-(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f
)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-
a*c)^(1/2)*d*g-(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)
/c)^(1/2)*ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*c-(-a
*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*e*
f-(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln(
(-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+a*e*g-c*d*f)/c)^(1/2)*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*c-(-a*c)^(1/2)*d*g
-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*e*g+(e*g)^(1/2)*(((-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*x*e*g+x
*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*
f)/(c*x+(-a*c)^(1/2)))*c*d*f-(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x
*c*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(
-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-
a*c)^(1/2)*d*g-(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(e*g*
x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)
/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*
(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*e*f
+(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(e*g*x^2+d*g*x+e*f*
x+d*f)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*a*e*g-(e*g)^(1/2)*ln((2*(-a*c)
^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)*(((-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*
e*g-c*d*f)/c)^(1/2)*c*d*f)/(e*g*x^2+d*g*x+e*f*x+d*f)^(1/2)/(-a*c)^(1/2)/c/(
e*g)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)/(-((-a
```

```
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{gx+f}}{cx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/(c*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/(a + c*x**2), x)
```

Giac [B] time = 36.9141, size = 4077, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(sqrt(g)*e^(3/2)*log((sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e -
 d*g*e + f*e^2))^2)/c - 2*(c^2*d^6*g^(13/2)*e^(11/2) - 4*(sqrt(x*e + d)*sqr
t(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d^4*g^(9/2)*e^(7/
2) + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2)
)^6*c^2*d^3*g^(7/2)*e^(5/2) - (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e +
d)*g*e - d*g*e + f*e^2))^8*c^2*d^2*g^(5/2)*e^(3/2) - 4*(sqrt(x*e + d)*sqrt(
g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d^3*f*g^(7/2)*e^(9/
```

2) + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^2*d^2*f*g^(5/2)*e^(7/2) - 9*c^2*d^4*f^2*g^(9/2)*e^(15/2) - 2*a*c*d^4*g^(13/2)*e^(15/2) - 16*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d^2*f^2*g^(5/2)*e^(11/2) - 28*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c*d^2*g^(9/2)*e^(11/2) + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^2*d*f^2*g^(3/2)*e^(9/2) + 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c*d*g^(7/2)*e^(9/2) - (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c^2*f^2*sqrt(g)*e^(7/2) - 2*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*a*c*g^(5/2)*e^(7/2) + 16*c^2*d^3*f^3*g^(7/2)*e^(17/2) + 8*a*c*d^3*f*g^(11/2)*e^(17/2) - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d*f^3*g^(3/2)*e^(13/2) - 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c*d*f*g^(7/2)*e^(13/2) + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^2*f^3*sqrt(g)*e^(11/2) + 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c*f*g^(5/2)*e^(11/2) - 9*c^2*d^2*f^4*g^(5/2)*e^(19/2) - 12*a*c*d^2*f^2*g^(9/2)*e^(19/2) - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*f^4*sqrt(g)*e^(15/2) - 28*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c*f^2*g^(5/2)*e^(15/2) - 32*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a^2*g^(9/2)*e^(15/2) + 8*a*c*d*f^3*g^(7/2)*e^(21/2) + c^2*f^6*sqrt(g)*e^(23/2) - 2*a*c*f^4*g^(5/2)*e^(23/2))*log(abs(c^2*d^4*g^4*e^4 - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c^2*d^3*g^3*e^3 + 6*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d^2*g^2*e^2 - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^2*d*g*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c^2 - 4*c^2*d^3*f*g^3*e^5 + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c^2*d^2*f*g^2*e^4 + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*d*f*g*e^3 - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^2*f*e^2 + 6*c^2*d^2*f^2*g^2*e^6 + 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c^2*d*f^2*g*e^5 + 6*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^2*f^2*e^4 + 16*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c*g^2*e^4 - 4*c^2*d*f^3*g*e^7 - 4*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c^2*f^3*e^6 + c^2*f^4*e^8))/(c^3*d^6*g^6*e^4 - 6*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^3*d^4*g^4*e^2 + 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^3*d^3*g^3*e - 3*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c^3*d^2*g^2 + 2*c^3*d^5*f*g^5*e^5 - 24*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^3*d^3*f*g^3*e^3 + 24*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^3*d^2*f*g^2*e^2 - 2*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c^3*d*f*g*e - 17*c^3*d^4*f^2*g^4*e^6 - 8*a*c^2*d^4*g^6*e^6 - 68*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^3*d^2*f^2*g^2*e^4 - 96*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c^2*d^2*g^4*e^4 + 24*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^3*d*f^2*g*e^3 + 32*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c^2*d*g^3*e^3 - 3*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c^3*f^2*e^2 - 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*a*c^2*g^2*e^2 + 28*c^3*d^3*f^3*g^3*e^7 + 32*a*c^2*d^3*f*g^5*e^7 - 24*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c^3*d*f^3*g*e^5 - 64*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*c^2*d*f*g^3*e^5 + 8*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c^3*f^3*e^4 + 32*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*c^2*f*g^2*e^4 - 17*c^3*d^2*f^4*g^2*e^8 - 48*a*c^2*d^2

```
*f^2*g^4*e^8 - 6*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*
e + f*e^2))^4*c^3*f^4*e^6 - 96*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e +
 d)*g*e - d*g*e + f*e^2))^4*a*c^2*f^2*g^2*e^6 - 128*(sqrt(x*e + d)*sqrt(g)*
e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a^2*c*g^4*e^6 + 2*c^3*d*f^
5*g*e^9 + 32*a*c^2*d*f^3*g^3*e^9 + c^3*f^6*e^10 - 8*a*c^2*f^4*g^2*e^10))*e^
(-1)
```

3.607 $\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}(a+cx^2)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\sqrt{\sqrt{-a}g+\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}$$

```
[Out] (Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*d - Sqrt[-a]*e]) - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Rubi [A] time = 0.340663, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {910, 93, 208}

$$\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\sqrt{\sqrt{-a}g+\sqrt{c}f}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*d - Sqrt[-a]*e]) - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}f-\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}f+\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$= \frac{1}{2}\left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx+\frac{1}{2}\left(\frac{af}{(-a)^{3/2}}+\frac{g}{\sqrt{c}}\right)\int \frac{}{\left(\sqrt{-a}+\sqrt{c}x\right)}$$

$$= \left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+\left(\frac{af}{(-a)^{3/2}}+\frac{g}{\sqrt{c}}\right.$$

$$= \frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{-a}e}}$$

Mathematica [A] time = 0.330259, size = 233, normalized size = 0.97

$$\frac{\frac{\sqrt{-\sqrt{-a}g-\sqrt{c}f}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e+\sqrt{c}d}}-\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e-\sqrt{c}d}}}{\sqrt{-a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] ((Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*S
qrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[Sqrt[c]*d
 + Sqrt[-a]*e] - (Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTan[(Sqrt[Sqrt[c]*f - Sqr
t[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/S
qrt[-(Sqrt[c]*d) + Sqrt[-a]*e])/(Sqrt[-a]*Sqrt[c])
```

Maple [B] time = 0.367, size = 1383, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x)
```

```
[Out] -1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*(ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+
2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/
c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a
*c*e^2*f*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(
-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e^2*g*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)+ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*
e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d
*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)
))*c^2*d^2*f*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln(
(2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d^2*g*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
```

```
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)-ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g
+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x
+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)
^(1/2)))*a*c*e^2*f*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2
)+ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-
a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*e^2*g*(-a*c)^(1/2)*(((-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)-ln((-2*(-a*c)^(1/2)*x*e*g+x
*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+
(-a*c)^(1/2)))*c^2*d^2*f*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c
)^(1/2)+ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a
*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*
d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*c*d^2*g*(-a*c)^(1/2)*(((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2))/((g*x+f)*(e*x+d))^(1
/2)/((-a*c)^(1/2)*e+c*d)/(-a*c)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a
*e*g+c*d*f)/c)^(1/2)/(-(-a*c)^(1/2)*e+c*d)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f+a*e*g-c*d*f)/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{(cx^2+a)\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*sqrt(e*x + d)), x)
```

Fricas [B] time = 79.1477, size = 3802, normalized size = 15.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*
f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a
^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e +
a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2
*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c
^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^
2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^
2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a
*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c
*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*
f - c*d^2*g - (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^
2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f
)*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*
```

```
c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*
e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d
^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x) - 1/4*sqrt(-(c*d*f
 + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a
*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(
e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g + (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-
(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^
2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*(c^2*d^
3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*s
qrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c
*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*
f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a
*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*f - c*d^2*g + (a*
c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2
*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f +
a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^
3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*
f*g - d*e*g^2)*x - (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f +
(c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^
4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+a),x)
```

```
[Out] Integral(sqrt(f + g*x)/((a + c*x**2)*sqrt(d + e*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.608 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=351

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(ae^2+cd^2\right)\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\cdots}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(ae^2+\cdots\right.}$$

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + ((c*d*f + a*e*g + Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f +
a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rubi [A] time = 2.15335, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {908, 37, 6725, 93, 208}

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(ae^2+cd^2\right)\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\cdots}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(ae^2+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + ((c*d*f + a*e*g + Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f +
a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{\int \left(\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\left(cd^2+ae^2\right)} - \frac{\left(cdf+\right.}{}$$

$$= -\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,\right.}{\sqrt{-a}\left(cd^2+ae^2\right)}$$

$$= -\frac{2e\sqrt{f+gx}}{\left(cd^2+ae^2\right)\sqrt{d+ex}} + \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cdf+\right.}{}$$

Mathematica [A] time = 0.67132, size = 267, normalized size = 0.76

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)} + \frac{\sqrt{-\sqrt{-a}g-\sqrt{c}f}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}} + \frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e-\sqrt{c}d\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (Sqrt[-(Sqrt[c]*f) -
 Sqrt[-a]*g]*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[S
qrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^
(3/2)) + (Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]
*(-(Sqrt[c]*d) + Sqrt[-a]*e)^(3/2))
```

Maple [B] time = 0.441, size = 5383, normalized size = 15.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.609 $\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=613

$$\frac{e\sqrt{f+gx}\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)(ef-dg)}-\frac{e\sqrt{f+gx}\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)(ef-dg)}+\frac{4eg}{3\sqrt{d+ex}\left(ae^2\right.}$$

```
[Out] (-2*e*Sqrt[f + g*x])/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (4*e*g*Sqrt[f +
g*x])/(3*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (e*(c*d*f + a*e*g - S
qrt[-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]
*e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) - (e*(c*d*f + a*e*g + Sqrt[-
a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[c]*(c*d*f + a*e*g + Sqrt[
-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x
])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d - Sq
rt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) + (Sqrt[c]*(S
qrt[-a]*c*d*f + Sqrt[-a]*a*e*g + a*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[
c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*
x])])/(a*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sq
rt[-a]*g])
```

Rubi [A] time = 3.1602, antiderivative size = 613, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {908, 45, 37, 6725, 96, 93, 208}

$$\frac{e\sqrt{f+gx}\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)(ef-dg)}-\frac{e\sqrt{f+gx}\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(ae^2+cd^2\right)(ef-dg)}+\frac{4eg}{3\sqrt{d+ex}\left(ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (4*e*g*Sqrt[f +
g*x])/(3*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (e*(c*d*f + a*e*g - S
qrt[-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]
*e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) - (e*(c*d*f + a*e*g + Sqrt[-
a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[c]*(c*d*f + a*e*g + Sqrt[
-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x
])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d - Sq
rt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) + (Sqrt[c]*(S
qrt[-a]*c*d*f + Sqrt[-a]*a*e*g + a*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[
c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*
x])])/(a*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sq
rt[-a]*g])
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
```

```
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}dx = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+cx^2\right)}dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{\int \left(\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\right)dx}{cd^2+ae^2} -$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-\right.}{2\sqrt{-}}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{-a}\sqrt{c}\right.}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+\right.}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{-a}\sqrt{c}\right.}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+\right.}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+aeg-\sqrt{-a}\sqrt{c}\right.}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+\right.}$$

Mathematica [A] time = 4.22092, size = 355, normalized size = 0.58

$$-\frac{2e\sqrt{f+gx}\left(ae^3(f+gx)+cd\left(-6d^2g+7def-5degx+6e^2fx\right)\right)}{3(d+ex)^{3/2}\left(ae^2+cd^2\right)^2(ef-dg)} + \frac{\sqrt{c}\sqrt{-\sqrt{-a}g-\sqrt{c}f}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\left(\sqrt{-a}\sqrt{c}d-ae\right)} + -$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x]*(a*e^3*(f + g*x) + c*d*(7*d*e*f - 6*d^2*g + 6*e^2*f*x -
 5*d*e*g*x)))/(3*(c*d^2 + a*e^2)^2*(e*f - d*g)*(d + e*x)^(3/2)) + (Sqrt[c]*
Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*Sqr
t[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/((Sqrt[c]*d + Sq
rt[-a]*e)^(3/2)*(Sqrt[-a]*Sqrt[c]*d - a*e)) + (Sqrt[c]*Sqrt[Sqrt[c]*f - Sqr
t[-a]*g]*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c
]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/((-(Sqrt[c]*d) + Sqrt[-a]*e)^(3/2)*(Sqr
t[-a]*Sqrt[c]*d + a*e))
```

Maple [B] time = 0.458, size = 14861, normalized size = 24.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\left(cx^2+a\right)(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.610 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=337

$$\frac{\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}+\frac{2e^{3/2}\,\mathrm{ta}}{}$$

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) + ((c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f -
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((
c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*
c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rubi [A] time = 2.46279, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {910, 63, 217, 206, 6725, 93, 208}

$$\frac{\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}+\frac{2e^{3/2}\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) + ((c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f -
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((
c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*
c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(a+cx^2\right)}\,dx = \int \left(\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}} + \frac{cd^2-ae^2+2cdex}{c\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{cd^2-ae^2+2cdex}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{c} + \frac{e^2\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{-2a\sqrt{c}de+\sqrt{-a}\left(cd^2-ae^2\right)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{2a\sqrt{c}de+\sqrt{-a}\left(cd^2-ae^2\right)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c} + \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d \cdots}\right.}{c}$$

$$= \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} - \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}c} - \frac{\left(cd \cdots\right.}{\cdots}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\mathrm{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx, x, \cdots\right.}{\sqrt{-a}c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cd^2+2\sqrt{-a}\cdots\right.}{\sqrt{-a}\cdots}$$

Mathematica [A] time = 1.03612, size = 343, normalized size = 1.02

$$\frac{\frac{\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(\sqrt{-a}\sqrt{c}d-ae\right)\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-\sqrt{-a}g-\sqrt{c}f}}-\frac{\sqrt{\sqrt{-a}e-\sqrt{c}d}\left(\sqrt{-a}\sqrt{c}d+ae\right)\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}f-\sqrt{-a}g}}}{a}+\frac{2(ef-dg)^{3/2}\left(\frac{e(f+gx)}{ef-dg}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{\sqrt{g}(f+gx)^{3/2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((2*(e*f - d*g)^(3/2)*((e*(f + g*x))/(e*f - d*g))^(3/2)*ArcSinh[(Sqrt[g]*Sq
rt[d + e*x])/Sqrt[e*f - d*g]])/(Sqrt[g]*(f + g*x)^(3/2)) + ((Sqrt[Sqrt[c]*d
 + Sqrt[-a]*e]*(Sqrt[-a]*Sqrt[c]*d - a*e)*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[
-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[-
(Sqrt[c]*f) - Sqrt[-a]*g] - (Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*(Sqrt[-a]*Sqrt
[c]*d + a*e)*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sq
rt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[Sqrt[c]*f - Sqrt[-a]*g])/a)/c
```

Maple [B] time = 0.385, size = 2336, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2
*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c
)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a^
2*e^2*g^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^
(1/2)-ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*((
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(1/2)*(-((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*x*
e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(
c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f+a*e*g-c*d*f)/c)^(1/2)-2*ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g
*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1
/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*d*e*
g^2*(e*g)^(1/2)*(-a*c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d
*f)/c)^(1/2)-ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(
1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d^2*f^2*(e*g)^(1/2
)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)-2*ln((2*(-a*c)
^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2
*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*e*f^2*(e*g)^(1/2)*(-a*c)^(1/2)*(-((-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)-ln((-2*(-a*c)^(1/2)*x*e*g+x*
c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*
((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(
-a*c)^(1/2)))*a^2*e^2*g^2*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a
*e*g+c*d*f)/c)^(1/2)+ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-
(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*d^2*g^2*
```

```
(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)-ln((-
2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1
/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*(((-a*c)^(1/2)
*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)-2*ln((-2*(-a*c)^(1/2)*x*e*g+x*c
*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(
(g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-
a*c)^(1/2)))*a*d*e*g^2*(e*g)^(1/2)*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)+ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*
(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d)
)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*c^
2*d^2*f^2*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(
1/2)-2*ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d
*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*c*d*e*f^2*(e*g)^(1/2)*(-a*
c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)+2*ln(1/2
*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^2
*g^2*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)
*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+2*ln(1/2*(2*e*g
*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2*(-
a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-((-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2))/((g*x+f)*(e*x+d))^(1/
2)/((-a*c)^(1/2)*g+c*f)/(-a*c)^(1/2)/(e*g)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)/(c*f-(-a*c)^(1/2)*g)/(-((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")

[Out] integrate((e*x + d)^(3/2)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/((a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.611 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}(a+cx^2)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

```
[Out] (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rubi [A] time = 0.333845, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {910, 93, 208}

$$\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}d-\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}d+\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$
$$=\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\int\frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx+\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}+\frac{e}{\sqrt{c}}\right)\int\frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\ldots}$$
$$=\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)x^2}\,dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+\left(\frac{ad}{(-a)^{3/2}}+\ldots\right.$$
$$=\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [A] time = 0.310259, size = 233, normalized size = 0.97

$$\frac{\frac{\sqrt{\sqrt{-a}e-\sqrt{c}d}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-\sqrt{-a}g-\sqrt{c}f}}}{\sqrt{-a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (-((Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*Sq
rt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/Sqrt[-(Sqrt[c]*
f) - Sqrt[-a]*g]) + (Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTan[(Sqrt[Sqrt[c]*f
 - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x
])])/Sqrt[Sqrt[c]*f - Sqrt[-a]*g])/(Sqrt[-a]*Sqrt[c])
```

Maple [B] time = 0.4, size = 1383, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] -1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+
2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/
c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a
*c*d*g^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(
-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e*g^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)+ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*
e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d
*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)
))*c^2*d*f^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln(
(2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*e*f^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
```

```
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)-ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g
+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x
+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)
^(1/2)))*a*c*d*g^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2
)+ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-
a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*e*g^2*(((-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)-ln((-2*(-a*c)^(1/2)*x*e*g+x
*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+
(-a*c)^(1/2)))*c^2*d*f^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c
)^(1/2)+ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a
*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*
d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*c*e*f^2*(((-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2))/((g*x+f)*(e*x+d))^(1
/2)/((-a*c)^(1/2)*g+c*f)/(-a*c)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a
*e*g+c*d*f)/c)^(1/2)/(c*f-(-a*c)^(1/2)*g)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+a*e*g-c*d*f)/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [B] time = 69.8372, size = 3812, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*
f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a
^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g +
a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2
*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c
^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^
2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^
2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^
2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*
c^2*f^2*g^2 + a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2
*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^
2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*
f^2 - c*d*f*g + (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*
g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f
*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^
```

```
2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e
*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g
 + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x) - 1/4*sqrt(-(c
*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2
)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*lo
g(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g - (a*c^2*f^2*g + a^2*c*g^3)*sq
rt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*
g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*
c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2
 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*c^2
*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3
)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 +
 a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(
-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4
)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*f^2 - c*d*f*g
 - (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f
^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*
d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)
/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2
*(e^2*f*g - d*e*g^2)*x + (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*
f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(
a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\left(a+cx^2\right)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/((a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.612 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx$

Optimal. Leaf size=230

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

```
[Out] ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt
[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(S
qrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rubi [A] time = 0.203994, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {912, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt
[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(S
qrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}dx=\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$=-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{2\sqrt{-a}}-\frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{2\sqrt{-a}}$$

$$=-\frac{\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}g\right)x^2}dx,x,\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}d+\sqrt{-a}e-\left(\sqrt{c}f+\sqrt{-a}g\right)\right.}{\sqrt{-a}}$$

$$=\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

Mathematica [A] time = 0.193762, size = 229, normalized size = 1.

$$\frac{-\frac{\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}-\frac{\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e-\sqrt{c}d}\sqrt{\sqrt{c}f-\sqrt{-a}g}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (-(ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d +
 Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[-(Sqrt[c]*f
) - Sqrt[-a]*g])) - ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sq
rt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]
*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]))/Sqrt[-a]
```

Maple [B] time = 0.413, size = 1415, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] -1/2*c^2*(ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2
)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2*(-((-a*c)^(1/
2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*x*e*g+x*c*
d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a
*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*
c)^(1/2)))*a*c*d^2*g^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)
^(1/2)+ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(
((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(-((-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)+ln((2*(-a*c)^(1/2)*x*e*g+x*c*d*g
+x*c*e*f+2*((g*x+f)*(e*x+d))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*
g+c*d*f)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x-(-a*c)^
```

```
(1/2)))*c^2*d^2*f^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1
/2)-ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-
(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a^2*e^2*g^2*(((-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)-ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+
x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+
f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^
(1/2)))*a*c*d^2*g^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/
2)-ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(
-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(((-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)-ln((-2*(-a*c)^(1/2)*x*e*g+x*c*d*g+x
*c*e*f+2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*((g*x+f
)*(e*x+d))^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*c*d*f)/(c*x+(-a*c)^(
1/2)))*c^2*d^2*f^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2
))*(g*x+f)^(1/2)*(e*x+d)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c
*d*f)/c)^(1/2)/(c*f-(-a*c)^(1/2)*g)/(-(-a*c)^(1/2)*e+c*d)/(((-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)/(-a*c)^(1/2)/((-a*c)^(1/2)*g+c*f)/
((-a*c)^(1/2)*e+c*d)/((g*x+f)*(e*x+d))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+a)\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Fricas [B] time = 141.759, size = 8408, normalized size = 36.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3
*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*
c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c
*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^
2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g +
 d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e +
 a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)
*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^
2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g + ((a*c^2
*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d
*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*
(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a
^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*
```

```
(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d
*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*s
qrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2
+ a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^
2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x) + 1/4*sqrt(-(c*d*f
- a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(
c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*
c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2
 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*
e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e + a^2*c*e^3)*f^3 +
(a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^
3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4
 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^
2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*s
qrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*
f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2
)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a
^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*
(e^2*f*g + d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e
^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c
*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 +
2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4
 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 +
 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x) - 1/4*sqrt(-(c*d*f - a*e*g - ((a*c^2*
d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*
e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(
a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4
*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^
3*e^2)*g^2))*log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 + 2*(c*d*e*f^2 - a*d*e*g^2
+ (c*d^2 - a*e^2)*f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c
*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3
)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e
^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2
*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 +
 a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*
a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a
^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^
2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2
)*x - (2*(c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*
d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*
f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^
2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*g^4)))/x) + 1/4*sqrt(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f
^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)
/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^
3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e
^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2))*log((
e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*
f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a
^2*c*d^2*e + a^3*e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*sqrt(-(c*e^2*f^2
 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*
f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*
d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)*sqr
t(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 +
```

```
a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2
+ (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/((a*c^2*d^2 + a^2*c*e^2)
*f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x - (2*(c^2*d^3
+ a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + a*c*e^3)*f
^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3)*f*g^2 + (a*c*d^3 +
 a^2*d*e^2)*g^3)*x)*sqrt(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)/((a*c^4*d^4
 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^
2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)))/x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.613 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=354

$$-\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$

```
[Out] -((e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d +
 e*x])) + (e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*
Sqrt[d + e*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*
x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d - S
qrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqrt[Sqr
t[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*
g])
```

Rubi [A] time = 0.613599, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {912, 96, 93, 208}

$$-\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] -((e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d +
 e*x])) + (e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*
Sqrt[d + e*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*
x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*(Sqrt[c]*d - S
qrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqrt[Sqr
t[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*
g])
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\right) dx$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} - \cdots$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} - \cdots$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}} + \cdots$$

Mathematica [A] time = 0.86199, size = 291, normalized size = 0.82

$$\frac{\frac{2\sqrt{-a}e^2\sqrt{f+gx}}{\sqrt{d+ex}\left(ae^2+cd^2\right)(dg-ef)} - \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\sqrt{-\sqrt{-a}g-\sqrt{c}f}} + \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\left(\sqrt{-a}e-\sqrt{c}d\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((2*Sqrt[-a]*e^2*Sqrt[f + g*x])/((c*d^2 + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*
x]) - (Sqrt[c]*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt
[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/((Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*S
qrt[-(Sqrt[c]*f) - Sqrt[-a]*g]) + (Sqrt[c]*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a
]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/((-(S
qrt[c]*d) + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]))/Sqrt[-a]
```

Maple [B] time = 0.451, size = 10977, normalized size = 31.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.614 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=625

$$\frac{2\sqrt{d+ex}(ef-dg)}{\sqrt{f+gx}\left(ag^2+cf^2\right)}-\frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\left(ag^2+cf^2\right)}-\frac{\sqrt{e}\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\left(ag^2+cf^2\right)}+\frac{\sqrt{e}(\cdots}{\cdots}$$

```
[Out] (2*(e*f - d*g)*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[e]*
(e*f - d*g)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt
[g]*(c*f^2 + a*g^2)) - (Sqrt[e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*
g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqr
t[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*
(e*f - d*g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqr
t[-a]*Sqrt[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d
*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-
a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-
a]*Sqrt[c]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - (Sqrt[Sqrt[c]*d
+ Sqrt[-a]*e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[
Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
 + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rubi [A] time = 2.53051, antiderivative size = 625, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {908, 47, 63, 217, 206, 6725, 105, 93, 208}

$$\frac{2\sqrt{d+ex}(ef-dg)}{\sqrt{f+gx}\left(ag^2+cf^2\right)}-\frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\left(ag^2+cf^2\right)}-\frac{\sqrt{e}\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}\left(ag^2+cf^2\right)}+\frac{\sqrt{e}(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[e]*
(e*f - d*g)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt
[g]*(c*f^2 + a*g^2)) - (Sqrt[e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*
g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqr
t[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*
(e*f - d*g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqr
t[-a]*Sqrt[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d
*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-
a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-
a]*Sqrt[c]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - (Sqrt[Sqrt[c]*d
+ Sqrt[-a]*e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[
Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
 + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
```

```
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}}\,dx}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\int \left(\frac{\left(-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)\right)\sqrt{d+ex}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{f+gx}} + \frac{\left(a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)\right)\sqrt{d+ex}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{f+gx}}\right) d}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(2(ef-dg))\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+ex}\right)}{cf^2+ag^2} - \frac{\left(cdf+aeg-\sqrt{\right.}{}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(2(ef-dg))\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{cf^2+ag^2} - \frac{\left(e\left(cdf+aeg-\sqrt{-a}\sqrt{\right.\right.}{2\sqrt{-a}\sqrt{}}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)S}{\sqrt{-a}\sqrt{c}}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} + \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(cdf+aeg-\sqrt{-}\right.}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}}}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\left(cf^2+ag^2\right)} - \frac{\sqrt{e}\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg\right.}{\sqrt{-a}\sqrt{c}\sqrt{g}\left(cf^2\right.}$$

Mathematica [A] time = 0.891688, size = 340, normalized size = 0.54

$$-\left(\frac{ad}{(-a)^{3/2}} - \frac{e}{\sqrt{c}}\right)\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)} + \frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\left(-\sqrt{-a}g-\sqrt{c}f\right)^{3/2}}\right) - \left(\frac{d}{\sqrt{-a}} - \frac{e}{\sqrt{c}}\right)\left(\frac{\sqrt{d+e}}{\sqrt{f+gx}\left(\sqrt{c}f\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -(((a*d)/(-a)^(3/2) - e/Sqrt[c])*(Sqrt[d + e*x]/((Sqrt[c]*f + Sqrt[-a]*g)*S
qrt[f + g*x]) + (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTan[(Sqrt[-(Sqrt[c]*f) - S
qrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(-
(Sqrt[c]*f) - Sqrt[-a]*g)^(3/2))) - (d/Sqrt[-a] - e/Sqrt[c])*(Sqrt[d + e*x]
/((Sqrt[c]*f - Sqrt[-a]*g)*Sqrt[f + g*x]) - (Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e
]*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) +
Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[c]*f - Sqrt[-a]*g)^(3/2))
```

Maple [B] time = 0.508, size = 8264, normalized size = 13.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.615 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=351

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)}+\frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\left(ag^2+cf^2\right)}-\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\mathrm{ta}}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g}}$$

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + ((c*d*f + a*e*g - Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - ((c*d*f +
a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rubi [A] time = 1.80615, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {908, 37, 6725, 93, 208}

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)}+\frac{\left(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\left(ag^2+cf^2\right)}-\frac{\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\mathrm{ta}}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + ((c*d*f + a*e*g - Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - ((c*d*f +
a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 908

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> -Dist[(g*(e*f - d*g))/(c*f^2 + a*g^2), Int[(d + e*x)^(m
- 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[(Simp[c*d*f + a*e*g
+ c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n + 1))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[
m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}\left(a+cx^2\right)}\,dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\int \left(\frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2\sqrt{-a}\left(cf^2+ag^2\right)} - \frac{(c}{}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{1}{-\sqrt{c}d+\sqrt{-a}e-\left(-\sqrt{c}f+\sqrt{-a}\right.}\right.}{\sqrt{-a}\left(cf^2+ag^2\right)}$$

$$= -\frac{2g\sqrt{d+ex}}{\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\left(cf^2+ag^2\right)} - \frac{(c}{}$$

Mathematica [A] time = 0.596949, size = 267, normalized size = 0.76

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)} + \frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(-\sqrt{-a}g-\sqrt{c}f\right)^{3/2}} + \frac{\sqrt{\sqrt{-a}e-\sqrt{c}d}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (Sqrt[Sqrt[c]*d + Sq
rt[-a]*e]*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt
[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(-(Sqrt[c]*f) - Sqrt[-a]*g)^
(3/2)) + (Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]
*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[
-a]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2))
```

Maple [B] time = 0.426, size = 5383, normalized size = 15.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+a\right)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.616 $\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=354

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}$$

```
[Out] (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f + g
*x]) - (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)*Sqr
t[f + g*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])
/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d -
Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c
]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x
])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rubi [A] time = 0.762631, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {912, 96, 93, 208}

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f + g
*x]) - (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)*Sqr
t[f + g*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])
/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d -
Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c
]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x
])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\left(\sqrt{c}f+\sqrt{-a}g\right)(ef-dg)\sqrt{f+gx}}$$

Mathematica [A] time = 0.843464, size = 291, normalized size = 0.82

$$\frac{\frac{2\sqrt{-a}g^2\sqrt{d+ex}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)} + \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(-\sqrt{-a}g-\sqrt{c}f\right)^{3/2}} - \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e-\sqrt{c}d}}\right)}{\sqrt{\sqrt{-a}e-\sqrt{c}d}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] ((2*Sqrt[-a]*g^2*Sqrt[d + e*x])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])
 + (Sqrt[c]*ArcTan[(Sqrt[-(Sqrt[c]*f) - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sq
rt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(-(Sqr
t[c]*f) - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTan[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g
]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-(
Sqrt[c]*d) + Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)))/Sqrt[-a]
```

Maple [B] time = 0.572, size = 10977, normalized size = 31.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+a)\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*(g*x + f)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.617 $\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx$

Optimal. Leaf size=549

$$-\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{g\sqrt{d+e}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}\right.}$$

```
[Out] -(e/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])) + e/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt
[f + g*x]) + (g*(2*Sqrt[-a]*e*g - Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(Sqrt
[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f
 + g*x]) + (g*(2*Sqrt[-a]*e*g + Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(Sqrt[-
a]*(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f +
 g*x]) + (c*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt
[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/
2)*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (c*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*
(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rubi [A] time = 1.32362, antiderivative size = 543, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {912, 104, 152, 12, 93, 208}

$$-\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{g\sqrt{d+ex}\,(}{a\sqrt{f+gx}\left(\sqrt{-a}e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -(e/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])) + e/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt
[f + g*x]) + (g*(2*a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*
(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*
x]) + (g*(2*a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*(Sqrt[c
]*d - Sqrt[-a]*e)*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*x]) + (
c*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sq
rt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2)*(Sqrt[c
]*f - Sqrt[-a]*g)^(3/2)) - (c*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d
 + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}} + \frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}(f+\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}}dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}}dx}{2\sqrt{-a}}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-d\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-d\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-d\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-d\right.}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e}{\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(ef-d\right.}$$

Mathematica [A] time = 3.24467, size = 525, normalized size = 0.96

$$\frac{\frac{e}{\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e-\sqrt{c}d\right)} + \frac{e}{\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)} + \frac{g\sqrt{d+ex}\left(2\sqrt{-a}eg+\sqrt{c}(dg+ef)\right)}{\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)} + \frac{c(ef-dg)\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-\sqrt{-a}g-\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\left(-\sqrt{-a}g-\sqrt{c}f\right)^{3/2}} + \frac{g\sqrt{d+ex}}{\sqrt{f+g}}}{\sqrt{-a}(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (e/((-(Sqrt[c]*d) + Sqrt[-a]*e)*Sqrt[d + e*x]*Sqrt[f + g*x]) + e/((Sqrt[c]*
d + Sqrt[-a]*e)*Sqrt[d + e*x]*Sqrt[f + g*x]) + (g*(2*Sqrt[-a]*e*g + Sqrt[c]
*(e*f + d*g))*Sqrt[d + e*x])/((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a
]*g)*(e*f - d*g)*Sqrt[f + g*x]) + (c*(e*f - d*g)*ArcTan[(Sqrt[-(Sqrt[c]*f)
- Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])
/((Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(-(Sqrt[c]*f) - Sqrt[-a]*g)^(3/2)) + ((g*(
2*Sqrt[-a]*e*g - Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/((Sqrt[c]*f - Sqrt[-a]
*g)*(e*f - d*g)*Sqrt[f + g*x]) + (c*(-(e*f) + d*g)*ArcTan[(Sqrt[Sqrt[c]*f -
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])
])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)))/(Sqrt[
c]*d - Sqrt[-a]*e))/(Sqrt[-a]*(e*f - d*g))
```

Maple [B] time = 0.652, size = 30648, normalized size = 55.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + a\right)(ex + d)^{\frac{3}{2}}\left(gx + f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^2\right)\left(d + ex\right)^{\frac{3}{2}}\left(f + gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*(f + g*x)**(3/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.618 $\int \frac{\sqrt{x}}{\sqrt{1+x}(1+x^2)}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{x+1}}\right)$$

```
[Out] -((1 - I)^(3/2)*ArcTanh[(Sqrt[1 - I]*Sqrt[x])/Sqrt[1 + x]])/2 - ((1 + I)^(3
/2)*ArcTanh[(Sqrt[1 + I]*Sqrt[x])/Sqrt[1 + x]])/2
```

Rubi [A] time = 0.048894, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {910, 93, 208}

$$-\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -((1 - I)^(3/2)*ArcTanh[(Sqrt[1 - I]*Sqrt[x])/Sqrt[1 + x]])/2 - ((1 + I)^(3
/2)*ArcTanh[(Sqrt[1 + I]*Sqrt[x])/Sqrt[1 + x]])/2
```

Rule 910

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\left(1+x^2\right)} dx = \int \left(-\frac{1}{2(i-x)\sqrt{x}\sqrt{1+x}} + \frac{1}{2\sqrt{x}(i+x)\sqrt{1+x}}\right) dx$$
$$= -\left(\frac{1}{2}\int \frac{1}{(i-x)\sqrt{x}\sqrt{1+x}} dx\right) + \frac{1}{2}\int \frac{1}{\sqrt{x}(i+x)\sqrt{1+x}} dx$$
$$= -\text{Subst}\left(\int \frac{1}{i-(1+i)x^2} dx, x, \frac{\sqrt{x}}{\sqrt{1+x}}\right) + \text{Subst}\left(\int \frac{1}{i+(1-i)x^2} dx, x, \frac{\sqrt{x}}{\sqrt{1+x}}\right)$$
$$= -\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{1+x}}\right) - \frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{1+x}}\right)$$

Mathematica [A] time = 0.0614122, size = 63, normalized size = 0.97

$$\frac{1}{2}\left(-(-1+i)^{3/2}\tan^{-1}\left(\sqrt{-1+i}\sqrt{\frac{x}{x+1}}\right) - (1+i)^{3/2}\tanh^{-1}\left(\sqrt{1+i}\sqrt{\frac{x}{x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] (-((-1 + I)^(3/2)*ArcTan[Sqrt[-1 + I]*Sqrt[x/(1 + x)]]) - (1 + I)^(3/2)*Arc
Tanh[Sqrt[1 + I]*Sqrt[x/(1 + x)]])/2
```

Maple [B] time = 0.231, size = 305, normalized size = 4.7

$$\frac{\left(\sqrt{2}-1+x\right)\sqrt{2}}{\left(12\sqrt{2}-16\right)\sqrt{1+\sqrt{2}}}\sqrt{\frac{x(1+x)}{\left(\sqrt{2}-1+x\right)^2}}\left(\sqrt{-2+2\sqrt{2}}\arctan\left(\frac{\sqrt{-2+2\sqrt{2}}\left(3+2\sqrt{2}\right)\left(3\sqrt{2}-4\right)\left(\sqrt{2}+1-x\right)\left(\sqrt{2}-1\right.}{(4+4x)x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)/(1+x)^(1/2),x)
```

```
[Out] 1/4/x^(1/2)/(1+x)^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2)^(1/2)*(2^(1/2)-1+x)*((-2+
2*2^(1/2))^(1/2)*arctan(1/4*((3*2^(1/2)-4)*x*(1+x)*(4+3*2^(1/2))/(2^(1/2)-1
+x)^2)^(1/2)*(-2+2*2^(1/2))^(1/2)*(3+2*2^(1/2))*(3*2^(1/2)-4)*(2^(1/2)+1-x)
*(2^(1/2)-1+x)/(1+x)/x)*(1+2^(1/2))^(1/2)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*ar
ctan(1/4*((3*2^(1/2)-4)*x*(1+x)*(4+3*2^(1/2))/(2^(1/2)-1+x)^2)^(1/2)*(-2+2*
2^(1/2))^(1/2)*(3+2*2^(1/2))*(3*2^(1/2)-4)*(2^(1/2)+1-x)*(2^(1/2)-1+x)/(1+x
)/x)*(1+2^(1/2))^(1/2)+4*arctanh(2^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2)^(1/2)/(1
+2^(1/2))^(1/2))*2^(1/2)-6*arctanh(2^(1/2)*((1+x)*x/(2^(1/2)-1+x)^2)^(1/2)/
(1+2^(1/2))^(1/2)))*2^(1/2)/(3*2^(1/2)-4)/(1+2^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(x^2+1\right)\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/((x^2 + 1)*sqrt(x + 1)), x)
```

Fricas [B] time = 2.76693, size = 2338, normalized size = 35.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-8*sqrt(x + 1)*x^(3/2) +
8*x^2 + 2*(2^(1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x
+ 1) - 2*x))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/8*2^(1/4)*sqrt(
2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-8*sqrt(x + 1)*x^(3/2) + 8*x^2 - 2*(2^(1/4
)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x + 1) - 2*x))*sqrt
(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*ar
ctan(1/7*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*sqrt(x + 1)*sqrt(x) - 1/
7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1/28*sqrt(-8*sqrt(x + 1)*x^(3/2)
+ 8*x^2 - 2*(2^(1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(
x + 1) - 2*x))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(2*sqrt(2)*(5*sqr
t(2) + 6) - (2^(3/4)*(3*sqrt(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(2*sqrt
(2) + 4) + 16*sqrt(2) + 8) - 1/7*sqrt(2)*(8*x + 3) - 1/14*((2^(3/4)*(3*sqrt
(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(x + 1)*sqrt(x) - 2^(3/4)*(sqrt(2)*
(3*x + 2) + 5*x + 1) - 2*2^(1/4)*(sqrt(2)*(x + 3) + 4*x - 2))*sqrt(2*sqrt(2
) + 4) - 4/7*x - 5/7) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*(sqrt(2
)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*sqrt(x + 1)*sqrt(x) + 1/7*sqrt(2)*(sqrt(
2)*(5*x + 1) + 6*x + 4) + 1/28*sqrt(-8*sqrt(x + 1)*x^(3/2) + 8*x^2 + 2*(2^(
1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x + 1) - 2*x))*s
qrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(
3/4)*(3*sqrt(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(2*sqrt(2) + 4) + 16*sq
rt(2) + 8) + 1/7*sqrt(2)*(8*x + 3) - 1/14*((2^(3/4)*(3*sqrt(2) + 5) + 2*2^(
1/4)*(sqrt(2) + 4))*sqrt(x + 1)*sqrt(x) - 2^(3/4)*(sqrt(2)*(3*x + 2) + 5*x
+ 1) - 2*2^(1/4)*(sqrt(2)*(x + 3) + 4*x - 2))*sqrt(2*sqrt(2) + 4) + 4/7*x +
 5/7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/(sqrt(x + 1)*(x**2 + 1)), x)
```

Giac [A] time = 1.50335, size = 1, normalized size = 0.02

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 0
```

3.619 $\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4} dx$

Optimal. Leaf size=80

$$\frac{(x+1)^4(f+g)^2}{5\left(1-x^2\right)^{5/2}}+\frac{(x+1)^3(f-9g)(f+g)}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(x+1)}{\sqrt{1-x^2}}-g^2\sin^{-1}(x)$$

```
[Out] ((f + g)^2*(1 + x)^4)/(5*(1 - x^2)^(5/2)) + ((f - 9*g)*(f + g)*(1 + x)^3)/(
15*(1 - x^2)^(3/2)) + (2*g^2*(1 + x))/Sqrt[1 - x^2] - g^2*ArcSin[x]
```

Rubi [A] time = 0.143246, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {853, 1635, 789, 653, 216}

$$\frac{(x+1)^4(f+g)^2}{5\left(1-x^2\right)^{5/2}}+\frac{(x+1)^3(f-9g)(f+g)}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(x+1)}{\sqrt{1-x^2}}-g^2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] ((f + g)^2*(1 + x)^4)/(5*(1 - x^2)^(5/2)) + ((f - 9*g)*(f + g)*(1 + x)^3)/(
15*(1 - x^2)^(3/2)) + (2*g^2*(1 + x))/Sqrt[1 - x^2] - g^2*ArcSin[x]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
```

```
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx &= \int \frac{(1+x)^4(f+gx)^2}{\left(1-x^2\right)^{7/2}}\,dx\\ &= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}}-\frac{1}{5}\int \frac{(1+x)^3\left(-f^2+8fg+4g^2+5g^2x\right)}{\left(1-x^2\right)^{5/2}}\,dx\\ &= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}}+\frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}}+g^2\int \frac{(1+x)^2}{\left(1-x^2\right)^{3/2}}\,dx\\ &= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}}+\frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(1+x)}{\sqrt{1-x^2}}-g^2\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}}+\frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(1+x)}{\sqrt{1-x^2}}-g^2\sin^{-1}(x)\end{aligned}$$

Mathematica [C] time = 0.151701, size = 91, normalized size = 1.14

$$\frac{\sqrt{1-x^2}\left((x+1)^{3/2}\left(f^2(x-4)+fg(2-8x)+g^2(x-4)\right)-20\sqrt{2}g^2(x-1)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)\right)}{15(x-1)^3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] (Sqrt[1 - x^2]*((f*g*(2 - 8*x) + f^2*(-4 + x) + g^2*(-4 + x))*(1 + x)^(3/2)
 - 20*Sqrt[2]*g^2*(-1 + x)*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - x)/2]))
/(15*(-1 + x)^3*Sqrt[1 + x])
```

Maple [A] time = 0.064, size = 125, normalized size = 1.6

$$g^2\left(\frac{1}{(-1+x)^2}\left(-(-1+x)^2+2-2x\right)^{\frac{3}{2}}+\sqrt{-(-1+x)^2+2-2x}-\arcsin(x)\right)+\frac{2g\left(f+g\right)}{3\left(-1+x\right)^3}\left(-(-1+x)^2+2-2x\right)^{\frac{3}{2}}+($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x)
```

```
[Out] g^2*(1/(-1+x)^2*(-(-1+x)^2+2-2*x)^(3/2)+(-(-1+x)^2+2-2*x)^(1/2)-arcsin(x))+
2/3*g*(f+g)/(-1+x)^3*(-(-1+x)^2+2-2*x)^(3/2)+(f^2+2*f*g+g^2)*(1/5/(-1+x)^4*
(-(-1+x)^2+2-2*x)^(3/2)-1/15/(-1+x)^3*(-(-1+x)^2+2-2*x)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^{2}\sqrt{-x^{2}+1}}{(x-1)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*sqrt(-x^2 + 1)/(x - 1)^4, x)
```

Fricas [B] time = 2.05811, size = 432, normalized size = 5.4

$$\frac{2\left(2f^{2}-fg+12g^{2}\right)x^{3}-6\left(2f^{2}-fg+12g^{2}\right)x^{2}-4f^{2}+2fg-24g^{2}+6\left(2f^{2}-fg+12g^{2}\right)x+30\left(g^{2}x^{3}-3g^{2}x^{2}+\right.}{15\left(x^{3}-3x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(2*(2*f^2 - f*g + 12*g^2)*x^3 - 6*(2*f^2 - f*g + 12*g^2)*x^2 - 4*f^2 +
 2*f*g - 24*g^2 + 6*(2*f^2 - f*g + 12*g^2)*x + 30*(g^2*x^3 - 3*g^2*x^2 + 3*
g^2*x - g^2)*arctan((sqrt(-x^2 + 1) - 1)/x) + ((f^2 - 8*f*g - 39*g^2)*x^2 -
 4*f^2 + 2*f*g - 24*g^2 - 3*(f^2 + 2*f*g - 19*g^2)*x)*sqrt(-x^2 + 1))/(x^3
- 3*x^2 + 3*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}\left(f+gx\right)^{2}}{(x-1)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-x**2+1)**(1/2)/(1-x)**4,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))*(f + g*x)**2/(x - 1)**4, x)
```

Giac [B] time = 1.3042, size = 359, normalized size = 4.49

$$-g^{2}\arcsin\left(x\right)+\frac{2\left(4f^{2}-2fg+24g^{2}+\frac{5f^{2}\left(\sqrt{-x^{2}+1}-1\right)}{x}-\frac{10fg\left(\sqrt{-x^{2}+1}-1\right)}{x}+\frac{105g^{2}\left(\sqrt{-x^{2}+1}-1\right)}{x}+\frac{25f^{2}\left(\sqrt{-x^{2}+1}-1\right)^{2}}{x^{2}}+\frac{10fg\left(\sqrt{\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="giac")
```

```
[Out] -g^2*arcsin(x) + 2/15*(4*f^2 - 2*f*g + 24*g^2 + 5*f^2*(sqrt(-x^2 + 1) - 1)/
x - 10*f*g*(sqrt(-x^2 + 1) - 1)/x + 105*g^2*(sqrt(-x^2 + 1) - 1)/x + 25*f^2
*(sqrt(-x^2 + 1) - 1)^2/x^2 + 10*f*g*(sqrt(-x^2 + 1) - 1)^2/x^2 + 165*g^2*(
sqrt(-x^2 + 1) - 1)^2/x^2 + 15*f^2*(sqrt(-x^2 + 1) - 1)^3/x^3 - 30*f*g*(sqr
t(-x^2 + 1) - 1)^3/x^3 + 75*g^2*(sqrt(-x^2 + 1) - 1)^3/x^3 + 15*f^2*(sqrt(-
x^2 + 1) - 1)^4/x^4 + 15*g^2*(sqrt(-x^2 + 1) - 1)^4/x^4)/((sqrt(-x^2 + 1) -
 1)/x + 1)^5
```

3.620 $\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rubi [A] time = 0.240769, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {853, 1654, 844, 216, 725, 204}

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1-ax)^2(c+dx)}\,dx &= \int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2}+\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}-\frac{(ac-d)^2\,\text{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx,x,\frac{d+a^2cx}{\sqrt{1-a^2x^2}}\right)}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}+\frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}
\end{aligned}
$$

Mathematica [C] time = 0.307408, size = 148, normalized size = 1.38

$$
-\frac{\frac{i(d-ac)^2\log\left(\frac{2d^3\left(\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}+ia^2cx+id\right)}{(d-ac)^2\sqrt{a^2c^2-d^2}(c+dx)}\right)}{\sqrt{a^2c^2-d^2}}+d\sqrt{1-a^2x^2}+(ac-2d)\sin^{-1}(ax)}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -((d*Sqrt[1 - a^2*x^2] + (a*c - 2*d)*ArcSin[a*x] + (I*(-(a*c) + d)^2*Log[(2
*d^3*(I*d + I*a^2*c*x + Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2]))/((-(a*c) +
d)^2*Sqrt[a^2*c^2 - d^2]*(c + d*x))])/Sqrt[a^2*c^2 - d^2])/d^2)
```

Maple [B] time = 0.289, size = 1178, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x)
```

```
[Out] 1/3*d/(a*c+d)^2*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(3/2)+
1/2/(a*c+d)^2*a^2*c*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1
/2)*x+3/2/(a*c+d)^2*a^2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x+c/d)^2*a^2+
2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))-1/d/(a*c+d)^2*(-(x+c/d)^2*a^2+2
*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2)*a^2*c^2+d/(a*c+d)^2*(-(x+c/d)^2*a
^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2)-1/d^2/(a*c+d)^2*a^4*c^3/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)
/d^2)^(1/2))-1/d^3/(a*c+d)^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2
)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a^2*
c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))*a^4*c^4+2/d/(a*c+d)^2/(-(a^2
*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^
2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2
))/(x+c/d))*a^2*c^2-d/(a*c+d)^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-
d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a
^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))-1/3*d/(a*c+d)^2*(-(x-1/a)
^2*a^2-2*a*(x-1/a))^(3/2)+1/2*d/(a*c+d)^2*a*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(1
/2)*x+1/2*d/(a*c+d)^2*a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x-1/a)^2*a^2-2*
a*(x-1/a))^(1/2))-1/a^2/(a*c+d)/(x-1/a)^2*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(5/2
)-1/(a*c+d)*(-(x-1/a)^2*a^2-2*a*(x-1/a))^(3/2)+3/2*a/(a*c+d)*(-(x-1/a)^2*a^
2-2*a*(x-1/a))^(1/2)*x+3/2*a/(a*c+d)/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x-
1/a)^2*a^2-2*a*(x-1/a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax-1)^2(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)), x)
```

Fricas [A] time = 3.10313, size = 679, normalized size = 6.35

$$\left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-\left(a^2c^2-d^2\right)\sqrt{-a^2x^2+1}-\left(acd+d^2+\left(a^3c^2+a^2cd\right)x+\sqrt{-a^2x^2+1}\left(acd+d^2\right)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac-2\,d)\arctan\left(\frac{\sqrt{-a}}{}\right.}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{(c+dx)(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/(-a*x+1)**2/(d*x+c),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((c + d*x)*(a*x - 1)**2), x)
```

Giac [B] time = 1.46747, size = 281, normalized size = 2.63

$$-\left(\frac{(ax-1)\sqrt{-\frac{2}{ax-1}-1}\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}\left(a\right)}{ad}-\frac{2\left(ac\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}\left(a\right)-2\,d\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}\left(a\right)\right)\arctan\left(\sqrt{-\frac{2}{ax-1}-1}\right)}{ad^2}+\frac{2\left(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -((a*x - 1)*sqrt(-2/(a*x - 1) - 1)*sgn(1/(a*x - 1))*sgn(a)/(a*d) - 2*(a*c*s
gn(1/(a*x - 1))*sgn(a) - 2*d*sgn(1/(a*x - 1))*sgn(a))*arctan(sqrt(-2/(a*x -
 1) - 1))/(a*d^2) + 2*(a^2*c^2*sgn(1/(a*x - 1))*sgn(a) - 2*a*c*d*sgn(1/(a*x
 - 1))*sgn(a) + d^2*sgn(1/(a*x - 1))*sgn(a))*arctan((a*c*sqrt(-2/(a*x - 1)
- 1) + d*sqrt(-2/(a*x - 1) - 1))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*
a*d^2))*abs(a)
```

3.621 $\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rubi [A] time = 0.179428, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 216, 725, 204}

$$\frac{(ac-d)^2\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2}+\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}-\frac{(ac-d)^2\,\text{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx,x,\frac{d+a^2cx}{\sqrt{1-a^2x^2}}\right)}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}+\frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}
\end{aligned}$$

Mathematica [A] time = 0.108653, size = 120, normalized size = 1.12

$$\frac{(ac-d)\sqrt{a^2c^2-d^2}\tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2(ac+d)}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-d)\sin^{-1}(ax)}{d^2}+\frac{\sin^{-1}(ax)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - d)*ArcSin[a*x])/d^2 + ArcSin[a*x]/d + ((a*
c - d)*Sqrt[a^2*c^2 - d^2]*ArcTan[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1
 - a^2*x^2])])/(d^2*(a*c + d))
```

Maple [B] time = 0.23, size = 524, normalized size = 4.9

$$-\frac{1}{d}\sqrt{-a^2x^2+1}-\frac{a^2c}{d^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}+2\frac{a}{d\sqrt{a^2}}\arctan\left(\frac{\sqrt{a^2}x}{\sqrt{-a^2x^2+1}}\right)-\frac{a^2c^2}{d^3}\ln\left(\left(-2\frac{a^2c^2-d^2}{d^2}+2\frac{a^2c}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/d-a^2/d^2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+2*a/d/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/d^3/
(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(
a^2*c^2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2
)^(1/2))/(x+c/d))*a^2*c^2+2/d^2/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-
d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(-(x+c/d)^2*a^2+2*a
^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))*a*c-1/d/(-(a^2*c^2-d^2)/d
```

```
^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)
^(1/2)*(-(x+c/d)^2*a^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.19296, size = 679, normalized size = 6.35

$$\left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}(acd+d^2)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac-2\,d)\arctan\left(\frac{\sqrt{-a\cdots}}{\cdots}\right.}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^2}{\sqrt{-(ax-1)(ax+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(d*x+c)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**2/(sqrt(-(a*x - 1)*(a*x + 1))*(c + d*x)), x)
```

Giac [A] time = 1.34627, size = 177, normalized size = 1.65

$$-\frac{\left(a^2c-2\,ad\right)\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{d^2|a|}-\frac{\sqrt{-a^2x^2+1}}{d}-\frac{2\left(a^3c^2-2\,a^2cd+ad^2\right)\arctan\left(\frac{d+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)c}{ax}}{\sqrt{a^2c^2-d^2}}\right)}{\sqrt{a^2c^2-d^2}\,d^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a^2*c - 2*a*d)*arcsin(a*x)*sgn(a)/(d^2*abs(a)) - sqrt(-a^2*x^2 + 1)/d - 2
*(a^3*c^2 - 2*a^2*c*d + a*d^2)*arctan((d + (sqrt(-a^2*x^2 + 1)*abs(a) + a)*
c/(a*x))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*d^2*abs(a))
```

3.622 $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=851

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e}+\frac{4\sqrt{-a}\left(3a^2e^2(26ef+231dg)g^4-9ac\left(6e^3f^3-33de^2gf^2+88d^2eg^2f+77d^3g^3\right)g^2-c^2\right.}{3465c^{3/2}}$$

```
[Out] (-2*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2)
 + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*
g^3 + 315*d^4*g^4))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3465*c^2*e*g^4) + (2*(d
 + e*x)^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(11*e) - (2*(2*a*e^2*g^2*(74*e*f -
 231*d*g) - c*(233*e^3*f^3 - 843*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g
^3))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(3465*c*g^4) + (2*e*(18*a*e^2*g^2 - c
*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(
693*c*g^4) + (2*e^2*(e*f - 3*d*g)*(f + g*x)^(7/2)*Sqrt[a + c*x^2])/(99*g^4)
 + (4*Sqrt[-a]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g) - c^2*f^2*(64*e^3*f^3 - 26
4*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) - 9*a*c*g^2*(6*e^3*f^3 - 33*
d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(3465*c^(3/2)*g^5*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(75*a^2*e^
3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) - c^2*f*(64*e^3*
f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3))*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3
465*c^(5/2)*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 2.69679, antiderivative size = 851, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {919, 1654, 844, 719, 424, 419}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e}+\frac{4\sqrt{-a}\left(3a^2e^2(26ef+231dg)g^4-9ac\left(6e^3f^3-33de^2gf^2+88d^2eg^2f+77d^3g^3\right)g^2-c^2\right.}{3465c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2)
 + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*
g^3 + 315*d^4*g^4))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3465*c^2*e*g^4) + (2*(d
 + e*x)^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(11*e) - (2*(2*a*e^2*g^2*(74*e*f -
 231*d*g) - c*(233*e^3*f^3 - 843*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g
^3))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(3465*c*g^4) + (2*e*(18*a*e^2*g^2 - c
*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(
693*c*g^4) + (2*e^2*(e*f - 3*d*g)*(f + g*x)^(7/2)*Sqrt[a + c*x^2])/(99*g^4)
 + (4*Sqrt[-a]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g) - c^2*f^2*(64*e^3*f^3 - 26
4*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) - 9*a*c*g^2*(6*e^3*f^3 - 33*
d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
```

```
[-a]*Sqrt[c]*f - a*g)])/(3465*c^(3/2)*g^5*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(75*a^2*e^
3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) - c^2*f*(64*e^3*
f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3))*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3
465*c^(5/2)*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 919

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(e*(2*m + 5)), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m*Simp[3*a*e*f
- a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3*c*d*g)*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{\int \frac{(d+ex)^3\left(a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{11e}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4} + \frac{2\int \frac{-\frac{1}{2}acg^2(7e^4f \cdots}{\cdots}}{\cdots}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} + \frac{2e\left(18ae^2g^2-c\left(29e^2f^2-96defg+81d^2g^2\right)\right)(f+ \cdots}{693cg^4}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+cx^2}}{11e} - \frac{2\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2f^2g \cdots\right.\right.}{3465c \cdots}$$

$$= -\frac{2\left(150a^2e^4g^4-6ace^2g^2\left(2e^2f^2-33defg+165d^2g^2\right)+c^2\left(187e^4f^4-732de^3f^3g \cdots\right.\right.}{3465c^2eg^4}$$

$$= -\frac{2\left(150a^2e^4g^4-6ace^2g^2\left(2e^2f^2-33defg+165d^2g^2\right)+c^2\left(187e^4f^4-732de^3f^3g \cdots\right.\right.}{3465c^2eg^4}$$

$$= -\frac{2\left(150a^2e^4g^4-6ace^2g^2\left(2e^2f^2-33defg+165d^2g^2\right)+c^2\left(187e^4f^4-732de^3f^3g \cdots\right.\right.}{3465c^2eg^4}$$

$$= -\frac{2\left(150a^2e^4g^4-6ace^2g^2\left(2e^2f^2-33defg+165d^2g^2\right)+c^2\left(187e^4f^4-732de^3f^3g \cdots\right.\right.}{3465c^2eg^4}$$

Mathematica [C] time = 10.5846, size = 1034, normalized size = 1.22

$$2\sqrt{f+gx}\left(\frac{2\left(-3a^2e^2(26ef+231dg)g^4+9ac\left(6e^3f^3-33de^2gf^2+88d^2eg^2f+77d^3g^3\right)g^2+c^2f^2\left(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d^3g^3\right)\right)\left(cx^2+a\right)g^2}{f+gx} - \left(cx^2 \cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*((2*g^2*(-3*a^2*e^2*g^4*(26*e*f + 231*d*g) + c^2*f^2*(64*e
^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 9*a*c*g^2*(6*e^
3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*(a + c*x^2))/(f + g*
x) - g^2*(a + c*x^2)*(150*a^2*e^3*g^4 - 2*a*c*e*g^2*(495*d^2*g^2 + 33*d*e*g
*(4*f + 7*g*x) + e^2*(-23*f^2 + 16*f*g*x + 45*g^2*x^2)) + c^2*(-231*d^3*g^3
*(f + 3*g*x) - 99*d^2*e*g^2*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) - 33*d*e^2*g*(8
*f^3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3) + e^3*(64*f^4 - 48*f^3*g*x + 4
```

```
0*f^2*g^2*x^2 - 35*f*g^3*x^3 - 315*g^4*x^4))) + (2*Sqrt[c]*((-I)*Sqrt[c]*f
+ Sqrt[a]*g)*(-3*a^2*e^2*g^4*(26*e*f + 231*d*g) + c^2*f^2*(64*e^3*f^3 - 264
*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 9*a*c*g^2*(6*e^3*f^3 - 33*d
*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x
))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x
]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqr
t[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]] + (2*Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(75*a^2*e^3*g^4 - (3*I)*a^
(3/2)*Sqrt[c]*e^2*g^3*(e*f + 231*d*g) - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g
 + 165*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 264*d*e^2*f^2*g - 396*d^2*e*f*g^2 +
231*d^3*g^3) + (3*I)*Sqrt[a]*c^(3/2)*g*(16*e^3*f^3 - 66*d*e^2*f^2*g + 99*d^
2*e*f*g^2 + 231*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3465*c
^2*g^6*Sqrt[a + c*x^2])
```

Maple [B] time = 0.479, size = 6457, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}(ex + d)^3 \sqrt{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\sqrt{cx^2 + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)*sqrt(g*x
 + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + cx^2}\,(d + ex)^3\,\sqrt{f + gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**3*sqrt(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}(ex + d)^3\sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

3.623 $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=635

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(3aeg^2(ef-10dg)+cf\left(21d^2g^2-24defg+8e^2f^2\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\right.}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[Out] (-2*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*d^2*e
*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*e*g^3) + (2*(d
+ e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*e) + (4*(7*a*e^2*g^2 - c*(8*e^2*
f^2 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*c*g^3
) + (2*e*(e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^3) + (4*Sqrt[
-a]*(21*a^2*e^2*g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2
*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^4*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(3*a*e*g^2*
(e*f - 10*d*g) + c*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[(Sqrt[c]*(
f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]
)/(315*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 1.62228, antiderivative size = 635, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {919, 1654, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(21a^2e^2g^4+3acg^2\left(-21d^2g^2-16defg+3e^2f^2\right)+c^2f^2\left(21d^2g^2-24defg+8e^2f^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{1\cdots}}{}\right.\right.}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```


```
[Out] (-2*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*d^2*e
*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*e*g^3) + (2*(d
+ e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*e) + (4*(7*a*e^2*g^2 - c*(8*e^2*
f^2 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*c*g^3
) + (2*e*(e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^3) + (4*Sqrt[
-a]*(21*a^2*e^2*g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2
*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^4*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(3*a*e*g^2*
(e*f - 10*d*g) + c*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[(Sqrt[c]*(
f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]
)/(315*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```


Rule 919

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(e*(2*m + 5)), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m*Simp[3*a*e*f
- a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3*c*d*g)*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{\int \frac{(d+ex)^2\left(a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{9e}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3} + \frac{2\int \frac{-\frac{1}{2}acg^2\left(5e^3f^3-15d \ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9e} + \frac{4\left(7ae^2g^2-c\left(8e^2f^2-24defg+21d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{\ldots}}{315cg^3}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+c\ldots}}{315ceg^3}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+c\ldots}}{315ceg^3}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+c\ldots}}{315ceg^3}$$

$$= -\frac{2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+c\ldots}}{315ceg^3}$$

Mathematica [C] time = 7.41708, size = 809, normalized size = 1.27

$$\sqrt{f+gx}\left(\frac{2(cx^2+a)\left(2ae(4ef+30dg+7egx)g^2+c\left(\left(8f^3-6gxf^2+5g^2x^2f+35g^3x^3\right)e^2+6dg\left(-4f^2+3gxf+15g^2x^2\right)e+21d^2g^2(f+3gx)\right)\right)}{cg^3} - \frac{4\left(\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(21a^2e^2g^4-\ldots\right.\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e*g^2*(4*e*f + 30*d*g + 7*e*g*x) + c*(2
1*d^2*g^2*(f + 3*g*x) + 6*d*e*g*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) + e^2*(8*f^
3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3))))/(c*g^3) - (4*(g^2*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21
*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2))*(a + c*x^2) -
 I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 -
 24*d*e*f*g + 21*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2
))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt
[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)*a^(3/2)*e^
2*g^3 - 3*a*Sqrt[c]*e*g^2*(e*f - 10*d*g) + c^(3/2)*f*(-8*e^2*f^2 + 24*d*e*f
```

```
*g - 21*d^2*g^2) - (3*I)*Sqrt[a]*c*g*(-2*e^2*f^2 + 6*d*e*f*g + 21*d^2*g^2))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)]))/(c^2*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(315*Sqrt
[a + c*x^2])
```

Maple [B] time = 0.28, size = 4351, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] -2/315*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-108*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f)
)^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/
2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a
*c*d*e*f^2*g^4-60*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/
2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f
))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*
g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a^2*d*e*g^6+54*(-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(
1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*
c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^
(1/2))*a^2*c*e^2*f^2*g^4-126*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^
(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-
a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d^2*f^2*g^4+12*(-(g*x+
f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f
))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x
+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*
f))^(1/2))*a*c^2*e^2*f^4*g^2-60*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-
((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*c*e^2*f^2*g^4+84*(-(g
*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+
c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g
+c*f))^(1/2))*a*c^2*d^2*f^2*g^4-34*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*
((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*e^2*f^4*g^2+48*(
-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)
*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE(
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2
)*g+c*f))^(1/2))*c^3*d*e*f^5*g+6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((
-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a
*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(
-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a^2*e^2*f*g
^5+42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c
)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ell
ipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*
c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c^2*d^2*f^3*g^3-14*x^2*a^2*c*e^2*g^6-8
*a^2*c*e^2*f^2*g^4-21*a*c^2*d^2*f^2*g^4-8*a*c^2*e^2*f^4*g^2-42*(-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1
```

/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(
1/2))*c^3*d^2*f^4*g^2-49*x^4*a*c^2*e^2*g^6+x^4*c^3*e^2*f^2*g^4-84*x^3*c^3*d
^2*f*g^5-2*x^3*c^3*e^2*f^3*g^3-63*x^2*a*c^2*d^2*g^6-21*x^2*c^3*d^2*f^2*g^4-
8*x^2*c^3*e^2*f^4*g^2-90*x^5*c^3*d*e*g^6-40*x^5*c^3*e^2*f*g^5-35*x^6*c^3*e^
2*g^6-63*x^4*c^3*d^2*g^6-60*a^2*c*d*e*f*g^5+24*a*c^2*d*e*f^3*g^3-126*(-(g*x
+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*
f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c
*f))^(1/2))*a^2*c*d^2*g^6+126*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)
^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((
-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*c*d^2*g^6-168*x^2*a*c^2
*d*e*f*g^5+6*x*a*c^2*d*e*f^2*g^4+42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
(-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2
),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^3*e^2*g^6-42*(-(g*x
+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*
f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c
*f))^(1/2))*a^3*e^2*g^6-16*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(
-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1
/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*
c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^3*e^2*f^6+16*(-(g*x+f)*c/((-
a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)
*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2
))*(-a*c)^(1/2)*c^2*e^2*f^5*g-108*x^4*c^3*d*e*f*g^5-150*x^3*a*c^2*d*e*g^6-6
2*x^3*a*c^2*e^2*f*g^5+6*x^3*c^3*d*e*f^2*g^4-7*x^2*a*c^2*e^2*f^2*g^4+24*x^2*
c^3*d*e*f^3*g^3-60*x*a^2*c*d*e*g^6-22*x*a^2*c*e^2*f*g^5-84*x*a*c^2*d^2*f*g^
5-2*x*a*c^2*e^2*f^3*g^3+42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(
-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1
/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*
c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*c*d^2*f*g^5+22*
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF
((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/
2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*c*e^2*f^3*g^3-48*(-(g*x+f)*c/((-a*c)^(1/2)
*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a
*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2
)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^
(1/2)*c^2*d*e*f^4*g^2-36*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a
*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)
^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*c*d*e*f*g^5-36*(-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/
2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1
/2))*a*c^2*d*e*f^3*g^3+96*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-
a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/
2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c
)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*c*d*e*f*g^5+144*(-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(
1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*
c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^
(1/2))*a*c^2*d*e*f^3*g^3)/c^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^5

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+a}\sqrt{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+cx^2}\,(d+ex)^2\,\sqrt{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

3.624 $\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=434

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(5aeg^2+cf(4ef-7dg)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+\cdots}}{\cdots}$$

[Out] (-2*Sqrt[f + g*x]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)
*Sqrt[a + c*x^2])/(105*c*g^2) + (2*e*Sqrt[f + g*x]*(a + c*x^2)^(3/2))/(7*c)
 - (4*Sqrt[-a]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(c*f^2
+ a*g^2)*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c
]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^
3*Sqrt[f + g*x]*Sqrt[a + c*x^2])

Rubi [A] time = 0.488274, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {833, 815, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(5aeg^2+cf(4ef-7dg)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]

[Out] (-2*Sqrt[f + g*x]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)
*Sqrt[a + c*x^2])/(105*c*g^2) + (2*e*Sqrt[f + g*x]*(a + c*x^2)^(3/2))/(7*c)
 - (4*Sqrt[-a]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(c*f^2
+ a*g^2)*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c
]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^
3*Sqrt[f + g*x]*Sqrt[a + c*x^2])

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2e\sqrt{f+gx}\left(a+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}(7cdf-aeg)+\frac{1}{2}c(ef+7dg)x\right)\sqrt{a+cx^2}}{\sqrt{f+gx}}dx}{7c}$$

$$= -\frac{2\sqrt{f+gx}\left(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x\right)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}\left(a+cx^2\right.}{7c}$$

$$= -\frac{2\sqrt{f+gx}\left(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x\right)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}\left(a+cx^2\right.}{7c}$$

$$= -\frac{2\sqrt{f+gx}\left(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x\right)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}\left(a+cx^2\right.}{7c}$$

$$= -\frac{2\sqrt{f+gx}\left(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x\right)\sqrt{a+cx^2}}{105cg^2} + \frac{2e\sqrt{f+gx}\left(a+cx^2\right.}{7c}$$

Mathematica [C] time = 4.64444, size = 610, normalized size = 1.41

$$\sqrt{f+gx}\left(\frac{2\left(a+cx^2\right)\left(10aeg^2+7cdg(f+3gx)+ce\left(-4f^2+3fgx+15g^2x^2\right)\right)}{cg^2} + \frac{4\left(\sqrt{a}g(f+gx)^{3/2}\left(-\sqrt{a}g+i\sqrt{c}f\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(3\sqrt{a}\sqrt{c}g(7dg+ef)+5iaeg^2+\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e*g^2 + 7*c*d*g*(f + 3*g*x) + c*e*(-4*
f^2 + 3*f*g*x + 15*g^2*x^2)))/(c*g^2) + (4*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*(a + c*x^2) + I*Sqrt
[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(c*f^2*(-4*e*f + 7*d*g) - a*g^2*(8*e*f + 21*d
*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sq
rt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] + Sqrt[a]*g*(I*Sqrt[c]*f - Sqrt[a]*g)*((5*I)*a*e*g^2 + I*c*f*(
4*e*f - 7*d*g) + 3*Sqrt[a]*Sqrt[c]*g*(e*f + 7*d*g))*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
+ g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

Maple [B] time = 0.447, size = 2551, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-28*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2)
)*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))
^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d*f^2*g^3-
24*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellipt
icE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^
(1/2)*g+c*f))^(1/2))*a*c^2*e*f^3*g^2-16*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*c*e*f*g^4+6*(
-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)
*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF(
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2
)*g+c*f))^(1/2))*a^2*c*e*f*g^4+42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*(
(-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-
a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),
(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d*f^2*g^3+6*(-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c
*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g
*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+
c*f))^(1/2))*a*c^2*e*f^3*g^2-14*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-
((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c^2*d*f^3*g^
2-14*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)
^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Elli
pticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c
)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*c*d*f*g^4+18*(-(g*x+f)*c/((-a*c)^(1/2
)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-
a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/
2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)
^(1/2)*a*c*e*f^2*g^3+15*x^5*c^3*e*g^5+21*x^4*c^3*d*g^5+18*x^4*c^3*e*f*g^4+1
4*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1
/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellipti
cE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(
1/2)*g+c*f))^(1/2))*c^3*d*f^4*g+10*a^2*c*e*f*g^4-4*a*c^2*e*f^3*g^2+42*(-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c
*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g
*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+
c*f))^(1/2))*a^2*c*d*g^5+10*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a
*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a^2*e*g^5-42*(-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-
(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*
g+c*f))^(1/2))*a^2*c*d*g^5+8*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^
(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-
a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c^2*e*f^4*g-x^3
*c^3*e*f^2*g^3+21*x^2*a*c^2*d*g^5+7*x^2*c^3*d*f^2*g^3-4*x^2*c^3*e*f^3*g^2+1
0*x*a^2*c*e*g^5+25*x^3*a*c^2*e*g^5+28*x^3*c^3*d*f*g^4+7*a*c^2*d*f^2*g^3-8*(
-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)
*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE(
```

```
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2
)*g+c*f))^(1/2))*c^3*e*f^5+28*x^2*a*c^2*e*f*g^4+28*x*a*c^2*d*f*g^4-x*a*c^2*
e*f^2*g^3)/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^4/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}(ex+d)\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^2+a}(ex+d)\sqrt{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+cx^2}\,(d+ex)\,\sqrt{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}(ex+d)\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

3.625 $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=362

$$-\frac{4\sqrt{-a}f\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(cf^2-3\right.}{15\sqrt{c}g^2\sqrt{a}}$$

```
[Out] (-4*f*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*g) + (2*(f + g*x)^(3/2)*Sqrt[a + c
*x^2])/(5*g) + (4*Sqrt[-a]*(c*f^2 - 3*a*g^2)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqr
t[-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*f*(c*f^2 + a*g^2)*Sqrt[(Sqr
t[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f -
 a*g)])/(15*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.317375, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {735, 833, 844, 719, 424, 419}

$$-\frac{4\sqrt{-a}f\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(cf^2-3ag^2\right)E}{15\sqrt{c}g^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-4*f*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*g) + (2*(f + g*x)^(3/2)*Sqrt[a + c
*x^2])/(5*g) + (4*Sqrt[-a]*(c*f^2 - 3*a*g^2)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqr
t[-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*f*(c*f^2 + a*g^2)*Sqrt[(Sqr
t[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f -
 a*g)])/(15*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{2\int \frac{(ag-cfx)\sqrt{f+gx}}{\sqrt{a+cx^2}}dx}{5g}$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{4\int \frac{2acfg-\frac{1}{2}c(cf^2-3ag^2)x}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{15cg}$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{1}{15}\left(2\left(3a-\frac{cf^2}{g^2}\right)\right)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}dx + \frac{1}{15}\Big($$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{\left(4a\left(3a-\frac{cf^2}{g^2}\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,\text{Subst}\right.}{15\sqrt{-a}\sqrt{c}\sqrt{\frac{c(f+gx}{cf-\frac{a\sqrt{c}}{\sqrt{-}}}}}$$

$$= -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} - \frac{4\sqrt{-a}\left(3a-\frac{cf^2}{g^2}\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin\right.}{15\sqrt{c}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a}}$$

Mathematica [C] time = 2.96802, size = 536, normalized size = 1.48

$$\frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(f+3gx)}{g} - \frac{4\left(-\sqrt{a}\sqrt{c}g(f+gx)^{3/2}(4i\sqrt{a}\sqrt{c}fg-3ag^2+cf^2)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+g^2\sqrt{-f}\right.}{}\right)}{15\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(f + 3*g*x)*(a + c*x^2))/g - (4*(g^2*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]]*(-3*a^2*g^2 + c^2*f^2*x^2 + a*c*(f^2 - 3*g^2*x^2)) + Sqrt[c]*
((-I)*c^(3/2)*f^3 + Sqrt[a]*c*f^2*g + (3*I)*a*Sqrt[c]*f*g^2 - 3*a^(3/2)*g^3
)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)] - Sqrt[a]*Sqrt[c]*g*(c*f^2 + (4*I)*Sqrt[a]*Sqrt[c]*f*g - 3*a*g^2)
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)]))/(c*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(15*Sqrt[a
+ c*x^2])
```

Maple [B] time = 0.29, size = 1162, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(6*a^2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))
^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2
))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f)
)^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*g^4+6*(-(g*x+f)
*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))
^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f
)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f)
)^(1/2))*a*c*f^2*g^2-2*(-a*c)^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
(-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2
),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*f*g^3-2*(-a*c)^(1/2
)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1
/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellipti
cF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(
1/2)*g+c*f))^(1/2))*c*f^3*g-6*a^2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*(
(-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-
a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),
(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*g^4-4*(-(g*x+f)*c/((-a*
c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*(
(c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a
*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))
*a*c*f^2*g^2+2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^
(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c
*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*f^4+3*x^4*c^2*g^4+4*x^3*c^2*f*g^3+3*x^
2*a*c*g^4+x^2*c^2*f^2*g^2+4*x*a*c*f*g^3+a*c*f^2*g^2)/c/(c*g*x^3+c*f*x^2+a*g
*x+a*f)/g^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}\sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+cx^2}\sqrt{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+a}\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

3.626 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=683

$$\frac{2\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(2ae^2g-\cdots\right.}{3\cdots}$$

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) - (2*Sqrt[-a]*Sqrt[c]*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqr
t[c]*f*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[f + g*x]*Sqrt[a + c*x^2
]) - (2*Sqrt[-a]*(2*a*e^2*g - 3*c*d*(e*f - d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(
Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c
]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-
a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 2.12211, antiderivative size = 683, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {919, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}+\frac{2\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}(ef-3dg)\sqrt{\cdots}}{3e^2g\sqrt{a+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) - (2*Sqrt[-a]*Sqrt[c]*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqr
t[c]*f*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[f + g*x]*Sqrt[a + c*x^2
]) - (2*Sqrt[-a]*(2*a*e^2*g - 3*c*d*(e*f - d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(
Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c
]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-
a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 919

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(e*(2*m + 5)), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m*Simp[3*a*e*f
- a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3*c*d*g)*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{\int \frac{a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{\int \left(\frac{2ae^2g-3cd(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{c(ef-3dg)x}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{3\left(cd^2+ae^2\right)(ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{(c(ef-3dg))\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e^2} + \frac{\left(\left(cd^2+ae^2\right)(ef-dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{\ldots}}}{e^3}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} + \frac{(c(ef-3dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3e^2g} - \frac{(cf(ef-3dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3e^2g} + \frac{\left(\left(cd^2+\ldots\right.\right.}{\ldots}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\left(2ae^2g-3cd(ef-dg)\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\ldots\right)}{3\sqrt{c}e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \ldots$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\sqrt{c}(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3e^2g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \ldots$$

Mathematica [C] time = 9.13253, size = 1216, normalized size = 1.78

$$\left(\frac{2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3}{(f+gx)^2}-\frac{4ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2}{f+gx}-\frac{6cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2}{(f+gx)^2}+2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f+\frac{12cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f}{f+gx}+\frac{2ae^2g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f}{(f+gx)^2}+\frac{2\sqrt{c}e\left(\sqrt{a}g\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) + ((f + g*x)^(3/2)*(2*c*e^2*f*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - 6*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] +
(2*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*c*d*e*f^2*g
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (2*a*e^2*f*g^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]])/(f + g*x)^2 - (4*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
g*x) + (12*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (2*Sqrt[
c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - 3*d*g)*Sqrt[1 - f/(f + g*x) - (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c
]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]
 + (2*e*(3*Sqrt[c]*d - I*Sqrt[a]*e)*g*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[1 -
 f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I
*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)])/Sqrt[f + g*x] + ((6*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*
x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((6*I)*a*e^2*g^2*Sqrt[1 - f
/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]
*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x
]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(
3*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + (c*(f + g*x)^2*(-1 + f/
(f + g*x))^2)/g^2])
```

Maple [B] time = 0.325, size = 2496, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x)
```

```
[Out] -2/3*(2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a
*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*E
llipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-
a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3+3*(-(g*x+f)*c/((-a*c)^(1/2
)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-
```

```
a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/
2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)
^(1/2)*c*d^2*g^3-(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2
))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f)
)^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g
-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g+3*(-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/
2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1
/2))*a*c*d*e*g^3-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1
/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*
f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)
*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e^2*f*g^2-3*(-(g*x+f)*c/((-a*c)^(1
/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(
1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*
d^2*f*g^2+3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/
((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/
2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)
/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*e*f^2*g-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c
*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^
(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-
c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*e*g^3
+(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/
2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Elliptic
E((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1
/2)*g+c*f))^(1/2))*a*c*e^2*f*g^2-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*
((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*e*f^2*g+(-(g*x+f
)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f)
)^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+
f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f
))^(1/2))*c^2*e^2*f^3-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*
c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)
*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(
1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2
))*(-a*c)^(1/2)*a*e^2*g^3-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*
c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^
(1/2))*(-a*c)^(1/2)*c*d^2*g^3+3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(
(-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*
f))^(1/2))*a*c*e^2*f*g^2+3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(
-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1
/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c
)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(
1/2))*c^2*d^2*f*g^2-x^3*c^2*e^2*g^3-x^2*c^2*e^2*f*g^2-x*a*c*e^2*g^3-a*c*e^2
*f*g^2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e^3/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}\sqrt{f + gx}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.627 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=650

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{3\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{Elli}\cdots}{e^2\sqrt{a+cx\cdots}}$$

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x))) - (3*Sqrt[-a]*Sqrt[c]*Sqrt
[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a
]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c]*f*Sq
rt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt
[c]*f - a*g)])/(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*(2*e
*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*
x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/
(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*
g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]
*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 1.66141, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {917, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(ae^2g-cd(2ef-3dg)\right)\cdots}{e^3\sqrt{a+cx^2}\sqrt{f+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x))) - (3*Sqrt[-a]*Sqrt[c]*Sqrt
[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a
]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c]*f*Sq
rt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt
[c]*f - a*g)])/(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*(2*e
*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*
x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/
(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*
g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]
*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 917

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)*Simp[a*g +
2*c*f*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{
a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && Inte
gerQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{\int \frac{ag+2cfx+3cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{\int \left(\frac{c(2ef-3dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{3cgx}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{ae^2g-cd(2ef-3dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{(3cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2} + \frac{(c(2ef-3dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^3} + \frac{\left(ae^2g-cd\right.}{}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} + \frac{(3c)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2e^2} - \frac{(3cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2} + \frac{\left(\left(ae^2g-cd(2ef-3dg)\right)\right.}{}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} - \frac{\sqrt{-a}\sqrt{c}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{3\sqrt{-a}\sqrt{c}}{}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{3\sqrt{-a}\sqrt{c}}{}$$

Mathematica [C] time = 7.17079, size = 1331, normalized size = 2.05

$$\sqrt{f+gx}\left(-\frac{(cx^2+a)e^2}{d+ex}-\frac{-3ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3+6ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2+3cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2-3ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f-6cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f+2icc}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

```
[Out] (Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-3*c*e^2*f^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]] + 3*c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a
*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]] + 6*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) -
6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 3*c*e^2*f*Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]*(f + g*x)^2 + 3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g
)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)] + e*(Sqrt[c]*f + I*Sqrt[a]*g)*(Sqrt[a]*e*g - I*Sqrt[c]*(2*e*f - 3
*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
 I*Sqrt[a]*g)] + (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] - (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f
 + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(
f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*
EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(
Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*
(f + g*x))))/(e^3*Sqrt[a + c*x^2])
```

Maple [B] time = 0.293, size = 6044, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.628 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3} \, dx$

Optimal. Leaf size=1205

result too large to display

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*e*(d + e*x)^2) - ((a*e^2*g - c*d*(2*e*f
 - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)*
(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x
]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[
2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*
g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (3
*Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt
[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (
-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
+ (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*
x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 -
 (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e
^2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*S
qrt[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c
]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^3*(c*d^2 +
 a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (c*(e*f - 3*d*g)*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-
a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))^2
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ell
ipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^3*((Sqrt[c]*d
)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 4.19892, antiderivative size = 1205, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {917, 6742, 719, 419, 940, 844, 424, 933, 168, 538, 537}

$$\frac{\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)\left(ae^2g-cd(2ef-3dg)\right)^2}{4e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\sin^{-1}\right.}{4e^2\left(cd^2+ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```



```
[Out] -(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*e*(d + e*x)^2) - ((a*e^2*g - c*d*(2*e*f
 - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)*
(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x
]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[
2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*
g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (3
*Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt
[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (
-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

```
+ (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*
x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 -
 (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e
^2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*S
qrt[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c
]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^3*(c*d^2 +
 a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (c*(e*f - 3*d*g)*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-
a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))^2
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ell
ipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^3*((Sqrt[c]*d
)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 917

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)*Simp[a*g +
2*c*f*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{
a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && Inte
gerQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{\int \frac{ag+2cfx+3cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{\int \left(\frac{3cg}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{ae^2g-cd(2ef-3dg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{2c(ef-3dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{4e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{(3cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e^3} + \frac{(c(ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^3} + \frac{\left(ae^2g \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\int \cdots}{8e^3\left(cd^2+a\cdots\right.}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\cdots}}{2e^3\sqrt{\cdots}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\cdots}}{2e^3\sqrt{\cdots}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\cdots}}{2e^3\sqrt{\cdots}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\cdots}}{2e^3\sqrt{\cdots}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\left(ae^2g-cd(2ef-3\cdots\right.}{4e^2\left(cd^2\cdots\right.}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\left(ae^2g-cd(2ef-3\cdots\right.}{4e^2\left(cd^2\cdots\right.}$$

Mathematica [C] time = 11.4464, size = 2703, normalized size = 2.24

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

```
[Out] Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-1/(2*e*(d + e*x)^2) + (2*c*d*e*f - 3*c*d^2*
g - a*e^2*g)/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x))) + ((f + g*x)^(3/2
)*(-2*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*c^2*d^2*e^2*f*g*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*c*e^4*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]] - 3*c^2*d^3*e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*c*d*e^3*g^2*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - (2*c^2*d*e^3*f^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]])/(f + g*x)^2 + (5*c^2*d^2*e^2*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/
(f + g*x)^2 + (a*c*e^4*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2
- (3*c^2*d^3*e*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a
*c*d*e^3*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (5*a*c*d^2
*e^2*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (a^2*e^4*f*g^3*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*c*d^3*e*g^4*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (a^2*d*e^3*g^4*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]])/(f + g*x)^2 + (4*c^2*d*e^3*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])
/(f + g*x) - (10*c^2*d^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g
*x) - (2*a*c*e^4*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (6*c^2
*d^3*e*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (2*a*c*d*e^3*f*g
^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (Sqrt[c]*e*((-I)*Sqrt[c]*f
 + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*Sqrt[1 - f/(f +
g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]
*g)/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sq
rt[f + g*x] + (e*(Sqrt[c]*d - I*Sqrt[a]*e)*g*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(
-(a*e^2*g) - (2*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g) + c*d*(-4*e*f + 3*d*g))*Sq
rt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x
) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)])/Sqrt[f + g*x] + ((4*I)*a*c*e^4*f^2*g*Sqrt[1 - f/(f + g*x) - (I*
Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[
c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g
)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
 I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - ((4*I)*c^2*d^3*e*
f*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/
(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d
*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/S
qrt[f + g*x] - ((12*I)*a*c*d*e^3*f*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*
x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((3*I)*c^2*d^4*g^3*Sqrt[1 -
 f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I
*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[
c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g
*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] +
 ((6*I)*a*c*d^2*e^2*g^3*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f +
g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi
[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)])/Sqrt[f + g*x] - (I*a^2*e^4*g^3*Sqrt[1 - f/(f + g*x) - (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c
]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)
), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(4*e^3*(c*d^2 + a*
e^2)*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)^2*Sqrt[a + (c*(f + g*x)
^2*(-1 + f/(f + g*x))^2)/g^2])
```

Maple [B] time = 0.317, size = 19180, normalized size = 15.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}\sqrt{gx + f}}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.629 $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=666

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(9ae^2g^2(2ef-5dg)-c\left(252d^2efg^2-105d^3g^3-216de^2f^2g+64e^3f^3\right)\right)\text{Ellip}}{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[Out] (-4*(9*a*e^2*g^2*(2*e*f - 5*d*g) + c*(76*e^3*f^3 - 204*d*e^2*f^2*g + 168*d^
2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*g^4) + (2*(d
 + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*g) + (4*e*(7*a*e^2*g^2 + c*(64*
e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*
c*g^4) - (4*e^2*(4*e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^4) +
 (4*Sqrt[-a]*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^
2*g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^
3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^5
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*S
qrt[-a]*(c*f^2 + a*g^2)*(9*a*e^2*g^2*(2*e*f - 5*d*g) - c*(64*e^3*f^3 - 216*
d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqr
t[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)
*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 1.57141, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {921, 1654, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(21a^2e^3g^4-3aceg^2\left(63d^2g^2-39defg+10e^2f^2\right)-c^2f\left(252d^2efg^2-105d^3g^3-216de^2f^2g+6\right.\right.}{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```


```
[Out] (-4*(9*a*e^2*g^2*(2*e*f - 5*d*g) + c*(76*e^3*f^3 - 204*d*e^2*f^2*g + 168*d^
2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*g^4) + (2*(d
 + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*g) + (4*e*(7*a*e^2*g^2 + c*(64*
e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*
c*g^4) - (4*e^2*(4*e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^4) +
 (4*Sqrt[-a]*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^
2*g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^
3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^5
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*S
qrt[-a]*(c*f^2 + a*g^2)*(9*a*e^2*g^2*(2*e*f - 5*d*g) - c*(64*e^3*f^3 - 216*
d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqr
t[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)
```

```
*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 921

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(2*(d + e*x)^m*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(
g*(2*m + 3)), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)^(m - 1)*Simp[2*a*(e
*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)*x - (2*c*(d*g*m - e*f*(m + 1)))*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[
m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} - \frac{\int \frac{(d+ex)^2\left(2a(3ef-4dg)+2(cdf-aeg)x+2c(4ef-3dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{9g}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} - \frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4} - \frac{2\int \frac{-acg^2\left(20e^3f^3-15de^2f\right.}{}}{}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}{9g} + \frac{4e\left(7ae^2g^2+c\left(64e^2f^2-111defg+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{}}{315cg^4}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+cx}}{315cg^4}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+cx}}{315cg^4}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+cx}}{315cg^4}$$

$$= -\frac{4\left(9ae^2g^2(2ef-5dg)+c\left(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{a+cx}}{315cg^4}$$

Mathematica [C] time = 8.1031, size = 864, normalized size = 1.3

$$\frac{2\sqrt{f+gx}\left(-c\left(cx^2+a\right)\left(c\left(\left(64f^3-48gxf^2+40g^2x^2f-35g^3x^3\right)e^3-27dg\left(8f^2-6gxf+5g^2x^2\right)e^2+63d^2g^2(4f-3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-(c*g^2*(a + c*x^2)*(-2*a*e^2*g^2*(-11*e*f + 45*d*g + 7*e
*g*x) + c*(-105*d^3*g^3 + 63*d^2*e*g^2*(4*f - 3*g*x) - 27*d*e^2*g*(8*f^2 -
6*f*g*x + 5*g^2*x^2) + e^3*(64*f^3 - 48*f^2*g*x + 40*f*g^2*x^2 - 35*g^3*x^3
)))) - (2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^3*g^4 - 3*a*c*e*g
^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2*
f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g^3))*(a + c*x^2) - Sqrt[c]*(I*Sqrt[c]*f
- Sqrt[a]*g)*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^
2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2*f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g
^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sq
rt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)*a^(3/2)*
```

```
e^3*g^3 - 9*a*Sqrt[c]*e^2*g^2*(2*e*f - 5*d*g) - (3*I)*Sqrt[a]*c*e*g*(16*e^2
*f^2 - 54*d*e*f*g + 63*d^2*g^2) + c^(3/2)*(64*e^3*f^3 - 216*d*e^2*f^2*g + 2
52*d^2*e*f*g^2 - 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)
]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elliptic
F[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I
*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(f + g*x))))/(315*c^2*g^6*Sqrt[a + c*x^2])
```

Maple [B] time = 0.287, size = 5079, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}(ex + d)^3}{\sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\sqrt{cx^2 + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)/sqrt(g*x
 + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}\,(d + ex)^3}{\sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**3/sqrt(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

3.630 $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=508

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(5ae^2g^2-c\left(35d^2g^2-56defg+24e^2f^2\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[Out] (4*(5*a*e^2*g^2 + c*(21*e^2*f^2 - 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*S
qrt[a + c*x^2])/(105*c*g^3) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
/(7*g) - (4*e*(3*e*f - 2*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*g^3) + (
4*Sqrt[-a]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d
^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c
]*g^4*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (4*Sqrt[-a]*(c*f^2 + a*g^2)*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e*f*g + 35
*d^2*g^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x
^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(
Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.977467, antiderivative size = 503, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {921, 1654, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(5ae^2g^2-c\left(35d^2g^2-56defg+24e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}}+4\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (4*(10*d^2 + e^2*((5*a)/c + (21*f^2)/g^2) - (34*d*e*f)/g)*Sqrt[f + g*x]*Sqr
t[a + c*x^2])/(105*g) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(7*g)
 - (4*e*(3*e*f - 2*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*g^3) + (4*Sqrt
[-a]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2
))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^4*
Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sq
rt[-a]*(c*f^2 + a*g^2)*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g
^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]
*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-
a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 921

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(2*(d + e*x)^m*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(
g*(2*m + 3)), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)^(m - 1)*Simp[2*a*(e
*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)*x - (2*c*(d*g*m - e*f*(m + 1)))*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
```

```
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[
m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g} - \frac{\int \frac{(d+ex)\left(2a(2ef-3dg)+2(cdf-aeg)x+2c(3ef-2dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{7g}$$

$$= \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g} - \frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3} - \frac{2\int \frac{-acg^2\left(9e^2f^2-16defg+15d^2\right.}{}}{}$$

$$= \frac{4\left(5ae^2g^2+c\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g}$$

$$= \frac{4\left(5ae^2g^2+c\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g}$$

$$= \frac{4\left(5ae^2g^2+c\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g}$$

$$= \frac{4\left(5ae^2g^2+c\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7g}$$

Mathematica [C] time = 5.35991, size = 712, normalized size = 1.4

$$2\sqrt{f+gx}\left(g^2\left(a+cx^2\right)\left(10ae^2g^2+c\left(35d^2g^2+14deg(3gx-4f)+3e^2\left(8f^2-6fgx+5g^2x^2\right)\right)\right) - \frac{2\left(\sqrt{a}g(f+gx)^{3/2}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\right.}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(g^2*(a + c*x^2)*(10*a*e^2*g^2 + c*(35*d^2*g^2 + 14*d*e*g*
(-4*f + 3*g*x) + 3*e^2*(8*f^2 - 6*f*g*x + 5*g^2*x^2))) - (2*(g^2*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]*(a^2*e*g^2*(13*e*f - 42*d*g) + c^2*f*(24*e^2*f^2 - 5
6*d*e*f*g + 35*d^2*g^2)*x^2 + a*c*(35*d^2*f*g^2 - 14*d*e*g*(4*f^2 + 3*g^2*x
^2) + e^2*(24*f^3 + 13*f*g^2*x^2))) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(
a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] + Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(5*a*e^2*g^2 + (6*I)*Sqrt[a]*Sqr
t[c]*e*g*(3*e*f - 7*d*g) + c*(-24*e^2*f^2 + 56*d*e*f*g - 35*d^2*g^2))*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*c*g^5*Sqrt[a + c*x
```

```
^2])
```

Maple [B] time = 0.259, size = 3278, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(-84*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2)
)*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))
^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*c*d*e*g^5+26
*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/
2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Elliptic
E((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1
/2)*g+c*f))^(1/2))*a^2*c*e^2*f*g^4+70*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/
2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g
/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d^2*f*g^4+74*
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE
((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/
2)*g+c*f))^(1/2))*a*c^2*e^2*f^3*g^2-112*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^3*d*e*f^4*g+84*
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF
((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/
2)*g+c*f))^(1/2))*a^2*c*d*e*g^5-36*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*
((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*c*e^2*f*g^4-36*(-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-
(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*
g+c*f))^(1/2))*a*c^2*e^2*f^3*g^2-70*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
(-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2
),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*c*d^2*
g^5+112*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a
*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*E
llipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-
a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*c*d*e*f*g^4+70*(-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c
*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c
)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c
^3*d^2*f^3*g^2-56*a*c^2*d*e*f^2*g^3+48*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*
g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^3*e^2*f^5-14*x*a
*c^2*d*e*f*g^4+35*a*c^2*d^2*f*g^4+24*a*c^2*e^2*f^3*g^2+42*x^4*c^3*d*e*g^5-3
*x^4*c^3*e^2*f*g^4+25*x^3*a*c^2*e^2*g^5+6*x^3*c^3*e^2*f^2*g^3+35*x^2*c^3*d^
2*f*g^4+24*x^2*c^3*e^2*f^3*g^2+10*x*a^2*c*e^2*g^5+35*x*a*c^2*d^2*g^5+15*x^5
*c^3*e^2*g^5+35*x^3*c^3*d^2*g^5+10*a^2*c*e^2*f*g^4+10*(-(g*x+f)*c/((-a*c)^(
```

```
1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x
+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a
*c)^(1/2)*a^2*e^2*g^5-70*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a
*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)
^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c^2*d^2*f^2*g^3-48*
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF
((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/
2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c^2*e^2*f^4*g+84*(-(g*x+f)*c/((-a*c)^(1/2)*g
-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c
)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*
g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d*e
*f^2*g^3-38*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/
((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/
2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)
/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*c*e^2*f^2*g^3+112*(-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(
1/2))*(-a*c)^(1/2)*c^2*d*e*f^3*g^2-196*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*
g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d*e*f^2*g^3-
14*x^3*c^3*d*e*f*g^4+42*x^2*a*c^2*d*e*g^5+7*x^2*a*c^2*e^2*f*g^4-56*x^2*c^3*
d*e*f^2*g^3+6*x*a*c^2*e^2*f^2*g^3)/g^5/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}(ex + d)^2}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2 + 2\,dex + d^2\right)\sqrt{cx^2 + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}\,(d+ex)^2}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**2/sqrt(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

3.631 $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=364

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)(4ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg}{15g^2}$$

```
[Out] (-2*Sqrt[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*Sqrt[a + c*x^2])/(15*g^2) - (4*
Sqrt[-a]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a
]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[
-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
+ Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(4*e*f - 5*d*g)*(c*f^2 + a*g^
2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.377145, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {815, 844, 719, 424, 419}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)(4ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-}{15g^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*Sqrt[a + c*x^2])/(15*g^2) - (4*
Sqrt[-a]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a
]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[
-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
+ Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(4*e*f - 5*d*g)*(c*f^2 + a*g^
2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} + \frac{4\int \frac{-\frac{1}{2}acg(ef-5dg)+\frac{1}{2}c\left(3aeg^2+cf(4ef-5dg)\right)x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{15cg^2}$$

$$= -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{\left(2(4ef-5dg)\left(cf^2+ag^2\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{15g^3} + \cdots$$

$$= -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} + \frac{\left(4a\left(3aeg^2+cf(4ef-5dg)\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\cdots}{15\sqrt{-a}\sqrt{c}g^3\sqrt{\frac{c(f+g\cdots}{cf-\frac{a\cdots}{\sqrt{\cdots}}}}\cdots}$$

$$= -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{4\sqrt{-a}\left(3aeg^2+cf(4ef-5dg)\right)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\cdots}{15\sqrt{c}g^3\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\cdots}$$

Mathematica [C] time = 4.93431, size = 545, normalized size = 1.5

$$\frac{\sqrt{f+gx}\left(\frac{2\left(a+cx^2\right)(5dg-4ef+3egx)}{g^2}+\frac{4\left(\sqrt{a}\sqrt{c}g(f+gx)^{3/2}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(\sqrt{c}(5dg-4ef)+3i\sqrt{a}eg\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f}{\sqrt{c}f}\right.\right.}{}\right.}{15\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(-4*e*f + 5*d*g + 3*e*g*x)*(a + c*x^2))/g^2 + (4*(g^2*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(a + c*x^2
) - Sqrt[c]*(I*Sqrt[c]*f - Sqrt[a]*g)*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((3*I)*Sqrt[a]*e*g + Sqr
t[c]*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-
(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
 + g*x))))/(15*Sqrt[a + c*x^2])
```

Maple [B] time = 0.302, size = 1828, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/
2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g
/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*e*g^4+6*(-(g*x+
f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f
))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x
+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*
f))^(1/2))*a*c*e*f^2*g^2-10*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a
*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*d*g^4+8*(-(g*x
+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*
f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c
*f))^(1/2))*(-a*c)^(1/2)*a*e*f*g^3-10*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/
2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g
/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d*f^
2*g^2+8*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a
*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*E
llipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-
```

```
a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*e*f^3*g-6*(-(g*x+f)*c/((-a*c)^(1/2
)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-
a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/
2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*e*
g^4+10*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*El
lipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a
*c)^(1/2)*g+c*f))^(1/2))*a*c*d*f*g^3-14*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e*f^2*g^2+10*
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE
((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/
2)*g+c*f))^(1/2))*c^2*d*f^3*g-8*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-
((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*e*f^4+3*x^4*c^2*e*g^4
+5*x^3*c^2*d*g^4-x^3*c^2*e*f*g^3+3*x^2*a*c*e*g^4+5*x^2*c^2*d*f*g^3-4*x^2*c^
2*e*f^2*g^2+5*x*a*c*d*g^4-x*a*c*e*f*g^3+5*a*c*d*f*g^3-4*a*c*e*f^2*g^2)/c/(c
*g*x^3+c*f*x^2+a*g*x+a*f)/g^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}\,(d+ex)}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)/sqrt(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

3.632 $\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=322

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin\ldots\right.}{3g^2\sqrt{a+cx^2}\sqrt{\ldots}}$$

[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) + (4*Sqrt[-a]*Sqrt[c]*f*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])

Rubi [A] time = 0.204486, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {735, 844, 719, 424, 419}

$$-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1\ldots}}{\ldots}\right.\right.}{3g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+g\ldots)}{\sqrt{-a}g+\sqrt{\ldots}}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]

[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) + (4*Sqrt[-a]*Sqrt[c]*f*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g}+\frac{2\int \frac{ag-cfx}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3g}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g}+\frac{1}{3}\left(2\left(a+\frac{cf^2}{g^2}\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx-\frac{(2cf)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g^2}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g}-\frac{\left(4a\sqrt{c}f\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{3\sqrt{-a}g^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}+\frac{\left(4a\left(a+\right.\right.}{}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g}+\frac{4\sqrt{-a}\sqrt{c}f\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}-\frac{4\sqrt{-a}\left(a+\frac{cf^2}{g^2}\right)}{}$$

Mathematica [C] time = 2.02741, size = 456, normalized size = 1.42

$$\frac{2\sqrt{f+gx}\left(g^2\left(a+cx^2\right)-\frac{2\left(-\sqrt{a}g(f+gx)^{3/2}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+fg^2\left(a+cx^2\right)\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}+\right.}{(f+gx)\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}\right)}{3g^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(g^2*(a + c*x^2) - (2*(f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]*(a + c*x^2) + Sqrt[c]*f*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a
])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))
]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt
[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqrt[a]*
g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*S
qrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
+ g*x))))/(3*g^3*Sqrt[a + c*x^2])
```

Maple [B] time = 0.299, size = 688, normalized size = 2.1

$$-\frac{2}{3\,c\left(cgx^3+cfx^2+agx+af\right)g^3}\sqrt{gx+f}\sqrt{cx^2+a}\left(2\sqrt{-\frac{c\left(gx+f\right)}{\sqrt{-ac}g-cf}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{\sqrt{-ac}g+cf}}\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{\sqrt{-ac}g-cf}}\mathrm{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] -2/3*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/
2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g
/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*g^3+
2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1
/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellipti
cF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(
1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*f^2*g-2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f)
)^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/
2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*f*g^2-2*(-
(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*
g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((
-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)
*g+c*f))^(1/2))*c^2*f^3-x^3*c^2*g^3-x^2*c^2*f*g^2-x*a*c*g^3-a*c*f*g^2)/c/(c
*g*x^3+c*f*x^2+a*g*x+a*f)/g^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/sqrt(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

3.633 $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=473

$$\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(dg+ef)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{I}}{e^2\sqrt{a+cx^2}\sqrt{f}}$$

```
[Out] (-2*Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])
/(e*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*Sqrt[c]*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt
[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*g*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqr
t[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e),
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f +
 Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rubi [A] time = 0.639387, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {923, 933, 168, 538, 537, 844, 719, 424, 419}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\left(ae^2+cd^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}+\frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(dg+ef)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F}{e^2g\sqrt{a+cx^2}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```


```
[Out] (-2*Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])
/(e*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*Sqrt[c]*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt
[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*g*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqr
t[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e),
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f +
 Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```


Rule 923

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)
]), x_Symbol] :> Dist[(c*d^2 + a*e^2)/e^2, Int[1/((d + e*x)*Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d - c*e*x)/(Sqrt[f + g*x]*Sqrt
[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &
```

```
& NeQ[c*d^2 + a*e^2, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx - \frac{\int \frac{cd-cex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2}$$

$$= \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{eg} - \frac{(c(ef+dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2 g} + \frac{\left(\left(a+\frac{cd^2}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)}}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{\sqrt{a+cx^2}} + \frac{(2a\sqrt{\cdots}}{}$$

$$= -\frac{2\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\sqrt{c}(ef+dg)\sqrt{\frac{\sqrt{c}(f+g\cdots}{\sqrt{c}f+\sqrt{-\cdots}}}}{e^2\cdots}$$

$$= -\frac{2\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}\sqrt{c}(ef+dg)\sqrt{\frac{\sqrt{c}(f+g\cdots}{\sqrt{c}f+\sqrt{-\cdots}}}}{e^2\cdots}$$

Mathematica [C] time = 4.42265, size = 1096, normalized size = 2.32

$$-\frac{2\left(-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3+2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2+cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f-2cdeg\sqrt{-f\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(-(c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]] - a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d
*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]*(f + g*x) - 2*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g
*x) - c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a
]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*
Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]
```

```
*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)] - I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g
*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellip
ticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[
c]*f + I*Sqrt[a]*g)]))/(e^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.325, size = 1216, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] 2*(EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)
/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3-EllipticF((-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1
/2))*(-a*c)^(1/2)*c*e^2*f^2*g+EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*e*g^3-Ellipt
icF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^
(1/2)*g+c*f))^(1/2))*a*c*e^2*f*g^2-EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d^2*f*g^2
+EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(
(-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*e*f^2*g-EllipticE((-(g*x+f)*c/((-a*c)^(1/
2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*
e*g^3+EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c
*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e^2*f*g^2-EllipticE((-(g*x+f)*c/((-a*c
)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c
^2*d*e*f^2*g+EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1
/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*e^2*f^3-EllipticPi((-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1
/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3-EllipticPi((
-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(
-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3+E
llipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/
(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e^2*f*g^2
+EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/
c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d^2*f*g
^2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*
g/((-a*c)^(1/2)*g+c*f))^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*(g*x+
f)^(1/2)*(c*x^2+a)^(1/2)/g^2/c/e^2/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)\sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.634 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=694

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-dg)}+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{Ellipti}}{e\sqrt{a+cx^2}\sqrt{f+}}$$

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x))) - (Sqrt[-a]*Sqrt
[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[
-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (
c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g
)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) - (Sqrt[-a]*Sqrt[c]*(2*e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*(e*f - d*g)*Sqrt[
f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*(f
 + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((S
qrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2
*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*
f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 1.72244, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {925, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-dg)}+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(ae^2g+cd(2ef-dg)\right)\text{I}}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}}{\sqrt{-}}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x))) - (Sqrt[-a]*Sqrt
[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[
-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (
c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g
)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) - (Sqrt[-a]*Sqrt[c]*(2*e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*(e*f - d*g)*Sqrt[
f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*(f
 + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((S
qrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2
*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*
```

```
f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/((m + 1)*(e*f - d*g)), x] - Dist[1/(2*(m + 1)*(e*f - d*g)), Int[((d + e*
x)^(m + 1)*Simp[a*g*(2*m + 3) + 2*(c*f)*x + c*g*(2*m + 5)*x^2, x])/(Sqrt[f
+ g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f
- d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \frac{-ag+2cfx+cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \left(\frac{c(2ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{cgx}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{-ae^2g-cd(2ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} + \frac{(c(2ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2(ef-dg)} - \frac{\left(ag+\frac{cd(2ef-\dots}{e^2}\right.}{}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} - \frac{\left(\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\int \dots}{2(ef-dg)\dots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)\dots}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{\sqrt{-a}\sqrt{c}f\dots}{\dots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{\sqrt{-a}\sqrt{c}f\dots}{\dots}$$

Mathematica [C] time = 6.83651, size = 1336, normalized size = 1.93

$$\sqrt{f+gx}\left(\frac{cx^2+a}{d+ex}-\frac{ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3-2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2-cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2+ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f+2cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f-2icdeg\sqrt{\frac{g\left(x+\frac{i\cdots}{\cdots}\right)}{f+g\cdots}}\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*((a + c*x^2)/(d + e*x) - (c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]] - c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*e^2*f*g^2*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] -
2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*c*d*e*f*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x)^2 - c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + I
*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
+ g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f
+ I*Sqrt[a]*g)*(Sqrt[a]*e*g + I*Sqrt[c]*(2*e*f - d*g))*Sqrt[(g*((I*Sqrt[a])
/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*
(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f
 + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (2*I)*c*d*
e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/S
qrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/
(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/S
qrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + I*c*d
^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))
/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a*
e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e^2
*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(f + g*x))))/((-(e*f) + d*g
)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.313, size = 6034, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)^2 \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.635 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1241

result too large to display

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*g
+ c*d*(2*e*f + d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e^2)*(e*f
 - d*g)^2*(d + e*x)) + (Sqrt[-a]*Sqrt[c]*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sq
rt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)*(
e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*
x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*
g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
t[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^2*(e*f - d*g)*Sqrt[f + g*
x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*f*(3*a*e^2*g + c*d*(2*e*f + d*g))*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqr
t[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c
*x^2]) + (Sqrt[-a]*Sqrt[c]*d*g*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*
g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
(c*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
(c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^2
*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((
a*e^2*g - c*d*(2*e*f - 3*d*g))*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)
/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]]
, (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^2*((Sqrt[c]*d)/Sqrt[-a] +
e)*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 4.23663, antiderivative size = 1241, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {925, 6742, 719, 419, 940, 844, 424, 933, 168, 538, 537}

$$\frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)\left(3age^2+cd(2ef+dg)\right)}{4e\left(cd^2+ae^2\right)(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}-\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}F\left(\cdots\right.}{4e\left(cd^2+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*g
+ c*d*(2*e*f + d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e^2)*(e*f
 - d*g)^2*(d + e*x)) + (Sqrt[-a]*Sqrt[c]*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sq
rt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)*(
e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*
x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*
g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
```

```
t[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^2*(e*f - d*g)*Sqrt[f + g*
x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*f*(3*a*e^2*g + c*d*(2*e*f + d*g))*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqr
t[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c
*x^2]) + (Sqrt[-a]*Sqrt[c]*d*g*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*
g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
(c*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
(c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sq
rt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^2
*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((
a*e^2*g - c*d*(2*e*f - 3*d*g))*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)
/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]]
, (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^2*((Sqrt[c]*d)/Sqrt[-a] +
e)*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/((m + 1)*(e*f - d*g)), x] - Dist[1/(2*(m + 1)*(e*f - d*g)), Int[((d + e*
x)^(m + 1)*Simp[a*g*(2*m + 3) + 2*(c*f)*x + c*g*(2*m + 5)*x^2, x])/(Sqrt[f
+ g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f
- d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \frac{-3ag+2cfx-cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \left(-\frac{cg}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{-3ae^2g-cd(2ef+dg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{2c(ef+dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e^2(ef-dg)} + \frac{(c(ef+dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e^2(ef-dg)} - \frac{\left(3ae^2g \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\int \cdots}{8e^2\left(cd^2+\cdots\right.}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1 \cdots}}{2e^2(ef-\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1 \cdots}}{2e^2(ef-\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1 \cdots}}{2e^2(ef-\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1 \cdots}}{2e^2(ef-\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\sqrt{c}\left(3ae^2g+cd(2ef\cdots\right.}{4e\left(cd^{\cdots}\right.}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\left(3ae^2g+cd(2ef+dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)} + \frac{\sqrt{-a}\sqrt{c}\left(3ae^2g+cd(2ef\cdots\right.}{4e\left(cd^{\cdots}\right.}$$

Mathematica [C] time = 11.0399, size = 2197, normalized size = 1.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] (c^2*d^2*f^3 - 3*a*c*e^2*f^3 - (2*c^2*d*e*f^4)/g + (c^2*d^3*f^2*g)/e + a*c*
d*e*f^2*g + a*c*d^2*f*g^2 - 3*a^2*e^2*f*g^2 + (a*c*d^3*g^3)/e + 3*a^2*d*e*g
^3 - 2*c^2*d^2*f^2*(f + g*x) + 6*a*c*e^2*f^2*(f + g*x) + (4*c^2*d*e*f^3*(f
+ g*x))/g - (2*c^2*d^3*f*g*(f + g*x))/e - 6*a*c*d*e*f*g*(f + g*x) + c^2*d^2
*f*(f + g*x)^2 - 3*a*c*e^2*f*(f + g*x)^2 - (2*c^2*d*e*f^2*(f + g*x)^2)/g +
(c^2*d^3*g*(f + g*x)^2)/e + 3*a*c*d*e*g*(f + g*x)^2 - ((e*f - d*g)*(f + g*x
)*(a + c*x^2)*(a*e^2*(2*e*f - 5*d*g - 3*e*g*x) - c*d*(3*d^2*g + 2*e^2*f*x +
 d*e*g*x)))/(d + e*x)^2 + (Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d
*g)*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f +
 g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*
f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*g*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]) + ((I*Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(-3*a*e^2*g
 - (6*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g) + c*d*(-4*e*f + d*g))*Sqrt[(g*((I*Sq
rt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g
*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + ((4*I)*a*c*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/S
qrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
 + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)
), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] +
 ((4*I)*c^2*d^3*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]
*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[
a]*g)])/(e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) - ((4*I)*a*c*d*e*f*g*Sqrt[(g*(
(I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(
f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f +
 I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]] + ((6*I)*a*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (I*c^2*d^4*g^2*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c
]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*
x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]) + ((3*I)*a^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(
3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(4*(c*d^2
 + a*e^2)*(e*f - d*g)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.357, size = 19170, normalized size = 15.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)^3 \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.636 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=531

$$-\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(25ae^2g^2-c\left(105d^2g^2-42defg+8e^2f^2\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{}{\sqrt{-}}\right.}{105c^{5/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[Out] (-2*e*(25*a*e^2*g^2 + c*(7*e^2*f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x
]*Sqrt[a + c*x^2])/(105*c^2*g^2) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(7*c) + (2*e^2*(e*f + 11*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*
c*g^2) + (2*Sqrt[-a]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^3*f^3 - 42*d*e^
2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]
*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-
a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
+ Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*e*(c*f^2 + a*g^2)*(25*a*e^2*g
^2 - c*(8*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sq
rt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(5/2
)*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 1.12241, antiderivative size = 527, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {942, 1654, 844, 719, 424, 419}

$$-\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(25ae^2g^2-c\left(105d^2g^2-42defg+8e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{105c^{5/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*(90*d^2 - e^2*((25*a)/c + (7*f^2)/g^2) - (12*d*e*f)/g)*Sqrt[f + g*x]*S
qrt[a + c*x^2])/(105*c) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(
7*c) + (2*e^2*(e*f + 11*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*c*g^2) +
(2*Sqrt[-a]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^3*f^3 - 42*d*e^2*f^2*g +
 105*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(105*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a
]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*e*(c*f^2 + a*g^2)*(25*a*e^2*g^2 - c*(8
*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(5/2)*g^3*Sqr
t[f + g*x]*Sqrt[a + c*x^2])
```

Rule 942

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)*Simp
```

```
[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(
4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d*g*(4*m - 1))*x^2, x])/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} - \frac{\int \frac{(d+ex)\left(-7cd^2f+ae(4ef+dg)+\left(5ae^2g-cd(12ef+7dg)\right)x-ce(ef+11dg)x^2\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{7c}$$

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2} - \frac{2\int \frac{-\frac{1}{2}cg^2\left(35cd^3fg-ae(3\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a\ldots}}{7c}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a\ldots}}{7c}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a\ldots}}{7c}$$

$$= -\frac{2e\left(25ae^2g^2+c\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a\ldots}}{7c}$$

Mathematica [C] time = 6.34307, size = 747, normalized size = 1.41

$$2\sqrt{f+gx}\left(\frac{g\sqrt{f+gx}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(25a^{3/2}e^3g^2+\sqrt{a}ce\left(-105d^2g^2+42defg-8e^2f^2\right)+3ia\sqrt{c}e^2g(2ef-63dg)+105ic^{3/2}d^3g^2\right)\text{EllipticF}\left(\ldots\right.}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-(g^2*(a + c*x^2)*(25*a*e^3*g^2 + c*e*(-105*d^2*g^2 - 21*
d*e*g*(f + 3*g*x) + e^2*(4*f^2 - 3*f*g*x - 15*g^2*x^2)))) + (g^2*(-(a^2*e^2
*g^2*(19*e*f + 189*d*g)) + c^2*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^
2 + 105*d^3*g^3)*x^2 + a*c*(105*d^2*e*f*g^2 + 105*d^3*g^3 - 21*d*e^2*g*(2*f
^2 + 9*g^2*x^2) + e^3*(8*f^3 - 19*f*g^2*x^2))))/(f + g*x) + I*c*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(-(a*e^2*g^2*(19*e*f + 189*d*g)) + c*(8*e^3*f^3 - 42*
d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f +
g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (g*(Sqrt[c]*f + I*Sqr
t[a]*g)*((105*I)*c^(3/2)*d^3*g^2 + 25*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2
*g*(2*e*f - 63*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 42*d*e*f*g - 105*d^2*g^2))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
 - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
```

```
]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(105*c^2*g^4*Sqrt[a + c*x^2])
```

Maple [B] time = 0.32, size = 3922, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/105*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(8*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^3*e^3*f^5-84*x*
a*c^2*d*e^2*f*g^4+42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^
(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-
c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/
2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*c*d*e^2*f*g^4+105*(-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-
(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*
g+c*f))^(1/2))*c^3*d^2*e*f^3*g^2-42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
(-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2
),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^3*d*e^2*f^4*g+189*(
-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)
*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF(
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2
)*g+c*f))^(1/2))*a^2*c*d*e^2*g^5-11*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
(-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2
),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*e^3*f^3*g^2+25*
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2
)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF
((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/
2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a^2*e^3*g^5-105*(-(g*x+f)*c/((-a*c)^(1/2)*g-
c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)
^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g
-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^3*d^3*f^
2*g^3+105*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)
*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(
(-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d^3*g^5+105*(-(g*x+f)*c/((-a*c)^(1/2)*g-c
*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^
(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-
c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^3*d^3*f^2
*g^3-6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*El
lipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a
*c)^(1/2)*g+c*f))^(1/2))*a^2*c*e^3*f*g^4-6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f)
)^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/
2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*e^3*f^3*
g^2-8*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c
)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ell
ipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*
c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c^2*e^3*f^4*g-189*(-(g*x+f)*c/((-a*c)^
```

```
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^
2*c*d*e^2*g^5-19*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2
))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f)
)^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g
-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*c*e^3*f*g^4+4*x^2*c^3*e^3*f^3*g^2-10
5*a*c^2*d^2*e*f*g^4-21*a*c^2*d*e^2*f^2*g^3+25*a^2*c*e^3*f*g^4+4*a*c^2*e^3*f
^3*g^2+25*x*a^2*c*e^3*g^5-63*x^4*c^3*d*e^2*g^5-18*x^4*c^3*e^3*f*g^4+10*x^3*
a*c^2*e^3*g^5-105*x^3*c^3*d^2*e*g^5+x^3*c^3*e^3*f^2*g^3-15*x^5*c^3*e^3*g^5-
105*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^
(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellip
ticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)
^(1/2)*g+c*f))^(1/2))*a*c^2*d^3*g^5+189*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d*e^2*f^2*g
^3-105*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*El
lipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a
*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*c*d^2*e*g^5+17*(-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-
a*c)^(1/2)*a*c*e^3*f^2*g^3-105*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c
*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c
)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-(
(-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c^2*d^2*e*f^2
*g^3+42*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a
*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*E
llipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-
a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c^2*d*e^2*f^3*g^2+105*(-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2
)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/
2))*a*c^2*d^2*e*f*g^4-231*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-
a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/
2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c
)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c^2*d*e^2*f^2*g^3-84*x^3*c^3*
d*e^2*f*g^4-63*x^2*a*c^2*d*e^2*g^5+7*x^2*a*c^2*e^3*f*g^4-105*x^2*c^3*d^2*e*
f*g^4-21*x^2*c^3*d*e^2*f^2*g^3-105*x*a*c^2*d^2*e*g^5+x*a*c^2*e^3*f^2*g^3)/(
c*g*x^3+c*f*x^2+a*g*x+a*f)/c^3/g^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^{3}x^{3}+3\,de^{2}x^{2}+3\,d^{2}ex+d^{3}\right)\sqrt{gx+f}}{\sqrt{cx^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(g*x + f)/sqrt(c*x^2
 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{3}\sqrt{f+gx}}{\sqrt{a+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^{3}\sqrt{gx+f}}{\sqrt{cx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.637 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=410

$$-\frac{4\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)(ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+g\cdots}}{\cdots}$$

```
[Out] (2*e*(e*f + 7*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g) + (2*e*(d + e*x)
*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c) + (2*Sqrt[-a]*(9*a*e^2*g^2 + c*(2*e^2
*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(15*c^(3/2)*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]
*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*e*(e*f - 5*d*g)*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f
 - a*g)])/(15*c^(3/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.56402, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {942, 1654, 844, 719, 424, 419}

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\left(9ae^2g^2+c\left(-15d^2g^2-10defg+2e^2f^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}-\frac{4\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*(e*f + 7*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g) + (2*e*(d + e*x)
*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c) + (2*Sqrt[-a]*(9*a*e^2*g^2 + c*(2*e^2
*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(15*c^(3/2)*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]
*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*e*(e*f - 5*d*g)*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f
 - a*g)])/(15*c^(3/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 942

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)*Simp
[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(
4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d*g*(4*m - 1))*x^2, x])/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c} - \frac{\int \frac{-5cd^2f+ae(2ef+dg)+\left(3ae^2g-cd(8ef+5dg)\right)x-ce(ef+7dg)x^2}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{5c}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c} - \frac{2\int \frac{-\frac{1}{2}cg^2\left(15cd^2f-ae(7ef+10dg\right.}{}}{}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c} - \frac{1}{15}\left(-15d^2+e^2\left(\frac{9a}{c}+\frac{2f^2}{g^2}\right.\right.$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c} - \frac{\left(2a\left(-15d^2+e^2\left(\frac{9a}{c}+\frac{2f^2}{g^2}\right.\right.\right.}{}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5c} - \frac{2\sqrt{-a}\left(15d^2-e^2\left(\frac{9a}{c}+\frac{2f^2}{g^2}\right.\right.}{}$$

Mathematica [C] time = 4.59343, size = 596, normalized size = 1.45

$$\frac{2\sqrt{f+gx}\left(\frac{\sqrt{c}g\sqrt{f+gx}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(2\sqrt{a}\sqrt{c}e(ef-5dg)-9iae^2g+15icd^2g\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}+\frac{g^2(-9a^2\cdots}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e*g^2*(a + c*x^2)*(10*d*g + e*(f + 3*g*x)) + (g^2*(-9*a
^2*e^2*g^2 + c^2*(-2*e^2*f^2 + 10*d*e*f*g + 15*d^2*g^2)*x^2 + a*c*(10*d*e*f
*g + 15*d^2*g^2 - e^2*(2*f^2 + 9*g^2*x^2))))/(f + g*x) - I*c*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]*(9*a*e^2*g^2 + c*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
 - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)] + (Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((15*I)*c*d^2*g - (9*I)*a*e^2*
g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(
f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*El
lipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]
*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]))/(15*c^2*g^3*Sqrt[a + c*x^2])
```

Maple [B] time = 0.297, size = 2470, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/15*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(15*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d^2*g^4+15*(-
(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*
g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((
-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)
*g+c*f))^(1/2))*c^2*d^2*f^2*g^2-15*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*
((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d^2*g^4+10*(-(g*x+
f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f
))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x
+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*
f))^(1/2))*a*c*d*e*f*g^3-10*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a
*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d*e*f^2*g^2-10
*a*c*d*e*f*g^3-10*x^3*c^2*d*e*g^4-4*x^3*c^2*e^2*f*g^3-3*x^2*a*c*e^2*g^4-x^2
*c^2*e^2*f^2*g^2-15*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(
1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c
*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2
)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d^2*f^2*g^2-2*(-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c
*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c
)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c
^2*e^2*f^4+9*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g
/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1
/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f
)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*e^2*g^4-9*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(
1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c
*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*e^2*g^4-
10*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellipt
icF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^
(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*d*e*g^4+2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c
*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^
(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-
c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2
)*a*e^2*f*g^3+2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2)
)*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))
^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-
c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*e^2*f^3*g+9*(-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2
)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/
2))*a*c*e^2*f^2*g^2-11*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c
)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*
g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(
1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e^2*f^2*g^2+10*(-(g*x+f)*c/((-
a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)
*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2
))*c^2*d*e*f^3*g-a*c*e^2*f^2*g^2-3*x^4*c^2*e^2*g^4-10*x*a*c*d*e*g^4-4*x*a*c
```

```
*e^2*f*g^3-10*x^2*c^2*d*e*f*g^3)/c^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{gx+f}}{\sqrt{cx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.638 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=331

$$\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(3dg+ef)E\left(\right.}{3\sqrt{c}g\sqrt{a+cx^2}\sqrt{}}$$

```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c) - (2*Sqrt[-a]*(e*f + 3*d*g)*Sqrt[
f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*e*(c*
f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*g*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.264131, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {833, 844, 719, 424, 419}

$$\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\left(ag^2+cf^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(3dg+ef)E\left(\sin^{-1}\left(\right.\right.}{3\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+g}{\sqrt{-a}g+\sqrt{}}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c) - (2*Sqrt[-a]*(e*f + 3*d*g)*Sqrt[
f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*e*(c*
f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*g*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}(3cdf-aeg)+\frac{1}{2}c(ef+3dg)x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} + \frac{(ef+3dg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g} - \frac{\left(e\left(cf^2+ag^2\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3cg}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} + \frac{\left(2a(ef+3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-}}{}\right.}{3\sqrt{-a}\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c} - \frac{2\sqrt{-a}(ef+3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 3.43348, size = 464, normalized size = 1.4

$$\frac{2\sqrt{f+gx}\left(\frac{i\sqrt{f+gx}\left(3\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}} + \frac{ic\sqrt{f+gx}\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(3dg+ef)\sqrt{\frac{g(}{}}}{}\right.}{3c\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(e*(a + c*x^2) + ((e*f + 3*d*g)*(a + c*x^2))/(f + g*x) + (
I*c*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f + 3*d*g)*Sqrt[(g*((I*Sqrt[a])/Sqr
t[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt
[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x
]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/g^2 + (I*(3*Sqrt[
c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f +
g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]])))/(3*c*Sqrt[a + c*x^2])
```

Maple [B] time = 0.265, size = 1286, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*
((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*e*g^3+(
-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)
*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF(
(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2
)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*e*f^2*g+3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))
^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2
))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f)
)^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*g^3+3*(-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-
(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*
g+c*f))^(1/2))*c^2*d*f^2*g-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^
(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-
a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*g^3-(-(g*x+f)*c/((-a*c
)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*
c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*
a*c*e*f*g^2-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*
g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(
1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*
f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*f^2*g-(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1
/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*e*f^3+x^3
*c^2*e*g^3+x^2*c^2*e*f*g^2+x*a*c*e*g^3+a*c*e*f*g^2)/(c*g*x^3+c*f*x^2+a*g*x+
a*f)/c^2/g^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.639 $\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

```
[Out] (-2*Sqrt[-a]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c
]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0573208, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {719, 424}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c
]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{\left(2a\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 0.46476, size = 294, normalized size = 2.16

$$\frac{2i\sqrt{f+gx}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(\sqrt{a}+i\sqrt{c}x\right)}{\sqrt{a}g-i\sqrt{c}f}}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right),\frac{\sqrt{c}f}{\sqrt{c}f}\right.\right.}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{g\left(\sqrt{c}x+i\sqrt{a}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

```
[Out] ((2*I)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqr
t[c]*f + Sqrt[a]*g)]*Sqrt[f + g*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(f
+ g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/
(Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt[a + c
*x^2])
```

Maple [B] time = 0.261, size = 396, normalized size = 2.9

$$2\frac{\sqrt{gx+f}\sqrt{cx^2+a}\left(cf-\sqrt{-acg}\right)}{g\left(cgx^3+cfx^2+agx+af\right)c^2}\sqrt{-\frac{c\left(gx+f\right)}{\sqrt{-acg}-cf}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{\sqrt{-acg}+cf}}\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{\sqrt{-acg}-cf}}\left(\sqrt{-ac}\text{EllipticF}\left(\sqrt{-\frac{c\left(gx+\right.}{\sqrt{-acg}-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(c*f-(-a*c)^(1/2)*g)*(-(g*x+f)*c/((-a*c)^(1
/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*((-a*c)^(1/2)*EllipticF((-(g*x+
f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f
))^(1/2))*g+f*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(
1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c-EllipticE((-(g*x+f)*c/((-a*c)^(1
/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c
)^(1/2)*g-EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)
*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*f)/g/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

3.640 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=319

$$-\frac{2\sqrt{-a}g\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin\right.}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}\right.}$$

```
[Out] (-2*Sqrt[-a]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
(c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
 (2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
 (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*
((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.487925, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {944, 719, 419, 933, 168, 538, 537}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}-\frac{2\sqrt{-a}g\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{\sqrt{c}e\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
(c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
 (2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
 (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*
((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
```

```
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e}$$

$$= \frac{\left((ef-dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{e\sqrt{a+cx^2}} + \frac{\left(2ag\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int -\right.}{\sqrt{-a}\sqrt{c}e\sqrt{f+}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left(2(ef-dg)\sqrt{1+\frac{cx^2}{a}}\right)\text{Sub}}{}$$

$$= -\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{\left(2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\right.}{}$$

$$= -\frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{2(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1}}{e\left(\frac{\sqrt{c}d}{\sqrt{-a}}\right.}$$

Mathematica [C] time = 1.21592, size = 300, normalized size = 0.94

$$-\frac{2i\sqrt{f+gx}\sqrt{\frac{g(\sqrt{a}+i\sqrt{c}x)}{\sqrt{a}g-i\sqrt{c}f}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\Pi\left(\frac{e\left(f-\frac{i\sqrt{a}g}{\sqrt{c}}\right)}{ef-dg};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{}}{\sqrt{c}f+i\sqrt{}}\right.\right.}{e\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{g(\sqrt{c}x+i\sqrt{a})}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*Sqrt
[f + g*x]*(EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sq
rt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Ellipti
cPi[(e*(f - (I*Sqrt[a]*g)/Sqrt[c]))/(e*f - d*g), I*ArcSinh[Sqrt[-((Sqrt[c]*
(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)]))/(e*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x)
)]*Sqrt[a + c*x^2])
```

Maple [A] time = 0.249, size = 439, normalized size = 1.4

$$2\frac{\sqrt{gx+f}\sqrt{cx^2+a}}{ce\left(cgx^3+cfx^2+agx+af\right)}\sqrt{-\frac{c\left(gx+f\right)}{\sqrt{-ac}g-cf}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{\sqrt{-ac}g+cf}}\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{\sqrt{-ac}g-cf}}\left(f\text{EllipticF}\left(\sqrt{-\frac{c\left(gx+f\right)}{\sqrt{-ac}g-cf}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g-c*f))^(1/2)*(f*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c-(-a*c)^(1/2)*Ellipti
cF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(
1/2)*g+c*f))^(1/2))*g-EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((
-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f
))^(1/2))*c*f+EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1
/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2)
)*(-a*c)^(1/2)*g)/c/e/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

3.641 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=698

$$\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd^2\right)}-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\right.}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+c\right.}$$

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (Sqrt[-a
]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 +
a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
+ (Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
 (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[
a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(c*d^2 + a*e^2)*Sqr
t[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/(
(Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*
d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 1.93269, antiderivative size = 698, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {946, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd^2\right)}-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}{e\sqrt{a+cx^2}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```


```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (Sqrt[-a
]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 +
a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
+ (Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
 (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[
a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(c*d^2 + a*e^2)*Sqr
t[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/(
(Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*
d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 946

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^
2])/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x)^(m + 1)*Simp[2*c*d*f*(m + 1) - e*(a*g) + 2*c*(d*g*(m + 1) - e*f
*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-2cdf-aeg-2cdgx-cegx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \left(-\frac{cdg}{e\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{cgx}{\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{-ae^2g-cd(2ef-dg)}{e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{2\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e\left(cd^2+ae^2\right)} + \frac{\left(ae^2g+cd(2ef-\ldots\right.}{2e\,(\ldots}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)} + \frac{\left(\left(ae^2g+cd(2ef-dg)\right)\sqrt{1-\ldots\right.}{2e\left(cd^2\ldots\right.}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e\left(cd^2+ae^2\right)\sqrt{f+gx}\sqrt{a+cx^2}} - \ldots$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{\sqrt{-a}\sqrt{c}\ldots}{\ldots}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\left(cd^2+ae^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{\sqrt{-a}\sqrt{c}\ldots}{\ldots}$$

Mathematica [C] time = 6.17204, size = 1330, normalized size = 1.91

$$\sqrt{f+gx}\left(-\frac{(cx^2+a)e^2}{d+ex}-\frac{-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3+2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2+cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f-2cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f-2icdeg\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-(c*e^2*f^3*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*e^2
*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c*d*
e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
+ g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((
I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f
 + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] +
e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a]
)/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]
*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[
f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (2*I)*c*d
*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))
/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + I*c*
d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a
*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g
))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(f + g*x))))/((c*d^2*e + a*e^3
)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.263, size = 5743, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

3.642 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=1246

result too large to display

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (e*(a*
e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e
^2)^2*(e*f - d*g)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g + c*d*(6*e*f - 5*
d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*
x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*
e^2)^2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[
a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqr
t[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a
]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e*(c*d^2 + a*e^2)*Sqr
t[f + g*x]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g + c*d*(6*e*f - 5
*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/
a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[(
Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*
f - a*g)])/(4*e*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]
)/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^
2]) + ((a*e^2*g + c*d*(6*e*f - 5*d*g))*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Elliptic
Pi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/
Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e*((Sqrt[c]*d)/Sqrt[
-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 4.40228, antiderivative size = 1246, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {946, 6742, 719, 419, 940, 844, 424, 933, 168, 538, 537}

$$-\frac{\left(age^2+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{cx^2+ae}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}-\frac{\sqrt{f+gx}\sqrt{cx^2+ae}}{2\left(cd^2+ae^2\right)(d+ex)^2}-\frac{\sqrt{-a}\sqrt{c}\left(age^2+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{\frac{cx^2}{a}}}{4\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (e*(a*
e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e
^2)^2*(e*f - d*g)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g + c*d*(6*e*f - 5*
d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*
x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*
e^2)^2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[
a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqr
t[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a
```

```
]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e*(c*d^2 + a*e^2)*Sqr
t[f + g*x]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g + c*d*(6*e*f - 5
*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/
a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[(
Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*
f - a*g)])/(4*e*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]
)/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^
2]) + ((a*e^2*g + c*d*(6*e*f - 5*d*g))*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Elliptic
Pi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/
Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e*((Sqrt[c]*d)/Sqrt[
-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 946

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*x^
2])/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x)^(m + 1)*Simp[2*c*d*f*(m + 1) - e*(a*g) + 2*c*(d*g*(m + 1) - e*f
*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
```

```
m] && LeQ[m, -2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{-4cdf-aeg+2c(ef-2dg)x+cegx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{\int \left(\frac{cg}{e\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{-ae^2g-cd(6ef-5dg)}{e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{2c(ef-3dg)}{e(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx}{4\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e\left(cd^2+ae^2\right)} - \frac{(c(ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2e\left(cd^2+ae^2\right)} + \frac{(ae}{}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} - \frac{\left(ae^2g+cd(6ef-5d\right.}{8e\left(cd\right.}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}}{2e\left(c\right.}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}}{2e\left(c\right.}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}}{2e\left(c\right.}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} + \frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}}{2e\left(c\right.}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\left(ae^2g+cd(6\right.}{4}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e\left(ae^2g+cd(6ef-5dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}\left(ae^2g+cd(6\right.}{4}$$

Mathematica [C] time = 11.3971, size = 2450, normalized size = 1.97

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (-11*c^2*d^2*e^2*f^3 + a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 5*c^2*d^3*e*f^2*
g + 5*a*c*d*e^3*f^2*g - 11*a*c*d^2*e^2*f*g^2 + a^2*e^4*f*g^2 + 5*a*c*d^3*e*
g^3 - a^2*d*e^3*g^3 + 22*c^2*d^2*e^2*f^2*(f + g*x) - 2*a*c*e^4*f^2*(f + g*x
) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 10*c^2*d^3*e*f*g*(f + g*x) + 2*a*c*d*e
^3*f*g*(f + g*x) - 11*c^2*d^2*e^2*f*(f + g*x)^2 + a*c*e^4*f*(f + g*x)^2 + (
6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 5*c^2*d^3*e*g*(f + g*x)^2 - a*c*d*e^3*g*(f
 + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d^2 + a*e^2)*(e*f
- d*g) + (a*e^2*g + c*d*(6*e*f - 5*d*g))*(d + e*x)))/(d + e*x)^2 + (Sqrt[c]
*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(6*e*f - 5*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + (e*(I*Sqrt[c]*d + Sqrt[a]*
e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(a*e^2*g + (2*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g)
 + c*d*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt
[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*Ar
cSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[
a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((8*I)
*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*S
qrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e
*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*
g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((4*I)*a*c*e^4*f^2*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x
))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt
[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c
]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]] - ((12*I)*c^2*d^3*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*
Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi
[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((12*I)*a*c*d*e^3*f*g*S
qrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
- g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqr
t[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + ((3*I)*c^2*d^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))
/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2
)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((10*I)*a*c
*d^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[
a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])
/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (I*a^2*e^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqr
t[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f +
 g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)),
 I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*
Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(4
*e*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.301, size = 20364, normalized size = 16.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

3.643 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=600

$$\frac{2\sqrt{-a}g\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(ae^2g^2+c\left(-3d^2g^2+6defg-2e^2f^2\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{a\sqrt{c}x}{(-a)^{3/2}}+1}}{\sqrt{2}}\right),\frac{2ag}{ag-\sqrt{-a}\sqrt{c}f}\right)}{3c^{3/2}e^3\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-ag}}{}$$

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) - (2*Sqrt[-a]*g*(7*e*f - 3*d*
g)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 + (a*Sqrt[c]*x
)/(-a)^(3/2)]/Sqrt[2]], (2*a*g)/(-(Sqrt[-a]*Sqrt[c]*f) + a*g)])/(3*Sqrt[c]*
e^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (
2*Sqrt[-a]*g*(a*e^2*g^2 + c*(-2*e^2*f^2 + 6*d*e*f*g - 3*d^2*g^2))*Sqrt[(Sqr
t[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 + (a*Sqrt[c]*x)/(-a)^(3/2)]/Sqrt[2]], (2*a*g)/(-(Sqrt[-a]*Sqrt[c
]*f) + a*g)])/(3*c^(3/2)*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (2*(e*f - d*g
)^2*Sqrt[(g*(Sqrt[-a] - Sqrt[c]*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[-((g*(Sq
rt[-a] + Sqrt[c]*x))/(Sqrt[c]*f - Sqrt[-a]*g))]*EllipticPi[(e*(f + (Sqrt[-a
]*g)/Sqrt[c]))/(e*f - d*g), ArcSin[Sqrt[c/(c*f + Sqrt[-a]*Sqrt[c]*g)]*Sqrt[
f + g*x]], (Sqrt[c]*f + Sqrt[-a]*g)/(Sqrt[c]*f - Sqrt[-a]*g)])/(e^3*Sqrt[c/
(c*f + Sqrt[-a]*Sqrt[c]*g)]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.946193, antiderivative size = 808, normalized size of antiderivative = 1.35, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {958, 719, 419, 933, 168, 538, 537, 424, 743, 844}

$$-\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)(ef-dg)^3}{e^3\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{cx^2+a}} - \frac{2\sqrt{-a}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{}}{\sqrt{}}}}{\sqrt{2}}\right.\right.}{\sqrt{c}e^3\sqrt{f+gx}\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) - (8*Sqrt[-a]*f*g*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*e*Sqrt[(Sqrt[c]*(f +
 g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*g*(e*f - d*
g)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
a]*g*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[
1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-
2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + (2*Sqrt[-a]*g*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*e*Sqrt[f +
 g*x]*Sqrt[a + c*x^2]) - (2*(e*f - d*g)^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a
] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqr
t[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a
```

```
 + c*x^2])
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
```

```
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{g(ef-dg)\sqrt{f+gx}}{e^2\sqrt{a+cx^2}} + \frac{g(f+gx)^{3/}}{e\sqrt{a+cx^2}} \right.$$

$$= \frac{g\int \frac{(f+gx)^{3/2}}{\sqrt{a+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{e^2} + \frac{\left(g(ef-dg)^2\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^3} + \frac{(ef-dg)^3 \int}{}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} + \frac{(2g)\int \frac{\frac{1}{2}\left(3cf^2-ag^2\right)+2cfgx}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3ce} + \frac{\left((ef-dg)^3\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}}{\sqrt{-}}}}}{e^3\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} - \frac{2\sqrt{-a}g(ef-dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} - \frac{2\sqrt{-a}g(ef-dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce} - \frac{8\sqrt{-a}fg\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-}}{}$$

Mathematica [C] time = 9.72578, size = 1440, normalized size = 2.4

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}g^2}{3ce}+\frac{2(f+gx)^{3/2}\left(\frac{7ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3}{(f+gx)^2}+\frac{3ice^2\sqrt{-\frac{f}{f+gx}-\frac{i\sqrt{a}g}{\sqrt{c}(f+gx)}+1}\sqrt{-\frac{f}{f+gx}+\frac{i\sqrt{a}g}{\sqrt{c}(f+gx)}+1}\Pi\left(\frac{\sqrt{c}(ef-dg)}{e\left(\sqrt{c}f+i\sqrt{a}g\right)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\right.}{\sqrt{f+gx}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) + (2*(f + g*x)^(3/2)*(7*c*e^2
*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]] + (7*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*c*d
*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (7*a*e^2*f*g^2*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]])/(f + g*x)^2 - (14*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]])/(f + g*x) + (6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) +
(Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(7*e*f - 3*d*g)*Sqrt[1 - f/(f + g*x
) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[
f + g*x] + (I*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(I*Sqrt[a]*e*g + Sqrt[c]*(6*e*f -
 3*d*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 -
 f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt
[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((3*I)*c*e^2*f^2*Sqrt[1 - f/(f + g*x
) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sq
rt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt
[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - ((6*I)*c*d
*e*f*g*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f
/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f -
d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/
Sqrt[f + g*x] + ((3*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt
[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(3*c*e^3*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]*Sqrt[a + (c*(f + g*x)^2*(-1 + f/(f + g*x))^2)/g^2])
```

Maple [B] time = 0.273, size = 3164, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/
2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g
```

/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*e*g^3-6*(-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c
f))^(1/2)((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g
*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+
c*f))^(1/2))*a*c*e^2*f*g^2-(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(
-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1
/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*
c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3-3*(-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c
f))^(1/2)((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g
*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+
c*f))^(1/2))*c^2*d^2*f*g^2+9*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^
(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-
a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*e*f^2*g-9*(-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(
1/2))*c^2*e^2*f^3+3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(
1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c
*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2
)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3-6*(-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(
1/2))*(-a*c)^(1/2)*c*d*e*f*g^2+2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((
-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a
*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(
-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g
-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ellipt
icE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^
(1/2)*g+c*f))^(1/2))*a*c*d*e*g^3+7*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*
((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e^2*f*g^2-3*(-(g*x
+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*
f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c
*f))^(1/2))*c^2*d*e*f^2*g+7*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(
1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a
*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*e^2*f^3+3*(-(g*x+f)*c/((-
a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)
*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/
2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d^2*f*g^2-6*(-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c
)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-
c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*e*f^2*g+3*(-(g*x+f)*c/((-a*c)^(1/2)
*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a
*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/
2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/
((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*e^2*f^3-3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f)
)^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/
2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)

```
^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3+6*(-(g*x+f)*c/((-a*c)^(1/2)*g-
c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)
^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*
g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-
a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d*e*f*g^2-3*(-(g*x+f)*c/((-a*c)^(1
/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*
f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g-x^3*c^2*e^2*g^3-x^
2*c^2*e^2*f*g^2-x*a*c*e^2*g^3-a*c*e^2*f*g^2)/e^3/c^2/(c*g*x^3+c*f*x^2+a*g*x
+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f+gx)^{\frac{5}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**(5/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.644 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=469

$$-\frac{2\sqrt{-a}g\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\Pi\left(\ldots\right.}{e^2\sqrt{a+cx^2}\sqrt{f+g\ldots}}$$

```
[Out] (-2*Sqrt[-a]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) -
 (2*Sqrt[-a]*g*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g
)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[
-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), Ar
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + S
qrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.637311, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {958, 719, 419, 933, 168, 538, 537, 424}

$$-\frac{2\sqrt{-a}g\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e^2\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin\ldots\right.}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```


```
[Out] (-2*Sqrt[-a]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) -
 (2*Sqrt[-a]*g*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g
)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[
-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), Ar
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + S
qrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```


Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int\left(\frac{g(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{g\sqrt{f+gx}}{e\sqrt{a+cx^2}}\right)dx$$

$$= \frac{g\int\frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int\frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2} + \frac{(ef-dg)^2\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e^2}$$

$$= \frac{\left((ef-dg)^2\sqrt{1+\frac{cx^2}{a}}\right)\int\frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{e^2\sqrt{a+cx^2}} + \frac{\left(2ag\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\cdots}}{\cdots}\right.}{\sqrt{-a}\sqrt{c}e\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{cg}}{\sqrt{-a}}}}\sqrt{\cdots}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}g(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1\cdots}}{\sqrt{c}e^2\sqrt{f\cdots}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}g(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1\cdots}}{\sqrt{c}e^2\sqrt{f\cdots}}$$

$$= -\frac{2\sqrt{-a}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} - \frac{2\sqrt{-a}g(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1\cdots}}{\sqrt{c}e^2\sqrt{f\cdots}}$$

Mathematica [C] time = 1.0958, size = 927, normalized size = 1.98

$$\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\left(-\frac{\sqrt{a}\sqrt{\frac{cx^2}{a}+1}\Pi\left(\frac{2\sqrt{a}e}{i\sqrt{c}d+\sqrt{a}e};\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)f^2}{i\sqrt{c}d+\sqrt{a}e} + \frac{2i\sqrt{a}g\sqrt{\frac{cx^2}{a}+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)f}{\sqrt{c}e} + \frac{2\sqrt{a}dg\sqrt{\frac{cx^2}{a}+1}\Pi\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g)]*(((2*I)*Sqrt[a]*f*g*
Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2
]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e) - (I*Sqrt[a]*d*g^
2*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt
[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e^2) + (g*Sqrt[(g*
(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*(I*Sqrt[a] + Sqrt[c]
*x)*((Sqrt[c]*f + I*Sqrt[a]*g)*EllipticE[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(S
qrt[c]*f - I*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] - I*Sqrt[a]*g*EllipticF[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I
*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*e*
```

```
Sqrt[(g*(Sqrt[a] - I*Sqrt[c]*x))/(I*Sqrt[c]*f + Sqrt[a]*g)]) - (Sqrt[a]*f^2
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), Ar
cSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f +
 Sqrt[a]*g)])/(I*Sqrt[c]*d + Sqrt[a]*e) + (2*Sqrt[a]*d*f*g*Sqrt[1 + (c*x^2)
/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1 - (I*
Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(I*
Sqrt[c]*d*e + Sqrt[a]*e^2) - (Sqrt[a]*d^2*g^2*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sq
rt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(e^2*(I*Sqrt[c]*
d + Sqrt[a]*e))))/(Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] time = 0.257, size = 959, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g-c*f))^(1/2)/c*(EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*d*g^2-Ell
ipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*
c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*e*f*g+EllipticF((-(g*x+f)*c/((-a*c)^(1
/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*e*g
^2-EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)
/((-a*c)^(1/2)*g+c*f))^(1/2))*c*d*f*g+2*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)
*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*e*f^2-
EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((
-a*c)^(1/2)*g+c*f))^(1/2))*a*e*g^2-EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*e*f^2-Ellip
ticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g
-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*d*g^
2+EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e
/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/
2)*e*f*g+EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g
-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*d
*f*g-EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f
)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*e*f^2
)/e^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f+gx)^{\frac{3}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**(3/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.645 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=457

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(ae^2g^2(7ef-15dg)-c\left(45d^2efg^2-15d^3g^3-30de^2f^2g+8e^3f^3\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\right.}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[Out] (-8*e^2*(e*f - 3*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g^2) + (2*e^2*(d
 + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c*g) + (2*Sqrt[-a]*e*(9*a*e^2*g^2
 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(15*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*(a*e^2*g^2*(7*e*f - 15*d*g)
- c*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d^3*g^3))*Sqrt[(Sqrt[
c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/(15*c^(3/2)*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.613127, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {931, 1654, 844, 719, 424, 419}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(ae^2g^2(7ef-15dg)-c\left(45d^2efg^2-15d^3g^3-30de^2f^2g+8e^3f^3\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```


```
[Out] (-8*e^2*(e*f - 3*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g^2) + (2*e^2*(d
 + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c*g) + (2*Sqrt[-a]*e*(9*a*e^2*g^2
 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(15*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*(a*e^2*g^2*(7*e*f - 15*d*g)
- c*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d^3*g^3))*Sqrt[(Sqrt[
c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/(15*c^(3/2)*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```


Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
```

```
m, 2]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\int \frac{-5cd^3g+ae^2(2ef+dg)+e\left(3ae^2g+cd(2ef-15dg)\right)x+4ce^2(ef-3dg)x^2}{\sqrt{f+gx}\sqrt{a+cx^2}}dx}{5cg}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{2\int \frac{-\frac{1}{2}cg^2\left(15cd^3g-ae^2(2ef\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\left(e\left(9ae^2g^2-c\left(8e^2f^2\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\left(2ae\left(9ae^2g^2-c\left(8e^2f\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} + \frac{2\sqrt{-ae}\left(9ae^2g^2-c\left(8e\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 4.56646, size = 625, normalized size = 1.37

$$\frac{2\sqrt{f+gx}\left(\frac{\sqrt{c}g\sqrt{f+gx}\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(9a^{3/2}e^3g^2+\sqrt{a}ce\left(-45d^2g^2+30defg-8e^2f^2\right)-ia\sqrt{c}e^2g(15dg+2ef)+15ic^{3/2}d^3g^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\cdots}}{\sqrt{f+\cdots}}\right.\right.}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e^2*g^2*(-4*e*f + 15*d*g + 3*e*g*x)*(a + c*x^2) + (e*g^
2*(-9*a^2*e^2*g^2 + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2)*x^2 + a*c*(-3
0*d*e*f*g + 45*d^2*g^2 + e^2*(8*f^2 - 9*g^2*x^2))))/(f + g*x) + I*c*e*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(8*e^2*f^2 - 30*d*e*f*g + 45*
d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*
g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)] + (Sqrt[c]*g*((15*I)*c^(3/2)*d^3*g^2 + 9*a^(3/2)*e^3*g^2 -
I*a*Sqrt[c]*e^2*g*(2*e*f + 15*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 30*d*e*f*g -
 45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt
[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(15*c^2*g^4*Sqrt[a
+ c*x^2])
```

Maple [B] time = 0.265, size = 2949, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2/15*(-15*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(
(-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2
)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/
((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d^3*f*g^3+15*(-(g*x+f)*c/((-a*c)^(1/2)*g-c
*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^
(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-
c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2
)*c*d^3*g^4-15*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^
(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c
*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*d*e^2*g^4+7*(-(g*x+f)*c/((-
a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)
*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2
))*(-a*c)^(1/2)*a*e^3*f*g^3-8*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)
^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((
-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*e^3*f^3*g-45
*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/
2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Elliptic
F((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1
/2)*g+c*f))^(1/2))*a*c*d^2*e*g^4-6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*
((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
,(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e^3*f^2*g^2+45*(-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-
(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*
g+c*f))^(1/2))*a*c*d^2*e*g^4-(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^
(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-
a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e^3*f^2*g^2+45*(-(g*x+f)
*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))
^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f
)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f)
)^(1/2))*c^2*d^2*e*f^2*g^2-30*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)
^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((
-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*e^2*f^3*g+x^3*c^2*e^3
*f*g^3-3*x^2*a*c*e^3*g^4+4*x^2*c^2*e^3*f^2*g^2-15*x^2*c^2*d*e^2*f*g^3-15*x^
3*c^2*d*e^2*g^4-15*a*c*d*e^2*f*g^3-3*x^4*c^2*e^3*g^4+9*(-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a^
2*e^3*g^4-9*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/
((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/
2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)
/((-a*c)^(1/2)*g+c*f))^(1/2))*a^2*e^3*g^4+8*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1
/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*e^3*f^4+4
*a*c*e^3*f^2*g^2+30*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(
1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c
*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2
)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d*e^2*f^2*g^2+45*(-(g*
```

```
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c
*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g
*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+
c*f))^(1/2))*a*c*d*e^2*f*g^3-30*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*
c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-
((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*e^2*f*g^3-45*(-(g*x
+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*
f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c
*f))^(1/2))*(-a*c)^(1/2)*c*d^2*e*f*g^3+x*a*c*e^3*f*g^3-15*x*a*c*d*e^2*g^4)*
(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/g^4/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+a}\sqrt{gx+f}}{cgx^3+cfx^2+agx+af},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)*sqrt(g*x
 + f)/(c*g*x^3 + c*f*x^2 + a*g*x + a*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.646 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=356

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(g^2\left(3cd^2-ae^2\right)+2cef(ef-3dg)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}e\sqrt{\cdots}}{\cdots}$$

```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*g) + (4*Sqrt[-a]*e*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
a]*((3*c*d^2 - a*e^2)*g^2 + 2*c*e*f*(e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))
/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3
/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.404872, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {931, 24, 844, 719, 424, 419}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\left(g^2\left(3cd^2-ae^2\right)+2cef(ef-3dg)\right)F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```


```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*g) + (4*Sqrt[-a]*e*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
a]*((3*c*d^2 - a*e^2)*g^2 + 2*c*e*f*(e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))
/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3
/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```


Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\int \frac{-d(3cd^2-ae^2)g+e(ae^2g+cd(2ef-9dg))x+2ce^2(ef-3dg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3cg}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\int \frac{-e^2(3cd^2-ae^2)g+2ce^3(ef-3dg)x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{3ce^2g}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{(2e(ef-3dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g^2} + \frac{1}{3}\left(3d^2 - \frac{ae^2}{c} + \frac{2ef(ef-3dg)}{g^2}\right)\int \frac{}{\sqrt{}}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} - \frac{\left(4ae(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \right.}{3\sqrt{-a}\sqrt{c}g^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg} + \frac{4\sqrt{-a}e(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{c}f-a}\right.}{3\sqrt{c}g^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 3.63422, size = 473, normalized size = 1.33

$$\frac{2\sqrt{f+gx}\left(\frac{g\sqrt{f+gx}\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(2\sqrt{a}\sqrt{c}e(ef-3dg)-iae^2g+3icd^2g\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right), \frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}} - \frac{2eg^2(a+cx^2)(ef-3dg)}{f+gx} - \right.}{3cg^3\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(e^2*g^2*(a + c*x^2) - (2*e*g^2*(e*f - 3*d*g)*(a + c*x^2))
/(f + g*x) - (2*I)*c*e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - 3*d*g)*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
 + (g*((3*I)*c*d^2*g - I*a*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 3*d*g))*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
)/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3*c*g^3*Sqrt[a + c*x^2])
```

Maple [B] time = 0.263, size = 1769, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2/3*((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)
^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Elli
pticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c
)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3-3*(-(g*x+f)*c/((-a*c)^(1/2)*g
-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c
)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*
g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1
/2)*c*d^2*g^3+6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2)
)*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))
^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-
c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*c*d*e*f*g^2-2*(-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2
)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/
2))*(-a*c)^(1/2)*c*e^2*f^2*g+6*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c
*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c
)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-(
(-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*e*g^3-3*(-(g*x+f)*c/
((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c
/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(
1/2))*a*c*e^2*f*g^2+3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)
^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g
-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1
/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d^2*f*g^2-6*(-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c
*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c
)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a
*c*d*e*g^3+2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g
/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1
/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f
)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e^2*f*g^2-6*(-(g*x+f)*c/((-a*c)^(1/2)*g-
c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)
^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g
-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*e*f^
2*g+2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c
)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*Ell
ipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*
c)^(1/2)*g+c*f))^(1/2))*c^2*e^2*f^3+x^3*c^2*e^2*g^3+x^2*c^2*e^2*f*g^2+x*a*c
*e^2*g^3+a*c*e^2*f*g^2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/g^3/(c*g*x^3+c*f*
x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^{2}x^{2}+2\,dex+d^{2}\right)\sqrt{cx^{2}+a}\sqrt{gx+f}}{cgx^{3}+cfx^{2}+agx+af},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)*sqrt(g*x + f)/(c*g*x^3 +
 c*f*x^2 + a*g*x + a*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{2}}{\sqrt{a+cx^{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^{2}}{\sqrt{cx^{2}+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.647 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}} dx$

Optimal. Leaf size=288

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\right.}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

```
[Out] (-2*Sqrt[-a]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[a +
 c*x^2])
```

Rubi [A] time = 0.163342, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {844, 719, 424, 419}

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[a +
 c*x^2])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{e\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{g} + \frac{(-ef+dg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{g}$$

$$= \frac{\left(2ae\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{a+cx^2}} + \frac{\left(2a(-ef+dg)\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{cg}}{\sqrt{-a}}}}\right.}{}$$

$$= -\frac{2\sqrt{-a}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}} + \frac{2\sqrt{-a}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}}{\sqrt{c}g}$$

Mathematica [C] time = 1.72132, size = 439, normalized size = 1.52

$$-\frac{2\left(\sqrt{c}g(f+gx)^{3/2}\left(\sqrt{a}e-i\sqrt{c}d\right)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right), \frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-eg^2\left(a+cx^2\right)\sqrt{-\right.}{cg^2\sqrt{a+cx^2}\sqrt{f+gx}\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*(-(e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a + c*x^2)) + I*Sqrt[c]*e*(S
qrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[
-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + Sqrt[c]*((-I)*Sqrt[c]*d + Sqrt[a]*e)*g*Sq
rt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)]))/(c*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[f + g*x]*Sqrt[a + c*x^
2])
```

Maple [B] time = 0.253, size = 520, normalized size = 1.8

$$2\frac{\sqrt{gx+f}\,\sqrt{cx^2+a}}{g^2c\left(cgx^3+cfx^2+agx+af\right)}\left(\text{EllipticF}\left(\sqrt{-\frac{c\left(gx+f\right)}{\sqrt{-ac}g-cf}},\sqrt{-\frac{\sqrt{-ac}g-cf}{\sqrt{-ac}g+cf}}\right)aeg^2+\text{EllipticF}\left(\sqrt{-\frac{c\left(gx+f\right)}{\sqrt{-ac}g-cf}},\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)
/((-a*c)^(1/2)*g+c*f))^(1/2))*a*e*g^2+EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g
-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*d*f*g-El
lipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a
*c)^(1/2)*g+c*f))^(1/2))*(-a*c)^(1/2)*d*g^2+EllipticF((-(g*x+f)*c/((-a*c)^(
1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*(-a*
c)^(1/2)*e*f*g-EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^
(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*e*g^2-EllipticE((-(g*x+f)*c/((-
a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2)
)*c*e*f^2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^
(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2
)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}{cgx^3+cfx^2+agx+af},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)/(c*g*x^3 + c*f*x^2 + a*g*x
 + a*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.648 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

```
[Out] (-2*Sqrt[-a]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0592399, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {719, 419}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{\left(2a\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}g}{\sqrt{-a}}\right)}}}dx, x, \frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] time = 0.226461, size = 186, normalized size = 1.37

$$\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right), \frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{g\sqrt{a+cx^2}\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + c*x^2])
```

Maple [A] time = 0.328, size = 200, normalized size = 1.5

$$2\,\frac{\left(cf-\sqrt{-ac}g\right)\sqrt{gx+f}\sqrt{cx^2+a}}{cg\left(cgx^3+cfx^2+agx+af\right)}\text{EllipticF}\left(\sqrt{-\frac{c\left(gx+f\right)}{\sqrt{-ac}g-cf}}, \sqrt{-\frac{\sqrt{-ac}g-cf}{\sqrt{-ac}g+cf}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{\sqrt{-ac}g-cf}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{\sqrt{-ac}g+cf}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(c*f-(-a*c)^(1/2)*g)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-
((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*((c*x+(-a*c)^(1/2))*g/((-
a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)
*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g/c/
(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{gx+f}}{cgx^3+cfx^2+agx+af},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(g*x + f)/(c*g*x^3 + c*f*x^2 + a*g*x + a*f), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

3.649 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}$$

```
[Out] (-2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*
EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/S
qrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.336181, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {933, 168, 538, 537}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```


```
[Out] (-2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*
EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/S
qrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```


Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{\sqrt{1+\frac{cx^2}{a}}\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}\,dx}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}g}{\sqrt{c}}-\frac{\sqrt{-a}gx^2}{\sqrt{c}}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right)\sqrt{1-\frac{\sqrt{-a}gx^2}{\sqrt{c}\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}}}\,dx, x, \sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] time = 0.965371, size = 311, normalized size = 1.86

$$-\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\Pi\left(\frac{\sqrt{c}(ef-dg)}{e\left(\sqrt{c}f+i\sqrt{a}g\right)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-\dots}{\sqrt{c}f+\dots}\right)\right)}{\sqrt{a+cx^2}\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] - EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g))
, I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I
*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(e*f - d*g)*Sqrt[a + c*x^2])
```

Maple [A] time = 0.296, size = 235, normalized size = 1.4

$$2\frac{\left(cf-\sqrt{-acg}\right)\sqrt{cx^{2}+a}\,\sqrt{gx+f}}{c\left(dg-ef\right)\left(cgx^{3}+cfx^{2}+agx+af\right)}\mathrm{EllipticPi}\left(\sqrt{-\frac{c\left(gx+f\right)}{\sqrt{-acg}-cf}},\frac{\left(\sqrt{-acg}-cf\right)e}{c\left(dg-ef\right)},\sqrt{-\frac{\sqrt{-acg}-cf}{\sqrt{-acg}+cf}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)}{\sqrt{-acg}-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(c*f-(-a*c)^(1/2)*g)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(
(-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*
f))^(1/2))*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^
(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2
)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

3.650 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=746

$$\frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right),-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd^2\right)(ef-dg)}-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,\text{EllipticF}\left(\text{si}\right.}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd\right.}$$

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x
))) - (Sqrt[-a]*Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sq
rt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*e*f*Sqrt[(Sqrt[c]*(f + g*x)
)/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (
Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^
2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d
*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a]
+ e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[
c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 2.10898, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {940, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}+\frac{\sqrt{-a}\sqrt{c}ef\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(ae^2+cd^2\right)(ef-dg)}-\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+\right.}{\sqrt{a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x
))) - (Sqrt[-a]*Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sq
rt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*e*f*Sqrt[(Sqrt[c]*(f + g*x)
)/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (
Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^
2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d
*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a]
+ e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[
```

```
c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
```

```
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\int \frac{ae^2g-2cd(ef-dg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\int\left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+cx^2}} - \frac{cegx}{\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{ae^2g-cd(2ef-3dg)}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right.}{2\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)} + \frac{(ceg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)} -$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} + \frac{(ce)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)} - \frac{(cef)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{2\left(cd^2+ae^2\right)(ef-dg)} -$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\right.}{\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\right.}{\left(cd^2+ae^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\sqrt{c}e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\right.}{\left(cd^2+ae^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

Mathematica [C] time = 6.99154, size = 1349, normalized size = 1.81

$$\sqrt{f+gx}\left(\frac{2\left(-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^3+2ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f^2+cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}f^2-ce^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)^2f-2cdeg\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)f-2icdeg\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*((-2*e^2*(a + c*x^2))/(d + e*x) + (2*(-(c*e^2*f^3*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*
e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c
*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*
```

```
Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g
*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)
/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
 + (Sqrt[c]*d - I*Sqrt[a]*e)*g*(Sqrt[a]*e*g + I*Sqrt[c]*(e*f - 2*d*g))*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*
g)] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[
c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)] + (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sq
rt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(
Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)] + I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*S
qrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[
(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
 I*Sqrt[a]*g)]))/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-(e*f) + d*g)*(f + g*
x))))/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.281, size = 5738, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^2 \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

3.651 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=1257

result too large to display

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(e*f - d*g)*(d + e*
x)^2) + (3*e^2*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*(d + e*x)) + (3*Sqrt[-a]*Sqrt[c]*e*(a
*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f
 - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt
[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(
f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]
)/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*Sqrt[-
a]*Sqrt[c]*e*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sq
rt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 +
 a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c
]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2
*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*
e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*(a*e^2*g - c
*d*(2*e*f - 3*d*g))^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[
1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)
])/(4*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g
*x]*Sqrt[a + c*x^2])
```

Rubi [A] time = 4.33319, antiderivative size = 1257, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {940, 6742, 719, 419, 844, 424, 933, 168, 538, 537}

$$\frac{3\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{cx^2+ae^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}-\frac{\sqrt{f+gx}\sqrt{cx^2+ae^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}+\frac{3\sqrt{-a}\sqrt{c}\left(ae^2g-cd(2ef-3dg)\right)\sqrt{\ldots}}{4\left(cd^2+ae^2\right)^2(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(e*f - d*g)*(d + e*
x)^2) + (3*e^2*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2
])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*(d + e*x)) + (3*Sqrt[-a]*Sqrt[c]*e*(a
*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f
 - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt
[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(
f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sq
```

```
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]
)/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*Sqrt[-
a]*Sqrt[c]*e*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sq
rt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[
c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 +
 a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c
]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2
*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*
e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*(a*e^2*g - c
*d*(2*e*f - 3*d*g))^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[
1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)
])/(4*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g
*x]*Sqrt[a + c*x^2])
```

Rule 940

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g)*(m +
 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m + 1) - c*e*f*(m + 2))*x - c*e^2*g*(
2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \frac{3ae^2g-4cd(ef-dg)+2ce(ef-2dg)x+ce^2gx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \left(\frac{cg}{\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{3\left(ae^2g-cd(2ef-3dg)\right)}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{2c(ef-3\cdots}{(d+ex)\sqrt{f+g\cdots}}\cdots\right.}{4\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)(ef-dg)} - \frac{(c(ef-3dg))\int \frac{\cdots}{(d+ex)\sqrt{f\cdots}}}{2\left(cd^2+ae^2\right)(ef\cdots}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} + \cdots$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} + \cdots$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} + \cdots$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} + \cdots$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} + \cdots$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} + \cdots$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)} + \cdots$$

Mathematica [C] time = 12.3437, size = 2990, normalized size = 2.38

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-e^2/(2*(c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x
)^2) + (3*e^2*(-2*c*d*e*f + 3*c*d^2*g + a*e^2*g))/(4*(c*d^2 + a*e^2)^2*(e*f
 - d*g)^2*(d + e*x))) - ((f + g*x)^(3/2)*(-6*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]] + 15*c^2*d^2*e^2*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 3*
a*c*e^4*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 9*c^2*d^3*e*g^2*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*d*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (
6*c^2*d*e^3*f^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (15*c^2*d^2
*e^2*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (3*a*c*e^4*f^3*g
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (9*c^2*d^3*e*f^2*g^2*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (9*a*c*d*e^3*f^2*g^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (15*a*c*d^2*e^2*f*g^3*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]])/(f + g*x)^2 + (3*a^2*e^4*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]])/(f + g*x)^2 - (9*a*c*d^3*e*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f
 + g*x)^2 - (3*a^2*d*e^3*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2
+ (12*c^2*d*e^3*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) - (30*c^2*d
^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) - (6*a*c*e^4*f^2*g
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (18*c^2*d^3*e*f*g^2*Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (6*a*c*d*e^3*f*g^2*Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]])/(f + g*x) + (3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f
- d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g
)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g
*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((I*S
qrt[c]*d + Sqrt[a]*e)*g*((3*I)*a^(3/2)*e^3*g^2 - 3*a*Sqrt[c]*e^2*g*(e*f - 2
*d*g) - I*Sqrt[a]*c*e*(2*e^2*f^2 - 6*d*e*f*g + d^2*g^2) + c^(3/2)*d*(4*e^2*
f^2 - 9*d*e*f*g + 8*d^2*g^2))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]
*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Elli
pticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f
 - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - ((8*I)*c^2*d^2*
e^2*f^2*g*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1
- f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f
 - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)
])/Sqrt[f + g*x] + ((4*I)*a*c*e^4*f^2*g*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g
)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g
*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((20*I)*c^2*d^3*e*f*g^2*Sq
rt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x
) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*
(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt
[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f +
g*x] - ((4*I)*a*c*d*e^3*f*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]
*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Elli
pticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt
[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - ((15*I)*c^2*d^4*g^3*Sqrt[1 - f/(f +
g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]
*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I
*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (S
qrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] - ((6*I)*
a*c*d^2*e^2*g^3*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*S
qrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c
]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt
```

```
[a]*g)])/Sqrt[f + g*x] - ((3*I)*a^2*e^4*g^3*Sqrt[1 - f/(f + g*x) - (I*Sqrt[
a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f
 + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(4*(c*d^2 + a*e^2)^2*g
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(-(e*f) + d*g)^2*Sqrt[a + (c*
(f + g*x)^2*(-1 + f/(f + g*x))^2)/g^2])
```

Maple [B] time = 0.332, size = 20365, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

3.652 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=387

$$\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}+\frac{2\sqrt{-a}\sqrt{c}g\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\left(ag^2+cf^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}-\frac{2e\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\cdots}{\sqrt{a+cx^2}\sqrt{f\cdots}}$$

```
[Out] (2*g^2*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) + (2*Sq
rt[-a]*Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
- (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e
*f - d*g)*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[a + c*x^2]) - (2*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[S
qrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a
]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rubi [A] time = 0.574465, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {958, 745, 21, 719, 424, 933, 168, 538, 537}

$$\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}+\frac{2\sqrt{-a}\sqrt{c}g\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\left(ag^2+cf^2\right)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}-\frac{2e\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\cdots}{\sqrt{a+cx^2}\sqrt{f\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) + (2*Sq
rt[-a]*Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
- (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e
*f - d*g)*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[a + c*x^2]) - (2*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[S
qrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a
]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
```

```
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{3/2}\sqrt{a+cx^2}} + \frac{e}{(ef-dg)(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\right)dx$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{(2cg)\int \frac{-\frac{f}{2}-\frac{gx}{2}}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{(ef-dg)\left(cf^2+ag^2\right)} + \frac{\left(e\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{}{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}\sqrt{}}}{(ef-dg)\sqrt{}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(cg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{(ef-dg)\left(cf^2+ag^2\right)} - \frac{\left(2e\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int -\right.}{}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(2e\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{}{\sqrt{2-x^2}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e-ex^2\right.}\right.}{(ef-dg)\sqrt{f+gx}\sqrt{a+}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\right.}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+c}}$$

Mathematica [C] time = 3.34869, size = 468, normalized size = 1.21

$$\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}}\left(\left(\sqrt{c}(dg-2ef)+i\sqrt{a}eg\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{ag}}{\sqrt{c}f+i\sqrt{ag}}\right)+\sqrt{c}(ef-dg)E\left(i\,\mathrm{si}\right.\right.}{\sqrt{a+cx^2}\left(\sqrt{c}f-i\sqrt{ag}\right)\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(Sqrt[c]*(e*f - d*g)*EllipticE[I*ArcSi
nh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*
g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (I*Sqrt[a]*e*g + Sqrt[c]*(-2*e*f + d*g))*El
lipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]
*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f - I*Sqrt[a]*g)*
EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(
Sqrt[c]*f + I*Sqrt[a]*g)]))/((Sqrt[c]*f - I*Sqrt[a]*g)*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]*(e*f - d*g)^2*Sqrt[a + c*x^2])
```

Maple [B] time = 0.345, size = 2011, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2*(EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)
*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e*f*
g^2*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^
(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)-Ellip
ticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g
-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*e*g^3*(-a*c)^(1
/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^
(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)+Ellip
ticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g
-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*e*f^3*(-(g*x+
f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f
))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)-EllipticPi((-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-
a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*e*f^2*g*(-a*c)^(1/2)*(-(g*
x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c
*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)-(-(g*x+f)*c/((
-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2
)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(
(-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/
2))*a*c*d*g^3+EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(
1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*e*f*g^2*(-(g*x+f)*c/((-a*c)^(1
/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+
(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)-(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f
))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1
/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*
f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*f^2*g+E
llipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-
a*c)^(1/2)*g+c*f))^(1/2))*c^2*e*f^3*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)
*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
(-a*c)^(1/2)*g-c*f))^(1/2)+(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(
-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1
/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*
c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*c*d*g^3-(-(g*x+f)*c/((-a*c)^
(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)
^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*
c*e*f*g^2+(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((
-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)
*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/(
(-a*c)^(1/2)*g+c*f))^(1/2))*c^2*d*f^2*g-(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))
*g/((-a*c)^(1/2)*g-c*f))^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c^2*e*f^3+x^2*c^2
*d*g^3-x^2*c^2*e*f*g^2+a*c*d*g^3-a*c*e*f*g^2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)
/c/(d*g-e*f)^2/(a*g^2+c*f^2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

3.653 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=818

$$-\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)e^2}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+}}$$

```
[Out] (2*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(3/2)) + (
8*c*f*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])
+ (2*e*g^2*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x]) +
 (8*Sqrt[-a]*c^(3/2)*f*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*
g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqrt[c]*e*g*Sqrt[f + g*x]*Sqrt
[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (
-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*
Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^
2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(S
qrt[-a]*Sqrt[c]*f - a*g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]*Sqr
t[a + c*x^2]) - (2*e^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*S
qrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt
[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g
)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]
)
```

Rubi [A] time = 0.996386, antiderivative size = 818, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {958, 745, 835, 844, 719, 424, 419, 21, 933, 168, 538, 537}

$$-\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)e^2}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\right.}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(3/2)) + (
8*c*f*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])
+ (2*e*g^2*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x]) +
 (8*Sqrt[-a]*c^(3/2)*f*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*
g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqrt[c]*e*g*Sqrt[f + g*x]*Sqrt
[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (
-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*
Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^
2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(S
```

```
qrt[-a]*Sqrt[c]*f - a*g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]*Sqr
t[a + c*x^2]) - (2*e^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*S
qrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt
[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g
)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]
)
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
```

```
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{5/2}\sqrt{a+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)^{3/2}\sqrt{a+cx^2}} + \frac{}{(ef-dg)^2(d} \right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{(2ceg)\int \frac{}{\sqrt{f}}}{(ef-dg)^2\,(}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{2}{(ef-dg)^2}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{2}{(ef-dg)^2}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{2}{(ef-dg)^2}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f+gx}} + \frac{2}{(ef-dg)^2}$$

Mathematica [C] time = 9.23841, size = 1917, normalized size = 2.34

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(a + c*x^2)*(a*g^2*(4*
e*f - d*g + 3*e*g*x) + c*f*(-(d*g*(5*f + 4*g*x)) + e*f*(8*f + 7*g*x))) - (f
 + g*x)*(7*c^2*e^2*f^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 11*c^2*d*e*f^4*g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*c^2*d^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]] + 10*a*c*e^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*a*c*d
*e*f^2*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*a*c*d^2*f*g^4*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + 3*a^2*e^2*f*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*
a^2*d*e*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*c^2*e^2*f^4*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 22*c^2*d*e*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]*(f + g*x) - 8*c^2*d^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
g*x) - 6*a*c*e^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 6*a*c
*d*e*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 7*c^2*e^2*f^3*Sqrt[
```

```
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 11*c^2*d*e*f^2*g*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]*(f + g*x)^2 + 4*c^2*d^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x)^2 + 3*a*c*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*
x)^2 - 3*a*c*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]
*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(3*a*e*g^2 + c*f*(7*e*f - 4*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)] + (Sqrt[c]*f + I*Sqrt[a]*g)*(3*a^(3/2)*e^2*g^3 + (3*I)*a*Sqrt[c]*e
*g^2*(2*e*f - d*g) + Sqrt[a]*c*g*(2*e^2*f^2 + 2*d*e*f*g - d^2*g^2) + (3*I)*
c^(3/2)*f*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
 x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3*I)*c^2*e^2*f^4*Sq
rt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt
[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (6*I)*a*c*e^2
*f^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g
))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3
*I)*a^2*e^2*g^4*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sq
rt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*
f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)])))/(3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)^3*(c*f^2 + a*g^2)^2*(
f + g*x)^(3/2)*Sqrt[a + c*x^2])
```

Maple [B] time = 0.334, size = 9409, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2}\,(d + ex)\left(f + gx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(5/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.654 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-c}f+g}}\Pi\left(\frac{2e}{\sqrt{-c}d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

```
[Out] (-2*Sqrt[(Sqrt[-c]*(f + g*x))/(Sqrt[-c]*f + g)]*EllipticPi[(2*e)/(Sqrt[-c]*
d + e), ArcSin[Sqrt[1 - Sqrt[-c]*x]/Sqrt[2]], (2*g)/(Sqrt[-c]*f + g)])/((Sq
rt[-c]*d + e)*Sqrt[f + g*x])
```

Rubi [A] time = 0.304273, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {932, 168, 538, 537}

$$-\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-c}f+g}}\Pi\left(\frac{2e}{\sqrt{-c}d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(Sqrt[-c]*(f + g*x))/(Sqrt[-c]*f + g)]*EllipticPi[(2*e)/(Sqrt[-c]*
d + e), ArcSin[Sqrt[1 - Sqrt[-c]*x]/Sqrt[2]], (2*g)/(Sqrt[-c]*f + g)])/((Sq
rt[-c]*d + e)*Sqrt[f + g*x])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{1-\sqrt{-c}x}\sqrt{1+\sqrt{-c}x}(d+ex)\sqrt{f+gx}}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\sqrt{-c}d+e-ex^2\right)\sqrt{f+\frac{g}{\sqrt{-c}}-\frac{gx^2}{\sqrt{-c}}}}\,dx, x, \sqrt{1-\sqrt{-c}x}\right)\right)$$
$$= -\frac{\left(2\sqrt{1+\frac{g\left(-1+\sqrt{-c}x\right)}{\sqrt{-c}f+g}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\sqrt{-c}d+e-ex^2\right)\sqrt{1-\frac{gx^2}{\sqrt{-c}\left(f+\frac{g}{\sqrt{-c}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c}x}\right)}{\sqrt{f+gx}}$$
$$= -\frac{2\sqrt{1-\frac{g\left(1-\sqrt{-c}x\right)}{\sqrt{-c}f+g}}\,\Pi\left(\frac{2e}{\sqrt{-c}d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}f+g}\right)}{\left(\sqrt{-c}d+e\right)\sqrt{f+gx}}$$

Mathematica [C] time = 0.902458, size = 261, normalized size = 2.37

$$-\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{ig}{\sqrt{c}}}{f+gx}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-ig}{\sqrt{c}f+ig}\right)-\Pi\left(\frac{\sqrt{c}(ef-dg)}{e\left(\sqrt{c}f+ig\right)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-ig}{\sqrt{c}f+ig}\right)\right)}{\sqrt{cx^2+1}\sqrt{-f-\frac{ig}{\sqrt{c}}}(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*(I/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*g)/Sqrt[c] - g*x)/(f
 + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)] - EllipticPi[(Sqrt[c]*(e*f - d*
g))/(e*(Sqrt[c]*f + I*g)), I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f + g*x]
], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)]))/(Sqrt[-f - (I*g)/Sqrt[c]]*(e*f -
d*g)*Sqrt[1 + c*x^2])
```

Maple [B] time = 0.342, size = 215, normalized size = 2.

$$2\,\frac{\left(g+f\sqrt{-c}\right)\sqrt{cx^2+1}\sqrt{gx+f}}{\sqrt{-c}\left(dg-ef\right)\left(cgx^3+cfx^2+gx+f\right)}\text{EllipticPi}\left(\sqrt{\frac{\left(gx+f\right)\sqrt{-c}}{g+f\sqrt{-c}}},-\frac{e\left(g+f\sqrt{-c}\right)}{\sqrt{-c}\left(dg-ef\right)},\sqrt{\frac{g+f\sqrt{-c}}{f\sqrt{-c}-g}}\right)\sqrt{-\frac{\left(-1+x\sqrt{-c}\right)g}{g+f\sqrt{-c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x)
```

```
[Out] 2*(g+f*(-c)^(1/2))/(-c)^(1/2)*EllipticPi(((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2
)))^(1/2),-(g+f*(-c)^(1/2))*e/(-c)^(1/2)/(d*g-e*f),((g+f*(-c)^(1/2))/(f*(-c
)^(1/2)-g))^(1/2))*(-(-1+x*(-c)^(1/2))*g/(g+f*(-c)^(1/2)))^(1/2)*(-(x*(-c)^
(1/2)+1)*g/(f*(-c)^(1/2)-g))^(1/2)*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))^(1
/2)*(c*x^2+1)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+c*f*x^2+g*x+f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+1}(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(c*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+1}(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

3.655 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=454

$$-\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt{f+gx}\sqrt[4]{ae^2+cd^2}}{\sqrt{d+ex}\sqrt[4]{ag^2+cf^2}}\right.\right.\cdots}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}}$$

```
[Out] -(((c*f^2 + a*g^2)^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/((c*f^2
 + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 +
a*g^2]*(d + e*x)))*Sqrt[(1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)
*(d + e*x)) + ((c*d^2 + a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2))/
(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 + a*g^2]*(d + e*x)))^2]*El
lipticF[2*ArcTan[((c*d^2 + a*e^2)^(1/4)*Sqrt[f + g*x])/((c*f^2 + a*g^2)^(1/
4)*Sqrt[d + e*x])], (1 + (c*d*f + a*e*g)/(Sqrt[c*d^2 + a*e^2]*Sqrt[c*f^2 +
a*g^2]))/2])/((c*d^2 + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + c*x^2]*Sqrt[1 - (2
*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)*(d + e*x)) + ((c*d^2 + a*e^2)*
(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2)]))
```

Rubi [A] time = 0.628851, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {936, 1103}

$$-\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cd^2+ae^2}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}\sqrt{d+ex}}\right)|\frac{1}{2}\left(\cdots\right.\right.}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```



```
[Out] -(((c*f^2 + a*g^2)^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/((c*f^2
 + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 +
a*g^2]*(d + e*x)))*Sqrt[(1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)
*(d + e*x)) + ((c*d^2 + a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2))/
(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 + a*g^2]*(d + e*x)))^2]*El
lipticF[2*ArcTan[((c*d^2 + a*e^2)^(1/4)*Sqrt[f + g*x])/((c*f^2 + a*g^2)^(1/
4)*Sqrt[d + e*x])], (1 + (c*d*f + a*e*g)/(Sqrt[c*d^2 + a*e^2]*Sqrt[c*f^2 +
a*g^2]))/2])/((c*d^2 + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + c*x^2]*Sqrt[1 - (2
*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)*(d + e*x)) + ((c*d^2 + a*e^2)*
(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2)]))
```


Rule 936

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*
(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/
((c*f^2 + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sqrt[a + c*x^2]), Subst[Int[1/
Sqrt[1 - ((2*c*d*f + 2*a*e*g)*x^2)/(c*f^2 + a*g^2) + ((c*d^2 + a*e^2)*x^4)/
```

```
(c*f^2 + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{\left(2(d+ex)\sqrt{\frac{(ef-dg)^2(a+cx^2)}{(cf^2+ag^2)(d+ex)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(2cdf+2aeg)x^2}{cf^2+ag^2}+\frac{(cd^2+ae^2)x^4}{cf^2+ag^2}}}\,dx, x, \frac{\sqrt{f+gx}}{\sqrt{d+ex}}\right)}{(ef-dg)\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt[4]{cf^2+ag^2}(d+ex)\sqrt{\frac{(ef-dg)^2(a+cx^2)}{(cf^2+ag^2)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2+ae^2}(f+gx)}{\sqrt{cf^2+ag^2}(d+ex)}\right)\sqrt{\frac{1-\frac{2(cdf+aeg)(f+gx)}{(cf^2+ag^2)(d+ex)}+\frac{(cd^2+ae^2 \ldots}{(cf^2+ag^2 \ldots}}{\left(1+\frac{\sqrt{cd^2+ae^2}(f+gx)}{\sqrt{cf^2+ag^2}(d+ex)}\right)^2}}}{\sqrt[4]{cd^2+ae^2}(ef-dg)\sqrt{a+cx^2}\sqrt{1-\frac{2(cdf+aeg) \ldots}{(cf^2+ag^2) \ldots}}}$$

Mathematica [C] time = 1.39044, size = 344, normalized size = 0.76

$$\frac{\sqrt{2}\left(\sqrt{c}x+i\sqrt{a}\right)\sqrt{d+ex}\sqrt{\frac{\frac{i\sqrt{c}dx}{\sqrt{a}}-\frac{i\sqrt{a}e}{\sqrt{c}}+d+ex}{d+ex}}\sqrt{\frac{(f+gx)\left(\sqrt{a}e+i\sqrt{c}d\right)}{(d+ex)\left(\sqrt{a}g+i\sqrt{c}f\right)}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{c}x+i\sqrt{a}\right)(ef-dg)}{(d+ex)\left(\sqrt{c}f-i\sqrt{a}g\right)}}\right),-\frac{\frac{i\sqrt{c}df}{\sqrt{a}}+\frac{i\sqrt{a}eg}{\sqrt{c}}+dg-ef}{2ef-2dg}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{\frac{\left(\sqrt{c}x+i\sqrt{a}\right)(ef-dg)}{(d+ex)\left(\sqrt{c}f-i\sqrt{a}g\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[2]*(I*Sqrt[a] + Sqrt[c]*x)*Sqrt[d + e*x]*Sqrt[(d - (I*Sqrt[a]*e)/Sqrt
[c] + (I*Sqrt[c]*d*x)/Sqrt[a] + e*x)/(d + e*x)]*Sqrt[((I*Sqrt[c]*d + Sqrt[a
]*e)*(f + g*x))/((I*Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*EllipticF[ArcSin[Sqr
t[((e*f - d*g)*(I*Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x
))]], -(((I*Sqrt[c]*d*f)/Sqrt[a] - e*f + d*g + (I*Sqrt[a]*e*g)/Sqrt[c])/(2*
e*f - 2*d*g))])/((Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[((e*f - d*g)*(I*Sqrt[a] + S
qrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x]*Sqrt[a + c*
x^2])
```

Maple [A] time = 0.624, size = 433, normalized size = 1.

$$2\frac{\left(ce^2fx^2-\sqrt{-ac}x^2e^2g+2xcdef-2\sqrt{-ac}xdeg+cd^2f-\sqrt{-ac}d^2g\right)\sqrt{ex+d}\sqrt{gx+f}\sqrt{cx^2+a}}{(dg-ef)\left(-\sqrt{-ac}e+cd\right)\sqrt{cegx^4+cdgx^3+cefx^3+aegx^2+cdfx^2+adgx+aefx+adf}}\text{EllipticF}\left(\sqrt{\frac{\left(\sqrt{-ac}e \ldots\right.}{\left(\sqrt{-ac}g \ldots\right.}} \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

```
[Out] 2*(c*e^2*f*x^2-(-a*c)^(1/2)*x^2*e^2*g+2*x*c*d*e*f-2*(-a*c)^(1/2)*x*d*e*g+c*
d^2*f-(-a*c)^(1/2)*d^2*g)*EllipticF((((-a*c)^(1/2)*e-c*d)*(g*x+f)/((-a*c)^(
1/2)*g-c*f)/(e*x+d))^(1/2),(((-a*c)^(1/2)*e+c*d)*((-a*c)^(1/2)*g-c*f)/((-a*
c)^(1/2)*g+c*f)/((-a*c)^(1/2)*e-c*d))^(1/2))*((d*g-e*f)*(c*x+(-a*c)^(1/2))/
((-a*c)^(1/2)*g-c*f)/(e*x+d))^(1/2)*((d*g-e*f)*(-c*x+(-a*c)^(1/2))/((-a*c)^
(1/2)*g+c*f)/(e*x+d))^(1/2)*(((-a*c)^(1/2)*e-c*d)*(g*x+f)/((-a*c)^(1/2)*g-c
*f)/(e*x+d))^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-1/c*(g*x+f
)*(e*x+d)*(-c*x+(-a*c)^(1/2))*(c*x+(-a*c)^(1/2)))^(1/2)/(d*g-e*f)/(-(-a*c)^
(1/2)*e+c*d)/(c*e*g*x^4+c*d*g*x^3+c*e*f*x^3+a*e*g*x^2+c*d*f*x^2+a*d*g*x+a*e
*f*x+a*d*f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}{cegx^4+\left(cef+cdg\right)x^3+adf+\left(cdf+aeg\right)x^2+\left(aef+adg\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c*e*f +
c*d*g)*x^3 + a*d*f + (c*d*f + a*e*g)*x^2 + (a*e*f + a*d*g)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}\sqrt{ex + d}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

3.656 $\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}} dx$

Optimal. Leaf size=52

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}\text{EllipticF}\left(\sin^{-1}(x),2\right)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]*Sqrt[
1 + x]*Sqrt[-1 + 2*x^2])
```

Rubi [A] time = 0.0463509, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {519, 421, 419}

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}F\left(\sin^{-1}(x)|2\right)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]*Sqrt[
1 + x]*Sqrt[-1 + 2*x^2])
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{-1+x^2}\int \frac{1}{\sqrt{-1+x^2}\sqrt{-1+2x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}}$$
$$= \frac{\left(\sqrt{1-2x^2}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{1-2x^2}\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}$$
$$= \frac{\left(\sqrt{1-2x^2}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-2x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}$$
$$= \frac{\sqrt{1-2x^2}\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}$$

Mathematica [B] time = 0.238413, size = 107, normalized size = 2.06

$$-\frac{2(x-1)^{3/2}\sqrt{\frac{x+1}{1-x}}\sqrt{\frac{1-2x^2}{(x-1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{1}{x-1}+\sqrt{2}+2}}{2^{3/4}}\right),4\left(3\sqrt{2}-4\right)\right)}{\sqrt{3+2\sqrt{2}}\sqrt{x+1}\sqrt{2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (-2*(-1 + x)^(3/2)*Sqrt[(1 + x)/(1 - x)]*Sqrt[(1 - 2*x^2)/(-1 + x)^2]*Ellip
ticF[ArcSin[Sqrt[2 + Sqrt[2] + (-1 + x)^(-1)]/2^(3/4)], 4*(-4 + 3*Sqrt[2])]
)/(Sqrt[3 + 2*Sqrt[2]]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2])
```

Maple [A] time = 0.115, size = 58, normalized size = 1.1

$$\frac{\text{EllipticF}\left(x,\sqrt{2}\right)}{2x^4-3x^2+1}\sqrt{-1+x}\sqrt{1+x}\sqrt{2x^2-1}\sqrt{-x^2+1}\sqrt{-2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x)
```

```
[Out] (-1+x)^(1/2)*(1+x)^(1/2)*(2*x^2-1)^(1/2)/(2*x^4-3*x^2+1)*(-x^2+1)^(1/2)*(-2
*x^2+1)^(1/2)*EllipticF(x,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}{2\,x^4-3\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)/(2*x^4 - 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/(1+x)**(1/2)/(2*x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x - 1)*sqrt(x + 1)*sqrt(2*x**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

3.657 $\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=269

$$\frac{12(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{35c^2d^2\sqrt{d+ex}}+\frac{16g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{35c^3d^3e}-\frac{16\sqrt{\cdot}}{}$$

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(35*c^4*d^4*e*Sqrt[d + e*x]) + (16*g*(c*d*f - a*e*
g)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3
*e) + (12*(c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(35*c^2*d^2*Sqrt[d + e*x]) + (2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d + e*x])
```

Rubi [A] time = 0.423284, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{12(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{35c^2d^2\sqrt{d+ex}}+\frac{16g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{35c^3d^3e}-\frac{16\sqrt{\cdot}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(35*c^4*d^4*e*Sqrt[d + e*x]) + (16*g*(c*d*f - a*e*
g)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3
*e) + (12*(c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(35*c^2*d^2*Sqrt[d + e*x]) + (2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d + e*x])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7cd\sqrt{d+ex}} + \frac{\left(6\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right)\int}{7cde^2}$$

$$= \frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^2d^2\sqrt{d+ex}} + \frac{2(f+gx)^3\sqrt{ade+\left(cd^2+\right.}}{7cd\sqrt{d+ex}}$$

$$= \frac{16g(cdf-aeg)^2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^3d^3e} + \frac{12(cdf-aeg)(f+gx)^2\sqrt{}}{35c^2d}$$

$$= -\frac{16(cdf-aeg)^2\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^4d^4e\sqrt{d+ex}} + \frac{16g(cdf}{}$$

Mathematica [A] time = 0.13536, size = 136, normalized size = 0.51

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2cde^2g^2(7f+gx)-16a^3e^3g^3-2ac^2d^2eg\left(35f^2+14fgx+3g^2x^2\right)+c^3d^3\left(35f^2gx+35f^3+21fg\right.\right.}{35c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(7*f
+ g*x) - 2*a*c^2*d^2*e*g*(35*f^2 + 14*f*g*x + 3*g^2*x^2) + c^3*d^3*(35*f^3
+ 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

Maple [A] time = 0.051, size = 188, normalized size = 0.7

$$-\frac{(2\,cdx+2\,ae)\left(-5\,g^3x^3c^3d^3+6\,ac^2d^2eg^3x^2-21\,c^3d^3fg^2x^2-8\,a^2cde^2g^3x+28\,ac^2d^2efg^2x-35\,c^3d^3f^2gx+16\,a^3e^3g^3-5\right.}{35\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-5*c^3*d^3*g^3*x^3+6*a*c^2*d^2*e*g^3*x^2-21*c^3*d^3*f*g^
2*x^2-8*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^2*x-35*c^3*d^3*f^2*g*x+16*a^3*
e^3*g^3-56*a^2*c*d*e^2*f*g^2+70*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)*(e*x+d)^(
1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.21868, size = 294, normalized size = 1.09

$$\frac{2\sqrt{cdx+ae}f^3}{cd}+\frac{2\left(c^2d^2x^2-acdex-2a^2e^2\right)f^2g}{\sqrt{cdx+ae}c^2d^2}+\frac{2\left(3c^3d^3x^3-ac^2d^2ex^2+4a^2cde^2x+8a^3e^3\right)fg^2}{5\sqrt{cdx+ae}c^3d^3}+\frac{2\left(5c^4d^4x^4-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^3/(c*d) + 2*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f^2
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*a
^2*c*d*e^2*x + 8*a^3*e^3)*f*g^2/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(5*c^4*d
^4*x^4 - a*c^3*d^3*e*x^3 + 2*a^2*c^2*d^2*e^2*x^2 - 8*a^3*c*d*e^3*x - 16*a^4
*e^4)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Fricas [A] time = 1.7912, size = 405, normalized size = 1.51

$$\frac{2\left(5c^3d^3g^3x^3+35c^3d^3f^3-70ac^2d^2ef^2g+56a^2cde^2fg^2-16a^3e^3g^3+3\left(7c^3d^3fg^2-2ac^2d^2eg^3\right)x^2+\left(35c^3d^3f^2g-\right.\right.}{35\left(c^4d^4ex+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 70*a*c^2*d^2*e*f^2*g + 56*a^2*c*
d*e^2*f*g^2 - 16*a^3*e^3*g^3 + 3*(7*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2
+ (35*c^3*d^3*f^2*g - 28*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^3}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^3/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

3.658 $\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=200

$$\frac{8g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{15c^2d^2e}-\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)\left(2ae^2g-cd(3ef-d\right.}{15c^3d^3e\sqrt{d+ex}}$$

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) + (8*g*(c*d*f - a*e*g)*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) +
(2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d +
 e*x])
```

Rubi [A] time = 0.232959, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{15c^2d^2e}-\frac{8\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)\left(2ae^2g-cd(3ef-d\right.}{15c^3d^3e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) + (8*g*(c*d*f - a*e*g)*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) +
(2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d +
 e*x])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5cd\sqrt{d+ex}} + \frac{\left(4\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right)\int}{5cde^2}$$

$$= \frac{8g(cdf-aeg)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^2d^2e} + \frac{2(f+gx)^2\sqrt{ade+\left(cd^2+a\right.}}{5cd\sqrt{d+ex}}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}} + \frac{8g(cdf-a}{}$$

Mathematica [A] time = 0.0830496, size = 89, normalized size = 0.44

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^2g^2-4acdeg(5f+gx)+c^2d^2\left(15f^2+10fgx+3g^2x^2\right)\right)}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(5*f + g*x) +
 c^2*d^2*(15*f^2 + 10*f*g*x + 3*g^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

Maple [A] time = 0.051, size = 116, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(3\,g^2x^2c^2d^2-4\,acdeg^2x+10\,c^2d^2fgx+8\,a^2e^2g^2-20\,acdefg+15\,f^2c^2d^2\right)}{15\,c^3d^3}\sqrt{ex+d}\frac{1}{\sqrt{cdex^2+ae^2x+cd^2x+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 2/15*(c*d*x+a*e)*(3*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x+10*c^2*d^2*f*g*x+8*a^2*
e^2*g^2-20*a*c*d*e*f*g+15*c^2*d^2*f^2)*(e*x+d)^(1/2)/c^3/d^3/(c*d*e*x^2+a*e
^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.19517, size = 180, normalized size = 0.9

$$\frac{2\sqrt{cdx+ae}f^2}{cd} + \frac{4\left(c^2d^2x^2-acdex-2\,a^2e^2\right)fg}{3\sqrt{cdx+ae}c^2d^2} + \frac{2\left(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3\right)g^2}{15\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*
a^2*c*d*e^2*x + 8*a^3*e^3)*g^2/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Fricas [A] time = 1.94361, size = 265, normalized size = 1.32

$$\frac{2\left(3c^2d^2g^2x^2+15c^2d^2f^2-20acdefg+8a^2e^2g^2+2\left(5c^2d^2fg-2acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{15\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g + 8*a^2*e^2*g^2 +
 2*(5*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d+ex}\left(f+gx\right)^2}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex+d}\left(gx+f\right)^2}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^2/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

3.659 $\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^2d^2e\sqrt{d+ex}}$$

```
[Out] (-2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c^2*d^2*e*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(3*c*d*e)
```

Rubi [A] time = 0.0917179, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^2d^2e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c^2*d^2*e*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(3*c*d*e)
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2g\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cde}+\frac{1}{3}\left(3f-\frac{dg}{e}-\frac{2aeg}{cd}\right)\int \frac{\sqrt{d}}{\sqrt{ade+\left(cd^2+\right.}}$$

$$=-\frac{2\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2e\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{ade+\left(cd\right.}}{3cde}$$

Mathematica [A] time = 0.0469351, size = 53, normalized size = 0.42

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(cd(3f+gx)-2aeg)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(3*f + g*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

Maple [A] time = 0.047, size = 67, normalized size = 0.5

$$-\frac{(2\,cdx+2\,ae)\left(-xcdg+2\,aeg-3\,cdf\right)}{3\,c^2d^2}\sqrt{ex+d}\frac{1}{\sqrt{cdex^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c*d*g*x+2*a*e*g-3*c*d*f)*(e*x+d)^(1/2)/c^2/d^2/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.11433, size = 88, normalized size = 0.7

$$\frac{2\,\sqrt{cdx+ae}f}{cd}+\frac{2\left(c^2d^2x^2-acdex-2\,a^2e^2\right)g}{3\,\sqrt{cdx+ae}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f/(c*d) + 2/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*g/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

Fricas [A] time = 2.00958, size = 158, normalized size = 1.26

$$\frac{2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx+3\,cdf-2\,aeg\right)\sqrt{ex+d}}{3\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*a*e*
g)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\,(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(gx+f)}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x), x)
```

3.660 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Rubi [A] time = 0.0206646, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.0167825, size = 35, normalized size = 0.76

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

Maple [A] time = 0.048, size = 50, normalized size = 1.1

$$2\frac{(cdx+ae)\sqrt{ex+d}}{cd\sqrt{cde x^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(e*x+d)^(1/2)/d/c/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.05088, size = 24, normalized size = 0.52

$$\frac{2\sqrt{cdx+ae}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)/(c*d)
```

Fricas [A] time = 1.57786, size = 107, normalized size = 2.33

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{cdex+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.661 $\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=80

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f
 - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*Sqrt[c*d*f - a*e*g])
```

Rubi [A] time = 0.13142, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {874, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x
]
```

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f
 - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*Sqrt[c*d*f - a*e*g])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\left(2e^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2gx^2}\,dx,x,\frac{\sqrt{ade+(c}}{}\right.$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

Mathematica [A] time = 0.0437619, size = 93, normalized size = 1.16

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[
c*d*f - a*e*g]])/(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

Maple [A] time = 0.319, size = 87, normalized size = 1.1

$$-2\,{\frac{\sqrt{cde{x}^{2}+a{e}^{2}x+c{d}^{2}x+ade}}{\sqrt{ex+d}\sqrt{cdx+ae}\sqrt{\left(aeg-cdf\right)g}}{\rm Artanh}\left({\frac{\sqrt{cdx+ae}g}{\sqrt{\left(aeg-cdf\right)g}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/(e*x+d)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(c*d*x+a*e)^(1/2)/
((a*e*g-c*d*f)*g)^(1/2)*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)), x)
```

Fricas [A] time = 1.65055, size = 556, normalized size = 6.95

$$\left[-\frac{\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{-cdfg+aeg^2}\sqrt{ex+d}}{egx^2+df+\left(ef+dg\right)x}\right)}{cdfg-aeg^2}, -\frac{2\arctan\left(\frac{\sqrt{cdex^2+\cdots}}{\cdots}\right)}{\sqrt{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [-sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e
*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x))/(
c*d*f*g - a*e*g^2), -2*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^
2)*g*x))/sqrt(c*d*f*g - a*e*g^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

3.662 $\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*
(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))
```

Rubi [A] time = 0.19132, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {872, 874, 205}

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*
(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{2(cdf-aeg)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{\left(cde^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2}\right.}{cdf-aeg}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}$$

Mathematica [A] time = 0.111714, size = 136, normalized size = 0.97

$$\frac{\sqrt{d+ex}\left(\sqrt{g}(ae+cdx)\sqrt{cdf-aeg}+cd(f+gx)\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{\sqrt{g}(f+gx)\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*Sqrt[a*e +
c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/
(Sqrt[g]*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))
```

Maple [A] time = 0.297, size = 168, normalized size = 1.2

$$\frac{1}{\left(aeg-cdf\right)\left(gx+f\right)}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\text{Artanh}\left(g\sqrt{cdx+ae}\frac{1}{\sqrt{\left(aeg-cdf\right)g}}\right)xcdg+\text{Artanh}\left(g\sqrt{cdx+ae}\frac{1}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)*g/((a*e*
g-c*d*f)*g)^(1/2))*x*c*d*g+arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1
/2))*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+
a*e)^(1/2)/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^2), x)
```

Fricas [B] time = 1.71952, size = 1478, normalized size = 10.56

$$\left[\frac{\left(cdegx^2 + cd^2f + \left(cdef + cd^2g\right)x\right)\sqrt{-cdfg + aeg^2}\log\left(-\frac{cdegx^2 - cd^2f + 2\,adeg - \left(cdef - \left(cd^2 + 2\,ae^2\right)g\right)x + 2\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{-cdf}}{egx^2 + df + \left(ef + dg\right)x}\right.}{2\left(c^2d^3f^3g - 2\,acd^2ef^2g^2 + a^2de^2fg^3 + \left(c^2d^2ef^2g^2 - 2\,acde^2fg^3 + a^2e^3g^4\right)x^2 + \left(c^2d^2ef^3g + a^2de^2g\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*g
^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*
g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^
2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g -
 2*a*c*d^2*e*f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f
*g^3 + a^2*e^3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a
*c*d*e^2)*f^2*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x), -((c*d*e*g*x^2 + c*d
^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*
g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g - 2*a*c*d^2*e*
f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^
3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f^2
*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.663 $\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=213

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4\sqrt{g}(cdf-aeg)^{5/2}}+\frac{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^2) + (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*
d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2))
```

Rubi [A] time = 0.313584, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {872, 874, 205}

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4\sqrt{g}(cdf-aeg)^{5/2}}+\frac{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```


```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^2) + (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*
d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2))
```


Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{(3cd)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4(cdf-aeg)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)}+\frac{(3c}{}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)}+\frac{(3c}{}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)}+\frac{3c^2}{}$$

Mathematica [C] time = 0.0475734, size = 77, normalized size = 0.36

$$\frac{2c^2d^2\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{d+ex}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*c^2*d^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[1/2, 3, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^3*Sqrt[d + e*x])
```

Maple [A] time = 0.329, size = 285, normalized size = 1.3

$$-\frac{1}{4\left(aeg-cdf\right)^2\left(gx+f\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^2c^2d^2g^2+6\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+ae}}{\sqrt{\left(aeg-cdf\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^2*d^2*g^2+6*arctanh((c*d*x+a*e)^(1/2)*g/((a
*e*g-c*d*f)*g)^(1/2))*x*c^2*d^2*f*g+3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c
*d*f)*g)^(1/2))*c^2*d^2*f^2-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c
*d*g+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g-5*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f
)^2/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^3), x)
```

Fricas [B] time = 1.88753, size = 2587, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(5*c^2*d^2*f^2*g - 7*a*c*d*e*f*g
^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g - 3*a*c^2*d^3*e*f^4*
g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 + (c^3*d^3*e*f^3*g^3 - 3*
a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^3 + (2*c^3*d^3
*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2
*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^5)*x^2
+ (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*
g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)
*f^2*g^4)*x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g +
 c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g
^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g
^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - (5*c^2*d
^2*f^2*g - 7*a*c*d*e*f*g^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3
)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^
5*g - 3*a*c^2*d^3*e*f^4*g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 +
 (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e
^4*g^6)*x^3 + (2*c^3*d^3*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2
*e^2)*f^3*g^3 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2
- 2*a^3*e^4)*f*g^5)*x^2 + (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4
 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*
a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.664 $\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=280

$$\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8\sqrt{g}(cdf-aeg)^{7/2}}+\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\cdots$$

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^3) + (5*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(8*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (
5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*Sqrt[g]*(c*d*f - a*e*g)^(7/2))
```

Rubi [A] time = 0.423973, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {872, 874, 205}

$$\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8\sqrt{g}(cdf-aeg)^{7/2}}+\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^3) + (5*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(8*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (
5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*Sqrt[g]*(c*d*f - a*e*g)^(7/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{(5cd)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{6(cdf-aeg)}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \ldots$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \ldots$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \ldots$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} + \ldots$$

Mathematica [C] time = 0.0456276, size = 77, normalized size = 0.28

$$\frac{2c^3d^3\sqrt{(d+ex)(ae+cdx)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{d+ex}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*Hypergeometric2F1[1/2, 4, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^4*Sqrt[d + e*x])
```

Maple [A] time = 0.378, size = 450, normalized size = 1.6

$$\frac{1}{24\left(aeg-cdf\right)^3\left(gx+f\right)^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^3c^3d^3g^3+45\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+\ldots}}{\sqrt{\left(aeg-c\ldots\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^3*d^3*g^3+45*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*x^2*c^3*d^3*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)*g/((
```

```
a*e*g-c*d*f)*g)^(1/2))*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e
*g-c*d*f)*g)^(1/2))*c^3*d^3*f^3-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2
)*x^2*c^2*d^2*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*g^
2-40*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g-8*((a*e*g-c*d*
f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x
+a*e)^(1/2)*a*c*d*e*f*g-33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^
2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^3/(g*x+f)^3/((a*e*g-c*
d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^4), x)
```

Fricas [B] time = 2.00879, size = 4030, normalized size = 14.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*
d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*
e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*(33*c^3*d^3*f^3*g - 59*a*c^2*d^2*e*f^2*g^2 + 34*a^2*c*d*e^2*
f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 10*(4*c^
3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g - 4*a*c^3*d^4*e*f^
6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3*c*d^2*e^3*f^4*g^4 + a^4*d*e^4*f^3
*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a^2*c^2*d^2*e^3*f^2
*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^4 + (3*c^4*d^4*e*f^5*g^3 + a^4*
d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^4 - 2*(2*a*c^3*d^4*e - 9*a^2
*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^6 - (4*a^
3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(c^4*d^4*e*f^6*g^2 + a^4*d*e^4*f*g^
7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*
e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^5 - (4*a^3*c*d^2
*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*
c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^
5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^4 - (12*a^3*c*d^2*e^3 -
 a^4*e^5)*f^3*g^5)*x), -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*
d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 +
(c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*
```

```
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/
(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) - (33*c^3*d^3*f^3*g - 59*a*c
^2*d^2*e*f^2*g^2 + 34*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3
 - a*c^2*d^2*e*g^4)*x^2 + 10*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2
*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))
/(c^4*d^5*f^7*g - 4*a*c^3*d^4*e*f^6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3
*c*d^2*e^3*f^4*g^4 + a^4*d*e^4*f^3*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e
^2*f^3*g^5 + 6*a^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)
*x^4 + (3*c^4*d^4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*
f^4*g^4 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^2*g^6 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(
c^4*d^4*e*f^6*g^2 + a^4*d*e^4*f*g^7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 -
 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a
^3*c*d*e^4)*f^3*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e
*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2
*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*
e^4)*f^4*g^4 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.665 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$\frac{16g^2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{5c^3d^3e}+\frac{12g(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5c^2d^2\sqrt{d+ex}}-16g\sqrt{x\left(ae^2+c\right.}$$

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (16*g*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^2*(c*d*
f - a*e*g)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^
3*d^3*e) + (12*g*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
5*c^2*d^2*Sqrt[d + e*x])
```

Rubi [A] time = 0.330761, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {866, 870, 794, 648}

$$\frac{16g^2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{5c^3d^3e}+\frac{12g(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5c^2d^2\sqrt{d+ex}}-16g\sqrt{x\left(ae^2+c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (16*g*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^2*(c*d*
f - a*e*g)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^
3*d^3*e) + (12*g*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
5*c^2*d^2*Sqrt[d + e*x])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
```

```
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(6g)\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{12g(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{16g^2(cdf-aeg)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right.}}{5c^3d^3e}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{16g(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ad}}{5c^4d^4e\sqrt{d+ex}}$$

Mathematica [A] time = 0.111001, size = 134, normalized size = 0.52

$$\frac{2\sqrt{d+ex}\left(8a^2cde^2g^2(gx-5f)+16a^3e^3g^3-2ac^2d^2eg\left(-15f^2+10fgx+g^2x^2\right)+c^3d^3\left(15f^2gx-5f^3+5fg^2x^2+g^3x^3\right)\right)}{5c^4d^4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(-5*f + g*x) - 2*a*c^2
*d^2*e*g*(-15*f^2 + 10*f*g*x + g^2*x^2) + c^3*d^3*(-5*f^3 + 15*f^2*g*x + 5*
f*g^2*x^2 + g^3*x^3)))/(5*c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.049, size = 187, normalized size = 0.7

$$\frac{(2\,cdx+2\,ae)\left(g^3x^3c^3d^3-2\,ac^2d^2eg^3x^2+5\,c^3d^3fg^2x^2+8\,a^2cde^2g^3x-20\,ac^2d^2efg^2x+15\,c^3d^3f^2gx+16\,a^3e^3g^3-40\,a^2c\right.}{5\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2/5*(c*d*x+a*e)*(c^3*d^3*g^3*x^3-2*a*c^2*d^2*e*g^3*x^2+5*c^3*d^3*f*g^2*x^2+
8*a^2*c*d*e^2*g^3*x-20*a*c^2*d^2*e*f*g^2*x+15*c^3*d^3*f^2*g*x+16*a^3*e^3*g^
3-40*a^2*c*d*e^2*f*g^2+30*a*c^2*d^2*e*f^2*g-5*c^3*d^3*f^3)*(e*x+d)^(3/2)/c^
4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [A] time = 1.2193, size = 223, normalized size = 0.87

$$-\frac{2\,f^3}{\sqrt{cdx+ae}cd}+\frac{6\,(cdx+2\,ae)f^2g}{\sqrt{cdx+ae}c^2d^2}+\frac{2\left(c^2d^2x^2-4\,acdex-8\,a^2e^2\right)fg^2}{\sqrt{cdx+ae}c^3d^3}+\frac{2\left(c^3d^3x^3-2\,ac^2d^2ex^2+8\,a^2cde^2x+16\,a^3e^3\right.}{5\,\sqrt{cdx+ae}c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^3/(sqrt(c*d*x + a*e)*c*d) + 6*(c*d*x + 2*a*e)*f^2*g/(sqrt(c*d*x + a*e)
*c^2*d^2) + 2*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*f*g^2/(sqrt(c*d*x + a
*e)*c^3*d^3) + 2/5*(c^3*d^3*x^3 - 2*a*c^2*d^2*e*x^2 + 8*a^2*c*d*e^2*x + 16*
a^3*e^3)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Fricas [A] time = 1.6162, size = 446, normalized size = 1.74

$$\frac{2\left(c^3d^3g^3x^3-5\,c^3d^3f^3+30\,ac^2d^2ef^2g-40\,a^2cde^2fg^2+16\,a^3e^3g^3+\left(5\,c^3d^3fg^2-2\,ac^2d^2eg^3\right)x^2+\left(15\,c^3d^3f^2g-20\,ac\right.\right.}{5\left(c^5d^5ex^2+ac^4d^5e+\left(c^5d^6+ac^4d^4e^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/5*(c^3*d^3*g^3*x^3 - 5*c^3*d^3*f^3 + 30*a*c^2*d^2*e*f^2*g - 40*a^2*c*d*e^
2*f*g^2 + 16*a^3*e^3*g^3 + (5*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 + (15*
c^3*d^3*f^2*g - 20*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x^2 + a*c^4*d^5*e +
(c^5*d^6 + a*c^4*d^4*e^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.666 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^3d^3e\sqrt{d+ex}}+\frac{8g^2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2e}-\frac{2\sqrt{d+\cdots}}{cd\sqrt{x\left(ae^2+\cdots\right.}}$$

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (8*g*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(3*c^3*d^3*e*Sqrt[d + e*x]) + (8*g^2*Sqrt[d + e*x]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e)
```

Rubi [A] time = 0.18466, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 794, 648}

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^3d^3e\sqrt{d+ex}}+\frac{8g^2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2e}-\frac{2\sqrt{d+\cdots}}{cd\sqrt{x\left(ae^2+\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (8*g*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(3*c^3*d^3*e*Sqrt[d + e*x]) + (8*g^2*Sqrt[d + e*x]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e)
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
```

```
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(4g)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{8g^2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2e} -$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+a\right.}}{3c^3d^3e\sqrt{d+ex}}$$

Mathematica [A] time = 0.07382, size = 88, normalized size = 0.49

$$\frac{2\sqrt{d+ex}\left(-8a^2e^2g^2-4acdeg(gx-3f)+c^2d^2\left(-3f^2+6fgx+g^2x^2\right)\right)}{3c^3d^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-8*a^2*e^2*g^2 - 4*a*c*d*e*g*(-3*f + g*x) + c^2*d^2*(-3*f
^2 + 6*f*g*x + g^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.052, size = 116, normalized size = 0.6

$$-\frac{(2\,cdx+2\,ae)\left(-g^2x^2c^2d^2+4\,acdeg^2x-6\,c^2d^2fgx+8\,a^2e^2g^2-12\,acdefg+3\,c^2d^2f^2\right)}{3\,c^3d^3}(ex+d)^{\frac{3}{2}}\left(cdex^2+ae^2x+cd^2x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c^2*d^2*g^2*x^2+4*a*c*d*e*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^
2*g^2-12*a*c*d*e*f*g+3*c^2*d^2*f^2)*(e*x+d)^(3/2)/c^3/d^3/(c*d*e*x^2+a*e^2*
x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [A] time = 1.17767, size = 132, normalized size = 0.73

$$-\frac{2\,f^2}{\sqrt{cdx+ae}cd} + \frac{4\,(cdx+2\,ae)fg}{\sqrt{cdx+ae}c^2d^2} + \frac{2\left(c^2d^2x^2-4\,acdex-8\,a^2e^2\right)g^2}{3\,\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^2/(sqrt(c*d*x + a*e)*c*d) + 4*(c*d*x + 2*a*e)*f*g/(sqrt(c*d*x + a*e)*c
^2*d^2) + 2/3*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*g^2/(sqrt(c*d*x + a*e
)*c^3*d^3)
```

Fricas [A] time = 1.59079, size = 308, normalized size = 1.7

$$\frac{2\left(c^2d^2g^2x^2-3\,c^2d^2f^2+12\,acdefg-8\,a^2e^2g^2+2\left(3\,c^2d^2fg-2\,acdeg^2\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{3\left(c^4d^4ex^2+ac^3d^4e+\left(c^4d^5+ac^3d^3e^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^2*d^2*g^2*x^2 - 3*c^2*d^2*f^2 + 12*a*c*d*e*f*g - 8*a^2*e^2*g^2 + 2*(
3*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.667 $\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```

Rubi [A] time = 0.143614, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {788, 648}

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2),x]
```


```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```


Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(2\left(-\frac{1}{2}e\left(2cdef-\left(cd^2+ae^2\right.\right.\right.\right.}{}$$

$$= -\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\left(2ae^2g-cd(ef+dg)\right)\sqrt{ac}}{c^2d^2\left(cd^2-ae\right.}$$

Mathematica [A] time = 0.0438532, size = 51, normalized size = 0.34

$$\frac{2\sqrt{d+ex}(2aeg+cd(gx-f))}{c^2d^2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(2*a*e*g + c*d*(-f + g*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

Maple [A] time = 0.046, size = 66, normalized size = 0.4

$$2\,{\frac{(cdx+ae)\left(xcdg+2\,aeg-cdf\right)(ex+d)^{3/2}}{c^2d^2\left(cdex^2+ae^2x+cd^2x+ade\right)^{3/2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(c*d*g*x+2*a*e*g-c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(c*d*e*x^2+a*e^
2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [A] time = 1.42288, size = 65, normalized size = 0.43

$$-\frac{2\,f}{\sqrt{cdx+ae}cd}+\frac{2\,(cdx+2\,ae)g}{\sqrt{cdx+ae}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] -2*f/(sqrt(c*d*x + a*e)*c*d) + 2*(c*d*x + 2*a*e)*g/(sqrt(c*d*x + a*e)*c^2*d
^2)
```

Fricas [A] time = 1.59341, size = 201, normalized size = 1.34

$$\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(cdgx-cdf+2\,aeg)\sqrt{ex+d}}{c^3d^3ex^2+ac^2d^3e+(c^3d^4+ac^2d^2e^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - c*d*f + 2*a*e*g)*s
qrt(e*x + d)/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] sage0*x
```

3.668 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0216481, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.0123299, size = 35, normalized size = 0.76

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.044, size = 50, normalized size = 1.1

$$-2\,\frac{(cdx+ae)\,(ex+d)^{3/2}}{cd\left(cdex^2+ae^2x+cd^2x+ade\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2*(c*d*x+a*e)*(e*x+d)^(3/2)/d/c/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [A] time = 1.25688, size = 24, normalized size = 0.52

$$-\frac{2}{\sqrt{cdx+ae}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] -2/(sqrt(c*d*x + a*e)*c*d)
```

Fricas [A] time = 1.54448, size = 157, normalized size = 3.41

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{c^2d^2ex^2+acd^2e+\left(c^2d^3+acde^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^2
 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^{\frac{3}{2}}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.669 $\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}$$

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(3/2)
```

Rubi [A] time = 0.176276, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {868, 874, 205}

$$-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)),x]
```


```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(3/2)
```


Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{g\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)}}}{cdf-aeg}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(2e^2g\right)\text{Subst}\left(\int \frac{}{-e\left(cd^2\right.}\right.}{}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(c}}{\sqrt{cdf-}}\right.}{(cdf-aeg)}$$

Mathematica [C] time = 0.0290364, size = 71, normalized size = 0.53

$$-\frac{2\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 1, 1/2, (g*(a*e + c*d*x))/(-(c*d*
f) + a*e*g)])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.278, size = 128, normalized size = 1.

$$-2\,\frac{\sqrt{cdex^2+ae^2x+cd^2x+ade}}{\sqrt{ex+d}\,(cdx+ae)\left(aeg-cdf\right)\sqrt{\left(aeg-cdf\right)g}}\left(g\text{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{\left(aeg-cdf\right)g}}\right)\sqrt{cdx+ae}-\sqrt{\left(aeg-cdf\right)g}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)-((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^
(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)), x)
```

Fricas [B] time = 1.74607, size = 1170, normalized size = 8.8

$$\left[-\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)\sqrt{-\frac{g}{cdf-aeg}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg+2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdf-aeg\right)\sqrt{ex+d}\sqrt{-\frac{g}{cdf-aeg}}-\left(cdef-\left(cd^2+2\,ae\right.\right.}{egx^2+df+\left(ef+dg\right)x}\right.}{acd^2ef-a^2de^2g+\left(c^2d^2ef-acde^2g\right)x^2+\left(\left(c^2d^3+acde^2\right)f-\left(acd^2e+a^2e^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] [-((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c
*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (
c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*d^2*e*f - a^2*d*e^2*g + (c
^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2
*e^3)*g)*x), -2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d*f - a*
e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*s
qrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^
2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*
d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d
*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] sage0*x
```

3.670 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{3g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}-\frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right.}{(cdf-aeg)^{5/2}}$$

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (3*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f
 - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) - (3*c*d*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/(c*d*f - a*e*g)^(5/2)
```

Rubi [A] time = 0.25665, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$-\frac{3g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}-\frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right.}{(cdf-aeg)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (3*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f
 - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) - (3*c*d*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/(c*d*f - a*e*g)^(5/2)
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
```

```
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{(3g)\int \frac{}{(f+gx)^2}}{}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{3g\sqrt{ade+(c}}{(cdf-aeg)}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{3g\sqrt{ade+(c}}{(cdf-aeg)}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{3g\sqrt{ade+(c}}{(cdf-aeg)}$$

Mathematica [C] time = 0.0357541, size = 73, normalized size = 0.36

$$-\frac{2cd\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] (-2*c*d*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 2, 1/2, (g*(a*e + c*d*x))/(-(
c*d*f) + a*e*g)])/((c*d*f - a*e*g)^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.342, size = 225, normalized size = 1.1

$$\frac{1}{(cdx+ae)\left(aeg-cdf\right)^2\left(gx+f\right)}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)\sqrt{cdx+aexcdg^2}+3\,\mathrm{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)*g/((a*
e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*x*c*d*g^2+3*arctanh((c*d*x+a*e)^(1/2
)*g/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c*d*f*g-3*((a*e*g-c*d*f)*g)^
(1/2)*x*c*d*g-((a*e*g-c*d*f)*g)^(1/2)*a*e*g-2*((a*e*g-c*d*f)*g)^(1/2)*c*d*f
)/(e*x+d)^(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^2), x)
```

Fricas [B] time = 1.83021, size = 2176, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 + a*c*d*e^
2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(-g/(c*d*f - a*e
*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) -
(c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + 2*c*d*f + a*e*g)*sqrt
(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*d*e^3*f*g^2 + (c^
3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x^3 + (c^3*d^3*e*f
^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^2*c*d*e^3)*f*g^2
+ (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c^3*d^4 + a*c^2*d^
2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*d^2*e^2 - a^3*e^4
)*f*g^2)*x), -(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 +
 a*c*d*e^2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(g/(c*d
*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a
*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2
 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x +
2*c*d*f + a*e*g)*sqrt(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g +
a^3*d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^
3)*x^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e
+ a^2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3
+ (c^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^
```

```
2*c*d^2*e^2 - a^3*e^4)*f*g^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.671 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}}-\frac{15cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-\frac{5g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}-\ldots$$

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) - (5*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) - (15*c*d*g*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (1
5*c^2*d^2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*(c*d*f - a*e*g)^(7/2))
```

Rubi [A] time = 0.352333, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$-\frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}}-\frac{15cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-\frac{5g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```


```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) - (5*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) - (15*c*d*g*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (1
5*c^2*d^2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*(c*d*f - a*e*g)^(7/2))
```


Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{(5g)\int \frac{}{(f+gx}}{}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ade+(}}{2(cdf-aeg}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ade+(}}{2(cdf-aeg}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ade+(}}{2(cdf-aeg}$$

$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ade+(}}{2(cdf-aeg}$$

Mathematica [C] time = 0.0390038, size = 77, normalized size = 0.28

$$-\frac{2c^2d^2\sqrt{d+ex}\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] (-2*c^2*d^2*Sqrt[d + e*x]*Hypergeometric2F1[-1/2, 3, 1/2, (g*(a*e + c*d*x))
/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.3, size = 379, normalized size = 1.4

$$-\frac{1}{\left(4\,cdx+4\,ae\right)\left(aeg-cdf\right)^{3}\left(gx+f\right)^{2}}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{\left(aeg-cdf\right)g}}\right)\sqrt{cdx+ae}\,x^{2}c^{2}d^{2}g^{3}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*x^2*c^2*d^2*g^3+30*arctanh((c*
d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g^2
+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*
c^2*d^2*f^2*g-15*((a*e*g-c*d*f)*g)^(1/2)*x^2*c^2*d^2*g^2-5*((a*e*g-c*d*f)*g
)^(1/2)*x*a*c*d*e*g^2-25*((a*e*g-c*d*f)*g)^(1/2)*x*c^2*d^2*f*g+2*((a*e*g-c*
d*f)*g)^(1/2)*a^2*e^2*g^2-9*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-8*((a*e*g-c
*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3/(g*x+
f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^3), x)
```

Fricas [B] time = 1.99486, size = 3673, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(15*(c^3*d^3*e*g^2*x^4 + a*c^2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^
4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d^4
 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)
*f^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g +
 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d
)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 +
d*f + (e*f + d*g)*x)) + 2*(15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*c*d*e*f
*g - 2*a^2*e^2*g^2 + 5*(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^
2*f^4*g + 3*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2
- 3*a*c^3*d^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4
```

```
+ (2*c^4*d^4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e
 - a^2*c^2*d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^
3*c*d^2*e^3 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d
^5 - a*c^3*d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (
3*a^2*c^2*d^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*
g^4)*x^2 - (2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*
e + 3*a^2*c^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 -
(5*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^3)*x), -1/4*(15*(c^3*d^3*e*g^2*x^4 + a*c^
2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*
d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d^4 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c
^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e^2)*f^2)*x)*sqrt(g/(c*d*f - a*e*g))*ar
ctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x
+ d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x))
 + (15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*c*d*e*f*g - 2*a^2*e^2*g^2 + 5*
(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g + 3*a^3*c*d^2*e
^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c^3*d^3*e^2*f^2*g
^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c^4*d^4*e*f^4*g +
(c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^2
*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*d^2*e^3 + a^4*e^5)*
g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5 - a*c^3*d^3*e^2)*f^4
*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*a^2*c^2*d^3*e^2 + 5*a
^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g^4)*x^2 - (2*a^4*d*e^4
*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*
f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 - (5*a^3*c*d^2*e^3 - a^4*
e^5)*f^2*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.672 $\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^4d^4e\sqrt{d+ex}}-\frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16g^3\sqrt{d+ex}\sqrt{x\left(ae^2\right.}}{3c^3}$$

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (4*g*Sqrt[d + e*x]*(f + g*x)^2)/(c^2*d^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) - (16*g^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^3*
Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e)
```

Rubi [A] time = 0.279648, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 794, 648}

$$-\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(3ef-dg)\right)}{3c^4d^4e\sqrt{d+ex}}-\frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16g^3\sqrt{d+ex}\sqrt{x\left(ae^2\right.}}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (4*g*Sqrt[d + e*x]*(f + g*x)^2)/(c^2*d^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) - (16*g^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^3*
Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e)
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(2g)\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} +$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} +$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} -$$

Mathematica [A] time = 0.113385, size = 131, normalized size = 0.55

$$\frac{2(d+ex)^{3/2}\left(24a^2cde^2g^2(f-gx)-16a^3e^3g^3-6ac^2d^2eg\left(f^2-6fgx+g^2x^2\right)+c^3d^3\left(-9f^2gx-f^3+9fg^2x^2+g^3x^3\right)\right)}{3c^4d^4((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(f - g*x) - 6*a*c^
2*d^2*e*g*(f^2 - 6*f*g*x + g^2*x^2) + c^3*d^3*(-f^3 - 9*f^2*g*x + 9*f*g^2*x
^2 + g^3*x^3)))/(3*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.048, size = 187, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-g^3x^3c^3d^3+6\,ac^2d^2eg^3x^2-9\,c^3d^3fg^2x^2+24\,a^2cde^2g^3x-36\,ac^2d^2efg^2x+9\,c^3d^3f^2gx+16\,a^3e^3g^3-\right.}{3\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c^3*d^3*g^3*x^3+6*a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f*g^2*x^
2+24*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2*x+9*c^3*d^3*f^2*g*x+16*a^3*e^3*
g^3-24*a^2*c*d*e^2*f*g^2+6*a*c^2*d^2*e*f^2*g+c^3*d^3*f^3)*(e*x+d)^(5/2)/c^4
/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [A] time = 1.29584, size = 296, normalized size = 1.24

$$-\frac{2\,(3\,cdx+2\,ae)f^2g}{\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}}+\frac{2\left(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2\right)fg^2}{\left(c^4d^4x+ac^3d^3e\right)\sqrt{cdx+ae}}+\frac{2\left(c^3d^3x^3-6\,ac^2d^2ex^2-24\,a^2cde^2x-16\,a^3e^3\right)g^3}{3\left(c^5d^5x+ac^4d^4e\right)\sqrt{cdx+ae}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -2*(3*c*d*x + 2*a*e)*f^2*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*f*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) + 2/3*(c^3*d^3*x^3 - 6*a*c^2*d^2*e*x^2 - 24*a^2*c*d*e
^2*x - 16*a^3*e^3)*g^3/((c^5*d^5*x + a*c^4*d^4*e)*sqrt(c*d*x + a*e)) - 2/3*
f^3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [A] time = 1.65549, size = 508, normalized size = 2.13

$$\frac{2\left(c^3d^3g^3x^3-c^3d^3f^3-6\,ac^2d^2ef^2g+24\,a^2cde^2fg^2-16\,a^3e^3g^3+3\left(3\,c^3d^3fg^2-2\,ac^2d^2eg^3\right)x^2-3\left(3\,c^3d^3f^2g-12\,ac^2d^2e\right.\right.}{3\left(c^6d^6ex^3+a^2c^4d^5e^2+\left(c^6d^7+2\,ac^5d^5e^2\right)x^2+\left(2\,ac^5d^6e+a^2c^4d^4e^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^3*d^3*g^3*x^3 - c^3*d^3*f^3 - 6*a*c^2*d^2*e*f^2*g + 24*a^2*c*d*e^2*f
*g^2 - 16*a^3*e^3*g^3 + 3*(3*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 - 3*(3*
c^3*d^3*f^2*g - 12*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^6*d^6*e*x^3 + a^2*c^4*d^5*e^
2 + (c^6*d^7 + 2*a*c^5*d^5*e^2)*x^2 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.673 $\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{3c^3d^3\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{8g(d+ex)^{3/2}(cdf-aeg)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d}{3cd\left(x\left(ae^2\right.\right.}$$

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^2)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (8*g*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(3*c^2*d^2*(c*d^2 - a*e
^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*(2*a*e^2*g - c*d*(e
*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*(c*d^2 -
 a*e^2)*Sqrt[d + e*x])
```

Rubi [A] time = 0.219514, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {866, 788, 648}

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{3c^3d^3\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{8g(d+ex)^{3/2}(cdf-aeg)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d}{3cd\left(x\left(ae^2\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^2)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (8*g*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(3*c^2*d^2*(c*d^2 - a*e
^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*(2*a*e^2*g - c*d*(e
*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*(c*d^2 -
 a*e^2)*Sqrt[d + e*x])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(4g)\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{8g(cdf-aeg)(d+ex)^{3/2}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right.}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{8g(cdf-aeg)(d+ex)^{3/2}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right.}}$$

Mathematica [A] time = 0.0746126, size = 87, normalized size = 0.41

$$\frac{2(d+ex)^{3/2}\left(8a^2e^2g^2-4acdeg(f-3gx)-c^2d^2\left(f^2+6fgx-3g^2x^2\right)\right)}{3c^3d^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(f - 3*g*x) - c^2*d^2*(f^2
+ 6*f*g*x - 3*g^2*x^2)))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.05, size = 116, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(3\,g^2x^2c^2d^2+12\,acdeg^2x-6\,c^2d^2fgx+8\,a^2e^2g^2-4\,acdefg-c^2d^2f^2\right)}{3\,c^3d^3}(ex+d)^{\frac{5}{2}}\left(cdex^2+ae^2x+cd^2x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(3*c^2*d^2*g^2*x^2+12*a*c*d*e*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e
^2*g^2-4*a*c*d*e*f*g-c^2*d^2*f^2)*(e*x+d)^(5/2)/c^3/d^3/(c*d*e*x^2+a*e^2*x+
c*d^2*x+a*d*e)^(5/2)
```

Maxima [A] time = 1.24443, size = 186, normalized size = 0.88

$$-\frac{4\,(3\,cdx+2\,ae)fg}{3\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}}+\frac{2\left(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2\right)g^2}{3\left(c^4d^4x+ac^3d^3e\right)\sqrt{cdx+ae}}-\frac{2\,f^2}{3\left(c^2d^2x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -4/3*(3*c*d*x + 2*a*e)*f*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2/3*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) - 2/3*f^2/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [A] time = 1.55553, size = 366, normalized size = 1.73

$$\frac{2\left(3\,c^2d^2g^2x^2 - c^2d^2f^2 - 4\,acdefg + 8\,a^2e^2g^2 - 6\left(c^2d^2fg - 2\,acdeg^2\right)x\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}}{3\left(c^5d^5ex^3 + a^2c^3d^4e^2 + \left(c^5d^6 + 2\,ac^4d^4e^2\right)x^2 + \left(2\,ac^4d^5e + a^2c^3d^3e^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(3*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 - 4*a*c*d*e*f*g + 8*a^2*e^2*g^2 - 6*(c
^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2
)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.674 $\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{2\sqrt{d+ex}\left(2ae^2g+cd(ef-3dg)\right)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(5/2))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*Sqrt[d
 + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])
```

Rubi [A] time = 0.133725, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {788, 648}

$$\frac{2\sqrt{d+ex}\left(2ae^2g+cd(ef-3dg)\right)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(5/2))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*Sqrt[d
 + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])
```

Rule 788

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((g*(c*d - b*e) + c*e*f)*(d + e*x)^m*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(2*c*d - b*e)), x] - Dist[(e*(m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g)))/(c*(p + 1)*(2*c*d - b*e)), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{\left(2ae^2g+cd(ef-3dg)\right)\int -}{3cd\left(cd^2-\right.}$$

$$= -\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(2ae^2g+cd(ef-3\right.}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(c\right.}}$$

Mathematica [A] time = 0.0498132, size = 52, normalized size = 0.34

$$-\frac{2(d+ex)^{3/2}(2aeg+cd(f+3gx))}{3c^2d^2((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(2*a*e*g + c*d*(f + 3*g*x)))/(3*c^2*d^2*((a*e + c*d*x)*
(d + e*x))^(3/2))
```

Maple [A] time = 0.046, size = 66, normalized size = 0.4

$$-\frac{(2\,cdx+2\,ae)\left(3\,xcdg+2\,aeg+cdf\right)}{3\,c^2d^2}(ex+d)^{\frac{5}{2}}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(3*c*d*g*x+2*a*e*g+c*d*f)*(e*x+d)^(5/2)/c^2/d^2/(c*d*e*x^2
+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [A] time = 1.17725, size = 99, normalized size = 0.64

$$-\frac{2\,(3\,cdx+2\,ae)g}{3\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}} - \frac{2\,f}{3\left(c^2d^2x+acde\right)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] -2/3*(3*c*d*x + 2*a*e)*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) - 2/
3*f/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [A] time = 1.56673, size = 270, normalized size = 1.75

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(3\,cdgx+cdf+2\,aeg\right)\sqrt{ex+d}}{3\left(c^4d^4ex^3+a^2c^2d^3e^2+\left(c^4d^5+2\,ac^3d^3e^2\right)x^2+\left(2\,ac^3d^4e+a^2c^2d^2e^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + c*d*f + 2*a*e
*g)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3
*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] sage0*x
```

3.675 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Rubi [A] time = 0.0215449, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.0312852, size = 37, normalized size = 0.77

$$-\frac{2(d+ex)^{3/2}}{3cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.045, size = 50, normalized size = 1.

$$-\frac{2\,cdx+2\,ae}{3\,cd}(ex+d)^{\frac{5}{2}}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(e*x+d)^(5/2)/d/c/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [A] time = 1.09454, size = 38, normalized size = 0.79

$$-\frac{2}{3\left(c^2d^2x + acde\right)\sqrt{cdx + ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] -2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [B] time = 1.50973, size = 221, normalized size = 4.6

$$-\frac{2\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}}{3\left(c^3d^3ex^3 + a^2cd^2e^2 + \left(c^3d^4 + 2\,ac^2d^2e^2\right)x^2 + \left(2\,ac^2d^3e + a^2cde^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x
^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a^2
*c*d*e^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.676 $\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}+\frac{2g\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) + (2*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)
^(5/2)
```

Rubi [A] time = 0.269652, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {868, 874, 205}

$$\frac{2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}+\frac{2g\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) + (2*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)
^(5/2)
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{g\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)\right.}}{cdf-aeg}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2g\sqrt{d}}{(cdf-aeg)^2\sqrt{ade+(}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2g\sqrt{d}}{(cdf-aeg)^2\sqrt{ade+(}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2g\sqrt{d}}{(cdf-aeg)^2\sqrt{ade+(}}$$

Mathematica [C] time = 0.0396192, size = 73, normalized size = 0.39

$$\frac{2(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 1, -1/2, (g*(a*e + c*d*x))/(-(c*
d*f) + a*e*g)])/(3*(-(c*d*f) + a*e*g)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.419, size = 219, normalized size = 1.2

$$-\frac{2}{3\,(cdx+ae)^2\left(aeg-cdf\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)\sqrt{cdx+ae}xcdg^2+3\,\mathrm{Artanh}\left(\frac{\sqrt{}}{\sqrt{(}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*x*c*d*g^2+3*arctanh((c*d*x+a*e)
^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*a*e*g^2*(c*d*x+a*e)^(1/2)-3*((a*e*g-c*d*f
)*g)^(1/2)*x*c*d*g-4*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*
c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)), x)
```

Fricas [B] time = 1.80263, size = 2083, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2
*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d
^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a
*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)
*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*e^2*
f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^
2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*
c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)
*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*
c*d*e^4)*f*g + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x), 2/3*(3*(c^2*d^2*e*g*x^3
 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*
x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*
d*e*g + (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*
c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*
c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e +
2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a
*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g
 + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] sage0*x
```

3.677 $\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$\frac{5g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5cdg^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}}+\frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(}$$

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)) + (10*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*x)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (5*c
*d*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(7/2)
```

Rubi [A] time = 0.340196, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$\frac{5g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5cdg^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}}+\frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)) + (10*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*x)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (5*c
*d*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(7/2)
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(5g)\int \frac{}{(f+gx}}{}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-aeg}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-aeg}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-aeg}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-aeg}$$

Mathematica [C] time = 0.0489586, size = 75, normalized size = 0.28

$$-\frac{2cd(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] (-2*c*d*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 2, -1/2, (g*(a*e + c*d*x))/
(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.31, size = 424, normalized size = 1.6

$$\frac{1}{3\left(cdx+ae\right)^{2}\left(aeg-cdf\right)^{3}\left(gx+f\right)}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+ae}g}{\sqrt{\left(aeg-cdf\right)g}}\right)\sqrt{cdx+ae}x^{2}c^{2}d^{2}g^{3}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*x^2*c^2*d^2*g^3+15*arctanh((c*d
*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*a*c*d*e*g^3*(c*d*x+a*e)^(1/2)+15
*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*x*c
^2*d^2*f*g^2+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*
e*f*g^2*(c*d*x+a*e)^(1/2)-15*((a*e*g-c*d*f)*g)^(1/2)*x^2*c^2*d^2*g^2-20*((a
*e*g-c*d*f)*g)^(1/2)*x*a*c*d*e*g^2-10*((a*e*g-c*d*f)*g)^(1/2)*x*c^2*d^2*f*g
-3*((a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f
*g+2*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*
g-c*d*f)^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+d\right)^{\frac{5}{2}}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^2), x)
```

Fricas [B] time = 1.9781, size = 3753, normalized size = 14.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d^
4 + 2*a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c^2
*d^3*e + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a^2*
c*d*e^3)*f*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt
(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*
g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 + 14*
a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2*f^4 - 3*
a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^
```

```
5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2
*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4
*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)
*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d
^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2
- a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a
^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^
3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2
*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x), 1/3*
(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d^4 + 2*
a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c^2*d^3*e
 + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a^2*c*d*e^
3)*f*g)*x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*
x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2
 + 14*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2*f^
4 - 3*a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (
c^5*d^5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c
^2*d^2*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*
(a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^
2*e^4)*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a
*c^4*d^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^
4*e^2 - a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3
- (2*a^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e +
a^2*c^3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a
^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.678 $\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=342

$$\frac{35c^2d^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}}+\frac{35cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^4}+\frac{35g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{6\sqrt{d+ex}(f+gx)^2(cdf-aeg)^3}$$

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)) + (14*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*
x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*g^2*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(6*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^
2) + (35*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a
*e*g)^4*Sqrt[d + e*x]*(f + g*x)) + (35*c^2*d^2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x]
)])/(4*(c*d*f - a*e*g)^(9/2))
```

Rubi [A] time = 0.538171, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {868, 872, 874, 205}

$$\frac{35c^2d^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}}+\frac{35cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^4}+\frac{35g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{6\sqrt{d+ex}(f+gx)^2(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)) + (14*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*
x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*g^2*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(6*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^
2) + (35*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a
*e*g)^4*Sqrt[d + e*x]*(f + g*x)) + (35*c^2*d^2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x]
)])/(4*(c*d*f - a*e*g)^(9/2))
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
```

```
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(7g)\int \frac{}{(f+g}}{}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-ae}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-ae}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-ae}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-ae}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf-ae}$$

Mathematica [C] time = 0.0540707, size = 79, normalized size = 0.23

$$-\frac{2c^2d^2(d+ex)^{3/2}\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] (-2*c^2*d^2*(d + e*x)^(3/2)*Hypergeometric2F1[-3/2, 3, -1/2, (g*(a*e + c*d*
x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^(3/
2))
```

Maple [B] time = 0.348, size = 670, normalized size = 2.

$$-\frac{1}{12\,(cdx+ae)^2\left(aeg-cdf\right)^4\left(gx+f\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(105\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^3c^3d^3g^4\sqrt{cdx+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -1/12*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(105*arctanh((c*d*x+a*e)^(1/2
)*g/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^3*d^3*g^4*(c*d*x+a*e)^(1/2)+105*arctanh(
(c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^2*a*c^2*d^2*e*g^4*(c*d*x+a*e
)^(1/2)+210*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^3*d^
3*f*g^3*(c*d*x+a*e)^(1/2)+210*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)
^(1/2))*x*a*c^2*d^2*e*f*g^3*(c*d*x+a*e)^(1/2)+105*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*x*c^3*d^3*f^2*g^2*(c*d*x+a*e)^(1/2)-105*((a*e*g
-c*d*f)*g)^(1/2)*x^3*c^3*d^3*g^3+105*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*
d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g^2*(c*d*x+a*e)^(1/2)-140*((a*e*g-c*d*f)*g)^
(1/2)*x^2*a*c^2*d^2*e*g^3-175*((a*e*g-c*d*f)*g)^(1/2)*x^2*c^3*d^3*f*g^2-21*
((a*e*g-c*d*f)*g)^(1/2)*x*a^2*c*d*e^2*g^3-238*((a*e*g-c*d*f)*g)^(1/2)*x*a*c
^2*d^2*e*f*g^2-56*((a*e*g-c*d*f)*g)^(1/2)*x*c^3*d^3*f^2*g+6*((a*e*g-c*d*f)*
g)^(1/2)*a^3*e^3*g^3-39*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-80*((a*e*
g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+8*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/
(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^3), x)
```

Fricas [B] time = 2.14563, size = 5742, normalized size = 16.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [1/24*(105*(c^4*d^4*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2
+ (c^4*d^5 + 2*a*c^3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 + 2*
a*c^3*d^3*e^2)*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2*c^
2*d^3*e^2*g^3 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e + a^2*
c^2*d^2*e^3)*f*g^2)*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e + a^2*c
^2*d^2*e^3)*f^2*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f
+ 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)
*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x
)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(105*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3
 + 80*a*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 + 35*(5*c^3*
d^3*f*g^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c^2*d^2*e*f*
g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*
e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*
g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e
^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c
^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*
c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2
*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f
^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c
^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*
a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*
g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^
6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*
g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^
4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2
*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*
a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d
*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x), 1/12*(105*(c^4*d^4
*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2 + (c^4*d^5 + 2*a*c^
3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 + 2*a*c^3*d^3*e^2)*f*g^
2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2*c^2*d^3*e^2*g^3 + (c^
4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f*g^2)
*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2*g)*
x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*
d*e*g + (c*d^2 + a*e^2)*g*x)) + (105*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3 + 80*a
*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 + 35*(5*c^3*d^3*f*g
^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c^2*d^2*e*f*g^2 + 3
*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4
*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4
*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^
5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*
e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5
*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^
5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*
c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*
e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3
*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^
3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 +
 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4
*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a
^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^
4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3
*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f
^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.679 $\int \frac{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=336

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{99c^2d^2(d+ex)^{3/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{231c^3d^3(d+ex)^{3/2}}+\ldots$$

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(3465*c^5*d^5*e*(d + e*x)^(3/2)) + (128*g*(c*
d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(1155*c^4*d^4
*e*Sqrt[d + e*x]) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(231*c^3*d^3*(d + e*x)^(3/2)) + (16*(c*d*f - a*e*
g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(99*c^2*d^2*(
d + e*x)^(3/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2))/(11*c*d*(d + e*x)^(3/2))
```

Rubi [A] time = 0.607251, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{99c^2d^2(d+ex)^{3/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{231c^3d^3(d+ex)^{3/2}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```


```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(3465*c^5*d^5*e*(d + e*x)^(3/2)) + (128*g*(c*
d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(1155*c^4*d^4
*e*Sqrt[d + e*x]) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(231*c^3*d^3*(d + e*x)^(3/2)) + (16*(c*d*f - a*e*
g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(99*c^2*d^2*(
d + e*x)^(3/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2))/(11*c*d*(d + e*x)^(3/2))
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}} + \frac{(8(cdf-aeg))\int \frac{(f+gx)^3}{}}{11}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}} + \frac{2(f+gx)^4}{}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{231c^3d^3(d+ex)^{3/2}} + \frac{16(cdf-}{}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{1155c^4d^4e\sqrt{d+ex}} + \frac{32(cdf-aeg)^2(}{}$$

$$= \frac{128(cdf-aeg)^3\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3465c^4d^4(d+ex)^{3/2}} +$$

Mathematica [A] time = 0.185146, size = 195, normalized size = 0.58

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(48a^2c^2d^2e^2g^2\left(33f^2+22fgx+5g^2x^2\right)-64a^3cde^3g^3(11f+3gx)+128a^4e^4g^4-8ac^3d^3eg\left(29\right.\right.}{3465c^5d^5(d+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
1*f + 3*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(33*f^2 + 22*f*g*x + 5*g^2*x^2) - 8*a
*c^3*d^3*e*g*(231*f^3 + 297*f^2*g*x + 165*f*g^2*x^2 + 35*g^3*x^3) + c^4*d^4
*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 1540*f*g^3*x^3 + 315*g^4*x^4
)))/(3465*c^5*d^5*(d + e*x)^(3/2))
```

Maple [A] time = 0.054, size = 283, normalized size = 0.8

$$\frac{(2\,cdx+2\,ae)\left(315\,g^4x^4c^4d^4-280\,ac^3d^3eg^4x^3+1540\,c^4d^4fg^3x^3+240\,a^2c^2d^2e^2g^4x^2-1320\,ac^3d^3efg^3x^2+2970\,c^4d^4f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3465*(c*d*x+a*e)*(315*c^4*d^4*g^4*x^4-280*a*c^3*d^3*e*g^4*x^3+1540*c^4*d^
4*f*g^3*x^3+240*a^2*c^2*d^2*e^2*g^4*x^2-1320*a*c^3*d^3*e*f*g^3*x^2+2970*c^4
*d^4*f^2*g^2*x^2-192*a^3*c*d*e^3*g^4*x+1056*a^2*c^2*d^2*e^2*f*g^3*x-2376*a*
c^3*d^3*e*f^2*g^2*x+2772*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-704*a^3*c*d*e^3*f*
g^3+1584*a^2*c^2*d^2*e^2*f^2*g^2-1848*a*c^3*d^3*e*f^3*g+1155*c^4*d^4*f^4)*(
c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^5/d^5/(e*x+d)^(1/2)
```

Maxima [A] time = 1.26439, size = 432, normalized size = 1.29

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f^4}{3\,cd}+\frac{8\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}\,f^3g}{15\,c^2d^2}+\frac{4\left(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3\right)\sqrt{cdx+a}}{35\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^4/(c*d) + 8/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2) + 4/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^
2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f^2*g^2/(c^3*d^3)
+ 8/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^3
*c*d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/3465*(315*c^
5*d^5*x^5 + 35*a*c^4*d^4*e*x^4 - 40*a^2*c^3*d^3*e^2*x^3 + 48*a^3*c^2*d^2*e^
3*x^2 - 64*a^4*c*d*e^4*x + 128*a^5*e^5)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Fricas [A] time = 1.61815, size = 798, normalized size = 2.38

$$\frac{2\left(315\,c^5d^5g^4x^5+1155\,ac^4d^4ef^4-1848\,a^2c^3d^3e^2f^3g+1584\,a^3c^2d^2e^3f^2g^2-704\,a^4cde^4fg^3+128\,a^5e^5g^4+35\left(44\,c^5d^5fg^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*g^4*x^5 + 1155*a*c^4*d^4*e*f^4 - 1848*a^2*c^3*d^3*e^2*f
^3*g + 1584*a^3*c^2*d^2*e^3*f^2*g^2 - 704*a^4*c*d*e^4*f*g^3 + 128*a^5*e^5*g
^4 + 35*(44*c^5*d^5*f*g^3 + a*c^4*d^4*e*g^4)*x^4 + 10*(297*c^5*d^5*f^2*g^2
+ 22*a*c^4*d^4*e*f*g^3 - 4*a^2*c^3*d^3*e^2*g^4)*x^3 + 6*(462*c^5*d^5*f^3*g
+ 99*a*c^4*d^4*e*f^2*g^2 - 44*a^2*c^3*d^3*e^2*f*g^3 + 8*a^3*c^2*d^2*e^3*g^4
)*x^2 + (1155*c^5*d^5*f^4 + 924*a*c^4*d^4*e*f^3*g - 792*a^2*c^3*d^3*e^2*f^2
*g^2 + 352*a^3*c^2*d^2*e^3*f*g^3 - 64*a^4*c*d*e^4*g^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.680 $\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=269

$$\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{21c^2d^2(d+ex)^{3/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{105c^3d^3e\sqrt{d+ex}}-\frac{16\left(x\left(ae^2+\right.\right.}{}$$

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2))/(315*c^4*d^4*e*(d + e*x)^(3/2)) + (16*g*(c*d*f
 - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*S
qrt[d + e*x]) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(21*c^2*d^2*(d + e*x)^(3/2)) + (2*(f + g*x)^3*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d*(d + e*x)^(3/2))
```

Rubi [A] time = 0.39, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{21c^2d^2(d+ex)^{3/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{105c^3d^3e\sqrt{d+ex}}-\frac{16\left(x\left(ae^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```



```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2))/(315*c^4*d^4*e*(d + e*x)^(3/2)) + (16*g*(c*d*f
 - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*S
qrt[d + e*x]) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(21*c^2*d^2*(d + e*x)^(3/2)) + (2*(f + g*x)^3*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d*(d + e*x)^(3/2))
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}} + \frac{\left(2\left(cde^2f+cd^2eg-e\left(\right.\right.\right.}{}$$

$$= \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21c^2d^2(d+ex)^{3/2}} + \frac{2(f+gx)^3\left(\right.}{}$$

$$= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105c^3d^3e\sqrt{d+ex}} + \frac{4(cdf-aeg)(f+}{}$$

$$= \frac{16(cdf-aeg)^2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{315c^3d^3(d+ex)^{3/2}} + \frac{1}{}$$

Mathematica [A] time = 0.124367, size = 136, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(24a^2cde^2g^2(3f+gx)-16a^3e^3g^3-6ac^2d^2eg\left(21f^2+18fgx+5g^2x^2\right)+c^3d^3\left(189f^2gx+105f\right.\right.}{315c^4d^4(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(3
*f + g*x) - 6*a*c^2*d^2*e*g*(21*f^2 + 18*f*g*x + 5*g^2*x^2) + c^3*d^3*(105*
f^3 + 189*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3)))/(315*c^4*d^4*(d + e*x)^(3
/2))
```

Maple [A] time = 0.052, size = 188, normalized size = 0.7

$$-\frac{(2\,cdx+2\,ae)\left(-35\,g^3x^3c^3d^3+30\,ac^2d^2eg^3x^2-135\,c^3d^3fg^2x^2-24\,a^2cde^2g^3x+108\,ac^2d^2efg^2x-189\,c^3d^3f^2gx+16\right.}{315\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(-35*c^3*d^3*g^3*x^3+30*a*c^2*d^2*e*g^3*x^2-135*c^3*d^3*
f*g^2*x^2-24*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-189*c^3*d^3*f^2*g*x+
16*a^3*e^3*g^3-72*a^2*c*d*e^2*f*g^2+126*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*
(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^4/d^4/(e*x+d)^(1/2)
```

Maxima [A] time = 1.23418, size = 294, normalized size = 1.09

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f^3}{3\,cd}+\frac{2\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}\,f^2g}{5\,c^2d^2}+\frac{2\left(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3\right)\sqrt{cdx+a}}{35\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^3/(c*d) + 2/5*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*
e^2)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2) + 2/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f*g^2/(c^3*d^3) + 2
/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^3*c*
d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

Fricas [A] time = 1.62027, size = 554, normalized size = 2.06

$$\frac{2\left(35\,c^4d^4g^3x^4+105\,ac^3d^3ef^3-126\,a^2c^2d^2e^2f^2g+72\,a^3cde^3fg^2-16\,a^4e^4g^3+5\left(27\,c^4d^4fg^2+ac^3d^3eg^3\right)x^3+3\left(63\,c^4d^4\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 126*a^2*c^2*d^2*e^2*f^2*g
 + 72*a^3*c*d*e^3*f*g^2 - 16*a^4*e^4*g^3 + 5*(27*c^4*d^4*f*g^2 + a*c^3*d^3*
e*g^3)*x^3 + 3*(63*c^4*d^4*f^2*g + 9*a*c^3*d^3*e*f*g^2 - 2*a^2*c^2*d^2*e^2*
g^3)*x^2 + (105*c^4*d^4*f^3 + 63*a*c^3*d^3*e*f^2*g - 36*a^2*c^2*d^2*e^2*f*g
^2 + 8*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.681 $\int \frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=200

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{35c^2d^2e\sqrt{d+ex}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)}{105c^3d^3e(d+ex)^{3/2}}$$

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*(d + e*x)^(3/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(35*c^2*d^2*e*Sqrt[d +
 e*x]) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c
*d*(d + e*x)^(3/2))
```

Rubi [A] time = 0.228383, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{35c^2d^2e\sqrt{d+ex}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)}{105c^3d^3e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*(d + e*x)^(3/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(35*c^2*d^2*e*Sqrt[d +
 e*x]) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c
*d*(d + e*x)^(3/2))
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
```

```
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}+\frac{\left(4\left(cde^2f+cd^2eg-e\left(\right.\right.\right.}{}$$

$$=\frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35c^2d^2e\sqrt{d+ex}}+\frac{2(f+gx)^2\left(ade+\left(\right.\right.}{7cd(}$$

$$=-\frac{8(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{105c^3d^3e(d+ex)^{3/2}}$$

Mathematica [A] time = 0.0859647, size = 90, normalized size = 0.45

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(8a^2e^2g^2-4acdeg(7f+3gx)+c^2d^2\left(35f^2+42fgx+15g^2x^2\right)\right)}{105c^3d^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(7*f + 3*g*
x) + c^2*d^2*(35*f^2 + 42*f*g*x + 15*g^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3/2
))
```

Maple [A] time = 0.049, size = 116, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(15\,g^2x^2c^2d^2-12\,acdeg^2x+42\,c^2d^2fgx+8\,a^2e^2g^2-28\,acdefg+35\,f^2c^2d^2\right)}{105\,c^3d^3}\sqrt{cdex^2+ae^2x+cd^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/105*(c*d*x+a*e)*(15*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+42*c^2*d^2*f*g*x+8*a
^2*e^2*g^2-28*a*c*d*e*f*g+35*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Maxima [A] time = 1.13287, size = 180, normalized size = 0.9

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f^2}{3\,cd}+\frac{4\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}fg}{15\,c^2d^2}+\frac{2\left(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3\right)\sqrt{cdx}}{105\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^2/(c*d) + 4/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) + 2/105*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*g^2/(c^3*d^3)
```

Fricas [A] time = 1.62733, size = 369, normalized size = 1.84

$$\frac{2\left(15\,c^3d^3g^2x^3+35\,ac^2d^2ef^2-28\,a^2cde^2fg+8\,a^3e^3g^2+3\left(14\,c^3d^3fg+ac^2d^2eg^2\right)x^2+\left(35\,c^3d^3f^2+14\,ac^2d^2efg-4\,a^2ca\right.}{105\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 28*a^2*c*d*e^2*f*g + 8*a^3
*e^3*g^2 + 3*(14*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 + 14*
a*c^2*d^2*e*f*g - 4*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\left(f+gx\right)^2}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2/sqrt(d + e*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.682 $\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(2ae^2g-cd(5ef-3dg)\right)}{15c^2d^2e(d+ex)^{3/2}}$$

```
[Out] (-2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2))/(15*c^2*d^2*e*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
```

Rubi [A] time = 0.0951645, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(2ae^2g-cd(5ef-3dg)\right)}{15c^2d^2e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2))/(15*c^2*d^2*e*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}+\frac{1}{5}\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\int \frac{\sqrt{ade+}}{}$$

$$=\frac{2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{15cd(d+ex)^{3/2}}+\frac{2g\left(ade+\left(cd^2+\right.\right.}{5cde}$$

Mathematica [A] time = 0.0534116, size = 54, normalized size = 0.43

$$\frac{2((d+ex)(ae+cdx))^{3/2}(cd(5f+3gx)-2aeg)}{15c^2d^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e*g + c*d*(5*f + 3*g*x)))/(15*c^2*
d^2*(d + e*x)^(3/2))
```

Maple [A] time = 0.047, size = 67, normalized size = 0.5

$$-\frac{(2\,cdx+2\,ae)\left(-3\,xcdg+2\,aeg-5\,cdf\right)}{15\,c^2d^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(-3*c*d*g*x+2*a*e*g-5*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Maxima [A] time = 1.10425, size = 88, normalized size = 0.7

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f}{3\,cd}+\frac{2\left(3\,c^2d^2x^2+acdex-2\,a^2e^2\right)\sqrt{cdx+ae}g}{15\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f/(c*d) + 2/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*e
^2)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

Fricas [A] time = 1.61304, size = 221, normalized size = 1.77

$$\frac{2\left(3\,c^2d^2gx^2+5\,acdef-2\,a^2e^2g+\left(5\,c^2d^2f+acdeg\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}}{15\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 2*a^2*e^2*g + (5*c^2*d^2*f + a*c*d*e*
g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*
```

```
x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}\,(f+gx)}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)/sqrt(d + e*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

3.683 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rubi [A] time = 0.0206349, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Mathematica [A] time = 0.0243028, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Maple [A] time = 0.044, size = 50, normalized size = 1.

$$\frac{2\,cdx+2\,ae}{3\,cd}\sqrt{cdex^2+ae^2x+cd^2x+ade}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/d/c/(e*x+d)^(1/2)
```

Maxima [A] time = 1.04918, size = 24, normalized size = 0.5

$$\frac{2\,(cdx + ae)^{\frac{3}{2}}}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

Fricas [A] time = 1.59384, size = 128, normalized size = 2.67

$$\frac{2\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}(cdx + ae)\sqrt{ex + d}}{3\left(cdex + cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
/(c*d*e*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d + ex)(ae + cdx)}}{\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/sqrt(e*x + d), x)
```

3.684 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx$

Optimal. Leaf size=124

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt
[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)
```

Rubi [A] time = 0.186361, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {864, 874, 205}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)),x
]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt
[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int \frac{\sqrt{d\cdots}}{(f+gx)\sqrt{ade+(c\cdots}}}{e^2g}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left(2e^2(cdf-aeg)\right)\mathrm{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+\cdots}\right.}{g}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}}$$

Mathematica [A] time = 0.135251, size = 101, normalized size = 0.81

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}-\frac{\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g] - (Sqrt[c*d*f - a*e*g]*ArcTan[(Sq
rt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/Sqrt[a*e + c*d*x]))/(g^(3/2)
*Sqrt[d + e*x])
```

Maple [A] time = 0.38, size = 153, normalized size = 1.2

$$-2\frac{\sqrt{cdex^2+ae^2x+cd^2x+ade}}{\sqrt{ex+d}\sqrt{cdx+aeg}\sqrt{\left(aeg-cdf\right)g}}\left(\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)aeg-\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)cdf-\sqrt{cdx+ae}\sqrt{(\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)*g/((a
*e*g-c*d*f)*g)^(1/2))*a*e*g-arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(
1/2))*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x
+a*e)^(1/2)/g/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)), x)
```

Fricas [A] time = 1.70312, size = 702, normalized size = 5.66

$$\left[\frac{(ex+d)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}g\sqrt{-\frac{cdf-aeg}{g}}-\left(cdef-\left(cd^2+2\,ae^2\right)g\right)x}{egx^2+df+\left(ef+dg\right)x}\right)+2\sqrt{cdex^2+ade+}}{egx+dg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((e*x + d)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*
g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*
d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
e*g*x + d*g), 2*((e*x + d)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqr
t((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g*x + d*g)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

3.685 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx$

Optimal. Leaf size=132

$$\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)))
+ (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c
*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rubi [A] time = 0.160969, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^2)
,x]
```


```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)))
+ (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c
*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}(f+gx)} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx}{2g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}(f+gx)} + \frac{\left(cde^2\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cde(ef+dg)+e^2gx^2}d\right.}{g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Mathematica [A] time = 0.198076, size = 110, normalized size = 0.83

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}}-\frac{\sqrt{g}}{f+gx}\right)}{g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^2),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]/(f + g*x)) + (c*d*ArcTan[(Sqrt[g]
*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x])))/(g^(3/2)*Sqrt[d + e*x])
```

Maple [A] time = 0.349, size = 161, normalized size = 1.2

$$\frac{1}{g\left(gx+f\right)}\left(-\text{Artanh}\left(g\sqrt{cdx+ae}\frac{1}{\sqrt{\left(aeg-cdf\right)g}}\right)xcdg-\text{Artanh}\left(g\sqrt{cdx+ae}\frac{1}{\sqrt{\left(aeg-cdf\right)g}}\right)cdf-\sqrt{cdx+ae}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x)
```

```
[Out] (-arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*c*d*g-arctanh((c*d
*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2))*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(1/2)/(c*d*x
+a*e)^(1/2)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^2), x)
```

Fricas [B] time = 1.769, size = 1204, normalized size = 9.12

$$\left[-\frac{\left(cdegx^2 + cd^2f + \left(cdef + cd^2g\right)x\right)\sqrt{-cdfg + aeg^2}\log\left(-\frac{cdegx^2 - cd^2f + 2\,adeg - \left(cdef - \left(cd^2 + 2\,ae^2\right)g\right)x - 2\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{-cdfg +}\cdots}{egx^2 + df + \left(ef + dg\right)x}\cdots\right.}{2\left(cd^2f^2g^2 - adefg^3 + \left(cdefg^3 - ae^2g^4\right)x^2 + \left(cdef^2g^2 - adeg^4 + \left(c\cdots\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*
g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)
*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g
^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2
- a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4
+ (c*d^2 - a*e^2)*f*g^3)*x), -((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)
*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 +
 a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*
g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2 - a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4
)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - a*e^2)*f*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**2/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.686 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx$

Optimal. Leaf size=207

$$\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*g*Sqrt[d + e*x]*(f + g*x)^2
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)*
Sqrt[d + e*x]*(f + g*x)) + (c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(3/2)*(c*d
*f - a*e*g)^(3/2))
```

Rubi [A] time = 0.273324, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^3)
,x]
```


```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*g*Sqrt[d + e*x]*(f + g*x)^2
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)*
Sqrt[d + e*x]*(f + g*x)) + (c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(3/2)*(c*d
*f - a*e*g)^(3/2))
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}+\frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{4g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)}+\frac{\left(c^2d^2\right)\int \cdots}{8 \cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)}+\frac{\left(c^2d^2e^2\right)\text{Sub}\cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)}+\frac{c^2d^2\tan^{-1}\left(\cdots\right.}{4g^{3/2}\cdots}$$

Mathematica [C] time = 0.0418726, size = 79, normalized size = 0.38

$$\frac{2c^2d^2((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d+ex)^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^3),x]
```

```
[Out] (2*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^3*(d + e*x)^(3/2))
```

Maple [A] time = 0.329, size = 285, normalized size = 1.4

$$\frac{1}{\left(4\,aeg-4\,cdf\right)g\left(gx+f\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\text{Artanh}\left(g\sqrt{cdx+ae}\frac{1}{\sqrt{\left(aeg-cdf\right)g}}\right)x^2c^2d^2g^2+2\,\text{Artanh}\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x)
```

```
[Out] 1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)*g/((
a*e*g-c*d*f)*g)^(1/2))*x^2*c^2*d^2*g^2+2*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*
g-c*d*f)*g)^(1/2))*x*c^2*d^2*f*g+arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)
*g)^(1/2))*c^2*d^2*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c*d*g-2*
((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*(c*
d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+
f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^3), x)
```

Fricas [B] time = 1.75227, size = 2148, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^
2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e
*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d
))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(c^2*d^2*f^2*g - 3*a*c*d*e*f*g^2 +
2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e*f^3*g^3 + a
^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2*e^3*g^6)*x^
3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^4
- 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a^2*d*e^2*f*g
^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*f^2*g^4)*x),
 -1/4*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x
^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2*f^2*g - 3*a*c*
d*e*f*g^2 + 2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e
*f^3*g^3 + a^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2
*e^3*g^6)*x^3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e
^2)*f^2*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a
^2*d*e^2*f*g^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*
f^2*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**3/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.687 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4} dx$

Optimal. Leaf size=277

$$\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(a\right.}}{3g}$$

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*g*Sqrt[d + e*x]*(f + g*x)^3
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)
*Sqrt[d + e*x]*(f + g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(8*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (c^3*d^3*ArcTan[
(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*
Sqrt[d + e*x])])/(8*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rubi [A] time = 0.349668, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(a\right.}}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^4)
,x]
```


```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*g*Sqrt[d + e*x]*(f + g*x)^3
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)
*Sqrt[d + e*x]*(f + g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(8*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (c^3*d^3*ArcTan[
(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*
Sqrt[d + e*x])])/(8*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
```

]

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{6g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{\left(c^2d^2\right)\int \frac{}{(f+}}{}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+}}{8g(cdf-a}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+}}{8g(cdf-a}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+}}{8g(cdf-a}$$

Mathematica [C] time = 0.047536, size = 79, normalized size = 0.29

$$\frac{2c^3d^3((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d+ex)^{3/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^4),x]
```

```
[Out] (2*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^4*(d + e*x)^(3/2))
```

Maple [A] time = 0.331, size = 453, normalized size = 1.6

$$-\frac{1}{24\left(gx+f\right)^3 g\left(aeg-cdf\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^3c^3d^3g^3+9\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+}}{\sqrt{(aeg-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^3*d^3*g^3+9*arctanh((c*d*x+a*e)^(1/2)*g/((
a*e*g-c*d*f)*g)^(1/2))*x^2*c^3*d^3*f*g^2+9*arctanh((c*d*x+a*e)^(1/2)*g/((a*
e*g-c*d*f)*g)^(1/2))*x*c^3*d^3*f^2*g+3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-
c*d*f)*g)^(1/2))*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x^
2*c^2*d^2*g^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*g^2-8*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g+8*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^
(1/2)*a*c*d*e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/
(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)^2/(c*d*x+a*
e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex+d}(gx+f)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^4), x)
```

Fricas [B] time = 1.92175, size = 3432, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*
d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*
e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
+ d*g)*x)) + 2*(3*c^3*d^3*f^3*g - 17*a*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d*e^2*f
*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 - 2*(4*c^3*d
^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^6*g^2 - 3*a*c^2*d^3*e*f^5
*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d*e^3*f^3*g^5 + (c^3*d^3*e*f^3*g^5 - 3
*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^4 + (3*c^3*d^
3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^
2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^7)*x^3 +
 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g
^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^
```

```
2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^3*d*e^3*f^2*g^6 + 3*(c^3*d^4 - a*c^2*
d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4 + (9*a^2*c*d^2*e
^2 - a^3*e^4)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c
^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2
 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^3*d^3*f^3*g - 17*a
*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^
3 - a*c^2*d^2*e*g^4)*x^2 - 2*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2
*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))
/(c^3*d^4*f^6*g^2 - 3*a*c^2*d^3*e*f^5*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d
*e^3*f^3*g^5 + (c^3*d^3*e*f^3*g^5 - 3*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3
*f*g^7 - a^3*e^4*g^8)*x^4 + (3*c^3*d^3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4
 - 9*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(
a^2*c*d^2*e^2 - a^3*e^4)*f*g^7)*x^3 + 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^
7 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3
*g^5 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^
3*d*e^3*f^2*g^6 + 3*(c^3*d^4 - a*c^2*d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e -
a^2*c*d*e^3)*f^4*g^4 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**4/(e*x+d)**(
1/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="giac")

[Out] Timed out
```

3.688 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$

Optimal. Leaf size=347

$$\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}+\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{96g\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{c}{2}$$

```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*g*Sqrt[d + e*x]*(f + g*x)^4
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*g*(c*d*f - a*e*g)
*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(96*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^3*d^3*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g*(c*d*f - a*e*g)^3*Sqrt[d
 + e*x]*(f + g*x)) + (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(3/2)*(c*d*f
- a*e*g)^(7/2))
```

Rubi [A] time = 0.454439, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}+\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{96g\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{c}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^5)
,x]
```


```
[Out] -Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*g*Sqrt[d + e*x]*(f + g*x)^4
) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*g*(c*d*f - a*e*g)
*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(96*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^3*d^3*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g*(c*d*f - a*e*g)^3*Sqrt[d
 + e*x]*(f + g*x)) + (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(3/2)*(c*d*f
- a*e*g)^(7/2))
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
```

```
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx = -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{8g}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{\left(5c^2d^2\right)\int \cdots}{\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c^2d^2\sqrt{ade\cdots}}{96g(cdf-\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c^2d^2\sqrt{ade\cdots}}{96g(cdf-\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c^2d^2\sqrt{ade\cdots}}{96g(cdf-\cdots}$$

$$= -\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c^2d^2\sqrt{ade\cdots}}{96g(cdf-\cdots}$$

Mathematica [C] time = 0.0483946, size = 79, normalized size = 0.23

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3(d+ex)^{3/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*(c*d*f - a*e*g)^5*(d + e*x)^(3/2))
```

Maple [B] time = 0.349, size = 696, normalized size = 2.

$$\frac{1}{192\left(gx+f\right)^{4}g\left(aeg-cdf\right)\left(a^{2}e^{2}g^{2}-2\,acdefg+c^{2}d^{2}f^{2}\right)}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x)
```

```
[Out] 1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*x^4*c^4*d^4*g^4+60*arctanh((c*d*x+a*e)^(1/2)*g/
((a*e*g-c*d*f)*g)^(1/2))*x^3*c^4*d^4*f*g^3+90*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*x^2*c^4*d^4*f^2*g^2+60*arctanh((c*d*x+a*e)^(1/2)*g/
((a*e*g-c*d*f)*g)^(1/2))*x*c^4*d^4*f^3*g-15*x^3*c^3*d^3*g^3*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)
^(1/2))*c^4*d^4*f^4+10*x^2*a*c^2*d^2*e*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)
*g)^(1/2)-55*x^2*c^3*d^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*
x*a^2*c*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*x*a*c^2*d^2*
e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-73*x*c^3*d^3*f^2*g*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^
(1/2)*a^3*e^3*g^3+136*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2
*f*g^2-118*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2)/(g*x+f)^4/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d
^2*f^2)/(c*d*x+a*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^5), x)
```

Fricas [B] time = 2.10394, size = 5183, normalized size = 14.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
```

```
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c
^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2*g^3 - 184*
a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x
^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*
x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2*d^2*e^2*f*
g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^2*d^3*e^2*f^
6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4*e*f^4*g^6 -
4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 +
 a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 16*
a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^7 + 2*(3
*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g
^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 6*a*c^3
*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^6 + 12*(a^2
*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^8
)*x^3 + 2*(2*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 8*a*c^3
*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a^2*c
^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g^6 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^7
)*x^2 + (c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 - a*c^3*d^3*e
^2)*f^7*g^3 - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^4 + 4*(6*a^2*c^2*
d^3*e^2 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^6)*x),
-1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(c
*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c
*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*
x)) + (15*c^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2
*g^3 - 184*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d
^3*e*g^5)*x^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^
2*e^2*g^5)*x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2
*d^2*e^2*f*g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^2
*d^3*e^2*f^6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4*e
*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*
e^4*f*g^9 + a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^
4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3
*g^7 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e^3 - a
^4*e^5)*f*g^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^
5 - 6*a*c^3*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^
6 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 3*a^4*e
^5)*f^2*g^8)*x^3 + 2*(2*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^
5 - 8*a*c^3*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^5 +
 2*(9*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g^6 - 2*(6*a^3*c*d^2*e^3 - a^4*e
^5)*f^3*g^7)*x^2 + (c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 -
a*c^3*d^3*e^2)*f^7*g^3 - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^4 + 4*
(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^
4*g^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**5/(e*x+d)**(
```

```
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.689 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=336

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{143c^2d^2(d+ex)^{5/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{429c^3d^3(d+ex)^{5/2}}+$$

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2))/(15015*c^5*d^5*e*(d + e*x)^(5/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3003*c^4*d^
4*e*(d + e*x)^(3/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(429*c^3*d^3*(d + e*x)^(5/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(143*c^2*d
^2*(d + e*x)^(5/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2))/(13*c*d*(d + e*x)^(5/2))
```

Rubi [A] time = 0.606542, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{143c^2d^2(d+ex)^{5/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{429c^3d^3(d+ex)^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```



```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2))/(15015*c^5*d^5*e*(d + e*x)^(5/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3003*c^4*d^
4*e*(d + e*x)^(3/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(429*c^3*d^3*(d + e*x)^(5/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(143*c^2*d
^2*(d + e*x)^(5/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2))/(13*c*d*(d + e*x)^(5/2))
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
```

```
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}} + \frac{(8(cdf-aeg))\int \frac{(f+g}{}}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}} + \frac{2(f+g}{}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{429c^3d^3(d+ex)^{5/2}} + \frac{16(cdf}{}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4e(d+ex)^{3/2}} + \frac{32(cdf-aeg)}{}$$

$$= \frac{128(cdf-aeg)^3\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{15015c^4d^4(d+ex)^{5/2}}$$

Mathematica [A] time = 0.245682, size = 195, normalized size = 0.58

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(16a^2c^2d^2e^2g^2\left(143f^2+130fgx+35g^2x^2\right)-64a^3cde^3g^3(13f+5gx)+128a^4e^4g^4-8ac^3d^3eg\left(\right.\right.}{15015c^5d^5(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
3*f + 5*g*x) + 16*a^2*c^2*d^2*e^2*g^2*(143*f^2 + 130*f*g*x + 35*g^2*x^2) -
8*a*c^3*d^3*e*g*(429*f^3 + 715*f^2*g*x + 455*f*g^2*x^2 + 105*g^3*x^3) + c^4
*d^4*(3003*f^4 + 8580*f^3*g*x + 10010*f^2*g^2*x^2 + 5460*f*g^3*x^3 + 1155*g
^4*x^4)))/(15015*c^5*d^5*(d + e*x)^(5/2))
```

Maple [A] time = 0.054, size = 283, normalized size = 0.8

$$\frac{(2\,cdx+2\,ae)\left(1155\,g^4x^4c^4d^4-840\,ac^3d^3eg^4x^3+5460\,c^4d^4fg^3x^3+560\,a^2c^2d^2e^2g^4x^2-3640\,ac^3d^3efg^3x^2+10010\,c^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/15015*(c*d*x+a*e)*(1155*c^4*d^4*g^4*x^4-840*a*c^3*d^3*e*g^4*x^3+5460*c^4*
d^4*f*g^3*x^3+560*a^2*c^2*d^2*e^2*g^4*x^2-3640*a*c^3*d^3*e*f*g^3*x^2+10010*
c^4*d^4*f^2*g^2*x^2-320*a^3*c*d*e^3*g^4*x+2080*a^2*c^2*d^2*e^2*f*g^3*x-5720
*a*c^3*d^3*e*f^2*g^2*x+8580*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-832*a^3*c*d*e^3
*f*g^3+2288*a^2*c^2*d^2*e^2*f^2*g^2-3432*a*c^3*d^3*e*f^3*g+3003*c^4*d^4*f^4
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^5/d^5/(e*x+d)^(3/2)
```

Maxima [A] time = 1.30834, size = 558, normalized size = 1.66

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}f^4}{5\,cd}+\frac{8\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}f^3g}{35\,c^2d^2}+\frac{4\left(35\,c^4d^4x^4+5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^4/(c*d) + 8/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^3*g/(c^2*d^2) + 4/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a
^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g^2
/(c^3*d^3) + 8/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d^3*
e^2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*d*x
+ a*e)*f*g^3/(c^4*d^4) + 2/15015*(1155*c^6*d^6*x^6 + 1470*a*c^5*d^5*e*x^5 +
 35*a^2*c^4*d^4*e^2*x^4 - 40*a^3*c^3*d^3*e^3*x^3 + 48*a^4*c^2*d^2*e^4*x^2 -
 64*a^5*c*d*e^5*x + 128*a^6*e^6)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Fricas [A] time = 1.6907, size = 1007, normalized size = 3.

$$\frac{2\left(1155\,c^6d^6g^4x^6+3003\,a^2c^4d^4e^2f^4-3432\,a^3c^3d^3e^3f^3g+2288\,a^4c^2d^2e^4f^2g^2-832\,a^5cde^5fg^3+128\,a^6e^6g^4+210\left(26\,c^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^6*d^6*g^4*x^6 + 3003*a^2*c^4*d^4*e^2*f^4 - 3432*a^3*c^3*d^3
*e^3*f^3*g + 2288*a^4*c^2*d^2*e^4*f^2*g^2 - 832*a^5*c*d*e^5*f*g^3 + 128*a^6
*e^6*g^4 + 210*(26*c^6*d^6*f*g^3 + 7*a*c^5*d^5*e*g^4)*x^5 + 35*(286*c^6*d^6
*f^2*g^2 + 208*a*c^5*d^5*e*f*g^3 + a^2*c^4*d^4*e^2*g^4)*x^4 + 20*(429*c^6*d
^6*f^3*g + 715*a*c^5*d^5*e*f^2*g^2 + 13*a^2*c^4*d^4*e^2*f*g^3 - 2*a^3*c^3*d
^3*e^3*g^4)*x^3 + 3*(1001*c^6*d^6*f^4 + 4576*a*c^5*d^5*e*f^3*g + 286*a^2*c^
4*d^4*e^2*f^2*g^2 - 104*a^3*c^3*d^3*e^3*f*g^3 + 16*a^4*c^2*d^2*e^4*g^4)*x^2
 + 2*(3003*a*c^5*d^5*e*f^4 + 858*a^2*c^4*d^4*e^2*f^3*g - 572*a^3*c^3*d^3*e^
3*f^2*g^2 + 208*a^4*c^2*d^2*e^4*f*g^3 - 32*a^5*c*d*e^5*g^4)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.690 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{33c^2d^2(d+ex)^{5/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{231c^3d^3e(d+ex)^{3/2}}-16\left(x\left(ae^2+\right.\right.$$

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*c^4*d^4*e*(d + e*x)^(5/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*c^3*d^3*e*
(d + e*x)^(3/2)) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2))/(33*c^2*d^2*(d + e*x)^(5/2)) + (2*(f + g*x)^3*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d*(d + e*x)^(5/2))
```

Rubi [A] time = 0.407257, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{33c^2d^2(d+ex)^{5/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{231c^3d^3e(d+ex)^{3/2}}-16\left(x\left(ae^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```


```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*c^4*d^4*e*(d + e*x)^(5/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*c^3*d^3*e*
(d + e*x)^(3/2)) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2))/(33*c^2*d^2*(d + e*x)^(5/2)) + (2*(f + g*x)^3*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d*(d + e*x)^(5/2))
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx = \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}} + \frac{(6(cdf-aeg))\int \frac{(f+g}{}}{}$$

$$= \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}} + \frac{2(f+gx}{}$$

$$= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3e(d+ex)^{3/2}} + \frac{4(cdf-aeg)(f}{}$$

$$= \frac{16(cdf-aeg)^2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155c^3d^3(d+ex)^{5/2}} +$$

Mathematica [A] time = 0.179156, size = 137, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(8a^2cde^2g^2(11f+5gx)-16a^3e^3g^3-2ac^2d^2eg\left(99f^2+110fgx+35g^2x^2\right)+c^3d^3\left(495f^2gx+2\right.\right.}{1155c^4d^4(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(11*f + 5*g*x) - 2*a*c^2*d^2*e*g*(99*f^2 + 110*f*g*x + 35*g^2*x^2) + c^3*d^3*(231*f^3 + 495*f^2*g*x + 385*f*g^2*x^2 + 105*g^3*x^3)))/(1155*c^4*d^4*(d + e*x)^(5/2))
```

Maple [A] time = 0.058, size = 188, normalized size = 0.7

$$-\frac{(2\,cdx+2\,ae)\left(-105\,g^3x^3c^3d^3+70\,ac^2d^2eg^3x^2-385\,c^3d^3fg^2x^2-40\,a^2cde^2g^3x+220\,ac^2d^2efg^2x-495\,c^3d^3f^2gx+1\right.}{1155\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-105*c^3*d^3*g^3*x^3+70*a*c^2*d^2*e*g^3*x^2-385*c^3*d^3*f*g^2*x^2-40*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-495*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-88*a^2*c*d*e^2*f*g^2+198*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^4/d^4/(e*x+d)^(3/2)
```

Maxima [A] time = 1.17583, size = 397, normalized size = 1.48

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}\,f^3}{5\,cd} + \frac{6\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}\,f^2g}{35\,c^2d^2} + \frac{2\left(35\,c^4d^4x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^3/(c*d) + 6/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^2*g/(c^2*d^2) + 2/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a
^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^2/(
c^3*d^3) + 2/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d^3*e^
2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*d*x +
a*e)*g^3/(c^4*d^4)
```

Fricas [A] time = 1.63672, size = 711, normalized size = 2.64

$$2\left(105\,c^5d^5g^3x^5 + 231\,a^2c^3d^3e^2f^3 - 198\,a^3c^2d^2e^3f^2g + 88\,a^4cde^4fg^2 - 16\,a^5e^5g^3 + 35\left(11\,c^5d^5fg^2 + 4\,ac^4d^4eg^3\right)x^4 + 5\,(99$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 198*a^3*c^2*d^2*e^3
*f^2*g + 88*a^4*c*d*e^4*f*g^2 - 16*a^5*e^5*g^3 + 35*(11*c^5*d^5*f*g^2 + 4*a
*c^4*d^4*e*g^3)*x^4 + 5*(99*c^5*d^5*f^2*g + 110*a*c^4*d^4*e*f*g^2 + a^2*c^3
*d^3*e^2*g^3)*x^3 + 3*(77*c^5*d^5*f^3 + 264*a*c^4*d^4*e*f^2*g + 11*a^2*c^3*
d^3*e^2*f*g^2 - 2*a^3*c^2*d^2*e^3*g^3)*x^2 + (462*a*c^4*d^4*e*f^3 + 99*a^2*
c^3*d^3*e^2*f^2*g - 44*a^3*c^2*d^2*e^3*f*g^2 + 8*a^4*c*d*e^4*g^3)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.691 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=200

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{63c^2d^2e(d+ex)^{3/2}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)\left(2ae^2g-cd(7ef-5dg\right.}{315c^3d^3e(d+ex)^{5/2}}$$

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/2))/(315*c^3*d^3*e*(d + e*x)^(5/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(63*c^2*d^2*e*(d + e*x
)^(3/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9
*c*d*(d + e*x)^(5/2))
```

Rubi [A] time = 0.232528, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{63c^2d^2e(d+ex)^{3/2}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)\left(2ae^2g-cd(7ef-5dg\right.}{315c^3d^3e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/2))/(315*c^3*d^3*e*(d + e*x)^(5/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(63*c^2*d^2*e*(d + e*x
)^(3/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9
*c*d*(d + e*x)^(5/2))
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
```

```
x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}} + \frac{\left(4\left(cde^2f+cd^2eg-e\left(c\right.\right.\right.}{}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63c^2d^2e(d+ex)^{3/2}} + \frac{2(f+gx)^2\left(ade+\left(c\right.\right.}{9cd(d}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/}}{315c^3d^3e(d+ex)^{5/2}}$$

Mathematica [A] time = 0.124801, size = 90, normalized size = 0.45

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(8a^2e^2g^2-4acdeg(9f+5gx)+c^2d^2\left(63f^2+90fgx+35g^2x^2\right)\right)}{315c^3d^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(9*f + 5*g*
x) + c^2*d^2*(63*f^2 + 90*f*g*x + 35*g^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5/2
))
```

Maple [A] time = 0.05, size = 116, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(35\,g^2x^2c^2d^2-20\,acdeg^2x+90\,c^2d^2fgx+8\,a^2e^2g^2-36\,acdefg+63\,f^2c^2d^2\right)}{315\,c^3d^3}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/315*(c*d*x+a*e)*(35*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+90*c^2*d^2*f*g*x+8*a
^2*e^2*g^2-36*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)
^(3/2)/c^3/d^3/(e*x+d)^(3/2)
```

Maxima [A] time = 1.18828, size = 259, normalized size = 1.3

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}\,f^2}{5\,cd} + \frac{4\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}\,fg}{35\,c^2d^2} + \frac{2\left(35\,c^4d^4x^4+50\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f*g/(c^2*d^2) + 2/315*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*a^2
*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*g^2/(c^3*
d^3)
```

Fricas [A] time = 1.61764, size = 481, normalized size = 2.4

$$\frac{2\left(35\,c^4d^4g^2x^4 + 63\,a^2c^2d^2e^2f^2 - 36\,a^3cde^3fg + 8\,a^4e^4g^2 + 10\left(9\,c^4d^4fg + 5\,ac^3d^3eg^2\right)x^3 + 3\left(21\,c^4d^4f^2 + 48\,ac^3d^3efg\right.\right.}{315\left(c^3d^3ex + c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 36*a^3*c*d*e^3*f*g + 8
*a^4*e^4*g^2 + 10*(9*c^4*d^4*f*g + 5*a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f
^2 + 48*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2
+ 9*a^2*c^2*d^2*e^2*f*g - 2*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.692 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(2ae^2g-cd(7ef-5dg)\right)}{35c^2d^2e(d+ex)^{5/2}}$$

```
[Out] (-2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(35*c^2*d^2*e*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))
```

Rubi [A] time = 0.100606, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(2ae^2g-cd(7ef-5dg)\right)}{35c^2d^2e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(35*c^2*d^2*e*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}+\frac{1}{7}\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\int \frac{\left(ade+\left(\right.\right.}{}$$

$$=\frac{2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35cd(d+ex)^{5/2}}+\frac{2g\left(ade+\left(cd^2+\right.\right.}{7cde(d}$$

Mathematica [A] time = 0.0780263, size = 54, normalized size = 0.43

$$\frac{2((d+ex)(ae+cdx))^{5/2}(cd(7f+5gx)-2aeg)}{35c^2d^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e*g + c*d*(7*f + 5*g*x)))/(35*c^2*
d^2*(d + e*x)^(5/2))
```

Maple [A] time = 0.052, size = 67, normalized size = 0.5

$$-\frac{(2\,cdx+2\,ae)\left(-5\,xcdg+2\,aeg-7\,cdf\right)}{35\,c^2d^2}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-5*c*d*g*x+2*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/c^2/d^2/(e*x+d)^(3/2)
```

Maxima [A] time = 1.14739, size = 144, normalized size = 1.15

$$\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}f}{5\,cd}+\frac{2\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}g}{35\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f/(c*d) + 2/35*
(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*x
+ a*e)*g/(c^2*d^2)
```

Fricas [A] time = 1.59321, size = 290, normalized size = 2.32

$$\frac{2\left(5\,c^3d^3gx^3+7\,a^2cde^2f-2\,a^3e^3g+\left(7\,c^3d^3f+8\,ac^2d^2eg\right)x^2+\left(14\,ac^2d^2ef+a^2cde^2g\right)x\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{35\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 2*a^3*e^3*g + (7*c^3*d^3*f + 8*a*
c^2*d^2*e*g)*x^2 + (14*a*c^2*d^2*e*f + a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 + a
```

```
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2
),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="giac")

[Out] Timed out
```

3.693 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rubi [A] time = 0.0232197, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Mathematica [A] time = 0.0332385, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Maple [A] time = 0.045, size = 50, normalized size = 1.

$$\frac{2\,cdx+2\,ae}{5\,cd}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 2/5*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/d/c/(e*x+d)^(3/2)
```

Maxima [A] time = 1.06495, size = 58, normalized size = 1.21

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdex+a^{2}e^{2}\right)\sqrt{cdx+ae}}{5\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)
```

Fricas [A] time = 1.6713, size = 161, normalized size = 3.35

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdex+a^{2}e^{2}\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}}{5\left(cdex+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.694 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=179

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{g^2\sqrt{d+ex}}+\frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+\right.}{3g(d+ex)^{3/2}}$$

```
[Out] (-2*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[
d + e*x]) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x
)^(3/2)) + (2*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rubi [A] time = 0.304446, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {864, 874, 205}

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{g^2\sqrt{d+ex}}+\frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+\right.}{3g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[
d + e*x]) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x
)^(3/2)) + (2*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int\frac{\sqrt{ade}}{}}{e^2g}$$

$$=-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{3g(d+ex)^{3/2}}$$

$$=-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{3g(d+ex)^{3/2}}$$

$$=-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{3g(d+ex)^{3/2}}$$

Mathematica [A] time = 0.269291, size = 132, normalized size = 0.74

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\left(\sqrt{g}\sqrt{ae+cdx}(4aeg+cd(gx-3f))+3(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{3g^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(4*a*e*g + c*
d*(-3*f + g*x)) + 3*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]]))/(3*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.332, size = 263, normalized size = 1.5

$$-\frac{2}{3\,g^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,{\rm Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)a^2e^2g^2-6\,{\rm Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)acdefg+3\,{\rm Art}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x)
```

```
[Out] -2/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*a^2*e^2*g^2-6*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g
-c*d*f)*g)^(1/2))*a*c*d*e*f*g+3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*
g)^(1/2))*c^2*d^2*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c*d*g-4*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/((a*e*g-c*d*f)*g
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex + d\right)^{\frac{3}{2}}\left(gx + f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)), x)
```

Fricas [A] time = 1.71638, size = 902, normalized size = 5.04

$$\left[-\frac{3\left(cd^2f - adeg + \left(cdef - ae^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}g\sqrt{-\frac{cdf-aeg}{g}}-\left(cdef-\left(cd^2+2\,ae^2\right)g\right.}{egx^2+df+\left(ef+dg\right)x}\right.}{3\left(eg^2x+dg^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [-1/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-(c*d*f - a*e*g)/
g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^
2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^
2*x + d*g^2), -2/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt((c*d
*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/(e*g^2*x + d*g^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] Timed out
```

3.695 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

Optimal. Leaf size=178

$$-\frac{3cd\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}$$

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]) - (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f + g*x))
- (3*c*d*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rubi [A] time = 0.250813, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 864, 874, 205}

$$-\frac{3cd\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^2),x]
```

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]) - (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f + g*x))
- (3*c*d*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx}{2g}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}-\frac{(3cd(}{}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}-\frac{(3cde}{}$$

$$=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}-\frac{3cd\sqrt{}}{}$$

Mathematica [C] time = 0.0617112, size = 75, normalized size = 0.42

$$\frac{2cd((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^2),x]
```

```
[Out] (2*c*d*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (g*(a
*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^2*(d + e*x)^(5/2))
```

Maple [A] time = 0.341, size = 306, normalized size = 1.7

$$\frac{1}{g^2\left(gx+f\right)}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(-3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)xacdeg^2+3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)xc^2d^2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(e*x+d)^(1/2)*(-3*arctanh((c*d*x+a*
e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*a*c*d*e*g^2+3*arctanh((c*d*x+a*e)^(1/
```

```
2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*c^2*d^2*f*g-3*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g+3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c
*d*f)*g)^(1/2))*c^2*d^2*f^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c
*d*g-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1
/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)/((a*e*g-c*d*f)*g
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^2), x)
```

Fricas [A] time = 1.95448, size = 975, normalized size = 5.48

$$\left[\frac{3\left(cdegx^2 + cd^2f + \left(cdef + cd^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2 - cd^2f + 2\,adeg - 2\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}g\sqrt{-\frac{cdf-aeg}{g}} - \left(cdef - \left(cd^2 + 2\,ae^2\right.\right.}{egx^2 + df + \left(ef + dg\right)x}\right.}{2\left(eg^3x^2 + dfg^2 + \left(efg^2 + dg^3\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-(c*d*f - a*e*
g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c
*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(e
*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x), (3*(c*d*e*g*x^2 + c*d^2*f + (c*d
*e*f + c*d^2*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*sqrt((c*d*f
 - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(
e*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] Timed out
```

3.696 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx$

Optimal. Leaf size=195

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(
f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*g*(d + e*x)^(3
/2)*(f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(5/2)*Sqrt[c*d*f
 - a*e*g])
```

Rubi [A] time = 0.256016, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^3),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(
f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*g*(d + e*x)^(3
/2)*(f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(5/2)*Sqrt[c*d*f
 - a*e*g])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx}{4g}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{(3c\cdots}{\cdots}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{(3c\cdots}{\cdots}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{3c^2\cdots}{\cdots}$$

Mathematica [A] time = 0.339809, size = 135, normalized size = 0.69

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}}-\frac{\sqrt{g}(2aeg+cd(3f+5gx))}{(f+gx)^2}\right)}{4g^{5/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^3),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(2*a*e*g + c*d*(3*f + 5*g*x)))/(
f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]])/Sqrt[c*d*f - a*
e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d + e*x])
```

Maple [A] time = 0.339, size = 276, normalized size = 1.4

$$-\frac{1}{4g^2\left(gx+f\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^2c^2d^2g^2+6\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)xc^2\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^2*d^2*g^2+6*arctanh((c*d*x+a*e)^(1/2)*g/((a
*e*g-c*d*f)*g)^(1/2))*x*c^2*d^2*f*g+3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c
*d*f)*g)^(1/2))*c^2*d^2*f^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c
*d*g+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)^
```

```
2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^3), x)
```

Fricas [B] time = 1.79288, size = 1750, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*f^2*g - a*c*d*e*f*g^2
 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*f^2*g^4 + (c*d
*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c*d^2 - 2*a*e^2
)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*e^2)*f^2*g^4)*
x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g
^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan
(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e
*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^2*d^2*f^2*g -
 a*c*d*e*f*g^2 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*
f^2*g^4 + (c*d*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c
*d^2 - 2*a*e^2)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*
e^2)*f^2*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="giac")
```

```
[Out] Timed out
```

3.697 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx$

Optimal. Leaf size=265

$$\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)(cdf-aeg)}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}-\frac{\left(x\left(ae^2+c\right.\right.}{3g(d}$$

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(f
+ g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*(c
*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(3*g*(d + e*x)^(3/2)*(f + g*x)^3) + (c^3*d^3*ArcTan[(Sqrt[g]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*
x])])/(8*g^(5/2)*(c*d*f - a*e*g)^(3/2))
```

Rubi [A] time = 0.349488, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)(cdf-aeg)}+\frac{c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}-\frac{\left(x\left(ae^2+c\right.\right.}{3g(d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^4),x]
```

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(f
+ g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*(c
*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(3*g*(d + e*x)^(3/2)*(f + g*x)^3) + (c^3*d^3*ArcTan[(Sqrt[g]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*
x])])/(8*g^(5/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
```

```
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{(cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx}{2g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{\left(c^2d^2\right.}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{(a}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{(a}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{(a}{}$$

Mathematica [C] time = 0.065176, size = 79, normalized size = 0.3

$$\frac{2c^3d^3((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^4),x]
```

```
[Out] (2*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^4*(d + e*x)^(5/2))
```

Maple [A] time = 0.333, size = 453, normalized size = 1.7

$$\frac{1}{\left(24\,aeg-24\,cdf\right)g^2\left(gx+f\right)^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^3c^3d^3g^3+9\,\mathrm{Artanh}\left(\frac{\sqrt{c}}{\sqrt{(ae}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x)
```

```
[Out] 1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^3*d^3*g^3+9*arctanh((c*d*x+a*e)^(1/2)*g/((a
*e*g-c*d*f)*g)^(1/2))*x^2*c^3*d^3*f*g^2+9*arctanh((c*d*x+a*e)^(1/2)*g/((a*e
*g-c*d*f)*g)^(1/2))*x*c^3*d^3*f^2*g+3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x^2
*c^2*d^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*g^2+8*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g-8*((a*e*g-c*d*f)*g)^
(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(
1/2)*a*c*d*e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(
e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g^2/(g*x+f)^3/((a*e*g-c*d*f)*g
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^4), x)
```

Fricas [B] time = 2.02307, size = 2890, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^
3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d
^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
 d*g)*x)) - 2*(3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2*f*g^3
 + 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^3*d^3*f
^2*g^2 - 11*a*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2*e*f^4*g^
4 + a^2*d*e^2*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 + a^2*e^3*g^
8)*x^4 + (3*c^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c*d*e^2)*f^2
*g^6 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*g^4 + a^2*d*
e^2*f*g^7 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a^2*e^3)*f^2*g
^6)*x^2 + (c^2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d^3 - 2*a*c*d*e
^2)*f^4*g^4 - (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*
```

```
x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^
2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*
g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*
g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) +
 (3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2*f*g^3 + 8*a^3*e^3*
g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^3*d^3*f^2*g^2 - 11*a
*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2*e*f^4*g^4 + a^2*d*e^2
*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 + a^2*e^3*g^8)*x^4 + (3*c
^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^6 - (2*a*c
*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*g^4 + a^2*d*e^2*f*g^7 + (
c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^6)*x^2 + (c^
2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g^4 -
 (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] Timed out
```

3.698 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

Optimal. Leaf size=335

$$\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}+\frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}-\frac{cd\sqrt{x}}{8}$$

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*Sqrt[d + e*x]*(f
+ g*x)^3) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^2*(
c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^3*d^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x))
 - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*g*(d + e*x)^(3/2)*(f +
g*x)^4) + (3*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(5/2)*(c*d*f - a*e*g)^(5
/2))
```

Rubi [A] time = 0.452313, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}+\frac{3c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}-\frac{cd\sqrt{x}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^5),x]
```

```
[Out] -(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*Sqrt[d + e*x]*(f
+ g*x)^3) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^2*(
c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^3*d^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x))
 - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*g*(d + e*x)^(3/2)*(f +
g*x)^4) + (3*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(5/2)*(c*d*f - a*e*g)^(5
/2))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
```

```
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx}{8g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+\frac{\left(c^2d^2\right.}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}-\frac{(a}{}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3c}{64}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3c}{64}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3c}{64}$$

Mathematica [C] time = 0.0635249, size = 79, normalized size = 0.24

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^5*(d + e*x)^(5/2))
```

Maple [B] time = 0.35, size = 665, normalized size = 2.

$$-\frac{1}{64\left(gx+f\right)^{4}g^{2}\left(aeg-cdf\right)^{2}}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\left(3\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^{4}c^{4}d^{4}g^{4}+12\,\mathrm{Artanh}\left(\frac{\sqrt{cdx}}{\sqrt{\left(aeg\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x)
```

```
[Out] -1/64*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*x^4*c^4*d^4*g^4+12*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*x^3*c^4*d^4*f*g^3+18*arctanh((c*d*x+a*e)^(1/2)*g/((
a*e*g-c*d*f)*g)^(1/2))*x^2*c^4*d^4*f^2*g^2+12*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*x*c^4*d^4*f^3*g-3*x^3*c^3*d^3*g^3*(c*d*x+a*e)^(1/2)
*((a*e*g-c*d*f)*g)^(1/2)+3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1
/2))*c^4*d^4*f^4+2*x^2*a*c^2*d^2*e*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)-11*x^2*c^3*d^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+24*x*a
^2*c*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-44*x*a*c^2*d^2*e*f
*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+11*x*c^3*d^3*f^2*g*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+16*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*a^3*e^3*g^3-24*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2*f*g
^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+3*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2)/(g*x+f)^4/g^2/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^5), x)
```

Fricas [B] time = 2.31297, size = 4467, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [-1/128*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
```

```
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^
4*d^4*f^4*g - a*c^3*d^3*e*f^3*g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*a^3*c*d
*e^3*f*g^4 - 16*a^4*e^4*g^5 - 3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 - (11
*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x^2 + (11*
c^4*d^4*f^3*g^2 - 55*a*c^3*d^3*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g^4 - 24*a^
3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
)/(c^3*d^4*f^7*g^3 - 3*a*c^2*d^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^5*g^5 - a^3*
d*e^3*f^4*g^6 + (c^3*d^3*e*f^3*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 + 3*a^2*c*d*e^
3*f*g^9 - a^3*e^4*g^10)*x^5 + (4*c^3*d^3*e*f^4*g^6 - a^3*d*e^3*g^10 + (c^3*
d^4 - 12*a*c^2*d^2*e^2)*f^3*g^7 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^8 +
 (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^9)*x^4 + 2*(3*c^3*d^3*e*f^5*g^5 - 2*a^3*
d*e^3*f*g^9 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g^6 - 3*(2*a*c^2*d^3*e - 3*
a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^8)*x^3 + 2*(2*c^
3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*f^2*g^8 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g
^5 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^6 + (9*a^2*c*d^2*e^2 - 2*a^3*e
^4)*f^3*g^7)*x^2 + (c^3*d^3*e*f^7*g^3 - 4*a^3*d*e^3*f^3*g^7 + (4*c^3*d^4 -
3*a*c^2*d^2*e^2)*f^6*g^4 - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^5 + (12*a^
2*c*d^2*e^2 - a^3*e^4)*f^4*g^6)*x), -1/64*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f
^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4
*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*
e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*
x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (3*c^4*d^4*f^4*g - a*c^3*d^3*e*f^3*
g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*a^3*c*d*e^3*f*g^4 - 16*a^4*e^4*g^5 -
3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 - (11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^
3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x^2 + (11*c^4*d^4*f^3*g^2 - 55*a*c^3*d^3
*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g^4 - 24*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^7*g^3 - 3*a*c^2*d
^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^5*g^5 - a^3*d*e^3*f^4*g^6 + (c^3*d^3*e*f^3
*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 + 3*a^2*c*d*e^3*f*g^9 - a^3*e^4*g^10)*x^5 +
(4*c^3*d^3*e*f^4*g^6 - a^3*d*e^3*g^10 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^
7 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^8 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)
*f*g^9)*x^4 + 2*(3*c^3*d^3*e*f^5*g^5 - 2*a^3*d*e^3*f*g^9 + (2*c^3*d^4 - 9*a
*c^2*d^2*e^2)*f^4*g^6 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^
2*c*d^2*e^2 - a^3*e^4)*f^2*g^8)*x^3 + 2*(2*c^3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*
f^2*g^8 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g^5 - 3*(3*a*c^2*d^3*e - 2*a^2*
c*d*e^3)*f^4*g^6 + (9*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^7)*x^2 + (c^3*d^3*e*
f^7*g^3 - 4*a^3*d*e^3*f^3*g^7 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^4 - 3*(
4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^5 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^6
)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] Timed out
```

3.699 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6} dx$

Optimal. Leaf size=405

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^2\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{80g^2\sqrt{d+ex}(f+gx)^3(cdf-aeg)}+\frac{3c^5d}{}$$

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(40*g^2*Sqrt[d + e*x]*
(f + g*x)^4) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(80*g^
2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)
^2) + (3*c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^2*(c*d
*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(5*g*(d + e*x)^(3/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
e*x])])/(128*g^(5/2)*(c*d*f - a*e*g)^(7/2))
```

Rubi [A] time = 0.563057, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^2\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{80g^2\sqrt{d+ex}(f+gx)^3(cdf-aeg)}+\frac{3c^5d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^6),x]
```


```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(40*g^2*Sqrt[d + e*x]*
(f + g*x)^4) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(80*g^
2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)
^2) + (3*c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^2*(c*d
*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(5*g*(d + e*x)^(3/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
e*x])])/(128*g^(5/2)*(c*d*f - a*e*g)^(7/2))
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
```

```
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx}{10g}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{\left(3c^2d^2\right.}{}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3}-\frac{\left(ade\right.}{}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3}{64g^2}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3}{64g^2}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3}{64g^2}$$

$$= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3}{64g^2}$$

Mathematica [C] time = 0.074745, size = 79, normalized size = 0.2

$$\frac{2c^5d^5((d+ex)(ae+cdx))^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5(d+ex)^{5/2}(cdf-aeg)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^6),x]
```

```
[Out] (2*c^5*d^5*((a*e + c*d*x)*(d + e*x))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*(c*d*f - a*e*g)^6*(d + e*x)^(5/2))
```

Maple [B] time = 0.366, size = 955, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x)
```

```
[Out] 1/640*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*x^5*c^5*d^5*g^5+75*arctanh((c*d*x+a*e)^(1/2)*g/
((a*e*g-c*d*f)*g)^(1/2))*x^4*c^5*d^5*f*g^4+150*arctanh((c*d*x+a*e)^(1/2)*g/
((a*e*g-c*d*f)*g)^(1/2))*x^3*c^5*d^5*f^2*g^3+150*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^5*d^5*f^3*g^2-15*x^4*c^4*d^4*g^4*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+75*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d
*f)*g)^(1/2))*x*c^5*d^5*f^4*g+10*x^3*a*c^3*d^3*e*g^4*(c*d*x+a*e)^(1/2)*((a*
e*g-c*d*f)*g)^(1/2)-70*x^3*c^4*d^4*f*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5
-8*x^2*a^2*c^2*d^2*e^2*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+46*x^2
*a*c^3*d^3*e*f*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-128*x^2*c^4*d^
4*f^2*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-176*x*a^3*c*d*e^3*g^4*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+512*x*a^2*c^2*d^2*e^2*f*g^3*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-466*x*a*c^3*d^3*e*f^2*g^2*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)+70*x*c^4*d^4*f^3*g*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)-128*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*e^4*g^4+33
6*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d*e^3*f*g^3-248*((a*e*g-c
*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*((a*e*g-c*d*f)*
g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^3*e*f^3*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*
d*x+a*e)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^5
/g^2/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^6), x)
```

Fricas [B] time = 2.33046, size = 6377, normalized size = 15.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6
*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3
*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 +
 (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*
g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d)
)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^
4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 - 464*a^4
*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4
 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 -
 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3*e^2*f*g^5
- 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^4*d^4*e*f^3*
g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5 + 88*a^4*c*d*
e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4
*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7*g^5 - 4*a^3*c*
d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 - 4*a*c^3*d^3*e^2*
f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^11 + a^4*e^5*g^12)
*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 - 20*a*c^3*d^3*e^2)
*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^9 + 2*(3*a^2*c^2*d^
3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^11)*x^
5 + 5*(2*c^4*d^4*e*f^6*g^6 + a^4*d*e^4*f*g^11 + (c^4*d^5 - 8*a*c^3*d^3*e^2)
*f^5*g^7 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^8 + 2*(3*a^2*c^2*d^3*e
^2 - 4*a^3*c*d*e^4)*f^3*g^9 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^10)*x^4 +
 10*(c^4*d^4*e*f^7*g^5 + a^4*d*e^4*f^2*g^10 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f
^6*g^6 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^7 + 2*(3*a^2*c^2*d^3*e
^2 - 2*a^3*c*d*e^4)*f^4*g^8 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^9)*x^3 + 5*
(c^4*d^4*e*f^8*g^4 + 2*a^4*d*e^4*f^3*g^9 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)*f^
7*g^5 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^6 + 4*(3*a^2*c^2*d^3*e^
2 - a^3*c*d*e^4)*f^5*g^7 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^8)*x^2 + (c^4*
d^4*e*f^9*g^3 + 5*a^4*d*e^4*f^4*g^8 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^8*g^4
 - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^5 + 2*(15*a^2*c^2*d^3*e^2 -
 2*a^3*c*d*e^4)*f^6*g^6 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^7)*x), -1/640*
(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^
5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c
^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^
5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*
g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e
*f^4*g^2 - 258*a^2*c^3*d^3*e^2*f^3*g^3 + 584*a^3*c^2*d^2*e^3*f^2*g^4 - 464*
a^4*c*d*e^4*f*g^5 + 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*
x^4 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^
3 - 2*(64*c^5*d^5*f^3*g^3 - 87*a*c^4*d^4*e*f^2*g^4 + 27*a^2*c^3*d^3*e^2*f*g
^5 - 4*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 268*a*c^4*d^4*e*f
^3*g^3 + 489*a^2*c^3*d^3*e^2*f^2*g^4 - 344*a^3*c^2*d^2*e^3*f*g^5 + 88*a^4*c
*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
c^4*d^5*f^9*g^3 - 4*a*c^3*d^4*e*f^8*g^4 + 6*a^2*c^2*d^3*e^2*f^7*g^5 - 4*a^3
*c*d^2*e^3*f^6*g^6 + a^4*d*e^4*f^5*g^7 + (c^4*d^4*e*f^4*g^8 - 4*a*c^3*d^3*e
^2*f^3*g^9 + 6*a^2*c^2*d^2*e^3*f^2*g^10 - 4*a^3*c*d*e^4*f*g^11 + a^4*e^5*g^
12)*x^6 + (5*c^4*d^4*e*f^5*g^7 + a^4*d*e^4*g^12 + (c^4*d^5 - 20*a*c^3*d^3*e
^2)*f^4*g^8 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^9 + 2*(3*a^2*c^2
```

```
*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^10 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^11)
*x^5 + 5*(2*c^4*d^4*e*f^6*g^6 + a^4*d*e^4*f*g^11 + (c^4*d^5 - 8*a*c^3*d^3*e
^2)*f^5*g^7 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^8 + 2*(3*a^2*c^2*d^
3*e^2 - 4*a^3*c*d*e^4)*f^3*g^9 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^10)*x^
4 + 10*(c^4*d^4*e*f^7*g^5 + a^4*d*e^4*f^2*g^10 + (c^4*d^5 - 4*a*c^3*d^3*e^2
)*f^6*g^6 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^7 + 2*(3*a^2*c^2*d^
3*e^2 - 2*a^3*c*d*e^4)*f^4*g^8 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^9)*x^3 +
 5*(c^4*d^4*e*f^8*g^4 + 2*a^4*d*e^4*f^3*g^9 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)
*f^7*g^5 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^6 + 4*(3*a^2*c^2*d^3
*e^2 - a^3*c*d*e^4)*f^5*g^7 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^8)*x^2 + (c
^4*d^4*e*f^9*g^3 + 5*a^4*d*e^4*f^4*g^8 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^8*
g^4 - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^5 + 2*(15*a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^6*g^6 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**6,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="giac")
```

```
[Out] Timed out
```

3.700 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=336

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{195c^2d^2(d+ex)^{7/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{715c^3d^3(d+ex)^{7/2}}+\frac{1\ldots}{\ldots}$$

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(7/2))/(45045*c^5*d^5*e*(d + e*x)^(7/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(6435*c^4*d^
4*e*(d + e*x)^(5/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(715*c^3*d^3*(d + e*x)^(7/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(195*c^2*d
^2*(d + e*x)^(7/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(7/2))/(15*c*d*(d + e*x)^(7/2))
```

Rubi [A] time = 0.618059, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{195c^2d^2(d+ex)^{7/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{715c^3d^3(d+ex)^{7/2}}+\frac{1\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```


```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(7/2))/(45045*c^5*d^5*e*(d + e*x)^(7/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(6435*c^4*d^
4*e*(d + e*x)^(5/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(715*c^3*d^3*(d + e*x)^(7/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(195*c^2*d
^2*(d + e*x)^(7/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(7/2))/(15*c*d*(d + e*x)^(7/2))
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
```

```
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}} + \frac{(8(cdf-aeg))\int \frac{(f+g}{}}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} + \frac{2(f+g}{}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{715c^3d^3(d+ex)^{7/2}} + \frac{16(cdf}{}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{6435c^4d^4e(d+ex)^{5/2}} + \frac{32(cdf-aeg)}{}$$

$$= \frac{128(cdf-aeg)^3\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{45045c^4d^4(d+ex)^{7/2}}$$

Mathematica [A] time = 0.22377, size = 205, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(48a^2c^2d^2e^2g^2\left(65f^2+70fgx+21g^2x^2\right)-64a^3cde^3g^3(15f+7gx)+128a^4e^4g^4-8ac\right.}{45045}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^4*g^4 - 64*a^3*
c*d*e^3*g^3*(15*f + 7*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(65*f^2 + 70*f*g*x + 21
*g^2*x^2) - 8*a*c^3*d^3*e*g*(715*f^3 + 1365*f^2*g*x + 945*f*g^2*x^2 + 231*g
^3*x^3) + c^4*d^4*(6435*f^4 + 20020*f^3*g*x + 24570*f^2*g^2*x^2 + 13860*f*g
^3*x^3 + 3003*g^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

Maple [A] time = 0.051, size = 283, normalized size = 0.8

$$(2\,cdx+2\,ae)\left(3003\,g^4x^4c^4d^4-1848\,ac^3d^3eg^4x^3+13860\,c^4d^4fg^3x^3+1008\,a^2c^2d^2e^2g^4x^2-7560\,ac^3d^3efg^3x^2+24570\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/45045*(c*d*x+a*e)*(3003*c^4*d^4*g^4*x^4-1848*a*c^3*d^3*e*g^4*x^3+13860*c^
4*d^4*f*g^3*x^3+1008*a^2*c^2*d^2*e^2*g^4*x^2-7560*a*c^3*d^3*e*f*g^3*x^2+245
70*c^4*d^4*f^2*g^2*x^2-448*a^3*c*d*e^3*g^4*x+3360*a^2*c^2*d^2*e^2*f*g^3*x-1
0920*a*c^3*d^3*e*f^2*g^2*x+20020*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-960*a^3*c*
d*e^3*f*g^3+3120*a^2*c^2*d^2*e^2*f^2*g^2-5720*a*c^3*d^3*e*f^3*g+6435*c^4*d^
4*f^4)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^5/d^5/(e*x+d)^(5/2)
```

Maxima [A] time = 1.25476, size = 672, normalized size = 2.

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}f^4}{7\,cd}+\frac{8\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)}{63\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^4/(c*d) + 8/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2)
+ 4/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3
*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f^2*g
^2/(c^3*d^3) + 8/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*c^4*
d^4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*d*e^5
*x - 16*a^6*e^6)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/45045*(3003*c^7*d^7*
x^7 + 7161*a*c^6*d^6*e*x^6 + 4473*a^2*c^5*d^5*e^2*x^5 + 35*a^3*c^4*d^4*e^3*
x^4 - 40*a^4*c^3*d^3*e^4*x^3 + 48*a^5*c^2*d^2*e^5*x^2 - 64*a^6*c*d*e^6*x +
128*a^7*e^7)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Fricas [A] time = 1.6954, size = 1214, normalized size = 3.61

$$2\left(3003\,c^7d^7g^4x^7+6435\,a^3c^4d^4e^3f^4-5720\,a^4c^3d^3e^4f^3g+3120\,a^5c^2d^2e^5f^2g^2-960\,a^6cde^6fg^3+128\,a^7e^7g^4+231\left(60\,c^7\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^7*d^7*g^4*x^7 + 6435*a^3*c^4*d^4*e^3*f^4 - 5720*a^4*c^3*d^3
*e^4*f^3*g + 3120*a^5*c^2*d^2*e^5*f^2*g^2 - 960*a^6*c*d*e^6*f*g^3 + 128*a^7
*e^7*g^4 + 231*(60*c^7*d^7*f*g^3 + 31*a*c^6*d^6*e*g^4)*x^6 + 63*(390*c^7*d^
7*f^2*g^2 + 540*a*c^6*d^6*e*f*g^3 + 71*a^2*c^5*d^5*e^2*g^4)*x^5 + 35*(572*c
^7*d^7*f^3*g + 1794*a*c^6*d^6*e*f^2*g^2 + 636*a^2*c^5*d^5*e^2*f*g^3 + a^3*c
^4*d^4*e^3*g^4)*x^4 + 5*(1287*c^7*d^7*f^4 + 10868*a*c^6*d^6*e*f^3*g + 8814*
a^2*c^5*d^5*e^2*f^2*g^2 + 60*a^3*c^4*d^4*e^3*f*g^3 - 8*a^4*c^3*d^3*e^4*g^4)
*x^3 + 3*(6435*a*c^6*d^6*e*f^4 + 14300*a^2*c^5*d^5*e^2*f^3*g + 390*a^3*c^4*
d^4*e^3*f^2*g^2 - 120*a^4*c^3*d^3*e^4*f*g^3 + 16*a^5*c^2*d^2*e^5*g^4)*x^2 +
 (19305*a^2*c^5*d^5*e^2*f^4 + 2860*a^3*c^4*d^4*e^3*f^3*g - 1560*a^4*c^3*d^3
*e^4*f^2*g^2 + 480*a^5*c^2*d^2*e^5*f*g^3 - 64*a^6*c*d*e^6*g^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.701 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=269

$$\frac{12(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{143c^2d^2(d+ex)^{7/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{429c^3d^3e(d+ex)^{5/2}}-\frac{16\left(x\left(ae^2\right.\right.}{}$$

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*c^4*d^4*e*(d + e*x)^(7/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*c^3*d^3*e*
(d + e*x)^(5/2)) + (12*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(143*c^2*d^2*(d + e*x)^(7/2)) + (2*(f + g*x)^3*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*(d + e*x)^(7/2))
```

Rubi [A] time = 0.396652, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{12(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{143c^2d^2(d+ex)^{7/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{429c^3d^3e(d+ex)^{5/2}}-\frac{16\left(x\left(ae^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```


```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*c^4*d^4*e*(d + e*x)^(7/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*c^3*d^3*e*
(d + e*x)^(5/2)) + (12*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(143*c^2*d^2*(d + e*x)^(7/2)) + (2*(f + g*x)^3*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*(d + e*x)^(7/2))
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}} + \frac{(6(cdf-aeg))\int \frac{(f+g}{}}{}$$

$$= \frac{12(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} + \frac{2(f+g}{}$$

$$= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3e(d+ex)^{5/2}} + \frac{12(cdf-aeg)(}{}$$

$$= \frac{16(cdf-aeg)^2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3003c^3d^3(d+ex)^{7/2}} +$$

Mathematica [A] time = 0.168285, size = 147, normalized size = 0.55

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(8a^2cde^2g^2(13f+7gx)-16a^3e^3g^3-2ac^2d^2eg\left(143f^2+182fgx+63g^2x^2\right)+c^3d^3\left(10\right.\right.}{3003c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c
*d*e^2*g^2*(13*f + 7*g*x) - 2*a*c^2*d^2*e*g*(143*f^2 + 182*f*g*x + 63*g^2*x
^2) + c^3*d^3*(429*f^3 + 1001*f^2*g*x + 819*f*g^2*x^2 + 231*g^3*x^3)))/(300
3*c^4*d^4*Sqrt[d + e*x])
```

Maple [A] time = 0.055, size = 188, normalized size = 0.7

$$-\frac{(2\,cdx+2\,ae)\left(-231\,g^3x^3c^3d^3+126\,ac^2d^2eg^3x^2-819\,c^3d^3fg^2x^2-56\,a^2cde^2g^3x+364\,ac^2d^2efg^2x-1001\,c^3d^3f^2gx+\right.}{3003\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-231*c^3*d^3*g^3*x^3+126*a*c^2*d^2*e*g^3*x^2-819*c^3*d
^3*f*g^2*x^2-56*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-1001*c^3*d^3*f^2*
g*x+16*a^3*e^3*g^3-104*a^2*c*d*e^2*f*g^2+286*a*c^2*d^2*e*f^2*g-429*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^4/d^4/(e*x+d)^(5/2)
```

Maxima [A] time = 1.24649, size = 489, normalized size = 1.82

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}\,f^3}{7\,cd} + \frac{2\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e\right.}{21\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^3/(c*d) + 2/21*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2)
+ 2/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3
*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f*g^2
/(c^3*d^3) + 2/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*c^4*d^
4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*d*e^5*x
 - 16*a^6*e^6)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

Fricas [A] time = 1.64143, size = 876, normalized size = 3.26

$$\frac{2\left(231\,c^6d^6g^3x^6+429\,a^3c^3d^3e^3f^3-286\,a^4c^2d^2e^4f^2g+104\,a^5cde^5fg^2-16\,a^6e^6g^3+63\left(13\,c^6d^6fg^2+9\,ac^5d^5eg^3\right)x^5+7\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3003*(231*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 286*a^4*c^2*d^2*e^4
*f^2*g + 104*a^5*c*d*e^5*f*g^2 - 16*a^6*e^6*g^3 + 63*(13*c^6*d^6*f*g^2 + 9*
a*c^5*d^5*e*g^3)*x^5 + 7*(143*c^6*d^6*f^2*g + 299*a*c^5*d^5*e*f*g^2 + 53*a^
2*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 + 2717*a*c^5*d^5*e*f^2*g + 1469*a
^2*c^4*d^4*e^2*f*g^2 + 5*a^3*c^3*d^3*e^3*g^3)*x^3 + 3*(429*a*c^5*d^5*e*f^3
+ 715*a^2*c^4*d^4*e^2*f^2*g + 13*a^3*c^3*d^3*e^3*f*g^2 - 2*a^4*c^2*d^2*e^4*
g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 + 143*a^3*c^3*d^3*e^3*f^2*g - 52*a^4*c
^2*d^2*e^4*f*g^2 + 8*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.702 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=200

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{99c^2d^2e(d+ex)^{5/2}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)}{693c^3d^3e(d+ex)^{7/2}}$$

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(7/2))/(693*c^3*d^3*e*(d + e*x)^(7/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(99*c^2*d^2*e*(d + e*x
)^(5/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(1
1*c*d*(d + e*x)^(7/2))
```

Rubi [A] time = 0.234938, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {870, 794, 648}

$$\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{99c^2d^2e(d+ex)^{5/2}}-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)}{693c^3d^3e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(7/2))/(693*c^3*d^3*e*(d + e*x)^(7/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(99*c^2*d^2*e*(d + e*x
)^(5/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(1
1*c*d*(d + e*x)^(7/2))
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
```

```
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}} + \frac{(4(cdf-aeg))\int \frac{(f+g}{}}{}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2e(d+ex)^{5/2}} + \frac{2(f+gx)^2\left(ade+\right.}{11c}$$

$$= \frac{8(cdf-aeg)\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693c^2d^2(d+ex)^{7/2}} + \cdots$$

Mathematica [A] time = 0.121441, size = 100, normalized size = 0.5

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^2g^2-4acdeg(11f+7gx)+c^2d^2\left(99f^2+154fgx+63g^2x^2\right)\right)}{693c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e
*g*(11*f + 7*g*x) + c^2*d^2*(99*f^2 + 154*f*g*x + 63*g^2*x^2)))/(693*c^3*d^
3*Sqrt[d + e*x])
```

Maple [A] time = 0.052, size = 116, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(63\,g^2x^2c^2d^2-28\,acdeg^2x+154\,c^2d^2fgx+8\,a^2e^2g^2-44\,acdefg+99\,f^2c^2d^2\right)}{693\,c^3d^3}\left(cdex^2+ae^2x+cd^2x+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/693*(c*d*x+a*e)*(63*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+154*c^2*d^2*f*g*x+8*
a^2*e^2*g^2-44*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(5/2)/c^3/d^3/(e*x+d)^(5/2)
```

Maxima [A] time = 1.16388, size = 328, normalized size = 1.64

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}f^2}{7\,cd} + \frac{4\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e\right.}{63\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f^2/(c*d) + 4/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) +
2/693*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 + 3*a
^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*g^2/(c^
3*d^3)
```

Fricas [A] time = 1.63472, size = 595, normalized size = 2.98

$$\frac{2\left(63\,c^5d^5g^2x^5+99\,a^3c^2d^2e^3f^2-44\,a^4cde^4fg+8\,a^5e^5g^2+7\left(22\,c^5d^5fg+23\,ac^4d^4eg^2\right)x^4+\left(99\,c^5d^5f^2+418\,ac^4d^4efg+\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/693*(63*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 44*a^4*c*d*e^4*f*g + 8
*a^5*e^5*g^2 + 7*(22*c^5*d^5*f*g + 23*a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^
2 + 418*a*c^4*d^4*e*f*g + 113*a^2*c^3*d^3*e^2*g^2)*x^3 + 3*(99*a*c^4*d^4*e*
f^2 + 110*a^2*c^3*d^3*e^2*f*g + a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d^3
*e^2*f^2 + 22*a^3*c^2*d^2*e^3*f*g - 4*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.703 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(2ae^2g-cd(9ef-7dg)\right)}{63c^2d^2e(d+ex)^{7/2}}$$

```
[Out] (-2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(63*c^2*d^2*e*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))
```

Rubi [A] time = 0.100152, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {794, 648}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(2ae^2g-cd(9ef-7dg)\right)}{63c^2d^2e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(63*c^2*d^2*e*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}+\frac{1}{9}\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\int \frac{(ade$$

$$=\frac{2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63cd(d+ex)^{7/2}}+\frac{2g\left(ade+(cd\right.}{9cd}$$

Mathematica [A] time = 0.0800597, size = 64, normalized size = 0.51

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}(cd(9f+7gx)-2aeg)}{63c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(9*f + 7*g
*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

Maple [A] time = 0.048, size = 67, normalized size = 0.5

$$-\frac{(2\,cdx+2\,ae)\left(-7\,xcdg+2\,aeg-9\,cdf\right)}{63\,c^2d^2}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{5}{2}}(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(-7*c*d*g*x+2*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/c^2/d^2/(e*x+d)^(5/2)
```

Maxima [A] time = 1.10633, size = 190, normalized size = 1.52

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}f}{7\,cd}+\frac{2\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{}}{63\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)*f/(c*d) + 2/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*d^
2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

Fricas [A] time = 1.59864, size = 362, normalized size = 2.9

$$\frac{2\left(7\,c^4d^4gx^4+9\,a^3cde^3f-2\,a^4e^4g+\left(9\,c^4d^4f+19\,ac^3d^3eg\right)x^3+3\left(9\,ac^3d^3ef+5\,a^2c^2d^2e^2g\right)x^2+\left(27\,a^2c^2d^2e^2f+a^3cde^3\right.}{63\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="fricas")
```

```
[Out] 2/63*(7*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 2*a^4*e^4*g + (9*c^4*d^4*f + 19*a
*c^3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f + 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2
```

```
*c^2*d^2*e^2*f + a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2
),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.704 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rubi [A] time = 0.0212911, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {648}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Mathematica [A] time = 0.044164, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Maple [A] time = 0.048, size = 50, normalized size = 1.

$$\frac{2\,cdx+2\,ae}{7\,cd}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{5}{2}}(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 2/7*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/d/c/(e*x+d)^(5/2)
```

Maxima [A] time = 1.05822, size = 81, normalized size = 1.69

$$\frac{2\left(c^3d^3x^3 + 3\,ac^2d^2ex^2 + 3\,a^2cde^2x + a^3e^3\right)\sqrt{cdx + ae}}{7\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
x + a*e)/(c*d)
```

Fricas [B] time = 1.72663, size = 193, normalized size = 4.02

$$\frac{2\left(c^3d^3x^3 + 3\,ac^2d^2ex^2 + 3\,a^2cde^2x + a^3e^3\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}}{7\left(cdex + cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="giac")
```

```
[Out] Exception raised: AttributeError
```

3.705 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx$

Optimal. Leaf size=236

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{g^3\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{3g^2(d+ex)^{3/2}}-\frac{2(cdf-aeg)^{5/2}\tan^{-1}\left(\cdots\right.}{g^{7\cdots}}$$

```
[Out] (2*(c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt
[d + e*x]) - (2*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(3*g^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2))/(5*g*(d + e*x)^(5/2)) - (2*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/g^(7/2)
```

Rubi [A] time = 0.466866, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {864, 874, 205}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{g^3\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{3g^2(d+ex)^{3/2}}-\frac{2(cdf-aeg)^{5/2}\tan^{-1}\left(\cdots\right.}{g^{7\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)),x]
```

```
[Out] (2*(c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt
[d + e*x]) - (2*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(3*g^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2))/(5*g*(d + e*x)^(5/2)) - (2*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/g^(7/2)
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int \frac{\left(ade+\left(cd^2\right.\right.}{(d+}}{e^2g}$$

$$= -\frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}} + \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{5g(d+ex)^{5/2}}$$

$$= \frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x\right.}{3g^2(d+ex)^{3/2}}$$

$$= \frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x\right.}{3g^2(d+ex)^{3/2}}$$

$$= \frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x\right.}{3g^2(d+ex)^{3/2}}$$

Mathematica [A] time = 0.39932, size = 145, normalized size = 0.61

$$\frac{((d+ex)(ae+cdx))^{5/2}\left(\frac{10(aeg-cdf)(4aeg+cd(gx-3f))}{3g^2(ae+cdx)^2} - \frac{10(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{g^{5/2}(ae+cdx)^{5/2}} + 2\right)}{5g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*(2 + (10*(-(c*d*f) + a*e*g)*(4*a*e*g + c*d
*(-3*f + g*x)))/(3*g^2*(a*e + c*d*x)^2) - (10*(c*d*f - a*e*g)^(5/2)*ArcTan[
(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(g^(5/2)*(a*e + c*d*x)^(5
/2))))/(5*g*(d + e*x)^(5/2))
```

Maple [B] time = 0.333, size = 431, normalized size = 1.8

$$-\frac{2}{15g^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)a^3e^3g^3 - 45\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)a^2cde^2fg^2 + 45\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x)
```

```
[Out] -2/15*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*a^3*e^3*g^3-45*arctanh((c*d*x+a*e)^(1/2)*g/((a*
e*g-c*d*f)*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)*g/((a*e
*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*
g-c*d*f)*g)^(1/2))*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*
x^2*c^2*d^2*g^2-11*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*g^2+
5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g-23*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+35*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a
*e)^(1/2)*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*
f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)), x)
```

Fricas [A] time = 1.96087, size = 1266, normalized size = 5.36

$$\left[\frac{15\left(c^2d^3f^2-2acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2acde^2fg+a^2e^3g^2\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2adeg-2\sqrt{cdex^2+ade+(\cdots}}{eg\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [1/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 -
 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x
^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)
/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 -
 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^3*x + d*g^3), 2
/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2
*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x +
d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (
3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c
^2*d^2*f*g - 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d))/(e*g^3*x + d*g^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.706 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx$

Optimal. Leaf size=235

$$-\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{g^3\sqrt{d+ex}}+\frac{5cd(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}-\frac{\left(x\left(ae^2+cd^2\right)+\right.}{g(d+ex)^{5/2}}$$

```
[Out] (-5*c*d*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*S
qrt[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g^
2*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d +
e*x)^(5/2)*(f + g*x)) + (5*c*d*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/g^(7/2)
```

Rubi [A] time = 0.377965, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 864, 874, 205}

$$-\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{g^3\sqrt{d+ex}}+\frac{5cd(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}-\frac{\left(x\left(ae^2+cd^2\right)+\right.}{g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^2),x]
```

```
[Out] (-5*c*d*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*S
qrt[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g^
2*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d +
e*x)^(5/2)*(f + g*x)) + (5*c*d*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/g^(7/2)
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
```

```
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx}{2g}$$

$$= \frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}-\frac{(5cd(\ldots}{}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}+\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cde\ldots\right.}{3g^2(d+ex)^{3/2}}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}+\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cde\ldots\right.}{3g^2(d+ex)^{3/2}}$$

$$= -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}+\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cde\ldots\right.}{3g^2(d+ex)^{3/2}}$$

Mathematica [C] time = 0.0759045, size = 75, normalized size = 0.32

$$\frac{2cd((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^2),x]
```

```
[Out] (2*c*d*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (g*(a
*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^2*(d + e*x)^(7/2))
```

Maple [B] time = 0.336, size = 523, normalized size = 2.2

$$-\frac{1}{3g^3(gx+f)}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{(aeg-cdf)g}}\right)xa^2cde^2g^3-30\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{(aeg-cdf)g}}\right)xac\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x)
```

```
[Out] -1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*x*a^2*c*d*e^2*g^3-30*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*x*a*c^2*d^2*e*f*g^2+15*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*a^2*c*d*e^2*f*g^2-30*arctanh((c*d*x+a*e)^(1/2)*g/((
a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)*g/((a
*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^3-2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*x^2*c^2*d^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*g
^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g+3*((a*e*g-c*d
*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-20*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^2), x)
```

Fricas [A] time = 2.30612, size = 1423, normalized size = 6.06

$$\left[-\frac{15\left(c^2d^3f^2-acd^2efg+\left(c^2d^2efg-acde^2g^2\right)x^2+\left(c^2d^2ef^2-acd^2eg^2+\left(c^2d^3-acde^2\right)fg\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^2*d^3*f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^
2 + (c^2*d^2*e*f^2 - a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt(-(c
*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c
*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(2*c^
2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*e^2*g^2 - 2*(5*c^2*
d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), -1/3*(15*(c^2*d^3*
f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^2 + (c^2*d^2*e*f^2
```

```
- a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arc
tan(sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)) - (2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*
e^2*g^2 - 2*(5*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.707 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{15c^2d^2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{7/2}}+\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade\right.}{4g^2(d+ex)^{3/2}(f+}$$

```
[Out] (15*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*Sqrt[d + e*
x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g^2*(d + e*x
)^(3/2)*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*g*(d
+ e*x)^(5/2)*(f + g*x)^2) - (15*c^2*d^2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
 e*x])])/(4*g^(7/2))
```

Rubi [A] time = 0.341862, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 864, 874, 205}

$$-\frac{15c^2d^2\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{7/2}}+\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade\right.}{4g^2(d+ex)^{3/2}(f+}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^3),x]
```


```
[Out] (15*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*Sqrt[d + e*
x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g^2*(d + e*x
)^(3/2)*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*g*(d
+ e*x)^(5/2)*(f + g*x)^2) - (15*c^2*d^2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
 e*x])])/(4*g^(7/2))
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
```

```
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx}{4g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}+\frac{(15c$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{(a$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{(a$$

$$= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{(a$$

Mathematica [C] time = 0.0747797, size = 79, normalized size = 0.32

$$\frac{2c^2d^2((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^3),x]
```

```
[Out] (2*c^2*d^2*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^3*(d + e*x)^(7/2))
```

Maple [B] time = 0.334, size = 526, normalized size = 2.1

$$-\frac{1}{4g^3\left(gx+f\right)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^2ac^2d^2eg^3-15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x)
```

```
[Out] -1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*x^2*a*c^2*d^2*e*g^3-15*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^3*d^3*f*g^2+30*arctanh((c*d*x+a*e)^(1/2)*
g/((a*e*g-c*d*f)*g)^(1/2))*x*a*c^2*d^2*e*f*g^2-30*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)*g/
((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^3-8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(
1/2)*x^2*c^2*d^2*g^2+9*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*
g^2-25*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g+2*((a*e*g-c*
d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^3), x)
```

Fricas [A] time = 4.34343, size = 1442, normalized size = 5.86

$$\left[\frac{15\left(c^2d^2eg^2x^3 + c^2d^3f^2 + \left(2c^2d^2efg + c^2d^3g^2\right)x^2 + \left(c^2d^2ef^2 + 2c^2d^3fg\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2adeg-2\sqrt{cdex^2 \cdots}}{\cdots}\right)\cdots}{8\left(eg^5 \cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [1/8*(15*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)
*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*
d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2
)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^
2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^5*x^3 + d*
f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x), 1/4*(15*(c^
```

```
2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^2 + (c^2*
d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(sqrt(e*x + d)*
sqrt((c*d*f - a*e*g)/g)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)) + (8*c
^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d
^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 +
2*d*f*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**3,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="giac")

[Out] Timed out
```

3.708 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx$

Optimal. Leaf size=253

$$-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-$$

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*Sqrt[d + e*
x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g
^2*(d + e*x)^(3/2)*(f + g*x)^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)/(3*g*(d + e*x)^(5/2)*(f + g*x)^3) + (5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])
/(8*g^(7/2)*Sqrt[c*d*f - a*e*g])
```

Rubi [A] time = 0.336287, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {862, 874, 205}

$$-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}+\frac{5c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^4),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*Sqrt[d + e*
x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g
^2*(d + e*x)^(3/2)*(f + g*x)^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)/(3*g*(d + e*x)^(5/2)*(f + g*x)^3) + (5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])
/(8*g^(7/2)*Sqrt[c*d*f - a*e*g])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{(5cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx}{6g}$$

$$=-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{(5c^2\cdots}{\cdots}$$

$$=-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\cdots$$

$$=-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\cdots$$

$$=-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\cdots$$

Mathematica [A] time = 0.390151, size = 171, normalized size = 0.68

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{15c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{ae+cdx}\sqrt{cdf-aeg}}-\frac{\sqrt{g}\left(8a^2e^2g^2+2acdeg(5f+13gx)+c^2d^2\left(15f^2+40fgx+33g^2x^2\right)\right)}{(f+gx)^3}\right)}{24g^{7/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^4),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(8*a^2*e^2*g^2 + 2*a*c*d*e*g*(5*
f + 13*g*x) + c^2*d^2*(15*f^2 + 40*f*g*x + 33*g^2*x^2)))/(f + g*x)^3) + (15
*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d
*f - a*e*g]*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x])
```

Maple [A] time = 0.328, size = 441, normalized size = 1.7

$$-\frac{1}{24g^3\left(gx+f\right)^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^3c^3d^3g^3+45\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^2\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^3*d^3*g^3+45*arctanh((c*d*x+a*e)^(1/2)*g/
```

```
((a*e*g-c*d*f)*g)^(1/2))*x^2*c^3*d^3*f*g^2+45*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*x*c^3*d^3*f^2*g+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*
e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^3+33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*x^2*c^2*d^2*g^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*a*c*d*e*g
^2+40*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*x*c^2*d^2*f*g+8*((a*e*g-c*d
*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*
x+a*e)^(1/2)*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d
^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\left(gx + f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^4), x)
```

Fricas [B] time = 1.90201, size = 2349, normalized size = 9.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [-1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d
*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f
 + d*g)*x)) + 2*(15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^2*c*d*e^2*f
*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(20*c^3
*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d*e*f^3*g^5
+ (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 + (c*d^2 - 3
*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a*e^2)*f^2*g
^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*f^3*g^5)*x),
 -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g
 + (c*d^2 + a*e^2)*g*x)) + (15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^
2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2
+ 2*(20*c^3*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d
*e*f^3*g^5 + (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 +
```

```
(c*d^2 - 3*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a
*e^2)*f^2*g^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*f
^3*g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] Timed out
```

3.709 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

Optimal. Leaf size=323

$$\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}(f+gx)(cdf-aeg)}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}+\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}-5c$$

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e
*x]*(f + g*x)^2) + (5*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(64*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(24*g^2*(d + e*x)^(3/2)*(f + g*x)^3) - (a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(4*g*(d + e*x)^(5/2)*(f + g*x)^4) +
 (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(7/2)*(c*d*f - a*e*g)^(3/2))
```

Rubi [A] time = 0.473358, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{5c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}(f+gx)(cdf-aeg)}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}+\frac{5c^4d^4\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}-5c$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^5),x]
```


```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e
*x]*(f + g*x)^2) + (5*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(64*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(24*g^2*(d + e*x)^(3/2)*(f + g*x)^3) - (a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(4*g*(d + e*x)^(5/2)*(f + g*x)^4) +
 (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(7/2)*(c*d*f - a*e*g)^(3/2))
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx}{8g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{(5c^2\cdots}{\cdots}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{(\cdots}{\cdots}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{5\cdots}{\cdots}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{5\cdots}{\cdots}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{5\cdots}{\cdots}$$

Mathematica [C] time = 0.0856832, size = 79, normalized size = 0.24

$$\frac{2c^4d^4((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^5),x]
```

```
[Out] (2*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^5*(d + e*x)^(7/2))
```

Maple [B] time = 0.345, size = 665, normalized size = 2.1

$$\frac{1}{192 g^3 \left(aeg - cdf\right) \left(gx + f\right)^4} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left(15 \,\mathrm{Artanh}\left(\frac{\sqrt{cdx + aeg}}{\sqrt{\left(aeg - cdf\right) g}}\right) x^4 c^4 d^4 g^4 + 60 \,\mathrm{Artanh}\left(\frac{\sqrt{c}}{\sqrt{(ae}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x)
```

```
[Out] 1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*x^4*c^4*d^4*g^4+60*arctanh((c*d*x+a*e)^(1/2)*g/
((a*e*g-c*d*f)*g)^(1/2))*x^3*c^4*d^4*f*g^3+90*arctanh((c*d*x+a*e)^(1/2)*g/(
(a*e*g-c*d*f)*g)^(1/2))*x^2*c^4*d^4*f^2*g^2+60*arctanh((c*d*x+a*e)^(1/2)*g/
((a*e*g-c*d*f)*g)^(1/2))*x*c^4*d^4*f^3*g-15*x^3*c^3*d^3*g^3*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)
^(1/2))*c^4*d^4*f^4-118*x^2*a*c^2*d^2*e*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f
)*g)^(1/2)+73*x^2*c^3*d^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1
36*x*a^2*c*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*x*a*c^2*d
^2*e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+55*x*c^3*d^3*f^2*g*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*
e)^(1/2)*a^3*e^3*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^
2*f*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/(c*d*x+
a*e)^(1/2)/g^3/(a*e*g-c*d*f)/(g*x+f)^4/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}} \left(gx + f\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^5), x)
```

Fricas [B] time = 2.05866, size = 3784, normalized size = 11.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5*
g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e*f
^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqrt(-
c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*
```

```
d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c
*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c
^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*f^2*g^3 - 56*a^3*c
*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x^3 +
(73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2*d^2*e^2*g^5)*x^2
+ (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*c^2*d^2*e^2*f*g^4
+ 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*e^2*f^4*g^6 + (c^2
*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 + (4*c^2*d^2*e*f^3*g
^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2*(a*c*d^2*e - 2*a^
2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*f*g^9 + 2*(c^2*d^3
 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g^8)*x^3 + 2*(2*c^2
*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^6 -
2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g^4 + 4*a^2*d*e^2*f
^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*e - a^2*e^3)*f^4*g^
6)*x), -1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c
^4*d^5*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*
d^4*e*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)
*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*
e^2)*g*x)) + (15*c^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*
f^2*g^3 - 56*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3
*d^3*e*g^5)*x^3 + (73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2
*d^2*e^2*g^5)*x^2 + (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*
c^2*d^2*e^2*f*g^4 + 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*
e^2*f^4*g^6 + (c^2*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 +
(4*c^2*d^2*e*f^3*g^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2
*(a*c*d^2*e - 2*a^2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*
f*g^9 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g
^8)*x^3 + 2*(2*c^2*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c
*d*e^2)*f^4*g^6 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g
^4 + 4*a^2*d*e^2*f^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*e
 - a^2*e^3)*f^4*g^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**5,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] Timed out
```

3.710 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

Optimal. Leaf size=393

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^3\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{3c^5d^5\text{ta}}{1}$$

```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*g^3*Sqrt[d + e*x
]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*
g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^4*d^4*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f +
 g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*
x)^(3/2)*(f + g*x)^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*g*
(d + e*x)^(5/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(128*g^(7
/2)*(c*d*f - a*e*g)^(5/2))
```

Rubi [A] time = 0.57166, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^3\sqrt{d+ex}(f+gx)(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{3c^5d^5\text{ta}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^6),x]
```


```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*g^3*Sqrt[d + e*x
]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*
g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^4*d^4*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f +
 g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*
x)^(3/2)*(f + g*x)^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*g*
(d + e*x)^(5/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(128*g^(7
/2)*(c*d*f - a*e*g)^(5/2))
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
```

```
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{(cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx}{2g}$$

$$= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{(3}{}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}-$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2}-$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+$$

Mathematica [C] time = 0.0858821, size = 79, normalized size = 0.2

$$\frac{2c^5d^5((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^6),x]
```

```
[Out] (2*c^5*d^5*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^6*(d + e*x)^(7/2))
```

Maple [B] time = 0.389, size = 924, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x)
```

```
[Out] -1/640*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh((c*d*x+a*e)^(1/2
)*g/((a*e*g-c*d*f)*g)^(1/2))*x^5*c^5*d^5*g^5+75*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*x^4*c^5*d^5*f*g^4+150*arctanh((c*d*x+a*e)^(1/2)*g
/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^5*d^5*f^2*g^3+150*arctanh((c*d*x+a*e)^(1/2)
*g/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^5*d^5*f^3*g^2-15*x^4*c^4*d^4*g^4*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+75*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*
d*f)*g)^(1/2))*x*c^5*d^5*f^4*g+10*x^3*a*c^3*d^3*e*g^4*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)-70*x^3*c^4*d^4*f*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^
5+248*x^2*a^2*c^2*d^2*e^2*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-466
*x^2*a*c^3*d^3*e*f*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+128*x^2*c^
4*d^4*f^2*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+336*x*a^3*c*d*e^3*g
^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-512*x*a^2*c^2*d^2*e^2*f*g^3*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+46*x*a*c^3*d^3*e*f^2*g^2*(c*d*x+a*e
)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+70*x*c^4*d^4*f^3*g*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2)+128*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*e^4*g^4
-176*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c*d*e^3*f*g^3+8*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^3*d^3*e*f^3*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^
5/g^3/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\left(gx + f\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^6), x)
```

Fricas [B] time = 2.23863, size = 5499, normalized size = 13.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^
6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^
3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2
+ (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e
*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d
))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f
^4*g^2 - 2*a^2*c^3*d^3*e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 + 304*a^4*
c*d*e^4*f*g^5 - 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4
- 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 +
2*(64*c^5*d^5*f^3*g^3 - 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^3*e^2*f*g^5
 - 124*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*c^4*d^4*e*f^
3*g^3 - 279*a^2*c^3*d^3*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^5 - 168*a^4*c
*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(
c^3*d^4*f^8*g^4 - 3*a*c^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^6*g^6 - a^3*d*e
^3*f^5*g^7 + (c^3*d^3*e*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10 + 3*a^2*c*d*e^3*
f*g^11 - a^3*e^4*g^12)*x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*e^3*g^12 + (c^3*d
^4 - 15*a*c^2*d^2*e^2)*f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^10 +
 (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^3*e*f^5*g^7 - a^3*d
*e^3*f*g^11 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^4*g^8 - 3*(a*c^2*d^3*e - 2*a^2*
c*d*e^3)*f^3*g^9 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^2*g^10)*x^4 + 10*(c^3*d^
3*e*f^6*g^6 - a^3*d*e^3*f^2*g^10 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^7 - 3*
(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^8 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^9)*
x^3 + 5*(c^3*d^3*e*f^7*g^5 - 2*a^3*d*e^3*f^3*g^9 + (2*c^3*d^4 - 3*a*c^2*d^2
*e^2)*f^6*g^6 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^7 + (6*a^2*c*d^2*e^2
- a^3*e^4)*f^4*g^8)*x^2 + (c^3*d^3*e*f^8*g^4 - 5*a^3*d*e^3*f^4*g^8 + (5*c^3
*d^4 - 3*a*c^2*d^2*e^2)*f^7*g^5 - 3*(5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^6 +
 (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^7)*x), -1/640*(15*(c^5*d^5*e*g^5*x^6 +
c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d^6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^
3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*f^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(
c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x^2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g
)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2
+ a*e^2)*g*x)) + (15*c^5*d^5*f^5*g - 5*a*c^4*d^4*e*f^4*g^2 - 2*a^2*c^3*d^3*
e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 + 304*a^4*c*d*e^4*f*g^5 - 128*a^5
*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4
 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g^6)*x^3 + 2*(64*c^5*d^5*f^3*g^3 -
 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^3*e^2*f*g^5 - 124*a^3*c^2*d^2*e^3*
g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*c^4*d^4*e*f^3*g^3 - 279*a^2*c^3*d^3
*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^5 - 168*a^4*c*d*e^4*g^6)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^8*g^4 - 3*a*c
^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^6*g^6 - a^3*d*e^3*f^5*g^7 + (c^3*d^3*e
*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10 + 3*a^2*c*d*e^3*f*g^11 - a^3*e^4*g^12)*
x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*e^3*g^12 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*
f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^10 + (3*a^2*c*d^2*e^2 - 5*a
^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^3*e*f^5*g^7 - a^3*d*e^3*f*g^11 + (c^3*d^4
- 6*a*c^2*d^2*e^2)*f^4*g^8 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^9 + (3*a
^2*c*d^2*e^2 - 2*a^3*e^4)*f^2*g^10)*x^4 + 10*(c^3*d^3*e*f^6*g^6 - a^3*d*e^3
*f^2*g^10 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^7 - 3*(a*c^2*d^3*e - a^2*c*d*
e^3)*f^4*g^8 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^9)*x^3 + 5*(c^3*d^3*e*f^7*
g^5 - 2*a^3*d*e^3*f^3*g^9 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^6 - 3*(2*a*
c^2*d^3*e - a^2*c*d*e^3)*f^5*g^7 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^8)*x^2
 + (c^3*d^3*e*f^8*g^4 - 5*a^3*d*e^3*f^4*g^8 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)
*f^7*g^5 - 3*(5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^6 + (15*a^2*c*d^2*e^2 - a^
```

```
3*e^4)*f^5*g^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**6,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="giac")

[Out] Timed out
```

3.711 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

Optimal. Leaf size=463

$$\frac{5c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512g^3\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{768g^3\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192g^3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}-\frac{c^2}{}$$

```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x
]*(f + g*x)^4) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192
*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^4*d^4*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(768*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f
+ g*x)^2) + (5*c^5*d^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*g^
3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)*(f + g*x)^5) - (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(6*g*(d + e*x)^(5/2)*(f + g*x)^6) + (5*c^
6*d^6*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*
d*f - a*e*g]*Sqrt[d + e*x])])/(512*g^(7/2)*(c*d*f - a*e*g)^(7/2))
```

Rubi [A] time = 0.716461, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {862, 872, 874, 205}

$$\frac{5c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512g^3\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{768g^3\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192g^3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}-\frac{c^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^7),x]
```


```
[Out] -(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x
]*(f + g*x)^4) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192
*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^4*d^4*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(768*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f
+ g*x)^2) + (5*c^5*d^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*g^
3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)*(f + g*x)^5) - (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(6*g*(d + e*x)^(5/2)*(f + g*x)^6) + (5*c^
6*d^6*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*
d*f - a*e*g]*Sqrt[d + e*x])])/(512*g^(7/2)*(c*d*f - a*e*g)^(7/2))
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx}{12g}$$

$$= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}+\frac{\left(c^2d^2\right.}{}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}-\frac{\left(ad\right.}{}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}-\frac{cd\,(}{}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c}{76}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c}{76}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c}{76}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{5c}{76}$$

Mathematica [C] time = 0.103569, size = 79, normalized size = 0.17

$$\frac{2c^6d^6((d+ex)(ae+cdx))^{7/2}\,{}_2F_1\left(\frac{7}{2},7;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7(d+ex)^{7/2}(cdf-aeg)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^7),x]
```

```
[Out] (2*c^6*d^6*((a*e + c*d*x)*(d + e*x))^(7/2)*Hypergeometric2F1[7/2, 7, 9/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*(c*d*f - a*e*g)^7*(d + e*x)^(7/2))
```

Maple [B] time = 0.351, size = 1261, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x)
```

```
[Out] 1/1536*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(56*x^3*a*c^4*d^4*e*f*g^4*((
a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+1272*x^2*a^2*c^3*d^3*e^2*f*g^4*((a*
e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-1188*x^2*a*c^4*d^4*e*f^2*g^3*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+56*x*a*c^4*d^4*e*f^3*g^2*((a*e*g-c*d*f)*g
)^(1/2)*(c*d*x+a*e)^(1/2)+1696*x*a^3*c^2*d^2*e^3*f*g^4*((a*e*g-c*d*f)*g)^(1
/2)*(c*d*x+a*e)^(1/2)+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2
))*x^6*c^6*d^6*g^6+15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*
c^6*d^6*f^6-256*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^5*e^5*g^5+15*((
a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^5*d^5*f^5+300*arctanh((c*d*x+a*e)
^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^6*d^6*f^3*g^3-432*((a*e*g-c*d*f)*g)
^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c^2*d^2*e^3*f^2*g^3-85*x^4*c^5*d^5*f*g^4*((a*e
*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-198*x^3*c^5*d^5*f^2*g^3*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)+198*x^2*c^5*d^5*f^3*g^2*((a*e*g-c*d*f)*g)^(1/2)
*(c*d*x+a*e)^(1/2)+85*x*c^5*d^5*f^4*g*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(
1/2)-1272*x*a^2*c^3*d^3*e^2*f^2*g^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)+225*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^6*d^6*f^4
*g^2+90*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*c^6*d^6*f^5*
g+90*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^5*c^6*d^6*f*g^5
+225*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^4*c^6*d^6*f^2*g
^4-15*x^5*c^5*d^5*g^5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-432*x^2*a^3
*c^2*d^2*e^3*g^5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+8*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^3*d^3*e^2*f^3*g^2+10*((a*e*g-c*d*f)*g)^(1
/2)*(c*d*x+a*e)^(1/2)*a*c^4*d^4*e*f^4*g+640*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+
a*e)^(1/2)*a^4*c*d*e^4*f*g^4-640*x*a^4*c*d*e^4*g^5*((a*e*g-c*d*f)*g)^(1/2)*
(c*d*x+a*e)^(1/2)+10*x^4*a*c^4*d^4*e*g^5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e
)^(1/2)-8*x^3*a^2*c^3*d^3*e^2*g^5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)
)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^6/g^3/(a*e*g-c*d*f)/(a^2*e^
2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex + d\right)^{\frac{5}{2}}\left(gx + f\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^7), x)
```

Fricas [B] time = 2.39786, size = 7769, normalized size = 16.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="fricas")
```

```
[Out] [1/3072*(15*(c^6*d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^7
*g^6)*x^6 + 3*(5*c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*e*
f^3*g^3 + 3*c^6*d^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^3*g
^3)*x^3 + 3*(2*c^6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^6 +
6*c^6*d^7*f^5*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f +
2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
 (e*f + d*g)*x)) - 2*(15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*a^2*c^4*
d^4*e^2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e^4*f^2*g^
5 - 896*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 - a*c^5*d^5
*e*g^7)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a^2*c^4*d^4*
e^2*g^7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5 + 32*a^2*c^4
*d^4*e^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^4*g^3 - 231*a
*c^5*d^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^3*d^3*e^3*f*g^
6 + 72*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a*c^5*d^5*e*f^4*
g^3 - 1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f^2*g^5 - 2336*a^
4*c^2*d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3*d^4*e*f^9*g^5 + 6
*a^2*c^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4*d*e^4*f^6*g^8 + (c
^4*d^4*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2*c^2*d^2*e^3*f^2*g^12 -
 4*a^3*c*d*e^4*f*g^13 + a^4*e^5*g^14)*x^7 + (6*c^4*d^4*e*f^5*g^9 + a^4*d*e^
4*g^14 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^10 - 4*(a*c^3*d^4*e - 9*a^2*c^2
*d^2*e^3)*f^3*g^11 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^2*g^12 - 2*(2*a^
3*c*d^2*e^3 - 3*a^4*e^5)*f*g^13)*x^6 + 3*(5*c^4*d^4*e*f^6*g^8 + 2*a^4*d*e^4
*f*g^13 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^9 - 2*(4*a*c^3*d^4*e - 15*a^
2*c^2*d^2*e^3)*f^4*g^10 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^11 -
(8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^12)*x^5 + 5*(4*c^4*d^4*e*f^7*g^7 + 3*a^
4*d*e^4*f^2*g^12 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^8 - 12*(a*c^3*d^4*e
 - 2*a^2*c^2*d^2*e^3)*f^5*g^9 + 2*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g
^10 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^11)*x^4 + 5*(3*c^4*d^4*e*f^8*g^6
+ 4*a^4*d*e^4*f^3*g^11 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*g^7 - 2*(8*a*c^3
*d^4*e - 9*a^2*c^2*d^2*e^3)*f^6*g^8 + 12*(2*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*
f^5*g^9 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^10)*x^3 + 3*(2*c^4*d^4*e*f^9
*g^5 + 5*a^4*d*e^4*f^4*g^10 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8*g^6 - 4*(5*
a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^7 + 2*(15*a^2*c^2*d^3*e^2 - 4*a^3*c*
d*e^4)*f^6*g^8 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^9)*x^2 + (c^4*d^4*e*f
^10*g^4 + 6*a^4*d*e^4*f^5*g^9 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9*g^5 - 6
*(4*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^6 + 4*(9*a^2*c^2*d^3*e^2 - a^3*c*d
*e^4)*f^7*g^7 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^8)*x), -1/1536*(15*(c^6*
d^6*e*g^6*x^7 + c^6*d^7*f^6 + (6*c^6*d^6*e*f*g^5 + c^6*d^7*g^6)*x^6 + 3*(5*
```

```
c^6*d^6*e*f^2*g^4 + 2*c^6*d^7*f*g^5)*x^5 + 5*(4*c^6*d^6*e*f^3*g^3 + 3*c^6*d
^7*f^2*g^4)*x^4 + 5*(3*c^6*d^6*e*f^4*g^2 + 4*c^6*d^7*f^3*g^3)*x^3 + 3*(2*c^
6*d^6*e*f^5*g + 5*c^6*d^7*f^4*g^2)*x^2 + (c^6*d^6*e*f^6 + 6*c^6*d^7*f^5*g)*
x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 +
a*e^2)*g*x)) + (15*c^6*d^6*f^6*g - 5*a*c^5*d^5*e*f^5*g^2 - 2*a^2*c^4*d^4*e^
2*f^4*g^3 - 440*a^3*c^3*d^3*e^3*f^3*g^4 + 1072*a^4*c^2*d^2*e^4*f^2*g^5 - 89
6*a^5*c*d*e^5*f*g^6 + 256*a^6*e^6*g^7 - 15*(c^6*d^6*f*g^6 - a*c^5*d^5*e*g^7
)*x^5 - 5*(17*c^6*d^6*f^2*g^5 - 19*a*c^5*d^5*e*f*g^6 + 2*a^2*c^4*d^4*e^2*g^
7)*x^4 - 2*(99*c^6*d^6*f^3*g^4 - 127*a*c^5*d^5*e*f^2*g^5 + 32*a^2*c^4*d^4*e
^2*f*g^6 - 4*a^3*c^3*d^3*e^3*g^7)*x^3 + 6*(33*c^6*d^6*f^4*g^3 - 231*a*c^5*d
^5*e*f^3*g^4 + 410*a^2*c^4*d^4*e^2*f^2*g^5 - 284*a^3*c^3*d^3*e^3*f*g^6 + 72
*a^4*c^2*d^2*e^4*g^7)*x^2 + (85*c^6*d^6*f^5*g^2 - 29*a*c^5*d^5*e*f^4*g^3 -
1328*a^2*c^4*d^4*e^2*f^3*g^4 + 2968*a^3*c^3*d^3*e^3*f^2*g^5 - 2336*a^4*c^2*
d^2*e^4*f*g^6 + 640*a^5*c*d*e^5*g^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^10*g^4 - 4*a*c^3*d^4*e*f^9*g^5 + 6*a^2*c
^2*d^3*e^2*f^8*g^6 - 4*a^3*c*d^2*e^3*f^7*g^7 + a^4*d*e^4*f^6*g^8 + (c^4*d^4
*e*f^4*g^10 - 4*a*c^3*d^3*e^2*f^3*g^11 + 6*a^2*c^2*d^2*e^3*f^2*g^12 - 4*a^3
*c*d*e^4*f*g^13 + a^4*e^5*g^14)*x^7 + (6*c^4*d^4*e*f^5*g^9 + a^4*d*e^4*g^14
 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^10 - 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e
^3)*f^3*g^11 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^2*g^12 - 2*(2*a^3*c*d^
2*e^3 - 3*a^4*e^5)*f*g^13)*x^6 + 3*(5*c^4*d^4*e*f^6*g^8 + 2*a^4*d*e^4*f*g^1
3 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^9 - 2*(4*a*c^3*d^4*e - 15*a^2*c^2*
d^2*e^3)*f^4*g^10 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^11 - (8*a^3
*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^12)*x^5 + 5*(4*c^4*d^4*e*f^7*g^7 + 3*a^4*d*e^
4*f^2*g^12 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^8 - 12*(a*c^3*d^4*e - 2*a
^2*c^2*d^2*e^3)*f^5*g^9 + 2*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g^10 -
4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^11)*x^4 + 5*(3*c^4*d^4*e*f^8*g^6 + 4*a^
4*d*e^4*f^3*g^11 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*g^7 - 2*(8*a*c^3*d^4*e
 - 9*a^2*c^2*d^2*e^3)*f^6*g^8 + 12*(2*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^
9 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^10)*x^3 + 3*(2*c^4*d^4*e*f^9*g^5 +
 5*a^4*d*e^4*f^4*g^10 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8*g^6 - 4*(5*a*c^3*
d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^7 + 2*(15*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)
*f^6*g^8 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^9)*x^2 + (c^4*d^4*e*f^10*g^
4 + 6*a^4*d*e^4*f^5*g^9 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9*g^5 - 6*(4*a*
c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^6 + 4*(9*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*
f^7*g^7 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^8)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**7,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="giac")
```

```
[Out] Timed out
```

3.712 $\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=313

$$\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}}+\frac{5(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{12c^2d^2\sqrt{d+ex}}+\frac{5\sqrt{d+}}{}$$

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*c^3*d^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^2*d^2*Sqrt[d + e*x]) + ((f + g*
x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x])
 + (5*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(7/2)*d^(7/2)*Sqrt[
g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.562478, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {870, 891, 63, 217, 206}

$$\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}}+\frac{5(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{12c^2d^2\sqrt{d+ex}}+\frac{5\sqrt{d+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*c^3*d^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^2*d^2*Sqrt[d + e*x]) + ((f + g*
x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x])
 + (5*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(7/2)*d^(7/2)*Sqrt[
g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
```

```
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{\left(5\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right)\int}{6cde^2}$$

$$= \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+\right.}}{3cd\sqrt{d+ex}}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{a}}{12c^2d^2}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{a}}{12c^2d^2}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{a}}{12c^2d^2}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{a}}{12c^2d^2}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{a}}{12c^2d^2}$$

Mathematica [A] time = 0.665308, size = 269, normalized size = 0.86

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\sqrt{ae+cdx}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\left(15a^2e^2g^2-10acdeg(4f+gx)+c^2d^2\left(33f^2+26fgx+8g^2\right.\right.\right.}{24c^{7/2}d^{7/2}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*(15*a^2*e^2*g^2 - 10*a
*c*d*e*g*(4*f + g*x) + c^2*d^2*(33*f^2 + 26*f*g*x + 8*g^2*x^2)) + 15*Sqrt[c
*d]*(c*d*f - a*e*g)^(5/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x
])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(24*c^(7/2)*d^(7/2)*Sqrt[g]*Sqrt[(a*e
 + c*d*x)*(d + e*x)]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)])
```

Maple [A] time = 0.404, size = 511, normalized size = 1.6

$$-\frac{1}{48\,c^3d^3}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\,\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}}{\sqrt{cdg}}\right)a^3e^3g^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*ln(1/2*(2*x
*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))*a^3*e^3*g^3-45*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*x*c*d*g+a*
e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2
*d^2*e*f^2*g-15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)
*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3-16*x^2*c^2*d^2*g^2*((c*d*x+a*e)*
(g*x+f))^(1/2)*(c*d*g)^(1/2)+20*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x
*a*c*d*e*g^2-52*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*c^2*d^2*f*g-30*
((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+80*((c*d*x+a*e)*(g*x+
f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-66*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^
(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/c^3/d^3/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{\frac{5}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(5/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

Fricas [A] time = 9.65544, size = 1798, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15*
a^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^
3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*
e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(
c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^
2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^
3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
+ (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g*x + c^4*d^5*g),
1/48*(2*(8*c^3*d^3*g^3*x^2 + 33*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 15*a
^2*c*d*e^2*g^3 + 2*(13*c^3*d^3*f*g^2 - 5*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^3
 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e
*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-
c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sq
rt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (
2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g*x + c^4*d^5*g)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{\frac{5}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(5/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

3.713 $\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=244

$$\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}}{}$$

```
[Out] (3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(4*c^2*d^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])])/(4*c^(5/2)*d^(5/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rubi [A] time = 0.366116, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {870, 891, 63, 217, 206}

$$\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```


```
[Out] (3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(4*c^2*d^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])])/(4*c^(5/2)*d^(5/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

```
&& !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2cd\sqrt{d+ex}} + \frac{\left(3\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\right.}{4cde^2}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2\right.}}{2cd\sqrt{d+e}}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2\right.}}{2cd\sqrt{d+e}}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2\right.}}{2cd\sqrt{d+e}}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2\right.}}{2cd\sqrt{d+e}}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2\right.}}{2cd\sqrt{d+e}}$$

Mathematica [A] time = 0.508078, size = 234, normalized size = 0.96

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\sqrt{ae+cdx}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}(cd(5f+2gx)-3aeg)+3\sqrt{cd}(cdf-aeg)^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}}{\sqrt{cd}\sqrt{c}}\right.\right.}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*(-3*a*e*g + c*d*(5*f +
 2*g*x)) + 3*Sqrt[c*d]*(c*d*f - a*e*g)^(3/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(4*c^(5/2)*d^(5/2)
*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)
])
```

Maple [A] time = 0.381, size = 328, normalized size = 1.3

$$\frac{1}{8\,c^2d^2}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\,(gx+f)}\sqrt{cdg}}{\sqrt{cdg}}\right)a^2e^2g^2-6\,\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/8*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*x*c*
d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
*a^2*e^2*g^2-6*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*
(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2
*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+4*(c
*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*c*d*g-6*(c*d*g)^(1/2)*((c*d*x+a*e
)*(g*x+f))^(1/2)*a*e*g+10*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*c*d*f)/
(e*x+d)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/c^2/d^2/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

Fricas [A] time = 7.39403, size = 1432, normalized size = 5.87

$$\left[\frac{4\left(2\,c^2d^2g^2x+5\,c^2d^2fg-3\,acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+3\left(c^2d^3f^2-2\,acd^2efg+a^2de^2g^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 -
2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^
3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e
*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*
a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g*x
 + c^3*d^4*g), 1/8*(2*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c
^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2
*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d
^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g*x + c^3*
d^4*g)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\left(gx+f\right)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

3.714 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e
*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.22248, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {870, 891, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```


```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e
*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\int \cdots}{2cde^2}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\sqrt{a\cdots}\right.}{2cde^2\sqrt{ade+\left(cd^2\cdots\right.}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\sqrt{a\cdots}\right.}{c^2d^2e^2\sqrt{\cdots}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left(\left(cde^2f+cd^2eg-e\left(cd^2+ae^2\right)g\right)\sqrt{a\cdots}\right.}{c^2d^2e^2\sqrt{ade\cdots}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}} + \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\cdots}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x\cdots}}$$

Mathematica [A] time = 0.201232, size = 213, normalized size = 1.26

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\sqrt{ae+cdx}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}+\sqrt{cd}\sqrt{cdf-aeg}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[f + g*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqr
t[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)] + Sqrt[c*d]*Sqrt[c*d*f
 - a*e*g]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sq
```

```
rt[c*d*f - a*e*g])]))/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d + e*x)
]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)])
```

Maple [A] time = 0.368, size = 201, normalized size = 1.2

$$-\frac{1}{2\,cd}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\ln\left(\frac{1}{2}\left(2\,xcdg+aeg+cdf+2\sqrt{(cdx+ae)\,(gx+f)}\sqrt{cdg}\right)\frac{1}{\sqrt{cdg}}\right)aeg-\ln\left(\frac{1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -1/2*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*x*c*d
*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*
a*e*g-ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))*c*d*f-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(e*
x+d)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/c/d/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{gx+f}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Fricas [A] time = 6.94317, size = 1156, normalized size = 6.84

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}cdg-\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{cdg}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2e\ldots}{\ldots}\right.}{4\left(c^2d^2\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(-(
8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c
*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2
)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*
e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g*x + c^2*d^3*g), 1/2*(2*sqrt(c
```

```
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g - (
c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/
(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(
c^2*d^2*e*g*x + c^2*d^3*g)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{(d+ex)(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{gx+f}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

3.715 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=105

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.118487, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {891, 63, 217, 206}

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&= \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx, x, \sqrt{ae+cdx}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&= \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&= \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}
\end{aligned}$$

Mathematica [A] time = 0.11336, size = 160, normalized size = 1.52

$$\frac{2\sqrt{cd}\sqrt{d+ex}\sqrt{ae+cdx}\sqrt{cdf-aeg}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (2*Sqrt[c*d]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Sqrt[(c*d*
(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A] time = 0.378, size = 120, normalized size = 1.1

$$\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\ln\left(\frac{1}{2}\left(2xcdg+aeg+cdf+2\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}\right)\frac{1}{\sqrt{cdg}}\right)\frac{1}{\sqrt{ex+d}}\frac{1}{\sqrt{cdg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/(e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*ln(1/
2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))/(c*d*g)^(1/2)/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g
*x + f)), x)
```

Fricas [A] time = 6.60677, size = 752, normalized size = 7.16

$$\left[\frac{\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2cdgx+cdf+aeg\right)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+(c}{ex+d}\right.}{2cdg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
+ a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
+ c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g
 + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d
*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d))/(c*d*g), -sqrt(-c*d*
g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e
*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*
d^2 + a*e^2)*g)*x))/(c*d*g)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g
*x + f)), x)
```

3.716 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)*Sqrt[d + e
*x]*Sqrt[f + g*x])
```

Rubi [A] time = 0.0652654, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)*Sqrt[d + e
*x]*Sqrt[f + g*x])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\sqrt{f+gx}}$$

Mathematica [A] time = 0.0342655, size = 50, normalized size = 0.82

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/((c*d*f - a*e*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])
```

Maple [A] time = 0.057, size = 63, normalized size = 1.

$$-2\,\frac{(cdx+ae)\sqrt{ex+d}}{\sqrt{gx+f}\,(aeg-cdf)\,\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2*(c*d*x+a*e)/(g*x+f)^(1/2)/(a*e*g-c*d*f)*(e*x+d)^(1/2)/(c*d*e*x^2+a*e^2*x
+c*d^2*x+a*d*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\,\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(3/2)), x)
```

Fricas [B] time = 1.70672, size = 242, normalized size = 3.97

$$\frac{2\,\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\,\sqrt{ex+d}\,\sqrt{gx+f}}{cd^{2}f^{2}-adefg+\left(cdefg-ae^{2}g^{2}\right)x^{2}+\left(cdef^{2}-adeg^{2}+\left(cd^{2}-ae^{2}\right)fg\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(
c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^
2 + (c*d^2 - a*e^2)*f*g)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(3/2)), x)
```

3.717 $\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rubi [A] time = 0.143316, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^{3/2}} + \frac{(2cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}d}{3(cdf-aeg)}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^{3/2}} + \frac{4cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

Mathematica [A] time = 0.0601594, size = 69, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(cd(3f+2gx)-aeg)}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e*g) + c*d*(3*f + 2*g*x)))/(3*(c*d*f
- a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

Maple [A] time = 0.051, size = 98, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-2\,xcdg+aeg-3\,cdf\right)}{3\,a^2e^2g^2-6\,acdefg+3\,c^2d^2f^2}\sqrt{ex+d}\left(gx+f\right)^{-\frac{3}{2}}\frac{1}{\sqrt{cdex^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-2*c*d*g*x+a*e*g-3*c*d*f)*(e*x+d)^(1/2)/(g*x+f)^(3/2)/(a^
2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(5/2)), x)
```

Fricas [B] time = 1.73931, size = 587, normalized size = 4.55

$$\frac{2\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(2\,cdgx + \right.}{3\left(c^2d^3f^4 - 2\,acd^2ef^3g + a^2de^2f^2g^2 + \left(c^2d^2ef^2g^2 - 2\,acde^2fg^3 + a^2e^3g^4\right)x^3 + \left(2\,c^2d^2ef^3g + a^2de^2g^4 + \left(c^2d^3 - 4\,acde^2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*
g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^4 - 2*a*c*d^2*e*f^3*g + a^2*d*e^2
*f^2*g^2 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^3*g^4)*x^3 + (2*c
^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^2 - 2*(a*c*d
^2*e - a^2*e^3)*f*g^3)*x^2 + (c^2*d^2*e*f^4 + 2*a^2*d*e^2*f*g^3 + 2*(c^2*d^
3 - a*c*d*e^2)*f^3*g - (4*a*c*d^2*e - a^2*e^3)*f^2*g^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + d}}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(gx + f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(5/2)), x)
```

3.718 $\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}+\frac{8cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(5/2)) + (8*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(15*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (16*c^2*d^2*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*
Sqrt[f + g*x])
```

Rubi [A] time = 0.218739, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}+\frac{8cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{15\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(5/2)) + (8*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(15*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (16*c^2*d^2*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*
Sqrt[f + g*x])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} + \frac{(4cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}}{5(cdf-aeg)}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/}}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/}}$$

Mathematica [A] time = 0.0924061, size = 105, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3a^2e^2g^2-2acdeg(5f+2gx)+c^2d^2\left(15f^2+20fgx+8g^2x^2\right)\right)}{15\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(5*f + 2*g*x)
 + c^2*d^2*(15*f^2 + 20*f*g*x + 8*g^2*x^2)))/(15*(c*d*f - a*e*g)^3*Sqrt[d +
 e*x]*(f + g*x)^(5/2))
```

Maple [A] time = 0.053, size = 169, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2g^2x^2-4\,acdeg^2x+20\,c^2d^2fgx+3\,a^2e^2g^2-10\,acdefg+15\,c^2d^2f^2\right)}{15\,a^3e^3g^3-45\,a^2cde^2fg^2+45\,ac^2d^2ef^2g-15\,c^3d^3f^3}\sqrt{ex+d}\left(gx+f\right)^{-\frac{5}{2}}\frac{}{\sqrt{cdex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x+20*c^2*d^2*f*g*x+3*a^2
*e^2*g^2-10*a*c*d*e*f*g+15*c^2*d^2*f^2)*(e*x+d)^(1/2)/(g*x+f)^(5/2)/(a^3*e^
3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2
*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(7/2)), x)
```

Fricas [B] time = 1.78278, size = 1123, normalized size = 5.67

$$\frac{}{15\left(c^3d^4f^6 - 3\,ac^2d^3ef^5g + 3\,a^2cd^2e^2f^4g^2 - a^3de^3f^3g^3 + \left(c^3d^3ef^3g^3 - 3\,ac^2d^2e^2f^2g^4 + 3\,a^2cde^3fg^5 - a^3e^4g^6\right)x^4 + \left(3\,c^3d^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 3*a^2*e^2*g^2 +
 4*(5*c^2*d^2*f*g - a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^6 - 3*a*c^2*d^3*e*f^5*g + 3*a^2
*c*d^2*e^2*f^4*g^2 - a^3*d*e^3*f^3*g^3 + (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d^2*e
^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^4 + (3*c^3*d^3*e*f^4*g^2
- a^3*d*e^3*g^6 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2*d^3*e - 3*
a^2*c*d*e^3)*f^2*g^4 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^5)*x^3 + 3*(c^3*d^3*
e*f^5*g - a^3*d*e^3*f*g^5 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3*(a*c^2*
d^3*e - a^2*c*d*e^3)*f^3*g^3 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4)*x^2 + (
c^3*d^3*e*f^6 - 3*a^3*d*e^3*f^2*g^4 + 3*(c^3*d^4 - a*c^2*d^2*e^2)*f^5*g - 3
*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^2 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^
3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(gx + f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(7/2)), x)
```

3.719 $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}+\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{12cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2}+$$

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(7/2)) + (12*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(35*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (16*c^2*d^2*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*(c*d*f - a*e*g)^3*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (32*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(35*(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rubi [A] time = 0.313017, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}+\frac{16c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{12cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(7/2)) + (12*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(35*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (16*c^2*d^2*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*(c*d*f - a*e*g)^3*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (32*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(35*(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{(6cd)\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} d}{7(cdf-aeg)}$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} +$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} +$$

$$= \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} +$$

Mathematica [A] time = 0.129423, size = 152, normalized size = 0.57

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3a^2cde^2g^2(7f+2gx)-5a^3e^3g^3-ac^2d^2eg\left(35f^2+28fgx+8g^2x^2\right)+c^3d^3\left(70f^2gx+35f^3+56fg\right.\right.}{35\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(7*f +
 2*g*x) - a*c^2*d^2*e*g*(35*f^2 + 28*f*g*x + 8*g^2*x^2) + c^3*d^3*(35*f^3 +
 70*f^2*g*x + 56*f*g^2*x^2 + 16*g^3*x^3)))/(35*(c*d*f - a*e*g)^4*Sqrt[d + e
*x]*(f + g*x)^(7/2))
```

Maple [A] time = 0.057, size = 260, normalized size = 1.

$$-\frac{(2\,cdx+2\,ae)\left(-16\,c^3d^3g^3x^3+8\,ac^2d^2eg^3x^2-56\,c^3d^3fg^2x^2-6\,a^2cde^2g^3x+28\,ac^2d^2efg^2x-70\,c^3d^3f^2gx+5\,a^3e^3g^3-\right.}{35\,g^4e^4a^4-140\,cdg^3fe^3a^3+210\,c^2d^2g^2f^2e^2a^2-140\,c^3d^3gf^3ea+35\,c^4d^4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+8*a*c^2*d^2*e*g^3*x^2-56*c^3*d^3*f*g
^2*x^2-6*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^2*x-70*c^3*d^3*f^2*g*x+5*a^3*
e^3*g^3-21*a^2*c*d*e^2*f*g^2+35*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)*(e*x+d)^(
1/2)/(g*x+f)^(7/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g
^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(9/2)), x)
```

Fricas [B] time = 1.84995, size = 1871, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 35*a*c^2*d^2*e*f^2*g + 21*a^2*c
*d*e^2*f*g^2 - 5*a^3*e^3*g^3 + 8*(7*c^3*d^3*f*g^2 - a*c^2*d^2*e*g^3)*x^2 +
2*(35*c^3*d^3*f^2*g - 14*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^
8 - 4*a*c^3*d^4*e*f^7*g + 6*a^2*c^2*d^3*e^2*f^6*g^2 - 4*a^3*c*d^2*e^3*f^5*g
^3 + a^4*d*e^4*f^4*g^4 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a
^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^5 + (4*c^4*d^
4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^4 - 4*(a*c
^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^
4)*f^2*g^6 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g^7)*x^4 + 2*(3*c^4*d^4*e*f^6*g^
2 + 2*a^4*d*e^4*f*g^7 + 2*(c^4*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^3 - 2*(4*a*c^3*
d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^4 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3
*g^5 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^6)*x^3 + 2*(2*c^4*d^4*e*f^7*g +
3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^6*g^2 - 12*(a*c^3*d^4
*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^4*g
^4 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x^2 + (c^4*d^4*e*f^8 + 4*a^4*d*
e^4*f^3*g^5 + 4*(c^4*d^5 - a*c^3*d^3*e^2)*f^7*g - 2*(8*a*c^3*d^4*e - 3*a^2*
c^2*d^2*e^3)*f^6*g^2 + 4*(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^3 - (16*a^
3*c*d^2*e^3 - a^4*e^5)*f^4*g^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(9/2)), x)
```

3.720 $\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{5g(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2c^2d^2\sqrt{d+ex}}+\frac{15g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4c^3d^3\sqrt{d+ex}}+\frac{15\sqrt{g}\sqrt{d+ex}\ldots}{4c^{7/2}\ldots}$$

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (15*g*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*c^3*d^3*Sqrt[d + e*x]) + (5*g*(f + g*x)^(3/2)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2*Sqrt[d + e*x]) + (15*Sqrt[g
]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a
*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*c^(7/2)*d^(7/2)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.468904, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{5g(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2c^2d^2\sqrt{d+ex}}+\frac{15g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4c^3d^3\sqrt{d+ex}}+\frac{15\sqrt{g}\sqrt{d+ex}\ldots}{4c^{7/2}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (15*g*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*c^3*d^3*Sqrt[d + e*x]) + (5*g*(f + g*x)^(3/2)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2*Sqrt[d + e*x]) + (15*Sqrt[g
]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a
*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*c^(7/2)*d^(7/2)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
```

```
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(5g)\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{2c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+a\right.}}{4c^3d^3\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+a\right.}}{4c^3d^3\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+a\right.}}{4c^3d^3\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+a\right.}}{4c^3d^3\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+a\right.}}{4c^3d^3\sqrt{d+ex}}$$

Mathematica [C] time = 0.124159, size = 100, normalized size = 0.33

$$-\frac{2\sqrt{d+ex}(f+gx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2)*Hypergeometric2F1[-5/2, -1/2, 1/2, (g*(a*
e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(
f + g*x))/(c*d*f - a*e*g))^(5/2))
```

Maple [B] time = 0.386, size = 648, normalized size = 2.2

$$\frac{1}{(8\,cdx+8\,ae)\,c^3d^3}\left(15\,\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\,(gx+f)}\sqrt{cdg}}{\sqrt{cdg}}\right)xa^2cde^2g^3-30\,\ln\left(1/2\,\frac{2\,xcdg+ae\ldots}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 1/8*(15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x*a^2*c*d*e^2*g^3-30*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2
*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*a*c^2*d^2*e*f*
g^2+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*x*c^3*d^3*f^2*g+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((
c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-30*ln(1
/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d
*g)^(1/2))*a^2*c*d*e^2*f*g^2+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e
)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g+4*x^2*c^2*
d^2*g^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-10*(c*d*g)^(1/2)*((c*d*x+
a*e)*(g*x+f))^(1/2)*x*a*c*d*e*g^2+18*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1
/2)*x*c^2*d^2*f*g-30*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+
50*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-16*((c*d*x+a*e)*(g
*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(
1/2)*(g*x+f)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/(c*d*x+a*e)/c^
3/d^3/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

Fricas [A] time = 7.50985, size = 2034, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 15*a^2*e^2*g
^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(a*c^2*d^3*e*f^2 - 2*a^2*c*d^2*e^
2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g + a^2*c*d*e^3*
g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e + a^2*c*d*e^3)*f
*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*
x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c
^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(
g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*
c^3*d^3*e^2)*x), 1/8*(2*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g
```

```
 - 15*a^2*e^2*g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a*c^2*d^3*e*f^2 -
 2*a^2*c*d^2*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g
 + a^2*c*d*e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e +
 a^2*c*d*e^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))*arctan
(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*
c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2
 + a*e^2)*g)*x)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2)*
x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.721 $\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=227

$$\frac{3g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}(f+}{cd\sqrt{x\left(ae^2+cd^2\right)+}$$

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (3*g*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(c^2*d^2*Sqrt[d + e*x]) + (3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqr
t[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*
x])])/(c^(5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.314217, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{3g\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}(f+}{cd\sqrt{x\left(ae^2+cd^2\right)+}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (3*g*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(c^2*d^2*Sqrt[d + e*x]) + (3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqr
t[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*
x])])/(c^(5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{(3g)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{c^2d^2\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{3g\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{c^2d^2\sqrt{d+ex}}$$

Mathematica [C] time = 0.087196, size = 100, normalized size = 0.44

$$-\frac{2\sqrt{d+ex}(f+gx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (g*(a*
e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(
f + g*x))/(c*d*f - a*e*g))^(3/2))
```

Maple [B] time = 0.379, size = 396, normalized size = 1.7

$$-\frac{1}{(2cdx+2ae)c^2d^2}\left(3\ln\left(1/2\frac{2xcdg+aeg+cdf+2\sqrt{(cdx+ae)(gx+f)}\sqrt{cdg}}{\sqrt{cdg}}\right)xacdeg^2-3\ln\left(1/2\frac{2xcdg+aeg+cd}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -1/2*(3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x*a*c*d*e*g^2-3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*
d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c^2*d^2*f*g+3*ln(1/
2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*a^2*e^2*g^2-3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f
))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g-2*(c*d*g)^(1/2)*((c*d*x+
a*e)*(g*x+f))^(1/2)*x*c*d*g-6*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a*e
*g+4*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*c*d*f)*(c*d*e*x^2+a*e^2*x+c*
d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*x+a*e)/(c
*d*g)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

Fricas [A] time = 7.00522, size = 1544, normalized size = 6.8

$$\left[\frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdgx - 2cdf + 3aeg)\sqrt{ex+d}\sqrt{gx+f} - 3\left(acd^2ef - a^2de^2g + (c^2d^2ef - acde^2g)x^2 + \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*
a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^
2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)
*g)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*
f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4
*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4
*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(
c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x), 1/2*(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*a*e*g)*sqrt(e*x
+ d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^
2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)*sqrt(-g/(
c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*
f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c
^2*d^2*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.722 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[
a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.196042, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {866, 891, 63, 217, 206}

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[
a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{g\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{cd}$$

$$=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(g\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2g\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}\cdots}}\cdots\right)}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x\cdots}}$$

$$=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(2g\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\cdots\right)}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+c\cdots}}$$

$$=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{g}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+\cdots}}{\sqrt{c}\sqrt{d}\sqrt{f\cdots}}\right)}{c^{3/2}d^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.39543, size = 176, normalized size = 1.09

$$\frac{2\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{cdf-aeg}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)-(cd)^{3/2}(f+gx)\right)}{(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-((c*d)^(3/2)*(f + g*x)) + Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[c
*d*f - a*e*g]*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSi
nh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*
g])]))/((c*d)^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A] time = 0.389, size = 210, normalized size = 1.3

$$\frac{1}{d\left(cdx+ae\right)c}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\ln\left(\frac{1}{2}\left(2xcdg+aeg+cdf+2\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}\right)\frac{1}{\sqrt{cdg}}\right)xcd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] (g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*x*c*d*g+a*
e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c*d
*g+ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2
))/(c*d*g)^(1/2))*a*e*g-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g
)^(1/2)/(c*d*x+a*e)/((c*d*x+a*e)*(g*x+f))^(1/2)/d/c/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+d\right)^{\frac{3}{2}}\sqrt{gx+f}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^(3/2), x)
```

Fricas [A] time = 6.66954, size = 1227, normalized size = 7.62

$$\left[\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)\sqrt{\frac{g}{cd}}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+8\left(c^2d^2efg+\left(c^2d^3+acde^2\right)g^2\right)x^2+4\left(2c^2d^2gx+c^2d^2f+acdeg\right)\sqrt{cd}}{ex+d}\right.}{2\left(c^2d^2ex^2+acd^2e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2
*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f
*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*
e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f
)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c
*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^
3 + a*c*d*e^2)*x), -((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d))
*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*
x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (
2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
```

```
t(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2
)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.723 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.0692246, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.0316041, size = 50, normalized size = 0.82

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

Maple [A] time = 0.052, size = 63, normalized size = 1.

$$2\,{\frac {(cdx+ae)\sqrt {gx+f}\,(ex+d)^{3/2}}{\left(aeg-cdf\right)\left(cde{x}^{2}+a{e}^{2}x+c{d}^{2}x+ade\right)^{3/2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] 2*(c*d*x+a*e)*(g*x+f)^(1/2)/(a*e*g-c*d*f)*(e*x+d)^(3/2)/(c*d*e*x^2+a*e^2*x+
c*d^2*x+a*d*e)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sq
rt(g*x + f)), x)
```

Fricas [B] time = 1.67096, size = 262, normalized size = 4.3

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}}{acd^2ef-a^2de^2g+\left(c^2d^2ef-acde^2g\right)x^2+\left(\left(c^2d^3+acde^2\right)f-\left(acd^2e+a^2e^3\right)g\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/
(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 +
a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sq
rt(g*x + f)), x)
```

3.724 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{4g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c
*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rubi [A] time = 0.154125, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$-\frac{4g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c
*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(2g)\int \frac{}{(f+gx)^{3/2}}}{c}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{4g\sqrt{ade+(cd}}{(cdf-aeg)^2}$$

Mathematica [A] time = 0.0523168, size = 64, normalized size = 0.52

$$-\frac{2\sqrt{d+ex}(aeg+cd(f+2gx))}{\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(a*e*g + c*d*(f + 2*g*x)))/((c*d*f - a*e*g)^2*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A] time = 0.049, size = 97, normalized size = 0.8

$$-2\,\frac{\left(2\,xcdg+aeg+cdf\right)(cdx+ae)(ex+d)^{3/2}}{\sqrt{gx+f}\left(a^2e^2g^2-2\,acdefg+c^2d^2f^2\right)\left(cdex^2+ae^2x+cd^2x+ade\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2*(c*d*x+a*e)*(2*c*d*g*x+a*e*g+c*d*f)*(e*x+d)^(3/2)/(g*x+f)^(1/2)/(a^2*e^2
*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(3/2)), x)
```

Fricas [B] time = 1.75026, size = 648, normalized size = 5.23

$$-\frac{2\sqrt{cdex^2+ade+\left(cd^2+a\right.}}{ac^2d^3ef^3-2a^2cd^2e^2f^2g+a^3de^3fg^2+\left(c^3d^3ef^2g-2ac^2d^2e^2fg^2+a^2cde^3g^3\right)x^3+\left(c^3d^3ef^3+\left(c^3d^4-ac^2d^2e^2\right)f^2g-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(e*x + d)*sqrt(g*x + f)/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*
d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x
^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^
2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c
^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*
d^2*e^2 - a^3*e^4)*f*g^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(3/2)), x)
```

3.725 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf$$

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (8*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) - (16*c*d*g*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rubi [A] time = 0.247417, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {868, 872, 860}

$$-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```


```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (8*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) - (16*c*d*g*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```


Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(4g)\int \frac{\ }{(f}}{\ }$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\sqrt{ade}}{3(cdf-}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8g\sqrt{ade}}{3(cdf-}$$

Mathematica [A] time = 0.0756248, size = 105, normalized size = 0.55

$$-\frac{2\sqrt{d+ex}\left(-a^2e^2g^2+2acdeg(3f+2gx)+c^2d^2\left(3f^2+12fgx+8g^2x^2\right)\right)}{3(f+gx)^{3/2}\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(-(a^2*e^2*g^2) + 2*a*c*d*e*g*(3*f + 2*g*x) + c^2*d^2*(3*
f^2 + 12*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d +
e*x)]*(f + g*x)^(3/2))
```

Maple [A] time = 0.053, size = 168, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(-8\,c^2d^2g^2x^2-4\,acdeg^2x-12\,c^2d^2fgx+a^2e^2g^2-6\,acdefg-3\,c^2d^2f^2\right)}{3\,a^3e^3g^3-9\,a^2cde^2fg^2+9\,ac^2d^2ef^2g-3\,c^3d^3f^3}(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{-\frac{3}{2}}\left(cdex^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-8*c^2*d^2*g^2*x^2-4*a*c*d*e*g^2*x-12*c^2*d^2*f*g*x+a^2*e
^2*g^2-6*a*c*d*e*f*g-3*c^2*d^2*f^2)*(e*x+d)^(3/2)/(g*x+f)^(3/2)/(a^3*e^3*g^
3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2*x+c
*d^2*x+a*d*e)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(5/2)), x)
```

Fricas [B] time = 1.86617, size = 1256, normalized size = 6.54

$$-\frac{}{3\left(ac^3d^4ef^5-3\,a^2c^2d^3e^2f^4g+3\,a^3cd^2e^3f^3g^2-a^4de^4f^2g^3+\left(c^4d^4ef^3g^2-3\,ac^3d^3e^2f^2g^3+3\,a^2c^2d^2e^3fg^4-a^3cde^4g^5\right)x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*c^2*d^2*g^2*x^2 + 3*c^2*d^2*f^2 + 6*a*c*d*e*f*g - a^2*e^2*g^2 + 4*(
3*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g + 3
*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c^3*d
^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c^4*d^
4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^2*c^2*
d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*d^2*e^3
 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5 - a*c^3*
d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*a^2*c^2*d
^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g^4)*x^2 -
(2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*e + 3*a^2*c
^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 - (5*a^3*c*d^
2*e^3 - a^4*e^5)*f^2*g^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(5/2)), x)
```

3.726 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{32c^2d^2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}-\frac{12g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2}-\frac{}{(f+}$$

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (12*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(5*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) - (16*c*d*g*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g
*x)^(3/2)) - (32*c^2*d^2*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rubi [A] time = 0.329366, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {868, 872, 860}

$$-\frac{32c^2d^2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}-\frac{12g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2}-\frac{}{(f+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```


```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (12*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(5*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) - (16*c*d*g*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g
*x)^(3/2)) - (32*c^2*d^2*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```


Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
```

```
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{(6g)\int \cdots}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{12g\sqrt{ade \cdots}}{5(cdf - \cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{12g\sqrt{ade \cdots}}{5(cdf - \cdots}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{12g\sqrt{ade \cdots}}{5(cdf - \cdots}$$

Mathematica [A] time = 0.104442, size = 150, normalized size = 0.57

$$-\frac{2\sqrt{d+ex}\left(-a^2cde^2g^2(5f+2gx)+a^3e^3g^3+ac^2d^2eg\left(15f^2+20fgx+8g^2x^2\right)+c^3d^3\left(30f^2gx+5f^3+40fg^2x^2+16g \cdots\right.\right.}{5(f+gx)^{5/2}\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(5*f + 2*g*x) + a*c^2*d^2*
e*g*(15*f^2 + 20*f*g*x + 8*g^2*x^2) + c^3*d^3*(5*f^3 + 30*f^2*g*x + 40*f*g^
2*x^2 + 16*g^3*x^3)))/(5*(c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f
 + g*x)^(5/2))
```

Maple [A] time = 0.055, size = 259, normalized size = 1.

$$-\frac{(2cdx+2ae)\left(16c^3d^3g^3x^3+8ac^2d^2eg^3x^2+40c^3d^3fg^2x^2-2a^2cde^2g^3x+20ac^2d^2efg^2x+30c^3d^3f^2gx+a^3e^3g^3- \cdots\right.}{5g^4e^4a^4-20cdg^3fe^3a^3+30c^2d^2g^2f^2e^2a^2-20c^3d^3gf^3ea+5c^4d^4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] -2/5*(c*d*x+a*e)*(16*c^3*d^3*g^3*x^3+8*a*c^2*d^2*e*g^3*x^2+40*c^3*d^3*f*g^2
*x^2-2*a^2*c*d*e^2*g^3*x+20*a*c^2*d^2*e*f*g^2*x+30*c^3*d^3*f^2*g*x+a^3*e^3*
g^3-5*a^2*c*d*e^2*f*g^2+15*a*c^2*d^2*e*f^2*g+5*c^3*d^3*f^3)*(e*x+d)^(3/2)/(
g*x+f)^(5/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a
*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} (gx+f)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(7/2)), x)
```

Fricas [B] time = 2.17315, size = 2063, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*c^3*d^3*g^3*x^3 + 5*c^3*d^3*f^3 + 15*a*c^2*d^2*e*f^2*g - 5*a^2*c*d
*e^2*f*g^2 + a^3*e^3*g^3 + 8*(5*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 2*(1
5*c^3*d^3*f^2*g + 10*a*c^2*d^2*e*f*g^2 - a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^4*d^5*e*f^7
- 4*a^2*c^3*d^4*e^2*f^6*g + 6*a^3*c^2*d^3*e^3*f^5*g^2 - 4*a^4*c*d^2*e^4*f^4
*g^3 + a^5*d*e^5*f^3*g^4 + (c^5*d^5*e*f^4*g^3 - 4*a*c^4*d^4*e^2*f^3*g^4 + 6
*a^2*c^3*d^3*e^3*f^2*g^5 - 4*a^3*c^2*d^2*e^4*f*g^6 + a^4*c*d*e^5*g^7)*x^5 +
 (3*c^5*d^5*e*f^5*g^2 + (c^5*d^6 - 11*a*c^4*d^4*e^2)*f^4*g^3 - 2*(2*a*c^4*d
^5*e - 7*a^2*c^3*d^3*e^3)*f^3*g^4 + 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f
^2*g^5 - (4*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^6 + (a^4*c*d^2*e^4 + a^5*e^6
)*g^7)*x^4 + (3*c^5*d^5*e*f^6*g + a^5*d*e^5*g^7 + 3*(c^5*d^6 - 3*a*c^4*d^4*
e^2)*f^5*g^2 - (11*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^4*g^3 + 2*(7*a^2*c^3*
d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g^4 - 3*(2*a^3*c^2*d^3*e^3 + 3*a^4*c*d*e^5
)*f^2*g^5 - (a^4*c*d^2*e^4 - 3*a^5*e^6)*f*g^6)*x^3 + (c^5*d^5*e*f^7 + 3*a^5
*d*e^5*f*g^6 + (3*c^5*d^6 - a*c^4*d^4*e^2)*f^6*g - 3*(3*a*c^4*d^5*e + 2*a^2
*c^3*d^3*e^3)*f^5*g^2 + 2*(3*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4)*f^4*g^3 +
 (6*a^3*c^2*d^3*e^3 - 11*a^4*c*d*e^5)*f^3*g^4 - 3*(3*a^4*c*d^2*e^4 - a^5*e^
6)*f^2*g^5)*x^2 + (3*a^5*d*e^5*f^2*g^5 + (c^5*d^6 + a*c^4*d^4*e^2)*f^7 - (a
*c^4*d^5*e + 4*a^2*c^3*d^3*e^3)*f^6*g - 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^
4)*f^5*g^2 + 2*(7*a^3*c^2*d^3*e^3 - 2*a^4*c*d*e^5)*f^4*g^3 - (11*a^4*c*d^2*
e^4 - a^5*e^6)*f^3*g^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(7/2)), x)
```

3.727 $\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=289

$$\frac{5g^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}}+\frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{10g\sqrt{d+e\ldots}}{3c^2d^2\sqrt{x\left(ae^2+c\ldots\right.}}$$

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (10*g*Sqrt[d + e*x]*(f + g*x)^(3/2))/(3*c^2*d^2*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*Sqrt[d + e*x]) + (5*g^(3/2)*(c*d*f -
a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/
(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(7/2)*d^(7/2)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.431764, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {866, 870, 891, 63, 217, 206}

$$\frac{5g^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}}+\frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{10g\sqrt{d+e\ldots}}{3c^2d^2\sqrt{x\left(ae^2+c\ldots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (10*g*Sqrt[d + e*x]*(f + g*x)^(3/2))/(3*c^2*d^2*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*Sqrt[d + e*x]) + (5*g^(3/2)*(c*d*f -
a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/
(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(7/2)*d^(7/2)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
```

```
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{(5g)\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{3cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \cdots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \cdots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \cdots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \cdots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \cdots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \cdots$$

Mathematica [C] time = 0.150803, size = 102, normalized size = 0.35

$$-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2)*Hypergeometric2F1[-5/2, -3/2, -1/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*
((c*d*(f + g*x))/(c*d*f - a*e*g))^(5/2))
```

Maple [B] time = 0.383, size = 652, normalized size = 2.3

$$-\frac{1}{6\,(cdx+ae)^2\,c^3d^3}\left(15\,\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}}{\sqrt{cdg}}\right)x^2ac^2d^2eg^3-15\,\ln\left(1/2\,\frac{2\,xcdg+aeg}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -1/6*(15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*x^2*a*c^2*d^2*e*g^3-15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*
f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x^2*c^3*d^3*f
*g^2+30*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x*a^2*c*d*e^2*g^3-30*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2
*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*a*c^2*d^2*e*f*
g^2+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*
x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2-6*x^2
*c^2*d^2*g^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-40*(c*d*g)^(1/2)*((c
*d*x+a*e)*(g*x+f))^(1/2)*x*a*c*d*e*g^2+28*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f
))^(1/2)*x*c^2*d^2*f*g-30*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2
*g^2+20*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+4*((c*d*x+a*e
)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(1/2)*(g*x+f)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/(c*d*x+a*e
)^2/c^3/d^3/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

Fricas [A] time = 7.14982, size = 2183, normalized size = 7.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*g
^2 - 2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*e^3
*g^2 + (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^
2)*f*g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^2*c*d
*e^3)*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d
^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e
*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*
d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a
*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 +
(c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x), 1/6
```

```
*(2*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*g^2 -
2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*e^3*g^2
+ (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*
g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^2*c*d*e^3)
*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g
/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g
)*x)))/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 +
 (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.728 $\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{2g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}$$

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (2*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(c^2*d^2*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(
5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.288746, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {866, 891, 63, 217, 206}

$$\frac{2g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (2*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(c^2*d^2*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(
5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a
 + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e*g*n)/(c*(p + 1)), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{g\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx}{cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \ldots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \ldots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \ldots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \ldots$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \ldots$$

Mathematica [C] time = 0.109813, size = 102, normalized size = 0.47

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2)*Hypergeometric2F1[-3/2, -3/2, -1/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*
((c*d*(f + g*x))/(c*d*f - a*e*g))^(3/2))
```

Maple [A] time = 0.388, size = 343, normalized size = 1.6

$$\frac{1}{3\,(cdx+ae)^2\,d^2c^2}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\,\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\,(gx+f)}\sqrt{cdg}}{\sqrt{cdg}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 1/3*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*x*c*
d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
*x^2*c^2*d^2*g^2+6*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*a*c*d*e*g^2+3*ln(1/2*(2*x*c*d*g+a*e*g+c
*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*e^2*g^
2-8*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*c*d*g-6*(c*d*g)^(1/2)*((c*d
*x+a*e)*(g*x+f))^(1/2)*a*e*g-2*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*c*
d*f)/(c*d*g)^(1/2)/(c*d*x+a*e)^2/((c*d*x+a*e)*(g*x+f))^(1/2)/d^2/c^2/(e*x+d
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

Fricas [A] time = 6.89768, size = 1607, normalized size = 7.34

$$\left[-\frac{4\,\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(4\,cdgx+cdf+3\,aeg\right)\sqrt{ex+d}\sqrt{gx+f}-3\,\left(c^2d^2egx^3+a^2de^2g+\left(c^2d^3+2\,acde^2\right)gx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*d*f + 3
*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c
^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(g/(c*d))*lo
g(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8
*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d
^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)
*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)
*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^3 + a^2*c^2
*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e
^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*
d*f + 3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2
*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(
c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*
f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 +
 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.729 $\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))
```

Rubi [A] time = 0.0663038, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A] time = 0.0334774, size = 52, normalized size = 0.83

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*((a*e + c*d*x)*(d +
 e*x))^(3/2))
```

Maple [A] time = 0.053, size = 63, normalized size = 1.

$$\frac{2\,cdx+2\,ae}{3\,aeg-3\,cdf}\left(gx+f\right)^{\frac{3}{2}}\left(ex+d\right)^{\frac{5}{2}}\left(cdex^{2}+ae^{2}x+cd^{2}x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(g*x+f)^(3/2)/(a*e*g-c*d*f)*(e*x+d)^(5/2)/(c*d*e*x^2+a*e^2*
x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex+d\right)^{\frac{5}{2}}\sqrt{gx+f}}{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^(5/2), x)
```

Fricas [B] time = 1.72344, size = 394, normalized size = 6.25

$$-\frac{2\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}\left(gx+f\right)^{\frac{3}{2}}}{3\left(a^{2}cd^{2}e^{2}f-a^{3}de^{3}g+\left(c^{3}d^{3}ef-ac^{2}d^{2}e^{2}g\right)x^{3}+\left(\left(c^{3}d^{4}+2\,ac^{2}d^{2}e^{2}\right)f-\left(ac^{2}d^{3}e+2\,a^{2}cde^{3}\right)g\right)x^{2}+\left(\left(2\,ac^{2}d^{3}e+a^{2}cde^{3}\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f)^(3
/2)/(a^2*c*d^2*e^2*f - a^3*d*e^3*g + (c^3*d^3*e*f - a*c^2*d^2*e^2*g)*x^3 +
((c^3*d^4 + 2*a*c^2*d^2*e^2)*f - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g)*x^2 + ((2
*a*c^2*d^3*e + a^2*c*d*e^3)*f - (2*a^2*c*d^2*e^2 + a^3*e^4)*g)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.730 $\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[Out] (-2*(d + e*x)^(3/2)*Sqrt[f + g*x])/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(3*(c*d*f - a
*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.141121, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*Sqrt[f + g*x])/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(3*(c*d*f - a
*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx = -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(2g)\int \frac{(}{\sqrt{f+gx}\left(ade+\left(\right.\right.}}{3(cdf}$$

$$= -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{4g\sqrt{d}}{3(cdf-aeg)^2\sqrt{ac}}$$

Mathematica [A] time = 0.0595597, size = 68, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}\sqrt{f+gx}(3aeg-cd(f-2gx))}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*Sqrt[f + g*x]*(3*a*e*g - c*d*(f - 2*g*x)))/(3*(c*d*f - a
*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A] time = 0.049, size = 99, normalized size = 0.8

$$\frac{(2\,cdx+2\,ae)\left(2\,xcdg+3\,aeg-cdf\right)}{3\,a^2e^2g^2-6\,acdefg+3\,c^2d^2f^2}\sqrt{gx+f}\,(ex+d)^{\frac{5}{2}}\left(cdex^2+ae^2x+cd^2x+ade\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] 2/3*(c*d*x+a*e)*(g*x+f)^(1/2)*(2*c*d*g*x+3*a*e*g-c*d*f)*(e*x+d)^(5/2)/(a^2*
e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sq
rt(g*x + f)), x)
```

Fricas [B] time = 1.77093, size = 641, normalized size = 5.01

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(2}{3\left(a^2c^2d^3e^2f^2 - 2a^3cd^2e^3fg + a^4de^4g^2 + \left(c^4d^4ef^2 - 2ac^3d^3e^2fg + a^2c^2d^2e^3g^2\right)x^3 + \left(\left(c^4d^5 + 2ac^3d^3e^2\right)f^2 - 2\left(ac^3d^4e + 2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - c*d*f + 3*a*e*
g)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g +
 a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2
)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e
^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^
2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g + (2*a^3*c*d^2
*e^3 + a^4*e^5)*g^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sq
rt(g*x + f)), x)
```

3.731 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}+\frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d}{3\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)\right.}$$

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (8*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*Sqrt[f
 + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rubi [A] time = 0.219604, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {868, 860}

$$\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3}+\frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d}{3\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (8*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*Sqrt[f
 + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(4g)\int \ldots}{\ldots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{3(cdf-a\ldots}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{\ldots}{3(cdf-a\ldots}$$

Mathematica [A] time = 0.0780404, size = 103, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}\left(3a^2e^2g^2+6acdeg(f+2gx)+c^2d^2\left(-f^2+4fgx+8g^2x^2\right)\right)}{3\sqrt{f+gx}((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(3*a^2*e^2*g^2 + 6*a*c*d*e*g*(f + 2*g*x) + c^2*d^2*(-f^2
 + 4*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^(3
/2)*Sqrt[f + g*x])
```

Maple [A] time = 0.052, size = 169, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2g^2x^2+12\,acdeg^2x+4\,c^2d^2fgx+3\,a^2e^2g^2+6\,acdefg-c^2d^2f^2\right)}{3\,a^3e^3g^3-9\,a^2cde^2fg^2+9\,ac^2d^2ef^2g-3\,c^3d^3f^3}(ex+d)^{\frac{5}{2}}\frac{1}{\sqrt{gx+f}}\left(cdex^2+ae^2x+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2+12*a*c*d*e*g^2*x+4*c^2*d^2*f*g*x+3*a^2*
e^2*g^2+6*a*c*d*e*f*g-c^2*d^2*f^2)*(e*x+d)^(5/2)/(g*x+f)^(1/2)/(a^3*e^3*g^3
-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c*d*e*x^2+a*e^2*x+c*
d^2*x+a*d*e)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(3/2)), x)
```

Fricas [B] time = 1.91612, size = 1281, normalized size = 6.6

$$\frac{\cdots}{3\left(a^2c^3d^4e^2f^4 - 3\,a^3c^2d^3e^3f^3g + 3\,a^4cd^2e^4f^2g^2 - a^5de^5fg^3 + \left(c^5d^5ef^3g - 3\,ac^4d^4e^2f^2g^2 + 3\,a^2c^3d^3e^3fg^3 - a^3c^2d^2e^4g^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 + 6*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 4*(c
^2*d^2*f*g + 3*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^3*d^4*e^2*f^4 - 3*a^3*c^2*d^3*e^3*f^3*g
+ 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^5*e*f^3*g - 3*a*c^4*d^
4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2*e^4*g^4)*x^4 + (c^5*d
^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4*d^5*e + a^2*c^3*d^3*e
^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)*f*g^3 - (a^3*c^2*d^3*
e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d^4*e^2)*f^4 - (a*c^4*d
^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f^2
*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a^4*c*d^2*e^4 + a^5*e^6
)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^4 + (5*a^
2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2*d^3*e^3 + a^4*c*d*e^5
)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(3/2)), x)
```

3.732 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=260

$$\frac{32cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}+\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cd}$$

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*(f + g
*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f +
g*x)^(3/2)) + (32*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(
c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rubi [A] time = 0.312868, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {868, 872, 860}

$$\frac{32cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}+\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*(f + g
*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f +
g*x)^(3/2)) + (32*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(
c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 868

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(c*e*f + c*d*g - b*e*g)), x] + Di
st[(e^2*g*(m - n - 2))/((p + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
```

```
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(2g)}{}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf}$$

Mathematica [A] time = 0.113361, size = 152, normalized size = 0.58

$$\frac{2(d+ex)^{3/2}\left(3a^2cde^2g^2(3f+2gx)-a^3e^3g^3+3ac^2d^2eg\left(3f^2+12fgx+8g^2x^2\right)+c^3d^3\left(6f^2gx-f^3+24fg^2x^2+16g^3x\right.\right.}{3(f+gx)^{3/2}((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-(a^3*e^3*g^3) + 3*a^2*c*d*e^2*g^2*(3*f + 2*g*x) + 3*a*
c^2*d^2*e*g*(3*f^2 + 12*f*g*x + 8*g^2*x^2) + c^3*d^3*(-f^3 + 6*f^2*g*x + 24
*f*g^2*x^2 + 16*g^3*x^3)))/(3*(c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x))^(
3/2)*(f + g*x)^(3/2))
```

Maple [A] time = 0.054, size = 258, normalized size = 1.

$$-\frac{(2cdx+2ae)\left(-16c^3d^3g^3x^3-24ac^2d^2eg^3x^2-24c^3d^3fg^2x^2-6a^2cde^2g^3x-36ac^2d^2efg^2x-6c^3d^3f^2gx+a^3e^3g^3-\right.}{3g^4e^4a^4-12cdg^3fe^3a^3+18c^2d^2g^2f^2e^2a^2-12c^3d^3gf^3ea+3c^4d^4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3-24*a*c^2*d^2*e*g^3*x^2-24*c^3*d^3*f*g
^2*x^2-6*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2*x-6*c^3*d^3*f^2*g*x+a^3*e^3
*g^3-9*a^2*c*d*e^2*f*g^2-9*a*c^2*d^2*e*f^2*g+c^3*d^3*f^3)*(e*x+d)^(5/2)/(g*
x+f)^(3/2)/(a^4*e^4*g^4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c
^3*d^3*e*f^3*g+c^4*d^4*f^4)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(5/2)), x)
```

Fricas [B] time = 2.07937, size = 2051, normalized size = 7.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*c^3*d^3*g^3*x^3 - c^3*d^3*f^3 + 9*a*c^2*d^2*e*f^2*g + 9*a^2*c*d*e^2
*f*g^2 - a^3*e^3*g^3 + 24*(c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 6*(c^3*d^
3*f^2*g + 6*a*c^2*d^2*e*f*g^2 + a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^4*d^5*e^2*f^6 - 4*a
^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3
+ a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*
c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 +
(2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e -
 a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g
^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2
*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e
^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e
^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f
^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g -
2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*
c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d
^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a
^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e
^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f
^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6
*e^7)*f^2*g^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(5/2)), x)
```

3.733 $\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=385

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}}-\frac{5(f+g}{}$$

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c^3*d^3*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*c^2*d^2*g*Sqrt[d + e*x]) +
(((a*e)/(c*d) - f/g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(24*Sqrt[d + e*x]) + ((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(4*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rubi [A] time = 0.718058, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}}-\frac{5(f+g}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```


```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c^3*d^3*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*c^2*d^2*g*Sqrt[d + e*x]) +
(((a*e)/(c*d) - f/g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(24*Sqrt[d + e*x]) + ((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(4*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```


Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}dx = \frac{(f+gx)^{7/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx}{\sqrt{ade+\left(cd^2+ae^2\right)x}}}{8g}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24\sqrt{d+ex}} + \frac{(f+gx)^{7/2}\sqrt{ade+}}{4g\sqrt{}}$$

$$= -\frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{96c^2d^2g\sqrt{d+ex}} + \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}} - \frac{5(cdf-aeg)^2(}{}$$

Mathematica [A] time = 1.20928, size = 300, normalized size = 0.78

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(-5a^2cde^2g^2(11f+2gx)+15a^3e^3g^3+ac^2d^2eg\left(73f^2+36fgx+8g^2x^2\right)+\right.\right.}{192c^{9/2}d^{9/2}g^{3/2}\sqrt{f+gx}\sqrt{(d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)*(
f + g*x)*(15*a^3*e^3*g^3 - 5*a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g
*(73*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(15*f^3 + 118*f^2*g*x + 136*f*g^
2*x^2 + 48*g^3*x^3)) - 15*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d
*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*
d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(192*c^(9/2)*d^(9/2)*g^(3/2)*Sqrt[
(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [B] time = 0.338, size = 870, normalized size = 2.3

$$-\frac{1}{384\,c^3d^3g}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(-96\,x^3c^3d^3g^3\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}+15\,\ln\left(1/2\,\frac{2\,xcdg+}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/384*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-96*x^3*c^3*d
^3*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*x
*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))
/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a
*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3
+90*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)
*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-60*ln(1/2*(2*x*c*d*g
+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a
*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-16*x^
2*a*c^2*d^2*e*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)-272
*x^2*c^3*d^3*f*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+20
*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*c*d*e^2*g^3-72
*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c^2*d^2*e*f*g^2-
236*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^3*d^3*f^2*g-3
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3+110*(c*
d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2-146*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-30*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1
/2)/g/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/c^3/d^3/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(gx + f\right)^{\frac{5}{2}}}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(5/2)/sqrt(
e*x + d), x)
```

Fricas [A] time = 15.9162, size = 2244, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d^3*e*f^2*g^2 -
 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^4*f*g^3 + a*c^
3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g^3 - 5*a^2*c^2
*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f
^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d
```

```
^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^
4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
 + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*g^2*x +
 c^4*d^5*g^2), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d
^3*e*f^2*g^2 - 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^
4*f*g^3 + a*c^3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g
^3 - 5*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2
*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f
^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^
3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*
f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*g^2*x + c^4*d
^5*g^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^{\frac{5}{2}}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(5/2)/sqrt(
e*x + d), x)
```

3.734 $\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=313

$$-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8c^2d^2g\sqrt{d+ex}}+\frac{(f+gx}{}$$

```
[Out] -((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(8*c^2*d^2*g*Sqrt[d + e*x]) + (((a*e)/(c*d) - f/g)*(f + g*x)^(3/2)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*Sqrt[d + e*x]) + ((f + g*x)^(5
/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x]) - ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(5/2)*d^(5/2)*g^(3/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.51782, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8c^2d^2g\sqrt{d+ex}}+\frac{(f+gx}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] -((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(8*c^2*d^2*g*Sqrt[d + e*x]) + (((a*e)/(c*d) - f/g)*(f + g*x)^(3/2)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*Sqrt[d + e*x]) + ((f + g*x)^(5
/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x]) - ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(5/2)*d^(5/2)*g^(3/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}(f}{\sqrt{ade+(cd^2+}}}{6g}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\sqrt{ad}}{3}$$

$$=-\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}+\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f-}{}$$

$$=-\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}+\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f-}{}$$

$$=-\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}+\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f-}{}$$

$$=-\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}+\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f-}{}$$

$$=-\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}+\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f-}{}$$

Mathematica [A] time = 0.874469, size = 255, normalized size = 0.81

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(-\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(3a^2e^2g^2-2acdeg(4f+gx)-c^2d^2\left(3f^2+14fgx+8g^2x^2\right)\right)-3\sqrt{ae+}\right.}{24c^{7/2}d^{7/2}g^{3/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)
*(f + g*x)*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(4*f + g*x) - c^2*d^2*(3*f^2 + 14*f
*g*x + 8*g^2*x^2))) - 3*(c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(
f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*
x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(24*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[(a*
e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [B] time = 0.328, size = 602, normalized size = 1.9

$$\frac{1}{48\,c^2d^2g}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}}{\sqrt{cdg}}\right)a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*x*c
*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^3*e^3*g^3-9*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*
g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*
ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*
d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*
f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c
^3*d^3*f^3+16*x^2*c^2*d^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*
g)^(1/2)+4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c*d*e*
g^2+28*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^2*d^2*f*g-
6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*e^2*g^2+16*a*c*
d*e*f*g*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+6*(c*d*g)^(1/
2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/g/(c*
d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/c^2/d^2/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\left(gx + f\right)^{\frac{3}{2}}}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(
e*x + d), x)
```

Fricas [A] time = 9.43747, size = 1790, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^2
*c*d*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a
*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 -
 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*d*g)*
log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 +
 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2), 1/4
8*(2*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^2*c*d
*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^3*d^4*f^3 - 3*a*c^2
*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 - 3*a
```

```
*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*d*g)*arc
tan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d
)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
a*e^2)*g)*x)))/(c^3*d^3*e*g^2*x + c^3*d^4*g^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^{\frac{3}{2}}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(
e*x + d), x)
```

3.735 $\int \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+c\ldots\right.}}{4\ldots}$$

```
[Out] (((a*e)/(c*d) - f/g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(4*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(2*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt
[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])])/(4*c^(3/2)*d^(3/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)
```

Rubi [A] time = 0.352413, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+c\ldots\right.}}{4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*
x],x]
```

```
[Out] (((a*e)/(c*d) - f/g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(4*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(2*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt
[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])])/(4*c^(3/2)*d^(3/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
```

```
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f\cdots}}{\sqrt{ade+\left(cd^2+ae^2\cdots\right.}}}{4g}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\cdots}}{2g\sqrt{\cdots}}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\cdots}}{2g\sqrt{\cdots}}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\cdots}}{2g\sqrt{\cdots}}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\cdots}}{2g\sqrt{\cdots}}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+\cdots}}{2g\sqrt{\cdots}}$$

Mathematica [A] time = 0.609267, size = 215, normalized size = 0.89

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)(aeg+cd(f+2gx))-\sqrt{ae+cdx}(cdf-aeg)^{5/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{}}{\sqrt{cd}\sqrt{cd}}\right.\right.}{4g^{3/2}(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[
d + e*x],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*
d*x)*(f + g*x)*(a*e*g + c*d*(f + 2*g*x)) - (c*d*f - a*e*g)^(5/2)*Sqrt[a*e +
 c*d*x]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt
[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(4*(c*d)^(5/2)*g^
(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A] time = 0.323, size = 385, normalized size = 1.6

$$-\frac{1}{8\,cdg}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\ln\left(\frac{1}{2}\left(2\,xcdg+aeg+cdf+2\,\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}\right)\frac{1}{\sqrt{cdg}}\right)a^{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*x*c*d
*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))*a^2*e^2*g^2-2*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*
x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+ln(1/2*(2*
x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)
)/(c*d*g)^(1/2))*c^2*d^2*f^2-4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e
*f)^(1/2)*x*c*d*g-2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a
*e*g-2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c*d*f)/(e*x+d)
^(1/2)/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/c/d/g/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{gx+f}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e*
x + d), x)
```

Fricas [A] time = 7.31236, size = 1427, normalized size = 5.92

$$\left[\frac{4\left(2\,c^2d^2g^2x + c^2d^2fg + acdeg^2\right)\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f} + \left(c^2d^3f^2 - 2\,acd^2efg + a^2de^2g^2 + \left(c\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f^2 - 2*a*c*
d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)
*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g +
 a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x +
 c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g
+ (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*
e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^2*x + c
^2*d^3*g^2), 1/8*(2*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f
^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a
^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a
*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^2*x + c^2*d^3*g^
2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{f+gx}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)/sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{gx+f}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e*
x + d), x)
```

3.736 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=167

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x
]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.209438, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {864, 891, 63, 217, 206}

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f + g*x
]),x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x
]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}}{2g}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{}{\sqrt{a}}}{2g\sqrt{ade+\left(cd^2+ae^2\right)x+c}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subs}}{cdg\sqrt{ade+\left(cd^2+\right.}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{\left((cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subs}}{cdg\sqrt{ade+\left(cd^2+ae^2\right)}}$$

$$= \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x}}$$

Mathematica [A] time = 0.835665, size = 173, normalized size = 1.04

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}(f+gx)-\frac{\sqrt{c}\sqrt{d}(cdf-aeg)^{3/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f
 + g*x]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x) - (Sqrt[c]*Sqrt[d]*(c*d*f
 - a*e*g)^(3/2)*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt
[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])/((c*d)^(3/
```

```
2)*Sqrt[a*e + c*d*x]))/(g^(3/2)*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Maple [A] time = 0.338, size = 198, normalized size = 1.2

$$\frac{1}{2g}\sqrt{cdex^2+ae^2x+cd^2x+ade}\sqrt{gx+f}\left(\ln\left(\frac{1}{2}\left(2xcdg+aeg+cdf+2\sqrt{(cdx+ae)(gx+f)}\sqrt{cdg}\right)\frac{1}{\sqrt{cdg}}\right)aeg-\ln\left(\frac{1}{2}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] 1/2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)/(e*x+d)^(1/2)*(ln
(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c
*d*g)^(1/2))*a*e*g-ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*
g)^(1/2))/((c*d*x+a*e)*(g*x+f))^(1/2)/g/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*sqrt(g
*x + f)), x)
```

Fricas [A] time = 6.83417, size = 1150, normalized size = 6.89

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}cdg-\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{cdg}\log\left(-\frac{8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2e}{}\right.}{4\left(cdeg\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(-(
8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c
*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2
)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*
e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^2*x + c*d^2*g^2), 1/2*(2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d*g + (c*
```

```
d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2
*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*
d*e*g^2*x + c*d^2*g^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*sqrt(g
*x + f)), x)
```

3.737 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] (-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f +
g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.188197, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f +
g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}+\frac{(cd)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{g}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}+\frac{\left(cd\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}\,dx}{g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}+\frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}} \ldots\right.}{g\sqrt{ade+\left(cd^2+ae^2\right)x+cd \ldots}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}+\frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx, x, \ldots\right.}{g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+c \ldots}}{\sqrt{c}\sqrt{d}\sqrt{f+ \ldots}}\right.}{g^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.801488, size = 169, normalized size = 1.07

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{cdf-aeg}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\sqrt{ae+cdx}}-\sqrt{g}\right)}{g^{3/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-Sqrt[g] + (Sqrt[c]*Sqrt[d]*Sqrt[c*d*f -
a*e*g]*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]])/(Sqrt[c*d]*Sqrt[a*e
 + c*d*x])))/(g^(3/2)*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Maple [A] time = 0.348, size = 197, normalized size = 1.3

$$\frac{1}{g}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\ln\left(\frac{1}{2}\left(2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\,(gx+f)}\sqrt{cdg}\right)\frac{1}{\sqrt{cdg}}\right)xcdg+\ln\left(\frac{1}{2}\left(2\,xcdg+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/2),x)
```

```
[Out] (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((
c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c*d*g+ln(1/2*(2*x
*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))*c*d*f-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/((c*d*
x+a*e)*(g*x+f))^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(3/2)), x)
```

Fricas [A] time = 6.66846, size = 1146, normalized size = 7.25

$$\left[\frac{\left(egx^2+df+\left(ef+dg\right)x\right)\sqrt{\frac{cd}{g}}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2efg+a^2de^2g^2+4\left(2\,cdg^2x+cdfg+aeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{\frac{cd}{g}}}{ex+d}\right.}{2\left(eg^2x^2+dfg+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((e*g*x^2 + d*f + (e*f + d*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3
 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g
 + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
(c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g + (e*f*g + d*g^2)*x), -((e*g*x^2 +
d*f + (e*f + d*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
```

```
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d
^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g +
 (e*f*g + d*g^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(3/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(3/2)), x)
```

3.738 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(3/2))
```

Rubi [A] time = 0.064531, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(5
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A] time = 0.0343866, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(5/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f +
 g*x)^(3/2))
```

Maple [A] time = 0.05, size = 63, normalized size = 1.

$$-\frac{2\,cdx+2\,ae}{3\,aeg-3\,cdf}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\left(gx+f\right)^{-\frac{3}{2}}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)/(g*x+f)^(3/2)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(1/2)/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(5/2)), x)
```

Fricas [B] time = 1.91476, size = 360, normalized size = 5.71

$$\frac{2\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}(cdx+ae)\sqrt{ex+d}\sqrt{gx+f}}{3\left(cd^{2}f^{3}-adef^{2}g+\left(cdefg^{2}-ae^{2}g^{3}\right)x^{3}+\left(2\,cdef^{2}g-adeg^{3}+\left(cd^{2}-2\,ae^{2}\right)fg^{2}\right)x^{2}+\left(cdef^{3}-2\,adefg^{2}+\left(2\,cd^{2}-ae\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d)
*sqrt(g*x + f)/(c*d^2*f^3 - a*d*e*f^2*g + (c*d*e*f*g^2 - a*e^2*g^3)*x^3 + (
2*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 - 2*a*e^2)*f*g^2)*x^2 + (c*d*e*f^3 - 2*a
*d*e*f*g^2 + (2*c*d^2 - a*e^2)*f^2*g)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(5/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(5/2)), x)
```

3.739 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rubi [A] time = 0.138755, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(7
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{(2cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}dx}{5(cdf-aeg)}$$
$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A] time = 0.0597125, size = 69, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{3/2}(cd(5f+2gx)-3aeg)}{15(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-3*a*e*g + c*d*(5*f + 2*g*x)))/(15*(c*d
*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2))
```

Maple [A] time = 0.052, size = 99, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-2\,xcdg+3\,aeg-5\,cdf\right)}{15\,a^2e^2g^2-30\,acdefg+15\,c^2d^2f^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(gx+f\right)^{-\frac{5}{2}}\frac{1}{\sqrt{ex+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/15*(c*d*x+a*e)*(-2*c*d*g*x+3*a*e*g-5*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/(g*x+f)^(5/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(7/2)), x)
```

Fricas [B] time = 2.08362, size = 813, normalized size = 6.3

$$\frac{2\left(2c^2d^2gx^2+\right.}{15\left(c^2d^3f^5-2acd^2ef^4g+a^2de^2f^3g^2+\left(c^2d^2ef^2g^3-2acde^2fg^4+a^2e^3g^5\right)x^4+\left(3c^2d^2ef^3g^2+a^2de^2g^5+\left(c^2d^3-6ac\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 3*a^2*e^2*g + (5*c^2*d^2*f - a*c*d*e*
g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f)/(c^2*d^3*f^5 - 2*a*c*d^2*e*f^4*g + a^2*d*e^2*f^3*g^2 + (c^2*d^2*e*f^2*g^
3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^4 + (3*c^2*d^2*e*f^3*g^2 + a^2*d*e^2
*g^5 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^3 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^4)*x
^3 + 3*(c^2*d^2*e*f^4*g + a^2*d*e^2*f*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^2
 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^3)*x^2 + (c^2*d^2*e*f^5 + 3*a^2*d*e^2*f^2*
g^3 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g - (6*a*c*d^2*e - a^2*e^3)*f^3*g^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(7/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(7/2)), x)
```

3.740 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(7/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(3/2))
```

Rubi [A] time = 0.219924, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(9
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(7/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{(4cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}dx}{7(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{(8}{}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{16}{1}$$

Mathematica [A] time = 0.105384, size = 105, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(15a^2e^2g^2-6acdeg(7f+2gx)+c^2d^2\left(35f^2+28fgx+8g^2x^2\right)\right)}{105(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(15*a^2*e^2*g^2 - 6*a*c*d*e*g*(7*f + 2*g
*x) + c^2*d^2*(35*f^2 + 28*f*g*x + 8*g^2*x^2)))/(105*(c*d*f - a*e*g)^3*(d +
 e*x)^(3/2)*(f + g*x)^(7/2))
```

Maple [A] time = 0.053, size = 169, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2g^2x^2-12\,acdeg^2x+28\,c^2d^2fgx+15\,a^2e^2g^2-42\,acdefg+35\,c^2d^2f^2\right)}{105\,a^3e^3g^3-315\,a^2cde^2fg^2+315\,ac^2d^2ef^2g-105\,c^3d^3f^3}\sqrt{cdex^2+ae^2x+cd^2x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/105*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+28*c^2*d^2*f*g*x+15*
a^2*e^2*g^2-42*a*c*d*e*f*g+35*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(1/2)/(g*x+f)^(7/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-
c^3*d^3*f^3)/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(9/2)), x)
```

Fricas [B] time = 2.09092, size = 1490, normalized size = 7.53

$$\frac{}{105\left(c^3d^4f^7 - 3\,ac^2d^3ef^6g + 3\,a^2cd^2e^2f^5g^2 - a^3de^3f^4g^3 + \left(c^3d^3ef^3g^4 - 3\,ac^2d^2e^2f^2g^5 + 3\,a^2cde^3fg^6 - a^3e^4g^7\right)x^5 + \left(4\,c^3d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 42*a^2*c*d*e^2*f*g + 15*a^3
*e^3*g^2 + 4*(7*c^3*d^3*f*g - a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 - 14*a
*c^2*d^2*e*f*g + 3*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^7 - 3*a*c^2*d^3*e*f^6*g + 3*
a^2*c*d^2*e^2*f^5*g^2 - a^3*d*e^3*f^4*g^3 + (c^3*d^3*e*f^3*g^4 - 3*a*c^2*d^
2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^5 + (4*c^3*d^3*e*f^4*g
^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e
- 4*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^6)*x^4 + 2*(3*
c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g
^3 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^4 + 3*(2*a^2*c*d^2*e^2 - a^3*e
^4)*f^2*g^5)*x^3 + 2*(2*c^3*d^3*e*f^6*g - 3*a^3*d*e^3*f^2*g^5 + 3*(c^3*d^4
- 2*a*c^2*d^2*e^2)*f^5*g^2 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^3 + (9
*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^4)*x^2 + (c^3*d^3*e*f^7 - 4*a^3*d*e^3*f^3
*g^4 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3
)*f^5*g^2 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(9/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}}{\sqrt{ex + d}\left(gx + f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(9/2)), x)
```

3.741 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^2}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(21*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(5/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(315*(c*d*f - a*e*g)^4*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rubi [A] time = 0.308033, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(1
1/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(21*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(5/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(315*(c*d*f - a*e*g)^4*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{(2cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx}{3(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{(8\cdots}{\cdots}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{16\cdots}{1\cdots}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{16\cdots}{1\cdots}$$

Mathematica [A] time = 0.154941, size = 152, normalized size = 0.57

$$\frac{2((d+ex)(ae+cdx))^{3/2}\left(15a^2cde^2g^2(9f+2gx)-35a^3e^3g^3-3ac^2d^2eg\left(63f^2+36fgx+8g^2x^2\right)+c^3d^3\left(126f^2gx+10\cdots\right.\right.}{315(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-35*a^3*e^3*g^3 + 15*a^2*c*d*e^2*g^2*(9
*f + 2*g*x) - 3*a*c^2*d^2*e*g*(63*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(10
5*f^3 + 126*f^2*g*x + 72*f*g^2*x^2 + 16*g^3*x^3)))/(315*(c*d*f - a*e*g)^4*(
d + e*x)^(3/2)*(f + g*x)^(9/2))
```

Maple [A] time = 0.054, size = 260, normalized size = 1.

$$-\frac{(2\,cdx+2\,ae)\left(-16\,c^3d^3g^3x^3+24\,ac^2d^2eg^3x^2-72\,c^3d^3fg^2x^2-30\,a^2cde^2g^3x+108\,ac^2d^2efg^2x-126\,c^3d^3f^2gx+35\,\cdots\right.}{315\,g^4e^4a^4-1260\,cdg^3fe^3a^3+1890\,c^2d^2g^2f^2e^2a^2-1260\,c^3d^3gf^3ea+3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+24*a*c^2*d^2*e*g^3*x^2-72*c^3*d^3*f
*g^2*x^2-30*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-126*c^3*d^3*f^2*g*x+3
5*a^3*e^3*g^3-135*a^2*c*d*e^2*f*g^2+189*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*
(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/(g*x+f)^(9/2)/(a^4*e^4*g^4-4*a^3*c*
d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(e*x
+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}{\sqrt{ex+d}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(11/2)), x)
```

Fricas [B] time = 1.99459, size = 2323, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 189*a^2*c^2*d^2*e^2*f^2*g
 + 135*a^3*c*d*e^3*f*g^2 - 35*a^4*e^4*g^3 + 8*(9*c^4*d^4*f*g^2 - a*c^3*d^3*
e*g^3)*x^3 + 6*(21*c^4*d^4*f^2*g - 6*a*c^3*d^3*e*f*g^2 + a^2*c^2*d^2*e^2*g^
3)*x^2 + (105*c^4*d^4*f^3 - 63*a*c^3*d^3*e*f^2*g + 27*a^2*c^2*d^2*e^2*f*g^2
 - 5*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e
*x + d)*sqrt(g*x + f)/(c^4*d^5*f^9 - 4*a*c^3*d^4*e*f^8*g + 6*a^2*c^2*d^3*e^
2*f^7*g^2 - 4*a^3*c*d^2*e^3*f^6*g^3 + a^4*d*e^4*f^5*g^4 + (c^4*d^4*e*f^4*g^
5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*d*e^4*f*g
^8 + a^4*e^5*g^9)*x^6 + (5*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^4*d^5 - 2
0*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^3*g^6 +
 2*(3*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 5*a^4*
e^5)*f*g^8)*x^5 + 5*(2*c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 - 8*a
*c^3*d^3*e^2)*f^5*g^4 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 + 2*(3*
a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^3*g^6 - 2*(2*a^3*c*d^2*e^3 - a^4*e^5)*f^
2*g^7)*x^4 + 10*(c^4*d^4*e*f^7*g^2 + a^4*d*e^4*f^2*g^7 + (c^4*d^5 - 4*a*c^3
*d^3*e^2)*f^6*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^5*g^4 + 2*(3*a^
2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^
6)*x^3 + 5*(c^4*d^4*e*f^8*g + 2*a^4*d*e^4*f^3*g^6 + 2*(c^4*d^5 - 2*a*c^3*d^
3*e^2)*f^7*g^2 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^3 + 4*(3*a^2*c
^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^4 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^5)*x^
2 + (c^4*d^4*e*f^9 + 5*a^4*d*e^4*f^4*g^5 + (5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^
8*g - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^2 + 2*(15*a^2*c^2*d^3*e^
2 - 2*a^3*c*d*e^4)*f^6*g^3 - (20*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(11/2)/(e*x+
d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x +
 f)^(11/2)), x)
```

3.742 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=382

$$\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3}}{}$$

```
[Out] (3*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(64*c^2*d^2*g^2*Sqrt[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*c*d*g^2*Sqrt[d + e*x]) - ((c
*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(8*g^2*Sqrt[d + e*x]) + ((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2))/(4*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(5/2)*d^(5/2)*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rubi [A] time = 0.711359, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{(f+gx)^{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```


```
[Out] (3*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(64*c^2*d^2*g^2*Sqrt[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*c*d*g^2*Sqrt[d + e*x]) - ((c
*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(8*g^2*Sqrt[d + e*x]) + ((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2))/(4*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(5/2)*d^(5/2)*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```


Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} - \frac{(3(cdf-aeg))\int \frac{(f+gx)}{}}{}$$

$$= -\frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\,(}{}$$

$$= \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(}{}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} + \frac{(cdf-aeg)^2(}{}$$

Mathematica [A] time = 1.23109, size = 302, normalized size = 0.79

$$\frac{\sqrt{cd}\sqrt{d+ex}\left(3\sqrt{ae+cdx}(cdf-aeg)^{9/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)-\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(-a^2cde^2g^2(1\right.\right.}{64c^{7/2}d^{7/2}g^{5/2}\sqrt{f+gx}\sqrt{(d+e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
d + e*x)^(3/2),x]
```

```
[Out] (Sqrt[c*d]*Sqrt[d + e*x]*(-(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*d*x)
*(f + g*x)*(3*a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(11*f + 2*g*x) - a*c^2*d^2*e*g*
(11*f^2 + 44*f*g*x + 24*g^2*x^2) + c^3*d^3*(3*f^3 - 2*f^2*g*x - 24*f*g^2*x^
2 - 16*g^3*x^3))) + 3*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqrt[(c*d*(f
+ g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]
)/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g]])]))/(64*c^(7/2)*d^(7/2)*g^(5/2)*Sqrt[(a*e
+ c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [B] time = 0.338, size = 870, normalized size = 2.3

$$\frac{1}{128\,g^2c^2d^2}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(32\,x^3c^3d^3g^3\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}+3\,\ln\left(1/2\,\frac{2\,xcdg+aeg}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] 1/128*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(32*x^3*c^3*d^3
*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+3*ln(1/2*(2*x*c*
d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c
*d*g)^(1/2))*a^4*e^4*g^4-12*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*
g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3+18
*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c
*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-12*ln(1/2*(2*x*c*d*g+a*
e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^
(1/2))*a*c^3*d^3*e*f^3*g+3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g
*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4+48*x^2*a*
c^2*d^2*e*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+48*x^2*
c^3*d^3*f*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+4*(c*d*
g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*c*d*e^2*g^3+88*(c*d*
g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c^2*d^2*e*f*g^2+4*(c*d
*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^3*d^3*f^2*g-6*(c*d*g)
^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3+22*(c*d*g)^(1/2)
*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2+22*(c*d*g)^(1/2)
*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-6*(c*d*g)^(1/2)*
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c^2/d^2/
g^2/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e*
x + d)^(3/2), x)
```

Fricas [A] time = 17.3003, size = 2217, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/256*(4*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^2 +
11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^3*d^
3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*e^2*
g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f) + 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4
*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3
```

```
*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*sqrt
(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e
^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
 a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d
^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
 + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^3*x + c^3*d^4*g
^3), 1/128*(2*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^
2 + 11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^
3*d^3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*
e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f) - 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2
 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2
*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*
sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e
*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^3*d^3*e*g^3*x + c^3*d^4*g^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} \left(gx + f\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e*
x + d)^(3/2), x)
```

3.743 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}}-\frac{(f+gx)^3}{}$$

```
[Out] ((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(8*c*d*g^2*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) + ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(3/2)*d^(3/2)*g^(5/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.533453, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}}-\frac{(f+gx)^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)
^(3/2),x]
```


```
[Out] ((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(8*c*d*g^2*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) + ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(3/2)*d^(3/2)*g^(5/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx = \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}} - \frac{(cdf-aeg)\int \frac{\sqrt{f+gx}\sqrt{}}{}}{2}$$

$$= -\frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f}{}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} - \frac{(cdf-aeg)(f}{}$$

Mathematica [A] time = 0.880542, size = 254, normalized size = 0.82

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(3a^2e^2g^2+2acdeg(4f+7gx)+c^2d^2\left(-3f^2+2fgx+8g^2x^2\right)\right)+3\sqrt{ae}\right.}{24g^{5/2}(cd)^{5/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d
 + e*x)^(3/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*
d*x)*(f + g*x)*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + 7*g*x) + c^2*d^2*(-3*f^2
 + 2*f*g*x + 8*g^2*x^2)) + 3*(c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x]*Sqrt[(
c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e +
 c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(24*(c*d)^(5/2)*g^(5/2)*Sqrt[(a
*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [B] time = 0.327, size = 602, normalized size = 1.9

$$-\frac{1}{48\,cdg^2}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}}{\sqrt{cdg}}\right)a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] -1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*x*
c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/
(c*d*g)^(1/2))*a^3*e^3*g^3-9*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e
*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9
*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c
*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-3*ln(1/2*(2*x*c*d*g+a*e*g+c*d
*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*
c^3*d^3*f^3-16*x^2*c^2*d^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d
*g)^(1/2)-28*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c*d*
e*g^2-4*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^2*d^2*f*g
-6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*e^2*g^2-16*a*c
*d*e*f*g*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+6*(c*d*g)^(1
/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/c/d/
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/g^2/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}} \sqrt{gx + f}}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(g*x + f)/(e*x
+ d)^(3/2), x)
```

Fricas [A] time = 9.61693, size = 1790, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^2
*c*d*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a
*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 -
 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*d*g)*
log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 -
 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*
sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3), 1/4
8*(2*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d
*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 - 3*a*c^2
*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f^3 - 3*a
```

```
*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*d*g)*arc
tan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d
)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
a*e^2)*g)*x)))/(c^2*d^2*e*g^3*x + c^2*d^3*g^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}\sqrt{gx + f}}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(g*x + f)/(e*x
+ d)^(3/2), x)
```

3.744 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=238

$$-\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4g^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}}{}$$

```
[Out] (-3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*g^2*Sqrt[d + e*x]) + (Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2))/(2*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(4*Sqrt[c]*Sqrt[d]*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])
```

Rubi [A] time = 0.345087, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {864, 891, 63, 217, 206}

$$-\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4g^2\sqrt{d+ex}}+\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sqrt[f +
 g*x]),x]
```


```
[Out] (-3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*g^2*Sqrt[d + e*x]) + (Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2))/(2*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(4*Sqrt[c]*Sqrt[d]*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])
```


Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

```
&& !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}} - \frac{(3(cdf-aeg))\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+\cdots}}{\sqrt{d+ex}\sqrt{f+gx}}\cdots}{4g}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade+\left(cd^2\cdots\right.\right.}{2g(d+\cdots}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade+\left(cd^2\cdots\right.\right.}{2g(d+\cdots}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade+\left(cd^2\cdots\right.\right.}{2g(d+\cdots}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade+\left(cd^2\cdots\right.\right.}{2g(d+\cdots}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}\left(ade+\left(cd^2\cdots\right.\right.}{2g(d+\cdots}$$

Mathematica [A] time = 0.802621, size = 193, normalized size = 0.81

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}(f+gx)(5aeg+cd(2gx-3f))+\frac{3\sqrt{c}\sqrt{d}(cdf-aeg)^{5/2}\sqrt{\frac{cd(f+gx)}{cdf-aeg}}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{(cd)^{3/2}\sqrt{ae+cdx}}\right)}{4g^{5/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x)*(5*a*e*g + c*d*(-3*f + 2*
g*x)) + (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^(5/2)*Sqrt[(c*d*(f + g*x))/(c*d*
f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*
Sqrt[c*d*f - a*e*g])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d
+ e*x]*Sqrt[f + g*x])
```

Maple [A] time = 0.345, size = 325, normalized size = 1.4

$$\frac{1}{8g^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\sqrt{gx+f}\left(3\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}}{\sqrt{cdg}}\right)a^2e^2g^2-6\ln\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)*(3*ln(1/2*(2*x*c*
d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))
*a^2*e^2*g^2-6*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*
(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2
*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+4*(c
*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*c*d*g+10*(c*d*g)^(1/2)*((c*d*x+a*
e)*(g*x+f))^(1/2)*a*e*g-6*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*c*d*f)/
(e*x+d)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/g^2/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*sq
rt(g*x + f)), x)
```

Fricas [A] time = 6.25638, size = 1427, normalized size = 6.

$$\left[\frac{4\left(2\,c^2d^2g^2x-3\,c^2d^2fg+5\,acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\sqrt{ex+d}\sqrt{gx+f}+3\left(c^2d^3f^2-2\,acd^2efg+a^2de^2g^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 -
2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^
3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e
*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*
a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^3*x +
 c*d^2*g^3), 1/8*(2*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2
*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f
*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2
*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^3*x + c*d^2*g^
3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex + d\right)^{\frac{3}{2}}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*sq
rt(g*x + f)), x)
```

3.745 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=222

$$\frac{3cd\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}-\frac{3\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(x\left(ae^2+cd^2\right.\right.}{g(d+ex}$$

```
[Out] (3*c*d*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt
[d + e*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(g*(d + e*x)
^(3/2)*Sqrt[f + g*x]) - (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
 + g*x])])/(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.303124, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{3cd\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}-\frac{3\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\left(x\left(ae^2+cd^2\right.\right.}{g(d+ex}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (3*c*d*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt
[d + e*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(g*(d + e*x)
^(3/2)*Sqrt[f + g*x]) - (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
 + g*x])])/(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}} + \frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{g}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$= \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

Mathematica [C] time = 0.16722, size = 102, normalized size = 0.46

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5cd(d+ex)^{5/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^(3/2)*
Hypergeometric2F1[3/2, 5/2, 7/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*
c*d*(d + e*x)^(5/2)*(f + g*x)^(3/2))
```

Maple [B] time = 0.351, size = 383, normalized size = 1.7

$$\frac{1}{2g^2}\left(3\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}}{\sqrt{cdg}}\right)xacdeg^2-3\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+a}}{\sqrt{cdg}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/2),x)
```

```
[Out] 1/2*(3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*x*a*c*d*e*g^2-3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d
*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c^2*d^2*f*g+3*ln(1/2
*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g
)^(1/2))*a*c*d*e*f*g-3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f)
)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+2*(c*d*g)^(1/2)*((c*d*x+a
*e)*(g*x+f))^(1/2)*x*c*d*g-4*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a*e*
g+6*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d
^2*x+a*d*e)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/g^2/(g*x+f)^(1/
2)/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(3/2)), x)
```

Fricas [A] time = 5.84913, size = 1442, normalized size = 6.5

$$\left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdgx+3\,cdf-2\,aeg\right)\sqrt{ex+d}\sqrt{gx+f}-3\left(cd^2f^2-adefg+\left(cdefg-ae^2g^2\right)x^2+\left(cd\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*
a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g
- a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g)*x)*sqrt(c*
d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*
g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^
2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*
f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^3*x^2 + d*f*g^2 + (e
*f*g^2 + d*g^3)*x), 1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d
*g*x + 3*c*d*f - 2*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c*d^2*f^2 - a*d*
e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e
^2)*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*
d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^3*x^2 + d*f*g^2 + (e*f*g^
2 + d*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(3/2)), x)
```

3.746 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[Out] (-2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*Sqr
t[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e
*x)^(3/2)*(f + g*x)^(3/2)) + (2*c^(3/2)*d^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/
(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.277769, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(5/2)),x]
```


```
[Out] (-2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*Sqr
t[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e
*x)^(3/2)*(f + g*x)^(3/2)) + (2*c^(3/2)*d^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/
(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{(cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{g}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{(c}{}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{(c}{}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{(2}{}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{(2}{}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{2c}{}$$

Mathematica [A] time = 1.09298, size = 188, normalized size = 0.88

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{3\sqrt{c}\sqrt{d}(cdf-aeg)^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\sqrt{ae+cdx}}-\sqrt{g}(aeg+cd(3f+4gx))\right)}{3g^{5/2}\sqrt{d+ex}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]*(a*e*g + c*d*(3*f + 4*g*x))) +
(3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^(3/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^
(3/2)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c
*d*f - a*e*g])])/(Sqrt[c*d]*Sqrt[a*e + c*d*x])))/(3*g^(5/2)*Sqrt[d + e*x]*(
f + g*x)^(3/2))
```

Maple [A] time = 0.405, size = 331, normalized size = 1.6

$$\frac{1}{3g^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}}{\sqrt{cdg}}\right)x^2c^2d^2g^2+6\ln\left(1/2\,\frac{2\,xcd}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/2),x)
```

```
[Out] 1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*
f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x^2*c^2*d^2*g
^2+6*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*x*c^2*d^2*f*g+3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x
+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2-8*(c*d*g)^(1
/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*c*d*g-2*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f
))^(1/2)*a*e*g-6*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*c*d*f)/(c*d*g)^(
1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/g^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(5/2)), x)
```

Fricas [A] time = 5.60675, size = 1499, normalized size = 7.

$$\left[-\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(4\,cdgx+3\,cdf+aeg\right)\sqrt{ex+d}\sqrt{gx+f}-3\left(cdeg^2x^3+cd^2f^2+\left(2\,cdefg+cd^2g^2\right)x^2+\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f +
 a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d
*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^2 + 2*c*d^2*f*g)*x)*sqrt(c*d/g)*log(-(8*
c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*
d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d
*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*
d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^4*x^3 + d*f^2*g^2 + (2*e*f*g^3 +
d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f + a*e*g)*sqrt(e*x + d)*sqrt(g*x + f)
 + 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^
2 + 2*c*d^2*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^
2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^3 + d*f^2*g^2
 + (2*e*f*g^3 + d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(5/2)), x)
```

3.747 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(5/2))
```

Rubi [A] time = 0.0702738, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

Mathematica [A] time = 0.0542479, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f +
 g*x)^(5/2))
```

Maple [A] time = 0.049, size = 63, normalized size = 1.

$$-\frac{2\,cdx+2\,ae}{5\,aeg-5\,cdf}\left(cdex^{2}+ae^{2}x+cd^{2}x+ade\right)^{\frac{3}{2}}\left(gx+f\right)^{-\frac{5}{2}}\left(ex+d\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/2),x)
```

```
[Out] -2/5*(c*d*x+a*e)/(g*x+f)^(5/2)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(3/2)/(e*x+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(7/2)), x)
```

Fricas [B] time = 1.47522, size = 482, normalized size = 7.65

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdex+a^{2}e^{2}\right)\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\sqrt{ex+d}\sqrt{g\cdots}}{5\left(cd^{2}f^{4}-adef^{3}g+\left(cdefg^{3}-ae^{2}g^{4}\right)x^{4}+\left(3\,cdef^{2}g^{2}-adeg^{4}+\left(cd^{2}-3\,ae^{2}\right)fg^{3}\right)x^{3}+3\left(cdef^{3}g-adefg^{3}+\left(cd^{2}-a\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^4 - a*d*e*f^3*g + (c*d*e*f*
g^3 - a*e^2*g^4)*x^4 + (3*c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - 3*a*e^2)*f*g
^3)*x^3 + 3*(c*d*e*f^3*g - a*d*e*f*g^3 + (c*d^2 - a*e^2)*f^2*g^2)*x^2 + (c*
d*e*f^4 - 3*a*d*e*f^2*g^2 + (3*c*d^2 - a*e^2)*f^3*g)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(7/2)), x)
```

3.748 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rubi [A] time = 0.147863, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}} + \frac{(2cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx}{7(cdf-aeg)}$$
$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{5/2}}$$

Mathematica [A] time = 0.0896059, size = 69, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{5/2}(cd(7f+2gx)-5aeg)}{35(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e*g + c*d*(7*f + 2*g*x)))/(35*(c*d
*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2))
```

Maple [A] time = 0.053, size = 99, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-2\,xcdg+5\,aeg-7\,cdf\right)}{35\,a^2e^2g^2-70\,acdefg+35\,c^2d^2f^2}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{3}{2}}\left(gx+f\right)^{-\frac{7}{2}}(ex+d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/2),x)
```

```
[Out] -2/35*(c*d*x+a*e)*(-2*c*d*g*x+5*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(3/2)/(g*x+f)^(7/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(9/2)), x)
```

Fricas [B] time = 1.52951, size = 1061, normalized size = 8.22

$$35\left(c^2d^3f^6 - 2\,acd^2ef^5g + a^2de^2f^4g^2 + \left(c^2d^2ef^2g^4 - 2\,acde^2fg^5 + a^2e^3g^6\right)x^5 + \left(4\,c^2d^2ef^3g^3 + a^2de^2g^6 + \left(c^2d^3 - 8\,ac\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 5*a^3*e^3*g + (7*c^3*d^3*f - a*c^
2*d^2*e*g)*x^2 + 2*(7*a*c^2*d^2*e*f - 4*a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^6 - 2*a*c
*d^2*e*f^5*g + a^2*d*e^2*f^4*g^2 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 +
 a^2*e^3*g^6)*x^5 + (4*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 8*a*c
*d*e^2)*f^2*g^4 - 2*(a*c*d^2*e - 2*a^2*e^3)*f*g^5)*x^4 + 2*(3*c^2*d^2*e*f^4
*g^2 + 2*a^2*d*e^2*f*g^5 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e
 - 3*a^2*e^3)*f^2*g^4)*x^3 + 2*(2*c^2*d^2*e*f^5*g + 3*a^2*d*e^2*f^2*g^4 + (
3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^2 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^3)*x^2 +
 (c^2*d^2*e*f^6 + 4*a^2*d*e^2*f^3*g^3 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g - (
8*a*c*d^2*e - a^2*e^3)*f^4*g^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(9/2)), x)
```

3.749 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(9/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(5/2))
```

Rubi [A] time = 0.227919, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(9/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{(4cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}} +$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}} +$$

Mathematica [A] time = 0.134648, size = 105, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(35a^2e^2g^2-10acdeg(9f+2gx)+c^2d^2\left(63f^2+36fgx+8g^2x^2\right)\right)}{315(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(35*a^2*e^2*g^2 - 10*a*c*d*e*g*(9*f + 2*
g*x) + c^2*d^2*(63*f^2 + 36*f*g*x + 8*g^2*x^2)))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(9/2))
```

Maple [A] time = 0.052, size = 169, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2g^2x^2-20\,acdeg^2x+36\,c^2d^2fgx+35\,a^2e^2g^2-90\,acdefg+63\,c^2d^2f^2\right)}{315\,a^3e^3g^3-945\,a^2cde^2fg^2+945\,ac^2d^2efg^2g-315\,c^3d^3f^3}\left(cdex^2+ae^2x+cd^2x+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11/2),x)
```

```
[Out] -2/315*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+36*c^2*d^2*f*g*x+35*
a^2*e^2*g^2-90*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(3/2)/(g*x+f)^(9/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-
c^3*d^3*f^3)/(e*x+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(11/2)), x)
```

Fricas [B] time = 1.75612, size = 1808, normalized size = 9.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(8*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 90*a^3*c*d*e^3*f*g + 35
*a^4*e^4*g^2 + 4*(9*c^4*d^4*f*g - a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f^2
- 6*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2 - 72
*a^2*c^2*d^2*e^2*f*g + 25*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^8 - 3*a*c^2*d^3*e*f^7
*g + 3*a^2*c*d^2*e^2*f^6*g^2 - a^3*d*e^3*f^5*g^3 + (c^3*d^3*e*f^3*g^5 - 3*a
*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^6 + (5*c^3*d^3*
e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*f^3*g^5 - 3*(a*c^2
*d^3*e - 5*a^2*c*d*e^3)*f^2*g^6 + (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^7)*x^5
+ 5*(2*c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^
4*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^2 - 2*a^3*
e^4)*f^2*g^6)*x^4 + 10*(c^3*d^3*e*f^6*g^2 - a^3*d*e^3*f^2*g^6 + (c^3*d^4 -
3*a*c^2*d^2*e^2)*f^5*g^3 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4 + (3*a^2*c
*d^2*e^2 - a^3*e^4)*f^3*g^5)*x^3 + 5*(c^3*d^3*e*f^7*g - 2*a^3*d*e^3*f^3*g^5
 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*
f^5*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^4)*x^2 + (c^3*d^3*e*f^8 - 5*a^3
*d*e^3*f^4*g^4 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^7*g - 3*(5*a*c^2*d^3*e - a
^2*c*d*e^3)*f^6*g^2 + (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\left(gx + f\right)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(11/2)), x)
```

3.750 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{33(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)^2}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(11/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(33*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*(c*d*f - a*e*g)^3*(
d + e*x)^(5/2)*(f + g*x)^(7/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(5/2))/(1155*(c*d*f - a*e*g)^4*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rubi [A] time = 0.323945, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{33(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(11/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(33*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*(c*d*f - a*e*g)^3*(
d + e*x)^(5/2)*(f + g*x)^(7/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(5/2))/(1155*(c*d*f - a*e*g)^4*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{(6cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}dx}{11(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}}$$

Mathematica [A] time = 0.186018, size = 152, normalized size = 0.57

$$\frac{2((d+ex)(ae+cdx))^{5/2}\left(35a^2cde^2g^2(11f+2gx)-105a^3e^3g^3-5ac^2d^2eg\left(99f^2+44fgx+8g^2x^2\right)+c^3d^3\left(198f^2gx+2\right.\right.}{1155(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-105*a^3*e^3*g^3 + 35*a^2*c*d*e^2*g^2*(
11*f + 2*g*x) - 5*a*c^2*d^2*e*g*(99*f^2 + 44*f*g*x + 8*g^2*x^2) + c^3*d^3*(
231*f^3 + 198*f^2*g*x + 88*f*g^2*x^2 + 16*g^3*x^3)))/(1155*(c*d*f - a*e*g)^
4*(d + e*x)^(5/2)*(f + g*x)^(11/2))
```

Maple [A] time = 0.061, size = 260, normalized size = 1.

$$-\frac{(2\,cdx+2\,ae)\left(-16\,c^3d^3g^3x^3+40\,ac^2d^2eg^3x^2-88\,c^3d^3fg^2x^2-70\,a^2cde^2g^3x+220\,ac^2d^2efg^2x-198\,c^3d^3f^2gx+105\right.}{1155\,g^4e^4a^4-4620\,cdg^3fe^3a^3+6930\,c^2d^2g^2f^2e^2a^2-4620\,c^3d^3gf^3ea+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13/2),x)
```

```
[Out] -2/1155*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+40*a*c^2*d^2*e*g^3*x^2-88*c^3*d^3*
f*g^2*x^2-70*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-198*c^3*d^3*f^2*g*x+
105*a^3*e^3*g^3-385*a^2*c*d*e^2*f*g^2+495*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3
)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/(g*x+f)^(11/2)/(a^4*e^4*g^4-4*a^3
*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4)/(
e*x+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(13/2)), x)
```

Fricas [B] time = 1.79803, size = 2820, normalized size = 10.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 495*a^3*c^2*d^2*e^3*
f^2*g + 385*a^4*c*d*e^4*f*g^2 - 105*a^5*e^5*g^3 + 8*(11*c^5*d^5*f*g^2 - a*c
^4*d^4*e*g^3)*x^4 + 2*(99*c^5*d^5*f^2*g - 22*a*c^4*d^4*e*f*g^2 + 3*a^2*c^3*
d^3*e^2*g^3)*x^3 + (231*c^5*d^5*f^3 - 99*a*c^4*d^4*e*f^2*g + 33*a^2*c^3*d^3
*e^2*f*g^2 - 5*a^3*c^2*d^2*e^3*g^3)*x^2 + 2*(231*a*c^4*d^4*e*f^3 - 396*a^2*
c^3*d^3*e^2*f^2*g + 275*a^3*c^2*d^2*e^3*f*g^2 - 70*a^4*c*d*e^4*g^3)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^
5*f^10 - 4*a*c^3*d^4*e*f^9*g + 6*a^2*c^2*d^3*e^2*f^8*g^2 - 4*a^3*c*d^2*e^3*
f^7*g^3 + a^4*d*e^4*f^6*g^4 + (c^4*d^4*e*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7
+ 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 + a^4*e^5*g^10)*x^7 + (6*
c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 24*a*c^3*d^3*e^2)*f^4*g^6 -
 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^7 + 6*(a^2*c^2*d^3*e^2 - 4*a^3*c
*d*e^4)*f^2*g^8 - 2*(2*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^9)*x^6 + 3*(5*c^4*d^4
*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^2)*f^5*g^5 - 2
*(4*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^4*g^6 + 4*(3*a^2*c^2*d^3*e^2 - 5*a^
3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^8)*x^5 + 5*(4*c^4*
d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 16*a*c^3*d^3*e^2)*f^6*g^
4 - 12*(a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a^2*c^2*d^3*e^2 - 8
*a^3*c*d*e^4)*f^4*g^6 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^7)*x^4 + 5*(3*c
^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 - 3*a*c^3*d^3*e^2)*f^7*
g^3 - 2*(8*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^6*g^4 + 12*(2*a^2*c^2*d^3*e^2
 - a^3*c*d*e^4)*f^5*g^5 - (16*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^4*g^6)*x^3 + 3*(
2*c^4*d^4*e*f^9*g + 5*a^4*d*e^4*f^4*g^6 + (5*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^8
*g^2 - 4*(5*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g^3 + 2*(15*a^2*c^2*d^3*e^
2 - 4*a^3*c*d*e^4)*f^6*g^4 - 2*(10*a^3*c*d^2*e^3 - a^4*e^5)*f^5*g^5)*x^2 +
(c^4*d^4*e*f^10 + 6*a^4*d*e^4*f^5*g^5 + 2*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^9
*g - 6*(4*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^2 + 4*(9*a^2*c^2*d^3*e^2 - a
^3*c*d*e^4)*f^7*g^3 - (24*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g
*x + f)^(13/2)), x)
```

3.751 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=448

$$-\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^4}{128c^2d^2g^3\sqrt{d+ex}}-\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^5\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-(f+gx)$$

```
[Out] (-3*(c*d*f - a*e*g)^4*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(128*c^2*d^2*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^3*(f + g*x)^(3/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (
(c*d*f - a*e*g)^2*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(16*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(5/2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(5/2
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)) - (3
*(c*d*f - a*e*g)^5*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(128*c^(5/2)*d^(5/2)*g^(7/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.888996, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{3\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^4}{128c^2d^2g^3\sqrt{d+ex}}-\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^5\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-(f+gx)$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```


```
[Out] (-3*(c*d*f - a*e*g)^4*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(128*c^2*d^2*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^3*(f + g*x)^(3/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (
(c*d*f - a*e*g)^2*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(16*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(5/2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(5/2
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)) - (3
*(c*d*f - a*e*g)^5*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(128*c^(5/2)*d^(5/2)*g^(7/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{(cdf-aeg)\int \frac{(f+gx)^{3/2}(}{}}{2}$$

$$= -\frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}} + \frac{(f+gx)^5}{}$$

$$= \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}} - \frac{(cdf-aeg)(}{}$$

$$= -\frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{(cdf-aeg}{}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3}{}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3}{}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3}{}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3}{}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}} - \frac{(cdf-aeg)^3}{}$$

Mathematica [B] time = 6.129, size = 974, normalized size = 2.17

$$\frac{2(cdf-aeg)(ae+cdx)((ae+cdx)(d+ex))^{5/2}\sqrt{f+gx}\left(\frac{cdg(ae+cdx)}{(cdf-aeg)\left(\frac{c^2d^2f}{cdf-aeg}-\frac{acdeg}{cdf-aeg}\right)}+1\right)^{5/2}\left(\frac{21(cdf-aeg)^4\left(\frac{16c^3d^3g^3(ae+cdx)^3}{15(cdf-aeg)^3\left(\frac{c^2d^2f}{cdf-aeg}-\frac{acdeg}{cdf-aeg}\right)^3}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(
d + e*x)^(5/2),x]
```

```
[Out] (2*(c*d*f - a*e*g)*(a*e + c*d*x)*((a*e + c*d*x)*(d + e*x))^(5/2)*Sqrt[f + g
*x]*(1 + (c*d*g*(a*e + c*d*x))/((c*d*f - a*e*g)*((c^2*d^2*f)/(c*d*f - a*e*g
) - (a*c*d*e*g)/(c*d*f - a*e*g))))^(5/2)*((7*(3/(8*(1 + (c*d*g*(a*e + c*d*x
))/((c*d*f - a*e*g)*((c^2*d^2*f)/(c*d*f - a*e*g) - (a*c*d*e*g)/(c*d*f - a*e
*g))))^2) + (1 + (c*d*g*(a*e + c*d*x))/((c*d*f - a*e*g)*((c^2*d^2*f)/(c*d*f
 - a*e*g) - (a*c*d*e*g)/(c*d*f - a*e*g))))^(-1)))/10 + (21*(c*d*f - a*e*g)^
4*((c^2*d^2*f)/(c*d*f - a*e*g) - (a*c*d*e*g)/(c*d*f - a*e*g))^4*((2*c*d*g*(
a*e + c*d*x))/((c*d*f - a*e*g)*((c^2*d^2*f)/(c*d*f - a*e*g) - (a*c*d*e*g)/(
c*d*f - a*e*g))) - (4*c^2*d^2*g^2*(a*e + c*d*x)^2)/(3*(c*d*f - a*e*g)^2*((c
^2*d^2*f)/(c*d*f - a*e*g) - (a*c*d*e*g)/(c*d*f - a*e*g))^2) + (16*c^3*d^3*g
^3*(a*e + c*d*x)^3)/(15*(c*d*f - a*e*g)^3*((c^2*d^2*f)/(c*d*f - a*e*g) - (a
*c*d*e*g)/(c*d*f - a*e*g))^3) - (2*Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x
]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d*f - a*e*g]*
Sqrt[(c^2*d^2*f)/(c*d*f - a*e*g) - (a*c*d*e*g)/(c*d*f - a*e*g)])])/(Sqrt[c*
d*f - a*e*g]*Sqrt[(c^2*d^2*f)/(c*d*f - a*e*g) - (a*c*d*e*g)/(c*d*f - a*e*g)
]*Sqrt[1 + (c*d*g*(a*e + c*d*x))/((c*d*f - a*e*g)*((c^2*d^2*f)/(c*d*f - a*e
*g) - (a*c*d*e*g)/(c*d*f - a*e*g)))]))))/(512*c^4*d^4*g^4*(a*e + c*d*x)^4*(1
 + (c*d*g*(a*e + c*d*x))/((c*d*f - a*e*g)*((c^2*d^2*f)/(c*d*f - a*e*g) - (a
*c*d*e*g)/(c*d*f - a*e*g))))^2)))/(7*c^2*d^2*((c*d)/((c^2*d^2*f)/(c*d*f - a
*e*g) - (a*c*d*e*g)/(c*d*f - a*e*g)))^(3/2)*(d + e*x)^(5/2)*Sqrt[(c*d*(f +
g*x))/(c*d*f - a*e*g)])
```

Maple [B] time = 0.41, size = 1191, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] 1/1280*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(256*x^4*c^4*d
^4*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+672*x^3*a*c^3*
d^3*e*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+352*x^3*c^4
*d^4*f*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*
(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a^5*e^5*g^5-75*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x
^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*c*d*e^4*f
*g^4+150*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^
(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c^2*d^2*e^3*f^2*g^3-150*ln(1/2*(2*x
*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))
/(c*d*g)^(1/2))*a^2*c^3*d^3*e^2*f^3*g^2+75*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*
(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^4
*d^4*e*f^4*g-15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+
a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^5*d^5*f^5+496*x^2*a^2*c^2*d^2*
e^2*g^4*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+1024*x^2*a*c^
3*d^3*e*f*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+16*x^2*
c^4*d^4*f^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+20*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^3*c*d*e^3*g^4+932*(
c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*c^2*d^2*e^2*f*g^
3+92*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c^3*d^3*e*f^
2*g^2-20*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^4*d^4*f^
3*g-30*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^4*e^4*g^4+14
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*c*d*e^3*f*g^3+2
56*a^2*c^2*d^2*e^2*f^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^
(1/2)-140*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^3*d^3*e
*f^3*g+30*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^4*d^4*f^4
)/(e*x+d)^(1/2)/c^2/d^2/g^3/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/(c*d*g)
```

^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\left(gx + f\right)^{\frac{3}{2}}}{\left(ex + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^(3/2)/(e*
x + d)^(5/2), x)
```

Fricas [A] time = 28.5776, size = 2781, normalized size = 6.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/2560*(4*(128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^2
 + 128*a^2*c^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^4*
g^5 + 16*(11*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^3 +
 64*a*c^4*d^4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*g^2
- 23*a*c^4*d^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e^3*g^
5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f) - 15*(c^5*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 1
0*a^3*c^2*d^3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^
5*e*f^5 - 5*a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d
^2*e^4*f^2*g^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(c*d*g)*log(-(8*
c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d
*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*
g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^4*x + c^3*d^4*g^4), 1/1280*(2*(
128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^2 + 128*a^2*c
^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^4*g^5 + 16*(11
*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^3 + 64*a*c^4*d^
4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*g^2 - 23*a*c^4*d
^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e^3*g^5)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^5
*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 10*a^3*c^2*d^
3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^5*e*f^5 - 5*
a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d^2*e^4*f^2*g
^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/
(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(
c^3*d^3*e*g^4*x + c^3*d^4*g^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\left(gx + f\right)^{\frac{3}{2}}}{\left(ex + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^(3/2)/(e*
x + d)^(5/2), x)
```

3.752 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}}+\frac{5(f+g\ldots}{}$$

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x]) - (5*(c*d*
f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
24*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2))/(4*g*(d + e*x)^(5/2)) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(64*c^(3/2)*d^(3/2)*g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])
```

Rubi [A] time = 0.678094, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {864, 870, 891, 63, 217, 206}

$$-\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}}+\frac{5(f+g\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)
^(5/2),x]
```


```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x]) - (5*(c*d*
f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
24*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2))/(4*g*(d + e*x)^(5/2)) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(64*c^(3/2)*d^(3/2)*g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])
```


Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}} - \frac{(5(cdf-aeg))\int \frac{\sqrt{f+gx}(a}{}}{8g}$$

$$= -\frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}} + \frac{(f+gx)^{3/}}{}$$

$$= \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)(}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2}{}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}} + \frac{5(cdf-aeg)^2}{}$$

Mathematica [A] time = 1.18936, size = 299, normalized size = 0.8

$$\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{cd}(f+gx)(ae+cdx)\left(a^2cde^2g^2(73f+118gx)+15a^3e^3g^3+ac^2d^2eg\left(-55f^2+36fgx+136g^2x\right.\right.\right.}{192g^{7/2}(cd)^{5/2}\sqrt{f+gx}\sqrt{(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d
+ e*x)^(5/2),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[c*d]*Sqrt[g]*(a*e + c*
d*x)*(f + g*x)*(15*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(73*f + 118*g*x) + a*c^2*d
^2*e*g*(-55*f^2 + 36*f*g*x + 136*g^2*x^2) + c^3*d^3*(15*f^3 - 10*f^2*g*x +
8*f*g^2*x^2 + 48*g^3*x^3)) - 15*(c*d*f - a*e*g)^(9/2)*Sqrt[a*e + c*d*x]*Sqr
t[(c*d*(f + g*x))/(c*d*f - a*e*g)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])]))/(192*(c*d)^(5/2)*g^(7/2)*Sqr
t[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [B] time = 0.358, size = 870, normalized size = 2.3

$$-\frac{1}{384\,cdg^3}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(-96\,x^3c^3d^3g^3\sqrt{cdgx^2+aegx+cdfx+aef}\sqrt{cdg}+15\,\ln\left(1/2\,\frac{2\,xcdg+ae}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] -1/384*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-96*x^3*c^3*d
^3*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*x
*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))
/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a
*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3
+90*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)
*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-60*ln(1/2*(2*x*c*d*g
+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a
*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-272*x
^2*a*c^2*d^2*e*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)-16
*x^2*c^3*d^3*f*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(c*d*g)^(1/2)-23
6*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*c*d*e^2*g^3-7
2*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*c^2*d^2*e*f*g^2
+20*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*c^3*d^3*f^2*g-3
0*(c*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*g^3-146*(c*
d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*c*d*e^2*f*g^2+110*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a*c^2*d^2*e*f^2*g-30*(c
*d*g)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1
/2)/c/d/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/g^3/(c*d*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx + f}}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)/(e*x
+ d)^(5/2), x)
```

Fricas [A] time = 13.457, size = 2244, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d^3*e*f^2*g^2 +
 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f*g^3 + 17*a*c^
3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^3 - 59*a^2*c^2
*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f
^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d
```

```
^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^
4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
 + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^4*x +
 c^2*d^3*g^4), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d
^3*e*f^2*g^2 + 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f
*g^3 + 17*a*c^3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^
3 - 59*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2
*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f
^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^
3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*
f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^2*d^2*e*g^4*x + c^2*d
^3*g^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx + f}}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)/(e*x
+ d)^(5/2), x)
```

3.753 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=304

$$\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8g^3\sqrt{d+ex}}-\frac{5\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{12g^2(d+ex)^{3/2}}-\frac{5\sqrt{d}}{}$$

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*g^3*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)) + (Sqrt[f + g*x]
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*x)^(5/2)) - (5*
(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
 + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*Sqrt[c]*Sqrt[d]*g^(7/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.487025, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {864, 891, 63, 217, 206}

$$\frac{5\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}{8g^3\sqrt{d+ex}}-\frac{5\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{12g^2(d+ex)^{3/2}}-\frac{5\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sqrt[f +
 g*x]),x]
```


```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*g^3*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)) + (Sqrt[f + g*x]
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*x)^(5/2)) - (5*
(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
 + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*Sqrt[c]*Sqrt[d]*g^(7/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
```

```
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}} - \frac{(5(cdf-aeg))\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{(d+ex)^{3/2}\sqrt{f+gx}}}{6g}$$

$$= -\frac{5(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}} + \frac{\sqrt{f+gx}\left(ade+\left(cd^2\right.\right.}{3g(d+}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}\left(ad\right.}{12g^2(}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}\left(ad\right.}{12g^2(}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}\left(ad\right.}{12g^2(}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}\left(ad\right.}{12g^2(}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}} - \frac{5(cdf-aeg)\sqrt{f+gx}\left(ad\right.}{12g^2(}$$

Mathematica [A] time = 1.04351, size = 229, normalized size = 0.75

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\sqrt{g}(f+gx)\left(33a^2e^2g^2+2acdeg(13gx-20f)+c^2d^2\left(15f^2-10fgx+8g^2x^2\right)\right)-\frac{15\sqrt{c}\sqrt{d}(cdf-aeg)^{7/2}\cdots}{\cdots}\right)}{24g^{7/2}\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[g]*(f + g*x)*(33*a^2*e^2*g^2 + 2*a*c*d
*e*g*(-20*f + 13*g*x) + c^2*d^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)) - (15*Sqrt
[c]*Sqrt[d]*(c*d*f - a*e*g)^(7/2)*Sqrt[(c*d*(f + g*x))/(c*d*f - a*e*g)]*Arc
Sinh[(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*
e*g])])/((c*d)^(3/2)*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Maple [A] time = 0.368, size = 508, normalized size = 1.7

$$\frac{1}{48\,g^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\sqrt{gx+f}\left(15\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)(gx+f)}\sqrt{cdg}}{\sqrt{cdg}}\right)a^3e^3g^3-45\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/48*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)*(15*ln(1/2*(2*x*
c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2
))*a^3*e^3*g^3-45*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/
2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*x*c*d*g+a*e
*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*
d^2*e*f^2*g-15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*
(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+16*x^2*c^2*d^2*g^2*((c*d*x+a*e)*(
g*x+f))^(1/2)*(c*d*g)^(1/2)+52*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*
a*c*d*e*g^2-20*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*c^2*d^2*f*g+66*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-80*((c*d*x+a*e)*(g*x+f
))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(
1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/g^3/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*sq
rt(g*x + f)), x)
```

Fricas [A] time = 8.05044, size = 1793, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33*
a^2*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4*f^
3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*
e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(
c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^
2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^
3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g
+ (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^4*x + c*d^2*g^4), 1/
48*(2*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33*a^2
*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^3*d^4*f^3 -
 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f
^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(-c*
d*g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt
(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*
c*d^2 + a*e^2)*g)*x)))/(c*d*e*g^4*x + c*d^2*g^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*sq
rt(g*x + f)), x)
```

3.754 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{5cd\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{15cd\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4g^3\sqrt{d+ex}}+\frac{15\sqrt{c}\sqrt{d}\sqrt{d+ex}}{4g^{7}}$$

```
[Out] (-15*c*d*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(4*g^3*Sqrt[d + e*x]) + (5*c*d*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(2*g^2*(d + e*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(5/2))/(g*(d + e*x)^(5/2)*Sqrt[f + g*x]) + (15*Sqrt[c]*
Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*g^(7/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.428359, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{5cd\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{15cd\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{4g^3\sqrt{d+ex}}+\frac{15\sqrt{c}\sqrt{d}\sqrt{d+ex}}{4g^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(3/2)),x]
```



```
[Out] (-15*c*d*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(4*g^3*Sqrt[d + e*x]) + (5*c*d*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(2*g^2*(d + e*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(5/2))/(g*(d + e*x)^(5/2)*Sqrt[f + g*x]) + (15*Sqrt[c]*
Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*g^(7/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])
```


Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{g}$$

$$= \frac{5cd\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}\left(ade+\left(\right.\right.}{2g^2(d}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}\left(ade+\left(\right.\right.}{2g^2(d}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}\left(ade+\left(\right.\right.}{2g^2(d}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}\left(ade+\left(\right.\right.}{2g^2(d}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}\left(ade+\left(\right.\right.}{2g^2(d}$$

Mathematica [C] time = 0.106331, size = 112, normalized size = 0.38

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*x))/(c*d*f -
a*e*g))^(3/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (g*(a*e + c*d*x))/(-(c*d*f)
+ a*e*g)])/(7*c*d*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

Maple [B] time = 0.348, size = 635, normalized size = 2.2

$$\frac{1}{8g^3}\left(15\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}}{\sqrt{cdg}}\right)xa^2cde^2g^3-30\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cd}}{\sqrt{cdg}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2),x)
```

```
[Out] 1/8*(15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x*a^2*c*d*e^2*g^3-30*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2
*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*a*c^2*d^2*e*f*
g^2+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*x*c^3*d^3*f^2*g+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((
c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2-3
0*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)
)/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d
*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+4*x^2*c^2*
d^2*g^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+18*(c*d*g)^(1/2)*((c*d*x+
a*e)*(g*x+f))^(1/2)*x*a*c*d*e*g^2-10*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1
/2)*x*c^2*d^2*f*g-16*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+
50*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g-30*((c*d*x+a*e)*(g
*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(
1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(1/2)/(e*x+d)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(3/2)), x)
```

Fricas [A] time = 6.4132, size = 1937, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g
^2 - (5*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g
+ a^2*d*e^2*f*g^2 + (c^2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2
 + (c^2*d^2*e*f^3 + a^2*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*
d^2*e - a^2*e^3)*f*g^2)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*
f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sq
rt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*
f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*
x + d)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), 1/8*(2*(2*c^2*d^2*g^2
*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 9
*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*
sqrt(g*x + f) - 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g + a^2*d*e^2*f*g^2 + (c^
```

```
2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2 + (c^2*d^2*e*f^3 + a^2
*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*d^2*e - a^2*e^3)*f*g^2)
*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g +
(c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^
4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(3/2)), x)
```

3.755 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=284

$$\frac{5c^2d^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}}-\frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{10cd\left(x\left(\right.\right.}{3}$$

```
[Out] (5*c^2*d^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*
Sqrt[d + e*x]) - (10*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*Sqrt[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(3*g*(d + e*x)^(5/2)*(f + g*x)^(3/2)) - (5*c^(3/2)*d^(3/2)*(c*d
*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])
```

Rubi [A] time = 0.403685, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {862, 864, 891, 63, 217, 206}

$$\frac{5c^2d^2\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}}-\frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{10cd\left(x\left(\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (5*c^2*d^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*
Sqrt[d + e*x]) - (10*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*Sqrt[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(3*g*(d + e*x)^(5/2)*(f + g*x)^(3/2)) - (5*c^(3/2)*d^(3/2)*(c*d
*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 864

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a
+ b*x + c*x^2)^p)/(g*(m - n - 1)), x] - Dist[(m*(c*e*f + c*d*g - b*e*g))/(e
^2*g*(m - n - 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ
[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(IntegerQ
[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}dx}{3g}$$

$$= -\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}$$

Mathematica [C] time = 0.126918, size = 112, normalized size = 0.39

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*x))/(c*d*f -
a*e*g))^(5/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (g*(a*e + c*d*x))/(-(c*d*f)
+ a*e*g)])/(7*c*d*Sqrt[d + e*x]*(f + g*x)^(5/2))
```

Maple [B] time = 0.385, size = 638, normalized size = 2.3

$$\frac{1}{6g^3}\left(15\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}}{\sqrt{cdg}}\right)x^2ac^2d^2eg^3-15\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2),x)
```

```
[Out] 1/6*(15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x^2*a*c^2*d^2*e*g^3-15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f
+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x^2*c^3*d^3*f*
g^2+30*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*x*a*c^2*d^2*e*f*g^2-30*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+
2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c^3*d^3*f^2*g
+15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/
2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c
*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+6*x^2*c^
2*d^2*g^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-28*(c*d*g)^(1/2)*((c*d*
x+a*e)*(g*x+f))^(1/2)*x*a*c*d*e*g^2+40*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^
(1/2)*x*c^2*d^2*f*g-4*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2
-20*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*f*g+30*((c*d*x+a*e)*(
g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(3/2)/(e*x+d)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(5/2)), x)
```

Fricas [A] time = 5.88059, size = 2059, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g - 2*a^2*e^2*g
^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^3*f^3 - a*c*d^2*e*f^2*g
 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2*e*f^2*g - a*c*d^2*e*g
^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*f^3 - 2*a*c*d^2*e*f*g^
2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3
 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g
 + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
(c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f
^2*g^3 + 2*d*f*g^4)*x), 1/6*(2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c
*d*e*f*g - 2*a^2*e^2*g^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3
```

```
*f^3 - a*c*d^2*e*f^2*g + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2
*e*f^2*g - a*c*d^2*e*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*
f^3 - 2*a*c*d^2*e*f*g^2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(-c*d/g)*ar
ctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2
 + a*e^2)*g)*x)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2
*g^3 + 2*d*f*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(5/2)), x)
```

3.756 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}+\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+ade+cde\right.}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[Out] (-2*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e*x]
*Sqrt[f + g*x]) - (2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*(f + g*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)*(f + g*x)^(5/2)) + (2*c^(5/2)*d^(5/2)*Sq
rt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*
Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])
```

Rubi [A] time = 0.36601, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {862, 891, 63, 217, 206}

$$-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}+\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+ade+cde\right.}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (-2*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e*x]
*Sqrt[f + g*x]) - (2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*(f + g*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)*(f + g*x)^(5/2)) + (2*c^(5/2)*d^(5/2)*Sq
rt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*
Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a +
 b*x + c*x^2)^p)/(g*(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

```
&& !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+\frac{(cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx}{g}$$

$$= -\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}+$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A] time = 1.42852, size = 224, normalized size = 0.82

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{15\sqrt{c}\sqrt{d}(cdf-aeg)^{5/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cd}\sqrt{cdf-aeg}}\right)}{\sqrt{cd}\sqrt{ae+cdx}}-\sqrt{g}\left(3a^2e^2g^2+acdeg(5f+11gx)+c^2d^2\left(15f^2+\right.\right.}{15g^{7/2}\sqrt{d+ex}(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]*(3*a^2*e^2*g^2 + a*c*d*e*g*(5*f
 + 11*g*x) + c^2*d^2*(15*f^2 + 35*f*g*x + 23*g^2*x^2))) + (15*Sqrt[c]*Sqrt[
d]*(c*d*f - a*e*g)^(5/2)*((c*d*(f + g*x))/(c*d*f - a*e*g))^(5/2)*ArcSinh[(S
qrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d*f - a*e*g])])
/(Sqrt[c*d]*Sqrt[a*e + c*d*x])))/(15*g^(7/2)*Sqrt[d + e*x]*(f + g*x)^(5/2))
```

Maple [B] time = 0.415, size = 511, normalized size = 1.9

$$\frac{1}{15g^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\ln\left(1/2\,\frac{2\,xcdg+aeg+cdf+2\,\sqrt{(cdx+ae)\left(gx+f\right)}\sqrt{cdg}}{\sqrt{cdg}}\right)x^3c^3d^3g^3+45\ln\left(1/2\,\frac{2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/2),x)
```

```
[Out] 1/15*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*ln(1/2*(2*x*c*d*g+a*e*g+c*
d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x^3*c^3*d^3
*g^3+45*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*x^2*c^3*d^3*f*g^2+45*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2
*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*x*c^3*d^3*f^2*g+
15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2
))/(c*d*g)^(1/2))*c^3*d^3*f^3-46*x^2*c^2*d^2*g^2*((c*d*x+a*e)*(g*x+f))^(1/2
)*(c*d*g)^(1/2)-22*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*a*c*d*e*g^2-
70*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*x*c^2*d^2*f*g-6*((c*d*x+a*e)*(
g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-10*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d
*g)^(1/2)*a*c*d*e*f*g-30*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*
f^2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(5/2)/(e*x+d)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(7/2)), x)
```

Fricas [A] time = 5.70247, size = 1974, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] [-1/30*(4*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2*
g^2 + (35*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^2*e*g^3*x^4 + c^2*d^3*f
^3 + (3*c^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d^3*f
*g^2)*x^2 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^
2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*
x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*
g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e
+ a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)
*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x), -1/15*(2
*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2*g^2 + (35
*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^2*e*g^3*x^4 + c^2*d^3*f^3 + (3*c
^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d^3*f*g^2)*x^2
 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(
2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e
*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^
2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(7/2)), x)
```

3.757 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(7/2))
```

Rubi [A] time = 0.0689705, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {860}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

Mathematica [A] time = 0.0826987, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f +
 g*x)^(7/2))
```

Maple [A] time = 0.049, size = 63, normalized size = 1.

$$-\frac{2\,cdx+2\,ae}{7\,aeg-7\,cdf}\left(cdex^{2}+ae^{2}x+cd^{2}x+ade\right)^{\frac{5}{2}}\left(gx+f\right)^{-\frac{7}{2}}(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/2),x)
```

```
[Out] -2/7*(c*d*x+a*e)/(g*x+f)^(7/2)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(5/2)/(e*x+d)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(9/2)), x)
```

Fricas [B] time = 1.52042, size = 625, normalized size = 9.92

$$\frac{2\left(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}ex^{2}+3\,a^{2}cde^{2}x+a^{3}e^{3}\right)\sqrt{c}}{7\left(cd^{2}f^{5}-adef^{4}g+\left(cdefg^{4}-ae^{2}g^{5}\right)x^{5}+\left(4\,cdef^{2}g^{3}-adeg^{5}+\left(cd^{2}-4\,ae^{2}\right)fg^{4}\right)x^{4}+2\left(3\,cdef^{3}g^{2}-2\,adefg^{4}+\left(2\,c\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^5 -
 a*d*e*f^4*g + (c*d*e*f*g^4 - a*e^2*g^5)*x^5 + (4*c*d*e*f^2*g^3 - a*d*e*g^5
 + (c*d^2 - 4*a*e^2)*f*g^4)*x^4 + 2*(3*c*d*e*f^3*g^2 - 2*a*d*e*f*g^4 + (2*c
*d^2 - 3*a*e^2)*f^2*g^3)*x^3 + 2*(2*c*d*e*f^4*g - 3*a*d*e*f^2*g^3 + (3*c*d^
2 - 2*a*e^2)*f^3*g^2)*x^2 + (c*d*e*f^5 - 4*a*d*e*f^3*g^2 + (4*c*d^2 - a*e^2
)*f^4*g)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\left(gx + f\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(9/2)), x)
```

3.758 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=129

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rubi [A] time = 0.148876, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{(2cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}$$
$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{7/2}}$$

Mathematica [A] time = 0.0874178, size = 79, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}(cd(9f+2gx)-7aeg)}{63\sqrt{d+ex}(f+gx)^{9/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-7*a*e*g + c*d*(9*f + 2*g
*x)))/(63*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(9/2))
```

Maple [A] time = 0.053, size = 99, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-2\,xcdg+7\,aeg-9\,cdf\right)}{63\,a^2e^2g^2-126\,acdefg+63\,c^2d^2f^2}\left(cdex^2+ae^2x+cd^2x+ade\right)^{\frac{5}{2}}\left(gx+f\right)^{-\frac{9}{2}}(ex+d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11/2),x)
```

```
[Out] -2/63*(c*d*x+a*e)*(-2*c*d*g*x+7*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(5/2)/(g*x+f)^(9/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(11/2)), x)
```

Fricas [B] time = 1.70142, size = 1276, normalized size = 9.89

$$\overline{63\left(c^2d^3f^7-2\,acd^2ef^6g+a^2de^2f^5g^2+\left(c^2d^2ef^2g^5-2\,acde^2fg^6+a^2e^3g^7\right)x^6+\left(5\,c^2d^2ef^3g^4+a^2de^2g^7+\left(c^2d^3-10\,ac\ldots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 7*a^4*e^4*g + (9*c^4*d^4*f - a*c^
3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f - 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2*c^
2*d^2*e^2*f - 19*a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^7 - 2*a*c*d^2*e*f^6*g + a^2*d*e^
2*f^5*g^2 + (c^2*d^2*e*f^2*g^5 - 2*a*c*d*e^2*f*g^6 + a^2*e^3*g^7)*x^6 + (5*
c^2*d^2*e*f^3*g^4 + a^2*d*e^2*g^7 + (c^2*d^3 - 10*a*c*d*e^2)*f^2*g^5 - (2*a
*c*d^2*e - 5*a^2*e^3)*f*g^6)*x^5 + 5*(2*c^2*d^2*e*f^4*g^3 + a^2*d*e^2*f*g^6
 + (c^2*d^3 - 4*a*c*d*e^2)*f^3*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f^2*g^5)*x^4 +
 10*(c^2*d^2*e*f^5*g^2 + a^2*d*e^2*f^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f^4*g^
3 - (2*a*c*d^2*e - a^2*e^3)*f^3*g^4)*x^3 + 5*(c^2*d^2*e*f^6*g + 2*a^2*d*e^2
*f^3*g^4 + 2*(c^2*d^3 - a*c*d*e^2)*f^5*g^2 - (4*a*c*d^2*e - a^2*e^3)*f^4*g^
3)*x^2 + (c^2*d^2*e*f^7 + 5*a^2*d*e^2*f^4*g^3 + (5*c^2*d^3 - 2*a*c*d*e^2)*f
^6*g - (10*a*c*d^2*e - a^2*e^3)*f^5*g^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(11/2)), x)
```

3.759 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx$

Optimal. Leaf size=198

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(11/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(99*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(693*(c*d*f - a*e*g)^3*(
d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rubi [A] time = 0.229637, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(11/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(99*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(693*(c*d*f - a*e*g)^3*(
d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{(4cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}dx}{11(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}$$

Mathematica [A] time = 0.118473, size = 115, normalized size = 0.58

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(63a^2e^2g^2-14acdeg(11f+2gx)+c^2d^2\left(99f^2+44fgx+8g^2x^2\right)\right)}{693\sqrt{d+ex}(f+gx)^{11/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(63*a^2*e^2*g^2 - 14*a*c*d
*e*g*(11*f + 2*g*x) + c^2*d^2*(99*f^2 + 44*f*g*x + 8*g^2*x^2)))/(693*(c*d*f
 - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^(11/2))
```

Maple [A] time = 0.053, size = 169, normalized size = 0.9

$$-\frac{(2\,cdx+2\,ae)\left(8\,c^2d^2g^2x^2-28\,acdeg^2x+44\,c^2d^2fgx+63\,a^2e^2g^2-154\,acdefg+99\,c^2d^2f^2\right)}{693\,a^3e^3g^3-2079\,a^2cde^2fg^2+2079\,ac^2d^2ef^2g-693\,c^3d^3f^3}\left(cdex^2+ae^2x+cd^2x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13/2),x)
```

```
[Out] -2/693*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+44*c^2*d^2*f*g*x+63*
a^2*e^2*g^2-154*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(5/2)/(g*x+f)^(11/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*
g-c^3*d^3*f^3)/(e*x+d)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{13}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(13/2)), x)
```

Fricas [B] time = 1.75339, size = 2198, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/693*(8*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 154*a^4*c*d*e^4*f*g + 6
3*a^5*e^5*g^2 + 4*(11*c^5*d^5*f*g - a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^2
- 22*a*c^4*d^4*e*f*g + 3*a^2*c^3*d^3*e^2*g^2)*x^3 + (297*a*c^4*d^4*e*f^2 -
330*a^2*c^3*d^3*e^2*f*g + 113*a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d^3*e
^2*f^2 - 418*a^3*c^2*d^2*e^3*f*g + 161*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^9 - 3*a*
c^2*d^3*e*f^8*g + 3*a^2*c*d^2*e^2*f^7*g^2 - a^3*d*e^3*f^6*g^3 + (c^3*d^3*e*
f^3*g^6 - 3*a*c^2*d^2*e^2*f^2*g^7 + 3*a^2*c*d*e^3*f*g^8 - a^3*e^4*g^9)*x^7
+ (6*c^3*d^3*e*f^4*g^5 - a^3*d*e^3*g^9 + (c^3*d^4 - 18*a*c^2*d^2*e^2)*f^3*g
^6 - 3*(a*c^2*d^3*e - 6*a^2*c*d*e^3)*f^2*g^7 + 3*(a^2*c*d^2*e^2 - 2*a^3*e^4
)*f*g^8)*x^6 + 3*(5*c^3*d^3*e*f^5*g^4 - 2*a^3*d*e^3*f*g^8 + (2*c^3*d^4 - 15
*a*c^2*d^2*e^2)*f^4*g^5 - 3*(2*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^3*g^6 + (6*a^
2*c*d^2*e^2 - 5*a^3*e^4)*f^2*g^7)*x^5 + 5*(4*c^3*d^3*e*f^6*g^3 - 3*a^3*d*e^
3*f^2*g^7 + 3*(c^3*d^4 - 4*a*c^2*d^2*e^2)*f^5*g^4 - 3*(3*a*c^2*d^3*e - 4*a^
2*c*d*e^3)*f^4*g^5 + (9*a^2*c*d^2*e^2 - 4*a^3*e^4)*f^3*g^6)*x^4 + 5*(3*c^3*
d^3*e*f^7*g^2 - 4*a^3*d*e^3*f^3*g^6 + (4*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^6*g^3
 - 3*(4*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^5*g^4 + 3*(4*a^2*c*d^2*e^2 - a^3*e^4
)*f^4*g^5)*x^3 + 3*(2*c^3*d^3*e*f^8*g - 5*a^3*d*e^3*f^4*g^5 + (5*c^3*d^4 -
6*a*c^2*d^2*e^2)*f^7*g^2 - 3*(5*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^6*g^3 + (15*
a^2*c*d^2*e^2 - 2*a^3*e^4)*f^5*g^4)*x^2 + (c^3*d^3*e*f^9 - 6*a^3*d*e^3*f^5*
g^4 + 3*(2*c^3*d^4 - a*c^2*d^2*e^2)*f^8*g - 3*(6*a*c^2*d^3*e - a^2*c*d*e^3)
*f^7*g^2 + (18*a^2*c*d^2*e^2 - a^3*e^4)*f^6*g^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(13/2)), x)
```

3.760 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx$

Optimal. Leaf size=267

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/}}{143(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg}$$

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(13/2)) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(7/2))/(143*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(11/2)) + (16*c
^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*(c*d*f - a*e*g)^
3*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(3003*(c*d*f - a*e*g)^4*(d + e*x)^(7/2)*(f + g*x)^(7/2
))
```

Rubi [A] time = 0.323646, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {872, 860}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/}}{143(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(15/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(13/2)) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(7/2))/(143*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(11/2)) + (16*c
^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*(c*d*f - a*e*g)^
3*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(3003*(c*d*f - a*e*g)^4*(d + e*x)^(7/2)*(f + g*x)^(7/2
))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 860

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] /;
FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 -
4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0
] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{(6cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx}{13(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/}}$$

Mathematica [A] time = 0.158708, size = 162, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(63a^2cde^2g^2(13f+2gx)-231a^3e^3g^3-7ac^2d^2eg\left(143f^2+52fgx+8g^2x^2\right)+c^3d^3\left(2\right.\right.}{3003\sqrt{d+ex}(f+gx)^{13/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(15/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-231*a^3*e^3*g^3 + 63*a^2
*c*d*e^2*g^2*(13*f + 2*g*x) - 7*a*c^2*d^2*e*g*(143*f^2 + 52*f*g*x + 8*g^2*x
^2) + c^3*d^3*(429*f^3 + 286*f^2*g*x + 104*f*g^2*x^2 + 16*g^3*x^3)))/(3003*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*(f + g*x)^(13/2))
```

Maple [A] time = 0.052, size = 260, normalized size = 1.

$$-\frac{(2\,cdx+2\,ae)\left(-16\,c^3d^3g^3x^3+56\,ac^2d^2eg^3x^2-104\,c^3d^3fg^2x^2-126\,a^2cde^2g^3x+364\,ac^2d^2efg^2x-286\,c^3d^3f^2gx+2\right.}{3003\,g^4e^4a^4-12012\,cdg^3fe^3a^3+18018\,c^2d^2g^2f^2e^2a^2-12012\,c^3d^3gf^3ea}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15/2),x)
```

```
[Out] -2/3003*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+56*a*c^2*d^2*e*g^3*x^2-104*c^3*d^3
*f*g^2*x^2-126*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-286*c^3*d^3*f^2*g*
x+231*a^3*e^3*g^3-819*a^2*c*d*e^2*f*g^2+1001*a*c^2*d^2*e*f^2*g-429*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/(g*x+f)^(13/2)/(a^4*e^4*g^4-4*
a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f^4
)/(e*x+d)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(15/2)), x)
```

Fricas [B] time = 1.95941, size = 3291, normalized size = 12.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 1001*a^4*c^2*d^2*e^4
*f^2*g + 819*a^5*c*d*e^5*f*g^2 - 231*a^6*e^6*g^3 + 8*(13*c^6*d^6*f*g^2 - a*
c^5*d^5*e*g^3)*x^5 + 2*(143*c^6*d^6*f^2*g - 26*a*c^5*d^5*e*f*g^2 + 3*a^2*c^
4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 - 143*a*c^5*d^5*e*f^2*g + 39*a^2*c^4*
d^4*e^2*f*g^2 - 5*a^3*c^3*d^3*e^3*g^3)*x^3 + (1287*a*c^5*d^5*e*f^3 - 2145*a
^2*c^4*d^4*e^2*f^2*g + 1469*a^3*c^3*d^3*e^3*f*g^2 - 371*a^4*c^2*d^2*e^4*g^3
)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 - 2717*a^3*c^3*d^3*e^3*f^2*g + 2093*a^4*c
^2*d^2*e^4*f*g^2 - 567*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^11 - 4*a*c^3*d^4*e*f^10*
g + 6*a^2*c^2*d^3*e^2*f^9*g^2 - 4*a^3*c*d^2*e^3*f^8*g^3 + a^4*d*e^4*f^7*g^4
 + (c^4*d^4*e*f^4*g^7 - 4*a*c^3*d^3*e^2*f^3*g^8 + 6*a^2*c^2*d^2*e^3*f^2*g^9
 - 4*a^3*c*d*e^4*f*g^10 + a^4*e^5*g^11)*x^8 + (7*c^4*d^4*e*f^5*g^6 + a^4*d*
e^4*g^11 + (c^4*d^5 - 28*a*c^3*d^3*e^2)*f^4*g^7 - 2*(2*a*c^3*d^4*e - 21*a^2
*c^2*d^2*e^3)*f^3*g^8 + 2*(3*a^2*c^2*d^3*e^2 - 14*a^3*c*d*e^4)*f^2*g^9 - (4
*a^3*c*d^2*e^3 - 7*a^4*e^5)*f*g^10)*x^7 + 7*(3*c^4*d^4*e*f^6*g^5 + a^4*d*e^
4*f*g^10 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^5*g^6 - 2*(2*a*c^3*d^4*e - 9*a^2*
c^2*d^2*e^3)*f^4*g^7 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^8 - (4*a^3
*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^9)*x^6 + 7*(5*c^4*d^4*e*f^7*g^4 + 3*a^4*d*e^4
*f^2*g^9 + (3*c^4*d^5 - 20*a*c^3*d^3*e^2)*f^6*g^5 - 6*(2*a*c^3*d^4*e - 5*a^
2*c^2*d^2*e^3)*f^5*g^6 + 2*(9*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^4*g^7 - (
12*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^3*g^8)*x^5 + 35*(c^4*d^4*e*f^8*g^3 + a^4*d*
e^4*f^3*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^7*g^4 - 2*(2*a*c^3*d^4*e - 3*a^
2*c^2*d^2*e^3)*f^6*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^5*g^6 - (4
*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^7)*x^4 + 7*(3*c^4*d^4*e*f^9*g^2 + 5*a^4*d*e
^4*f^4*g^7 + (5*c^4*d^5 - 12*a*c^3*d^3*e^2)*f^8*g^3 - 2*(10*a*c^3*d^4*e - 9
*a^2*c^2*d^2*e^3)*f^7*g^4 + 6*(5*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^6*g^5 -
 (20*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^5*g^6)*x^3 + 7*(c^4*d^4*e*f^10*g + 3*a^4*
d*e^4*f^5*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^9*g^2 - 6*(2*a*c^3*d^4*e -
a^2*c^2*d^2*e^3)*f^8*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^7*g^4 -
(12*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^5)*x^2 + (c^4*d^4*e*f^11 + 7*a^4*d*e^4*f
^6*g^5 + (7*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^10*g - 2*(14*a*c^3*d^4*e - 3*a^2*c
^2*d^2*e^3)*f^9*g^2 + 2*(21*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^8*g^3 - (28*
a^3*c*d^2*e^3 - a^4*e^5)*f^7*g^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}\left(gx + f\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g
*x + f)^(15/2)), x)
```

3.761 $\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(d+ex)^{5/2}(f+gx)^{n+1}(ae+cdx)\,_2F_1\left(1,n-\frac{1}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}$$

```
[Out] -(((a*e + c*d*x)*(d + e*x)^(5/2)*(f + g*x)^(1 + n)*Hypergeometric2F1[1, -1/
2 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/((c*d*f - a*e*g)*(1 + n)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)))
```

Rubi [A] time = 0.119212, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((g*(a*e +
 c*d*x))/(c*d*f - a*e*g))])/(3*c*d*(a*e + c*d*x)*((c*d*(f + g*x))/(c*d*f -
a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{(ae+cdx)^{5/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$=\frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{5/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$=-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.085808, size = 100, normalized size = 0.96

$$-\frac{2(d+ex)^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, (g*(a*e +
 c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*((c*d*
(f + g*x))/(c*d*f - a*e*g))^n)
```

Maple [F] time = 1.763, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^n(ex+d)^{\frac{5}{2}}\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}\left(gx+f\right)^n}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex + d}\left(gx + f\right)^n}{c^3d^3ex^4 + a^3de^3 + \left(c^3d^4 + 3\,ac^2d^2e^2\right)x^3 + 3\,\left(ac^2d^3e + a^2cde^3\right)x^2 + \left(3\,a^2cd^2e^2 + a^3e^4\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c^3*d^3*e*x^4 + a^3*d*e^3 + (c^3*d^4 + 3*a*c^2*d^2*e^2)*x^3 + 3*(a*c^2
*d^3*e + a^2*c*d*e^3)*x^2 + (3*a^2*c*d^2*e^2 + a^3*e^4)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.762 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(d+ex)^{3/2}(f+gx)^{n+1}(ae+cdx)\,{}_2F_1\left(1,n+\frac{1}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

```
[Out] -(((a*e + c*d*x)*(d + e*x)^(3/2)*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 1/2
 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/((c*d*f - a*e*g)*(1 + n)*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)))
```

Rubi [A] time = 0.110115, antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((g*(a*e +
c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g))^n*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{(ae+cdx)^{3/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$=\frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{3/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$=-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A] time = 0.0467142, size = 98, normalized size = 0.94

$$-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, (g*(a*e + c*
d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*
x))/(c*d*f - a*e*g))^n)
```

Maple [F] time = 1.721, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^n(ex+d)^{\frac{3}{2}}\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^n}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{ex+d}\left(gx+f\right)^n}{c^2d^2ex^3 + a^2de^2 + \left(c^2d^3 + 2\,acde^2\right)x^2 + \left(2\,acd^2e + a^2e^3\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c^2*d^2*e*x^3 + a^2*d*e^2 + (c^2*d^3 + 2*a*c*d*e^2)*x^2 + (2*a*c*d^2*e
 + a^2*e^3)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] sage0*x
```

3.763 $\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{d+ex}(f+gx)^{n+1}(ae+cdx)\,{}_2F_1\left(1,n+\frac{3}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

```
[Out] -(((a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 3/2 +
 n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/((c*d*f - a*e*g)*(1 + n)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]))
```

Rubi [A] time = 0.106365, antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (2*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
-((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g
))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx &= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{\sqrt{ae+cdx}}\,dx}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&= \frac{\left(\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{\sqrt{ae+cdx}}\,dx}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\\
&= \frac{2(ae+cdx)\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}
\end{aligned}
$$

Mathematica [A] time = 0.0465375, size = 98, normalized size = 0.94

$$
\frac{2(f+gx)^n\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{d+ex}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2
, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[d + e*x]*((c*d*(f + g*x)
)/(c*d*f - a*e*g))^n)
```

Maple [F] time = 1.765, size = 0, normalized size = 0.

$$
\int (gx+f)^n\sqrt{ex+d}\frac{1}{\sqrt{ade+(ae^2+cd^2)x+cdex^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\,(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

3.764 $\int \frac{(f+gx)^n \sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}} \, dx$

Optimal. Leaf size=104

$$-\frac{(f+gx)^{n+1}(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,{}_2F_1\left(1,n+\frac{5}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\sqrt{d+ex}(cdf-aeg)}$$

```
[Out] -(((a*e + c*d*x)*(f + g*x)^(1 + n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]*Hypergeometric2F1[1, 5/2 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/
((c*d*f - a*e*g)*(1 + n)*Sqrt[d + e*x]))
```

Rubi [A] time = 0.102002, antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2(f+gx)^n(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (2*(a*e + c*d*x)*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*Hy
pergeometric2F1[3/2, -n, 5/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(3*c*d
*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(f+gx)^n\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\int \sqrt{ae+cdx}(f+gx)^n\,dx}{\sqrt{ae+cdx}\sqrt{d+ex}}$$
$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\right)\int \sqrt{ae+cdx}\left(\frac{cd}{cdf-}\right.}{\sqrt{ae+cdx}\sqrt{d+ex}}$$
$$= \frac{2(ae+cdx)(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,{}_2F_1\left(\frac{3}{2},-\right.}{3cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.0721161, size = 100, normalized size = 0.96

$$\frac{2(f+gx)^n((d+ex)(ae+cdx))^{3/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x)^n*Hypergeometric2F1[3/2, -n, 5
/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*(d + e*x)^(3/2)*((c*d*(f
 + g*x))/(c*d*f - a*e*g))^n)
```

Maple [F] time = 1.722, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^n\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}\frac{1}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^n}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^n}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x +
 d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^n}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
+ d), x)
```

3.765 $\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(f+gx)^{n+1}(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}{}_2F_1\left(1,n+\frac{7}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(d+ex)^{3/2}(cdf-aeg)}$$

```
[Out] -(((a*e + c*d*x)*(f + g*x)^(1 + n)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)*Hypergeometric2F1[1, 7/2 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)]
)/((c*d*f - a*e*g)*(1 + n)*(d + e*x)^(3/2)))
```

Rubi [A] time = 0.109082, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2(f+gx)^n(ae+cdx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (2*(a*e + c*d*x)^2*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*
Hypergeometric2F1[5/2, -n, 7/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(5*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\int (ae+cdx)^{3/2}(f+gx)^n\,dx}{\sqrt{ae+cdx}\sqrt{d+ex}}$$
$$= \frac{\left((f+gx)^n \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\right)\int (ae+cdx)^{\cdot}}{\sqrt{ae+cdx}\sqrt{d+ex}}$$
$$= \frac{2(ae+cdx)^2(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,{}_2F}{5cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.108329, size = 100, normalized size = 0.96

$$\frac{2(f+gx)^n((d+ex)(ae+cdx))^{5/2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(f + g*x)^n*Hypergeometric2F1[5/2, -n, 7
/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*c*d*(d + e*x)^(5/2)*((c*d*(f
+ g*x))/(c*d*f - a*e*g))^n)
```

Maple [F] time = 1.656, size = 0, normalized size = 0.

$$\int (gx+f)^n \left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^{\frac{3}{2}}(ex+d)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdx + ae)(gx + f)^n}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*(g*x + f
)^n/sqrt(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(gx + f)^n}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
d)^(3/2), x)
```

3.766 $\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(f+gx)^{n+1}(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}{}_2F_1\left(1,n+\frac{9}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(d+ex)^{5/2}(cdf-aeg)}$$

```
[Out] -(((a*e + c*d*x)*(f + g*x)^(1 + n)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2)*Hypergeometric2F1[1, 9/2 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)]
)/((c*d*f - a*e*g)*(1 + n)*(d + e*x)^(5/2)))
```

Rubi [A] time = 0.111715, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2(f+gx)^n(ae+cdx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{7}{2},-n;\frac{9}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*
Hypergeometric2F1[7/2, -n, 9/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(7*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\int (ae+cdx)^{5/2}(f+gx)^n\,dx}{\sqrt{ae+cdx}\sqrt{d+ex}}$$
$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\right)\int (ae+cdx)^{5/2}\left(\right.}{\sqrt{ae+cdx}\sqrt{d+ex}}$$
$$= \frac{2(ae+cdx)^3(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,{}_2F_1\left(\right.}{7cd\sqrt{d+ex}}$$

Mathematica [A] time = 0.0723055, size = 110, normalized size = 1.06

$$\frac{2(f+gx)^n(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{7}{2},-n;\frac{9}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric
2F1[7/2, -n, 9/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*c*d*Sqrt[d + e*
x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Maple [F] time = 1.705, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^n\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^{\frac{5}{2}}(ex+d)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^n}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(e*x +
d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^n}{\sqrt{ex+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\left(gx+f\right)^n}{\left(ex+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(e*x +
d)^(5/2), x)
```

3.767 $\int (d+ex)^m (f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal. Leaf size=103

$$-\frac{(d+ex)^m (f+gx)^{n+1} (ae+cdx) \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} {}_2F_1\left(1,-m+n+2;n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(cdf-aeg)}$$

```
[Out] -(((a*e + c*d*x)*(d + e*x)^m*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 2 - m +
 n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/((c*d*f - a*e*g)*(1 + n)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m))
```

Rubi [A] time = 0.0854224, antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {891, 70, 69}

$$\frac{(d+ex)^m (f+gx)^{n+1} \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(m,n+1;n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{g(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hyp
ergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 + n
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int$$
$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-}\right.$$
$$= \frac{\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+\right.}{g(1+n)}$$

Mathematica [A] time = 0.0545406, size = 95, normalized size = 0.92

$$\frac{(d+ex)^m(f+gx)^{n+1}((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(m,n+1;n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{g(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] (((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hyp
ergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 + n
)*((a*e + c*d*x)*(d + e*x))^m)
```

Maple [F] time = 1.717, size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^n (ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m\left(gx+f\right)^n}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m\left(gx+f\right)^n}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m\left(gx+f\right)^n}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

3.768 $\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal. Leaf size=343

$$-\frac{6(d+ex)^{m-1}(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}+\frac{6g(d+ex)^m(cdf-}{c^3d}$$

```
[Out] (-6*(c*d*f - a*e*g)^2*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)
^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^4*d^4*e*(1 -
m)*(2 - m)*(3 - m)*(4 - m)) + (6*g*(c*d*f - a*e*g)^2*(d + e*x)^m*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(2 - m)*(3 - m)*(4 - m))
+ (3*(c*d*f - a*e*g)*(d + e*x)^(-1 + m)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*(3 - m)*(4 - m)) + ((d + e*x)^(-1 + m)*(
f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(4 - m))
```

Rubi [A] time = 0.449159, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {870, 794, 648}

$$-\frac{6(d+ex)^{m-1}(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}+\frac{6g(d+ex)^m(cdf-}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```


```
[Out] (-6*(c*d*f - a*e*g)^2*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)
^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^4*d^4*e*(1 -
m)*(2 - m)*(3 - m)*(4 - m)) + (6*g*(c*d*f - a*e*g)^2*(d + e*x)^m*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(2 - m)*(3 - m)*(4 - m))
+ (3*(c*d*f - a*e*g)*(d + e*x)^(-1 + m)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*(3 - m)*(4 - m)) + ((d + e*x)^(-1 + m)*(
f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(4 - m))
```


Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(4-m)} + \frac{(3$$

$$= \frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cde}{c^2d^2(3-m)(4-m)}$$

$$= \frac{6g(cdf-aeg)^2(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3e(2-m)(3-m)(4-m)} +$$

$$= -\frac{6(cdf-aeg)^2\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-}}{c^4d^4e(1-m)(2-m)(3-m)(4}$$

Mathematica [A] time = 0.16738, size = 134, normalized size = 0.39

$$\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\left(\frac{3g^2(ae+cdx)^2(aeg-cdf)}{m-3}-\frac{3g(ae+cdx)(cdf-aeg)^2}{m-2}-\frac{(cdf-aeg)^3}{m-1}-\frac{g^3(ae+cdx)^3}{m-4}\right)}{c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(-((c*d*f - a*e*g)^3/
(-1 + m)) - (3*g*(c*d*f - a*e*g)^2*(a*e + c*d*x))/(-2 + m) + (3*g^2*(-(c*d*
f) + a*e*g)*(a*e + c*d*x)^2)/(-3 + m) - (g^3*(a*e + c*d*x)^3)/(-4 + m)))/(c
^4*d^4)
```

Maple [A] time = 0.051, size = 527, normalized size = 1.5

$$-\frac{(ex+d)^m\left(c^3d^3g^3m^3x^3+3c^3d^3fg^2m^3x^2-6c^3d^3g^3m^2x^3+3ac^2d^2eg^3m^2x^2+3c^3d^3f^2gm^3x-21c^3d^3fg^2m^2x^2+11c^3d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] -(e*x+d)^m*(c^3*d^3*g^3*m^3*x^3+3*c^3*d^3*f*g^2*m^3*x^2-6*c^3*d^3*g^3*m^2*x
^3+3*a*c^2*d^2*e*g^3*m^2*x^2+3*c^3*d^3*f^2*g*m^3*x-21*c^3*d^3*f*g^2*m^2*x^2
+11*c^3*d^3*g^3*m*x^3+6*a*c^2*d^2*e*f*g^2*m^2*x-9*a*c^2*d^2*e*g^3*m*x^2+c^3
*d^3*f^3*m^3-24*c^3*d^3*f^2*g*m^2*x+42*c^3*d^3*f*g^2*m*x^2-6*c^3*d^3*g^3*x^
3+6*a^2*c*d*e^2*g^3*m*x+3*a*c^2*d^2*e*f^2*g*m^2-30*a*c^2*d^2*e*f*g^2*m*x+6*
a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f^3*m^2+57*c^3*d^3*f^2*g*m*x-24*c^3*d^3*f*g^2
*x^2+6*a^2*c*d*e^2*f*g^2*m-6*a^2*c*d*e^2*g^3*x-21*a*c^2*d^2*e*f^2*g*m+24*a*
c^2*d^2*e*f*g^2*x+26*c^3*d^3*f^3*m-36*c^3*d^3*f^2*g*x+6*a^3*e^3*g^3-24*a^2*
c*d*e^2*f*g^2+36*a*c^2*d^2*e*f^2*g-24*c^3*d^3*f^3)*(c*d*x+a*e)/((c*d*e*x^2+
```

```
a*e^2*x+c*d^2*x+a*d*e)^m)/c^4/d^4/(m^4-10*m^3+35*m^2-50*m+24)
```

Maxima [A] time = 1.15708, size = 447, normalized size = 1.3

$$-\frac{(cdx+ae)f^3}{(cdx+ae)^m cd(m-1)}-\frac{3\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)f^2g}{\left(m^2-3m+2\right)(cdx+ae)^m c^2d^2}-\frac{3\left(\left(m^2-3m+2\right)c^3d^3x^3+\left(m^2-m\right)ac^2d^2ex^2+2\right.}{\left(m^3-6m^2+11m-6\right)(cdx+ae\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^3/((c*d*x + a*e)^m*c*d*(m - 1)) - 3*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*e*m*x + a^2*e^2)*f^2*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - 3
*((m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m
*x + 2*a^3*e^3)*f*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3) -
((m^3 - 6*m^2 + 11*m - 6)*c^4*d^4*x^4 + (m^3 - 3*m^2 + 2*m)*a*c^3*d^3*e*x^3
 + 3*(m^2 - m)*a^2*c^2*d^2*e^2*x^2 + 6*a^3*c*d*e^3*m*x + 6*a^4*e^4)*g^3/((m
^4 - 10*m^3 + 35*m^2 - 50*m + 24)*(c*d*x + a*e)^m*c^4*d^4)
```

Fricas [B] time = 1.4557, size = 1396, normalized size = 4.07

$$-\frac{\left(ac^3d^3ef^3m^3-24\,ac^3d^3ef^3+36\,a^2c^2d^2e^2f^2g-24\,a^3cde^3fg^2+6\,a^4e^4g^3+\left(c^4d^4g^3m^3-6\,c^4d^4g^3m^2+11\,c^4d^4g^3m-6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(a*c^3*d^3*e*f^3*m^3 - 24*a*c^3*d^3*e*f^3 + 36*a^2*c^2*d^2*e^2*f^2*g - 24*
a^3*c*d*e^3*f*g^2 + 6*a^4*e^4*g^3 + (c^4*d^4*g^3*m^3 - 6*c^4*d^4*g^3*m^2 +
11*c^4*d^4*g^3*m - 6*c^4*d^4*g^3)*x^4 - (24*c^4*d^4*f*g^2 - (3*c^4*d^4*f*g^
2 + a*c^3*d^3*e*g^3)*m^3 + 3*(7*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m^2 - 2*(2
1*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m)*x^3 - 3*(3*a*c^3*d^3*e*f^3 - a^2*c^2*
d^2*e^2*f^2*g)*m^2 - 3*(12*c^4*d^4*f^2*g - (c^4*d^4*f^2*g + a*c^3*d^3*e*f*g
^2)*m^3 + (8*c^4*d^4*f^2*g + 5*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m^2
 - (19*c^4*d^4*f^2*g + 4*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m)*x^2 +
(26*a*c^3*d^3*e*f^3 - 21*a^2*c^2*d^2*e^2*f^2*g + 6*a^3*c*d*e^3*f*g^2)*m - (
24*c^4*d^4*f^3 - (c^4*d^4*f^3 + 3*a*c^3*d^3*e*f^2*g)*m^3 + 3*(3*c^4*d^4*f^3
 + 7*a*c^3*d^3*e*f^2*g - 2*a^2*c^2*d^2*e^2*f*g^2)*m^2 - 2*(13*c^4*d^4*f^3 +
 18*a*c^3*d^3*e*f^2*g - 12*a^2*c^2*d^2*e^2*f*g^2 + 3*a^3*c*d*e^3*g^3)*m)*x)
*(e*x + d)^m/((c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m
 + 24*c^4*d^4)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.42207, size = 2732, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((x*e + d)^m*c^4*d^4*g^3*m^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(x*e + d))
+ 3*(x*e + d)^m*c^4*d^4*f*g^2*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d)) - 6*(x*e + d)^m*c^4*d^4*g^3*m^2*x^4*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) + (x*e + d)^m*a*c^3*d^3*g^3*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*
e + d) + 1) + 3*(x*e + d)^m*c^4*d^4*f^2*g*m^3*x^2*e^(-m*log(c*d*x + a*e) -
m*log(x*e + d)) - 21*(x*e + d)^m*c^4*d^4*f*g^2*m^2*x^3*e^(-m*log(c*d*x + a*
e) - m*log(x*e + d)) + 11*(x*e + d)^m*c^4*d^4*g^3*m*x^4*e^(-m*log(c*d*x + a
*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a*c^3*d^3*f*g^2*m^3*x^2*e^(-m*log(c*d
*x + a*e) - m*log(x*e + d) + 1) - 3*(x*e + d)^m*a*c^3*d^3*g^3*m^2*x^3*e^(-m
*log(c*d*x + a*e) - m*log(x*e + d) + 1) + (x*e + d)^m*c^4*d^4*f^3*m^3*x*e^(
-m*log(c*d*x + a*e) - m*log(x*e + d)) - 24*(x*e + d)^m*c^4*d^4*f^2*g*m^2*x^
2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 42*(x*e + d)^m*c^4*d^4*f*g^2*m
*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - 6*(x*e + d)^m*c^4*d^4*g^3*x
^4*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a*c^3*d^3*f^2*g
*m^3*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 15*(x*e + d)^m*a*c^3*
d^3*f*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) + 2*(x*e + d
)^m*a*c^3*d^3*g^3*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 9*(x
*e + d)^m*c^4*d^4*f^3*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 57*(
x*e + d)^m*c^4*d^4*f^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - 2
4*(x*e + d)^m*c^4*d^4*f*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) +
3*(x*e + d)^m*a^2*c^2*d^2*g^3*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 2) + (x*e + d)^m*a*c^3*d^3*f^3*m^3*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d) + 1) - 21*(x*e + d)^m*a*c^3*d^3*f^2*g*m^2*x*e^(-m*log(c*d*x + a*e) - m
*log(x*e + d) + 1) + 12*(x*e + d)^m*a*c^3*d^3*f*g^2*m*x^2*e^(-m*log(c*d*x +
 a*e) - m*log(x*e + d) + 1) + 26*(x*e + d)^m*c^4*d^4*f^3*m*x*e^(-m*log(c*d*
x + a*e) - m*log(x*e + d)) - 36*(x*e + d)^m*c^4*d^4*f^2*g*x^2*e^(-m*log(c*d
*x + a*e) - m*log(x*e + d)) + 6*(x*e + d)^m*a^2*c^2*d^2*f*g^2*m^2*x*e^(-m*l
og(c*d*x + a*e) - m*log(x*e + d) + 2) - 3*(x*e + d)^m*a^2*c^2*d^2*g^3*m*x^2
*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) - 9*(x*e + d)^m*a*c^3*d^3*f^3
*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) + 36*(x*e + d)^m*a*c^3*d^
3*f^2*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 24*(x*e + d)^m*c
^4*d^4*f^3*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a^2*c
^2*d^2*f^2*g*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) - 24*(x*e + d
)^m*a^2*c^2*d^2*f*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) + 26
*(x*e + d)^m*a*c^3*d^3*f^3*m*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) +
 6*(x*e + d)^m*a^3*c*d*g^3*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 3)
 - 21*(x*e + d)^m*a^2*c^2*d^2*f^2*g*m*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 2) - 24*(x*e + d)^m*a*c^3*d^3*f^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
 d) + 1) + 6*(x*e + d)^m*a^3*c*d*f*g^2*m*e^(-m*log(c*d*x + a*e) - m*log(x*e
 + d) + 3) + 36*(x*e + d)^m*a^2*c^2*d^2*f^2*g*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 2) - 24*(x*e + d)^m*a^3*c*d*f*g^2*e^(-m*log(c*d*x + a*e) - m*l
og(x*e + d) + 3) + 6*(x*e + d)^m*a^4*g^3*e^(-m*log(c*d*x + a*e) - m*log(x*e
 + d) + 4))/(c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m +
```

```
24*c^4*d^4)
```

3.769 $\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=246

$$-\frac{2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^3d^3e(1-m)(2-m)(3-m)}+\frac{2g(d+ex)^m(cdf-aeg}{c^2d^2}$$

```
[Out] (-2*(c*d*f - a*e*g)*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(
-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(1 - m)
*(2 - m)*(3 - m)) + (2*g*(c*d*f - a*e*g)*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(2 - m)*(3 - m)) + ((d + e*x)^(-1 + m
)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(3 - m)
)
```

Rubi [A] time = 0.204594, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {870, 794, 648}

$$-\frac{2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^3d^3e(1-m)(2-m)(3-m)}+\frac{2g(d+ex)^m(cdf-aeg}{c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(
-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(1 - m)
*(2 - m)*(3 - m)) + (2*g*(c*d*f - a*e*g)*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(2 - m)*(3 - m)) + ((d + e*x)^(-1 + m
)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(3 - m)
)
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
```

```
x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}dx=\frac{(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(3-m)}+$$
$$=\frac{2g(cdf-aeg)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(2-m)(3-m)}$$
$$=-\frac{2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex}{c^3d^3e(1-m)(2-m)(3}$$

Mathematica [A] time = 0.11471, size = 131, normalized size = 0.53

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\left(2a^2e^2g^2+2acdeg(f(m-3)+g(m-1)x)+c^2d^2\left(f^2\left(m^2-5m+6\right)+2fg\left(m^2-\right.\right.\right.}{c^3d^3(m-3)(m-2)(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(2*a^2*e^2*g^2 + 2*
a*c*d*e*g*(f*(-3 + m) + g*(-1 + m)*x) + c^2*d^2*(f^2*(6 - 5*m + m^2) + 2*f*
g*(3 - 4*m + m^2)*x + g^2*(2 - 3*m + m^2)*x^2)))/(c^3*d^3*(-3 + m)*(-2 + m)
*(-1 + m)))
```

Maple [A] time = 0.055, size = 235, normalized size = 1.

$$-\frac{\left(c^2d^2g^2m^2x^2+2c^2d^2fgm^2x-3c^2d^2g^2mx^2+2acdeg^2mx+c^2d^2f^2m^2-8c^2d^2fgmx+2g^2x^2c^2d^2+2acdefgm-2\right.}{\left(cdex^2+ae^2x+cd^2x+ade\right)^mc^3d^3\left(m^3-6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] -(c*d*x+a*e)*(c^2*d^2*g^2*m^2*x^2+2*c^2*d^2*f*g*m^2*x-3*c^2*d^2*g^2*m*x^2+2
*a*c*d*e*g^2*m*x+c^2*d^2*f^2*m^2-8*c^2*d^2*f*g*m*x+2*c^2*d^2*g^2*x^2+2*a*c*
d*e*f*g*m-2*a*c*d*e*g^2*x-5*c^2*d^2*f^2*m+6*c^2*d^2*f*g*x+2*a^2*e^2*g^2-6*a
*c*d*e*f*g+6*c^2*d^2*f^2)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)/c
^3/d^3/(m^3-6*m^2+11*m-6)
```

Maxima [A] time = 1.10489, size = 261, normalized size = 1.06

$$-\frac{(cdx+ae)f^2}{(cdx+ae)^mcd(m-1)}-\frac{2\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)fg}{\left(m^2-3m+2\right)(cdx+ae)^mc^2d^2}-\frac{\left(\left(m^2-3m+2\right)c^3d^3x^3+\left(m^2-m\right)ac^2d^2ex^2+2a^2\right.}{\left(m^3-6m^2+11m-6\right)(cdx+ae)^mc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^2/((c*d*x + a*e)^m*c*d*(m - 1)) - 2*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*e*m*x + a^2*e^2)*f*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - ((m
^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m*x +
 2*a^3*e^3)*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3)
```

Fricas [A] time = 1.34981, size = 695, normalized size = 2.83

$$-\frac{\left(ac^2d^2ef^2m^2 + 6\,ac^2d^2ef^2 - 6\,a^2cde^2fg + 2\,a^3e^3g^2 + \left(c^3d^3g^2m^2 - 3\,c^3d^3g^2m + 2\,c^3d^3g^2\right)x^3 + \left(6\,c^3d^3fg + \left(2\,c^3d^3fg + a\right.\right.\right.}{\left(c^3d^3m^3 - 6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(a*c^2*d^2*e*f^2*m^2 + 6*a*c^2*d^2*e*f^2 - 6*a^2*c*d*e^2*f*g + 2*a^3*e^3*g
^2 + (c^3*d^3*g^2*m^2 - 3*c^3*d^3*g^2*m + 2*c^3*d^3*g^2)*x^3 + (6*c^3*d^3*f
*g + (2*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*m^2 - (8*c^3*d^3*f*g + a*c^2*d^2*e*g
^2)*m)*x^2 - (5*a*c^2*d^2*e*f^2 - 2*a^2*c*d*e^2*f*g)*m + (6*c^3*d^3*f^2 + (
c^3*d^3*f^2 + 2*a*c^2*d^2*e*f*g)*m^2 - (5*c^3*d^3*f^2 + 6*a*c^2*d^2*e*f*g -
 2*a^2*c*d*e^2*g^2)*m)*x)*(e*x + d)^m/((c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^
3*d^3*m - 6*c^3*d^3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.32125, size = 1324, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((x*e + d)^m*c^3*d^3*g^2*m^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d))
+ 2*(x*e + d)^m*c^3*d^3*f*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) - 3*(x*e + d)^m*c^3*d^3*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) + (x*e + d)^m*a*c^2*d^2*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 1) + (x*e + d)^m*c^3*d^3*f^2*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e
```

```
+ d)) - 8*(x*e + d)^m*c^3*d^3*f*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) + 2*(x*e + d)^m*c^3*d^3*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d)) + 2*(x*e + d)^m*a*c^2*d^2*f*g*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d) + 1) - (x*e + d)^m*a*c^2*d^2*g^2*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 1) - 5*(x*e + d)^m*c^3*d^3*f^2*m*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d)) + 6*(x*e + d)^m*c^3*d^3*f*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d)) + (x*e + d)^m*a*c^2*d^2*f^2*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*
e + d) + 1) - 6*(x*e + d)^m*a*c^2*d^2*f*g*m*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) + 6*(x*e + d)^m*c^3*d^3*f^2*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d)) + 2*(x*e + d)^m*a^2*c*d*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 2) - 5*(x*e + d)^m*a*c^2*d^2*f^2*m*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) + 2*(x*e + d)^m*a^2*c*d*f*g*m*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 2) + 6*(x*e + d)^m*a*c^2*d^2*f^2*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) - 6*(x*e + d)^m*a^2*c*d*f*g*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 2) + 2*(x*e + d)^m*a^3*g^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 3))/(c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^3*d^3*m - 6*c^3*d^3)
```

3.770 $\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=150

$$\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2\right.}{c^2d^2e(1-m)(2-m)}$$

```
[Out] -(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g*(
d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 - m))
```

Rubi [A] time = 0.0816185, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {794, 648}

$$\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2\right.}{c^2d^2e(1-m)(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g*(
d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 - m))
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{g(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{\left(ae^2g+cd(dg(\right.}{}$$

$$=-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2\right.\right.}{c^2d^2e(1-m)(2-m)}$$

Mathematica [A] time = 0.0626045, size = 67, normalized size = 0.45

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}(aeg+cd(f(m-2)+g(m-1)x))}{c^2d^2(m-2)(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(a*e*g + c*d*(f*(-2
 + m) + g*(-1 + m)*x)))/(c^2*d^2*(-2 + m)*(-1 + m)))
```

Maple [A] time = 0.048, size = 89, normalized size = 0.6

$$-\frac{(ex+d)^m\left(cdgmx+cdfm-xcdg+aeg-2\,cdf\right)(cdx+ae)}{\left(cdex^2+ae^2x+cd^2x+ade\right)^m c^2d^2\left(m^2-3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] -(e*x+d)^m*(c*d*g*m*x+c*d*f*m-c*d*g*x+a*e*g-2*c*d*f)*(c*d*x+a*e)/((c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^m)/c^2/d^2/(m^2-3*m+2)
```

Maxima [A] time = 1.06504, size = 127, normalized size = 0.85

$$-\frac{(cdx+ae)f}{(cdx+ae)^m cd(m-1)}-\frac{\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)g}{\left(m^2-3\,m+2\right)(cdx+ae)^m c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
```

```
[Out] -(c*d*x + a*e)*f/((c*d*x + a*e)^m*c*d*(m - 1)) - (c^2*d^2*(m - 1)*x^2 + a*c
*d*e*m*x + a^2*e^2)*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2)
```

Fricas [A] time = 1.36285, size = 292, normalized size = 1.95

$$-\frac{\left(acdefm-2\,acdef+a^2e^2g+\left(c^2d^2gm-c^2d^2g\right)x^2-\left(2\,c^2d^2f-\left(c^2d^2f+acdeg\right)m\right)x\right)(ex+d)^m}{\left(c^2d^2m^2-3\,c^2d^2m+2\,c^2d^2\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
```

```
[Out] -(a*c*d*e*f*m - 2*a*c*d*e*f + a^2*e^2*g + (c^2*d^2*g*m - c^2*d^2*g)*x^2 - (
2*c^2*d^2*f - (c^2*d^2*f + a*c*d*e*g)*m)*x)*(e*x + d)^m/((c^2*d^2*m^2 - 3*c
^2*d^2*m + 2*c^2*d^2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27108, size = 498, normalized size = 3.32

$$-\frac{(xe+d)^m c^2 d^2 g m x^2 e^{(-m\log(cdx+ae)-m\log(xe+d))} + (xe+d)^m c^2 d^2 f m x e^{(-m\log(cdx+ae)-m\log(xe+d))} - (xe+d)^m c^2 d^2 g x^2 e^{(-m\log(cdx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] -((x*e + d)^m*c^2*d^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x
*e + d)^m*c^2*d^2*f*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - (x*e + d
)^m*c^2*d^2*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*a*
c*d*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 2*(x*e + d)^m*c^2*
d^2*f*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*a*c*d*f*m*e^
(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 2*(x*e + d)^m*a*c*d*f*e^(-m*lo
g(c*d*x + a*e) - m*log(x*e + d) + 1) + (x*e + d)^m*a^2*g*e^(-m*log(c*d*x +
a*e) - m*log(x*e + d) + 2))/(c^2*d^2*m^2 - 3*c^2*d^2*m + 2*c^2*d^2)
```

3.771 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=54

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

```
[Out] ((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(
1 - m))
```

Rubi [A] time = 0.0149937, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {648}

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(
1 - m))
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

Mathematica [A] time = 0.0214816, size = 42, normalized size = 0.78

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}}{cd(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

Maple [A] time = 0.046, size = 57, normalized size = 1.1

$$-\frac{(cdx+ae)\,(ex+d)^m}{cd\,(-1+m)\left(cdex^2+ae^2x+cd^2x+ade\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] -(c*d*x+a*e)/c/d/(-1+m)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

Maxima [A] time = 1.01867, size = 45, normalized size = 0.83

$$-\frac{cdx + ae}{(cdx + ae)^m cd(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="max
ima")
```

```
[Out] -(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

Fricas [A] time = 1.40221, size = 116, normalized size = 2.15

$$-\frac{(cdx + ae)(ex + d)^m}{(cdm - cd)\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fri
cas")
```

```
[Out] -(c*d*x + a*e)*(e*x + d)^m/((c*d*m - c*d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31714, size = 117, normalized size = 2.17

$$-\frac{(xe + d)^m cdxe^{(-m\log(cdx+ae) - m\log(xe+d))} + (xe + d)^m ae^{(-m\log(cdx+ae) - m\log(xe+d)+1)}}{cdm - cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gia
c")
```

```
[Out] -((x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*
a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1))/(c*d*m - c*d)
```

3.772 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx$

Optimal. Leaf size=99

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(1,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

```
[Out] ((a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[1, 1 - m, 2 - m, -((g*(a*e + c
*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)*(1 - m)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^m)
```

Rubi [A] time = 0.0612081, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {891, 68}

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(1,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[1, 1 - m, 2 - m, -((g*(a*e + c
*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)*(1 - m)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae+cdx)^{-}}{f+gx}$$

$$= \frac{(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(1,1-m;2-\right.}{(cdf-aeg)(1-m)}$$

Mathematica [A] time = 0.0280362, size = 82, normalized size = 0.83

$$-\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\,{}_2F_1\left(1,1-m;2-m;\frac{g(ae+cdx)}{aeg-cdf}\right)}{(m-1)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2F1[1
, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)*(-1
 + m)))
```

Maple [F] time = 1.767, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(gx+f\right)\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(gx+f\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\left(gx+f\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
```

```
[Out] integral((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

3.773 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx$

Optimal. Leaf size=101

$$\frac{cd(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(2,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

```
[Out] (c*d*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[2, 1 - m, 2 - m, -((g*(a*e
 + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^2*(1 - m)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^m)
```

Rubi [A] time = 0.0601161, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {891, 68}

$$\frac{cd(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(2,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] (c*d*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[2, 1 - m, 2 - m, -((g*(a*e
 + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^2*(1 - m)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx=\left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae+c\ldots}{(f+g\ldots}$$

$$=\frac{cd(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(2,1-\ldots\right.}{(cdf-aeg)^2(1-m)}$$

Mathematica [A] time = 0.0339749, size = 84, normalized size = 0.83

$$-\frac{cd(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\,{}_2F_1\left(2,1-m;2-m;\frac{g(ae+cdx)}{aeg-cdf}\right)}{(m-1)(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m),x]
```

```
[Out] -((c*d*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2
F1[2, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)
^2*(-1 + m)))
```

Maple [F] time = 1.73, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^2\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^2\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\left(g^2x^2+2\,fgx+f^2\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)^m), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

3.774 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$

Optimal. Leaf size=105

$$\frac{c^2d^2(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(3,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

```
[Out] (c^2*d^2*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[3, 1 - m, 2 - m, -((g*
(a*e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^3*(1 - m)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rubi [A] time = 0.0747857, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {891, 68}

$$\frac{c^2d^2(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(3,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] (c^2*d^2*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[3, 1 - m, 2 - m, -((g*
(a*e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^3*(1 - m)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx=\left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae+cdx)}{(f+gx)^3}$$

$$=\frac{c^2d^2(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(3,1-m\right.}{(cdf-aeg)^3(1-m)}$$

Mathematica [A] time = 0.0374521, size = 88, normalized size = 0.84

$$-\frac{c^2d^2(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}\,{}_2F_1\left(3,1-m;2-m;\frac{g(ae+cdx)}{aeg-cdf}\right)}{(m-1)(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m),x]
```

```
[Out] -((c^2*d^2*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeomet
ric2F1[3, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*
e*g)^3*(-1 + m)))
```

Maple [F] time = 1.732, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^3\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^3\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\left(g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^3\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

3.775 $\int (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=105

$$\frac{2(f+gx)^{5/2}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rubi [A] time = 0.080604, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2(f+gx)^{5/2}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \left((ae+cdx)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right) \int (\ldots$$
$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \ldots\right.$$
$$= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m (f+gx)^{5/2} \left(ade+\left(cd^2+ae^2\right)x+\ldots\right.}{5g}$$

Mathematica [A] time = 0.058219, size = 93, normalized size = 0.89

$$\frac{2(f+gx)^{5/2}(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*((a*e +
c*d*x)*(d + e*x))^m)
```

Maple [F] time = 1.638, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}(gx+f)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx+f\right)^{\frac{3}{2}}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^{\frac{3}{2}}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

3.776 $\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=105

$$\frac{2(f+gx)^{3/2}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m{}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rubi [A] time = 0.0761664, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2(f+gx)^{3/2}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m{}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x
]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int$$
$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right.$$
$$= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+\right.}{3g}$$

Mathematica [A] time = 0.0389244, size = 93, normalized size = 0.89

$$\frac{2(f+gx)^{3/2}(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^m,x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e +
c*d*x)*(d + e*x))^m)
```

Maple [F] time = 1.662, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{gx+f}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{gx+f}(ex+d)^m}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

3.777 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{f+gx}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rubi [A] time = 0.0751331, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$\frac{2\sqrt{f+gx}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x
]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \left((ae+cdx)^m(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae+cdx)}{\sqrt{f+gx}}$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{\left(-\frac{ae}{cdf}\right.}{}$$

$$= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\ {}_2F_1}{g}$$

Mathematica [A] time = 0.0334533, size = 91, normalized size = 0.88

$$\frac{2\sqrt{f+gx}(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m\ {}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m),x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*
x)*(d + e*x))^m)
```

Maple [F] time = 1.671, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}\frac{1}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{gx+f}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\sqrt{gx+f}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{gx+f}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

3.778 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*Sqrt[f + g*x]*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rubi [A] time = 0.0754757, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$-\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*Sqrt[f + g*x]*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae+c\ldots}{(f+g\ldots}$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \ldots$$

$$= -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} {}_2F_1\left(-\ldots\right.}{g\sqrt{f+gx}}$$

Mathematica [A] time = 0.0337344, size = 91, normalized size = 0.88

$$-\frac{2(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m),x]
```

```
[Out] (-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*x)*(d + e*x)
)^m*Sqrt[f + g*x])
```

Maple [F] time = 1.753, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}\left(gx+f\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(gx+f\right)^{\frac{3}{2}}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{gx+f}(ex+d)^m}{(g^2x^2+2fgx+f^2)\left(cdex^2+ade+(cd^2+ae^2)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}\left(cdex^2+ade+(cd^2+ae^2)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

3.779 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(f + g*x)^(3/2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rubi [A] time = 0.0755164, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {891, 70, 69}

$$-\frac{2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(f + g*x)^(3/2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = \left((ae+cdx)^m(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{(ae+cdx)}{(f+gx)^{5/}}$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\right)\int \frac{\left(-\frac{ae}{cdf-}\right.}{}$$

$$= -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} {}_2F_1\left(-\frac{3}{2},m\right.}{3g(f+gx)^{3/2}}$$

Mathematica [A] time = 0.0373216, size = 93, normalized size = 0.89

$$-\frac{2(d+ex)^m((d+ex)(ae+cdx))^{-m}\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m),x]
```

```
[Out] (-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e + c*d*x)*(d + e
*x))^m*(f + g*x)^(3/2))
```

Maple [F] time = 1.655, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}(gx+f)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{gx+f}(ex+d)^m}{\left(g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3\right)\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^
3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(5/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

3.780 $\int (ae+cdx)^n(d+ex)^m \left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{-m} dx$

Optimal. Leaf size=65

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}(ae+cdx)^n}{cd(-m+n+1)}$$

```
[Out] ((a*e + c*d*x)^n*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1 - m))/(c*d*(1 - m + n))
```

Rubi [A] time = 0.0442912, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {858}

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}(ae+cdx)^n}{cd(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] ((a*e + c*d*x)^n*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1 - m))/(c*d*(1 - m + n))
```

Rule 858

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] /; FreeQ[{a, b, c, d, e, f, g
, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0] && EqQ[c*e*f + c*d*g - b
*e*g, 0] && NeQ[m - n - 1, 0]
```

Rubi steps

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}dx=\frac{(ae+cdx)^n(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m+n)}$$

Mathematica [A] time = 0.0297271, size = 53, normalized size = 0.82

$$\frac{(d+ex)^m((d+ex)(ae+cdx))^{-m}(ae+cdx)^{n+1}}{-cdm+cdn+cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
```

```
[Out] ((a*e + c*d*x)^(1 + n)*(d + e*x)^m)/((c*d - c*d*m + c*d*n)*((a*e + c*d*x)*(
d + e*x))^m)
```

Maple [A] time = 0.049, size = 64, normalized size = 1.

$$-\frac{(cdx+ae)^{1+n}(ex+d)^{m}}{cd(-1+m-n)\left(cde x^{2}+a e^{2}x+c d^{2}x+ade\right)^{m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] -(c*d*x+a*e)^(1+n)/c/d/(-1+m-n)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^m)
```

Maxima [A] time = 1.47024, size = 66, normalized size = 1.02

$$-\frac{(cdx+ae)e^{(-m\log(cdx+ae)+n\log(cdx+ae))}}{cd(m-n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] -(c*d*x + a*e)*e^(-m*log(c*d*x + a*e) + n*log(c*d*x + a*e))/(c*d*(m - n - 1
))
```

Fricas [A] time = 1.36945, size = 144, normalized size = 2.22

$$-\frac{(cdx+ae)(cdx+ae)^{n}(ex+d)^{m}e^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cdn-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] -(c*d*x + a*e)*(c*d*x + a*e)^n*(e*x + d)^m*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d))/(c*d*m - c*d*n - c*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18307, size = 154, normalized size = 2.37

$$-\frac{(cdx+ae)^n(xe+d)^m cdxe^{(-m\log(cdx+ae)-m\log(xe+d))}+(cdx+ae)^n(xe+d)^m ae^{(-m\log(cdx+ae)-m\log(xe+d)+1)}}{cdm-cdn-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] -((c*d*x + a*e)^n*(x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) + (c*d*x + a*e)^n*(x*e + d)^m*a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) +
 1))/(c*d*m - c*d*n - c*d)
```

3.781 $\int (d+ex)^m \left(cd^2eg - e\left(cd^2+ae^2\right)g - cde^2gx\right)^{-1+m} \left(ade + \right.$

Optimal. Leaf size=78

$$-\frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \log(ae+cdx)\left(-ae^3g-cde^2gx\right)^m}{cde^2g}$$

```
[Out] -(((d + e*x)^m*(-(a*e^3*g) - c*d*e^2*g*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m))
```

Rubi [A] time = 0.150091, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.041, Rules used = {891, 23, 31}

$$-\frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \log(ae+cdx)\left(-ae^3g-cde^2gx\right)^m}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1 + m))/
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^m*(-(a*e^3*g) - c*d*e^2*g*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m))
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \left((ae+cdx)^m(d+ex)^m \left(ade \right.\right.$$

$$= \left((d+ex)^m \left(cd^2eg - e\left(cd^2 + \right.\right.\right.$$

$$= -\frac{(d+ex)^m \left(-ae^3g-cde^2gx\right.}{}$$

Mathematica [A] time = 0.0356854, size = 64, normalized size = 0.82

$$-\frac{(d+ex)^m((d+ex)(ae+cdx))^{-m}\log(ae+cdx)\left(-e^2g(ae+cdx)\right)^m}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1
+ m))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((-(e^2*g*(a*e + c*d*x)))^m*(d + e*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*((a*
e + c*d*x)*(d + e*x))^m))
```

Maple [F] time = 1.881, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m\left(cd^2eg-e\left(ae^2+cd^2\right)g-cde^2gx\right)^{-1+m}}{\left(ade+\left(ae^2+cd^2\right)x+cdex^2\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [A] time = 1.26411, size = 43, normalized size = 0.55

$$-\frac{e^{2\,m-2}\left(-g\right)^m\log\left(cdx+ae\right)}{cdg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="maxima")
```

```
[Out] -e^(2*m - 2)*(-g)^m*log(c*d*x + a*e)/(c*d*g)
```

Fricas [A] time = 1.34499, size = 62, normalized size = 0.79

$$-\frac{\log\left(cdx+ae\right)}{cde^2g\left(-\frac{1}{e^2g}\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fricas")
```

```
[Out] -log(c*d*x + a*e)/(c*d*e^2*g*(-1/(e^2*g))^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*d**2*e*g-e*(a*e**2+c*d**2)*g-c*d*e**2*g*x)**(-1+m)/
((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-cde^2gx + cd^2eg - \left(cd^2 + ae^2\right)eg\right)^{m-1}(ex + d)^m}{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="giac")
```

```
[Out] integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m - 1)*(e*x + d
)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)
```

3.782 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt{d+ex}(f+gx)^{n+1}(ae+cdx)\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\,{}_2F_1\left(1,n+\frac{3}{2};n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{cdg(n+1)(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}+\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae\right.}}{cdg(2n+}$$

```
[Out] (2*e*(f + g*x)^(1 + n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*
(3 + 2*n)*Sqrt[d + e*x]) + ((2*a*e^2*g*(1 + n) + c*d*(e*f - d*g*(3 + 2*n)))
*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 3/2 + n
, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(c*d*g*(c*d*f - a*e*g)*(1 + n)*(
3 + 2*n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rubi [A] time = 0.278404, antiderivative size = 222, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 891, 70, 69}

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right.}{c^2d^2g(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (2*e*(f + g*x)^(1 + n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*
(3 + 2*n)*Sqrt[d + e*x]) - (2*(2*a*e^2*g*(1 + n) + c*d*(e*f - d*g*(3 + 2*n)
))*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
-((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c^2*d^2*g*(3 + 2*n)*((c*d*(f + g*x)
)/(c*d*f - a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rule 891

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left(2ae^2g(1+n)+cd(ef-dg(3+\right.}{cdg(3-}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left(\left(2ae^2g(1+n)+cd(ef-dg(3+\right.\right.}{cdg(3+2n)\sqrt{ade}}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left(\left(2ae^2g(1+n)+cd(ef-dg(3+\right.\right.}{cd}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{2\left(2ae^2g(1+n)+cd(ef-dg(3+\right.}{c^2}$$

Mathematica [A] time = 0.135152, size = 145, normalized size = 0.68

$$\frac{(f+gx)^n\sqrt{(d+ex)(ae+cdx)}\left(\left(cd(dg(2n+3)-ef)-2ae^2g(n+1)\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)+cde(f+gx\right.}{c^2d^2g\left(n+\frac{3}{2}\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*(c*d*e*(f + g*x) + ((-2*a*e^2*g*
(1 + n) + c*d*(-(e*f) + d*g*(3 + 2*n)))*Hypergeometric2F1[1/2, -n, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*(f + g*x))/(c*d*f - a*e*g))^n))/(
c^2*d^2*g*(3/2 + n)*Sqrt[d + e*x])
```

Maple [F] time = 1.633, size = 0, normalized size = 0.

$$\int \left(gx+f\right)^{n}(ex+d)^{\frac{3}{2}}\frac{1}{\sqrt{ade+\left(ae^2+cd^2\right)x+cdex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}(gx+f)^n}{cdx+ae},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f
)^n/(c*d*x + a*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

3.783 $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=501

$$-\frac{2(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(10ae^2g+cd(ef-11dg)\right)}{99c^2d^2g\sqrt{d+ex}}-\frac{16(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-ae}{693c^3d^3g\sqrt{d+ex}}$$

```
[Out] (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(2*a*e^2*g - c*d*(
3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^6*d^6*e*
g*Sqrt[d + e*x]) - (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))
*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^5*d^5*e
) - (32*(c*d*f - a*e*g)^2*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1155*c^4*d^4*g*Sqrt[d + e*x]) -
(16*(c*d*f - a*e*g)*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^3*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(693*c^3*d^3*g*Sqrt[d + e*x]) - (2*(10*
a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(99*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^5*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(11*c*d*g*Sqrt[d + e*x])
```

Rubi [A] time = 0.893591, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(10ae^2g+cd(ef-11dg)\right)}{99c^2d^2g\sqrt{d+ex}}-\frac{16(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-ae}{693c^3d^3g\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```


```
[Out] (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(2*a*e^2*g - c*d*(
3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^6*d^6*e*
g*Sqrt[d + e*x]) - (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))
*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^5*d^5*e
) - (32*(c*d*f - a*e*g)^2*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1155*c^4*d^4*g*Sqrt[d + e*x]) -
(16*(c*d*f - a*e*g)*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^3*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(693*c^3*d^3*g*Sqrt[d + e*x]) - (2*(10*
a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(99*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^5*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(11*c*d*g*Sqrt[d + e*x])
```


Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{11cdg\sqrt{d+ex}} - \frac{1}{11}\left(-11d+\frac{10ae^2}{cd}+\frac{ef}{g}\right)\int \frac{\ldots}{\sqrt{ade \ldots}}$$

$$= -\frac{2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{99c^2d^2g\sqrt{d+ex}} + \frac{2e(f+ \ldots}{\ldots}$$

$$= -\frac{16(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex \ldots}}{693c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{32(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cde \ldots}}{1155c^4d^4g\sqrt{d+ex}}$$

$$= -\frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cde \ldots}}{3465c^5d^5e}$$

$$= \frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2 \ldots\right.}}{3465c^6d^6eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.436734, size = 246, normalized size = 0.49

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(3465\left(cd^2-ae^2\right)(cdf-aeg)^4-385g^3(ae+cdx)^4\left(5ae^2g-cd(dg+4ef)\right)+990g^2(ae+cdx)^3(c \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3465*(c*d^2 - a*e^2)*(c*d*f - a*e*g)^4 +
1155*(c*d*f - a*e*g)^3*(-5*a*e^2*g + c*d*(e*f + 4*d*g))*(a*e + c*d*x) + 138
6*g*(c*d*f - a*e*g)^2*(-5*a*e^2*g + c*d*(2*e*f + 3*d*g))*(a*e + c*d*x)^2 +
990*g^2*(c*d*f - a*e*g)*(-5*a*e^2*g + c*d*(3*e*f + 2*d*g))*(a*e + c*d*x)^3
- 385*g^3*(5*a*e^2*g - c*d*(4*e*f + d*g))*(a*e + c*d*x)^4 + 315*e*g^4*(a*e
+ c*d*x)^5))/(3465*c^6*d^6*Sqrt[d + e*x])
```

Maple [A] time = 0.055, size = 641, normalized size = 1.3

$$-\frac{(2\,cdx+2\,ae)\left(-315\,eg^4x^5c^5d^5+350\,ac^4d^4e^2g^4x^4-385\,c^5d^6g^4x^4-1540\,c^5d^5efg^3x^4-400\,a^2c^3d^3e^3g^4x^3+440\,ac^4d^5eg^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/3465*(c*d*x+a*e)*(-315*c^5*d^5*e*g^4*x^5+350*a*c^4*d^4*e^2*g^4*x^4-385*c
^5*d^6*g^4*x^4-1540*c^5*d^5*e*f*g^3*x^4-400*a^2*c^3*d^3*e^3*g^4*x^3+440*a*c
^4*d^5*e*g^4*x^3+1760*a*c^4*d^4*e^2*f*g^3*x^3-1980*c^5*d^6*f*g^3*x^3-2970*c
^5*d^5*e*f^2*g^2*x^3+480*a^3*c^2*d^2*e^4*g^4*x^2-528*a^2*c^3*d^4*e^2*g^4*x^
2-2112*a^2*c^3*d^3*e^3*f*g^3*x^2+2376*a*c^4*d^5*e*f*g^3*x^2+3564*a*c^4*d^4*
e^2*f^2*g^2*x^2-4158*c^5*d^6*f^2*g^2*x^2-2772*c^5*d^5*e*f^3*g*x^2-640*a^4*c
*d*e^5*g^4*x+704*a^3*c^2*d^3*e^3*g^4*x+2816*a^3*c^2*d^2*e^4*f*g^3*x-3168*a^
2*c^3*d^4*e^2*f*g^3*x-4752*a^2*c^3*d^3*e^3*f^2*g^2*x+5544*a*c^4*d^5*e*f^2*g
^2*x+3696*a*c^4*d^4*e^2*f^3*g*x-4620*c^5*d^6*f^3*g*x-1155*c^5*d^5*e*f^4*x+1
280*a^5*e^6*g^4-1408*a^4*c*d^2*e^4*g^4-5632*a^4*c*d*e^5*f*g^3+6336*a^3*c^2*
d^3*e^3*f*g^3+9504*a^3*c^2*d^2*e^4*f^2*g^2-11088*a^2*c^3*d^4*e^2*f^2*g^2-73
92*a^2*c^3*d^3*e^3*f^3*g+9240*a*c^4*d^5*e*f^3*g+2310*a*c^4*d^4*e^2*f^4-3465
*c^5*d^6*f^4)*(e*x+d)^(1/2)/c^6/d^6/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.71883, size = 936, normalized size = 1.87

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f^4}{3\,\sqrt{cdx+ae}c^2d^2}+\frac{8\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x^2-\left(5\,ac^2\right.\right.}{15\,\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^4/(sqrt(c*d*x + a*e)*c^2*d^2) + 8/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f^3*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 4/35*(15*c^4*d^4*e*x^4 + 56*a^3*
c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
 - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f^2*g^
2/(sqrt(c*d*x + a*e)*c^4*d^4) + 8/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4
 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2
*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^
3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*f*g^3/(sqrt(c*d*x + a*e)*c^5*d^5) + 2/346
```

```
5*(315*c^6*d^6*e*x^6 + 1408*a^5*c*d^2*e^5 - 1280*a^6*e^7 + 35*(11*c^6*d^7 -
 a*c^5*d^5*e^2)*x^5 - 5*(11*a*c^5*d^6*e - 10*a^2*c^4*d^4*e^3)*x^4 + 8*(11*a
^2*c^4*d^5*e^2 - 10*a^3*c^3*d^3*e^4)*x^3 - 16*(11*a^3*c^3*d^4*e^3 - 10*a^4*
c^2*d^2*e^5)*x^2 + 64*(11*a^4*c^2*d^3*e^4 - 10*a^5*c*d*e^6)*x)*g^4/(sqrt(c*
d*x + a*e)*c^6*d^6)
```

Fricas [A] time = 1.46703, size = 1246, normalized size = 2.49

$$\frac{2\left(315\,c^5d^5eg^4x^5+1155\left(3\,c^5d^6-2\,ac^4d^4e^2\right)f^4-1848\left(5\,ac^4d^5e-4\,a^2c^3d^3e^3\right)f^3g+1584\left(7\,a^2c^3d^4e^2-6\,a^3c^2d^2e^4\right)f^2g\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*e*g^4*x^5 + 1155*(3*c^5*d^6 - 2*a*c^4*d^4*e^2)*f^4 - 18
48*(5*a*c^4*d^5*e - 4*a^2*c^3*d^3*e^3)*f^3*g + 1584*(7*a^2*c^3*d^4*e^2 - 6*
a^3*c^2*d^2*e^4)*f^2*g^2 - 704*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*f*g^3 +
128*(11*a^4*c*d^2*e^4 - 10*a^5*e^6)*g^4 + 35*(44*c^5*d^5*e*f*g^3 + (11*c^5*
d^6 - 10*a*c^4*d^4*e^2)*g^4)*x^4 + 10*(297*c^5*d^5*e*f^2*g^2 + 22*(9*c^5*d^
6 - 8*a*c^4*d^4*e^2)*f*g^3 - 4*(11*a*c^4*d^5*e - 10*a^2*c^3*d^3*e^3)*g^4)*x
^3 + 6*(462*c^5*d^5*e*f^3*g + 99*(7*c^5*d^6 - 6*a*c^4*d^4*e^2)*f^2*g^2 - 44
*(9*a*c^4*d^5*e - 8*a^2*c^3*d^3*e^3)*f*g^3 + 8*(11*a^2*c^3*d^4*e^2 - 10*a^3
*c^2*d^2*e^4)*g^4)*x^2 + (1155*c^5*d^5*e*f^4 + 924*(5*c^5*d^6 - 4*a*c^4*d^4
*e^2)*f^3*g - 792*(7*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^2*g^2 + 352*(9*a^2*
c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*f*g^3 - 64*(11*a^3*c^2*d^3*e^3 - 10*a^4*c*
d*e^5)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c
^6*d^6*e*x + c^6*d^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}\left(gx+f\right)^4}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^4/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

3.784 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=412

$$-\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(8ae^2g+cd(ef-9dg)\right)}{63c^2d^2g\sqrt{d+ex}}-\frac{4(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-ae}{105c^3d^3g\sqrt{d+ex}}$$

```
[Out] (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(2*a*e^2*g - c*d*(3*e
*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^5*d^5*e*g*Sq
rt[d + e*x]) - (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*Sqrt[d
 + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^4*d^4*e) - (4*(
c*d*f - a*e*g)*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*g*Sqrt[d + e*x]) - (2*(8*a*e^2*g
+ c*d*(e*f - 9*d*g))*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(63*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(9*c*d*g*Sqrt[d + e*x])
```

Rubi [A] time = 0.627098, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(8ae^2g+cd(ef-9dg)\right)}{63c^2d^2g\sqrt{d+ex}}-\frac{4(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-ae}{105c^3d^3g\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(2*a*e^2*g - c*d*(3*e
*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^5*d^5*e*g*Sq
rt[d + e*x]) - (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*Sqrt[d
 + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^4*d^4*e) - (4*(
c*d*f - a*e*g)*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*g*Sqrt[d + e*x]) - (2*(8*a*e^2*g
+ c*d*(e*f - 9*d*g))*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(63*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(9*c*d*g*Sqrt[d + e*x])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
```

```
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{9cdg\sqrt{d+ex}} - \frac{1}{9}\left(-9d+\frac{8ae^2}{cd}+\frac{ef}{g}\right)\int \frac{\sqrt{d+e}}{\sqrt{ade+\left(cd^2\right.}}$$

$$= -\frac{2\left(8ae^2g+cd(ef-9dg)\right)(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{63c^2d^2g\sqrt{d+ex}} + \frac{2e(f+gx)^4\sqrt{}}{}$$

$$= -\frac{4(cdf-aeg)\left(8ae^2g+cd(ef-9dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{105c^3d^3g\sqrt{d+ex}} - \frac{2}{}$$

$$= -\frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{315c^4d^4e} - \frac{4}{}$$

$$= \frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)}}{315c^5d^5eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.278514, size = 264, normalized size = 0.64

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(24a^2c^2d^2e^2g\left(3dg(7f+gx)+e\left(21f^2+9fgx+2g^2x^2\right)\right)-16a^3cde^3g^2(9dg+27ef+4egx)+128a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^5*g^3 - 16*a^3*c*d*e^3*g^2*(27*
e*f + 9*d*g + 4*e*g*x) + 24*a^2*c^2*d^2*e^2*g*(3*d*g*(7*f + g*x) + e*(21*f^
2 + 9*f*g*x + 2*g^2*x^2)) - 2*a*c^3*d^3*e*(9*d*g*(35*f^2 + 14*f*g*x + 3*g^2
```

```
*x^2) + e*(105*f^3 + 126*f^2*g*x + 81*f*g^2*x^2 + 20*g^3*x^3)) + c^4*d^4*(9
*d*(35*f^3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3) + e*x*(105*f^3 + 189*f^
2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3))))/(315*c^5*d^5*Sqrt[d + e*x])
```

Maple [A] time = 0.051, size = 425, normalized size = 1.

$$(2\,cdx+2\,ae)\left(35\,eg^3x^4c^4d^4-40\,ac^3d^3e^2g^3x^3+45\,c^4d^5g^3x^3+135\,c^4d^4efg^2x^3+48\,a^2c^2d^2e^3g^3x^2-54\,ac^3d^4eg^3x^2-16\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 2/315*(c*d*x+a*e)*(35*c^4*d^4*e*g^3*x^4-40*a*c^3*d^3*e^2*g^3*x^3+45*c^4*d^5
*g^3*x^3+135*c^4*d^4*e*f*g^2*x^3+48*a^2*c^2*d^2*e^3*g^3*x^2-54*a*c^3*d^4*e*
g^3*x^2-162*a*c^3*d^3*e^2*f*g^2*x^2+189*c^4*d^5*f*g^2*x^2+189*c^4*d^4*e*f^2
*g*x^2-64*a^3*c*d*e^4*g^3*x+72*a^2*c^2*d^3*e^2*g^3*x+216*a^2*c^2*d^2*e^3*f*
g^2*x-252*a*c^3*d^4*e*f*g^2*x-252*a*c^3*d^3*e^2*f^2*g*x+315*c^4*d^5*f^2*g*x
+105*c^4*d^4*e*f^3*x+128*a^4*e^5*g^3-144*a^3*c*d^2*e^3*g^3-432*a^3*c*d*e^4*
f*g^2+504*a^2*c^2*d^3*e^2*f*g^2+504*a^2*c^2*d^2*e^3*f^2*g-630*a*c^3*d^4*e*f
^2*g-210*a*c^3*d^3*e^2*f^3+315*c^4*d^5*f^3)*(e*x+d)^(1/2)/c^5/d^5/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.67286, size = 653, normalized size = 1.58

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f^3}{3\,\sqrt{cdx+ae}c^2d^2}+\frac{2\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x^2-\left(5\,a\right.\right.}{5\,\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^3/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^
3)*x)*f^2*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(15*c^4*d^4*e*x^4 + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f*g^2/(
sqrt(c*d*x + a*e)*c^4*d^4) + 2/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4 +
128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2*c^
3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^3*c
^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*g^3/(sqrt(c*d*x + a*e)*c^5*d^5)
```

Fricas [A] time = 1.53569, size = 837, normalized size = 2.03

$$2\left(35\,c^4d^4eg^3x^4+105\left(3\,c^4d^5-2\,ac^3d^3e^2\right)f^3-126\left(5\,ac^3d^4e-4\,a^2c^2d^2e^3\right)f^2g+72\left(7\,a^2c^2d^3e^2-6\,a^3cde^4\right)fg^2-16\left(9\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*e*g^3*x^4 + 105*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^3 - 126*(
5*a*c^3*d^4*e - 4*a^2*c^2*d^2*e^3)*f^2*g + 72*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*
d*e^4)*f*g^2 - 16*(9*a^3*c*d^2*e^3 - 8*a^4*e^5)*g^3 + 5*(27*c^4*d^4*e*f*g^2
 + (9*c^4*d^5 - 8*a*c^3*d^3*e^2)*g^3)*x^3 + 3*(63*c^4*d^4*e*f^2*g + 9*(7*c^
4*d^5 - 6*a*c^3*d^3*e^2)*f*g^2 - 2*(9*a*c^3*d^4*e - 8*a^2*c^2*d^2*e^3)*g^3)
*x^2 + (105*c^4*d^4*e*f^3 + 63*(5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^2*g - 36*(7*
a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f*g^2 + 8*(9*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e
^4)*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*
d^5*e*x + c^5*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^3}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^3/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

3.785 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=321

$$-\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g+cd(ef-7dg)\right)}{35c^2d^2g\sqrt{d+ex}}-\frac{8\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg}{105c^3d^3e}$$

```
[Out] (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(2*a*e^2*g - c*d*(3*e*f
- d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^4*d^4*e*g*Sqrt[
d + e*x]) - (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*Sqrt[d + e*x
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*e) - (2*(6*a*e^
2*g + c*d*(e*f - 7*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(35*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(7*c*d*g*Sqrt[d + e*x])
```

Rubi [A] time = 0.419845, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {880, 870, 794, 648}

$$-\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g+cd(ef-7dg)\right)}{35c^2d^2g\sqrt{d+ex}}-\frac{8\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg}{105c^3d^3e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```


```
[Out] (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(2*a*e^2*g - c*d*(3*e*f
- d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^4*d^4*e*g*Sqrt[
d + e*x]) - (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*Sqrt[d + e*x
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*e) - (2*(6*a*e^
2*g + c*d*(e*f - 7*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(35*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(7*c*d*g*Sqrt[d + e*x])
```


Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rule 870

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(
a + b*x + c*x^2)^(p + 1))/(c*(m - n - 1)), x] - Dist[(n*(c*e*f + c*d*g - b*
e*g))/(c*e*(m - n - 1)), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] &
& NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&
```

```
EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Intege
rQ[n])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7cdg\sqrt{d+ex}} - \frac{1}{7}\left(-7d+\frac{6ae^2}{cd}+\frac{ef}{g}\right)\int \frac{\sqrt{d+e}}{\sqrt{ade+\left(cd^2\right.}}$$

$$= -\frac{2\left(6ae^2g+cd(ef-7dg)\right)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^2d^2g\sqrt{d+ex}} + \frac{2e(f+gx)^3}{}$$

$$= -\frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{105c^3d^3e} - \frac{2(}{}$$

$$= \frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x}}{105c^4d^4eg\sqrt{d+ex}}$$

Mathematica [A] time = 0.18984, size = 169, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2cde^2g(7dg+14ef+3egx)-48a^3e^4g^2-2ac^2d^2e\left(14dg(5f+gx)+e\left(35f^2+28fgx+9g^2x^2\right)\right)\right.}{105c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*a^3*e^4*g^2 + 8*a^2*c*d*e^2*g*(14*e*f
 + 7*d*g + 3*e*g*x) - 2*a*c^2*d^2*e*(14*d*g*(5*f + g*x) + e*(35*f^2 + 28*f*
g*x + 9*g^2*x^2)) + c^3*d^3*(7*d*(15*f^2 + 10*f*g*x + 3*g^2*x^2) + e*x*(35*
f^2 + 42*f*g*x + 15*g^2*x^2))))/(105*c^4*d^4*Sqrt[d + e*x])
```

Maple [A] time = 0.053, size = 255, normalized size = 0.8

$$-\frac{(2\,cdx+2\,ae)\left(-15\,eg^2x^3c^3d^3+18\,ac^2d^2e^2g^2x^2-21\,c^3d^4g^2x^2-42\,c^3d^3efgx^2-24\,a^2cde^3g^2x+28\,ac^2d^3eg^2x+56\,ac^2d^2e\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/105*(c*d*x+a*e)*(-15*c^3*d^3*e*g^2*x^3+18*a*c^2*d^2*e^2*g^2*x^2-21*c^3*d
^4*g^2*x^2-42*c^3*d^3*e*f*g*x^2-24*a^2*c*d*e^3*g^2*x+28*a*c^2*d^3*e*g^2*x+5
6*a*c^2*d^2*e^2*f*g*x-70*c^3*d^4*f*g*x-35*c^3*d^3*e*f^2*x+48*a^3*e^4*g^2-56
*a^2*c*d^2*e^2*g^2-112*a^2*c*d*e^3*f*g+140*a*c^2*d^3*e*f*g+70*a*c^2*d^2*e^2
*f^2-105*c^3*d^4*f^2)*(e*x+d)^(1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(1/2)
```

Maxima [A] time = 1.45183, size = 417, normalized size = 1.3

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f^2}{3\,\sqrt{cdx+ae}c^2d^2}+\frac{4\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x^2-\left(5\,a\right.\right.}{15\,\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^2/(sqrt(c*d*x + a*e)*c^2*d^2) + 4/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/105*(15*c^4*d^4*e*x^4 + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*g^2/(sq
rt(c*d*x + a*e)*c^4*d^4)
```

Fricas [A] time = 1.65022, size = 533, normalized size = 1.66

$$\frac{2\left(15\,c^3d^3eg^2x^3+35\left(3\,c^3d^4-2\,ac^2d^2e^2\right)f^2-28\left(5\,ac^2d^3e-4\,a^2cde^3\right)fg+8\left(7\,a^2cd^2e^2-6\,a^3e^4\right)g^2+3\left(14\,c^3d^3efg+\right.\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*e*g^2*x^3 + 35*(3*c^3*d^4 - 2*a*c^2*d^2*e^2)*f^2 - 28*(5*
a*c^2*d^3*e - 4*a^2*c*d*e^3)*f*g + 8*(7*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^2 + 3*
(14*c^3*d^3*e*f*g + (7*c^3*d^4 - 6*a*c^2*d^2*e^2)*g^2)*x^2 + (35*c^3*d^3*e*
f^2 + 14*(5*c^3*d^4 - 4*a*c^2*d^2*e^2)*f*g - 4*(7*a*c^2*d^3*e - 6*a^2*c*d*e
^3)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*
d^4*e*x + c^4*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^2}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)^2/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

3.786 $\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=209

$$-\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(4ae^2g-cd(5ef-dg)\right)}{15c^2d^2e}-\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(4ae^2g\right.}{15c^3d^3e\sqrt{d+ex}}$$

```
[Out] (-4*(c*d^2 - a*e^2)*(4*a*e^2*g - c*d*(5*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) - (2*(4*a*e^2*g - c*d*(5
*e*f - d*g))*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15
*c^2*d^2*e) + (2*g*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(5*c*d*e)
```

Rubi [A] time = 0.197591, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {794, 656, 648}

$$-\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(4ae^2g-cd(5ef-dg)\right)}{15c^2d^2e}-\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(4ae^2g\right.}{15c^3d^3e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```


```
[Out] (-4*(c*d^2 - a*e^2)*(4*a*e^2*g - c*d*(5*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) - (2*(4*a*e^2*g - c*d*(5
*e*f - d*g))*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15
*c^2*d^2*e) + (2*g*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(5*c*d*e)
```


Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
```

```
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2g(d+ex)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5cde}+\frac{1}{5}\left(5f-\frac{dg}{e}-\frac{4aeg}{cd}\right)\int \frac{(d}{\sqrt{ade+\left(cd^2\right.}}$$

$$= -\frac{2\left(4ae^2g-cd(5ef-dg)\right)\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^2d^2e}+\frac{2g(d+ex)^{3/2}}{}$$

$$= -\frac{4\left(cd^2-ae^2\right)\left(4ae^2g-cd(5ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}}-\frac{2\left(4ae^2g-\right.}{}$$

Mathematica [A] time = 0.0993486, size = 96, normalized size = 0.46

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(8a^2e^3g-2acde(5dg+5ef+2egx)+c^2d^2(5d(3f+gx)+ex(5f+3gx))\right)}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^3*g - 2*a*c*d*e*(5*e*f + 5*d*g +
2*e*g*x) + c^2*d^2*(5*d*(3*f + g*x) + e*x*(5*f + 3*g*x))))/(15*c^3*d^3*Sqrt
[d + e*x])
```

Maple [A] time = 0.05, size = 131, normalized size = 0.6

$$\frac{(2\,cdx+2\,ae)\left(3\,egx^2c^2d^2-4\,acde^2gx+5\,c^2d^3gx+5\,c^2d^2efx+8\,a^2e^3g-10\,acd^2eg-10\,acde^2f+15\,d^3fc^2\right)}{15\,c^3d^3}\frac{\sqrt{ex+d}}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 2/15*(c*d*x+a*e)*(3*c^2*d^2*e*g*x^2-4*a*c*d*e^2*g*x+5*c^2*d^3*g*x+5*c^2*d^2
*e*f*x+8*a^2*e^3*g-10*a*c*d^2*e*g-10*a*c*d*e^2*f+15*c^2*d^3*f)*(e*x+d)^(1/2
)/c^3/d^3/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.54379, size = 227, normalized size = 1.09

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)f}{3\,\sqrt{cdx+ae}c^2d^2}+\frac{2\left(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+\left(5\,c^3d^4-ac^2d^2e^2\right)x^2-\left(5\,ac^2d^{\dots}\right.\right.}{15\,\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 + 8
*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^3
)*x)*g/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Fricas [A] time = 1.54105, size = 300, normalized size = 1.44

$$\frac{2\left(3\,c^2d^2egx^2+5\left(3\,c^2d^3-2\,acde^2\right)f-2\left(5\,acd^2e-4\,a^2e^3\right)g+\left(5\,c^2d^2ef+\left(5\,c^2d^3-4\,acde^2\right)g\right)x\right)\sqrt{cdex^2+ade+\left(cd^2\right.}}{15\left(c^3d^3ex+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*e*g*x^2 + 5*(3*c^2*d^3 - 2*a*c*d*e^2)*f - 2*(5*a*c*d^2*e -
4*a^2*e^3)*g + (5*c^2*d^2*e*f + (5*c^2*d^3 - 4*a*c*d*e^2)*g)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

3.787 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=109

$$\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

```
[Out] (4*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*
Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*c*d)
```

Rubi [A] time = 0.0594681, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {656, 648}

$$\frac{4\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (4*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*
Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*c*d)
```

Rule 656

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[(Simplify[m + p]*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \frac{2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3cd}+\frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx}{3d}\\ &= \frac{4\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{3cd}\end{aligned}$$

Mathematica [A] time = 0.0443115, size = 54, normalized size = 0.5

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(cd(3d+ex)-2ae^2\right)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

Maple [A] time = 0.049, size = 69, normalized size = 0.6

$$-\frac{(2\,cdx+2\,ae)\left(-cdex+2\,ae^2-3\,cd^2\right)}{3\,c^2d^2}\sqrt{ex+d}\frac{1}{\sqrt{cdex^2+ae^2x+cd^2x+ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2/3*(c*d*x+a*e)*(-c*d*e*x+2*a*e^2-3*c*d^2)*(e*x+d)^(1/2)/c^2/d^2/(c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Maxima [A] time = 1.42706, size = 88, normalized size = 0.81

$$\frac{2\left(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+\left(3\,c^2d^3-acde^2\right)x\right)}{3\,\sqrt{cdx+ae}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

Fricas [A] time = 1.55217, size = 158, normalized size = 1.45

$$\frac{2\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(cdex+3\,cd^2-2\,ae^2\right)\sqrt{ex+d}}{3\left(c^2d^2ex+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e^
2)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

3.788 $\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

```
[Out] (2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*Sqrt[d + e*x]) - (
2*(e*f - d*g)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rubi [A] time = 0.19195, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {880, 874, 205}

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] (2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*Sqrt[d + e*x]) - (
2*(e*f - d*g)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
 + 1)*(a + b*x + c*x^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{\left(2\left(\frac{1}{2}cde^2f-\frac{1}{2}cd^2eg\right)\right)\int \frac{\sqrt{d+}}{(f+gx)\sqrt{ade+(cd}}}{cdeg}$$

$$= \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{\left(2e^2(ef-dg)\right)\text{Subst}\left(\int \frac{1}{-e\left(cd^2+ae^2\right)g+cd}\right.}{g}$$

$$= \frac{2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right.}{g^{3/2}\sqrt{cdf-aeg}}$$

Mathematica [A] time = 0.110902, size = 140, normalized size = 1.01

$$\frac{2\sqrt{d+ex}\left(e\sqrt{g}(ae+cdx)\sqrt{cdf-aeg}+cd(dg-ef)\sqrt{ae+cdx}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{cdg^{3/2}\sqrt{(d+ex)(ae+cdx)}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*Sqrt[d + e*x]*(e*Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*(-(e*f)
 + d*g)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]]))/(c*d*g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A] time = 0.329, size = 163, normalized size = 1.2

$$-2\frac{\sqrt{cdex^2+ae^2x+cd^2x+ade}}{\sqrt{ex+d}\sqrt{cdx+ae}dcg\sqrt{\left(aeg-cdf\right)g}}\left(\text{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)cd^2g-\text{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)cdef-e\sqrt{cdx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh((c*d*x+a*e)^(1/2)*g/((a
*e*g-c*d*f)*g)^(1/2))*c*d^2*g-arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)
^(1/2))*c*d*e*f-e*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/
(c*d*x+a*e)^(1/2)/d/c/g/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)), x)
```

Fricas [A] time = 1.62759, size = 1085, normalized size = 7.81

$$\left[\frac{\left(cd^2ef - cd^3g + \left(cde^2f - cd^2eg\right)x\right)\sqrt{-cdfg + aeg^2}\log\left(-\frac{cdegx^2 - cd^2f + 2\,adeg - \left(cdef - \left(cd^2 + 2\,ae^2\right)g\right)x - 2\,\sqrt{cdex^2 + ade + \left(cd^2 + ae^2\right)x}\sqrt{-cdf}}{egx^2 + df + \left(ef + dg\right)x}\right.}{c^2d^3fg^2 - acd^2eg^3 + \left(c^2d^2efg^2 - acde^2g^3\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((c*d^2*e*f - c*d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(-c*d*f*g + a*e*g^2
)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)
*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)
*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(c*d*e*f*g - a*e^2*g^2
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*g^2
 - a*c*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x), 2*((c*d^2*e*f - c*
d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c
*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*f*g - a*e^2*g^2)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*g^2 - a*c
*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

3.789 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=170

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

```
[Out] -(((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f - a*e
*g)*Sqrt[d + e*x]*(f + g*x))) - ((2*a*e^2*g - c*d*(e*f + d*g))*ArcTan[(Sqrt
[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[
d + e*x])])/(g^(3/2)*(c*d*f - a*e*g)^(3/2))
```

Rubi [A] time = 0.233627, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {878, 874, 205}

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -(((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f - a*e
*g)*Sqrt[d + e*x]*(f + g*x))) - ((2*a*e^2*g - c*d*(e*f + d*g))*ArcTan[(Sqrt
[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[
d + e*x])])/(g^(3/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{\left(e\left(\frac{1}{2}cde^2f+\frac{3}{2}cd^2eg-e\left(\right.\right.\right.}{g\left(cde^2f\right.}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{\left(e^2\left(2ae^2g-cd(ef+dg)\right)\right)}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{\left(2ae^2g-cd(ef+dg)\right)\tan}{g^{3/2}(cdf}$$

Mathematica [A] time = 0.157788, size = 154, normalized size = 0.91

$$\frac{\sqrt{d+ex}\left(\frac{\sqrt{g}(ef-dg)(ae+cdx)}{f+gx}-\frac{\sqrt{ae+cdx}\left(cd(dg+ef)-2ae^2g\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{(d+ex)(ae+cdx)}(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*((Sqrt[g]*(e*f - d*g)*(a*e + c*d*x))/(f + g*x) - ((-2*a*e^2*
g + c*d*(e*f + d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/S
qrt[c*d*f - a*e*g]])/Sqrt[c*d*f - a*e*g]))/(g^(3/2)*(-(c*d*f) + a*e*g)*Sqrt
[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] time = 0.331, size = 347, normalized size = 2.

$$\frac{1}{\left(aeg-cdf\right)g\left(gx+f\right)}\left(-2\,\mathrm{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)xae^2g^2+\mathrm{Artanh}\left(g\sqrt{cdx+ae}\frac{1}{\sqrt{\left(aeg-cdf\right)g}}\right)xcd^2g^2+\mathrm{Arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] (-2*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*a*e^2*g^2+arctan
h((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*c*d^2*g^2+arctanh((c*d*x+a
*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*c*d*e*f*g-2*arctanh((c*d*x+a*e)^(1/2
)*g/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*f*g+arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-
c*d*f)*g)^(1/2))*c*d^2*f*g+arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1
/2))*c*d*e*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*d*g+((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*e*f)/(e*x+d)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a
*d*e)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^2), x)
```

Fricas [B] time = 1.58205, size = 1863, normalized size = 10.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2*e*f^2 + (c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2
*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*
e^2)*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
 d*g)*x)) + 2*(c*d*e*f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^
2*e*f^2*g^3 + a^2*d*e^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^
2*e^3*g^5)*x^2 + (c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^
2)*f^2*g^3 - (2*a*c*d^2*e - a^2*e^3)*f*g^4)*x), -((c*d^2*e*f^2 + (c*d^3 - 2
*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2
+ 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*e^2)*g^2)*x)*sqrt(c*d*f*g - a*e*
g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*
g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e*
f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^2*e*f^2*g^3 + a^2*d*e
^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^2 + (c^2
*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g^3 - (2*a*c*d
^2*e - a^2*e^3)*f*g^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.790 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{cd\left(4ae^2g-cd(3dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+}}{4g\sqrt{d}}$$

```
[Out] -((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x)^2) - ((4*a*e^2*g - c*d*(e*f + 3*d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(
f + g*x)) - (c*d*(4*a*e^2*g - c*d*(e*f + 3*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4
*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rubi [A] time = 0.355034, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {878, 872, 874, 205}

$$-\frac{cd\left(4ae^2g-cd(3dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+}}{4g\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```


```
[Out] -((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x)^2) - ((4*a*e^2*g - c*d*(e*f + 3*d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(
f + g*x)) - (c*d*(4*a*e^2*g - c*d*(e*f + 3*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4
*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```


Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{\left(e\left(\frac{1}{2}cde^2f+\frac{7}{2}cd^2eg-2e\right.\right.}{2g\left(cde^2\right.}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+3dg)\right)\sqrt{}}{4g(cdf-aeg)^2}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+3dg)\right)\sqrt{}}{4g(cdf-aeg)^2}$$

$$=-\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2}-\frac{\left(4ae^2g-cd(ef+3dg)\right)\sqrt{}}{4g(cdf-aeg)^2}$$

Mathematica [A] time = 0.464311, size = 189, normalized size = 0.72

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd\left(2ae^2g-\frac{1}{2}cd(3dg+ef)\right)\left(\frac{cdf-aeg}{cdf+cdgx}+\frac{\sqrt{cdf-aeg}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{ae+cdx}}\right)}{(cdf-aeg)^2}+\frac{ef-dg}{(f+gx)^2}\right)}{2g\sqrt{d+ex}(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((e*f - d*g)/(f + g*x)^2 + (c*d*(2*a*e^2*g -
 (c*d*(e*f + 3*d*g))/2)*((c*d*f - a*e*g)/(c*d*f + c*d*g*x) + (Sqrt[c*d*f -
a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[g]*Sq
rt[a*e + c*d*x])))/(c*d*f - a*e*g)^2))/(2*g*(-(c*d*f) + a*e*g)*Sqrt[d + e*x
])
```

Maple [B] time = 0.345, size = 673, normalized size = 2.6

$$\frac{1}{4\left(gx+f\right)^{2}g\left(aeg-cdf\right)^{2}}\sqrt{cdex^{2}+ae^{2}x+cd^{2}x+ade}\left(4\operatorname{Artanh}\left(\frac{\sqrt{cdx+aeg}}{\sqrt{\left(aeg-cdf\right)g}}\right)x^{2}acde^{2}g^{3}-3\operatorname{Artanh}\left(\frac{\sqrt{cdx+a}}{\sqrt{\left(aeg-cd\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] 1/4*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(4*arctanh((c*d*x+a*e)^(1/2)*g/
((a*e*g-c*d*f)*g)^(1/2))*x^2*a*c*d*e^2*g^3-3*arctanh((c*d*x+a*e)^(1/2)*g/((
a*e*g-c*d*f)*g)^(1/2))*x^2*c^2*d^3*g^3-arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-
c*d*f)*g)^(1/2))*x^2*c^2*d^2*e*f*g^2+8*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-
c*d*f)*g)^(1/2))*x*a*c*d*e^2*f*g^2-6*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*
d*f)*g)^(1/2))*x*c^2*d^3*f*g^2-2*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)
*g)^(1/2))*x*c^2*d^2*e*f^2*g+4*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g
)^(1/2))*a*c*d*e^2*f^2*g-3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1
/2))*c^2*d^3*f^2*g-arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*c^2
*d^2*e*f^3-4*x*a*e^2*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*x*c*d^
2*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+x*c*d*e*f*g*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-2*a*d*e*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)-2*a*e^2*f*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+5*c*d^2*f*g*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-c*d*e*f^2*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^2/g/(a*e*g-c
*d*f)^2/(c*d*x+a*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^{2}+ade+\left(cd^{2}+ae^{2}\right)x}\left(gx+f\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^3), x)
```

Fricas [B] time = 1.64706, size = 3411, normalized size = 13.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*g
^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2*d
```

```
^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*g^3)*x^2 + (c^2*d^2
*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*d^4 - 4*a*c*d^2*e^2
)*f*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*
g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f +
d*g)*x)) - 2*(c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g^4 - (5*c^2*d^3 - a*c*d*e^2)*f
^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c^2*d^2*e*f^2*g^2 + (3*c^2*d^3
- 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e^3)*g^4)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g^2 - 3*a*c^2*d^3*e*f
^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*f^2*g^5 + (c^3*d^3*e*f^3*g^4 -
 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^3 + (2*c^3*
d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*
c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^6)*x
^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)
*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^4 + (6*a^2*c*d^2*e^2 - a^3
*e^4)*f^2*g^5)*x), -1/4*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g
 + (c^2*d^2*e^2*f*g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e
^2*f^2*g + (7*c^2*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*
g^3)*x^2 + (c^2*d^2*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*
d^4 - 4*a*c*d^2*e^2)*f*g^2)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e
*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g
^4 - (5*c^2*d^3 - a*c*d*e^2)*f^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c
^2*d^2*e*f^2*g^2 + (3*c^2*d^3 - 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e
^3)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3
*d^4*f^5*g^2 - 3*a*c^2*d^3*e*f^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*
f^2*g^5 + (c^3*d^3*e*f^3*g^4 - 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^
6 - a^3*e^4*g^7)*x^3 + (2*c^3*d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*
a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c
*d^2*e^2 - 2*a^3*e^4)*f*g^6)*x^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 +
 (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^
3*g^4 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")

[Out] Timed out
```

3.791 $\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=351

$$-\frac{c^2d^2\left(6ae^2g-cd(5dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g-cd(5dg+ef)\right)}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-$$

```
[Out] -((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x)^3) - ((6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*
(f + g*x)^2) - (c*d*(6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(8*g*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c^
2*d^2*(6*a*e^2*g - c*d*(e*f + 5*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*g^(3/2)*(c
*d*f - a*e*g)^(7/2))
```

Rubi [A] time = 0.557881, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {878, 872, 874, 205}

$$-\frac{c^2d^2\left(6ae^2g-cd(5dg+ef)\right)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(6ae^2g-cd(5dg+ef)\right)}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```


```
[Out] -((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x)^3) - ((6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*
(f + g*x)^2) - (c*d*(6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(8*g*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c^
2*d^2*(6*a*e^2*g - c*d*(e*f + 5*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*g^(3/2)*(c
*d*f - a*e*g)^(7/2))
```


Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p + 1))/(g*(n + 1)*(c*e*f + c*d*g - b*
e*g)), x] - Dist[(e*(b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3)))/
(g*(n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(
```

```
n + 1)*(a + b*x + c*x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g - b*e*g)), x] - D
ist[(c*e*(m - n - 2))/((n + 1)*(c*e*f + c*d*g - b*e*g)), Int[(d + e*x)^m*(f
 + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p
]
```

Rule 874

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}+\frac{\left(e\left(\frac{1}{2}cde^2f+\frac{11}{2}cd^2eg-3e\right.\right.}{3g\left(cde\right.}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}-\frac{\left(6ae^2g-cd(ef+5dg)\right)\sqrt{}}{12g(cdf-aeg)^2}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}-\frac{\left(6ae^2g-cd(ef+5dg)\right)\sqrt{}}{12g(cdf-aeg)^2}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}-\frac{\left(6ae^2g-cd(ef+5dg)\right)\sqrt{}}{12g(cdf-aeg)^2}$$

$$= -\frac{(ef-dg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3}-\frac{\left(6ae^2g-cd(ef+5dg)\right)\sqrt{}}{12g(cdf-aeg)^2}$$

Mathematica [C] time = 0.117978, size = 132, normalized size = 0.38

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{ef-dg}{(f+gx)^3}-\frac{c^2d^2\left(cd(5dg+ef)-6ae^2g\right){}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{g(ae+cdx)}{aeg-cdf}\right)}{(cdf-aeg)^3}\right)}{3g\sqrt{d+ex}(aeg-cdf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((e*f - d*g)/(f + g*x)^3 - (c^2*d^2*(-6*a*e^
2*g + c*d*(e*f + 5*d*g))*Hypergeometric2F1[1/2, 3, 3/2, (g*(a*e + c*d*x))/(
-(c*d*f) + a*e*g)])/(c*d*f - a*e*g)^3))/(3*g*(-(c*d*f) + a*e*g)*Sqrt[d + e*
```

```
x])
```

Maple [B] time = 0.35, size = 1142, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] -1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-15*arctanh((c*d*x+a*e)^(1/2
)*g/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^3*d^4*g^4+4*a^2*e^3*f*g^2*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)-50*x*a*c*d*e^2*f*g^2*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)-15*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*c^3
*d^4*f^3*g-3*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*e
*f^4-16*a*c*d*e^2*f^2*g*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+54*arctan
h((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*a*c^2*d^2*e^2*f^2*g^2-18*x
^2*a*c*d*e^2*g^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+3*x^2*c^2*d^2*e*
f*g^2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)-10*x*a*c*d^2*e*g^3*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+8*x*c^2*d^2*e*f^2*g*((a*e*g-c*d*f)*g)^(1/
2)*(c*d*x+a*e)^(1/2)-26*a*c*d^2*e*f*g^2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)
^(1/2)-9*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^3*d^3*e
*f^2*g^2-9*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*c^3*d^3*e
*f^3*g+18*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e^
2*f^3*g+40*x*c^2*d^3*f*g^2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+18*arc
tanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^3*a*c^2*d^2*e^2*g^4-3*a
rctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^3*d^3*e*f*g^3+54*
arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^2*a*c^2*d^2*e^2*f*g^
3+12*x*a^2*e^3*g^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+8*a^2*d*e^2*g^
3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)+33*c^2*d^3*f^2*g*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)-3*c^2*d^2*e*f^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+
a*e)^(1/2)-45*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^3*
d^4*f*g^3-45*arctanh((c*d*x+a*e)^(1/2)*g/((a*e*g-c*d*f)*g)^(1/2))*x*c^3*d^4
*f^2*g^2+15*x^2*c^2*d^3*g^3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2))/(e*x
+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*
a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^4), x)
```

Fricas [B] time = 1.78885, size = 5411, normalized size = 15.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (c^3*d^3*e
^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e^2*f^2*g^
2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)
*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f^2*g^2 +
(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4*c^3*d^4*
e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(
3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*a*c^2*d^2*e^2)*f^3*g
^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*a^2*c*d^2*e^2 - 2*a^
3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*c^2*d^2*e^2)*f*g^4 -
 (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^3*e*f^3*g^2 + (20*c^
3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 31*a^2*c*d*e^3)*f*g^4
 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4*e*f^6*g^3 + 6*a^2*c
^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^4*f^3*g^6 + (c^4*d^4
*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*
d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^
4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f
^3*g^6 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 3
*a^4*e^5)*f*g^8)*x^3 + 3*(c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 -
4*a*c^3*d^3*e^2)*f^5*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 +
2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^6 - (4*a^3*c*d^2*e^3 - a^4*e^5)
*f^2*g^7)*x^2 + (c^4*d^4*e*f^7*g^2 + 3*a^4*d*e^4*f^2*g^7 + (3*c^4*d^5 - 4*a
*c^3*d^3*e^2)*f^6*g^3 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^4 + 2*(9*
a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^3
*g^6)*x), -1/24*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (
c^3*d^3*e^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e
^2*f^2*g^2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2
*d^3*e^2)*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f
^2*g^2 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4
*c^3*d^4*e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2 + a*d*e*g + (c*d^
2 + a*e^2)*g*x)) + (3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*
a*c^2*d^2*e^2)*f^3*g^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*
a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*
c^2*d^2*e^2)*f*g^4 - (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^
3*e*f^3*g^2 + (20*c^3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 3
1*a^2*c*d*e^3)*f*g^4 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4
*e*f^6*g^3 + 6*a^2*c^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^
4*f^3*g^6 + (c^4*d^4*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^
3*f^2*g^7 - 4*a^3*c*d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 +
 a^4*d*e^4*g^9 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e -
9*a^2*c^2*d^2*e^3)*f^3*g^6 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^2*g^7 -
(4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^8)*x^3 + 3*(c^4*d^4*e*f^6*g^3 + a^4*d*e^4
*f*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2
*d^2*e^3)*f^4*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^6 - (4*a^3*
c*d^2*e^3 - a^4*e^5)*f^2*g^7)*x^2 + (c^4*d^4*e*f^7*g^2 + 3*a^4*d*e^4*f^2*g^
7 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^3 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*
e^3)*f^5*g^4 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (12*a^3*c*d^
2*e^3 - a^4*e^5)*f^3*g^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

3.792 $\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=324

$$-\frac{x\sqrt{1-d^2x^2}\left(24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)}{16d^6}-\frac{b\sqrt{1-d^2x^2}\left(45a^2d^4+60acd^2+10b^2d^2+24c^2\right)}{15d^6}+\frac{\text{si}}{}$$

```
[Out] -(b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2 + 45*a^2*d^4)*Sqrt[1 - d^2*x^2])/(15*
d^6) - ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4)
*x*Sqrt[1 - d^2*x^2])/(16*d^6) - (b*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*S
qrt[1 - d^2*x^2])/(15*d^4) - (c*(5*c^2 + 18*b^2*c*d^2 + 18*a*c*d^2)*x^3*Sqrt[
1 - d^2*x^2])/(24*d^4) - (3*b*c^2*x^4*Sqrt[1 - d^2*x^2])/(5*d^2) - (c^3*x^5
*Sqrt[1 - d^2*x^2])/(6*d^2) + ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*
b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(16*d^7)
```

Rubi [A] time = 0.933511, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {899, 1815, 641, 216}

$$-\frac{x\sqrt{1-d^2x^2}\left(24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)}{16d^6}-\frac{b\sqrt{1-d^2x^2}\left(45a^2d^4+60acd^2+10b^2d^2+24c^2\right)}{15d^6}+\frac{\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```


```
[Out] -(b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2 + 45*a^2*d^4)*Sqrt[1 - d^2*x^2])/(15*
d^6) - ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4)
*x*Sqrt[1 - d^2*x^2])/(16*d^6) - (b*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*S
qrt[1 - d^2*x^2])/(15*d^4) - (c*(5*c^2 + 18*b^2*c*d^2 + 18*a*c*d^2)*x^3*Sqrt[
1 - d^2*x^2])/(24*d^4) - (3*b*c^2*x^4*Sqrt[1 - d^2*x^2])/(5*d^2) - (c^3*x^5
*Sqrt[1 - d^2*x^2])/(6*d^2) + ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*
b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(16*d^7)
```


Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(a+bx+cx^2)^3}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} - \frac{\int \frac{-6a^3d^2-18a^2bd^2x-18a(b^2+ac)d^2x^2-6b(b^2+6ac)d^2x^3-c(5c^2+18b^2d^2+18acd^2)x^4-18bc^2d^2x^5}{\sqrt{1-d^2x^2}}\,dx}{6d^2}$$

$$= -\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} + \frac{\int \frac{30a^3d^4+90a^2bd^4x+90a(b^2+ac)d^4x^2+6bd^2(12c^2+5b^2d^2+30acd^2)x^3+5c\cdots}{\sqrt{1-d^2x^2}}}{30d^4}$$

$$= -\frac{c(5c^2+18b^2d^2+18acd^2)x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} - \int \frac{-120a^3d^6-360\cdots}{\cdots}$$

$$= -\frac{b(12c^2+5b^2d^2+30acd^2)x^2\sqrt{1-d^2x^2}}{15d^4} - \frac{c(5c^2+18b^2d^2+18acd^2)x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1\cdots}}{5d\cdots}$$

$$= -\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)x\sqrt{1-d^2x^2}}{16d^6} - \frac{b(12c^2+5b^2d^2+30acd^2)x^2\sqrt{\cdots}}{15d^4}$$

$$= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6} - \frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a\cdots}{16d^6}$$

$$= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6} - \frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a\cdots}{16d^6}$$

Mathematica [A] time = 0.262266, size = 229, normalized size = 0.71

$$\frac{15\sin^{-1}(dx)\left(24a^2cd^4+16a^3d^6+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3\right)-d\sqrt{1-d^2x^2}\left(48b\left(15a^2d^4+10acd^2\left(d^2x^2+2\right)+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(80*b^3*d^2*(2 + d^2*x^2) + 90*b^2*d^2*x*(4*a*d^2 +
c*(3 + 2*d^2*x^2)) + 48*b*(15*a^2*d^4 + 10*a*c*d^2*(2 + d^2*x^2) + c^2*(8 +
 4*d^2*x^2 + 3*d^4*x^4)) + 5*c*x*(72*a^2*d^4 + 18*a*c*d^2*(3 + 2*d^2*x^2) +
 c^2*(15 + 10*d^2*x^2 + 8*d^4*x^4)))) + 15*(5*c^3 + 18*b^2*c*d^2 + 18*a*c^2
*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(240*d^7)
```

Maple [C] time = 0.183, size = 602, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/240*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(40*csgn(d)*x^5*c^3*d^5*(-d^2*x^2+1)^(1
/2)+144*csgn(d)*x^4*b*c^2*d^5*(-d^2*x^2+1)^(1/2)+180*csgn(d)*x^3*a*c^2*d^5*
(-d^2*x^2+1)^(1/2)+180*csgn(d)*x^3*b^2*c*d^5*(-d^2*x^2+1)^(1/2)+480*csgn(d)
*x^2*a*b*c*d^5*(-d^2*x^2+1)^(1/2)+80*csgn(d)*x^2*b^3*d^5*(-d^2*x^2+1)^(1/2)
+50*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^3*c^3+360*csgn(d)*d^5*(-d^2*x^2+1)^(1/
2)*x*a^2*c+360*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*x*a*b^2+192*csgn(d)*d^3*(-d^2
*x^2+1)^(1/2)*x^2*b*c^2+720*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*a^2*b-240*arctan
(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^3*d^6+270*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)
*x*a*c^2+270*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*b^2*c+960*csgn(d)*d^3*(-d^2*x
^2+1)^(1/2)*a*b*c+160*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*b^3-360*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*a^2*c*d^4-360*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*a*b^2*d^4+75*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c^3+384*csgn(d)*d*(-d^2*x^2+
1)^(1/2)*b*c^2-270*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*c^2*d^2-270*arc
tan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b^2*c*d^2-75*arctan(csgn(d)*d*x/(-d^2*x
^2+1)^(1/2))*c^3)*csgn(d)/d^7/(-d^2*x^2+1)^(1/2)
```

Maxima [A] time = 1.80824, size = 554, normalized size = 1.71

$$-\frac{\sqrt{-d^2x^2+1}c^3x^5}{6d^2} - \frac{3\sqrt{-d^2x^2+1}bc^2x^4}{5d^2} + \frac{a^3\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{5\sqrt{-d^2x^2+1}c^3x^3}{24d^4} - \frac{3\sqrt{-d^2x^2+1}\left(b^2c+ac^2\right)x^3}{4d^2} - 3\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*sqrt(-d^2*x^2 + 1)*c^3*x^5/d^2 - 3/5*sqrt(-d^2*x^2 + 1)*b*c^2*x^4/d^2
+ a^3*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 5/24*sqrt(-d^2*x^2 + 1)*c^3*x^3/d
^4 - 3/4*sqrt(-d^2*x^2 + 1)*(b^2*c + a*c^2)*x^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*
a^2*b/d^2 - 4/5*sqrt(-d^2*x^2 + 1)*b*c^2*x^2/d^4 - 1/3*sqrt(-d^2*x^2 + 1)*(
b^3 + 6*a*b*c)*x^2/d^2 - 3/2*sqrt(-d^2*x^2 + 1)*(a*b^2 + a^2*c)*x/d^2 + 3/2
*(a*b^2 + a^2*c)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) - 5/16*sqrt(-d^2*x
^2 + 1)*c^3*x/d^6 - 9/8*sqrt(-d^2*x^2 + 1)*(b^2*c + a*c^2)*x/d^4 - 8/5*sqrt
(-d^2*x^2 + 1)*b*c^2/d^6 - 2/3*sqrt(-d^2*x^2 + 1)*(b^3 + 6*a*b*c)/d^4 + 5/1
6*c^3*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^6) + 9/8*(b^2*c + a*c^2)*arcsin(
d^2*x/sqrt(d^2))/(sqrt(d^2)*d^4)
```

Fricas [A] time = 1.39823, size = 570, normalized size = 1.76

$$-\frac{\left(40\,c^3d^5x^5 + 144\,bc^2d^5x^4 + 720\,a^2bd^5 + 384\,bc^2d + 160\left(b^3 + 6\,abc\right)d^3 + 10\left(5\,c^3d^3 + 18\left(b^2c + ac^2\right)d^5\right)x^3 + 16\left(12\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*((40*c^3*d^5*x^5 + 144*b*c^2*d^5*x^4 + 720*a^2*b*d^5 + 384*b*c^2*d +
 160*(b^3 + 6*a*b*c)*d^3 + 10*(5*c^3*d^3 + 18*(b^2*c + a*c^2)*d^5)*x^3 + 16
*(12*b*c^2*d^3 + 5*(b^3 + 6*a*b*c)*d^5)*x^2 + 15*(24*(a*b^2 + a^2*c)*d^5 +
5*c^3*d + 18*(b^2*c + a*c^2)*d^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(16*
a^3*d^6 + 24*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 18*(b^2*c + a*c^2)*d^2)*arctan((
sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^7
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.58294, size = 518, normalized size = 1.6

$$-\frac{\left(720\,a^2bd^{41} - 360\,ab^2d^{40} - 360\,a^2cd^{40} + 240\,b^3d^{39} + 1440\,abcd^{39} - 450\,b^2cd^{38} - 450\,ac^2d^{38} + 720\,bc^2d^{37} - 165\,c^3d^{36} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/21626880*((720*a^2*b*d^41 - 360*a*b^2*d^40 - 360*a^2*c*d^40 + 240*b^3*d^
39 + 1440*a*b*c*d^39 - 450*b^2*c*d^38 - 450*a*c^2*d^38 + 720*b*c^2*d^37 - 1
65*c^3*d^36 + (360*a*b^2*d^40 + 360*a^2*c*d^40 - 160*b^3*d^39 - 960*a*b*c*d
^39 + 810*b^2*c*d^38 + 810*a*c^2*d^38 - 960*b*c^2*d^37 + 425*c^3*d^36 + 2*(
40*b^3*d^39 + 240*a*b*c*d^39 - 270*b^2*c*d^38 - 270*a*c^2*d^38 + 528*b*c^2*
d^37 - 275*c^3*d^36 + (90*b^2*c*d^38 + 90*a*c^2*d^38 - 288*b*c^2*d^37 + 225
*c^3*d^36 + 4*(5*(d*x + 1)*c^3*d^36 + 18*b*c^2*d^37 - 25*c^3*d^36)*(d*x + 1
))*(d*x + 1))*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(16*a
^3*d^42 + 24*a*b^2*d^40 + 24*a^2*c*d^40 + 18*b^2*c*d^38 + 18*a*c^2*d^38 + 5
*c^3*d^36)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d
```

3.793 $\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=166

$$\frac{\sin^{-1}(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{8d^5}-\frac{x\sqrt{1-d^2x^2}\left(c\left(8a+\frac{3c}{d^2}\right)+4b^2\right)}{8d^2}-\frac{2b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)}{3d^4}-\frac{2bcx^2\sqrt{1-}}{3d^2}$$

```
[Out] (-2*b*(2*c + 3*a*d^2)*Sqrt[1 - d^2*x^2])/(3*d^4) - ((4*b^2 + c*(8*a + (3*c)
/d^2))*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (2*b*c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2)
 - (c^2*x^3*Sqrt[1 - d^2*x^2])/(4*d^2) + ((3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 +
8*a^2*d^4)*ArcSin[d*x])/(8*d^5)
```

Rubi [A] time = 0.31886, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {899, 1815, 641, 216}

$$\frac{\sin^{-1}(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{8d^5}-\frac{x\sqrt{1-d^2x^2}\left(c\left(8a+\frac{3c}{d^2}\right)+4b^2\right)}{8d^2}-\frac{2b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)}{3d^4}-\frac{2bcx^2\sqrt{1-}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-2*b*(2*c + 3*a*d^2)*Sqrt[1 - d^2*x^2])/(3*d^4) - ((4*b^2 + c*(8*a + (3*c)
/d^2))*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (2*b*c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2)
 - (c^2*x^3*Sqrt[1 - d^2*x^2])/(4*d^2) + ((3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 +
8*a^2*d^4)*ArcSin[d*x])/(8*d^5)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(a+bx+cx^2)^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} - \frac{\int \frac{-4a^2d^2-8abd^2x-(3c^2+4b^2d^2+8acd^2)x^2-8bcd^2x^3}{\sqrt{1-d^2x^2}}\,dx}{4d^2}$$
$$= -\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} + \frac{\int \frac{12a^2d^4+8bd^2(2c+3ad^2)x+3d^2(3c^2+4b^2d^2+8acd^2)x^2}{\sqrt{1-d^2x^2}}\,dx}{12d^4}$$
$$= -\frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} - \int \frac{-3d^2(3c^2+4b^2d^2+8acd}{\sqrt{}}$$
$$= -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-}}{4d^2}$$
$$= -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-}}{4d^2}$$

Mathematica [A] time = 0.118725, size = 114, normalized size = 0.69

$$\frac{3\sin^{-1}(dx)(8a^2d^4+8acd^2+4b^2d^2+3c^2) - d\sqrt{1-d^2x^2}\left(16b(3ad^2+cd^2x^2+2c)+3cx(8ad^2+2cd^2x^2+3c)+12b^2d^2x\right.}{24d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(12*b^2*d^2*x + 16*b*(2*c + 3*a*d^2 + c*d^2*x^2) + 3
*c*x*(3*c + 8*a*d^2 + 2*c*d^2*x^2))) + 3*(3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 + 8
*a^2*d^4)*ArcSin[d*x])/(24*d^5)
```

Maple [C] time = 0.171, size = 291, normalized size = 1.8

$$-\frac{\text{csgn}(d)}{24\,d^5}\sqrt{-dx+1}\sqrt{dx+1}\left(6\,\text{csgn}(d)\,x^3c^2d^3\sqrt{-d^2x^2+1}+16\,\text{csgn}(d)\,x^2bcd^3\sqrt{-d^2x^2+1}+24\,\text{csgn}(d)\,d^3\sqrt{-d^2x^2+1}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*csgn(d)*x^3*c^2*d^3*(-d^2*x^2+1)^(1/2
)+16*csgn(d)*x^2*b*c*d^3*(-d^2*x^2+1)^(1/2)+24*csgn(d)*d^3*(-d^2*x^2+1)^(1/
2)*x*a*c+12*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*b^2+48*csgn(d)*d^3*(-d^2*x^2+1
)^(1/2)*a*b-24*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^2*d^4+9*csgn(d)*d*(
-d^2*x^2+1)^(1/2)*x*c^2+32*csgn(d)*d*(-d^2*x^2+1)^(1/2)*b*c-24*arctan(csgn(
d)*d*x/(-d^2*x^2+1)^(1/2))*a*c*d^2-12*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)
)*b^2*d^2-9*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c^2)*csgn(d)/d^5/(-d^2*x
^2+1)^(1/2)
```

Maxima [A] time = 1.99035, size = 275, normalized size = 1.66

$$-\frac{\sqrt{-d^2x^2+1}c^2x^3}{4\,d^2}-\frac{2\,\sqrt{-d^2x^2+1}bcx^2}{3\,d^2}+\frac{a^2\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,\sqrt{-d^2x^2+1}ab}{d^2}-\frac{\sqrt{-d^2x^2+1}\left(b^2+2\,ac\right)x}{2\,d^2}-\frac{3\,\sqrt{-d^2x^2}}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-d^2*x^2 + 1)*c^2*x^3/d^2 - 2/3*sqrt(-d^2*x^2 + 1)*b*c*x^2/d^2 +
a^2*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 2*sqrt(-d^2*x^2 + 1)*a*b/d^2 - 1/2*
sqrt(-d^2*x^2 + 1)*(b^2 + 2*a*c)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*c^2*x/d^4 +
 1/2*(b^2 + 2*a*c)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) - 4/3*sqrt(-d^2*
x^2 + 1)*b*c/d^4 + 3/8*c^2*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^4)
```

Fricas [A] time = 1.39013, size = 313, normalized size = 1.89

$$-\frac{\left(6\,c^2d^3x^3+16\,bcd^3x^2+48\,abd^3+32\,bcd+3\left(4\left(b^2+2\,ac\right)d^3+3\,c^2d\right)x\right)\sqrt{dx+1}\sqrt{-dx+1}+6\left(8\,a^2d^4+4\left(b^2+2\,ac\right.\right.}{24\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*((6*c^2*d^3*x^3 + 16*b*c*d^3*x^2 + 48*a*b*d^3 + 32*b*c*d + 3*(4*(b^2
+ 2*a*c)*d^3 + 3*c^2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*a^2*d^4 + 4*
(b^2 + 2*a*c)*d^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))
)/d^5
```

Sympy [C] time = 141.49, size = 631, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a**2*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + a**2*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))
/(4*pi**(3/2)*d) - I*a*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/
4, 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(2*pi**(3/2)*d**2) - a*b*meijerg(((
-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_
polar(-2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**2) - I*a*c*meijerg(((-3/4, -1/4
), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/
(2*pi**(3/2)*d**3) + a*c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-
5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(2*pi**(3/2
)*d**3) - I*b**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1
/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) + b**2*meijerg(((-3
/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_p
olar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*c*meijerg(((-5/4, -3/4)
```

```
, (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d**2*x**2))
/(2*pi**(3/2)*d**4) - b*c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7
/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(2*pi**(3/
2)*d**4) - I*c**2*meijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4,
-3/2, -5/4, -1, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**5) + c**2*meijerg((
(-5/2, -9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -2, -2, 0)), ex
p_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**5)
```

Giac [A] time = 1.7079, size = 243, normalized size = 1.46

$$-\frac{\left(48\,abd^{19}-12\,b^{2}d^{18}-24\,acd^{18}+48\,bcd^{17}-15\,c^{2}d^{16}+\left(12\,b^{2}d^{18}+24\,acd^{18}-32\,bcd^{17}+27\,c^{2}d^{16}+2\left(3\,(dx+1)c^{2}d^{16}+\right.\right.}{344}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/344064*((48*a*b*d^19 - 12*b^2*d^18 - 24*a*c*d^18 + 48*b*c*d^17 - 15*c^2*
d^16 + (12*b^2*d^18 + 24*a*c*d^18 - 32*b*c*d^17 + 27*c^2*d^16 + 2*(3*(d*x +
 1)*c^2*d^16 + 8*b*c*d^17 - 9*c^2*d^16)*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)
*sqrt(-d*x + 1) - 6*(8*a^2*d^20 + 4*b^2*d^18 + 8*a*c*d^18 + 3*c^2*d^16)*arc
sin(1/2*sqrt(2)*sqrt(d*x + 1)))/d
```

3.794 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rubi [A] time = 0.0627523, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{(-c-2ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.0344346, size = 45, normalized size = 0.71

$$\frac{(2ad^2+c)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*(2*b + c*x)*Sqrt[1 - d^2*x^2]) + (c + 2*a*d^2)*ArcSin[d*x])/(2*d^3)
```

Maple [C] time = 0.156, size = 117, normalized size = 1.9

$$-\frac{\mathrm{csgn}(d)}{2d^3}\sqrt{-dx+1}\sqrt{dx+1}\left(\mathrm{csgn}(d)\,d\sqrt{-d^2x^2+1}xc - 2\arctan\left(\frac{\mathrm{csgn}(d)\,dx}{\sqrt{-d^2x^2+1}}\right)ad^2 + 2\,\mathrm{csgn}(d)\,d\sqrt{-d^2x^2+1}b - \mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Maxima [A] time = 1.90411, size = 105, normalized size = 1.67

$$\frac{a\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d^2*x/sqrt(d^2))/sqrt(d^2) - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt
(-d^2*x^2 + 1)*b/d^2 + 1/2*c*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.46189, size = 167, normalized size = 2.65

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\left(2\,ad^2+c\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [C] time = 26.4147, size = 282, normalized size = 4.48

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - b*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

Giac [A] time = 1.55981, size = 97, normalized size = 1.54

$$-\frac{\left((dx+1)cd^4+2\,bd^5-cd^4\right)\sqrt{dx+1}\sqrt{-dx+1}-2\left(2\,ad^6+cd^4\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/192*(((d*x + 1)*c*d^4 + 2*b*d^5 - c*d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) -
2*(2*a*d^6 + c*d^4)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d
```

3.795 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}(a+bx+cx^2)}\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt{2}c\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\sqrt{2}c\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

```
[Out] -((Sqrt[2]*c*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*
c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt
[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2])) + (
Sqrt[2]*c*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2
 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^
2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2])
```

Rubi [A] time = 0.52237, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {899, 985, 725, 206}

$$\frac{\sqrt{2}c\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\sqrt{2}c\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```



```
[Out] -((Sqrt[2]*c*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*
c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt
[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2])) + (
Sqrt[2]*c*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2
 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^
2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2])
```


Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c^2-\left(b-\sqrt{b^2-4ac}\right)^2d^2-x^2}\,dx, x, \frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}} + \frac{(2c)\,\mathrm{Subst}\left(\int \right.}{}$$

$$= -\frac{\sqrt{2}c\tanh^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}} + \frac{\sqrt{2}c\tanh^{-1}\left(\frac{2c}{\sqrt{2}\sqrt{2c^2+2ac}}\right.}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2}}$$

Mathematica [A] time = 0.588563, size = 260, normalized size = 0.92

$$\frac{2\sqrt{2}c\left(\frac{\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{1-d^2x^2}\sqrt{-2bd^2\left(\sqrt{b^2-4ac}+b\right)+4acd^2+4c^2}}\right)}{2\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{2\sqrt{bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*Sqrt[2]*c*(-ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2 +
4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]/(2*Sqrt[2
*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2]) + ArcTanh[(2*c + (b + S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*
c])*d^2]*Sqrt[1 - d^2*x^2])]/(2*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2])))/Sqrt[b^2 - 4*a*c]
```

Maple [C] time = 0.51, size = 1759, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -32*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*c^2*(ln(2*(b*d^2*x-(-4*a*c+b^2)^
(1/2)*x*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d
^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b
^2)^(1/2)))*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c
+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+b*d^2*
x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*
c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))
*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2
)-2*a*c+b^2)/a^2/c^2)^(1/2)+2*ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*
x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^
(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))*a*c*d^
2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-
b^2)/a^2/c^2)^(1/2)-ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*x^2+1)^(1/
2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*
c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))*b^2*d^2*(-(b*(-4
*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c
^2)^(1/2)-2*ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+b*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*
(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^
2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*a*c*d^2*(-(b*(-4*a*c+b^2)
^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2
)+ln(2*((-4*a*c+b^2)^(1/2)*x*d^2+b*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^
2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/
2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*b^2*d^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a
*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)+ln(2*(b*
d^2*x-(-4*a*c+b^2)^(1/2)*x*d^2+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2
*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)
/(b+2*c*x-(-4*a*c+b^2)^(1/2)))*c^2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^
2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/
2)*x*d^2+b*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a
^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a
*c+b^2)^(1/2)))*c^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*
c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2))/(-d^2*x^2+1)^(1/2)/(b*d-d*(-4*a*c+b
^2)^(1/2)+2*c)/(d*(-4*a*c+b^2)^(1/2)+b*d+2*c)/(b*d-d*(-4*a*c+b^2)^(1/2)-2*c
)/(-4*a*c+b^2)^(1/2)/(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a
*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)/(d*(-4*a*c+b^2)^(1/2)+b*d-2*c)/(-(b
*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a
^2/c^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)\sqrt{dx + 1}\sqrt{-dx + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

Fricas [B] time = 2.52302, size = 8660, normalized size = 30.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")

[Out] 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b
^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^
2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*
a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 -
6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^
3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)
*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*
c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((
a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^
4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^
6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c
- 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2
*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b
^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*
c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(
-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3
- (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8
- 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*
a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 -
 ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^
2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c +
8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24
*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*
a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4
*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(
b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*
a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^
2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^
2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(
2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c
 + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4
/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2
*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b
^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3
*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^
3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*
d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 +
b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2
*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*
c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(
-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3
- (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8
- 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*
a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^
2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^
4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c
)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^
6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3
 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a
```

```
*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4
*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3
*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2
*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - ((
a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d
^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*
c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*
a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*
b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((
a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*
d^2)))/x) + 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^
3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*
d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 +
b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2
*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*
c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sq
rt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4
 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqr
t(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*
d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d
^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(2)*(((a^3*b^3 - 4
*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d
^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^
5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 +
(b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*
c^3 + 8*a^2*c^4)*d^2))*x - ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d
^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^
2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4
*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5
 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b
^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (
b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-dx+1}\sqrt{dx+1}\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.796 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=571

$$-\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2a}}\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

```
[Out] -(((b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)*Sqrt
[1 - d^2*x^2])/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^2)))
 - (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(5*b^
2 - b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c]
)*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*S
qrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b
*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) + (c*(4*c^3 + 12*a
*c^2*d^2 - 2*a*b^2*d^4 - b*(b + Sqrt[b^2 - 4*a*c])*d^2*(c - a*d^2) - 4*c*d^
2*(b^2 - 2*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]
*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rubi [A] time = 5.23475, antiderivative size = 571, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 975, 1034, 725, 206}

$$-\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2a}}\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(((b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)*Sqrt
[1 - d^2*x^2])/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^2)))
 - (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(5*b^
2 - b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c]
)*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*S
qrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b
*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) + (c*(4*c^3 + 12*a
*c^2*d^2 - 2*a*b^2*d^4 - b*(b + Sqrt[b^2 - 4*a*c])*d^2*(c - a*d^2) - 4*c*d^
2*(b^2 - 2*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]
*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
```

```
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)^2}\,dx=\int \frac{1}{\left(a+bx+cx^2\right)^2\sqrt{1-d^2x^2}}\,dx$$

$$=-\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{-2c^3-6ac\cdots}{\cdots}}{(b\cdots}$$

$$=-\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)}+\frac{\left(c\left(4c^3+\cdots\right.\right.}{\cdots}$$

$$=-\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)}-\frac{\left(c\left(4c^3+\cdots\right.\right.}{\cdots}$$

$$=-\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)}-\frac{c\left(4c^3+1\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.40914, size = 508, normalized size = 0.89

$$\frac{\frac{c\left(cd^2\left(8a^2d^2+b\sqrt{b^2-4ac}-5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2}}+\frac{c\left(cd^2\left(-8a^2d^2+b\sqrt{b^2-4ac}+5b^2\right)+abd^4\left(b-\cdots\right.\right.}{\sqrt{2}\sqrt{b\cdots}}}{\left(b^2-4ac\right)\left(\left(ad^2+c\right)^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] (((b^3*d^2 - b*c*(c + 3*a*d^2) + b^2*c*d^2*x - 2*c^2*(c + a*d^2)*x)*Sqrt[1
- d^2*x^2])/(a + x*(b + c*x)) + (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^
2 - 4*a*c])*d^4 + c*d^2*(-5*b^2 + b*Sqrt[b^2 - 4*a*c] + 8*a^2*d^2))*ArcTanh
[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2 + 4*a*c*d^2 + 2*b*(-b +
Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2]) + (c*(-4*c^3 - 12*a*
c^2*d^2 + a*b*(b - Sqrt[b^2 - 4*a*c])*d^4 + c*d^2*(5*b^2 + b*Sqrt[b^2 - 4*a
*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[4*c^2
 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[
2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2
]))/((b^2 - 4*a*c)*(-(b^2*d^2) + (c + a*d^2)^2))
```

Maple [C] time = 1.205, size = 41837, normalized size = 73.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)^2 \sqrt{dx + 1}\sqrt{-dx + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

3.797 $\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{x(ad^2+c)(a^2d^4+2acd^2+3b^2d^2+c^2)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}-\frac{3\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b\right.}{8d^7}$$

```
[Out] (b*(3*a^2 + (3*c^2)/d^4 + b^2/d^2 + (6*a*c)/d^2)*d^4 + (c + a*d^2)*(c^2 + 3
*b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^6*Sqrt[1 - d^2*x^2]) + (b*(5*c^2 + b^
2*d^2 + 6*a*c*d^2)*Sqrt[1 - d^2*x^2])/d^6 + (c*(7*c^2 + 12*b^2*d^2 + 12*a*c
*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^6) + (b*c^2*x^2*Sqrt[1 - d^2*x^2])/d^4 + (c
^3*x^3*Sqrt[1 - d^2*x^2])/(4*d^4) - (3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2
 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcSin[d*x])/(8*d^7)
```

Rubi [A] time = 0.597837, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 1814, 1815, 641, 216}

$$\frac{x(ad^2+c)(a^2d^4+2acd^2+3b^2d^2+c^2)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}-\frac{3\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b\right.}{8d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b*(3*a^2 + (3*c^2)/d^4 + b^2/d^2 + (6*a*c)/d^2)*d^4 + (c + a*d^2)*(c^2 + 3
*b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^6*Sqrt[1 - d^2*x^2]) + (b*(5*c^2 + b^
2*d^2 + 6*a*c*d^2)*Sqrt[1 - d^2*x^2])/d^6 + (c*(7*c^2 + 12*b^2*d^2 + 12*a*c
*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^6) + (b*c^2*x^2*Sqrt[1 - d^2*x^2])/d^4 + (c
^3*x^3*Sqrt[1 - d^2*x^2])/(4*d^4) - (3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2
 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcSin[d*x])/(8*d^7)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
```

```
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx=\int \frac{\left(a+bx+cx^2\right)^3}{\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$=\frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}-\int \frac{\frac{c^3+3ac^2d^2+3ab^2d^4+}{d^6}}{}$$

$$=\frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}+$$

$$=\frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}+\frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4}+$$

$$=\frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}+\frac{c\left(7c^2+12b^2d^2+\right.}{8}$$

$$=\frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}+\frac{b\left(5c^2+b^2d^2+6a\right.}{d^6}$$

$$=\frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+\left(c+ad^2\right)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}+\frac{b\left(5c^2+b^2d^2+6a\right.}{d^6}$$

Mathematica [A] time = 0.24352, size = 239, normalized size = 0.87

$$\frac{-3\sqrt{1-d^2x^2}\sin^{-1}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3\right)-8b\left(-3a^2d^5+6acd^3\left(d^2x^2-2\right)+c^2d\left(d^4x^4+4d^2x\right.\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (-8*b^3*d^3*(-2 + d^2*x^2) - 12*b^2*d^3*x*(-2*a*d^2 + c*(-3 + d^2*x^2)) + d
*x*(24*a^2*c*d^4 + 8*a^3*d^6 - 12*a*c^2*d^2*(-3 + d^2*x^2) + c^3*(15 - 5*d^
2*x^2 - 2*d^4*x^4)) - 8*b*(-3*a^2*d^5 + 6*a*c*d^3*(-2 + d^2*x^2) + c^2*d*(-
8 + 4*d^2*x^2 + d^4*x^4)) - 3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2 + 8*a*b^
2*d^4 + 8*a^2*c*d^4)*Sqrt[1 - d^2*x^2]*ArcSin[d*x])/(8*d^7*Sqrt[1 - d^2*x^2
])
```

Maple [C] time = 0.225, size = 755, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] 1/8*(-d*x+1)^(1/2)*(15*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c^3-15*csgn(d
)*d*(-d^2*x^2+1)^(1/2)*x*c^3-64*csgn(d)*d*(-d^2*x^2+1)^(1/2)*b*c^2+2*csgn(d
)*x^5*c^3*d^5*(-d^2*x^2+1)^(1/2)+48*csgn(d)*x^2*a*b*c*d^5*(-d^2*x^2+1)^(1/2
)-8*csgn(d)*d^7*(-d^2*x^2+1)^(1/2)*x*a^3-24*arctan(csgn(d)*d*x/(-d^2*x^2+1)
^(1/2))*x^2*a^2*c*d^6-24*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*x^2*a*b^2*d
^6-36*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*x^2*a*c^2*d^4-36*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*x^2*b^2*c*d^4-15*arctan(csgn(d)*d*x/(-d^2*x^2+1)^
(1/2))*x^2*c^3*d^2+8*csgn(d)*x^2*b^3*d^5*(-d^2*x^2+1)^(1/2)+5*csgn(d)*d^3*(
-d^2*x^2+1)^(1/2)*x^3*c^3-24*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*a^2*b+36*arctan
(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*c^2*d^2+36*arctan(csgn(d)*d*x/(-d^2*x^2+
1)^(1/2))*b^2*c*d^2-16*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*b^3+24*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*a^2*c*d^4+24*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)
)*a*b^2*d^4+8*csgn(d)*x^4*b*c^2*d^5*(-d^2*x^2+1)^(1/2)+12*csgn(d)*x^3*a*c^2
*d^5*(-d^2*x^2+1)^(1/2)+12*csgn(d)*x^3*b^2*c*d^5*(-d^2*x^2+1)^(1/2)-24*csgn
(d)*d^5*(-d^2*x^2+1)^(1/2)*x*a^2*c-24*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*x*a*b^
2+32*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^2*b*c^2-36*csgn(d)*d^3*(-d^2*x^2+1)^(
1/2)*x*a*c^2-36*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*b^2*c-96*csgn(d)*d^3*(-d^2
*x^2+1)^(1/2)*a*b*c)*csgn(d)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^7/(d*x+1)^(1/2)
```

Maxima [A] time = 1.67893, size = 549, normalized size = 1.99

$$-\frac{c^3x^5}{4\sqrt{-d^2x^2+1}d^2}-\frac{bc^2x^4}{\sqrt{-d^2x^2+1}d^2}+\frac{a^3x}{\sqrt{-d^2x^2+1}}-\frac{5c^3x^3}{8\sqrt{-d^2x^2+1}d^4}-\frac{3\left(b^2c+ac^2\right)x^3}{2\sqrt{-d^2x^2+1}d^2}+\frac{3a^2b}{\sqrt{-d^2x^2+1}d^2}-\frac{4bc^2x}{\sqrt{-d^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/4*c^3*x^5/(sqrt(-d^2*x^2 + 1)*d^2) - b*c^2*x^4/(sqrt(-d^2*x^2 + 1)*d^2)
+ a^3*x/sqrt(-d^2*x^2 + 1) - 5/8*c^3*x^3/(sqrt(-d^2*x^2 + 1)*d^4) - 3/2*(b^
2*c + a*c^2)*x^3/(sqrt(-d^2*x^2 + 1)*d^2) + 3*a^2*b/(sqrt(-d^2*x^2 + 1)*d^2
) - 4*b*c^2*x^2/(sqrt(-d^2*x^2 + 1)*d^4) - (b^3 + 6*a*b*c)*x^2/(sqrt(-d^2*x
^2 + 1)*d^2) + 3*(a*b^2 + a^2*c)*x/(sqrt(-d^2*x^2 + 1)*d^2) - 3*(a*b^2 + a^
2*c)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) + 15/8*c^3*x/(sqrt(-d^2*x^2 +
1)*d^6) + 9/2*(b^2*c + a*c^2)*x/(sqrt(-d^2*x^2 + 1)*d^4) - 15/8*c^3*arcsin(
d^2*x/sqrt(d^2))/(sqrt(d^2)*d^6) - 9/2*(b^2*c + a*c^2)*arcsin(d^2*x/sqrt(d^
2))/(sqrt(d^2)*d^4) + 8*b*c^2/(sqrt(-d^2*x^2 + 1)*d^6) + 2*(b^3 + 6*a*b*c)/
(sqrt(-d^2*x^2 + 1)*d^4)
```

Fricas [A] time = 1.71622, size = 802, normalized size = 2.91

$$-\frac{24a^2bd^5+64bc^2d+16\left(b^3+6abc\right)d^3-8\left(3a^2bd^7+8bc^2d^3+2\left(b^3+6abc\right)d^5\right)x^2-\left(2c^3d^5x^5+8bc^2d^5x^4-24a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/8*(24*a^2*b*d^5 + 64*b*c^2*d + 16*(b^3 + 6*a*b*c)*d^3 - 8*(3*a^2*b*d^7 +
 8*b*c^2*d^3 + 2*(b^3 + 6*a*b*c)*d^5)*x^2 - (2*c^3*d^5*x^5 + 8*b*c^2*d^5*x^
4 - 24*a^2*b*d^5 - 64*b*c^2*d - 16*(b^3 + 6*a*b*c)*d^3 + (5*c^3*d^3 + 12*(b
^2*c + a*c^2)*d^5)*x^3 + 8*(4*b*c^2*d^3 + (b^3 + 6*a*b*c)*d^5)*x^2 - (8*a^3
*d^7 + 24*(a*b^2 + a^2*c)*d^5 + 15*c^3*d + 36*(b^2*c + a*c^2)*d^3)*x)*sqrt(
d*x + 1)*sqrt(-d*x + 1) + 6*(8*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 12*(b^2*c + a*
c^2)*d^2 - (8*(a*b^2 + a^2*c)*d^6 + 5*c^3*d^2 + 12*(b^2*c + a*c^2)*d^4)*x^2
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^9*x^2 - d^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57657, size = 961, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/86016*(8*a*b^2*d^39 + 8*a^2*c*d^39 + 12*b^2*c*d^37 + 12*a*c^2*d^37 + 5*c
^3*d^35)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - 1/516096*(4*a^3*d^41 + 12*a^2*
b*d^40 + 12*a*b^2*d^39 + 12*a^2*c*d^39 + 20*b^3*d^38 + 120*a*b*c*d^38 - 12*
b^2*c*d^37 - 12*a*c^2*d^37 + 108*b*c^2*d^36 - 14*c^3*d^35 - (8*b^3*d^38 + 4
8*a*b*c*d^38 - 36*b^2*c*d^37 - 36*a*c^2*d^37 + 80*b*c^2*d^36 - 35*c^3*d^35
+ (12*b^2*c*d^37 + 12*a*c^2*d^37 - 32*b*c^2*d^36 + 25*c^3*d^35 + 2*((d*x +
1)*c^3*d^35 + 4*b*c^2*d^36 - 5*c^3*d^35)*(d*x + 1))*(d*x + 1))*(d*x + 1))*s
qrt(d*x + 1)*sqrt(-d*x + 1)/(d*x - 1) + 1/4*(a^3*d^6*(sqrt(2) - sqrt(-d*x +
 1))/sqrt(d*x + 1) - 3*a^2*b*d^5*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) +
 3*a*b^2*d^4*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a^2*c*d^4*(sqrt(2
) - sqrt(-d*x + 1))/sqrt(d*x + 1) - b^3*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt
(d*x + 1) - 6*a*b*c*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*b^2*c*
d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a*c^2*d^2*(sqrt(2) - sqrt(
-d*x + 1))/sqrt(d*x + 1) - 3*b*c^2*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x +
1) + c^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^7 - 1/4*(a^3*d^6 - 3*a
^2*b*d^5 + 3*a*b^2*d^4 + 3*a^2*c*d^4 - b^3*d^3 - 6*a*b*c*d^3 + 3*b^2*c*d^2
+ 3*a*c^2*d^2 - 3*b*c^2*d + c^3)*sqrt(d*x + 1)/(d^7*(sqrt(2) - sqrt(-d*x +
1)))
```

3.798 $\int \frac{\left(a+bx+cx^2\right)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{x\left(a^2d^4+2acd^2+b^2d^2+c^2\right)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\frac{\sin^{-1}(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{2d^3}+\frac{2bc\sqrt{1-d^2x^2}}{d^4}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

```
[Out] (2*b*(a + c/d^2)*d^2 + (c^2 + b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^4*Sqrt[1
 - d^2*x^2]) + (2*b*c*Sqrt[1 - d^2*x^2])/d^4 + (c^2*x*Sqrt[1 - d^2*x^2])/(2
*d^4) - ((2*b^2 + c*(4*a + (3*c)/d^2))*ArcSin[d*x])/(2*d^3)
```

Rubi [A] time = 0.194578, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 1814, 1815, 641, 216}

$$\frac{x\left(a^2d^4+2acd^2+b^2d^2+c^2\right)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\frac{\sin^{-1}(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{2d^3}+\frac{2bc\sqrt{1-d^2x^2}}{d^4}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (2*b*(a + c/d^2)*d^2 + (c^2 + b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^4*Sqrt[1
 - d^2*x^2]) + (2*b*c*Sqrt[1 - d^2*x^2])/d^4 + (c^2*x*Sqrt[1 - d^2*x^2])/(2
*d^4) - ((2*b^2 + c*(4*a + (3*c)/d^2))*ArcSin[d*x])/(2*d^3)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{\left(a+bx+cx^2\right)^2}{\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} - \int \frac{\frac{c^2+b^2d^2+2acd^2}{d^4}+\frac{2bcx}{d^2}+\frac{c^2x^2}{d^2}}{\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} + \frac{\int \frac{-2b^2-c\left(4a+\frac{3c}{d^2}\right)-4bcx}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} - \frac{\left(2b^2+c\right.}{}$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} - \frac{\left(2b^2+c\right.}{}$$

Mathematica [A] time = 0.124909, size = 127, normalized size = 0.94

$$\frac{dx\left(2a^2d^4+4acd^2+c^2\left(3-d^2x^2\right)\right)-\sqrt{1-d^2x^2}\sin^{-1}(dx)\left(4acd^2+2b^2d^2+3c^2\right)+4bd\left(ad^2+c\left(2-d^2x^2\right)\right)+2b^2d^3x}{2d^5\sqrt{1-d^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (2*b^2*d^3*x + 4*b*d*(a*d^2 + c*(2 - d^2*x^2)) + d*x*(4*a*c*d^2 + 2*a^2*d^4
 + c^2*(3 - d^2*x^2)) - (3*c^2 + 2*b^2*d^2 + 4*a*c*d^2)*Sqrt[1 - d^2*x^2]*A
rcSin[d*x])/(2*d^5*Sqrt[1 - d^2*x^2])
```

Maple [C] time = 0.174, size = 380, normalized size = 2.8

$$\frac{\mathrm{csgn}(d)}{(2dx-2)d^5}\sqrt{-dx+1}\left(\mathrm{csgn}(d)x^3c^2d^3\sqrt{-d^2x^2+1}-2\,\mathrm{csgn}(d)d^5\sqrt{-d^2x^2+1}xa^2+4\,\mathrm{csgn}(d)x^2bcd^3\sqrt{-d^2x^2+1}-4\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] 1/2*(-d*x+1)^(1/2)*(csgn(d)*x^3*c^2*d^3*(-d^2*x^2+1)^(1/2)-2*csgn(d)*d^5*(-
d^2*x^2+1)^(1/2)*x*a^2+4*csgn(d)*x^2*b*c*d^3*(-d^2*x^2+1)^(1/2)-4*arctan(cs
gn(d)*d*x/(-d^2*x^2+1)^(1/2))*x^2*a*c*d^4-2*arctan(csgn(d)*d*x/(-d^2*x^2+1)
^(1/2))*x^2*b^2*d^4-4*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*a*c-2*csgn(d)*d^3*(-
d^2*x^2+1)^(1/2)*x*b^2-4*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*a*b-3*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*x^2*c^2*d^2-3*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c^2-
```

```
8*csgn(d)*d*(-d^2*x^2+1)^(1/2)*b*c+4*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))
*a*c*d^2+2*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b^2*d^2+3*arctan(csgn(d)*
d*x/(-d^2*x^2+1)^(1/2))*c^2)*csgn(d)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^5/(d*x+1)
^(1/2)
```

Maxima [A] time = 1.51753, size = 270, normalized size = 2.

$$\frac{a^2x}{\sqrt{-d^2x^2+1}}-\frac{c^2x^3}{2\sqrt{-d^2x^2+1}d^2}-\frac{2\,bcx^2}{\sqrt{-d^2x^2+1}d^2}+\frac{2\,ab}{\sqrt{-d^2x^2+1}d^2}+\frac{\left(b^2+2\,ac\right)x}{\sqrt{-d^2x^2+1}d^2}-\frac{\left(b^2+2\,ac\right)\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}d^2}+\frac{}{2\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] a^2*x/sqrt(-d^2*x^2 + 1) - 1/2*c^2*x^3/(sqrt(-d^2*x^2 + 1)*d^2) - 2*b*c*x^2
/(sqrt(-d^2*x^2 + 1)*d^2) + 2*a*b/(sqrt(-d^2*x^2 + 1)*d^2) + (b^2 + 2*a*c)*
x/(sqrt(-d^2*x^2 + 1)*d^2) - (b^2 + 2*a*c)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^
2)*d^2) + 3/2*c^2*x/(sqrt(-d^2*x^2 + 1)*d^4) - 3/2*c^2*arcsin(d^2*x/sqrt(d^
2))/(sqrt(d^2)*d^4) + 4*b*c/(sqrt(-d^2*x^2 + 1)*d^4)
```

Fricas [A] time = 1.67216, size = 443, normalized size = 3.28

$$-\frac{4\,abd^3+8\,bcd-4\left(abd^5+2\,bcd^3\right)x^2-\left(c^2d^3x^3+4\,bcd^3x^2-4\,abd^3-8\,bcd-\left(2\,a^2d^5+2\left(b^2+2\,ac\right)d^3+3\,c^2d\right)x\right)\sqrt{dx}}{2\left(d^7x^2-d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*(4*a*b*d^3 + 8*b*c*d - 4*(a*b*d^5 + 2*b*c*d^3)*x^2 - (c^2*d^3*x^3 + 4*
b*c*d^3*x^2 - 4*a*b*d^3 - 8*b*c*d - (2*a^2*d^5 + 2*(b^2 + 2*a*c)*d^3 + 3*c^
2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*(b^2 + 2*a*c)*d^2 - (2*(b^2 + 2
*a*c)*d^4 + 3*c^2*d^2)*x^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) -
1)/(d*x)))/(d^7*x^2 - d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35141, size = 509, normalized size = 3.77

$$-\frac{1}{384}\left(2\,b^2d^{17}+4\,acd^{17}+3\,c^2d^{15}\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)-\frac{\left(a^2d^{19}+2\,abd^{18}+b^2d^{17}+2\,acd^{17}+10\,bcd^{16}-c^2d^{15}-\left((dx\right.\right.}{768\,(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/384*(2*b^2*d^17 + 4*a*c*d^17 + 3*c^2*d^15)*arcsin(1/2*sqrt(2)*sqrt(d*x +
 1)) - 1/768*(a^2*d^19 + 2*a*b*d^18 + b^2*d^17 + 2*a*c*d^17 + 10*b*c*d^16 -
 c^2*d^15 - ((d*x + 1)*c^2*d^15 + 4*b*c*d^16 - 3*c^2*d^15)*(d*x + 1))*sqrt(
d*x + 1)*sqrt(-d*x + 1)/(d*x - 1) + 1/4*(a^2*d^4*(sqrt(2) - sqrt(-d*x + 1))
/sqrt(d*x + 1) - 2*a*b*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + b^2*d
^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 2*a*c*d^2*(sqrt(2) - sqrt(-d*
x + 1))/sqrt(d*x + 1) - 2*b*c*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) +
c^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^5 - 1/4*(a^2*d^4 - 2*a*b*d^
3 + b^2*d^2 + 2*a*c*d^2 - 2*b*c*d + c^2)*sqrt(d*x + 1)/(d^5*(sqrt(2) - sqrt
(-d*x + 1)))
```

3.799 $\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}} dx$

Optimal. Leaf size=40

$$\frac{x\left(ad^2+c\right)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\sin^{-1}(dx)}{d^3}$$

```
[Out] (b + (c + a*d^2)*x)/(d^2*Sqrt[1 - d^2*x^2]) - (c*ArcSin[d*x])/d^3
```

Rubi [A] time = 0.0511869, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1814, 12, 216}

$$\frac{x\left(ad^2+c\right)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\sin^{-1}(dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b + (c + a*d^2)*x)/(d^2*Sqrt[1 - d^2*x^2]) - (c*ArcSin[d*x])/d^3
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{a+bx+cx^2}{\left(1-d^2x^2\right)^{3/2}}\,dx$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \int \frac{c}{d^2\sqrt{1-d^2x^2}}\,dx$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \frac{c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{d^2}$$
$$= \frac{b+\left(c+ad^2\right)x}{d^2\sqrt{1-d^2x^2}} - \frac{c\sin^{-1}(dx)}{d^3}$$

Mathematica [A] time = 0.0494655, size = 39, normalized size = 0.98

$$\frac{\frac{d\left(x\left(ad^2+c\right)+b\right)}{\sqrt{1-d^2x^2}} - c\sin^{-1}(dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] ((d*(b + (c + a*d^2)*x))/Sqrt[1 - d^2*x^2] - c*ArcSin[d*x])/d^3
```

Maple [C] time = 0.23, size = 151, normalized size = 3.8

$$\frac{\mathrm{csgn}(d)}{(dx-1)\,d^3}\left(-\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)\,d^3xa - \arctan\left(\mathrm{csgn}(d)\,dx\frac{1}{\sqrt{-(dx+1)(dx-1)}}\right)x^2cd^2 - \mathrm{csgn}(d)\,d\sqrt{-d^2x^2+1}xc - \mathrm{cs}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)
```

```
[Out] (-(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*x*a-arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^
(1/2))*x^2*c*d^2-csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c-csgn(d)*d*(-d^2*x^2+1)^(1
/2)*b+arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2))*c)*(-d*x+1)^(1/2)*csgn(d
)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^3/(d*x+1)^(1/2)
```

Maxima [A] time = 1.50029, size = 99, normalized size = 2.48

$$\frac{ax}{\sqrt{-d^2x^2+1}} + \frac{cx}{\sqrt{-d^2x^2+1}d^2} - \frac{c\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}d^2} + \frac{b}{\sqrt{-d^2x^2+1}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] a*x/sqrt(-d^2*x^2 + 1) + c*x/(sqrt(-d^2*x^2 + 1)*d^2) - c*arcsin(d^2*x/sqrt
(d^2))/(sqrt(d^2)*d^2) + b/(sqrt(-d^2*x^2 + 1)*d^2)
```

Fricas [B] time = 1.60634, size = 215, normalized size = 5.38

$$\frac{bd^3x^2 - \left(bd + \left(ad^3 + cd\right)x\right)\sqrt{dx+1}\sqrt{-dx+1} - bd + 2\left(cd^2x^2 - c\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{d^5x^2 - d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d^3*x^2 - (b*d + (a*d^3 + c*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - b*d + 2
*(c*d^2*x^2 - c)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*x^2
 - d^3)
```

Sympy [C] time = 159.678, size = 255, normalized size = 6.38

$$a\left(-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d}\right) + b\left(-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{4},\frac{3}{4},1 & 0,1,\frac{3}{2} \\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] a*(-I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,))
, 1/(d**2*x**2))/(2*pi**(3/2)*d) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()
), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(2*pi**(3
/2)*d)) + b*(-I*meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3
/2), (0,)), 1/(d**2*x**2))/(2*pi**(3/2)*d**2) - meijerg(((-1, -1/2, -1/4, 0
, 1/4, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2
*x**2))/(2*pi**(3/2)*d**2)) + c*(I*meijerg(((-1/4, 1/4), (-1/2, 1/2, 1, 1))
, ((-1/4, 0, 1/4, 1/2, 1, 0), ()), 1/(d**2*x**2))/(2*pi**(3/2)*d**3) + meij
erg(((-3/2, -1, -3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0))
, exp_polar(-2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**3))
```

Giac [B] time = 1.18268, size = 246, normalized size = 6.15

$$-\frac{2\,c\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^3} + \frac{\frac{ad^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}} - \frac{bd\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}} + \frac{c\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}}{4\,d^3} - \frac{\left(ad^2 - bd + c\right)\sqrt{dx+1}}{4\,d^3\left(\sqrt{2} - \sqrt{-dx+1}\right)} - \frac{\left(ad^5 + bd^4 +\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2*c*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3 + 1/4*(a*d^2*(sqrt(2) - sqrt(-d*
x + 1))/sqrt(d*x + 1) - b*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + c*(s
qrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^3 - 1/4*(a*d^2 - b*d + c)*sqrt(d*
x + 1)/(d^3*(sqrt(2) - sqrt(-d*x + 1))) - 1/2*(a*d^5 + b*d^4 + c*d^3)*sqrt(
d*x + 1)*sqrt(-d*x + 1)/((d*x - 1)*d^6)
```

3.800 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=443

$$\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2\right.}}$$

```
[Out] (d^2*(b - (c + a*d^2)*x))/((b^2*d^2 - (c + a*d^2)^2)*Sqrt[1 - d^2*x^2]) + (
c*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b - S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2
- 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) - (c
*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b + Sq
rt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rubi [A] time = 1.44267, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {899, 976, 1034, 725, 206}

$$\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```


```
[Out] (d^2*(b - (c + a*d^2)*x))/((b^2*d^2 - (c + a*d^2)^2)*Sqrt[1 - d^2*x^2]) + (
c*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b - S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2
- 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) - (c
*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b + Sq
rt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```


Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
```

```
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} - \frac{\int \frac{2d^2\left(c^2-b^2d^2+acd^2\right)-2bcd^4x}{\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}}\,dx}{2d^2\left(b^2d^2-\left(c+ad^2\right)^2\right)}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{\left(c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2\right) \cdots\right.}{\sqrt{b^2-4ac}\left(b^2d^2-\left(c \cdots\right.\right.}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} - \frac{\left(c\left(2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2\right) \cdots\right.}{\sqrt{b^2-\cdots}}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2\right) \cdots}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(\cdots\right.}}$$

Mathematica [A] time = 2.50816, size = 335, normalized size = 0.76

$$\frac{d^2\left(x\left(ad^2+c\right)-b\right)}{\sqrt{1-d^2x^2}\left(a^2d^4+2acd^2-b^2d^2+c^2\right)}-\frac{2\sqrt{2}c^3\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{1-d^2x^2}\sqrt{2bd^2\left(\sqrt{b^2-4ac}-b\right)+4acd^2+4c^2}}\right)}{\sqrt{b^2-4ac}\left(bd^2\left(\sqrt{b^2-4ac}-b\right)+2acd^2+2c^2\right)^{3/2}}+\frac{2\sqrt{2}c^3\tanh^{-1}\left(\frac{d^2x}{\sqrt{1-d^2x^2}\sqrt{-2}}\right)}{\sqrt{b^2-4ac}\left(-bd^2\left(\sqrt{b^2-4a}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (d^2*(-b + (c + a*d^2)*x))/((c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*Sqrt[1 -
d^2*x^2]) - (2*Sqrt[2]*c^3*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(S
qrt[4*c^2 + 4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2]
)])/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2)
^(3/2)) + (2*Sqrt[2]*c^3*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqr
t[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])
/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)^(3/
2))
```

Maple [C] time = 0.477, size = 11142, normalized size = 25.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)(dx+1)^{\frac{3}{2}}(-dx+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.801 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=939

$$-\frac{\left(b\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)-\left(2c^4-d^2\left(b^2+6a^2d^2\right)c^2-\left(4a^3d^6+6ab^2d^4\right)c+b^2d^4\left(2b^2+a^2d^2\right)\right)x\right)}{\left(b^2-4ac\right)\left(ad^2-bd+c\right)^2\left(ad^2+bd+c\right)^2\sqrt{1-d^2x^2}}$$

```
[Out] -((d^2*(b*(c^3 + 2*b^2*c*d^2 - 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) -
 (2*c^4 + b^2*d^4*(2*b^2 + a^2*d^2) - c^2*d^2*(b^2 + 6*a^2*d^2) - c*(6*a*b^
2*d^4 + 4*a^3*d^6))*x))/((b^2 - 4*a*c)*(c - b*d + a*d^2)^2*(c + b*d + a*d^2
)^2*Sqrt[1 - d^2*x^2])) - (b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d
^2 + 2*a*c*d^2)*x)/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^
2)*Sqrt[1 - d^2*x^2]) + (c*(4*c^5 + 24*a*c^4*d^2 + 3*a*b^3*(b + Sqrt[b^2 -
4*a*c])*d^6 - c^3*d^2*(9*b^2 - b*Sqrt[b^2 - 4*a*c] - 36*a^2*d^2) - 2*a*c^2*
d^4*(7*b^2 + 5*b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2) + b*c*d^4*(2*b^3 + 2*b^2*Sq
rt[b^2 - 4*a*c] - 17*a^2*b*d^2 - 11*a^2*Sqrt[b^2 - 4*a*c]*d^2))*ArcTanh[(2*
c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b -
 Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*
Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*
a*c*d^2 + a^2*d^4)^2) + (c*(b*(b + Sqrt[b^2 - 4*a*c])*d^4*(c^3 + 2*b^2*c*d^
2 - 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) - 2*(2*c^5*d^2 + 12*a*c^4*d^
4 + 3*a*b^4*d^8 + 2*b^2*c*d^6*(b^2 - 7*a^2*d^2) - c^3*(4*b^2*d^4 - 18*a^2*d
^6) - 4*c^2*(3*a*b^2*d^6 - 2*a^3*d^8)))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*
c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]
*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*d^2*Sqrt[2*c^2 + 2*a*c*d
^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)^2
)
```

Rubi [A] time = 11.8441, antiderivative size = 938, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {899, 975, 1062, 1034, 725, 206}

$$-\frac{\left(b\left(3ab^2d^4-11a^2cd^4-10ac^2d^2+2b^2cd^2+c^3\right)-\left(2c^4-d^2\left(b^2+6a^2d^2\right)c^2-\left(4a^3d^6+6ab^2d^4\right)c+b^2d^4\left(2b^2+a^2d^2\right)\right)x\right)}{\left(b^2-4ac\right)\left(ad^2-bd+c\right)^2\left(ad^2+bd+c\right)^2\sqrt{1-d^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((d^2*(b*(c^3 + 2*b^2*c*d^2 - 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) -
 (2*c^4 + b^2*d^4*(2*b^2 + a^2*d^2) - c^2*d^2*(b^2 + 6*a^2*d^2) - c*(6*a*b^
2*d^4 + 4*a^3*d^6))*x))/((b^2 - 4*a*c)*(c - b*d + a*d^2)^2*(c + b*d + a*d^2
)^2*Sqrt[1 - d^2*x^2])) - (b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d
^2 + 2*a*c*d^2)*x)/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^
2)*Sqrt[1 - d^2*x^2]) + (c*(4*c^5 + 24*a*c^4*d^2 + 3*a*b^3*(b + Sqrt[b^2 -
4*a*c])*d^6 - c^3*d^2*(9*b^2 - b*Sqrt[b^2 - 4*a*c] - 36*a^2*d^2) - 2*a*c^2*
d^4*(7*b^2 + 5*b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2) + b*c*d^4*(2*b^3 + 2*b^2*Sq
rt[b^2 - 4*a*c] - 17*a^2*b*d^2 - 11*a^2*Sqrt[b^2 - 4*a*c]*d^2))*ArcTanh[(2*
c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b -
```

```
 Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*
Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*
a*c*d^2 + a^2*d^4)^2) - (c*(4*c^5*d^2 + 24*a*c^4*d^4 + 6*a*b^4*d^8 + 4*b^2*
c*d^6*(b^2 - 7*a^2*d^2) - b*(b + Sqrt[b^2 - 4*a*c])*d^4*(c^3 + 2*b^2*c*d^2
- 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) - 4*c^3*(2*b^2*d^4 - 9*a^2*d^6
) - 8*c^2*(3*a*b^2*d^6 - 2*a^3*d^8))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])
*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sq
rt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*d^2*Sqrt[2*c^2 + 2*a*c*d^2
- b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)^2)
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1062

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d
+ e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*(2*a*
f)) + c*(A*(2*c^2*d - c*(2*a*f)) - B*(-2*a*c*e) + C*(-2*a*(c*d - a*f)))*x))
/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((-4*a*c)*(a*c*e
^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*S
imp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1) + (2*(A*c*(c*
d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*
c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2) -
(2*f*((A*c - a*C)*(2*a*c*e) + (-(a*B))*(2*c^2*d - c*((Plus[2])*a*f)))*(p +
q + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(-(c*e*(2*p + q
+ 4))))*x - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*
q + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2
- 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !Intege
rQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}} - \int \frac{-2c^3-6ac^2d^2+a}{}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d^4\left(2b^2+a^2\right.\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)^2\sqrt{}}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d^4\left(2b^2+a^2\right.\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)^2\sqrt{}}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d^4\left(2b^2+a^2\right.\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)^2\sqrt{}}$$

$$= -\frac{d^2\left(b\left(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4\right)-\left(2c^4+b^2d^4\left(2b^2+a^2\right.\right.\right.}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)^2\sqrt{}}$$

Mathematica [A] time = 9.33107, size = 890, normalized size = 0.95

$$\frac{\frac{\sqrt{2}\left(-3ab\left(b+\sqrt{b^2-4ac}\right)d^4+20ac^2d^2-c\left(7b^2-3\sqrt{b^2-4ac}b-16a^2d^2\right)d^2+4c^3\right)\tanh^{-1}\left(\frac{\left(b-\sqrt{b^2-4ac}\right)xd^2+2c}{\sqrt{2}\sqrt{2c^2+2ad^2c-b\left(b-\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}}\right)c^3}{\sqrt{b^2-4ac}\left(2c^2+2ad^2c-b\left(b-\sqrt{b^2-4ac}\right)d^2\right)^{3/2}} - \sqrt{2}\left(-3ab\left(b-\sqrt{b^2-4ac}\right)d^4+20ac^2d^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((-(b^3*d^2) + b*c*(c + 3*a*d^2) + c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)/((b^
2 - 4*a*c)*(-(b^2*d^2) + (c + a*d^2)^2)*(a + b*x + c*x^2)*Sqrt[1 - d^2*x^2]
)) + ((2*d^2*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)/Sqrt[1 - d^2*x^2] - (c*(3*b*d
^2*(c - a*d^2) + (4*c^3 - 7*b^2*c*d^2 + 20*a*c^2*d^2 - 3*a*b^2*d^4 + 16*a^2
```

```
*c*d^4)/Sqrt[b^2 - 4*a*c])*(2*c - (b - Sqrt[b^2 - 4*a*c])*d^2*x))/((4*c^2 -
 (b - Sqrt[b^2 - 4*a*c])^2*d^2)*Sqrt[1 - d^2*x^2]) - (c*(3*b*d^2*(c - a*d^2
) - (4*c^3 - 7*b^2*c*d^2 + 20*a*c^2*d^2 - 3*a*b^2*d^4 + 16*a^2*c*d^4)/Sqrt[
b^2 - 4*a*c])*(2*c - (b + Sqrt[b^2 - 4*a*c])*d^2*x))/((4*c^2 - (b + Sqrt[b^
2 - 4*a*c])^2*d^2)*Sqrt[1 - d^2*x^2]) + (Sqrt[2]*c^3*(4*c^3 + 20*a*c^2*d^2
- 3*a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(7*b^2 - 3*b*Sqrt[b^2 - 4*a*c]
- 16*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[
2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sq
rt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)^(3/2))
- (Sqrt[2]*c^3*(4*c^3 + 20*a*c^2*d^2 - 3*a*b*(b - Sqrt[b^2 - 4*a*c])*d^4 -
c*d^2*(7*b^2 + 3*b*Sqrt[b^2 - 4*a*c] - 16*a^2*d^2))*ArcTanh[(2*c + (b + Sqr
t[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^2 - 4*a*c]*(2*c^2 + 2*a*c*d^2 - b
*(b + Sqrt[b^2 - 4*a*c])*d^2)^(3/2)))/((b^2 - 4*a*c)*(-(b^2*d^2) + (c + a*d
^2)^2))
```

Maple [C] time = 4.968, size = 108969, normalized size = 116.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^2 (dx+1)^{\frac{3}{2}} (-dx+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

3.802 $\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=54

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)$$

```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (c
*x^2)/a)^p
```

Rubi [A] time = 0.0448021, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {517, 430, 429}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (c
*x^2)/a)^p
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx &= \int \left(a+cx^2\right)^p\left(1-e^2x^2\right)^m\,dx\\ &= \left(\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{cx^2}{a}\right)^p\left(1-e^2x^2\right)^m\,dx\\ &= x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)\end{aligned}$$

Mathematica [B] time = 0.20552, size = 167, normalized size = 3.09

$$\frac{3ax\left(1-e^2x^2\right)^m\left(a+cx^2\right)^pF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)}{2x^2\left(cpF_1\left(\frac{3}{2};1-p,-m;\frac{5}{2};-\frac{cx^2}{a},e^2x^2\right)-ae^2mF_1\left(\frac{3}{2};-p,1-m;\frac{5}{2};-\frac{cx^2}{a},e^2x^2\right)\right)+3aF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (3*a*x*(a + c*x^2)^p*(1 - e^2*x^2)^m*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a
), e^2*x^2])/(3*a*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2] + 2*x^2
*(c*p*AppellF1[3/2, 1 - p, -m, 5/2, -((c*x^2)/a), e^2*x^2] - a*e^2*m*Appell
F1[3/2, -p, 1 - m, 5/2, -((c*x^2)/a), e^2*x^2]))
```

Maple [F] time = 0.937, size = 0, normalized size = 0.

$$\int (-ex+1)^m (ex+1)^m \left(cx^2+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

```
[Out] int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p (ex+1)^m (-ex+1)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+a\right)^p (ex+1)^m (-ex+1)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)**m*(e*x+1)**m*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p (ex+1)^m (-ex+1)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

3.803 $\int (d-ex)^m (d+ex)^m \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=89

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d-ex)^m(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[Out] (x*(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^
2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rubi [A] time = 0.077514, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {519, 430, 429}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d-ex)^m(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^
2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx = \left((d-ex)^m(d+ex)^m\left(d^2-e^2x^2\right)^{-m}\right)\int \left(a+cx^2\right)^p\left(d^2-e^2x^2\right)^m\,dx$$
$$= \left((d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d^2-e^2x^2\right)^{-m}\right)\int \left(1+\frac{cx^2}{a}\right)^p\left(d^2-e\right.$$
$$= \left((d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\right)\int \left(1+\frac{cx^2}{a}\right)^p\left(1-\frac{e^2}{d}\right.$$
$$= x(d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a}\right.$$

Mathematica [F] time = 0.100643, size = 0, normalized size = 0.

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p, x]
```

Maple [F] time = 0.987, size = 0, normalized size = 0.

$$\int (-ex+d)^m(ex+d)^m\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

```
[Out] int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)**m*(e*x+d)**m*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^p (ex + d)^m (-ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

3.804 $\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=92

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}(df-efx)^mF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[Out] (x*(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((
c*x^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rubi [A] time = 0.0841984, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {519, 430, 429}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}(df-efx)^mF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((
c*x^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx = \left((d+ex)^m(df-efx)^m\left(d^2f-e^2fx^2\right)^{-m}\right)\int \left(a+cx^2\right)^p\left(d^2f-e^2fx^2\right)^m\,dx$$
$$= \left((d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d^2f-e^2fx^2\right)^{-m}\right)\int \left(1+\frac{cx^2}{a}\right)^p\left(\right.$$
$$= \left((d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\right)\int \left(1+\frac{cx^2}{a}\right)^p\left(1-\right.$$
$$= x(d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\right.$$

Mathematica [F] time = 0.0800779, size = 0, normalized size = 0.

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

```
[Out] Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p, x]
```

Maple [F] time = 1.041, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(-efx+df\right)^m\left(cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-efx+df\right)^m\left(cx^2+a\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-efx+df\right)^m\left(cx^2+a\right)^p(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(-e*f*x+d*f)**m*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-efx+df\right)^{m}\left(cx^{2}+a\right)^{p}(ex+d)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

3.805 $\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=275

$$\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{g^6(n+2)}-\frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c\left(7d^2g^2-20defg+10e^2\right.\right.}{g^6(n+3)}$$

```
[Out] -(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^6*(1 + n
))) + ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(
f + g*x)^(2 + n))/(g^6*(2 + n)) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2
 - 20*d*e*f*g + 7*d^2*g^2))*(f + g*x)^(3 + n))/(g^6*(3 + n)) + (e^2*(a*e*g^
2 + c*(10*e^2*f^2 - 20*d*e*f*g + 9*d^2*g^2))*(f + g*x)^(4 + n))/(g^6*(4 + n
)) - (5*c*e^3*(e*f - d*g)*(f + g*x)^(5 + n))/(g^6*(5 + n)) + (c*e^4*(f + g*
x)^(6 + n))/(g^6*(6 + n))
```

Rubi [A] time = 0.263294, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {947}

$$\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c\left(2d^2g^2-10defg+5e^2f^2\right)\right)}{g^6(n+2)}-\frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c\left(7d^2g^2-20defg+10e^2\right.\right.}{g^6(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^6*(1 + n
))) + ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(
f + g*x)^(2 + n))/(g^6*(2 + n)) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2
 - 20*d*e*f*g + 7*d^2*g^2))*(f + g*x)^(3 + n))/(g^6*(3 + n)) + (e^2*(a*e*g^
2 + c*(10*e^2*f^2 - 20*d*e*f*g + 9*d^2*g^2))*(f + g*x)^(4 + n))/(g^6*(4 + n
)) - (5*c*e^3*(e*f - d*g)*(f + g*x)^(5 + n))/(g^6*(5 + n)) + (c*e^4*(f + g*
x)^(6 + n))/(g^6*(6 + n))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^3(f+gx)^n\left(a+2cdx+cex^2\right)dx=\int\left(\frac{(ef-dg)^3\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^5}+\frac{(ef-dg)^2\left(3aeg^2+c\left(5e^2\right.\right.}{}\right.$$

$$=-\frac{(ef-dg)^3\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^6(1+n)}+\frac{(ef-dg)^2\left(3aeg^2+c\left(5e^2f\right.\right.}{g}$$

Mathematica [A] time = 0.330989, size = 249, normalized size = 0.91

$$\frac{(f+gx)^{n+1}\left(\frac{e^2(f+gx)^3\left(aeg^2+c\left(9d^2g^2-20defg+10e^2f^2\right)\right)}{n+4}-\frac{e(f+gx)^2(ef-dg)\left(3aeg^2+c\left(7d^2g^2-20defg+10e^2f^2\right)\right)}{n+3}+\frac{(f+gx)(ef-dg)^2\left(3aeg^2+c\left(2d^2g^2-10de\right.\right.}{n+2}\right.}{g^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(-(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n))
+ ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(f +
g*x))/(2 + n) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g + 7*
d^2*g^2))*(f + g*x)^2)/(3 + n) + (e^2*(a*e*g^2 + c*(10*e^2*f^2 - 20*d*e*f*g
 + 9*d^2*g^2))*(f + g*x)^3)/(4 + n) - (5*c*e^3*(e*f - d*g)*(f + g*x)^4)/(5
+ n) + (c*e^4*(f + g*x)^5)/(6 + n)))/g^6
```

Maple [B] time = 0.06, size = 2017, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(1+n)*(c*e^4*g^5*n^5*x^5+5*c*d*e^3*g^5*n^5*x^4+15*c*e^4*g^5*n^4*x^5
+9*c*d^2*e^2*g^5*n^5*x^3+80*c*d*e^3*g^5*n^4*x^4-5*c*e^4*f*g^4*n^4*x^4+85*c*
e^4*g^5*n^3*x^5+a*e^3*g^5*n^5*x^3+7*c*d^3*e*g^5*n^5*x^2+153*c*d^2*e^2*g^5*n
^4*x^3-20*c*d*e^3*f*g^4*n^4*x^3+475*c*d*e^3*g^5*n^3*x^4-50*c*e^4*f*g^4*n^3*
x^4+225*c*e^4*g^5*n^2*x^5+3*a*d*e^2*g^5*n^5*x^2+17*a*e^3*g^5*n^4*x^3+2*c*d^
4*g^5*n^5*x+126*c*d^3*e*g^5*n^4*x^2-27*c*d^2*e^2*f*g^4*n^4*x^2+963*c*d^2*e^
2*g^5*n^3*x^3-240*c*d*e^3*f*g^4*n^3*x^3+1300*c*d*e^3*g^5*n^2*x^4+20*c*e^4*f
^2*g^3*n^3*x^3-175*c*e^4*f*g^4*n^2*x^4+274*c*e^4*g^5*n*x^5+3*a*d^2*e*g^5*n^
5*x+54*a*d*e^2*g^5*n^4*x^2-3*a*e^3*f*g^4*n^4*x^2+107*a*e^3*g^5*n^3*x^3+38*c
*d^4*g^5*n^4*x-14*c*d^3*e*f*g^4*n^4*x+847*c*d^3*e*g^5*n^3*x^2-378*c*d^2*e^2
*f*g^4*n^3*x^2+2763*c*d^2*e^2*g^5*n^2*x^3+60*c*d*e^3*f^2*g^3*n^3*x^2-940*c*
d*e^3*f*g^4*n^2*x^3+1620*c*d*e^3*g^5*n*x^4+120*c*e^4*f^2*g^3*n^2*x^3-250*c*
e^4*f*g^4*n*x^4+120*c*e^4*g^5*x^5+a*d^3*g^5*n^5+57*a*d^2*e*g^5*n^4*x-6*a*d*
e^2*f*g^4*n^4*x+363*a*d*e^2*g^5*n^3*x^2-42*a*e^3*f*g^4*n^3*x^2+307*a*e^3*g^
5*n^2*x^3-2*c*d^4*f*g^4*n^4+274*c*d^4*g^5*n^3*x-224*c*d^3*e*f*g^4*n^3*x+260
4*c*d^3*e*g^5*n^2*x^2+54*c*d^2*e^2*f^2*g^3*n^3*x-1755*c*d^2*e^2*f*g^4*n^2*x
^2+3564*c*d^2*e^2*g^5*n*x^3+540*c*d*e^3*f^2*g^3*n^2*x^2-1440*c*d*e^3*f*g^4*
n*x^3+720*c*d*e^3*g^5*x^4-60*c*e^4*f^3*g^2*n^2*x^2+220*c*e^4*f^2*g^3*n*x^3-
120*c*e^4*f*g^4*x^4+20*a*d^3*g^5*n^4-3*a*d^2*e*f*g^4*n^4+411*a*d^2*e*g^5*n^
3*x-96*a*d*e^2*f*g^4*n^3*x+1116*a*d*e^2*g^5*n^2*x^2+6*a*e^3*f^2*g^3*n^3*x-1
95*a*e^3*f*g^4*n^2*x^2+396*a*e^3*g^5*n*x^3-36*c*d^4*f*g^4*n^3+922*c*d^4*g^5
*n^2*x+14*c*d^3*e*f^2*g^3*n^3-1246*c*d^3*e*f*g^4*n^2*x+3556*c*d^3*e*g^5*n*x
^2+648*c*d^2*e^2*f^2*g^3*n^2*x-3024*c*d^2*e^2*f*g^4*n*x^2+1620*c*d^2*e^2*g^
5*x^3-120*c*d*e^3*f^3*g^2*n^2*x+1200*c*d*e^3*f^2*g^3*n*x^2-720*c*d*e^3*f*g^
4*x^3-180*c*e^4*f^3*g^2*n*x^2+120*c*e^4*f^2*g^3*x^3+155*a*d^3*g^5*n^3-54*a*
d^2*e*f*g^4*n^3+1383*a*d^2*e*g^5*n^2*x+6*a*d*e^2*f^2*g^3*n^3-534*a*d*e^2*f*
g^4*n^2*x+1524*a*d*e^2*g^5*n*x^2+72*a*e^3*f^2*g^3*n^2*x-336*a*e^3*f*g^4*n*x
^2+180*a*e^3*g^5*x^3-238*c*d^4*f*g^4*n^2+1404*c*d^4*g^5*n*x+210*c*d^3*e*f^2
*g^3*n^2-2716*c*d^3*e*f*g^4*n*x+1680*c*d^3*e*g^5*x^2-54*c*d^2*e^2*f^3*g^2*n
^2+2214*c*d^2*e^2*f^2*g^3*n*x-1620*c*d^2*e^2*f*g^4*x^2-840*c*d*e^3*f^3*g^2*
n*x+720*c*d*e^3*f^2*g^3*x^2+120*c*e^4*f^4*g*n*x-120*c*e^4*f^3*g^2*x^2+580*a
*d^3*g^5*n^2-357*a*d^2*e*f*g^4*n^2+2106*a*d^2*e*g^5*n*x+90*a*d*e^2*f^2*g^3*
n^2-1164*a*d*e^2*f*g^4*n*x+720*a*d*e^2*g^5*x^2-6*a*e^3*f^3*g^2*n^2+246*a*e^
3*f^2*g^3*n*x-180*a*e^3*f*g^4*x^2-684*c*d^4*f*g^4*n+720*c*d^4*g^5*x+1036*c*
d^3*e*f^2*g^3*n-1680*c*d^3*e*f*g^4*x-594*c*d^2*e^2*f^3*g^2*n+1620*c*d^2*e^2
*f^2*g^3*x+120*c*d*e^3*f^4*g*n-720*c*d*e^3*f^3*g^2*x+120*c*e^4*f^4*g*x+1044
*a*d^3*g^5*n-1026*a*d^2*e*f*g^4*n+1080*a*d^2*e*g^5*x+444*a*d*e^2*f^2*g^3*n-
720*a*d*e^2*f*g^4*x-66*a*e^3*f^3*g^2*n+180*a*e^3*f^2*g^3*x-720*c*d^4*f*g^4+
```

```
1680*c*d^3*e*f^2*g^3-1620*c*d^2*e^2*f^3*g^2+720*c*d*e^3*f^4*g-120*c*e^4*f^5
+720*a*d^3*g^5-1080*a*d^2*e*f*g^4+720*a*d*e^2*f^2*g^3-180*a*e^3*f^3*g^2)/g^
6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10233, size = 4350, normalized size = 15.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")

[Out] (a*d^3*f*g^5*n^5 - 120*c*e^4*f^6 + 720*c*d*e^3*f^5*g + 720*a*d^3*f*g^5 - 18
0*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 240*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 360
*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4 + (c*e^4*g^6*n^5 + 15*c*e^4*g^6*n^4 + 85*c*e
^4*g^6*n^3 + 225*c*e^4*g^6*n^2 + 274*c*e^4*g^6*n + 120*c*e^4*g^6)*x^6 + (72
0*c*d*e^3*g^6 + (c*e^4*f*g^5 + 5*c*d*e^3*g^6)*n^5 + 10*(c*e^4*f*g^5 + 8*c*d
*e^3*g^6)*n^4 + 5*(7*c*e^4*f*g^5 + 95*c*d*e^3*g^6)*n^3 + 50*(c*e^4*f*g^5 +
26*c*d*e^3*g^6)*n^2 + 12*(2*c*e^4*f*g^5 + 135*c*d*e^3*g^6)*n)*x^5 + (20*a*d
^3*f*g^5 - (2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n^4 + (180*(9*c*d^2*e^2 + a*e^3)*
g^6 + (5*c*d*e^3*f*g^5 + (9*c*d^2*e^2 + a*e^3)*g^6)*n^5 - (5*c*e^4*f^2*g^4
- 60*c*d*e^3*f*g^5 - 17*(9*c*d^2*e^2 + a*e^3)*g^6)*n^4 - (30*c*e^4*f^2*g^4
- 235*c*d*e^3*f*g^5 - 107*(9*c*d^2*e^2 + a*e^3)*g^6)*n^3 - (55*c*e^4*f^2*g^
4 - 360*c*d*e^3*f*g^5 - 307*(9*c*d^2*e^2 + a*e^3)*g^6)*n^2 - 6*(5*c*e^4*f^2
*g^4 - 30*c*d*e^3*f*g^5 - 66*(9*c*d^2*e^2 + a*e^3)*g^6)*n)*x^4 + (155*a*d^3
*f*g^5 + 2*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 18*(2*c*d^4 + 3*a*d^2*e)*f^2*g
^4)*n^3 + (240*(7*c*d^3*e + 3*a*d*e^2)*g^6 + ((9*c*d^2*e^2 + a*e^3)*f*g^5 +
 (7*c*d^3*e + 3*a*d*e^2)*g^6)*n^5 - 2*(10*c*d*e^3*f^2*g^4 - 7*(9*c*d^2*e^2
+ a*e^3)*f*g^5 - 9*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^4 + (20*c*e^4*f^3*g^3 - 1
80*c*d*e^3*f^2*g^4 + 65*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 121*(7*c*d^3*e + 3*a*
d*e^2)*g^6)*n^3 + 4*(15*c*e^4*f^3*g^3 - 100*c*d*e^3*f^2*g^4 + 28*(9*c*d^2*e
^2 + a*e^3)*f*g^5 + 93*(7*c*d^3*e + 3*a*d*e^2)*g^6)*n^2 + 4*(10*c*e^4*f^3*g
^3 - 60*c*d*e^3*f^2*g^4 + 15*(9*c*d^2*e^2 + a*e^3)*f*g^5 + 127*(7*c*d^3*e +
 3*a*d*e^2)*g^6)*n)*x^3 + (580*a*d^3*f*g^5 - 6*(9*c*d^2*e^2 + a*e^3)*f^4*g^
2 + 30*(7*c*d^3*e + 3*a*d*e^2)*f^3*g^3 - 119*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4)
*n^2 + (360*(2*c*d^4 + 3*a*d^2*e)*g^6 + ((7*c*d^3*e + 3*a*d*e^2)*f*g^5 + (2
*c*d^4 + 3*a*d^2*e)*g^6)*n^5 - (3*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 16*(7*c*d
^3*e + 3*a*d*e^2)*f*g^5 - 19*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^4 + (60*c*d*e^3*f
^3*g^3 - 36*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 + 89*(7*c*d^3*e + 3*a*d*e^2)*f*g^
5 + 137*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^3 - (60*c*e^4*f^4*g^2 - 420*c*d*e^3*f^
3*g^3 + 123*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 194*(7*c*d^3*e + 3*a*d*e^2)*f*g
^5 - 461*(2*c*d^4 + 3*a*d^2*e)*g^6)*n^2 - 6*(10*c*e^4*f^4*g^2 - 60*c*d*e^3*
f^3*g^3 + 15*(9*c*d^2*e^2 + a*e^3)*f^2*g^4 - 20*(7*c*d^3*e + 3*a*d*e^2)*f*g
^5 - 117*(2*c*d^4 + 3*a*d^2*e)*g^6)*n)*x^2 + 2*(60*c*d*e^3*f^5*g + 522*a*d^
3*f*g^5 - 33*(9*c*d^2*e^2 + a*e^3)*f^4*g^2 + 74*(7*c*d^3*e + 3*a*d*e^2)*f^3
```

```
*g^3 - 171*(2*c*d^4 + 3*a*d^2*e)*f^2*g^4)*n + (720*a*d^3*g^6 + (a*d^3*g^6 +
 (2*c*d^4 + 3*a*d^2*e)*f*g^5)*n^5 + 2*(10*a*d^3*g^6 - (7*c*d^3*e + 3*a*d*e^
2)*f^2*g^4 + 9*(2*c*d^4 + 3*a*d^2*e)*f*g^5)*n^4 + (155*a*d^3*g^6 + 6*(9*c*d
^2*e^2 + a*e^3)*f^3*g^3 - 30*(7*c*d^3*e + 3*a*d*e^2)*f^2*g^4 + 119*(2*c*d^4
 + 3*a*d^2*e)*f*g^5)*n^3 - 2*(60*c*d*e^3*f^4*g^2 - 290*a*d^3*g^6 - 33*(9*c*
d^2*e^2 + a*e^3)*f^3*g^3 + 74*(7*c*d^3*e + 3*a*d*e^2)*f^2*g^4 - 171*(2*c*d^
4 + 3*a*d^2*e)*f*g^5)*n^2 + 12*(10*c*e^4*f^5*g - 60*c*d*e^3*f^4*g^2 + 87*a*
d^3*g^6 + 15*(9*c*d^2*e^2 + a*e^3)*f^3*g^3 - 20*(7*c*d^3*e + 3*a*d*e^2)*f^2
*g^4 + 30*(2*c*d^4 + 3*a*d^2*e)*f*g^5)*n)*x)*(g*x + f)^n/(g^6*n^6 + 21*g^6*
n^5 + 175*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2822, size = 5076, normalized size = 18.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^6*n^5*x^6*e^4 + 5*(g*x + f)^n*c*d*g^6*n^5*x^5*e^3 + 9*(g*x
 + f)^n*c*d^2*g^6*n^5*x^4*e^2 + 7*(g*x + f)^n*c*d^3*g^6*n^5*x^3*e + 2*(g*x
+ f)^n*c*d^4*g^6*n^5*x^2 + (g*x + f)^n*c*f*g^5*n^5*x^5*e^4 + 15*(g*x + f)^n
*c*g^6*n^4*x^6*e^4 + 5*(g*x + f)^n*c*d*f*g^5*n^5*x^4*e^3 + 80*(g*x + f)^n*c
*d*g^6*n^4*x^5*e^3 + 9*(g*x + f)^n*c*d^2*f*g^5*n^5*x^3*e^2 + 153*(g*x + f)^
n*c*d^2*g^6*n^4*x^4*e^2 + 7*(g*x + f)^n*c*d^3*f*g^5*n^5*x^2*e + 126*(g*x +
f)^n*c*d^3*g^6*n^4*x^3*e + 2*(g*x + f)^n*c*d^4*f*g^5*n^5*x + 38*(g*x + f)^n
*c*d^4*g^6*n^4*x^2 + 10*(g*x + f)^n*c*f*g^5*n^4*x^5*e^4 + 85*(g*x + f)^n*c*
g^6*n^3*x^6*e^4 + 60*(g*x + f)^n*c*d*f*g^5*n^4*x^4*e^3 + (g*x + f)^n*a*g^6*
n^5*x^4*e^3 + 475*(g*x + f)^n*c*d*g^6*n^3*x^5*e^3 + 126*(g*x + f)^n*c*d^2*f
*g^5*n^4*x^3*e^2 + 3*(g*x + f)^n*a*d*g^6*n^5*x^3*e^2 + 963*(g*x + f)^n*c*d^
2*g^6*n^3*x^4*e^2 + 112*(g*x + f)^n*c*d^3*f*g^5*n^4*x^2*e + 3*(g*x + f)^n*a
*d^2*g^6*n^5*x^2*e + 847*(g*x + f)^n*c*d^3*g^6*n^3*x^3*e + 36*(g*x + f)^n*c
*d^4*f*g^5*n^4*x + (g*x + f)^n*a*d^3*g^6*n^5*x + 274*(g*x + f)^n*c*d^4*g^6*
n^3*x^2 - 5*(g*x + f)^n*c*f^2*g^4*n^4*x^4*e^4 + 35*(g*x + f)^n*c*f*g^5*n^3*
x^5*e^4 + 225*(g*x + f)^n*c*g^6*n^2*x^6*e^4 - 20*(g*x + f)^n*c*d*f^2*g^4*n^
4*x^3*e^3 + (g*x + f)^n*a*f*g^5*n^5*x^3*e^3 + 235*(g*x + f)^n*c*d*f*g^5*n^3
*x^4*e^3 + 17*(g*x + f)^n*a*g^6*n^4*x^4*e^3 + 1300*(g*x + f)^n*c*d*g^6*n^2*
x^5*e^3 - 27*(g*x + f)^n*c*d^2*f^2*g^4*n^4*x^2*e^2 + 3*(g*x + f)^n*a*d*f*g^
5*n^5*x^2*e^2 + 585*(g*x + f)^n*c*d^2*f*g^5*n^3*x^3*e^2 + 54*(g*x + f)^n*a*
d*g^6*n^4*x^3*e^2 + 2763*(g*x + f)^n*c*d^2*g^6*n^2*x^4*e^2 - 14*(g*x + f)^n
*c*d^3*f^2*g^4*n^4*x*e + 3*(g*x + f)^n*a*d^2*f*g^5*n^5*x*e + 623*(g*x + f)^
n*c*d^3*f*g^5*n^3*x^2*e + 57*(g*x + f)^n*a*d^2*g^6*n^4*x^2*e + 2604*(g*x +
f)^n*c*d^3*g^6*n^2*x^3*e - 2*(g*x + f)^n*c*d^4*f^2*g^4*n^4 + (g*x + f)^n*a*
d^3*f*g^5*n^5 + 238*(g*x + f)^n*c*d^4*f*g^5*n^3*x + 20*(g*x + f)^n*a*d^3*g^
```

6*n^4*x + 922*(g*x + f)^n*c*d^4*g^6*n^2*x^2 - 30*(g*x + f)^n*c*f^2*g^4*n^3*x^4*e^4 + 50*(g*x + f)^n*c*f*g^5*n^2*x^5*e^4 + 274*(g*x + f)^n*c*g^6*n*x^6*e^4 - 180*(g*x + f)^n*c*d*f^2*g^4*n^3*x^3*e^3 + 14*(g*x + f)^n*a*f*g^5*n^4*x^3*e^3 + 360*(g*x + f)^n*c*d*f*g^5*n^2*x^4*e^3 + 107*(g*x + f)^n*a*g^6*n^3*x^4*e^3 + 1620*(g*x + f)^n*c*d*g^6*n*x^5*e^3 - 324*(g*x + f)^n*c*d^2*f^2*g^4*n^3*x^2*e^2 + 48*(g*x + f)^n*a*d*f*g^5*n^4*x^2*e^2 + 1008*(g*x + f)^n*c*d^2*f*g^5*n^2*x^3*e^2 + 363*(g*x + f)^n*a*d*g^6*n^3*x^3*e^2 + 3564*(g*x + f)^n*c*d^2*g^6*n*x^4*e^2 - 210*(g*x + f)^n*c*d^3*f^2*g^4*n^3*x*e + 54*(g*x + f)^n*a*d^2*f*g^5*n^4*x*e + 1358*(g*x + f)^n*c*d^3*f*g^5*n^2*x^2*e + 411*(g*x + f)^n*a*d^2*g^6*n^3*x^2*e + 3556*(g*x + f)^n*c*d^3*g^6*n*x^3*e - 36*(g*x + f)^n*c*d^4*f^2*g^4*n^3 + 20*(g*x + f)^n*a*d^3*f*g^5*n^4 + 684*(g*x + f)^n*c*d^4*f*g^5*n^2*x + 155*(g*x + f)^n*a*d^3*g^6*n^3*x + 1404*(g*x + f)^n*c*d^4*g^6*n*x^2 + 20*(g*x + f)^n*c*f^3*g^3*n^3*x^3*e^4 - 55*(g*x + f)^n*c*f^2*g^4*n^2*x^4*e^4 + 24*(g*x + f)^n*c*f*g^5*n*x^5*e^4 + 120*(g*x + f)^n*c*g^6*x^6*e^4 + 60*(g*x + f)^n*c*d*f^3*g^3*n^3*x^2*e^3 - 3*(g*x + f)^n*a*f^2*g^4*n^4*x^2*e^3 - 400*(g*x + f)^n*c*d*f^2*g^4*n^2*x^3*e^3 + 65*(g*x + f)^n*a*f*g^5*n^3*x^3*e^3 + 180*(g*x + f)^n*c*d*f*g^5*n*x^4*e^3 + 307*(g*x + f)^n*a*g^6*n^2*x^4*e^3 + 720*(g*x + f)^n*c*d*g^6*x^5*e^3 + 54*(g*x + f)^n*c*d^2*f^3*g^3*n^3*x*e^2 - 6*(g*x + f)^n*a*d*f^2*g^4*n^4*x*e^2 - 1107*(g*x + f)^n*c*d^2*f^2*g^4*n^2*x^2*e^2 + 267*(g*x + f)^n*a*d*f*g^5*n^3*x^2*e^2 + 540*(g*x + f)^n*c*d^2*f*g^5*n*x^3*e^2 + 1116*(g*x + f)^n*a*d*g^6*n^2*x^3*e^2 + 1620*(g*x + f)^n*c*d^2*g^6*x^4*e^2 + 14*(g*x + f)^n*c*d^3*f^3*g^3*n^3*e - 3*(g*x + f)^n*a*d^2*f^2*g^4*n^4*e - 1036*(g*x + f)^n*c*d^3*f^2*g^4*n^2*x*e + 357*(g*x + f)^n*a*d^2*f*g^5*n^3*x*e + 840*(g*x + f)^n*c*d^3*f*g^5*n*x^2*e + 1383*(g*x + f)^n*a*d^2*g^6*n^2*x^2*e + 1680*(g*x + f)^n*c*d^3*g^6*x^3*e - 238*(g*x + f)^n*c*d^4*f^2*g^4*n^2 + 155*(g*x + f)^n*a*d^3*f*g^5*n^3 + 720*(g*x + f)^n*c*d^4*f*g^5*n*x + 580*(g*x + f)^n*a*d^3*g^6*n^2*x + 720*(g*x + f)^n*c*d^4*g^6*x^2 + 60*(g*x + f)^n*c*f^3*g^3*n^2*x^3*e^4 - 30*(g*x + f)^n*c*f^2*g^4*n*x^4*e^4 + 420*(g*x + f)^n*c*d*f^3*g^3*n^2*x^2*e^3 - 36*(g*x + f)^n*a*f^2*g^4*n^3*x^2*e^3 - 240*(g*x + f)^n*c*d*f^2*g^4*n*x^3*e^3 + 112*(g*x + f)^n*a*f*g^5*n^2*x^3*e^3 + 396*(g*x + f)^n*a*g^6*n*x^4*e^3 + 594*(g*x + f)^n*c*d^2*f^3*g^3*n^2*x*e^2 - 90*(g*x + f)^n*a*d*f^2*g^4*n^3*x*e^2 - 810*(g*x + f)^n*c*d^2*f^2*g^4*n*x^2*e^2 + 582*(g*x + f)^n*a*d*f*g^5*n^2*x^2*e^2 + 1524*(g*x + f)^n*a*d*g^6*n*x^3*e^2 + 210*(g*x + f)^n*c*d^3*f^3*g^3*n^2*e - 54*(g*x + f)^n*a*d^2*f^2*g^4*n^3*e - 1680*(g*x + f)^n*c*d^3*f^2*g^4*n*x*e + 1026*(g*x + f)^n*a*d^2*f*g^5*n^2*x*e + 2106*(g*x + f)^n*a*d^2*g^6*n*x^2*e - 684*(g*x + f)^n*c*d^4*f^2*g^4*n + 580*(g*x + f)^n*a*d^3*f*g^5*n^2 + 1044*(g*x + f)^n*a*d^3*g^6*n*x - 60*(g*x + f)^n*c*f^4*g^2*n^2*x^2*e^4 + 40*(g*x + f)^n*c*f^3*g^3*n*x^3*e^4 - 120*(g*x + f)^n*c*d*f^4*g^2*n^2*x*e^3 + 6*(g*x + f)^n*a*f^3*g^3*n^3*x*e^3 + 360*(g*x + f)^n*c*d*f^3*g^3*n*x^2*e^3 - 123*(g*x + f)^n*a*f^2*g^4*n^2*x^2*e^3 + 60*(g*x + f)^n*a*f*g^5*n*x^3*e^3 + 180*(g*x + f)^n*a*g^6*x^4*e^3 - 54*(g*x + f)^n*c*d^2*f^4*g^2*n^2*e^2 + 6*(g*x + f)^n*a*d*f^3*g^3*n^3*e^2 + 1620*(g*x + f)^n*c*d^2*f^3*g^3*n*x*e^2 - 444*(g*x + f)^n*a*d*f^2*g^4*n^2*x*e^2 + 360*(g*x + f)^n*a*d*f*g^5*n*x^2*e^2 + 720*(g*x + f)^n*a*d*g^6*x^3*e^2 + 1036*(g*x + f)^n*c*d^3*f^3*g^3*n*e - 357*(g*x + f)^n*a*d^2*f^2*g^4*n^2*e + 1080*(g*x + f)^n*a*d^2*f*g^5*n*x*e + 1080*(g*x + f)^n*a*d^2*g^6*x^2*e - 720*(g*x + f)^n*c*d^4*f^2*g^4 + 1044*(g*x + f)^n*a*d^3*f*g^5*n + 720*(g*x + f)^n*a*d^3*g^6*x - 60*(g*x + f)^n*c*f^4*g^2*n*x^2*e^4 - 720*(g*x + f)^n*c*d*f^4*g^2*n*x*e^3 + 66*(g*x + f)^n*a*f^3*g^3*n^2*x*e^3 - 90*(g*x + f)^n*a*f^2*g^4*n*x^2*e^3 - 594*(g*x + f)^n*c*d^2*f^4*g^2*n*e^2 + 90*(g*x + f)^n*a*d*f^3*g^3*n^2*e^2 - 720*(g*x + f)^n*a*d*f^2*g^4*n*x*e^2 + 1680*(g*x + f)^n*c*d^3*f^3*g^3*e - 1026*(g*x + f)^n*a*d^2*f^2*g^4*n*e + 720*(g*x + f)^n*a*d^3*f*g^5 + 120*(g*x + f)^n*c*f^5*g*n*x*e^4 + 120*(g*x + f)^n*c*d*f^5*g*n*e^3 - 6*(g*x + f)^n*a*f^4*g^2*n^2*e^3 + 180*(g*x + f)^n*a*f^3*g^3*n*x*e^3 - 1620*(g*x + f)^n*c*d^2*f^4*g^2*e^2 + 444*(g*x + f)^n*a*d*f^3*g^3*n*e^2 - 1080*(g*x + f)^n*a*d^2*f^2*g^4*e + 720*(g*x + f)^n*c*d*f^5*g*e^3 - 66*(g*x + f)^n*a*f^4*g^2*n*e^3 + 720*(g*x + f)^n*a*d*f^3*g^3*e^2 - 120*(g*x + f)^n*c*f^6*e^4 - 180*(g*x + f)^n*a*f^4*g^2*e^3)/(g^6*n^6 + 21*g^6*n^5 + 175*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^

6)

3.806 $\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=208

$$-\frac{2(ef-dg)(f+gx)^{n+2}\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{g^5(n+2)}+\frac{e(f+gx)^{n+3}\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{g^5(n+3)}+\frac{(ef-\ldots}{}$$

```
[Out] ((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n))
 - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x)
^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g
^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^(4 +
 n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))
```

Rubi [A] time = 0.187682, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {947}

$$-\frac{2(ef-dg)(f+gx)^{n+2}\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{g^5(n+2)}+\frac{e(f+gx)^{n+3}\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{g^5(n+3)}+\frac{(ef-\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n))
 - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x)
^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g
^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^(4 +
 n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^2(f+gx)^n\left(a+2cdx+cex^2\right)dx=\int\left(\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^4}+\frac{2(ef-dg)\left(-aeg^2-c\left(2e^2\ldots\right.\right.}{}\right.$$

$$=\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^5(1+n)}-\frac{2(ef-dg)\left(aeg^2+c\left(2e^2f^2-\ldots\right.\right.}{g^5(2\ldots}$$

Mathematica [A] time = 0.213148, size = 187, normalized size = 0.9

$$\frac{(f+gx)^{n+1}\left(\frac{e(f+gx)^2\left(aeg^2+c\left(5d^2g^2-12defg+6e^2f^2\right)\right)}{n+3}-\frac{2(f+gx)(ef-dg)\left(aeg^2+c\left(d^2g^2-4defg+2e^2f^2\right)\right)}{n+2}+\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)}{n+1}-\frac{4ce^2(f+gx)^3(e\ldots}{n+4}\right.}{g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g)))/(1 + n) - (
2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x))/(2
 + n) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g^2))*(f + g*x)^2)/
(3 + n) - (4*c*e^2*(e*f - d*g)*(f + g*x)^3)/(4 + n) + (c*e^3*(f + g*x)^4)/(
5 + n)))/g^5
```

Maple [B] time = 0.055, size = 1048, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(1+n)*(c*e^3*g^4*n^4*x^4+4*c*d*e^2*g^4*n^4*x^3+10*c*e^3*g^4*n^3*x^4
+5*c*d^2*e*g^4*n^4*x^2+44*c*d*e^2*g^4*n^3*x^3-4*c*e^3*f*g^3*n^3*x^3+35*c*e^
3*g^4*n^2*x^4+a*e^2*g^4*n^4*x^2+2*c*d^3*g^4*n^4*x+60*c*d^2*e*g^4*n^3*x^2-12
*c*d*e^2*f*g^3*n^3*x^2+164*c*d*e^2*g^4*n^2*x^3-24*c*e^3*f*g^3*n^2*x^3+50*c*
e^3*g^4*n*x^4+2*a*d*e*g^4*n^4*x+12*a*e^2*g^4*n^3*x^2+26*c*d^3*g^4*n^3*x-10*
c*d^2*e*f*g^3*n^3*x+245*c*d^2*e*g^4*n^2*x^2-96*c*d*e^2*f*g^3*n^2*x^2+244*c*
d*e^2*g^4*n*x^3+12*c*e^3*f^2*g^2*n^2*x^2-44*c*e^3*f*g^3*n*x^3+24*c*e^3*g^4*
x^4+a*d^2*g^4*n^4+26*a*d*e*g^4*n^3*x-2*a*e^2*f*g^3*n^3*x+49*a*e^2*g^4*n^2*x
^2-2*c*d^3*f*g^3*n^3+118*c*d^3*g^4*n^2*x-100*c*d^2*e*f*g^3*n^2*x+390*c*d^2*
e*g^4*n*x^2+24*c*d*e^2*f^2*g^2*n^2*x-204*c*d*e^2*f*g^3*n*x^2+120*c*d*e^2*g^
4*x^3+36*c*e^3*f^2*g^2*n*x^2-24*c*e^3*f*g^3*x^3+14*a*d^2*g^4*n^3-2*a*d*e*f*
g^3*n^3+118*a*d*e*g^4*n^2*x-20*a*e^2*f*g^3*n^2*x+78*a*e^2*g^4*n*x^2-24*c*d^
3*f*g^3*n^2+214*c*d^3*g^4*n*x+10*c*d^2*e*f^2*g^2*n^2-290*c*d^2*e*f*g^3*n*x+
200*c*d^2*e*g^4*x^2+144*c*d*e^2*f^2*g^2*n*x-120*c*d*e^2*f*g^3*x^2-24*c*e^3*
f^3*g*n*x+24*c*e^3*f^2*g^2*x^2+71*a*d^2*g^4*n^2-24*a*d*e*f*g^3*n^2+214*a*d*
e*g^4*n*x+2*a*e^2*f^2*g^2*n^2-58*a*e^2*f*g^3*n*x+40*a*e^2*g^4*x^2-94*c*d^3*
f*g^3*n+120*c*d^3*g^4*x+90*c*d^2*e*f^2*g^2*n-200*c*d^2*e*f*g^3*x-24*c*d*e^2
*f^3*g*n+120*c*d*e^2*f^2*g^2*x-24*c*e^3*f^3*g*x+154*a*d^2*g^4*n-94*a*d*e*f*
g^3*n+120*a*d*e*g^4*x+18*a*e^2*f^2*g^2*n-40*a*e^2*f*g^3*x-120*c*d^3*f*g^3+2
00*c*d^2*e*f^2*g^2-120*c*d*e^2*f^3*g+24*c*e^3*f^4+120*a*d^2*g^4-120*a*d*e*f
*g^3+40*a*e^2*f^2*g^2)/g^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98973, size = 2392, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d^2*f*g^4*n^4 + 24*c*e^3*f^5 - 120*c*d*e^2*f^4*g + 120*a*d^2*f*g^4 + 40*
(5*c*d^2*e + a*e^2)*f^3*g^2 - 120*(c*d^3 + a*d*e)*f^2*g^3 + (c*e^3*g^5*n^4
+ 10*c*e^3*g^5*n^3 + 35*c*e^3*g^5*n^2 + 50*c*e^3*g^5*n + 24*c*e^3*g^5)*x^5
+ (120*c*d*e^2*g^5 + (c*e^3*f*g^4 + 4*c*d*e^2*g^5)*n^4 + 2*(3*c*e^3*f*g^4 +
 22*c*d*e^2*g^5)*n^3 + (11*c*e^3*f*g^4 + 164*c*d*e^2*g^5)*n^2 + 2*(3*c*e^3*
f*g^4 + 122*c*d*e^2*g^5)*n)*x^4 + 2*(7*a*d^2*f*g^4 - (c*d^3 + a*d*e)*f^2*g^
3)*n^3 + (40*(5*c*d^2*e + a*e^2)*g^5 + (4*c*d*e^2*f*g^4 + (5*c*d^2*e + a*e^
2)*g^5)*n^4 - 4*(c*e^3*f^2*g^3 - 8*c*d*e^2*f*g^4 - 3*(5*c*d^2*e + a*e^2)*g^
5)*n^3 - (12*c*e^3*f^2*g^3 - 68*c*d*e^2*f*g^4 - 49*(5*c*d^2*e + a*e^2)*g^5)
*n^2 - 2*(4*c*e^3*f^2*g^3 - 20*c*d*e^2*f*g^4 - 39*(5*c*d^2*e + a*e^2)*g^5)*
n)*x^3 + (71*a*d^2*f*g^4 + 2*(5*c*d^2*e + a*e^2)*f^3*g^2 - 24*(c*d^3 + a*d*
e)*f^2*g^3)*n^2 + (120*(c*d^3 + a*d*e)*g^5 + ((5*c*d^2*e + a*e^2)*f*g^4 + 2
*(c*d^3 + a*d*e)*g^5)*n^4 - 2*(6*c*d*e^2*f^2*g^3 - 5*(5*c*d^2*e + a*e^2)*f*
g^4 - 13*(c*d^3 + a*d*e)*g^5)*n^3 + (12*c*e^3*f^3*g^2 - 72*c*d*e^2*f^2*g^3
+ 29*(5*c*d^2*e + a*e^2)*f*g^4 + 118*(c*d^3 + a*d*e)*g^5)*n^2 + 2*(6*c*e^3*
f^3*g^2 - 30*c*d*e^2*f^2*g^3 + 10*(5*c*d^2*e + a*e^2)*f*g^4 + 107*(c*d^3 +
a*d*e)*g^5)*n)*x^2 - 2*(12*c*d*e^2*f^4*g - 77*a*d^2*f*g^4 - 9*(5*c*d^2*e +
a*e^2)*f^3*g^2 + 47*(c*d^3 + a*d*e)*f^2*g^3)*n + (120*a*d^2*g^5 + (a*d^2*g^
5 + 2*(c*d^3 + a*d*e)*f*g^4)*n^4 + 2*(7*a*d^2*g^5 - (5*c*d^2*e + a*e^2)*f^2
*g^3 + 12*(c*d^3 + a*d*e)*f*g^4)*n^3 + (24*c*d*e^2*f^3*g^2 + 71*a*d^2*g^5 -
 18*(5*c*d^2*e + a*e^2)*f^2*g^3 + 94*(c*d^3 + a*d*e)*f*g^4)*n^2 - 2*(12*c*e
^3*f^4*g - 60*c*d*e^2*f^3*g^2 - 77*a*d^2*g^5 + 20*(5*c*d^2*e + a*e^2)*f^2*g
^3 - 60*(c*d^3 + a*d*e)*f*g^4)*n)*x)*(g*x + f)^n/(g^5*n^5 + 15*g^5*n^4 + 85
*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

Sympy [A] time = 25.9905, size = 11628, normalized size = 55.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + c*d**3*x**2 + 5*c*
d**2*e*x**3/3 + c*d*e**2*x**4 + c*e**3*x**5/5), Eq(g, 0)), (-3*a*d**2*f**2*
g**4/(12*f**6*g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3
 + 12*f**2*g**9*x**4) - 2*a*d*e*f**3*g**3/(12*f**6*g**5 + 48*f**5*g**6*x +
72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**4) - 8*a*d*e*f**2*g
**4*x/(12*f**6*g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**
3 + 12*f**2*g**9*x**4) + 4*a*e**2*f*g**5*x**3/(12*f**6*g**5 + 48*f**5*g**6*
x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**4) + a*e**2*g**
6*x**4/(12*f**6*g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x*
*3 + 12*f**2*g**9*x**4) - 2*c*d**3*f**3*g**3/(12*f**6*g**5 + 48*f**5*g**6*x
 + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**4) - 8*c*d**3*f*
*2*g**4*x/(12*f**6*g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8
*x**3 + 12*f**2*g**9*x**4) + 20*c*d**2*e*f*g**5*x**3/(12*f**6*g**5 + 48*f**
5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**4) + 5*c
*d**2*e*g**6*x**4/(12*f**6*g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f
**3*g**8*x**3 + 12*f**2*g**9*x**4) + 12*c*d*e**2*f*g**5*x**4/(12*f**6*g**5
+ 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**
4) + 12*c*e**3*f**6*log(f/g + x)/(12*f**6*g**5 + 48*f**5*g**6*x + 72*f**4*g
**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**4) + 7*c*e**3*f**6/(12*f**6*
g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**
```

```
9*x**4) + 48*c*e**3*f**5*g*x*log(f/g + x)/(12*f**6*g**5 + 48*f**5*g**6*x +
72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**4) + 16*c*e**3*f**5
*g*x/(12*f**6*g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3
 + 12*f**2*g**9*x**4) + 72*c*e**3*f**4*g**2*x**2*log(f/g + x)/(12*f**6*g**5
 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x*
*4) + 48*c*e**3*f**3*g**3*x**3*log(f/g + x)/(12*f**6*g**5 + 48*f**5*g**6*x
+ 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**4) - 24*c*e**3*f*
*3*g**3*x**3/(12*f**6*g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g
**8*x**3 + 12*f**2*g**9*x**4) + 12*c*e**3*f**2*g**4*x**4*log(f/g + x)/(12*f
**6*g**5 + 48*f**5*g**6*x + 72*f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2
*g**9*x**4) - 18*c*e**3*f**2*g**4*x**4/(12*f**6*g**5 + 48*f**5*g**6*x + 72*
f**4*g**7*x**2 + 48*f**3*g**8*x**3 + 12*f**2*g**9*x**4), Eq(n, -5)), (-a*d*
*2*f*g**4/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3)
- a*d*e*f**2*g**3/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**
8*x**3) - 3*a*d*e*f*g**4*x/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2
+ 3*f*g**8*x**3) + a*e**2*g**5*x**3/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g
**7*x**2 + 3*f*g**8*x**3) - c*d**3*f**2*g**3/(3*f**4*g**5 + 9*f**3*g**6*x +
 9*f**2*g**7*x**2 + 3*f*g**8*x**3) - 3*c*d**3*f*g**4*x/(3*f**4*g**5 + 9*f**
3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) + 5*c*d**2*e*g**5*x**3/(3*f**4
*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) + 12*c*d*e**2*f**
4*g*log(f/g + x)/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8
*x**3) + 10*c*d*e**2*f**4*g/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2
 + 3*f*g**8*x**3) + 36*c*d*e**2*f**3*g**2*x*log(f/g + x)/(3*f**4*g**5 + 9*f
**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) + 18*c*d*e**2*f**3*g**2*x/(3
*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) + 36*c*d*e**
2*f**2*g**3*x**2*log(f/g + x)/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x*
*2 + 3*f*g**8*x**3) + 12*c*d*e**2*f*g**4*x**3*log(f/g + x)/(3*f**4*g**5 + 9
*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) - 12*c*d*e**2*f*g**4*x**3/
(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) - 12*c*e**
3*f**5*log(f/g + x)/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g
**8*x**3) - 10*c*e**3*f**5/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2
+ 3*f*g**8*x**3) - 36*c*e**3*f**4*g*x*log(f/g + x)/(3*f**4*g**5 + 9*f**3*g*
*6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) - 18*c*e**3*f**4*g*x/(3*f**4*g**5
+ 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) - 36*c*e**3*f**3*g**2*x
**2*log(f/g + x)/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8
*x**3) - 12*c*e**3*f**2*g**3*x**3*log(f/g + x)/(3*f**4*g**5 + 9*f**3*g**6*x
 + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) + 12*c*e**3*f**2*g**3*x**3/(3*f**4*g**
5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3) + 3*c*e**3*f*g**4*x**
4/(3*f**4*g**5 + 9*f**3*g**6*x + 9*f**2*g**7*x**2 + 3*f*g**8*x**3), Eq(n, -
4)), (-a*d**2*g**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 2*a*d*e*f*g**
3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4*a*d*e*g**4*x/(2*f**2*g**5 +
4*f*g**6*x + 2*g**7*x**2) + 2*a*e**2*f**2*g**2*log(f/g + x)/(2*f**2*g**5 +
4*f*g**6*x + 2*g**7*x**2) + 3*a*e**2*f**2*g**2/(2*f**2*g**5 + 4*f*g**6*x +
2*g**7*x**2) + 4*a*e**2*f*g**3*x*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2
*g**7*x**2) + 4*a*e**2*f*g**3*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) +
2*a*e**2*g**4*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) -
2*c*d**3*f*g**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4*c*d**3*g**4*x/
(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 10*c*d**2*e*f**2*g**2*log(f/g +
x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 15*c*d**2*e*f**2*g**2/(2*f**2
*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 20*c*d**2*e*f*g**3*x*log(f/g + x)/(2*f*
*2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 20*c*d**2*e*f*g**3*x/(2*f**2*g**5 + 4
*f*g**6*x + 2*g**7*x**2) + 10*c*d**2*e*g**4*x**2*log(f/g + x)/(2*f**2*g**5
+ 4*f*g**6*x + 2*g**7*x**2) - 24*c*d*e**2*f**3*g*log(f/g + x)/(2*f**2*g**5
+ 4*f*g**6*x + 2*g**7*x**2) - 36*c*d*e**2*f**3*g/(2*f**2*g**5 + 4*f*g**6*x
+ 2*g**7*x**2) - 48*c*d*e**2*f**2*g**2*x*log(f/g + x)/(2*f**2*g**5 + 4*f*g*
*6*x + 2*g**7*x**2) - 48*c*d*e**2*f**2*g**2*x/(2*f**2*g**5 + 4*f*g**6*x + 2
*g**7*x**2) - 24*c*d*e**2*f*g**3*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*
x + 2*g**7*x**2) + 8*c*d*e**2*g**4*x**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*
x**2) + 12*c*e**3*f**4*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2
```

) + 18*c*e**3*f**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 24*c*e**3*f**3*g*x*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 24*c*e**3*f**3*g*x/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + 12*c*e**3*f**2*g**2*x**2*log(f/g + x)/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) - 4*c*e**3*f*g**3*x**3/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2) + c*e**3*g**4*x**4/(2*f**2*g**5 + 4*f*g**6*x + 2*g**7*x**2), Eq(n, -3)), (-3*a*d**2*g**4/(3*f*g**5 + 3*g**6*x) + 6*a*d*e*f*g**3*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*a*d*e*f*g**3/(3*f*g**5 + 3*g**6*x) + 6*a*d*e*g**4*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 6*a*e**2*f**2*g**2*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 6*a*e**2*f**2*g**2/(3*f*g**5 + 3*g**6*x) - 6*a*e**2*f*g**3*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 3*a*e**2*g**4*x**2/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*f*g**3*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*f*g**3/(3*f*g**5 + 3*g**6*x) + 6*c*d**3*g**4*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f**2*g**2*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f**2*g**2/(3*f*g**5 + 3*g**6*x) - 30*c*d**2*e*f*g**3*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 15*c*d**2*e*g**4*x**2/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**3*g*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**3*g/(3*f*g**5 + 3*g**6*x) + 36*c*d*e**2*f**2*g**2*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 18*c*d*e**2*f*g**3*x**2/(3*f*g**5 + 3*g**6*x) + 6*c*d*e**2*g**4*x**3/(3*f*g**5 + 3*g**6*x) - 12*c*e**3*f**4*log(f/g + x)/(3*f*g**5 + 3*g**6*x) - 12*c*e**3*f**4/(3*f*g**5 + 3*g**6*x) - 12*c*e**3*f**3*g*x*log(f/g + x)/(3*f*g**5 + 3*g**6*x) + 6*c*e**3*f**2*g**2*x**2/(3*f*g**5 + 3*g**6*x) - 2*c*e**3*f*g**3*x**3/(3*f*g**5 + 3*g**6*x) + c*e**3*g**4*x**4/(3*f*g**5 + 3*g**6*x), Eq(n, -2)), (a*d**2*log(f/g + x)/g - 2*a*d*e*f*log(f/g + x)/g**2 + 2*a*d*e*x/g + a*e**2*f**2*log(f/g + x)/g**3 - a*e**2*f*x/g**2 + a*e**2*x**2/(2*g) - 2*c*d**3*f*log(f/g + x)/g**2 + 2*c*d**3*x/g + 5*c*d**2*e*f**2*log(f/g + x)/g**3 - 5*c*d**2*e*f*x/g**2 + 5*c*d**2*e*x**2/(2*g) - 4*c*d*e**2*f**3*log(f/g + x)/g**4 + 4*c*d*e**2*f**2*x/g**3 - 2*c*d*e**2*f*x**2/g**2 + 4*c*d*e**2*x**3/(3*g) + c*e**3*f**4*log(f/g + x)/g**5 - c*e**3*f**3*x/g**4 + c*e**3*f**2*x**2/(2*g**3) - c*e**3*f*x**3/(3*g**2) + c*e**3*x**4/(4*g), Eq(n, -1)), (a*d**2*f*g**4*n**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 14*a*d**2*f*g**4*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 71*a*d**2*f*g**4*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 154*a*d**2*f*g**4*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d**2*f*g**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + a*d**2*g**5*n**4*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 14*a*d**2*g**5*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 71*a*d**2*g**5*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 154*a*d**2*g**5*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d**2*g**5*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 2*a*d*e*f**2*g**3*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*a*d*e*f**2*g**3*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 94*a*d*e*f**2*g**3*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 120*a*d*e*f**2*g**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2*a*d*e*f*g**4*n**4*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 24*a*d*e*f*g**4*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 94*a*d*e*f*g**4*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 120*a*d*e*f*g**4*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2*a*d*e*g**5*n**4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n*

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*2 + 274*g**5*n + 120*g**5) + 26*a*d*e*g**5*n**3*x**2*(f + g*x)**n/(g**5*n*
*5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
 118*a*d*e*g**5*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*
n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 214*a*d*e*g**5*n*x**2*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) + 120*a*d*e*g**5*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2*a*e**2*f**3*g*
*2*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n*
*2 + 274*g**5*n + 120*g**5) + 18*a*e**2*f**3*g**2*n*(f + g*x)**n/(g**5*n**5
 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 4
0*a*e**2*f**3*g**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 +
225*g**5*n**2 + 274*g**5*n + 120*g**5) - 2*a*e**2*f**2*g**3*n**3*x*(f + g*x
)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n
+ 120*g**5) - 18*a*e**2*f**2*g**3*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*
n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 40*a*e**2*f*
*2*g**3*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**
5*n**2 + 274*g**5*n + 120*g**5) + a*e**2*f*g**4*n**4*x**2*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) + 10*a*e**2*f*g**4*n**3*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85
*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 29*a*e**2*f*g**4*n**2
*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
 + 274*g**5*n + 120*g**5) + 20*a*e**2*f*g**4*n*x**2*(f + g*x)**n/(g**5*n**5
 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + a
*e**2*g**5*n**4*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3
+ 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 12*a*e**2*g**5*n**3*x**3*(f + g*
x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n
 + 120*g**5) + 49*a*e**2*g**5*n**2*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n
**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 78*a*e**2*g**
5*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n
**2 + 274*g**5*n + 120*g**5) + 40*a*e**2*g**5*x**3*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 2*
c*d**3*f**2*g**3*n**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3
 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*d**3*f**2*g**3*n**2*(f + g
*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*
n + 120*g**5) - 94*c*d**3*f**2*g**3*n*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**
4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 120*c*d**3*f**2
*g**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
 + 274*g**5*n + 120*g**5) + 2*c*d**3*f*g**4*n**4*x*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 24
*c*d**3*f*g**4*n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3
 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 94*c*d**3*f*g**4*n**2*x*(f + g*
x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n
 + 120*g**5) + 120*c*d**3*f*g**4*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 2*c*d**3*g**5*n*
*4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n*
*2 + 274*g**5*n + 120*g**5) + 26*c*d**3*g**5*n**3*x**2*(f + g*x)**n/(g**5*n
**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
+ 118*c*d**3*g**5*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 214*c*d**3*g**5*n*x**2*(f
 + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g
**5*n + 120*g**5) + 120*c*d**3*g**5*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*
n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 10*c*d**2*e*
f**3*g**2*n**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*
g**5*n**2 + 274*g**5*n + 120*g**5) + 90*c*d**2*e*f**3*g**2*n*(f + g*x)**n/(
g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*
g**5) + 200*c*d**2*e*f**3*g**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*
g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 10*c*d**2*e*f**2*g**3*
n**3*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**
2 + 274*g**5*n + 120*g**5) - 90*c*d**2*e*f**2*g**3*n**2*x*(f + g*x)**n/(g**
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5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) - 200*c*d**2*e*f**2*g**3*n*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85
*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 5*c*d**2*e*f*g**4*n**
4*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**
2 + 274*g**5*n + 120*g**5) + 50*c*d**2*e*f*g**4*n**3*x**2*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) + 145*c*d**2*e*f*g**4*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 +
 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 100*c*d**2*e*f*g**
4*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n
2 + 274*g5*n + 120*g**5) + 5*c*d**2*e*g**5*n**4*x**3*(f + g*x)**n/(g**5
*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5
) + 60*c*d**2*e*g**5*n**3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*
g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 245*c*d**2*e*g**5*n**2
*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
 + 274*g**5*n + 120*g**5) + 390*c*d**2*e*g**5*n*x**3*(f + g*x)**n/(g**5*n**
5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
200*c*d**2*e*g**5*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**
3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*d*e**2*f**4*g*n*(f + g*x)
n/(g5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n +
 120*g**5) - 120*c*d*e**2*f**4*g*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 8
5*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 24*c*d*e**2*f**3*g**
2*n**2*x*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n
2 + 274*g5*n + 120*g**5) + 120*c*d*e**2*f**3*g**2*n*x*(f + g*x)**n/(g**
5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**
5) - 12*c*d*e**2*f**2*g**3*n**3*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4
 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 72*c*d*e**2*f**2
*g**3*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225
*g**5*n**2 + 274*g**5*n + 120*g**5) - 60*c*d*e**2*f**2*g**3*n*x**2*(f + g*x
)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n
+ 120*g**5) + 4*c*d*e**2*f*g**4*n**4*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5
*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 32*c*d*e**2
*f*g**4*n**3*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 2
25*g**5*n**2 + 274*g**5*n + 120*g**5) + 68*c*d*e**2*f*g**4*n**2*x**3*(f + g
*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*
n + 120*g**5) + 40*c*d*e**2*f*g**4*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5
*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 4*c*d*e**2*
g**5*n**4*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*
g**5*n**2 + 274*g**5*n + 120*g**5) + 44*c*d*e**2*g**5*n**3*x**4*(f + g*x)**
n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 1
20*g**5) + 164*c*d*e**2*g**5*n**2*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n*
*4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 244*c*d*e**2*g
5*n*x4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5
*n**2 + 274*g**5*n + 120*g**5) + 120*c*d*e**2*g**5*x**4*(f + g*x)**n/(g**5*
n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5)
 + 24*c*e**3*f**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 2
25*g**5*n**2 + 274*g**5*n + 120*g**5) - 24*c*e**3*f**4*g*n*x*(f + g*x)**n/(
g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*
g**5) + 12*c*e**3*f**3*g**2*n**2*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**
4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 12*c*e**3*f**3*
g**2*n*x**2*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**
5*n**2 + 274*g**5*n + 120*g**5) - 4*c*e**3*f**2*g**3*n**3*x**3*(f + g*x)**n
/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 12
0*g**5) - 12*c*e**3*f**2*g**3*n**2*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n
4 + 85*g5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) - 8*c*e**3*f**2
*g**3*n*x**3*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g*
*5*n**2 + 274*g**5*n + 120*g**5) + c*e**3*f*g**4*n**4*x**4*(f + g*x)**n/(g*
*5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g*
*5) + 6*c*e**3*f*g**4*n**3*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85
*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 11*c*e**3*f*g**4*n**2

```
*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
 + 274*g**5*n + 120*g**5) + 6*c*e**3*f*g**4*n*x**4*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + c*
e**3*g**5*n**4*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 +
 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 10*c*e**3*g**5*n**3*x**5*(f + g*x
)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n
+ 120*g**5) + 35*c*e**3*g**5*n**2*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n*
*4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 50*c*e**3*g**5
*n*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n*
*2 + 274*g**5*n + 120*g**5) + 24*c*e**3*g**5*x**5*(f + g*x)**n/(g**5*n**5 +
 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5), True
))
```

Giac [B] time = 1.50858, size = 2854, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^5*n^4*x^5*e^3 + 4*(g*x + f)^n*c*d*g^5*n^4*x^4*e^2 + 5*(g*x
 + f)^n*c*d^2*g^5*n^4*x^3*e + 2*(g*x + f)^n*c*d^3*g^5*n^4*x^2 + (g*x + f)^n
*c*f*g^4*n^4*x^4*e^3 + 10*(g*x + f)^n*c*g^5*n^3*x^5*e^3 + 4*(g*x + f)^n*c*d
*f*g^4*n^4*x^3*e^2 + 44*(g*x + f)^n*c*d*g^5*n^3*x^4*e^2 + 5*(g*x + f)^n*c*d
^2*f*g^4*n^4*x^2*e + 60*(g*x + f)^n*c*d^2*g^5*n^3*x^3*e + 2*(g*x + f)^n*c*d
^3*f*g^4*n^4*x + 26*(g*x + f)^n*c*d^3*g^5*n^3*x^2 + 6*(g*x + f)^n*c*f*g^4*n
^3*x^4*e^3 + 35*(g*x + f)^n*c*g^5*n^2*x^5*e^3 + 32*(g*x + f)^n*c*d*f*g^4*n^
3*x^3*e^2 + (g*x + f)^n*a*g^5*n^4*x^3*e^2 + 164*(g*x + f)^n*c*d*g^5*n^2*x^4
*e^2 + 50*(g*x + f)^n*c*d^2*f*g^4*n^3*x^2*e + 2*(g*x + f)^n*a*d*g^5*n^4*x^2
*e + 245*(g*x + f)^n*c*d^2*g^5*n^2*x^3*e + 24*(g*x + f)^n*c*d^3*f*g^4*n^3*x
 + (g*x + f)^n*a*d^2*g^5*n^4*x + 118*(g*x + f)^n*c*d^3*g^5*n^2*x^2 - 4*(g*x
 + f)^n*c*f^2*g^3*n^3*x^3*e^3 + 11*(g*x + f)^n*c*f*g^4*n^2*x^4*e^3 + 50*(g*
x + f)^n*c*g^5*n*x^5*e^3 - 12*(g*x + f)^n*c*d*f^2*g^3*n^3*x^2*e^2 + (g*x +
f)^n*a*f*g^4*n^4*x^2*e^2 + 68*(g*x + f)^n*c*d*f*g^4*n^2*x^3*e^2 + 12*(g*x +
 f)^n*a*g^5*n^3*x^3*e^2 + 244*(g*x + f)^n*c*d*g^5*n*x^4*e^2 - 10*(g*x + f)^
n*c*d^2*f^2*g^3*n^3*x*e + 2*(g*x + f)^n*a*d*f*g^4*n^4*x*e + 145*(g*x + f)^n
*c*d^2*f*g^4*n^2*x^2*e + 26*(g*x + f)^n*a*d*g^5*n^3*x^2*e + 390*(g*x + f)^n
*c*d^2*g^5*n*x^3*e - 2*(g*x + f)^n*c*d^3*f^2*g^3*n^3 + (g*x + f)^n*a*d^2*f*
g^4*n^4 + 94*(g*x + f)^n*c*d^3*f*g^4*n^2*x + 14*(g*x + f)^n*a*d^2*g^5*n^3*x
 + 214*(g*x + f)^n*c*d^3*g^5*n*x^2 - 12*(g*x + f)^n*c*f^2*g^3*n^2*x^3*e^3 +
 6*(g*x + f)^n*c*f*g^4*n*x^4*e^3 + 24*(g*x + f)^n*c*g^5*x^5*e^3 - 72*(g*x +
 f)^n*c*d*f^2*g^3*n^2*x^2*e^2 + 10*(g*x + f)^n*a*f*g^4*n^3*x^2*e^2 + 40*(g*
x + f)^n*c*d*f*g^4*n*x^3*e^2 + 49*(g*x + f)^n*a*g^5*n^2*x^3*e^2 + 120*(g*x
+ f)^n*c*d*g^5*x^4*e^2 - 90*(g*x + f)^n*c*d^2*f^2*g^3*n^2*x*e + 24*(g*x + f
)^n*a*d*f*g^4*n^3*x*e + 100*(g*x + f)^n*c*d^2*f*g^4*n*x^2*e + 118*(g*x + f)
^n*a*d*g^5*n^2*x^2*e + 200*(g*x + f)^n*c*d^2*g^5*x^3*e - 24*(g*x + f)^n*c*d
^3*f^2*g^3*n^2 + 14*(g*x + f)^n*a*d^2*f*g^4*n^3 + 120*(g*x + f)^n*c*d^3*f*g
^4*n*x + 71*(g*x + f)^n*a*d^2*g^5*n^2*x + 120*(g*x + f)^n*c*d^3*g^5*x^2 + 1
2*(g*x + f)^n*c*f^3*g^2*n^2*x^2*e^3 - 8*(g*x + f)^n*c*f^2*g^3*n*x^3*e^3 + 2
4*(g*x + f)^n*c*d*f^3*g^2*n^2*x*e^2 - 2*(g*x + f)^n*a*f^2*g^3*n^3*x*e^2 - 6
0*(g*x + f)^n*c*d*f^2*g^3*n*x^2*e^2 + 29*(g*x + f)^n*a*f*g^4*n^2*x^2*e^2 +
78*(g*x + f)^n*a*g^5*n*x^3*e^2 + 10*(g*x + f)^n*c*d^2*f^3*g^2*n^2*e - 2*(g*
x + f)^n*a*d*f^2*g^3*n^3*e - 200*(g*x + f)^n*c*d^2*f^2*g^3*n*x*e + 94*(g*x
+ f)^n*a*d*f*g^4*n^2*x*e + 214*(g*x + f)^n*a*d*g^5*n*x^2*e - 94*(g*x + f)^n
*c*d^3*f^2*g^3*n + 71*(g*x + f)^n*a*d^2*f*g^4*n^2 + 154*(g*x + f)^n*a*d^2*g
^5*n*x + 12*(g*x + f)^n*c*f^3*g^2*n*x^2*e^3 + 120*(g*x + f)^n*c*d*f^3*g^2*n
```

```
*x*e^2 - 18*(g*x + f)^n*a*f^2*g^3*n^2*x*e^2 + 20*(g*x + f)^n*a*f*g^4*n*x^2*
e^2 + 40*(g*x + f)^n*a*g^5*x^3*e^2 + 90*(g*x + f)^n*c*d^2*f^3*g^2*n*e - 24*
(g*x + f)^n*a*d*f^2*g^3*n^2*e + 120*(g*x + f)^n*a*d*f*g^4*n*x*e + 120*(g*x
+ f)^n*a*d*g^5*x^2*e - 120*(g*x + f)^n*c*d^3*f^2*g^3 + 154*(g*x + f)^n*a*d^
2*f*g^4*n + 120*(g*x + f)^n*a*d^2*g^5*x - 24*(g*x + f)^n*c*f^4*g*n*x*e^3 -
24*(g*x + f)^n*c*d*f^4*g*n*e^2 + 2*(g*x + f)^n*a*f^3*g^2*n^2*e^2 - 40*(g*x
+ f)^n*a*f^2*g^3*n*x*e^2 + 200*(g*x + f)^n*c*d^2*f^3*g^2*e - 94*(g*x + f)^n
*a*d*f^2*g^3*n*e + 120*(g*x + f)^n*a*d^2*f*g^4 - 120*(g*x + f)^n*c*d*f^4*g*
e^2 + 18*(g*x + f)^n*a*f^3*g^2*n*e^2 - 120*(g*x + f)^n*a*d*f^2*g^3*e + 24*(
g*x + f)^n*c*f^5*e^3 + 40*(g*x + f)^n*a*f^3*g^2*e^2)/(g^5*n^5 + 15*g^5*n^4
+ 85*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

3.807 $\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=146

$$\frac{(f+gx)^{n+2}\left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^4(n+2)}-\frac{(ef-dg)(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)}-\frac{3ce(ef-dg)(f+gx)^{\prime}}{g^4(n+3)}$$

```
[Out] -((((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^4*(1 + n))
) + ((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(2 + n))/(
g^4*(2 + n)) - (3*c*e*(e*f - d*g)*(f + g*x)^(3 + n))/(g^4*(3 + n)) + (c*e^2
*(f + g*x)^(4 + n))/(g^4*(4 + n))
```

Rubi [A] time = 0.112171, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {771}

$$\frac{(f+gx)^{n+2}\left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^4(n+2)}-\frac{(ef-dg)(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)}-\frac{3ce(ef-dg)(f+gx)^{\prime}}{g^4(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -((((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^4*(1 + n))
) + ((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(2 + n))/(
g^4*(2 + n)) - (3*c*e*(e*f - d*g)*(f + g*x)^(3 + n))/(g^4*(3 + n)) + (c*e^2
*(f + g*x)^(4 + n))/(g^4*(4 + n))
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx = \int \left(\frac{(ef-dg)\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^3}+\frac{\left(aeg^2+c\left(3e^2f^2-6def\right.\right.}{g}\right.$$

$$= -\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^4(1+n)}+\frac{\left(aeg^2+c\left(3e^2f^2-6defg\right.\right.}{g^4(2+}$$

Mathematica [A] time = 0.291619, size = 141, normalized size = 0.97

$$\frac{(f+gx)^{n+1}\left(\frac{2(f+gx)\left(aeg^2(n+3)+c\left(-d^2g^2n-6defg+3e^2f^2\right)\right)}{g^2(n+2)}+\frac{6(dg-ef)\left(ag^2+cf(ef-2dg)\right)}{g^2(n+1)}+(a+cx(2d+ex))(dg(n+6)-3ef+eg(n\right.}{g^2(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*((6*(-(e*f) + d*g)*(a*g^2 + c*f*(e*f - 2*d*g)))/(g^2*(1
+ n)) + (2*(a*e*g^2*(3 + n) + c*(3*e^2*f^2 - 6*d*e*f*g - d^2*g^2*n))*(f + g
*x))/(g^2*(2 + n)) + (-3*e*f + d*g*(6 + n) + e*g*(3 + n)*x)*(a + c*x*(2*d +
 e*x))))/(g^2*(3 + n)*(4 + n))
```

Maple [B] time = 0.05, size = 449, normalized size = 3.1

$$\frac{\left(gx+f\right)^{1+n}\left(ce^2g^3n^3x^3+3\,cdeg^3n^3x^2+6\,ce^2g^3n^2x^3+2\,cd^2g^3n^3x+21\,cdeg^3n^2x^2-3\,ce^2fg^2n^2x^2+11\,ce^2g^3nx^3+aeg^3n^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(1+n)*(c*e^2*g^3*n^3*x^3+3*c*d*e*g^3*n^3*x^2+6*c*e^2*g^3*n^2*x^3+2*
c*d^2*g^3*n^3*x+21*c*d*e*g^3*n^2*x^2-3*c*e^2*f*g^2*n^2*x^2+11*c*e^2*g^3*n*x
^3+a*e*g^3*n^3*x+16*c*d^2*g^3*n^2*x-6*c*d*e*f*g^2*n^2*x+42*c*d*e*g^3*n*x^2-
9*c*e^2*f*g^2*n*x^2+6*c*e^2*g^3*x^3+a*d*g^3*n^3+8*a*e*g^3*n^2*x-2*c*d^2*f*g
^2*n^2+38*c*d^2*g^3*n*x-30*c*d*e*f*g^2*n*x+24*c*d*e*g^3*x^2+6*c*e^2*f^2*g*n
*x-6*c*e^2*f*g^2*x^2+9*a*d*g^3*n^2-a*e*f*g^2*n^2+19*a*e*g^3*n*x-14*c*d^2*f*
g^2*n+24*c*d^2*g^3*x+6*c*d*e*f^2*g*n-24*c*d*e*f*g^2*x+6*c*e^2*f^2*g*x+26*a*
d*g^3*n-7*a*e*f*g^2*n+12*a*e*g^3*x-24*c*d^2*f*g^2+24*c*d*e*f^2*g-6*c*e^2*f^
3+24*a*d*g^3-12*a*e*f*g^2)/g^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64494, size = 1173, normalized size = 8.03

$$\frac{\left(adfg^3n^3-6\,ce^2f^4+24\,cdef^3g+24\,adfg^3-12\left(2\,cd^2+ae\right)f^2g^2+\left(ce^2g^4n^3+6\,ce^2g^4n^2+11\,ce^2g^4n+6\,ce^2g^4\right)x^4+\left(24\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*f*g^3*n^3 - 6*c*e^2*f^4 + 24*c*d*e*f^3*g + 24*a*d*f*g^3 - 12*(2*c*d^2
+ a*e)*f^2*g^2 + (c*e^2*g^4*n^3 + 6*c*e^2*g^4*n^2 + 11*c*e^2*g^4*n + 6*c*e^
2*g^4)*x^4 + (24*c*d*e*g^4 + (c*e^2*f*g^3 + 3*c*d*e*g^4)*n^3 + 3*(c*e^2*f*g
^3 + 7*c*d*e*g^4)*n^2 + 2*(c*e^2*f*g^3 + 21*c*d*e*g^4)*n)*x^3 + (9*a*d*f*g^
3 - (2*c*d^2 + a*e)*f^2*g^2)*n^2 + (12*(2*c*d^2 + a*e)*g^4 + (3*c*d*e*f*g^3
 + (2*c*d^2 + a*e)*g^4)*n^3 - (3*c*e^2*f^2*g^2 - 15*c*d*e*f*g^3 - 8*(2*c*d^
2 + a*e)*g^4)*n^2 - (3*c*e^2*f^2*g^2 - 12*c*d*e*f*g^3 - 19*(2*c*d^2 + a*e)*
g^4)*n)*x^2 + (6*c*d*e*f^3*g + 26*a*d*f*g^3 - 7*(2*c*d^2 + a*e)*f^2*g^2)*n
```

```
+ (24*a*d*g^4 + (a*d*g^4 + (2*c*d^2 + a*e)*f*g^3)*n^3 - (6*c*d*e*f^2*g^2 -
9*a*d*g^4 - 7*(2*c*d^2 + a*e)*f*g^3)*n^2 + 2*(3*c*e^2*f^3*g - 12*c*d*e*f^2*
g^2 + 13*a*d*g^4 + 6*(2*c*d^2 + a*e)*f*g^3)*n)*x)*(g*x + f)^n/(g^4*n^4 + 10
*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 24*g^4)
```

Sympy [A] time = 9.54863, size = 4908, normalized size = 33.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d*x + a*e*x**2/2 + c*d**2*x**2 + c*d*e*x**3 + c*e**2*x**
4/4), Eq(g, 0)), (-2*a*d*f*g**3/(6*f**4*g**4 + 18*f**3*g**5*x + 18*f**2*g**
6*x**2 + 6*f*g**7*x**3) - a*e*f**2*g**2/(6*f**4*g**4 + 18*f**3*g**5*x + 18*
f**2*g**6*x**2 + 6*f*g**7*x**3) - 3*a*e*f*g**3*x/(6*f**4*g**4 + 18*f**3*g**
5*x + 18*f**2*g**6*x**2 + 6*f*g**7*x**3) - 2*c*d**2*f**2*g**2/(6*f**4*g**4
+ 18*f**3*g**5*x + 18*f**2*g**6*x**2 + 6*f*g**7*x**3) - 6*c*d**2*f*g**3*x/(
6*f**4*g**4 + 18*f**3*g**5*x + 18*f**2*g**6*x**2 + 6*f*g**7*x**3) + 6*c*d*e
*g**4*x**3/(6*f**4*g**4 + 18*f**3*g**5*x + 18*f**2*g**6*x**2 + 6*f*g**7*x**
3) + 6*c*e**2*f**4*log(f/g + x)/(6*f**4*g**4 + 18*f**3*g**5*x + 18*f**2*g**
6*x**2 + 6*f*g**7*x**3) + 5*c*e**2*f**4/(6*f**4*g**4 + 18*f**3*g**5*x + 18*
f**2*g**6*x**2 + 6*f*g**7*x**3) + 18*c*e**2*f**3*g*x*log(f/g + x)/(6*f**4*g
**4 + 18*f**3*g**5*x + 18*f**2*g**6*x**2 + 6*f*g**7*x**3) + 9*c*e**2*f**3*g
*x/(6*f**4*g**4 + 18*f**3*g**5*x + 18*f**2*g**6*x**2 + 6*f*g**7*x**3) + 18*
c*e**2*f**2*g**2*x**2*log(f/g + x)/(6*f**4*g**4 + 18*f**3*g**5*x + 18*f**2*
g**6*x**2 + 6*f*g**7*x**3) + 6*c*e**2*f*g**3*x**3*log(f/g + x)/(6*f**4*g**4
 + 18*f**3*g**5*x + 18*f**2*g**6*x**2 + 6*f*g**7*x**3) - 6*c*e**2*f*g**3*x*
*3/(6*f**4*g**4 + 18*f**3*g**5*x + 18*f**2*g**6*x**2 + 6*f*g**7*x**3), Eq(n
, -4)), (-a*d*g**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - a*e*f*g**2/(2
*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 2*a*e*g**3*x/(2*f**2*g**4 + 4*f*g*
*5*x + 2*g**6*x**2) - 2*c*d**2*f*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x*
*2) - 4*c*d**2*g**3*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*c*d*e*f*
*2*g*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 9*c*d*e*f**2*g
/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x*log(f/g + x)/
(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x/(2*f**2*g**4 +
 4*f*g**5*x + 2*g**6*x**2) + 6*c*d*e*g**3*x**2*log(f/g + x)/(2*f**2*g**4 +
4*f*g**5*x + 2*g**6*x**2) - 6*c*e**2*f**3*log(f/g + x)/(2*f**2*g**4 + 4*f*g
**5*x + 2*g**6*x**2) - 9*c*e**2*f**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**
2) - 12*c*e**2*f**2*g*x*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**
2) - 12*c*e**2*f**2*g*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 6*c*e**2
*f*g**2*x**2*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 2*c*e*
*2*g**3*x**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2), Eq(n, -3)), (-2*a*d*
g**3/(2*f*g**4 + 2*g**5*x) + 2*a*e*f*g**2*log(f/g + x)/(2*f*g**4 + 2*g**5*x
) + 2*a*e*f*g**2/(2*f*g**4 + 2*g**5*x) + 2*a*e*g**3*x*log(f/g + x)/(2*f*g**
4 + 2*g**5*x) + 4*c*d**2*f*g**2*log(f/g + x)/(2*f*g**4 + 2*g**5*x) + 4*c*d*
*2*f*g**2/(2*f*g**4 + 2*g**5*x) + 4*c*d**2*g**3*x*log(f/g + x)/(2*f*g**4 +
2*g**5*x) - 12*c*d*e*f**2*g*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 12*c*d*e*f
**2*g/(2*f*g**4 + 2*g**5*x) - 12*c*d*e*f*g**2*x*log(f/g + x)/(2*f*g**4 + 2*
g**5*x) + 6*c*d*e*g**3*x**2/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3*log(f/g +
 x)/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*
f**2*g*x*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 3*c*e**2*f*g**2*x**2/(2*f*g**
4 + 2*g**5*x) + c*e**2*g**3*x**3/(2*f*g**4 + 2*g**5*x), Eq(n, -2)), (a*d*lo
g(f/g + x)/g - a*e*f*log(f/g + x)/g**2 + a*e*x/g - 2*c*d**2*f*log(f/g + x)/
g**2 + 2*c*d**2*x/g + 3*c*d*e*f**2*log(f/g + x)/g**3 - 3*c*d*e*f*x/g**2 + 3
*c*d*e*x**2/(2*g) - c*e**2*f**3*log(f/g + x)/g**4 + c*e**2*f**2*x/g**3 - c*
```

e**2*f*x**2/(2*g**2) + c*e**2*x**3/(3*g), Eq(n, -1)), (a*d*f*g**3*n**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 9*a*d*f*g**3*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 26*a*d*f*g**3*n*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*f*g**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*d*g**4*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 9*a*d*g**4*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 26*a*d*g**4*n*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*g**4*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - a*e*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 7*a*e*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 12*a*e*f**2*g**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*f*g**3*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 7*a*e*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*f*g**3*n*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*g**4*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 8*a*e*g**4*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 19*a*e*g**4*n*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*g**4*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 2*c*d**2*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 14*c*d**2*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 24*c*d**2*f**2*g**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 2*c*d**2*f*g**3*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 14*c*d**2*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d**2*f*g**3*n*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 2*c*d**2*g**4*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 16*c*d**2*g**4*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 38*c*d**2*g**4*n*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d**2*g**4*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*d*e*f**3*g*n*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d*e*f**3*g*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 6*c*d*e*f**2*g**2*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 24*c*d*e*f**2*g**2*n*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 3*c*d*e*f*g**3*n**3*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 15*c*d*e*f*g**3*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 12*c*d*e*f*g**3*n*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 3*c*d*e*g**4*n**3*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 21*c*d*e*g**4*n**2*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 42*c*d*e*g**4*n*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*c*d*e*g**4*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 6*c*e**2*f**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*e**2*f**3*g*n*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 3*c*e**2*f**2*g**2*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 3*c*e**2*f**2*g**2*n*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + c*e**2*f*g**3*n**3*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) +

```
3*c*e**2*f*g**3*n**2*x**3*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 2*c*e**2*f*g**3*n*x**3*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + c*e**2*g**4*n**3*x
**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*
g**4) + 6*c*e**2*g**4*n**2*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35
*g**4*n**2 + 50*g**4*n + 24*g**4) + 11*c*e**2*g**4*n*x**4*(f + g*x)**n/(g**
4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 6*c*e**2*g**4
*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 2
4*g**4), True))
```

Giac [B] time = 1.47953, size = 1374, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^4*n^3*x^4*e^2 + 3*(g*x + f)^n*c*d*g^4*n^3*x^3*e + 2*(g*x +
 f)^n*c*d^2*g^4*n^3*x^2 + (g*x + f)^n*c*f*g^3*n^3*x^3*e^2 + 6*(g*x + f)^n*c
*g^4*n^2*x^4*e^2 + 3*(g*x + f)^n*c*d*f*g^3*n^3*x^2*e + 21*(g*x + f)^n*c*d*g
^4*n^2*x^3*e + 2*(g*x + f)^n*c*d^2*f*g^3*n^3*x + 16*(g*x + f)^n*c*d^2*g^4*n
^2*x^2 + 3*(g*x + f)^n*c*f*g^3*n^2*x^3*e^2 + 11*(g*x + f)^n*c*g^4*n*x^4*e^2
 + 15*(g*x + f)^n*c*d*f*g^3*n^2*x^2*e + (g*x + f)^n*a*g^4*n^3*x^2*e + 42*(g
*x + f)^n*c*d*g^4*n*x^3*e + 14*(g*x + f)^n*c*d^2*f*g^3*n^2*x + (g*x + f)^n*
a*d*g^4*n^3*x + 38*(g*x + f)^n*c*d^2*g^4*n*x^2 - 3*(g*x + f)^n*c*f^2*g^2*n^
2*x^2*e^2 + 2*(g*x + f)^n*c*f*g^3*n*x^3*e^2 + 6*(g*x + f)^n*c*g^4*x^4*e^2 -
 6*(g*x + f)^n*c*d*f^2*g^2*n^2*x*e + (g*x + f)^n*a*f*g^3*n^3*x*e + 12*(g*x
+ f)^n*c*d*f*g^3*n*x^2*e + 8*(g*x + f)^n*a*g^4*n^2*x^2*e + 24*(g*x + f)^n*c
*d*g^4*x^3*e - 2*(g*x + f)^n*c*d^2*f^2*g^2*n^2 + (g*x + f)^n*a*d*f*g^3*n^3
+ 24*(g*x + f)^n*c*d^2*f*g^3*n*x + 9*(g*x + f)^n*a*d*g^4*n^2*x + 24*(g*x +
f)^n*c*d^2*g^4*x^2 - 3*(g*x + f)^n*c*f^2*g^2*n*x^2*e^2 - 24*(g*x + f)^n*c*d
*f^2*g^2*n*x*e + 7*(g*x + f)^n*a*f*g^3*n^2*x*e + 19*(g*x + f)^n*a*g^4*n*x^2
*e - 14*(g*x + f)^n*c*d^2*f^2*g^2*n + 9*(g*x + f)^n*a*d*f*g^3*n^2 + 26*(g*x
 + f)^n*a*d*g^4*n*x + 6*(g*x + f)^n*c*f^3*g*n*x*e^2 + 6*(g*x + f)^n*c*d*f^3
*g*n*e - (g*x + f)^n*a*f^2*g^2*n^2*e + 12*(g*x + f)^n*a*f*g^3*n*x*e + 12*(g
*x + f)^n*a*g^4*x^2*e - 24*(g*x + f)^n*c*d^2*f^2*g^2 + 26*(g*x + f)^n*a*d*f
*g^3*n + 24*(g*x + f)^n*a*d*g^4*x + 24*(g*x + f)^n*c*d*f^3*g*e - 7*(g*x + f
)^n*a*f^2*g^2*n*e + 24*(g*x + f)^n*a*d*f*g^3 - 6*(g*x + f)^n*c*f^4*e^2 - 12
*(g*x + f)^n*a*f^2*g^2*e)/(g^4*n^4 + 10*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 2
4*g^4)
```

3.808 $\int (f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=84

$$\frac{(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^3(n+1)}-\frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)}+\frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

```
[Out] ((a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^3*(1 + n)) - (2*c*(e*f -
 d*g)*(f + g*x)^(2 + n))/(g^3*(2 + n)) + (c*e*(f + g*x)^(3 + n))/(g^3*(3 +
n))
```

Rubi [A] time = 0.0634219, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {698}

$$\frac{(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^3(n+1)}-\frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)}+\frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^3*(1 + n)) - (2*c*(e*f -
 d*g)*(f + g*x)^(2 + n))/(g^3*(2 + n)) + (c*e*(f + g*x)^(3 + n))/(g^3*(3 +
n))
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (f+gx)^n\left(a+2cdx+cex^2\right)dx&=\int\left(\frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^2}+\frac{2c(-ef+dg)(f+gx)^{1+n}}{g^2}+\frac{ce(f+gx)^{2+n}}{g^2}\right)dx\\&=\frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^3(1+n)}-\frac{2c(ef-dg)(f+gx)^{2+n}}{g^3(2+n)}+\frac{ce(f+gx)^{3+n}}{g^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.105957, size = 73, normalized size = 0.87

$$\frac{(f+gx)^{n+1}\left(\frac{ag^2+cf(ef-2dg)}{n+1}-\frac{2c(f+gx)(ef-dg)}{n+2}+\frac{ce(f+gx)^2}{n+3}\right)}{g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*((a*g^2 + c*f*(e*f - 2*d*g))/(1 + n) - (2*c*(e*f - d*g)*
(f + g*x))/(2 + n) + (c*e*(f + g*x)^2)/(3 + n)))/g^3
```

Maple [A] time = 0.048, size = 147, normalized size = 1.8

$$\frac{(gx+f)^{1+n}\left(ceg^2n^2x^2+2\,cdg^2n^2x+3\,ceg^2nx^2+8\,cdg^2nx-2\,cefgnx+2\,cex^2g^2+ag^2n^2-2\,cdfgn+6\,cdg^2x-2\,ce\right.}{g^3\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] (g*x+f)^(1+n)*(c*e*g^2*n^2*x^2+2*c*d*g^2*n^2*x+3*c*e*g^2*n*x^2+8*c*d*g^2*n*
x-2*c*e*f*g*n*x+2*c*e*g^2*x^2+a*g^2*n^2-2*c*d*f*g*n+6*c*d*g^2*x-2*c*e*f*g*x
+5*a*g^2*n-6*c*d*f*g+2*c*e*f^2+6*a*g^2)/g^3/(n^3+6*n^2+11*n+6)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57113, size = 462, normalized size = 5.5

$$\frac{\left(afg^2n^2+2\,cef^3-6\,cdf^2g+6\,afg^2+\left(ceg^3n^2+3\,ceg^3n+2\,ceg^3\right)x^3+\left(6\,cdg^3+\left(cefg^2+2\,cdg^3\right)n^2+\left(cefg^2+8\,cd\right.\right.\right.}{g^3n^3+6\,g^3n^2+11\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*f*g^2*n^2 + 2*c*e*f^3 - 6*c*d*f^2*g + 6*a*f*g^2 + (c*e*g^3*n^2 + 3*c*e*g
^3*n + 2*c*e*g^3)*x^3 + (6*c*d*g^3 + (c*e*f*g^2 + 2*c*d*g^3)*n^2 + (c*e*f*g
^2 + 8*c*d*g^3)*n)*x^2 - (2*c*d*f^2*g - 5*a*f*g^2)*n + (6*a*g^3 + (2*c*d*f*
g^2 + a*g^3)*n^2 - (2*c*e*f^2*g - 6*c*d*f*g^2 - 5*a*g^3)*n)*x)*(g*x + f)^n/
(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^3)
```

Sympy [A] time = 3.98552, size = 1532, normalized size = 18.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*x + c*d*x**2 + c*e*x**3/3), Eq(g, 0)), (-a*g**2/(2*f**2*
g**3 + 4*f*g**4*x + 2*g**5*x**2) - 2*c*d*f*g/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) - 4*c*d*g**2*x/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 2*c*e*
f**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 3*c*e*f**2/(2*
f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x*log(f/g + x)/(2*f**2*g*
*3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) + 2*c*e*g**2*x**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**
5*x**2), Eq(n, -3)), (a*g**3*x/(f**2*g**3 + f*g**4*x) + 2*c*d*f**2*g*log(f/
g + x)/(f**2*g**3 + f*g**4*x) + 2*c*d*f*g**2*x*log(f/g + x)/(f**2*g**3 + f*
g**4*x) - 2*c*d*f*g**2*x/(f**2*g**3 + f*g**4*x) - 2*c*e*f**3*log(f/g + x)/(
f**2*g**3 + f*g**4*x) - 2*c*e*f**2*g*x*log(f/g + x)/(f**2*g**3 + f*g**4*x)
+ 2*c*e*f**2*g*x/(f**2*g**3 + f*g**4*x) + c*e*f*g**2*x**2/(f**2*g**3 + f*g*
*4*x), Eq(n, -2)), (a*log(f/g + x)/g - 2*c*d*f*log(f/g + x)/g**2 + 2*c*d*x/
g + c*e*f**2*log(f/g + x)/g**3 - c*e*f*x/g**2 + c*e*x**2/(2*g), Eq(n, -1)),
 (a*f*g**2*n**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3)
 + 5*a*f*g**2*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3)
 + 6*a*f*g**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) +
 a*g**3*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3)
+ 5*a*g**3*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3)
+ 6*a*g**3*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) -
2*c*d*f**2*g*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3)
- 6*c*d*f**2*g*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3)
+ 2*c*d*f*g**2*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6
*g**3) + 6*c*d*f*g**2*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n
 + 6*g**3) + 2*c*d*g**3*n**2*x**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 1
1*g**3*n + 6*g**3) + 8*c*d*g**3*n*x**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**
2 + 11*g**3*n + 6*g**3) + 6*c*d*g**3*x**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*
n**2 + 11*g**3*n + 6*g**3) + 2*c*e*f**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n*
*2 + 11*g**3*n + 6*g**3) - 2*c*e*f**2*g*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**
3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n**2*x**2*(f + g*x)**n/(g**3*n**3
 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n*x**2*(f + g*x)**n/(g**3
*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*g**3*n**2*x**3*(f + g*x)**n
/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 3*c*e*g**3*n*x**3*(f + g*
x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e*g**3*x**3*(f +
 g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3), True))
```

Giac [B] time = 1.10491, size = 504, normalized size = 6.

$$\frac{(gx+f)^n cg^3n^2x^3e + 2(gx+f)^n cdg^3n^2x^2 + (gx+f)^n cfg^2n^2x^2e + 3(gx+f)^n cg^3nx^3e + 2(gx+f)^n cdfg^2n^2x + 8(gx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^3*n^2*x^3*e + 2*(g*x + f)^n*c*d*g^3*n^2*x^2 + (g*x + f)^n*
c*f*g^2*n^2*x^2*e + 3*(g*x + f)^n*c*g^3*n*x^3*e + 2*(g*x + f)^n*c*d*f*g^2*n
^2*x + 8*(g*x + f)^n*c*d*g^3*n*x^2 + (g*x + f)^n*c*f*g^2*n*x^2*e + 2*(g*x +
 f)^n*c*g^3*x^3*e + 6*(g*x + f)^n*c*d*f*g^2*n*x + (g*x + f)^n*a*g^3*n^2*x +
 6*(g*x + f)^n*c*d*g^3*x^2 - 2*(g*x + f)^n*c*f^2*g*n*x*e - 2*(g*x + f)^n*c*
d*f^2*g*n + (g*x + f)^n*a*f*g^2*n^2 + 5*(g*x + f)^n*a*g^3*n*x - 6*(g*x + f)
^n*c*d*f^2*g + 5*(g*x + f)^n*a*f*g^2*n + 6*(g*x + f)^n*a*g^3*x + 2*(g*x + f
)^n*c*f^3*e + 6*(g*x + f)^n*a*f*g^2)/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^
3)
```

3.809 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx$

Optimal. Leaf size=114

$$\frac{(cd^2-ae)(f+gx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)}-\frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)}+\frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

```
[Out] -((c*(e*f - d*g)*(f + g*x)^(1 + n))/(e*g^2*(1 + n))) + (c*(f + g*x)^(2 + n)
)/(g^2*(2 + n)) + ((c*d^2 - a*e)*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)*(1 + n))
```

Rubi [A] time = 0.15287, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {951, 80, 68}

$$\frac{(cd^2-ae)(f+gx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)}-\frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)}+\frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

```
[Out] -((c*(e*f - d*g)*(f + g*x)^(1 + n))/(e*g^2*(1 + n))) + (c*(f + g*x)^(2 + n)
)/(g^2*(2 + n)) + ((c*d^2 - a*e)*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)*(1 + n))
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(f+gx)^n \left(a+2cdx+cex^2\right)}{d+ex}\,dx = \frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{\int \frac{(f+gx)^n(-eg(cdf-ag)(2+n)-ceg(ef-dg)(2+n)x)}{d+ex}\,dx}{eg^2(2+n)}$$
$$= -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)} + \frac{c(f+gx)^{2+n}}{g^2(2+n)} - \frac{\left(cd^2-ae\right)\int \frac{(f+gx)^n}{d+ex}\,dx}{e}$$
$$= -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)} + \frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{\left(cd^2-ae\right)(f+gx)^{1+n}\,{}_2F_1\left(1,1+n;2+n\right.}{e(ef-dg)(1+n)}$$

Mathematica [A] time = 0.157708, size = 93, normalized size = 0.82

$$\frac{(f+gx)^{n+1}\left(\frac{\left(cd^2-ae\right){}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{ef-dg}+\frac{c(dg(n+2)-ef+eg(n+1)x)}{g^2(n+2)}\right)}{e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

```
[Out] ((f + g*x)^(1 + n)*((c*(-(e*f) + d*g*(2 + n) + e*g*(1 + n)*x))/(g^2*(2 + n)
) + ((c*d^2 - a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f -
d*g)])/(e*f - d*g)))/(e*(1 + n))
```

Maple [F] time = 0.669, size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^n \left(ce x^2+2\,cdx+a\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ce x^2+2\,cdx+a\right)\left(gx+f\right)^n}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ce x^2+2\,cdx+a\right)\left(gx+f\right)^n}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2+2\,cdx+a)(gx+f)^n}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d), x)
```

3.810 $\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=88

$$\frac{c(f+gx)^{n+1}}{eg(n+1)}-\frac{g\left(cd^2-ae\right)(f+gx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

```
[Out] (c*(f + g*x)^(1 + n))/(e*g*(1 + n)) - ((c*d^2 - a*e)*g*(f + g*x)^(1 + n)*Hy
pergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)
^2*(1 + n))
```

Rubi [A] time = 0.0830026, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {947, 68}

$$\frac{c(f+gx)^{n+1}}{eg(n+1)}-\frac{g\left(cd^2-ae\right)(f+gx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

```
[Out] (c*(f + g*x)^(1 + n))/(e*g*(1 + n)) - ((c*d^2 - a*e)*g*(f + g*x)^(1 + n)*Hy
pergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)
^2*(1 + n))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^2}\,dx &= \int \left(\frac{c(f+gx)^n}{e}+\frac{\left(-cd^2+ae\right)(f+gx)^n}{e(d+ex)^2}\right)dx\\ &= \frac{c(f+gx)^{1+n}}{eg(1+n)}+\frac{\left(-cd^2+ae\right)\int \frac{(f+gx)^n}{(d+ex)^2}\,dx}{e}\\ &= \frac{c(f+gx)^{1+n}}{eg(1+n)}-\frac{\left(cd^2-ae\right)g(f+gx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)^2(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0926034, size = 83, normalized size = 0.94

$$\frac{(f+gx)^{n+1}\left(g^2\left(ae-cd^2\right){}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\right)}{eg(n+1)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

```
[Out] ((f + g*x)^(1 + n)*(c*(e*f - d*g)^2 + (-(c*d^2) + a*e)*g^2*Hypergeometric2F
1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*g*(e*f - d*g)^2*(1 + n))
```

Maple [F] time = 0.692, size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^n\left(cex^2+2\,cdx+a\right)}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f+gx\right)^n\left(a+2cdx+cex^2\right)}{\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**2,x)
```

```
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cex^2 + 2\,cdx + a\right)\left(gx + f\right)^n}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)
```

3.811 $\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=193

$$\frac{(f+gx)^{n+1}\left(aeg^2(1-n)n-c\left(d^2g^2\left(-n^2+n+2\right)-4defg+2e^2f^2\right)\right)\,_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{2e(n+1)(ef-dg)^3}-\frac{g(1-n)\left(cd^2-ae\right.}{2e(d+ex)(e}$$

```
[Out] -((a - (c*d^2)/e)*(f + g*x)^(1 + n))/(2*(e*f - d*g)*(d + e*x)^2) - ((c*d^2
- a*e)*g*(1 - n)*(f + g*x)^(1 + n))/(2*e*(e*f - d*g)^2*(d + e*x)) + ((a*e*g
^2*(1 - n)*n - c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2*(2 + n - n^2)))*(f + g*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(2*
e*(e*f - d*g)^3*(1 + n))
```

Rubi [A] time = 0.224053, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {949, 78, 68}

$$\frac{(f+gx)^{n+1}\left(aeg^2(1-n)n-c\left(d^2g^2\left(-n^2+n+2\right)-4defg+2e^2f^2\right)\right)\,_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{2e(n+1)(ef-dg)^3}-\frac{g(1-n)\left(cd^2-ae\right.}{2e(d+ex)(e}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```



```
[Out] -((a - (c*d^2)/e)*(f + g*x)^(1 + n))/(2*(e*f - d*g)*(d + e*x)^2) - ((c*d^2
- a*e)*g*(1 - n)*(f + g*x)^(1 + n))/(2*e*(e*f - d*g)^2*(d + e*x)) + ((a*e*g
^2*(1 - n)*n - c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2*(2 + n - n^2)))*(f + g*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(2*
e*(e*f - d*g)^3*(1 + n))
```


Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2} - \frac{\int \frac{(f+gx)^n\left(ag(1-n)-\frac{cd(2ef-dg(1+n))}{e}-2c(ef-dg)x\right)}{(d+ex)^2}dx}{2(ef-dg)}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2} - \frac{\left(cd^2-ae\right)g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)} - \frac{\left(aeg^2(1-n)n-c\left(2e^2f^2\right.\right.}{}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2} - \frac{\left(cd^2-ae\right)g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)} + \frac{\left(aeg^2(1-n)n-c\left(2e^2f^2\right.\right.}{}$$

Mathematica [A] time = 0.102173, size = 106, normalized size = 0.55

$$-\frac{(f+gx)^{n+1}\left(g^2\left(ae-cd^2\right)\,{}_2F_1\left(3,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)\right)}{e(n+1)(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

```
[Out] -(((f + g*x)^(1 + n)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + n, 2 + n, (e
*(f + g*x))/(e*f - d*g)] + (-(c*d^2) + a*e)*g^2*Hypergeometric2F1[3, 1 + n,
 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*(e*f - d*g)^3*(1 + n)))
```

Maple [F] time = 0.723, size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^n\left(cex^2+2\,cdx+a\right)}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2
*e*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(f+gx\right)^n\left(a+2cdx+cex^2\right)}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**3,x)
```

```
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^3, x)
```

3.812 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^4}\,dx$

Optimal. Leaf size=197

$$\frac{g(f+gx)^{n+1}\left(aeg^2\left(n^2-3n+2\right)+c\left(d^2g^2\left(-n^2+3n+4\right)-12defg+6e^2f^2\right)\right)\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{6e(n+1)(ef-dg)^4}-\frac{g(2-n)\left(c\right.}{6e(d+}$$

```
[Out] -((a - (c*d^2)/e)*(f + g*x)^(1 + n))/(3*(e*f - d*g)*(d + e*x)^3) - ((c*d^2
- a*e)*g*(2 - n)*(f + g*x)^(1 + n))/(6*e*(e*f - d*g)^2*(d + e*x)^2) + (g*(a
*e*g^2*(2 - 3*n + n^2) + c*(6*e^2*f^2 - 12*d*e*f*g + d^2*g^2*(4 + 3*n - n^2
)))*(f + g*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f
 - d*g)])/(6*e*(e*f - d*g)^4*(1 + n))
```

Rubi [A] time = 0.233455, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {949, 78, 68}

$$\frac{g(f+gx)^{n+1}\left(aeg^2\left(n^2-3n+2\right)+c\left(d^2g^2\left(-n^2+3n+4\right)-12defg+6e^2f^2\right)\right)\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{6e(n+1)(ef-dg)^4}-\frac{g(2-n)\left(c\right.}{6e(d+}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

```
[Out] -((a - (c*d^2)/e)*(f + g*x)^(1 + n))/(3*(e*f - d*g)*(d + e*x)^3) - ((c*d^2
- a*e)*g*(2 - n)*(f + g*x)^(1 + n))/(6*e*(e*f - d*g)^2*(d + e*x)^2) + (g*(a
*e*g^2*(2 - 3*n + n^2) + c*(6*e^2*f^2 - 12*d*e*f*g + d^2*g^2*(4 + 3*n - n^2
)))*(f + g*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f
 - d*g)])/(6*e*(e*f - d*g)^4*(1 + n))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3}-\frac{\int \frac{(f+gx)^n\left(ag(2-n)-\frac{cd(3ef-dg(1+n))}{e}-3c(ef-dg)x\right)}{(d+ex)^3}\,dx}{3(ef-dg)}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3}-\frac{\left(cd^2-ae\right)g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2}+\frac{\left(aeg^2\left(2-3n+n^2\right)+\right.}{}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3}-\frac{\left(cd^2-ae\right)g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2}+\frac{g\left(aeg^2\left(2-3n+n^2\right)\right.}{}$$

Mathematica [A] time = 0.111262, size = 106, normalized size = 0.54

$$\frac{g(f+gx)^{n+1}\left(g^2\left(ae-cd^2\right){}_2F_1\left(4,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)\right)}{e(n+1)(ef-dg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

```
[Out] (g*(f + g*x)^(1 + n)*(c*(e*f - d*g)^2*Hypergeometric2F1[2, 1 + n, 2 + n, (e
*(f + g*x))/(e*f - d*g)] + (-(c*d^2) + a*e)*g^2*Hypergeometric2F1[4, 1 + n,
 2 + n, (e*(f + g*x))/(e*f - d*g)]))/(e*(e*f - d*g)^4*(1 + n))
```

Maple [F] time = 0.767, size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^n\left(cex^2+2\,cdx+a\right)}{\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
```

```
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cex^2+2\,cdx+a\right)\left(gx+f\right)^n}{\left(ex+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cex^2+2\,cdx+a)(gx+f)^n}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2
*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**4,x)
```

```
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(cex^2+2\,cdx+a)(gx+f)^n}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^4, x)
```

3.813 $\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right)\,dx$

Optimal. Leaf size=231

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}(g(m+n+2)(aeg(m+n+3)-cd(dg(n+1)+ef(m+2)))+c(m+2)(ef-dg)(dg(\cdots}{e^2g^2(m+1)(m+n+2)(m+n+3)}$$

```
[Out] -((c*(e*f - d*g)*(2 + m)*(d + e*x)^(1 + m)*(f + g*x)^(1 + n))/(e*g^2*(2 + m
 + n)*(3 + m + n))) + (c*(d + e*x)^(2 + m)*(f + g*x)^(1 + n))/(e*g*(3 + m +
 n)) + ((c*(e*f - d*g)*(2 + m)*(e*f*(1 + m) + d*g*(1 + n)) + g*(2 + m + n)*
(a*e*g*(3 + m + n) - c*d*(e*f*(2 + m) + d*g*(1 + n))))*(d + e*x)^(1 + m)*(f
 + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g))]
)/(e^2*g^2*(1 + m)*(2 + m + n)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n)
```

Rubi [A] time = 0.261532, antiderivative size = 227, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {951, 80, 70, 69}

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(aeg(m+n+3)+\frac{c(m+2)(ef-dg)(dg(n+1)+ef(m+1))}{g(m+n+2)}-cd(dg(n+1)+ef(m+2))\right){}_2F_1\left(m\cdots\right.}{e^2g(m+1)(m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -((c*(e*f - d*g)*(2 + m)*(d + e*x)^(1 + m)*(f + g*x)^(1 + n))/(e*g^2*(2 + m
 + n)*(3 + m + n))) + (c*(d + e*x)^(2 + m)*(f + g*x)^(1 + n))/(e*g*(3 + m +
 n)) + ((a*e*g*(3 + m + n) + (c*(e*f - d*g)*(2 + m)*(e*f*(1 + m) + d*g*(1 +
 n)))/(g*(2 + m + n)) - c*d*(e*f*(2 + m) + d*g*(1 + n)))*(d + e*x)^(1 + m)*
(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g)
)])/(e^2*g*(1 + m)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n)
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
```

```
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right)dx=\frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)}+\frac{\int (d+ex)^m(f+gx)^n\left(e(aeg(3+m+n)-cd(ef\right.}{e^2g(3-}$$

$$=-\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)}+\frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)}+$$

$$=-\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)}+\frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)}+$$

$$=-\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)}+\frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)}+$$

Mathematica [A] time = 0.17647, size = 179, normalized size = 0.77

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(e\left(ag^2+cf(ef-2dg)\right){}_2F_1\left(m+1,-n;m+2;\frac{g(d+ex)}{dg-ef}\right)+c(ef-dg)^2{}_2F_1\left(m+1,-n-2;\right.\right.}{e^2g^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -2
 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] - 2*c*(e*f - d*g)^2*Hypergeometr
ic2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(a*g^2 + c*f*
(e*f - 2*d*g))*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d + e*x))/(-(e*f) +
d*g)]))/(e^2*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

Maple [F] time = 0.683, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(gx+f\right)^n\left(ce x^2+2\,cdx+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cex^2 + 2\,cdx + a\right)(ex + d)^m \left(gx + f\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cex^2 + 2\,cdx + a\right)(ex + d)^m \left(gx + f\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cex^2 + 2\,cdx + a\right)(ex + d)^m \left(gx + f\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

3.814 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)} dx$

Optimal. Leaf size=83

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

```
[Out] (c*x)/(e*g) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) - ((
c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Rubi [A] time = 0.100025, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {893}

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) - ((
c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx &= \int \left(\frac{c}{eg}+\frac{cd^2-bde+ae^2}{e(ef-dg)(d+ex)}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)(f+gx)}\right)dx\\ &= \frac{cx}{eg}+\frac{\left(cd^2-bde+ae^2\right)\log(d+ex)}{e^2(ef-dg)}-\frac{\left(cf^2-bfg+ag^2\right)\log(f+gx)}{g^2(ef-dg)}\end{aligned}$$

Mathematica [A] time = 0.0538467, size = 85, normalized size = 1.02

$$-\frac{\log(d+ex)\left(-ae^2+bde-cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) - ((-(c*d^2) + b*d*e - a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) -
 ((c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Maple [A] time = 0.051, size = 142, normalized size = 1.7

$$\frac{cx}{eg} - \frac{\ln(ex+d)\,a}{dg-ef} + \frac{\ln(ex+d)\,bd}{(dg-ef)\,e} - \frac{\ln(ex+d)\,cd^2}{(dg-ef)\,e^2} + \frac{\ln(gx+f)\,a}{dg-ef} - \frac{\ln(gx+f)\,bf}{(dg-ef)\,g} + \frac{\ln(gx+f)\,cf^2}{g^2\,(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x)
```

```
[Out] c*x/e/g-1/(d*g-e*f)*ln(e*x+d)*a+1/(d*g-e*f)/e*ln(e*x+d)*b*d-1/(d*g-e*f)/e^2
*ln(e*x+d)*c*d^2+1/(d*g-e*f)*ln(g*x+f)*a-1/g/(d*g-e*f)*ln(g*x+f)*b*f+1/g^2/
(d*g-e*f)*ln(g*x+f)*c*f^2
```

Maxima [A] time = 1.01263, size = 117, normalized size = 1.41

$$\frac{(cd^2 - bde + ae^2)\log(ex+d)}{e^3f - de^2g} - \frac{(cf^2 - bfg + ag^2)\log(gx+f)}{efg^2 - dg^3} + \frac{cx}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*log(e*x + d)/(e^3*f - d*e^2*g) - (c*f^2 - b*f*g + a
*g^2)*log(g*x + f)/(e*f*g^2 - d*g^3) + c*x/(e*g)
```

Fricas [A] time = 1.30229, size = 198, normalized size = 2.39

$$\frac{(cd^2 - bde + ae^2)g^2\log(ex+d) + (ce^2fg - cdeg^2)x - (ce^2f^2 - be^2fg + ae^2g^2)\log(gx+f)}{e^3fg^2 - de^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*g^2*log(e*x + d) + (c*e^2*f*g - c*d*e*g^2)*x - (c*
e^2*f^2 - b*e^2*f*g + a*e^2*g^2)*log(g*x + f))/(e^3*f*g^2 - d*e^2*g^3)
```

Sympy [B] time = 11.337, size = 420, normalized size = 5.06

$$\frac{cx}{eg} + \frac{(ag^2 - bfg + cf^2)\log\left(x + \frac{adeg^2 + ae^2fg - 2bdefg + cd^2fg + cdef^2 - \frac{d^2eg(ag^2 - bfg + cf^2)}{dg-ef} + \frac{2de^2f(ag^2 - bfg + cf^2)}{dg-ef} - \frac{e^3f^2(ag^2 - bfg + cf^2)}{g(dg-ef)}}{2ae^2g^2 - bdeg^2 - be^2fg + cd^2g^2 + ce^2f^2}\right)}{g^2(dg-ef)} - \frac{(ae^2 - b \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f),x)
```

```
[Out] c*x/(e*g) + (a*g**2 - b*f*g + c*f**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*
b*d*e*f*g + c*d**2*f*g + c*d*e*f**2 - d**2*e*g*(a*g**2 - b*f*g + c*f**2)/(d
*g - e*f) + 2*d*e**2*f*(a*g**2 - b*f*g + c*f**2)/(d*g - e*f) - e**3*f**2*(a
*g**2 - b*f*g + c*f**2)/(g*(d*g - e*f)))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e*
*2*f*g + c*d**2*g**2 + c*e**2*f**2))/(g**2*(d*g - e*f)) - (a*e**2 - b*d*e +
 c*d**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*b*d*e*f*g + c*d**2*f*g + c*d*
e*f**2 + d**2*g**3*(a*e**2 - b*d*e + c*d**2)/(e*(d*g - e*f)) - 2*d*f*g**2*(
a*e**2 - b*d*e + c*d**2)/(d*g - e*f) + e*f**2*g*(a*e**2 - b*d*e + c*d**2)/(
d*g - e*f))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e**2*f*g + c*d**2*g**2 + c*e**2
*f**2))/(e**2*(d*g - e*f))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.815 $\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)} \, dx$

Optimal. Leaf size=184

$$\frac{x\left(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{e^3g^3}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^2}{e^4(ef-dg)}-\frac{\log(f+gx)\left(ag^2 \right.}{g^4(ef\ldots}$$

```
[Out] ((b^2*e^2*g^2 - 2*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g +
d^2*g^2))*x)/(e^3*g^3) - (c*(c*e*f + c*d*g - 2*b*e*g)*x^2)/(2*e^2*g^2) + (c
^2*x^3)/(3*e*g) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/(e^4*(e*f - d*g)
) - ((c*f^2 - b*f*g + a*g^2)^2*Log[f + g*x])/(g^4*(e*f - d*g))
```

Rubi [A] time = 0.312182, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {893}

$$\frac{x\left(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{e^3g^3}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^2}{e^4(ef-dg)}-\frac{\log(f+gx)\left(ag^2 \right.}{g^4(ef\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((b^2*e^2*g^2 - 2*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g +
d^2*g^2))*x)/(e^3*g^3) - (c*(c*e*f + c*d*g - 2*b*e*g)*x^2)/(2*e^2*g^2) + (c
^2*x^3)/(3*e*g) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/(e^4*(e*f - d*g)
) - ((c*f^2 - b*f*g + a*g^2)^2*Log[f + g*x])/(g^4*(e*f - d*g))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)}\,dx &= \int \left(\frac{b^2e^2g^2-2ceg(bef+bdg-aeg)+c^2\left(e^2f^2+defg+d^2g^2\right)}{e^3g^3}-\frac{c(cef+cdg-2beg)x}{e^2g^2}+\frac{c^2x \ldots}{eg \ldots}\right.\\ &= \frac{\left(b^2e^2g^2-2ceg(bef+bdg-aeg)+c^2\left(e^2f^2+defg+d^2g^2\right)\right)x}{e^3g^3}-\frac{c(cef+cdg-2beg)x^2}{2e^2g^2}+\frac{c^2 \ldots}{3\ldots}\end{aligned}$$

Mathematica [A] time = 0.160047, size = 177, normalized size = 0.96

$$-\frac{egx(dg-ef)\left(6ceg(2aeg+b(-2dg-2ef+egx))+6b^2e^2g^2+c^2\left(6d^2g^2-3deg(gx-2f)+e^2\left(6f^2-3fgx+2g^2x^2\right)\right)\right.\ldots}{6e^4g^4(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] -(e*g*(-(e*f) + d*g)*x*(6*b^2*e^2*g^2 + 6*c*e*g*(2*a*e*g + b*(-2*e*f - 2*d*
g + e*g*x)) + c^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x
+ 2*g^2*x^2))) - 6*(c*d^2 + e*(-(b*d) + a*e))^2*g^4*Log[d + e*x] + 6*e^4*(c
*f^2 + g*(-(b*f) + a*g))^2*Log[f + g*x])/(6*e^4*g^4*(e*f - d*g))
```

Maple [B] time = 0.056, size = 444, normalized size = 2.4

$$\frac{c^2x^3}{3\,eg}+\frac{bcx^2}{eg}-\frac{c^2x^2d}{2\,e^2g}-\frac{c^2x^2f}{2\,eg^2}+2\,\frac{acx}{eg}+\frac{b^2x}{eg}-2\,\frac{bcdx}{e^2g}-2\,\frac{bcfx}{eg^2}+\frac{c^2d^2x}{e^3g}+\frac{c^2dfx}{e^2g^2}+\frac{c^2f^2x}{eg^3}-\frac{\ln(ex+d)\,a^2}{dg-ef}+2\,\frac{\ln(ex+}{(dg-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x)
```

```
[Out] 1/3*c^2*x^3/e/g+1/e/g*x^2*b*c-1/2/e^2/g*x^2*c^2*d-1/2/e/g^2*x^2*c^2*f+2/e/g
*a*c*x+1/e/g*b^2*x-2/e^2/g*b*c*d*x-2/e/g^2*b*c*f*x+1/e^3/g*c^2*d^2*x+1/e^2/
g^2*c^2*d*f*x+1/e/g^3*c^2*f^2*x-1/(d*g-e*f)*ln(e*x+d)*a^2+2/e/(d*g-e*f)*ln(
e*x+d)*a*b*d-2/e^2/(d*g-e*f)*ln(e*x+d)*a*c*d^2-1/e^2/(d*g-e*f)*ln(e*x+d)*b^
2*d^2+2/e^3/(d*g-e*f)*ln(e*x+d)*d^3*b*c-1/e^4/(d*g-e*f)*ln(e*x+d)*c^2*d^4+1
/(d*g-e*f)*ln(g*x+f)*a^2-2/g/(d*g-e*f)*ln(g*x+f)*a*b*f+2/g^2/(d*g-e*f)*ln(g
*x+f)*a*c*f^2+1/g^2/(d*g-e*f)*ln(g*x+f)*b^2*f^2-2/g^3/(d*g-e*f)*ln(g*x+f)*b
*c*f^3+1/g^4/(d*g-e*f)*ln(g*x+f)*c^2*f^4
```

Maxima [A] time = 0.973157, size = 344, normalized size = 1.87

$$\frac{\left(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+\left(b^2+2\,ac\right)d^2e^2\right)\log(ex+d)}{e^5f-de^4g}-\frac{\left(c^2f^4-2\,bcf^3g-2\,abfg^3+a^2g^4+\left(b^2+2\,ac\right)f^2g^2\right)\mathrm{lo}}{efg^4-dg^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log
(e*x + d)/(e^5*f - d*e^4*g) - (c^2*f^4 - 2*b*c*f^3*g - 2*a*b*f*g^3 + a^2*g^
4 + (b^2 + 2*a*c)*f^2*g^2)*log(g*x + f)/(e*f*g^4 - d*g^5) + 1/6*(2*c^2*e^2*
g^2*x^3 - 3*(c^2*e^2*f*g + (c^2*d*e - 2*b*c*e^2)*g^2)*x^2 + 6*(c^2*e^2*f^2
+ (c^2*d*e - 2*b*c*e^2)*f*g + (c^2*d^2 - 2*b*c*d*e + (b^2 + 2*a*c)*e^2)*g^2
)*x)/(e^3*g^3)
```

Fricas [A] time = 2.99518, size = 639, normalized size = 3.47

$$\frac{6\left(c^2d^4-2\,bcd^3e-2\,abde^3+a^2e^4+\left(b^2+2\,ac\right)d^2e^2\right)g^4\log(ex+d)+2\left(c^2e^4fg^3-c^2de^3g^4\right)x^3-3\left(c^2e^4f^2g^2-2\,bce^4fg^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e
^2)*g^4*log(e*x + d) + 2*(c^2*e^4*f*g^3 - c^2*d*e^3*g^4)*x^3 - 3*(c^2*e^4*f
^2*g^2 - 2*b*c*e^4*f*g^3 - (c^2*d^2*e^2 - 2*b*c*d*e^3)*g^4)*x^2 + 6*(c^2*e^
4*f^3*g - 2*b*c*e^4*f^2*g^2 + (b^2 + 2*a*c)*e^4*f*g^3 - (c^2*d^3*e - 2*b*c*
d^2*e^2 + (b^2 + 2*a*c)*d*e^3)*g^4)*x - 6*(c^2*e^4*f^4 - 2*b*c*e^4*f^3*g -
2*a*b*e^4*f*g^3 + a^2*e^4*g^4 + (b^2 + 2*a*c)*e^4*f^2*g^2)*log(g*x + f))/(e
^5*f*g^4 - d*e^4*g^5)
```

Sympy [B] time = 98.6109, size = 989, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)/(g*x+f),x)
```

```
[Out] c**2*x**3/(3*e*g) + (a*g**2 - b*f*g + c*f**2)**2*log(x + (a**2*d*e**3*g**4
+ a**2*e**4*f*g**3 - 4*a*b*d*e**3*f*g**3 + 2*a*c*d**2*e**2*f*g**3 + 2*a*c*d
*e**3*f**2*g**2 + b**2*d**2*e**2*f*g**3 + b**2*d*e**3*f**2*g**2 - 2*b*c*d**
3*e*f*g**3 - 2*b*c*d*e**3*f**3*g + c**2*d**4*f*g**3 + c**2*d*e**3*f**4 - d*
*2*e**3*g*(a*g**2 - b*f*g + c*f**2)**2/(d*g - e*f) + 2*d*e**4*f*(a*g**2 - b
*f*g + c*f**2)**2/(d*g - e*f) - e**5*f**2*(a*g**2 - b*f*g + c*f**2)**2/(g*(
d*g - e*f)))/(2*a**2*e**4*g**4 - 2*a*b*d*e**3*g**4 - 2*a*b*e**4*f*g**3 + 2*
a*c*d**2*e**2*g**4 + 2*a*c*e**4*f**2*g**2 + b**2*d**2*e**2*g**4 + b**2*e**4
*f**2*g**2 - 2*b*c*d**3*e*g**4 - 2*b*c*e**4*f**3*g + c**2*d**4*g**4 + c**2*
e**4*f**4))/(g**4*(d*g - e*f)) + x**2*(2*b*c*e*g - c**2*d*g - c**2*e*f)/(2*
e**2*g**2) + x*(2*a*c*e**2*g**2 + b**2*e**2*g**2 - 2*b*c*d*e*g**2 - 2*b*c*e
**2*f*g + c**2*d**2*g**2 + c**2*d*e*f*g + c**2*e**2*f**2)/(e**3*g**3) - (a*
e**2 - b*d*e + c*d**2)**2*log(x + (a**2*d*e**3*g**4 + a**2*e**4*f*g**3 - 4*
a*b*d*e**3*f*g**3 + 2*a*c*d**2*e**2*f*g**3 + 2*a*c*d*e**3*f**2*g**2 + b**2*
d**2*e**2*f*g**3 + b**2*d*e**3*f**2*g**2 - 2*b*c*d**3*e*f*g**3 - 2*b*c*d*e*
*3*f**3*g + c**2*d**4*f*g**3 + c**2*d*e**3*f**4 + d**2*g**5*(a*e**2 - b*d*e
 + c*d**2)**2/(e*(d*g - e*f)) - 2*d*f*g**4*(a*e**2 - b*d*e + c*d**2)**2/(d*
g - e*f) + e*f**2*g**3*(a*e**2 - b*d*e + c*d**2)**2/(d*g - e*f))/(2*a**2*e*
*4*g**4 - 2*a*b*d*e**3*g**4 - 2*a*b*e**4*f*g**3 + 2*a*c*d**2*e**2*g**4 + 2*
a*c*e**4*f**2*g**2 + b**2*d**2*e**2*g**4 + b**2*e**4*f**2*g**2 - 2*b*c*d**3
*e*g**4 - 2*b*c*e**4*f**3*g + c**2*d**4*g**4 + c**2*e**4*f**4))/(e**4*(d*g
- e*f))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.816 $\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)} \, dx$

Optimal. Leaf size=531

$$-\frac{x\left(-3ce^2g^2\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)+b^2e^3g^3(-3aeg+bdg+bef)-3c^2eg\left(aeg\left(d^2g^2+def\right.\right.\right.}{e^5g^5}$$

```
[Out] -(((b^2*e^3*g^3*(b*e*f + b*d*g - 3*a*e*g) - c^3*(e^4*f^4 + d*e^3*f^3*g + d^
2*e^2*f^2*g^2 + d^3*e*f*g^3 + d^4*g^4) - 3*c*e^2*g^2*(a^2*e^2*g^2 - 2*a*b*e
*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) - 3*c^2*e*g*(a*e*g*(e^2
*f^2 + d*e*f*g + d^2*g^2) - b*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^
3)))*x)/(e^5*g^5)) + ((b^3*e^3*g^3 - 3*b*c*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g
) - c^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3) - 3*c^2*e*g*(a*e*g*
(e*f + d*g) - b*(e^2*f^2 + d*e*f*g + d^2*g^2)))*x^2)/(2*e^4*g^4) + (c*(3*b^
2*e^2*g^2 - 3*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g + d^2*
g^2))*x^3)/(3*e^3*g^3) - (c^2*(c*e*f + c*d*g - 3*b*e*g)*x^4)/(4*e^2*g^2) +
(c^3*x^5)/(5*e*g) + ((c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/(e^6*(e*f - d*
g)) - ((c*f^2 - b*f*g + a*g^2)^3*Log[f + g*x])/(g^6*(e*f - d*g))
```

Rubi [A] time = 0.988291, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {893}

$$-\frac{x\left(-3ce^2g^2\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)+b^2e^3g^3(-3aeg+bdg+bef)-3c^2eg\left(aeg\left(d^2g^2+def\right.\right.\right.}{e^5g^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```


```
[Out] -(((b^2*e^3*g^3*(b*e*f + b*d*g - 3*a*e*g) - c^3*(e^4*f^4 + d*e^3*f^3*g + d^
2*e^2*f^2*g^2 + d^3*e*f*g^3 + d^4*g^4) - 3*c*e^2*g^2*(a^2*e^2*g^2 - 2*a*b*e
*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) - 3*c^2*e*g*(a*e*g*(e^2
*f^2 + d*e*f*g + d^2*g^2) - b*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^
3)))*x)/(e^5*g^5)) + ((b^3*e^3*g^3 - 3*b*c*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g
) - c^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3) - 3*c^2*e*g*(a*e*g*
(e*f + d*g) - b*(e^2*f^2 + d*e*f*g + d^2*g^2)))*x^2)/(2*e^4*g^4) + (c*(3*b^
2*e^2*g^2 - 3*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g + d^2*
g^2))*x^3)/(3*e^3*g^3) - (c^2*(c*e*f + c*d*g - 3*b*e*g)*x^4)/(4*e^2*g^2) +
(c^3*x^5)/(5*e*g) + ((c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/(e^6*(e*f - d*
g)) - ((c*f^2 - b*f*g + a*g^2)^3*Log[f + g*x])/(g^6*(e*f - d*g))
```


Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{(d+ex)(f+gx)}\,dx = \int \left(\frac{-b^2e^3g^3(bef+bdg-3aeg)+c^3\left(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4\right)+3ce^2g^2\left(\right.}{} \right.$$
$$= -\frac{\left(b^2e^3g^3(bef+bdg-3aeg)-c^3\left(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4\right)-3ce^2g^2\left(a^2e^2\right.\right.}{}$$

Mathematica [A] time = 0.465518, size = 476, normalized size = 0.9

$$-\frac{egx\left(-30ce^2g^2(ef-dg)\left(6a^2e^2g^2+6abeg(-2dg-2ef+egx)+b^2\left(6d^2g^2-3deg(gx-2f)+e^2\left(6f^2-3fgx+2g^2x^2\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -(e*g*x*(-30*b^2*e^3*g^3*(e*f - d*g)*(6*a*e*g + b*(-2*e*f - 2*d*g + e*g*x))
 + c^3*(60*d^5*g^5 - 30*d^4*e*g^5*x + 20*d^3*e^2*g^5*x^2 - 15*d^2*e^3*g^5*x
^3 + 12*d*e^4*g^5*x^4 + e^5*f*(-60*f^4 + 30*f^3*g*x - 20*f^2*g^2*x^2 + 15*f
*g^3*x^3 - 12*g^4*x^4)) - 30*c*e^2*g^2*(e*f - d*g)*(6*a^2*e^2*g^2 + 6*a*b*e
*g*(-2*e*f - 2*d*g + e*g*x) + b^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(
6*f^2 - 3*f*g*x + 2*g^2*x^2))) + 15*c^2*e*g*(-2*a*e*g*(e*f - d*g)*(6*d^2*g^
2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)) + b*(-12*d^4*
g^4 + 6*d^3*e*g^4*x - 4*d^2*e^2*g^4*x^2 + 3*d*e^3*g^4*x^3 + e^4*f*(12*f^3 -
 6*f^2*g*x + 4*f*g^2*x^2 - 3*g^3*x^3)))) - 60*(c*d^2 + e*(-(b*d) + a*e))^3*
g^6*Log[d + e*x] + 60*e^6*(c*f^2 + g*(-(b*f) + a*g))^3*Log[f + g*x])/(60*e^
6*g^6*(e*f - d*g))
```

Maple [B] time = 0.068, size = 1232, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x)
```

```
[Out] -1/e^6/(d*g-e*f)*ln(e*x+d)*c^3*d^6+1/e^5/g*c^3*d^4*x+3/4/e/g*x^4*b*c^2-1/4/
e^2/g*x^4*c^3*d-1/4/e/g^2*x^4*c^3*f+1/3/e^3/g*x^3*c^3*d^2+1/3/e/g^3*x^3*c^3
*f^2-1/2/e^4/g*x^2*c^3*d^3+1/e/g*x^3*a*c^2+1/e/g*x^3*b^2*c+1/e/g^5*c^3*f^4*
x-1/e^2/g*b^3*d*x+1/e^3/(d*g-e*f)*ln(e*x+d)*b^3*d^3-1/(d*g-e*f)*ln(e*x+d)*a
^3+1/(d*g-e*f)*ln(g*x+f)*a^3-1/g^3/(d*g-e*f)*ln(g*x+f)*b^3*f^3+1/g^6/(d*g-e
*f)*ln(g*x+f)*c^3*f^6+1/2/e/g*x^2*b^3-1/e/g^2*b^3*f*x-1/2/e/g^4*x^2*c^3*f^3
+3/e/g*a^2*c*x+3/e/g*a*b^2*x+3/2/e^2/g^2*x^2*b*c^2*d*f-6/e^2/g*a*b*c*d*x-6/
e/g^2*a*b*c*f*x+3/e^2/g^2*a*c^2*d*f*x+3/e^2/g^2*b^2*c*d*f*x-3/e^3/g^2*b*c^2
*d^2*f*x-3/e^2/g^3*b*c^2*d*f^2*x+6/e^3/(d*g-e*f)*ln(e*x+d)*a*b*c*d^3-6/g^3/
(d*g-e*f)*ln(g*x+f)*a*b*c*f^3+1/e^2/g^4*c^3*d*f^3*x-1/e^2/g*x^3*b*c^2*d-1/e
/g^2*x^3*b*c^2*f+1/3/e^2/g^2*x^3*c^3*d*f+3/e/g*x^2*a*b*c-3/2/e^2/g*x^2*a*c^
2*d-3/2/e/g^2*x^2*a*c^2*f-3/2/e^2/g*x^2*b^2*c*d+3/e/(d*g-e*f)*ln(e*x+d)*a^2
*b*d-3/e^2/(d*g-e*f)*ln(e*x+d)*a^2*c*d^2-3/e^2/(d*g-e*f)*ln(e*x+d)*a*b^2*d^
2-3/e^4/(d*g-e*f)*ln(e*x+d)*a*c^2*d^4-3/e^4/(d*g-e*f)*ln(e*x+d)*b^2*c*d^4+1
/5*c^3*x^5/e/g+3/e^5/(d*g-e*f)*ln(e*x+d)*b*c^2*d^5-3/g/(d*g-e*f)*ln(g*x+f)*
a^2*b*f+3/g^2/(d*g-e*f)*ln(g*x+f)*a^2*c*f^2+3/g^2/(d*g-e*f)*ln(g*x+f)*a*b^2
*f^2+3/g^4/(d*g-e*f)*ln(g*x+f)*a*c^2*f^4+3/g^4/(d*g-e*f)*ln(g*x+f)*b^2*c*f^
```

```
4-3/g^5/(d*g-e*f)*ln(g*x+f)*b*c^2*f^5-3/2/e/g^2*x^2*b^2*c*f+3/2/e^3/g*x^2*b
*c^2*d^2+3/2/e/g^3*x^2*b*c^2*f^2-1/2/e^3/g^2*x^2*c^3*d^2*f-1/2/e^2/g^3*x^2*
c^3*d*f^2+3/e^3/g*a*c^2*d^2*x+3/e/g^3*a*c^2*f^2*x+3/e^3/g*b^2*c*d^2*x+3/e/g
^3*b^2*c*f^2*x-3/e^4/g*b*c^2*d^3*x-3/e/g^4*b*c^2*f^3*x+1/e^4/g^2*c^3*d^3*f*
x+1/e^3/g^3*c^3*d^2*f^2*x
```

Maxima [A] time = 1.03556, size = 973, normalized size = 1.83

$$\frac{\left(c^3d^6 - 3\,bc^2d^5e - 3\,a^2bde^5 + a^3e^6 + 3\left(b^2c + ac^2\right)d^4e^2 - \left(b^3 + 6\,abc\right)d^3e^3 + 3\left(ab^2 + a^2c\right)d^2e^4\right)\log\left(ex + d\right)}{e^7f - de^6g} - \frac{\left(c^3f^6 - 3\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*
e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(e*x + d)/(e^
7*f - d*e^6*g) - (c^3*f^6 - 3*b*c^2*f^5*g - 3*a^2*b*f*g^5 + a^3*g^6 + 3*(b^
2*c + a*c^2)*f^4*g^2 - (b^3 + 6*a*b*c)*f^3*g^3 + 3*(a*b^2 + a^2*c)*f^2*g^4)
*log(g*x + f)/(e*f*g^6 - d*g^7) + 1/60*(12*c^3*e^4*g^4*x^5 - 15*(c^3*e^4*f*
g^3 + (c^3*d*e^3 - 3*b*c^2*e^4)*g^4)*x^4 + 20*(c^3*e^4*f^2*g^2 + (c^3*d*e^3
 - 3*b*c^2*e^4)*f*g^3 + (c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^
4)*g^4)*x^3 - 30*(c^3*e^4*f^3*g + (c^3*d*e^3 - 3*b*c^2*e^4)*f^2*g^2 + (c^3*
d^2*e^2 - 3*b*c^2*d*e^3 + 3*(b^2*c + a*c^2)*e^4)*f*g^3 + (c^3*d^3*e - 3*b*c
^2*d^2*e^2 + 3*(b^2*c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*g^4)*x^2 + 60*(
c^3*e^4*f^4 + (c^3*d*e^3 - 3*b*c^2*e^4)*f^3*g + (c^3*d^2*e^2 - 3*b*c^2*d*e^
3 + 3*(b^2*c + a*c^2)*e^4)*f^2*g^2 + (c^3*d^3*e - 3*b*c^2*d^2*e^2 + 3*(b^2*
c + a*c^2)*d*e^3 - (b^3 + 6*a*b*c)*e^4)*f*g^3 + (c^3*d^4 - 3*b*c^2*d^3*e +
3*(b^2*c + a*c^2)*d^2*e^2 - (b^3 + 6*a*b*c)*d*e^3 + 3*(a*b^2 + a^2*c)*e^4)*
g^4)*x)/(e^5*g^5)
```

Fricas [A] time = 20.5178, size = 1465, normalized size = 2.76

$$\frac{60\left(c^3d^6 - 3\,bc^2d^5e - 3\,a^2bde^5 + a^3e^6 + 3\left(b^2c + ac^2\right)d^4e^2 - \left(b^3 + 6\,abc\right)d^3e^3 + 3\left(ab^2 + a^2c\right)d^2e^4\right)g^6\log\left(ex + d\right) + 12\left(c^3e^{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/60*(60*(c^3*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*
c^2)*d^4*e^2 - (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*g^6*log
(e*x + d) + 12*(c^3*e^6*f*g^5 - c^3*d*e^5*g^6)*x^5 - 15*(c^3*e^6*f^2*g^4 -
3*b*c^2*e^6*f*g^5 - (c^3*d^2*e^4 - 3*b*c^2*d*e^5)*g^6)*x^4 + 20*(c^3*e^6*f^
3*g^3 - 3*b*c^2*e^6*f^2*g^4 + 3*(b^2*c + a*c^2)*e^6*f*g^5 - (c^3*d^3*e^3 -
3*b*c^2*d^2*e^4 + 3*(b^2*c + a*c^2)*d*e^5)*g^6)*x^3 - 30*(c^3*e^6*f^4*g^2 -
 3*b*c^2*e^6*f^3*g^3 + 3*(b^2*c + a*c^2)*e^6*f^2*g^4 - (b^3 + 6*a*b*c)*e^6*
f*g^5 - (c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c + a*c^2)*d^2*e^4 - (b^3 +
 6*a*b*c)*d*e^5)*g^6)*x^2 + 60*(c^3*e^6*f^5*g - 3*b*c^2*e^6*f^4*g^2 + 3*(b^
2*c + a*c^2)*e^6*f^3*g^3 - (b^3 + 6*a*b*c)*e^6*f^2*g^4 + 3*(a*b^2 + a^2*c)*
e^6*f*g^5 - (c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c + a*c^2)*d^3*e^3 - (b^3
 + 6*a*b*c)*d^2*e^4 + 3*(a*b^2 + a^2*c)*d*e^5)*g^6)*x - 60*(c^3*e^6*f^6 - 3
*b*c^2*e^6*f^5*g - 3*a^2*b*e^6*f*g^5 + a^3*e^6*g^6 + 3*(b^2*c + a*c^2)*e^6*
f^4*g^2 - (b^3 + 6*a*b*c)*e^6*f^3*g^3 + 3*(a*b^2 + a^2*c)*e^6*f^2*g^4)*log(
```

```
g*x + f))/(e^7*f*g^6 - d*e^6*g^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.817 $\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)}\,dx$

Optimal. Leaf size=246

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)}{2\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2\log(d+e}{(ef-dg)\left(ae^2-b\right.}$$

```
[Out] -(((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g)))) + (e^2*Log[d + e*x])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)) -
(g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) - ((c*e*f + c*d*g
- b*e*g)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f -
 a*g)))
```

Rubi [A] time = 0.467849, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {893, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)}{2\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2\log(d+e}{(ef-dg)\left(ae^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g)))) + (e^2*Log[d + e*x])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)) -
(g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) - ((c*e*f + c*d*g
- b*e*g)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f -
 a*g)))
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)}dx = \int\left(-\frac{e^3}{\left(cd^2-bde+ae^2\right)(-ef+dg)(d+ex)} - \frac{g^3}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+\right.$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)} + \frac{\int \frac{c^2df+b^2eg-c(bef-}{}}{\left(cd^2-bde+a\right.}$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)} + \frac{(-cef-cdg-}{2\left(cd^2-bde+a\right.}$$

$$= \frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)} - \frac{(cef+cdg-be}{2\left(cd^2-bde+a\right.}$$

$$= -\frac{\left(2c^2df+b^2eg-c(bef+bdg+2aeg)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)} + \frac{e^2\log(d+}{\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.377274, size = 246, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)\left(g(ag-bf)+cf^2\right)} + \frac{e^2\log(d+ex)}{(ef-dg)\left(e(ae-bd)+cd^2\right)} - \frac{\log(a+x(b+cx))(-beg+c}{2\left(e(ae-bd)+cd^2\right)\left(g(ag-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] ((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(c*f^2 + g
*(-(b*f) + a*g))) + (e^2*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))*(e*f - d
*g)) - (g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) - ((c*e*
f + c*d*g - b*e*g)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))*(c*f
^2 + g*(-(b*f) + a*g)))
```

Maple [B] time = 0.166, size = 606, normalized size = 2.5

$$\frac{\ln\left(cx^2+bx+a\right)beg}{\left(2ae^2-2bde+2cd^2\right)\left(ag^2-bfg+cf^2\right)} - \frac{c\ln\left(cx^2+bx+a\right)dg}{\left(2ae^2-2bde+2cd^2\right)\left(ag^2-bfg+cf^2\right)} - \frac{c\ln\left(cx^2+bx+a\right)e}{\left(2ae^2-2bde+2cd^2\right)\left(ag^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)*ln(c*x^2+b*x+a)*b*e*g-1/2/(a*e^
2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)*c*ln(c*x^2+b*x+a)*d*g-1/2/(a*e^2-b*d*e+c
*d^2)/(a*g^2-b*f*g+c*f^2)*c*ln(c*x^2+b*x+a)*e*f-2/(a*e^2-b*d*e+c*d^2)/(a*g^
2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c*e*
g+1/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*b^2*e*g-1/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*g-1/(a*e^2-b*d*e+
c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b*c*e*f+2/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d*f-e^2/(d*g-e*f)/(a*e^2-b*d*e+c*d^2
)*ln(e*x+d)+g^2/(a*g^2-b*f*g+c*f^2)/(d*g-e*f)*ln(g*x+f)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1308, size = 529, normalized size = 2.15

$$\frac{g^3\log\left(|gx+f|\right)}{cdf^2g^2 - bdfg^3 + adg^4 - cf^3ge + bf^2g^2e - afg^3e} - \frac{(cdg + cfe - bge)\log\left(cx^2 + bx + a\right)}{2\,(c^2d^2f^2 - bcd^2fg + acd^2g^2 - bcdf^2e + b^2dfge - abdg^2e + acf^2e^2 -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] g^3*log(abs(g*x + f))/(c*d*f^2*g^2 - b*d*f*g^3 + a*d*g^4 - c*f^3*g*e + b*f^
2*g^2*e - a*f*g^3*e) - 1/2*(c*d*g + c*f*e - b*g*e)*log(c*x^2 + b*x + a)/(c^
2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b^2*d*f*g*e - a*b*d*g
^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2) - e^3*log(abs(x*e + d))/(c*
d^3*g*e - c*d^2*f*e^2 - b*d^2*g*e^2 + b*d*f*e^3 + a*d*g*e^3 - a*f*e^4) + (2
*c^2*d*f - b*c*d*g - b*c*f*e + b^2*g*e - 2*a*c*g*e)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((c^2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b
^2*d*f*g*e - a*b*d*g^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2)*sqrt(-b
^2 + 4*a*c))
```

3.818 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=644

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg\left(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2\right)+b^2e^2g^2(-2aeg+bdg+bef)-c^2\left(4ade^2fg^2-b\left(5d^2efg\right.\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag)\right)^2}$$

```
[Out] -((b^3*e*g - b^2*c*(e*f + d*g) + 2*a*c^2*(e*f + d*g) + b*c*(c*d*f - 3*a*e*g
) + c*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x)/((b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a*g))*(a + b*x + c*x^2))) + (2*c
*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g))) + ((b^2*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*c^3*d*f*(e^2*f^2
 + d*e*f*g + d^2*g^2) + 2*c*e*g*(a^2*e^2*g^2 + a*b*e*g*(e*f + d*g) - b^2*(e
*f + d*g)^2) - c^2*(4*a*d*e^2*f*g^2 - b*(e^3*f^3 + 5*d*e^2*f^2*g + 5*d^2*e*
f*g^2 + d^3*g^3)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2) + (e^4*Log[d + e*x]
)/((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)
*(c*f^2 - b*f*g + a*g^2)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^
2) + e*g*(2*a*e*g - b*(e*f + d*g)))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e
 + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2)
```

Rubi [A] time = 2.05232, antiderivative size = 644, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {893, 638, 618, 206, 634, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg\left(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2\right)+b^2e^2g^2(-2aeg+bdg+bef)-c^2\left(4ade^2fg^2-b\left(5d^2efg\right.\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```


```
[Out] -((b^3*e*g - b^2*c*(e*f + d*g) + 2*a*c^2*(e*f + d*g) + b*c*(c*d*f - 3*a*e*g
) + c*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x)/((b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a*g))*(a + b*x + c*x^2))) + (2*c
*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g))) + ((b^2*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*c^3*d*f*(e^2*f^2
 + d*e*f*g + d^2*g^2) + 2*c*e*g*(a^2*e^2*g^2 + a*b*e*g*(e*f + d*g) - b^2*(e
*f + d*g)^2) - c^2*(4*a*d*e^2*f*g^2 - b*(e^3*f^3 + 5*d*e^2*f^2*g + 5*d^2*e*
f*g^2 + d^3*g^3)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2) + (e^4*Log[d + e*x]
)/((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)
*(c*f^2 - b*f*g + a*g^2)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^
2) + e*g*(2*a*e*g - b*(e*f + d*g)))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e
 + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2)
```


Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \int\left(-\frac{e^5}{\left(cd^2-bde+ae^2\right)^2(-ef+dg)(d+ex)} - \frac{g^5}{(ef-dg)\left(cf^2-bfg+ag^2\right)^2(\right.$$

$$= \frac{e^4\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2(ef-dg)} - \frac{g^4\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)^2} + \frac{\int \frac{-b^2e^2g^2(bef+}{}}{}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2eg-\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+bx+\right.}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2eg-\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+bx+\right.}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c\left(2c^2df+b^2eg-\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+bx+\right.}$$

Mathematica [A] time = 3.23609, size = 710, normalized size = 1.1

$$\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-2c^3\left(2a^2eg\left(d^2g^2-5defg+e^2f^2\right)+ab\left(11d^2efg^2+3d^3g^3+11de^2f^2g+3e^3f^3\right)-4b^2d^2ef^2g\right)+c^2\left(-6a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*(f -
 g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x)))/((b^2 - 4*a*c)*(-(c*d^
2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*(a + x*(b + c*x))) + ((4*c^5
*d^3*f^3 + b^4*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*b^2*c*e*g*(-6*a^2*e^2*
g^2 + 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) + 2*c^4*d*
f*(-3*b*d*f*(e*f + d*g) + 2*a*(3*e^2*f^2 + d*e*f*g + 3*d^2*g^2)) + c^2*(-12
*a^3*e^3*g^3 - 6*a^2*b*e^2*g^2*(e*f + d*g) + 12*a*b^2*e*g*(e^2*f^2 + d*e*f*
g + d^2*g^2) + b^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3)) - 2*c^3
*(-4*b^2*d^2*e*f^2*g + 2*a^2*e*g*(e^2*f^2 - 5*d*e*f*g + d^2*g^2) + a*b*(3*e
^3*f^3 + 11*d*e^2*f^2*g + 11*d^2*e*f*g^2 + 3*d^3*g^3)))*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2*(c*
f^2 + g*(-(b*f) + a*g))^2) + (e^4*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))
^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g)
)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(
e*f + d*g)))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^2*(c*f^2 +
 g*(-(b*f) + a*g))^2)
```

Maple [B] time = 0.256, size = 9103, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.819 $\int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=287

$$-\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg+2bef\right.}{5g^6}$$

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*
g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^(3/2))
/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2
- 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6) - (2*e*(e*g*(4*b*e*f - 3*b
*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/2))/(
7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6)
```

Rubi [A] time = 0.495004, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {897, 1153}

$$-\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg+2bef\right.}{5g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```


```
[Out] (-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*
g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^(3/2))
/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2
- 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6) - (2*e*(e*g*(4*b*e*f - 3*b
*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/2))/(
7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6)
```


Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3 (a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3 \left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(-ef+dg)^3(cf^2-bfg+ag^2)}{g^5}+\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))x^2}{g^5}+\frac{(ef-dg)(3eg(2\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{2(ef-dg)^3 (cf^2-bfg+ag^2)\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-ba\cdots}{3g^6}$$

Mathematica [A] time = 0.433423, size = 249, normalized size = 0.87

$$2\sqrt{f+gx}\left(-495e(f+gx)^3\left(c\left(-3d^2g^2+12defg-10e^2f^2\right)-eg(aeg+3bdg-4bef)\right)+693(f+gx)^2(ef-dg)\left(-3eg(a\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-3465*(e*f - d*g)^3*(c*f^2 + g*(-(b*f) + a*g)) + 1155*(e*
f - d*g)^2*(c*f*(5*e*f - 2*d*g) + g*(-4*b*e*f + b*d*g + 3*a*e*g))*(f + g*x)
 + 693*(e*f - d*g)*(-3*e*g*(-2*b*e*f + b*d*g + a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^2 - 495*e*(-(e*g*(-4*b*e*f + 3*b*d*g + a*e*g))
 + c*(-10*e^2*f^2 + 12*d*e*f*g - 3*d^2*g^2))*(f + g*x)^3 - 385*e^2*(5*c*e*f
 - 3*c*d*g - b*e*g)*(f + g*x)^4 + 315*c*e^3*(f + g*x)^5))/(3465*g^6)
```

Maple [B] time = 0.053, size = 540, normalized size = 1.9

$$630\,e^3cx^5g^5+770\,be^3g^5x^4+2310\,cde^2g^5x^4-700\,ce^3fg^4x^4+990\,ae^3g^5x^3+2970\,bde^2g^5x^3-880\,be^3fg^4x^3+2970\,cd^2\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3465*(g*x+f)^(1/2)*(315*c*e^3*g^5*x^5+385*b*e^3*g^5*x^4+1155*c*d*e^2*g^5*
x^4-350*c*e^3*f*g^4*x^4+495*a*e^3*g^5*x^3+1485*b*d*e^2*g^5*x^3-440*b*e^3*f*
g^4*x^3+1485*c*d^2*e*g^5*x^3-1320*c*d*e^2*f*g^4*x^3+400*c*e^3*f^2*g^3*x^3+2
079*a*d*e^2*g^5*x^2-594*a*e^3*f*g^4*x^2+2079*b*d^2*e*g^5*x^2-1782*b*d*e^2*f
*g^4*x^2+528*b*e^3*f^2*g^3*x^2+693*c*d^3*g^5*x^2-1782*c*d^2*e*f*g^4*x^2+158
4*c*d*e^2*f^2*g^3*x^2-480*c*e^3*f^3*g^2*x^2+3465*a*d^2*e*g^5*x-2772*a*d*e^2
*f*g^4*x+792*a*e^3*f^2*g^3*x+1155*b*d^3*g^5*x-2772*b*d^2*e*f*g^4*x+2376*b*d
*e^2*f^2*g^3*x-704*b*e^3*f^3*g^2*x-924*c*d^3*f*g^4*x+2376*c*d^2*e*f^2*g^3*x
-2112*c*d*e^2*f^3*g^2*x+640*c*e^3*f^4*g*x+3465*a*d^3*g^5-6930*a*d^2*e*f*g^4
+5544*a*d*e^2*f^2*g^3-1584*a*e^3*f^3*g^2-2310*b*d^3*f*g^4+5544*b*d^2*e*f^2*
g^3-4752*b*d*e^2*f^3*g^2+1408*b*e^3*f^4*g+1848*c*d^3*f^2*g^3-4752*c*d^2*e*f
^3*g^2+4224*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6
```

Maxima [A] time = 0.997175, size = 579, normalized size = 2.02

$$2\left(315\left(gx+f\right)^{\frac{11}{2}}ce^3-385\left(5\,ce^3f-\left(3\,cde^2+be^3\right)g\right)\left(gx+f\right)^{\frac{9}{2}}+495\left(10\,ce^3f^2-4\left(3\,cde^2+be^3\right)fg+\left(3\,cd^2e+3\,bde\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g
)*(g*x + f)^(9/2) + 495*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^
2*e + 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(7/2) - 693*(10*c*e^3*f^3 - 6*(3*c*
d*e^2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3
*b*d^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 4*(3*c*d*e
^2 + b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*b*d^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*(g*x + f)^(3/2) - 3
465*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e + 3*b*d
*e^2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*d^3 +
3*a*d^2*e)*f*g^4)*sqrt(g*x + f))/g^6
```

Fricas [A] time = 2.13294, size = 971, normalized size = 3.38

$$\frac{2\left(315\,ce^3g^5x^5 - 1280\,ce^3f^5 + 3465\,ad^3g^5 + 1408\left(3\,cde^2 + be^3\right)f^4g - 1584\left(3\,cd^2e + 3\,bde^2 + ae^3\right)f^3g^2 + 1848\left(cd^3 + 3\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 3465*a*d^3*g^5 + 1408*(3*c*d*e
^2 + b*e^3)*f^4*g - 1584*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 + 1848*(c*
d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 - 2310*(b*d^3 + 3*a*d^2*e)*f*g^4 - 35*
(10*c*e^3*f*g^4 - 11*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + 5*(80*c*e^3*f^2*g^3 - 8
8*(3*c*d*e^2 + b*e^3)*f*g^4 + 99*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 -
 3*(160*c*e^3*f^3*g^2 - 176*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 198*(3*c*d^2*e +
3*b*d*e^2 + a*e^3)*f*g^4 - 231*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (
640*c*e^3*f^4*g - 704*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 792*(3*c*d^2*e + 3*b*d*
e^2 + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 1155*(b*
d^3 + 3*a*d^2*e)*g^5)*x)*sqrt(g*x + f)/g^6
```

Sympy [A] time = 169.264, size = 1544, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d**3*f/sqrt(f + g*x) + 2*a*d**3*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) + 6*a*d**2*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 6*a*d**2*e*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 6*a*d*e**2
*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 6*a
*d*e**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
(f + g*x)**(5/2)/5)/g**2 + 2*a*e**3*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f
+ g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 + 2*a*e**3*(f**4/sqr
t(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)
**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 + 2*b*d**3*f*(-f/sqrt(f + g*x) - sqrt(
f + g*x))/g + 2*b*d**3*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)*
*(3/2)/3)/g + 6*b*d**2*e*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g
```

```
*x)**(3/2)/3)/g**2 + 6*b*d**2*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x)
 + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 6*b*d*e**2*f*(-f**3/sqrt
(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)
/g**3 + 6*b*d*e**2*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f +
 g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 + 2*b*e**3
*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4
*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 + 2*b*e**3*(-f**5/sqrt(f +
 g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x
)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 + 2*c*d**3*f*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*d**
3*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g
*x)**(5/2)/5)/g**2 + 6*c*d**2*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*
x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 + 6*c*d**2*e*(f**4/sqrt(
f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**
(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 + 6*c*d*e**2*f*(f**4/sqrt(f + g*x) + 4*f
**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f +
 g*x)**(7/2)/7)/g**4 + 6*c*d*e**2*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*
x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**
(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 + 2*c*e**3*f*(-f**5/sqrt(f + g*x) - 5*f*
*4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5
*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**5 + 2*c*e**3*(f**6/sqrt(f +
g*x) + 6*f**5*sqrt(f + g*x) - 5*f**4*(f + g*x)**(3/2) + 4*f**3*(f + g*x)**(
5/2) - 15*f**2*(f + g*x)**(7/2)/7 + 2*f*(f + g*x)**(9/2)/3 - (f + g*x)**(11
/2)/11)/g**5)/g, Ne(g, 0)), ((a*d**3*x + c*e**3*x**6/6 + x**5*(b*e**3 + 3*c
*d*e**2)/5 + x**4*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 +
 3*b*d**2*e + c*d**3)/3 + x**2*(3*a*d**2*e + b*d**3)/2)/sqrt(f), True))
```

Giac [B] time = 1.14787, size = 763, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*b*d^3/g + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d^2*e/g + 231*(3*
(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^3/g^2 +
693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d^2
*e/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x +
 f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 297*(5*(g*x + f)^(7/2)
- 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*d
*e^2/g^3 + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2
)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x
 + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqr
t(g*x + f)*f^4)*c*d*e^2/g^4 + 11*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*
f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f
^4)*b*e^3/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x +
 f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*s
qrt(g*x + f)*f^5)*c*e^3/g^5)/g
```

3.820 $\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=212

$$-\frac{2(f+gx)^{5/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5}-\frac{2(f+g}{}$$

```
[Out] (2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^5 - (2*(e*f - d*g
)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^(3/2))/(3
*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^
2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*
x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rubi [A] time = 0.339492, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {897, 1153}

$$-\frac{2(f+gx)^{5/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5}-\frac{2(f+g}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^5 - (2*(e*f - d*g
)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^(3/2))/(3
*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^
2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*
x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{(-ef+dg)^2\left(cf^2-bfg+ag^2\right)}{g^4}+\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))x^2}{g^4}+\frac{\left(-eg(3bef-2b\right.}{}\right.\right.}{g}$$

$$= \frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg}{3g^5}$$

Mathematica [A] time = 0.361759, size = 184, normalized size = 0.87

$$\frac{2\sqrt{f+gx}\left(-63(f+gx)^2\left(-eg(aeg+2bdg-3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)+315(ef-dg)^2\left(g(ag-bf)+cf^2\right)-\right.}{315}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(315*(e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g)) - 105*(e*f -
 d*g)*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*(f + g*x) - 63
*(-(e*g*(-3*b*e*f + 2*b*d*g + a*e*g)) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 45*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^3 + 35*c*e^2*(f
+ g*x)^4))/(315*g^5)
```

Maple [A] time = 0.059, size = 315, normalized size = 1.5

$$\frac{70\,ce^2x^4g^4+90\,be^2g^4x^3+180\,cdeg^4x^3-80\,ce^2fg^3x^3+126\,ae^2g^4x^2+252\,bdeg^4x^2-108\,be^2fg^3x^2+126\,cd^2g^4x^2-21}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/315*(g*x+f)^(1/2)*(35*c*e^2*g^4*x^4+45*b*e^2*g^4*x^3+90*c*d*e*g^4*x^3-40*
c*e^2*f*g^3*x^3+63*a*e^2*g^4*x^2+126*b*d*e*g^4*x^2-54*b*e^2*f*g^3*x^2+63*c*
d^2*g^4*x^2-108*c*d*e*f*g^3*x^2+48*c*e^2*f^2*g^2*x^2+210*a*d*e*g^4*x-84*a*e
^2*f*g^3*x+105*b*d^2*g^4*x-168*b*d*e*f*g^3*x+72*b*e^2*f^2*g^2*x-84*c*d^2*f*
g^3*x+144*c*d*e*f^2*g^2*x-64*c*e^2*f^3*g*x+315*a*d^2*g^4-420*a*d*e*f*g^3+16
8*a*e^2*f^2*g^2-210*b*d^2*f*g^3+336*b*d*e*f^2*g^2-144*b*e^2*f^3*g+168*c*d^2
*f^2*g^2-288*c*d*e*f^3*g+128*c*e^2*f^4)/g^5
```

Maxima [A] time = 0.982756, size = 352, normalized size = 1.66

$$\frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^2-45\left(4\,ce^2f-\left(2\,cde+be^2\right)g\right)(gx+f)^{\frac{7}{2}}+63\left(6\,ce^2f^2-3\left(2\,cde+be^2\right)fg+\left(cd^2+2\,bde+ae^2\right)g^2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 45*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*x
 + f)^(7/2) + 63*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
+ a*e^2)*g^2)*(g*x + f)^(5/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f^2*
g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*(g*x + f)^(3
/2) + 315*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2*b*d
*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)*sqrt(g*x + f))/g^5
```

Fricas [A] time = 1.76131, size = 586, normalized size = 2.76

$$\frac{2\left(35\,ce^2g^4x^4+128\,ce^2f^4+315\,ad^2g^4-144\left(2\,cde+be^2\right)f^3g+168\left(cd^2+2\,bde+ae^2\right)f^2g^2-210\left(bd^2+2\,ade\right)fg^3-5\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 + 315*a*d^2*g^4 - 144*(2*c*d*e + b*
e^2)*f^3*g + 168*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 - 210*(b*d^2 + 2*a*d*e)*
f*g^3 - 5*(8*c*e^2*f*g^3 - 9*(2*c*d*e + b*e^2)*g^4)*x^3 + 3*(16*c*e^2*f^2*g
^2 - 18*(2*c*d*e + b*e^2)*f*g^3 + 21*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 - (
64*c*e^2*f^3*g - 72*(2*c*d*e + b*e^2)*f^2*g^2 + 84*(c*d^2 + 2*b*d*e + a*e^2
)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/g^5
```

Sympy [A] time = 96.045, size = 1001, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d**2*f/sqrt(f + g*x) + 2*a*d**2*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) + 4*a*d*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 4*a*d*e*(f**2/s
qrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 2*a*e**2*f*(f**2
/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*a*e**2*(-
f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)*
*(5/2)/5)/g**2 + 2*b*d**2*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 2*b*d**2
*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 4*b*d*e*
f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 4*b*
d*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
 g*x)**(5/2)/5)/g**2 + 2*b*e**2*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*
x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 + 2*b*e**2*(f**4/sqrt(f
+ g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5
/2)/5 - (f + g*x)**(7/2)/7)/g**3 + 2*c*d**2*f*(f**2/sqrt(f + g*x) + 2*f*sqr
t(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*d**2*(-f**3/sqrt(f + g*x) - 3*f
**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 4*c*d*e
*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
g*x)**(5/2)/5)/g**3 + 4*c*d*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) -
2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3
 + 2*c*e**2*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)
**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 + 2*c*e**2*(-f*
*5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f*
*2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4)/g,
 Ne(g, 0)), ((a*d**2*x + c*e**2*x**5/5 + x**4*(b*e**2 + 2*c*d*e)/4 + x**3*(
```

```
a*e**2 + 2*b*d*e + c*d**2)/3 + x**2*(2*a*d*e + b*d**2)/2)/sqrt(f), True))
```

Giac [A] time = 1.12253, size = 490, normalized size = 2.31

$$2\left(315\sqrt{gx+f}ad^2 + \frac{105\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}f\right)bd^2}{g} + \frac{210\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}f\right)ade}{g} + \frac{21\left(3(gx+f)^{\frac{5}{2}} - 10(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}f^2\right)cd^2}{g^2} + \frac{42\left(3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 105*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
b*d^2/g + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d*e/g + 21*(3*(g*x +
f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^2/g^2 + 42*(3*(
g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d*e/g^2 + 2
1*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*e^2/g
^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2
- 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + 9*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/
2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*e^2/g^3 + (35*(g*x
+ f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f
)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

3.821 $\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=137

$$-\frac{2\sqrt{f+gx}(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4}+\frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4}-\frac{2(f+gx)^{5/2}(-beg-cd}{5g^4}$$

```
[Out] (-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(c*f*(3*e*f
 - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^(3/2))/(3*g^4) - (2*(3*c
*e*f - c*d*g - b*e*g)*(f + g*x)^(5/2))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7
*g^4)
```

Rubi [A] time = 0.109117, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {771}

$$-\frac{2\sqrt{f+gx}(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4}+\frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4}-\frac{2(f+gx)^{5/2}(-beg-cd}{5g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(c*f*(3*e*f
 - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^(3/2))/(3*g^4) - (2*(3*c
*e*f - c*d*g - b*e*g)*(f + g*x)^(5/2))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7
*g^4)
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx=\int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3\sqrt{f+gx}}+\frac{(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+gx}}{g^3}+(-$$

$$=-\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))(f+g}{3g^4}$$

Mathematica [A] time = 0.208884, size = 131, normalized size = 0.96

$$\frac{2\sqrt{f+gx}\left(7g\left(5ag(3dg-2ef+egx)+5bdg(gx-2f)+be\left(8f^2-4fgx+3g^2x^2\right)\right)+c\left(7dg\left(8f^2-4fgx+3g^2x^2\right)-3e(-\right.\right.}{105g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(7*g*(5*b*d*g*(-2*f + g*x) + 5*a*g*(-2*e*f + 3*d*g + e*g*x
) + b*e*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + c*(7*d*g*(8*f^2 - 4*f*g*x + 3*g^2*
x^2) - 3*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3))))/(105*g^4)
```

Maple [A] time = 0.05, size = 144, normalized size = 1.1

$$\frac{30\,ce x^3 g^3 + 42\,beg^3x^2 + 42\,cdg^3x^2 - 36\,cefg^2x^2 + 70\,aeg^3x + 70\,bdg^3x - 56\,befg^2x - 56\,cdfg^2x + 48\,cef^2gx + 210\,ad}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(15*c*e*g^3*x^3+21*b*e*g^3*x^2+21*c*d*g^3*x^2-18*c*e*f*
g^2*x^2+35*a*e*g^3*x+35*b*d*g^3*x-28*b*e*f*g^2*x-28*c*d*f*g^2*x+24*c*e*f^2*
g*x+105*a*d*g^3-70*a*e*f*g^2-70*b*d*f*g^2+56*b*e*f^2*g+56*c*d*f^2*g-48*c*e*
f^3)/g^4
```

Maxima [A] time = 0.967548, size = 174, normalized size = 1.27

$$\frac{2\left(15\left(gx+f\right)^{\frac{7}{2}}ce - 21\left(3\,cef - (cd+be)g\right)\left(gx+f\right)^{\frac{5}{2}} + 35\left(3\,cef^2 - 2\,(cd+be)fg + (bd+ae)g^2\right)\left(gx+f\right)^{\frac{3}{2}} - 105\left(cef\right.}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(5/2
) + 35*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*(g*x + f)^(3/2) -
105*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)*sqrt(g*x +
f))/g^4
```

Fricas [A] time = 1.7422, size = 297, normalized size = 2.17

$$\frac{2\left(15\,ceg^3x^3 - 48\,cef^3 + 105\,adg^3 + 56\,(cd+be)f^2g - 70\,(bd+ae)fg^2 - 3\left(6\,cefg^2 - 7\,(cd+be)g^3\right)x^2 + \left(24\,cef^2g - 2\right.\right.}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 105*a*d*g^3 + 56*(c*d + b*e)*f^2*g - 7
0*(b*d + a*e)*f*g^2 - 3*(6*c*e*f*g^2 - 7*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2
*g - 28*(c*d + b*e)*f*g^2 + 35*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/g^4
```

Sympy [A] time = 67.7122, size = 549, normalized size = 4.01

$$\begin{cases} -\frac{\frac{2adf}{\sqrt{f+gx}}+2ad\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)+\frac{2aef\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}+\frac{2ae\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}+\frac{2bdf\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}+\frac{2bd\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}+\frac{2bef\left(\frac{f^2}{\sqrt{f+gx}}+\cdots\right.}{}}{} \\ \frac{adx+\frac{cex^4}{4}+\frac{x^3(be+cd)}{3}+\frac{x^2(ae+bd)}{2}}{\sqrt{f}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*d*f/sqrt(f + g*x) + 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x
)) + 2*a*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 2*a*e*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 2*b*d*f*(-f/sqrt(f + g*x)
- sqrt(f + g*x))/g + 2*b*d*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g
*x)**(3/2)/3)/g + 2*b*e*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*
x)**(3/2)/3)/g**2 + 2*b*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(
f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 2*c*d*f*(f**2/sqrt(f + g*x) +
2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*d*(-f**3/sqrt(f + g*x) -
 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 2*c
*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f
+ g*x)**(5/2)/5)/g**3 + 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) -
2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3
)/g, Ne(g, 0)), ((a*d*x + c*e*x**4/4 + x**3*(b*e + c*d)/3 + x**2*(a*e + b*d
)/2)/sqrt(f), True))
```

Giac [A] time = 1.13976, size = 269, normalized size = 1.96

$$\frac{2\left(105\sqrt{gx+f}\,ad+\frac{35\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)bd}{g}+\frac{35\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ae}{g}+\frac{7\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)cd}{g^2}+\frac{7\left(3(gx+f)^{\frac{5}{2}}-10\left(\cdots\right.\right.}{}\right.}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b*d
/g + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e/g + 7*(3*(g*x + f)^(5/2)
- 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d/g^2 + 7*(3*(g*x + f)^(5/
2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*e/g^2 + 3*(5*(g*x + f)^
(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^
3)*c*e/g^3)/g
```

3.822 $\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{f+gx}\left(ag^2-bfg+cf^2\right)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

```
[Out] (2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)^
(3/2))/(3*g^3) + (2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rubi [A] time = 0.042033, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{2\sqrt{f+gx}\left(ag^2-bfg+cf^2\right)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)^
(3/2))/(3*g^3) + (2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx &= \int \left(\frac{cf^2-bfg+ag^2}{g^2\sqrt{f+gx}}+\frac{(-2cf+bg)\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx\\ &= \frac{2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^3}-\frac{2(2cf-bg)(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A] time = 0.0490043, size = 54, normalized size = 0.74

$$\frac{2\sqrt{f+gx}\left(5g(3ag-2bf+bgx)+c\left(8f^2-4fgx+3g^2x^2\right)\right)}{15g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(5*g*(-2*b*f + 3*a*g + b*g*x) + c*(8*f^2 - 4*f*g*x + 3*g^2
*x^2)))/(15*g^3)
```

Maple [A] time = 0.046, size = 53, normalized size = 0.7

$$\frac{6\,c x^{2} g^{2}+10\,b g^{2} x-8\,c f g x+30\,a g^{2}-20\,b f g+16\,c f^{2}}{15\,g^{3}}\sqrt{g x+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(3*c*g^2*x^2+5*b*g^2*x-4*c*f*g*x+15*a*g^2-10*b*f*g+8*c*f
^2)/g^3
```

Maxima [A] time = 0.956599, size = 104, normalized size = 1.42

$$\frac{2\left(15\sqrt{g x+f}\,a+\frac{5\left(\left(g x+f\right)^{\frac{3}{2}}-3\sqrt{g x+f}\,f\right)b}{g}+\frac{\left(3\left(g x+f\right)^{\frac{5}{2}}-10\left(g x+f\right)^{\frac{3}{2}}f+15\sqrt{g x+f}\,f^{2}\right)c}{g^{2}}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

Fricas [A] time = 1.71499, size = 127, normalized size = 1.74

$$\frac{2\left(3\,c g^{2} x^{2}+8\,c f^{2}-10\,b f g+15\,a g^{2}-\left(4\,c f g-5\,b g^{2}\right)x\right)\sqrt{g x+f}}{15\,g^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*g^2*x^2 + 8*c*f^2 - 10*b*f*g + 15*a*g^2 - (4*c*f*g - 5*b*g^2)*x)*
sqrt(g*x + f)/g^3
```

Sympy [A] time = 10.5038, size = 223, normalized size = 3.05

$$\begin{cases} - \frac{\frac{2af}{\sqrt{f+gx}}+2a\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)+\frac{2bf\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}+\frac{2b\left(\frac{f^{2}}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}+\frac{2cf\left(\frac{f^{2}}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^{2}}+\frac{2c\left(-\frac{f^{3}}{\sqrt{f+gx}}-3f^{2}\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx \cdots}{5}\cdots\right.}{g^{2}}\cdots}{g} \\ \frac{ax+\frac{bx^{2}}{2}+\frac{cx^{3}}{3}}{\sqrt{f}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*f/sqrt(f + g*x) + 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) +
 2*b*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 2*b*(f**2/sqrt(f + g*x) + 2*f
*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sq
rt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*
sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0))
, ((a*x + b*x**2/2 + c*x**3/3)/sqrt(f), True))
```

Giac [A] time = 1.16665, size = 104, normalized size = 1.42

$$\frac{2\left(15\sqrt{gx+f}a+\frac{5\left(\left(gx+f\right)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)b}{g}+\frac{\left(3\left(gx+f\right)^{\frac{5}{2}}-10\left(gx+f\right)^{\frac{3}{2}}f+15\sqrt{gx+f}f^{2}\right)c}{g^{2}}\right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

3.823 $\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=116

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2\sqrt{f+gx}(beg-c(dg+ef))}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

```
[Out] (2*(b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))
/(3*e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqr
t[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])
```

Rubi [A] time = 0.168351, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {897, 1153, 208}

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2\sqrt{f+gx}(beg-c(dg+ef))}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*(b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))
/(3*e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqr
t[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{beg-c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2-bde+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{e^2g}$$

$$= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2\left(cd^2-bde+ae^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A] time = 0.199735, size = 118, normalized size = 1.02

$$\frac{2\left(-\frac{g^2\left(cd^2-e(bd-ae)\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{\sqrt{f+gx}(beg-c(dg+ef))}{e^2}+\frac{c(f+gx)^{3/2}}{3e}\right)}{g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*(((b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/e^2 + (c*(f + g*x)^(3/2))/(3*e)
 - ((c*d^2 - e*(b*d - a*e))*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
d*g]])/(e^(5/2)*Sqrt[e*f - d*g])))/g^2
```

Maple [A] time = 0.217, size = 189, normalized size = 1.6

$$\frac{2c}{3eg^2}(gx+f)^{\frac{3}{2}}+2\frac{b\sqrt{gx+f}}{eg}-2\frac{\sqrt{gx+f}cd}{ge^2}-2\frac{\sqrt{gx+f}cf}{eg^2}+2\frac{a}{\sqrt{(dg-ef)e}}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)-2\frac{bd}{e\sqrt{(dg-ef)e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2+2/g/e*b*(g*x+f)^(1/2)-2/g/e^2*(g*x+f)^(1/2)*c*d-2
/g^2/e*(g*x+f)^(1/2)*c*f+2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g
-e*f)*e)^(1/2))*a-2/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)
*e)^(1/2))*b*d+2/e^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*
e)^(1/2))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85776, size = 714, normalized size = 6.16

$$\left[\frac{3\left(cd^2-bde+ae^2\right)\sqrt{e^2f-deg}g^2\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2\,ce^3f^2+\left(cde^2-3\,be^3\right)fg-3\left(cd^2e-bde^2\right)g^2-\left(\right.\right.}{3\left(e^4fg^2-de^3g^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(c*d^2 - b*d*e + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f
- d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 +
(c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)*g^2 - (c*e^3*f*g - c*d*e^2*
g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3), 2/3*(3*(c*d^2 - b*d*e + a*e
^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g
*x + e*f)) - (2*c*e^3*f^2 + (c*d*e^2 - 3*b*e^3)*f*g - 3*(c*d^2*e - b*d*e^2)
*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*g^2 - d*e^3*g^3)]
```

Sympy [A] time = 31.4552, size = 112, normalized size = 0.97

$$\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3eg^2}-\frac{2\left(ae^2-bde+cd^2\right)\operatorname{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{e^2\sqrt{\frac{e}{dg-ef}}\left(dg-ef\right)}+\frac{2\sqrt{f+gx}\left(beg-cdg-cef\right)}{e^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*e*g**2) - 2*(a*e**2 - b*d*e + c*d**2)*atan(1/(sqrt(
e/(d*g - e*f))*sqrt(f + g*x)))/(e**2*sqrt(e/(d*g - e*f))*(d*g - e*f)) + 2*s
qrt(f + g*x)*(b*e*g - c*d*g - c*e*f)/(e**2*g**2)
```

Giac [A] time = 1.12425, size = 173, normalized size = 1.49

$$\frac{2\left(cd^2-bde+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)e^{(-2)}}{\sqrt{dge-fe^2}}-\frac{2\left(3\sqrt{gx+f}cdg^5e-\left(gx+f\right)^{\frac{3}{2}}cg^4e^2+3\sqrt{gx+f}cfg^4e^2-3\sqrt{gx+f}bg^5e^2\right)}{3g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))*e^(-2
)/sqrt(d*g*e - f*e^2) - 2/3*(3*sqrt(g*x + f)*c*d*g^5*e - (g*x + f)^(3/2)*c*
```

```
g^4*e^2 + 3*sqrt(g*x + f)*c*f*g^4*e^2 - 3*sqrt(g*x + f)*b*g^5*e^2)*e^(-3)/g
^6
```

3.824 $\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e
*f - d*g)*(d + e*x)) + ((c*d*(4*e*f - 3*d*g) - e*(2*b*e*f - b*d*g - a*e*g))
*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/
2))
```

Rubi [A] time = 0.290021, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {897, 1157, 388, 208}

$$-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{(d+ex)(ef-dg)}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e
*f - d*g)*(d + e*x)) + ((c*d*(4*e*f - 3*d*g) - e*(2*b*e*f - b*d*g - a*e*g))
*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/
2))
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{\mathrm{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{2cf^2}{g^2}+\frac{2bf}{g}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} - \frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\,\mathrm{Subst}\left(\int \frac{\ldots}{\frac{-ef\ldots}{g}\ldots}\right.}{e^2g(ef-dg)}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{\ldots}}{\sqrt{ef\ldots}}\right.}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A] time = 0.647903, size = 150, normalized size = 1.07

$$\frac{\sqrt{f+gx}\left(eg(bd-ae)+c\left(-3d^2g+2de(f-gx)+2e^2fx\right)\right)}{e^2g(d+ex)(ef-dg)} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-aeg-bdg+2bef)+cd(3dg-4ef\ldots}{e^{5/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(e*(b*d - a*e)*g + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x)
)))/(e^2*g*(e*f - d*g)*(d + e*x)) - ((c*d*(-4*e*f + 3*d*g) + e*(2*b*e*f - b
*d*g - a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(
e*f - d*g)^(3/2))
```

Maple [B] time = 0.263, size = 371, normalized size = 2.7

$$2\frac{c\sqrt{gx+f}}{e^2g} + \frac{ag}{\left(dg-ef\right)\left(egx+dg\right)}\sqrt{gx+f} - \frac{bdg}{\left(dg-ef\right)e\left(egx+dg\right)}\sqrt{gx+f} + \frac{cd^2g}{e^2\left(dg-ef\right)\left(egx+dg\right)}\sqrt{gx+f} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e^2/g+g/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*a-g/e/(d*g-e*
f)*(g*x+f)^(1/2)/(e*g*x+d*g)*b*d+g/e^2/(d*g-e*f)*(g*x+f)^(1/2)/(e*g*x+d*g)*
c*d^2+g/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^
(1/2))*a+g/e/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f
)*e)^(1/2))*b*d-2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*
```

```
g-e*f)*e)^(1/2))*b*f-3*g/e^2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)
^(1/2)/((d*g-e*f)*e)^(1/2))*c*d^2+4/e/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(
e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*d*c*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88276, size = 1300, normalized size = 9.29

$$\left[-\frac{\sqrt{e^2f-deg}\left(2\left(2\,cd^2e-bde^2\right)fg-\left(3\,cd^3-bd^2e-ade^2\right)g^2+\left(2\left(2\,cde^2-be^3\right)fg-\left(3\,cd^2e-bde^2-ae^3\right)g^2\right)x\right)\log\left(\frac{egx+2}{}\right.}{2\left(de^5f^2g-2\,d^2e^4fg^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(e^2*f - d*e*g)*(2*(2*c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*
e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^
3)*g^2)*x)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/
(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 + a*e^4)*f*g + (3*c*
d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2
*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6
*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -(sqrt(-e^2*f + d*e*g)*(2*(2*c*d^
2*e - b*d*e^2)*f*g - (3*c*d^3 - b*d^2*e - a*d*e^2)*g^2 + (2*(2*c*d*e^2 - b*
e^3)*f*g - (3*c*d^2*e - b*d*e^2 - a*e^3)*g^2)*x)*arctan(sqrt(-e^2*f + d*e*g
)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*d*e^3*f^2 - (5*c*d^2*e^2 - b*d*e^3 +
a*e^4)*f*g + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e
^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 +
d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1144, size = 236, normalized size = 1.69

$$\frac{2\sqrt{gx+f}ce^{(-2)}}{g}-\frac{\left(3\,cd^2g-4\,cdfe-bdge+2\,bfe^2-age^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{\left(dge^2-fe^3\right)\sqrt{dge-fe^2}}+\frac{\sqrt{gx+f}cd^2g-\sqrt{gx+f}bdge+\sqrt{gx}}{\left(dge^2-fe^3\right)\left(dg+\left(gx+f\right)e-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(g*x + f)*c*e^(-2)/g - (3*c*d^2*g - 4*c*d*f*e - b*d*g*e + 2*b*f*e^2 -
 a*g*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d*g*e^2 - f*e^3)*sq
rt(d*g*e - f*e^2)) + (sqrt(g*x + f)*c*d^2*g - sqrt(g*x + f)*b*d*g*e + sqrt(
g*x + f)*a*g*e^2)/((d*g*e^2 - f*e^3)*(d*g + (g*x + f)*e - f*e))
```

3.825 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=206

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(eg(-3aeg-bdg+4bef)-c\left(3d^2g^2-8defg+8e^2f^2\right)\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}(cd(8ef-\ldots}{4e^2(\ldots}$$

```
[Out] -((a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(2*(e*f - d*g)*(d + e*x)^2) + ((
c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/(4*e^2*
(e*f - d*g)^2*(d + e*x)) + ((e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2*f^2
 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rubi [A] time = 0.386082, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {897, 1157, 385, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(eg(-3aeg-bdg+4bef)-c\left(3d^2g^2-8defg+8e^2f^2\right)\right)}{4e^{5/2}(ef-dg)^{5/2}}-\frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{2(d+ex)^2(ef-dg)}+\frac{\sqrt{f+gx}(cd(8ef-\ldots}{4e^2(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```


```
[Out] -((a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(2*(e*f - d*g)*(d + e*x)^2) + ((
c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/(4*e^2*
(e*f - d*g)^2*(d + e*x)) + ((e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2*f^2
 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(5/2)*(e*f - d*g)^(5/2))
```


Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\mathrm{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{4cf^2}{g^2}+\frac{4bf}{g}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{2(ef-dg)}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{\left(eg(4bef-b\right.}{}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{\left(eg(4bef-b\right.}{}$$

Mathematica [A] time = 0.725939, size = 297, normalized size = 1.44

$$\frac{-\frac{2\sqrt{e}\sqrt{f+gx}\left(e(ae-bd)+cd^2\right)}{(d+ex)^2(ef-dg)}-\frac{3g\left(e(ae-bd)+cd^2\right)\left(g(d+ex)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)-\sqrt{e}\sqrt{f+gx}\sqrt{ef-dg}\right)}{(d+ex)(ef-dg)^{5/2}}-\frac{4\sqrt{e}\sqrt{f+gx}(be-2cd)}{(d+ex)(ef-dg)}-\frac{4g(2cd-be)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{(ef-dg)^{3/2}}}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] ((-2*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f + g*x])/((e*f - d*g)*(d + e*
x)^2) - (4*Sqrt[e]*(-2*c*d + b*e)*Sqrt[f + g*x])/((e*f - d*g)*(d + e*x)) -
(4*(2*c*d - b*e)*g*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e*f -
 d*g)^(3/2) - (8*c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e
*f - d*g] - (3*(c*d^2 + e*(-(b*d) + a*e))*g*(-(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt
[f + g*x]) + g*(d + e*x)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]))
/((e*f - d*g)^(5/2)*(d + e*x)))/(4*e^(5/2))
```

Maple [B] time = 0.245, size = 538, normalized size = 2.6

$$2\,\frac{1}{\left(\left(gx+f\right)e+dg-ef\right)^2}\left(1/8\,\frac{g\left(3\,ae^2g+bdeg-4\,be^2f-5\,cd^2g+8\,decf\right)\left(gx+f\right)^{3/2}}{e\left(d^2g^2-2\,defg+e^2f^2\right)}+1/8\,\frac{\left(5\,ae^2g-bdeg-4\,be^2f-\right.}{e^2\left(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

```
[Out] 2*(1/8*g*(3*a*e^2*g+b*d*e*g-4*b*e^2*f-5*c*d^2*g+8*c*d*e*f)/e/(d^2*g^2-2*d*e
*f*g+e^2*f^2)*(g*x+f)^(3/2)+1/8*(5*a*e^2*g-b*d*e*g-4*b*e^2*f-3*c*d^2*g+8*c*
d*e*f)/e^2*g/(d*g-e*f)*(g*x+f)^(1/2))/((g*x+f)*e+d*g-e*f)^2+3/4/(d^2*g^2-2*
d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(
1/2))*a*g^2+1/4/(d^2*g^2-2*d*e*f*g+e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*arctan(e*
(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*b*d*g^2-1/(d^2*g^2-2*d*e*f*g+e^2*f^2)/((
d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*b*f*g+3/4/(d^
2*g^2-2*d*e*f*g+e^2*f^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d
*g-e*f)*e)^(1/2))*c*d^2*g^2-2/(d^2*g^2-2*d*e*f*g+e^2*f^2)/e/((d*g-e*f)*e)^(
1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d*f*g+2/(d^2*g^2-2*d*e*f
*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
*c*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78061, size = 2240, normalized size = 10.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d^4 + b*d^3*e
 + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g + (3*c*d^2*e
^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^2*e^2 + b*d*
e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*
log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d))
+ 2*(2*(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3 - 7*a
*d*e^4)*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*e^4 - b
*e^5)*f^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2 - b*d^2
*e^3 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3
*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 -
 d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*
e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 4*(2*c*d^3*e + b*d^2*e^2)*f*g + (3*c*d
^4 + b*d^3*e + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 4*(2*c*d*e^3 + b*e^4)*f*g
+ (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 4*(2*c*d^
2*e^2 + b*d*e^3)*f*g + (3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*g^2)*x)*sqrt(-e^
2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*
(3*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^2 - (9*c*d^3*e^2 - b*d^2*e^3 - 7*a*d*e^4)
*f*g + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*g^2 + (4*(2*c*d*e^4 - b*e^5)*f
^2 - (13*c*d^2*e^3 - 5*b*d*e^4 - 3*a*e^5)*f*g + (5*c*d^3*e^2 - b*d^2*e^3 -
3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^2*g + 3*d^4*e^
4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6*f*g^2 - d^3*e^
```

```
5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g^2 - d^4*e^4*g^3
)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18936, size = 504, normalized size = 2.45

$$\frac{\left(3\,cd^2g^2 - 8\,cdfge + bdg^2e + 8\,cf^2e^2 - 4\,bfge^2 + 3\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{4\left(d^2g^2e^2 - 2\,dfge^3 + f^2e^4\right)\sqrt{dge-fe^2}} - \frac{3\sqrt{gx+f}cd^3g^3 + 5\left(gx+f\right)^{\frac{3}{2}}cd^2g^2e -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*g^2 - 8*c*d*f*g*e + b*d*g^2*e + 8*c*f^2*e^2 - 4*b*f*g*e^2 + 3*
a*g^2*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e^2 - 2*d*
f*g*e^3 + f^2*e^4)*sqrt(d*g*e - f*e^2)) - 1/4*(3*sqrt(g*x + f)*c*d^3*g^3 +
5*(g*x + f)^(3/2)*c*d^2*g^2*e - 11*sqrt(g*x + f)*c*d^2*f*g^2*e + sqrt(g*x +
 f)*b*d^2*g^3*e - 8*(g*x + f)^(3/2)*c*d*f*g*e^2 + 8*sqrt(g*x + f)*c*d*f^2*g
*e^2 - (g*x + f)^(3/2)*b*d*g^2*e^2 + 3*sqrt(g*x + f)*b*d*f*g^2*e^2 - 5*sqrt
(g*x + f)*a*d*g^3*e^2 + 4*(g*x + f)^(3/2)*b*f*g*e^3 - 4*sqrt(g*x + f)*b*f^2
*g*e^3 - 3*(g*x + f)^(3/2)*a*g^2*e^3 + 5*sqrt(g*x + f)*a*f*g^2*e^3)/((d^2*g
^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

3.826 $\int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{2e(f+gx)^{5/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}+\frac{2(f+gx)^{3/2}(ef-dg)\left(3eg(-aeg-bdg+2bef\right.}{3g^6}$$

```
[Out] (2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d
*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/
g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g
- a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(5*g^6
) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*
(f + g*x)^(9/2))/(9*g^6)
```

Rubi [A] time = 0.405313, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {897, 1261}

$$-\frac{2e(f+gx)^{5/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}+\frac{2(f+gx)^{3/2}(ef-dg)\left(3eg(-aeg-bdg+2bef\right.}{3g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d
*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/
g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g
- a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(5*g^6
) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*
(f + g*x)^(9/2))/(9*g^6)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\text{Subst}\left(\int\left(\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))}{g^5}+\frac{(-ef+dg)^3\left(cf^2-bfg+ag^2\right)}{g^5x^2}+\frac{(ef-dg)\left(3eg(2be\right.}{}\right.\right.}{}$$

$$= \frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)}{g^6\sqrt{f+gx}} + \frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))}{g^6}$$

Mathematica [A] time = 0.747241, size = 249, normalized size = 0.87

$$\frac{2\left(-63e(f+gx)^3\left(c\left(-3d^2g^2+12defg-10e^2f^2\right)-eg(aeg+3bdg-4bef)\right)+105(f+gx)^2(ef-dg)\left(-3eg(aeg+bdg\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(315*(e*f - d*g)^3*(c*f^2 + g*(-(b*f) + a*g)) + 315*(e*f - d*g)^2*(c*f*(
5*e*f - 2*d*g) + g*(-4*b*e*f + b*d*g + 3*a*e*g))*(f + g*x) + 105*(e*f - d*g
)*(-3*e*g*(-2*b*e*f + b*d*g + a*e*g) - c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2)
)*(f + g*x)^2 - 63*e*(-(e*g*(-4*b*e*f + 3*b*d*g + a*e*g)) + c*(-10*e^2*f^2
+ 12*d*e*f*g - 3*d^2*g^2))*(f + g*x)^3 - 45*e^2*(5*c*e*f - 3*c*d*g - b*e*g)
*(f + g*x)^4 + 35*c*e^3*(f + g*x)^5))/(315*g^6*Sqrt[f + g*x])
```

Maple [B] time = 0.051, size = 540, normalized size = 1.9

$$-\frac{-70\,e^3cx^5g^5-90\,be^3g^5x^4-270\,cde^2g^5x^4+100\,ce^3fg^4x^4-126\,ae^3g^5x^3-378\,bde^2g^5x^3+144\,be^3fg^4x^3-378\,cd^2eg^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/315/(g*x+f)^(1/2)*(-35*c*e^3*g^5*x^5-45*b*e^3*g^5*x^4-135*c*d*e^2*g^5*x^
4+50*c*e^3*f*g^4*x^4-63*a*e^3*g^5*x^3-189*b*d*e^2*g^5*x^3+72*b*e^3*f*g^4*x^
3-189*c*d^2*e*g^5*x^3+216*c*d*e^2*f*g^4*x^3-80*c*e^3*f^2*g^3*x^3-315*a*d*e^
2*g^5*x^2+126*a*e^3*f*g^4*x^2-315*b*d^2*e*g^5*x^2+378*b*d*e^2*f*g^4*x^2-144
*b*e^3*f^2*g^3*x^2-105*c*d^3*g^5*x^2+378*c*d^2*e*f*g^4*x^2-432*c*d*e^2*f^2*
g^3*x^2+160*c*e^3*f^3*g^2*x^2-945*a*d^2*e*g^5*x+1260*a*d*e^2*f*g^4*x-504*a*
e^3*f^2*g^3*x-315*b*d^3*g^5*x+1260*b*d^2*e*f*g^4*x-1512*b*d*e^2*f^2*g^3*x+5
76*b*e^3*f^3*g^2*x+420*c*d^3*f*g^4*x-1512*c*d^2*e*f^2*g^3*x+1728*c*d*e^2*f^
3*g^2*x-640*c*e^3*f^4*g*x+315*a*d^3*g^5-1890*a*d^2*e*f*g^4+2520*a*d*e^2*f^2
*g^3-1008*a*e^3*f^3*g^2-630*b*d^3*f*g^4+2520*b*d^2*e*f^2*g^3-3024*b*d*e^2*f
^3*g^2+1152*b*e^3*f^4*g+840*c*d^3*f^2*g^3-3024*c*d^2*e*f^3*g^2+3456*c*d*e^2
*f^4*g-1280*c*e^3*f^5)/g^6
```

Maxima [A] time = 0.978479, size = 590, normalized size = 2.07

$$2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^3-45\,\left(5\,ce^3f-\left(3\,cde^2+be^3\right)g\right)(gx+f)^{\frac{7}{2}}+63\,\left(10\,ce^3f^2-4\,\left(3\,cde^2+be^3\right)fg+\left(3\,cd^2e+3\,bde^2+ae^3\right)g^2\right)(gx+f)^{\frac{5}{2}}-105\,\left(10\,ce^3f^3-6\,\left(3\,cde^2+be^3\right)f^2g+3\,(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - (3*c*d*e^2 + b*e^3)*g)*(
g*x + f)^(7/2) + 63*(10*c*e^3*f^2 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^2*e
+ 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(5/2) - 105*(10*c*e^3*f^3 - 6*(3*c*d*e^
2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 4*(3*c*d*e^2 +
b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*sqrt(g*x + f))/g^5 + 315
*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*c*d^2*e + 3*b*d*e^
2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + (b*d^3 + 3*a
*d^2*e)*f*g^4)/(sqrt(g*x + f)*g^5))/g
```

Fricas [A] time = 1.55055, size = 971, normalized size = 3.41

$$\frac{2\left(35\,ce^3g^5x^5+1280\,ce^3f^5-315\,ad^3g^5-1152\left(3\,cde^2+be^3\right)f^4g+1008\left(3\,cd^2e+3\,bde^2+ae^3\right)f^3g^2-840\left(cd^3+3\,bd^2e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 315*a*d^3*g^5 - 1152*(3*c*d*e^2
+ b*e^3)*f^4*g + 1008*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 - 840*(c*d^3
+ 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 + 630*(b*d^3 + 3*a*d^2*e)*f*g^4 - 5*(10*c*
e^3*f*g^4 - 9*(3*c*d*e^2 + b*e^3)*g^5)*x^4 + (80*c*e^3*f^2*g^3 - 72*(3*c*d*
e^2 + b*e^3)*f*g^4 + 63*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 - (160*c*e
^3*f^3*g^2 - 144*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 126*(3*c*d^2*e + 3*b*d*e^2 +
 a*e^3)*f*g^4 - 105*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f
^4*g - 576*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 504*(3*c*d^2*e + 3*b*d*e^2 + a*e^3
)*f^2*g^3 - 420*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f*g^4 + 315*(b*d^3 + 3*a*d^
2*e)*g^5)*x)*sqrt(g*x + f)/(g^7*x + f*g^6)
```

Sympy [A] time = 159.296, size = 452, normalized size = 1.59

$$\frac{2ce^3\left(f+gx\right)^{\frac{9}{2}}}{9g^6}+\frac{\left(f+gx\right)^{\frac{7}{2}}\left(2be^3g+6cde^2g-10ce^3f\right)}{7g^6}+\frac{\left(f+gx\right)^{\frac{5}{2}}\left(2ae^3g^2+6bde^2g^2-8be^3fg+6cd^2eg^2-24cde^2fg\right.}{5g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**3*(f + g*x)**(9/2)/(9*g**6) + (f + g*x)**(7/2)*(2*b*e**3*g + 6*c*d*e
**2*g - 10*c*e**3*f)/(7*g**6) + (f + g*x)**(5/2)*(2*a*e**3*g**2 + 6*b*d*e**
```

```
2*g**2 - 8*b*e**3*f*g + 6*c*d**2*e*g**2 - 24*c*d*e**2*f*g + 20*c*e**3*f**2)
/(5*g**6) + (f + g*x)**(3/2)*(6*a*d*e**2*g**3 - 6*a*e**3*f*g**2 + 6*b*d**2*
e*g**3 - 18*b*d*e**2*f*g**2 + 12*b*e**3*f**2*g + 2*c*d**3*g**3 - 18*c*d**2*
e*f*g**2 + 36*c*d*e**2*f**2*g - 20*c*e**3*f**3)/(3*g**6) + sqrt(f + g*x)*(6
*a*d**2*e*g**4 - 12*a*d*e**2*f*g**3 + 6*a*e**3*f**2*g**2 + 2*b*d**3*g**4 -
12*b*d**2*e*f*g**3 + 18*b*d*e**2*f**2*g**2 - 8*b*e**3*f**3*g - 4*c*d**3*f*g
**3 + 18*c*d**2*e*f**2*g**2 - 24*c*d*e**2*f**3*g + 10*c*e**3*f**4)/g**6 - 2
*(d*g - e*f)**3*(a*g**2 - b*f*g + c*f**2)/(g**6*sqrt(f + g*x))
```

Giac [B] time = 1.18099, size = 903, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^3*f^2*g^3 - b*d^3*f*g^4 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e + 3*b*d^2*f
^2*g^3*e - 3*a*d^2*f*g^4*e + 3*c*d*f^4*g*e^2 - 3*b*d*f^3*g^2*e^2 + 3*a*d*f^
2*g^3*e^2 - c*f^5*e^3 + b*f^4*g*e^3 - a*f^3*g^2*e^3)/(sqrt(g*x + f)*g^6) +
2/315*(105*(g*x + f)^(3/2)*c*d^3*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^51 + 31
5*sqrt(g*x + f)*b*d^3*g^52 + 189*(g*x + f)^(5/2)*c*d^2*g^50*e - 945*(g*x +
f)^(3/2)*c*d^2*f*g^50*e + 2835*sqrt(g*x + f)*c*d^2*f^2*g^50*e + 315*(g*x +
f)^(3/2)*b*d^2*g^51*e - 1890*sqrt(g*x + f)*b*d^2*f*g^51*e + 945*sqrt(g*x +
f)*a*d^2*g^52*e + 135*(g*x + f)^(7/2)*c*d*g^49*e^2 - 756*(g*x + f)^(5/2)*c*
d*f*g^49*e^2 + 1890*(g*x + f)^(3/2)*c*d*f^2*g^49*e^2 - 3780*sqrt(g*x + f)*c
*d*f^3*g^49*e^2 + 189*(g*x + f)^(5/2)*b*d*g^50*e^2 - 945*(g*x + f)^(3/2)*b*
d*f*g^50*e^2 + 2835*sqrt(g*x + f)*b*d*f^2*g^50*e^2 + 315*(g*x + f)^(3/2)*a*
d*g^51*e^2 - 1890*sqrt(g*x + f)*a*d*f*g^51*e^2 + 35*(g*x + f)^(9/2)*c*g^48*
e^3 - 225*(g*x + f)^(7/2)*c*f*g^48*e^3 + 630*(g*x + f)^(5/2)*c*f^2*g^48*e^3
 - 1050*(g*x + f)^(3/2)*c*f^3*g^48*e^3 + 1575*sqrt(g*x + f)*c*f^4*g^48*e^3
+ 45*(g*x + f)^(7/2)*b*g^49*e^3 - 252*(g*x + f)^(5/2)*b*f*g^49*e^3 + 630*(g
*x + f)^(3/2)*b*f^2*g^49*e^3 - 1260*sqrt(g*x + f)*b*f^3*g^49*e^3 + 63*(g*x
+ f)^(5/2)*a*g^50*e^3 - 315*(g*x + f)^(3/2)*a*f*g^50*e^3 + 945*sqrt(g*x + f
)*a*f^2*g^50*e^3)/g^54
```

3.827 $\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{2(f+gx)^{3/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(ef-d}{}$$

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*Sqrt[f + g*x]) - (2*(e*f -
d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x])/g
^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g
^2))*(f + g*x)^(3/2))/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^
(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rubi [A] time = 0.288764, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {897, 1261}

$$-\frac{2(f+gx)^{3/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(ef-d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*Sqrt[f + g*x]) - (2*(e*f -
d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x])/g
^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g
^2))*(f + g*x)^(3/2))/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^
(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))}{g^4}+\frac{(-ef+dg)^2\left(cf^2-bfg+ag^2\right)}{g^4x^2}+\frac{\left(-eg(3bef-2bdg\right.}{}\right.\right.}{g}$$

$$= -\frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)}{g^5\sqrt{f+gx}}-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))}{g^5}$$

Mathematica [A] time = 0.376082, size = 184, normalized size = 0.88

$$\frac{2\left(-35(f+gx)^2\left(-eg(aeg+2bdg-3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)-105(ef-dg)^2\left(g(ag-bf)+cf^2\right)-105(f+g\right.}{105g^5\sqrt{f+g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-105*(e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g)) - 105*(e*f - d*g)*(2*c*f*
(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*(f + g*x) - 35*(-(e*g*(-3*b
*e*f + 2*b*d*g + a*e*g)) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^2
 - 21*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^3 + 15*c*e^2*(f + g*x)^4))/(1
05*g^5*Sqrt[f + g*x])
```

Maple [A] time = 0.051, size = 315, normalized size = 1.5

$$-\frac{-30\,ce^2x^4g^4-42\,be^2g^4x^3-84\,cdeg^4x^3+48\,ce^2fg^3x^3-70\,ae^2g^4x^2-140\,bdeg^4x^2+84\,be^2fg^3x^2-70\,cd^2g^4x^2+168\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/105/(g*x+f)^(1/2)*(-15*c*e^2*g^4*x^4-21*b*e^2*g^4*x^3-42*c*d*e*g^4*x^3+2
4*c*e^2*f*g^3*x^3-35*a*e^2*g^4*x^2-70*b*d*e*g^4*x^2+42*b*e^2*f*g^3*x^2-35*c
*d^2*g^4*x^2+84*c*d*e*f*g^3*x^2-48*c*e^2*f^2*g^2*x^2-210*a*d*e*g^4*x+140*a*
e^2*f*g^3*x-105*b*d^2*g^4*x+280*b*d*e*f*g^3*x-168*b*e^2*f^2*g^2*x+140*c*d^2
*f*g^3*x-336*c*d*e*f^2*g^2*x+192*c*e^2*f^3*g*x+105*a*d^2*g^4-420*a*d*e*f*g^
3+280*a*e^2*f^2*g^2-210*b*d^2*f*g^3+560*b*d*e*f^2*g^2-336*b*e^2*f^3*g+280*c
*d^2*f^2*g^2-672*c*d*e*f^3*g+384*c*e^2*f^4)/g^5
```

Maxima [A] time = 0.96979, size = 363, normalized size = 1.73

$$\frac{2\left(\frac{15\left(gx+f\right)^{\frac{7}{2}}ce^2-21\left(4\,ce^2f-\left(2\,cde+be^2\right)g\right)\left(gx+f\right)^{\frac{5}{2}}+35\left(6\,ce^2f^2-3\left(2\,cde+be^2\right)fg+\left(cd^2+2\,bde+ae^2\right)g^2\right)\left(gx+f\right)^{\frac{3}{2}}-105\left(4\,ce^2f^3-3\left(2\,cde+be^2\right)f^2g+2\left(cd^2+2\right.\right.}{g^4}\right.}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 21*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*
x + f)^(5/2) + 35*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
 + a*e^2)*g^2)*(g*x + f)^(3/2) - 105*(4*c*e^2*f^3 - 3*(2*c*d*e + b*e^2)*f^2
*g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*sqrt(g*x +
f))/g^4 - 105*(c*e^2*f^4 + a*d^2*g^4 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2
*b*d*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)/(sqrt(g*x + f)*g^4))/g
```

Fricas [A] time = 1.48136, size = 603, normalized size = 2.87

$$\frac{2\left(15\,ce^2g^4x^4 - 384\,ce^2f^4 - 105\,ad^2g^4 + 336\left(2\,cde + be^2\right)f^3g - 280\left(cd^2 + 2\,bde + ae^2\right)f^2g^2 + 210\left(bd^2 + 2\,ade\right)fg^3 - 3\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 - 105*a*d^2*g^4 + 336*(2*c*d*e + b*
e^2)*f^3*g - 280*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g^2 + 210*(b*d^2 + 2*a*d*e)*
f*g^3 - 3*(8*c*e^2*f*g^3 - 7*(2*c*d*e + b*e^2)*g^4)*x^3 + (48*c*e^2*f^2*g^2
 - 42*(2*c*d*e + b*e^2)*f*g^3 + 35*(c*d^2 + 2*b*d*e + a*e^2)*g^4)*x^2 - (19
2*c*e^2*f^3*g - 168*(2*c*d*e + b*e^2)*f^2*g^2 + 140*(c*d^2 + 2*b*d*e + a*e^
2)*f*g^3 - 105*(b*d^2 + 2*a*d*e)*g^4)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

Sympy [A] time = 62.753, size = 272, normalized size = 1.3

$$\frac{2ce^2\left(f+gx\right)^{\frac{7}{2}}}{7g^5} + \frac{\left(f+gx\right)^{\frac{5}{2}}\left(2be^2g + 4cdeg - 8ce^2f\right)}{5g^5} + \frac{\left(f+gx\right)^{\frac{3}{2}}\left(2ae^2g^2 + 4bdeg^2 - 6be^2fg + 2cd^2g^2 - 12cdefg + 12c\cdots\right.}{3g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + (f + g*x)**(5/2)*(2*b*e**2*g + 4*c*d*e
*g - 8*c*e**2*f)/(5*g**5) + (f + g*x)**(3/2)*(2*a*e**2*g**2 + 4*b*d*e*g**2
- 6*b*e**2*f*g + 2*c*d**2*g**2 - 12*c*d*e*f*g + 12*c*e**2*f**2)/(3*g**5) +
sqrt(f + g*x)*(4*a*d*e*g**3 - 4*a*e**2*f*g**2 + 2*b*d**2*g**3 - 8*b*d*e*f*g
**2 + 6*b*e**2*f**2*g - 4*c*d**2*f*g**2 + 12*c*d*e*f**2*g - 8*c*e**2*f**3)/
g**5 - 2*(d*g - e*f)**2*(a*g**2 - b*f*g + c*f**2)/(g**5*sqrt(f + g*x))
```

Giac [B] time = 1.12825, size = 545, normalized size = 2.6

$$-\frac{2\left(cd^2f^2g^2 - bd^2fg^3 + ad^2g^4 - 2\,cdf^3ge + 2\,bdf^2g^2e - 2\,adfg^3e + cf^4e^2 - bf^3ge^2 + af^2g^2e^2\right)}{\sqrt{gx+f}g^5} + \frac{2\left(35\left(gx+f\right)^{\frac{3}{2}}cd^2g^{32}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*f^2*g^2 - b*d^2*f*g^3 + a*d^2*g^4 - 2*c*d*f^3*g*e + 2*b*d*f^2*g^2
*e - 2*a*d*f*g^3*e + c*f^4*e^2 - b*f^3*g*e^2 + a*f^2*g^2*e^2)/(sqrt(g*x + f
)*g^5) + 2/105*(35*(g*x + f)^(3/2)*c*d^2*g^32 - 210*sqrt(g*x + f)*c*d^2*f*g
^32 + 105*sqrt(g*x + f)*b*d^2*g^33 + 42*(g*x + f)^(5/2)*c*d*g^31*e - 210*(g
*x + f)^(3/2)*c*d*f*g^31*e + 630*sqrt(g*x + f)*c*d*f^2*g^31*e + 70*(g*x + f
)^(3/2)*b*d*g^32*e - 420*sqrt(g*x + f)*b*d*f*g^32*e + 210*sqrt(g*x + f)*a*d
*g^33*e + 15*(g*x + f)^(7/2)*c*g^30*e^2 - 84*(g*x + f)^(5/2)*c*f*g^30*e^2 +
 210*(g*x + f)^(3/2)*c*f^2*g^30*e^2 - 420*sqrt(g*x + f)*c*f^3*g^30*e^2 + 21
*(g*x + f)^(5/2)*b*g^31*e^2 - 105*(g*x + f)^(3/2)*b*f*g^31*e^2 + 315*sqrt(g
*x + f)*b*f^2*g^31*e^2 + 35*(g*x + f)^(3/2)*a*g^32*e^2 - 210*sqrt(g*x + f)*
a*f*g^32*e^2)/g^35
```

3.828 $\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4}-\frac{2(f+gx)^{3/2}(-beg-cdg+3cef)}{3g^4}+$$

```
[Out] (2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(c*f*(3*e*
f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*Sqrt[f + g*x])/g^4 - (2*(3*c*e*f
- c*d*g - b*e*g)*(f + g*x)^(3/2))/(3*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rubi [A] time = 0.098059, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {771}

$$\frac{2(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4}-\frac{2(f+gx)^{3/2}(-beg-cdg+3cef)}{3g^4}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(c*f*(3*e*
f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*Sqrt[f + g*x])/g^4 - (2*(3*c*e*f
- c*d*g - b*e*g)*(f + g*x)^(3/2))/(3*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx=\int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3(f+gx)^{3/2}}+\frac{cf(3ef-2dg)-g(2bef-bdg-aeg)}{g^3\sqrt{f+gx}}+\frac{(-3cef+cd}{}\right.$$

$$=\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+gx}}{g^4}-\frac{2(3ce}{}$$

Mathematica [A] time = 0.179613, size = 128, normalized size = 0.95

$$\frac{2\left(5g\left(3ag(-dg+2ef+egx)+3bdg(2f+gx)+be\left(-8f^2-4fgx+g^2x^2\right)\right)+c\left(5dg\left(-8f^2-4fgx+g^2x^2\right)+3e\left(8f^2gx+\right.\right.\right.}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(5*g*(3*b*d*g*(2*f + g*x) + 3*a*g*(2*e*f - d*g + e*g*x) + b*e*(-8*f^2 -
4*f*g*x + g^2*x^2)) + c*(5*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e*(16*f^3 +
 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))))/(15*g^4*Sqrt[f + g*x])
```

Maple [A] time = 0.047, size = 144, normalized size = 1.1

$$-\frac{-6\,ce x^3 g^3 - 10\,be g^3 x^2 - 10\,cd g^3 x^2 + 12\,cef g^2 x^2 - 30\,ae g^3 x - 30\,bd g^3 x + 40\,bef g^2 x + 40\,cdf g^2 x - 48\,cef^2 g x + 30\,ad}{15\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/15/(g*x+f)^(1/2)*(-3*c*e*g^3*x^3-5*b*e*g^3*x^2-5*c*d*g^3*x^2+6*c*e*f*g^2
*x^2-15*a*e*g^3*x-15*b*d*g^3*x+20*b*e*f*g^2*x+20*c*d*f*g^2*x-24*c*e*f^2*g*x
+15*a*d*g^3-30*a*e*f*g^2-30*b*d*f*g^2+40*b*e*f^2*g+40*c*d*f^2*g-48*c*e*f^3)
/g^4
```

Maxima [A] time = 0.961014, size = 185, normalized size = 1.37

$$\frac{2\left(\frac{3\,(gx+f)^{\frac{5}{2}}ce-5\left(3\,cef-(cd+be)g\right)(gx+f)^{\frac{3}{2}}+15\left(3\,cef^2-2\,(cd+be)fg+(bd+ae)g^2\right)\sqrt{gx+f}}{g^3}+\frac{15\left(cef^3-adg^3-(cd+be)f^2g+(bd+ae)fg^2\right)}{\sqrt{gx+f}\,g^3}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^(3/2)
+ 15*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*sqrt(g*x + f))/g^3 +
 15*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)/(sqrt(g*x +
 f)*g^3))/g
```

Fricas [A] time = 1.50451, size = 306, normalized size = 2.27

$$\frac{2\left(3\,ceg^3x^3 + 48\,cef^3 - 15\,adg^3 - 40\,(cd + be)f^2g + 30\,(bd + ae)fg^2 - \left(6\,cefg^2 - 5\,(cd + be)g^3\right)x^2 + \left(24\,cef^2g - 20\,(cd\right.\right.}{15\left(g^5x + fg^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 15*a*d*g^3 - 40*(c*d + b*e)*f^2*g + 30*(
b*d + a*e)*f*g^2 - (6*c*e*f*g^2 - 5*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2*g -
20*(c*d + b*e)*f*g^2 + 15*(b*d + a*e)*g^3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

Sympy [A] time = 26.8792, size = 141, normalized size = 1.04

$$\frac{2ce\left(f + gx\right)^{\frac{5}{2}}}{5g^4} + \frac{\left(f + gx\right)^{\frac{3}{2}}\left(2beg + 2cdg - 6cef\right)}{3g^4} + \frac{\sqrt{f + gx}\left(2aeg^2 + 2bdg^2 - 4befg - 4cdfg + 6cef^2\right)}{g^4} - \frac{2\left(dg - ef\right.}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e*(f + g*x)**(5/2)/(5*g**4) + (f + g*x)**(3/2)*(2*b*e*g + 2*c*d*g - 6*c
*e*f)/(3*g**4) + sqrt(f + g*x)*(2*a*e*g**2 + 2*b*d*g**2 - 4*b*e*f*g - 4*c*d
*f*g + 6*c*e*f**2)/g**4 - 2*(d*g - e*f)*(a*g**2 - b*f*g + c*f**2)/(g**4*sqr
t(f + g*x))
```

Giac [A] time = 1.12876, size = 275, normalized size = 2.04

$$-\frac{2\left(cdf^2g - bdfg^2 + adg^3 - cf^3e + bf^2ge - afg^2e\right)}{\sqrt{gx + f}g^4} + \frac{2\left(5\left(gx + f\right)^{\frac{3}{2}}cdg^{17} - 30\sqrt{gx + f}cdfg^{17} + 15\sqrt{gx + f}bdg^{18} + 3\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d*f^2*g - b*d*f*g^2 + a*d*g^3 - c*f^3*e + b*f^2*g*e - a*f*g^2*e)/(sqr
t(g*x + f)*g^4) + 2/15*(5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f
*g^17 + 15*sqrt(g*x + f)*b*d*g^18 + 3*(g*x + f)^(5/2)*c*g^16*e - 15*(g*x +
f)^(3/2)*c*f*g^16*e + 45*sqrt(g*x + f)*c*f^2*g^16*e + 5*(g*x + f)^(3/2)*b*g
^17*e - 30*sqrt(g*x + f)*b*f*g^17*e + 15*sqrt(g*x + f)*a*g^18*e)/g^20
```

3.829 $\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g^3*Sqrt[f + g*x]) - (2*(2*c*f - b*g)*Sqrt[f
+ g*x])/g^3 + (2*c*(f + g*x)^(3/2))/(3*g^3)
```

Rubi [A] time = 0.0422612, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g^3*Sqrt[f + g*x]) - (2*(2*c*f - b*g)*Sqrt[f
+ g*x])/g^3 + (2*c*(f + g*x)^(3/2))/(3*g^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{cf^2-bfg+ag^2}{g^2(f+gx)^{3/2}}+\frac{-2cf+bg}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx\\ &= -\frac{2\left(cf^2-bfg+ag^2\right)}{g^3\sqrt{f+gx}}-\frac{2(2cf-bg)\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A] time = 0.0573995, size = 54, normalized size = 0.76

$$\frac{6g(-ag+2bf+bgx)+2c\left(-8f^2-4fgx+g^2x^2\right)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (6*g*(2*b*f - a*g + b*g*x) + 2*c*(-8*f^2 - 4*f*g*x + g^2*x^2))/(3*g^3*Sqrt[
f + g*x])
```

Maple [A] time = 0.048, size = 53, normalized size = 0.8

$$-\frac{-2\,c x^2 g^2 - 6\,b g^2 x + 8\,c f g x + 6\,a g^2 - 12\,b f g + 16\,c f^2}{3\,g^3}\frac{1}{\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(3/2),x)
```

```
[Out] -2/3/(g*x+f)^(1/2)*(-c*g^2*x^2-3*b*g^2*x+4*c*f*g*x+3*a*g^2-6*b*f*g+8*c*f^2)
/g^3
```

Maxima [A] time = 0.959989, size = 89, normalized size = 1.25

$$\frac{2\left(\frac{(gx+f)^{\frac{3}{2}}c-3\,(2\,cf-bg)\sqrt{gx+f}}{g^2}-\frac{3\,(cf^2-bfg+ag^2)}{\sqrt{gx+f}g^2}\right)}{3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 3*(2*c*f - b*g)*sqrt(g*x + f))/g^2 - 3*(c*f^2 - b
*f*g + a*g^2)/(sqrt(g*x + f)*g^2))/g
```

Fricas [A] time = 1.45533, size = 136, normalized size = 1.92

$$\frac{2\left(cg^2x^2 - 8\,cf^2 + 6\,bfg - 3\,ag^2 - \left(4\,cfg - 3\,bg^2\right)x\right)\sqrt{gx+f}}{3\left(g^4x + fg^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 8*c*f^2 + 6*b*f*g - 3*a*g^2 - (4*c*f*g - 3*b*g^2)*x)*sqrt(
g*x + f)/(g^4*x + f*g^3)
```

Sympy [A] time = 10.0877, size = 70, normalized size = 0.99

$$\frac{2c\left(f+gx\right)^{\frac{3}{2}}}{3g^3}+\frac{\sqrt{f+gx}\left(2bg-4cf\right)}{g^3}-\frac{2\left(ag^2-bfg+cf^2\right)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*g**3) + sqrt(f + g*x)*(2*b*g - 4*c*f)/g**3 - 2*(a*g
**2 - b*f*g + c*f**2)/(g**3*sqrt(f + g*x))
```

Giac [A] time = 1.1739, size = 100, normalized size = 1.41

$$-\frac{2\left(cf^2 - bfg + ag^2\right)}{\sqrt{gx+f}g^3} + \frac{2\left(\left(gx+f\right)^{\frac{3}{2}}cg^6 - 6\sqrt{gx+f}cfg^6 + 3\sqrt{gx+f}bg^7\right)}{3g^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 - b*f*g + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6
 - 6*sqrt(g*x + f)*c*f*g^6 + 3*sqrt(g*x + f)*b*g^7)/g^9
```

3.830 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f +
 g*x])/(e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rubi [A] time = 0.221045, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {897, 1261, 208}

$$-\frac{2\left(ae^2-bde+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f +
 g*x])/(e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}dx, x, \sqrt{f+gx}\right)}{g}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2-bde+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$
$$= \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{(2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}dx, x, \sqrt{f+\dots}\right.}{e(ef-dg)}$$
$$= \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2-bde+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

Mathematica [A] time = 0.319113, size = 124, normalized size = 1.02

$$\frac{2\left(-\frac{g^2(cd^2-e(bd-ae))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{cf^2-g(bf-ag)}{\sqrt{f+gx}(ef-dg)}+\frac{c\sqrt{f+gx}}{e}\right)}{g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*((c*f^2 - g*(b*f - a*g))/((e*f - d*g)*Sqrt[f + g*x]) + (c*Sqrt[f + g*x])
/e - ((c*d^2 - e*(b*d - a*e))*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f
- d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))))/g^2
```

Maple [B] time = 0.236, size = 237, normalized size = 1.9

$$2\,\frac{c\sqrt{gx+f}}{eg^2}-2\,\frac{ae}{(dg-ef)\sqrt{(dg-ef)e}}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+2\,\frac{bd}{(dg-ef)\sqrt{(dg-ef)e}}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)-\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x)
```

```
[Out] 2*c*(g*x+f)^(1/2)/e/g^2-2/(d*g-e*f)*e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^
(1/2)/((d*g-e*f)*e)^(1/2))*a+2/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+
f)^(1/2)/((d*g-e*f)*e)^(1/2))*b*d-2/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(
e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d^2-2/(d*g-e*f)/(g*x+f)^(1/2)*a+2/g/
(d*g-e*f)/(g*x+f)^(1/2)*b*f-2/g^2/(d*g-e*f)/(g*x+f)^(1/2)*c*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77166, size = 1112, normalized size = 9.11

$$\left[-\frac{\left(\left(cd^2-bde+ae^2\right)g^3x+\left(cd^2-bde+ae^2\right)fg^2\right)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2\,ce^3f^3-ade^2g^3-\left(3\,c\right.\right.}{e^4f^3g^2-2\,de^3f^2g^3+d^2e^2fg^4+\left(e^4f^2g^3-2\,de^3f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*d^2 - b*d*e + a*e^2)*g^3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(e^2
*f - d*e*g)*log((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))
/(e*x + d)) - 2*(2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c*d*e^2 + b*e^3)*f^2*g + (c
*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*
g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^
4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x), 2*(((c*d^2 - b*d*e + a*e^2)*g^
3*x + (c*d^2 - b*d*e + a*e^2)*f*g^2)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*
f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*e^3*f^3 - a*d*e^2*g^3 - (3*c
*d*e^2 + b*e^3)*f^2*g + (c*d^2*e + b*d*e^2 + a*e^3)*f*g^2 + (c*e^3*f^2*g -
2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 - 2*d*e^3*f^2
*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*g^5)*x)]
```

Sympy [A] time = 34.8475, size = 116, normalized size = 0.95

$$\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2\left(ag^2-bfg+cf^2\right)}{g^2\sqrt{f+gx}\left(dg-ef\right)}-\frac{2\left(ae^2-bde+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}\left(dg-ef\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*sqrt(f + g*x)/(e*g**2) - 2*(a*g**2 - b*f*g + c*f**2)/(g**2*sqrt(f + g*x
)*(d*g - e*f)) - 2*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g -
 e*f)/e))/(e**2*sqrt((d*g - e*f)/e)*(d*g - e*f))
```

Giac [A] time = 1.14185, size = 151, normalized size = 1.24

$$-\frac{2\left(cd^2-bde+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{\left(dge-fe^2\right)^{\frac{3}{2}}}+\frac{2\sqrt{gx+f}ce^{(-1)}}{g^2}-\frac{2\left(cf^2-bfg+ag^2\right)}{\left(dg^3-fg^2e\right)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/(d*g
*e - f*e^2)^(3/2) + 2*sqrt(g*x + f)*c*e^(-1)/g^2 - 2*(c*f^2 - b*f*g + a*g^2
)/((d*g^3 - f*g^2*e)*sqrt(g*x + f))
```

3.831 $\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2-bfg+cf^2\right)}{g\sqrt{f+gx}(ef-dg)^2}$$

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 - b*
d*e + a*e^2)*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((c*d*(4*e*f - d*
g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

Rubi [A] time = 0.366759, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {897, 1259, 453, 208}

$$-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{e(d+ex)(ef-dg)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{e^{3/2}(ef-dg)^{5/2}}-\frac{2\left(ag^2-bfg+cf^2\right)}{g\sqrt{f+gx}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 - b*
d*e + a*e^2)*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((c*d*(4*e*f - d*
g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\,\mathrm{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(e(bd-ae)g^2+c\left(2e^2f^2-4defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\right.}{e^2(ef-dg)^2}$$

$$= -\frac{2\left(cf^2-bfg+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{(cd(4ef-dg)-e(2bef+bdg-3a}{eg(}$$

$$= -\frac{2\left(cf^2-bfg+ag^2\right)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(cd(4ef-dg)-e(2bef+bdg-3a}{e^{3/2}(ef-dg)^5}$$

Mathematica [A] time = 0.465803, size = 176, normalized size = 1.07

$$-\frac{eg(2adg+ae(f+3gx)-b(3df+dgx+2efx))+c\left(d^2g(f+gx)+2def^2+2e^2f^2x\right)}{eg(d+ex)\sqrt{f+gx}(ef-dg)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-3aeg+}{e^{3/2}(ef-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] -((c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x)) + e*g*(2*a*d*g + a*e*(f +
3*g*x) - b*(3*d*f + 2*e*f*x + d*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f
+ g*x])) - ((c*d*(-4*e*f + d*g) + e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(S
qrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

Maple [B] time = 0.244, size = 418, normalized size = 2.5

$$-2\,\frac{ag}{\left(dg-ef\right)^2\sqrt{gx+f}}+2\,\frac{bf}{\left(dg-ef\right)^2\sqrt{gx+f}}-2\,\frac{cf^2}{g\left(dg-ef\right)^2\sqrt{gx+f}}-\frac{aeg}{\left(dg-ef\right)^2\left(egx+dg\right)}\sqrt{gx+f}+\frac{}{\left(dg-ef\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x)
```

```
[Out] -2*g/(d*g-e*f)^2/(g*x+f)^(1/2)*a+2/(d*g-e*f)^2/(g*x+f)^(1/2)*b*f-2/g/(d*g-e
*f)^2/(g*x+f)^(1/2)*c*f^2-g/(d*g-e*f)^2*e*(g*x+f)^(1/2)/(e*g*x+d*g)*a+g/(d*
g-e*f)^2*(g*x+f)^(1/2)/(e*g*x+d*g)*b*d-g/(d*g-e*f)^2/e*(g*x+f)^(1/2)/(e*g*x
+d*g)*c*d^2-3*g/(d*g-e*f)^2*e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((
d*g-e*f)*e)^(1/2))*a+g/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/
2)/((d*g-e*f)*e)^(1/2))*b*d+2/(d*g-e*f)^2*e/((d*g-e*f)*e)^(1/2)*arctan(e*(g
*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*b*f+g/(d*g-e*f)^2/e/((d*g-e*f)*e)^(1/2)*ar
ctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d^2-4/(d*g-e*f)^2/((d*g-e*f)*e)
^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*d*c*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78997, size = 2184, normalized size = 13.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((2*(2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2
+ (2*(2*c*d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 + (
2*(2*c*d*e^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d^3
+ b*d^2*e - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g
 + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*a
*d^2*e^2*g^3 - (c*d^2*e^2 + 3*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e
^2 - a*d*e^3)*f*g^2 + (2*c*e^4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2
 + b*d*e^3 + 3*a*e^4)*f*g^2 - (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqr
t(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*
f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2
 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x), -((2*(
2*c*d^2*e - b*d*e^2)*f^2*g - (c*d^3 + b*d^2*e - 3*a*d*e^2)*f*g^2 + (2*(2*c*
d*e^2 - b*e^3)*f*g^2 - (c*d^2*e + b*d*e^2 - 3*a*e^3)*g^3)*x^2 + (2*(2*c*d*e
^2 - b*e^3)*f^2*g + 3*(c*d^2*e - b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + b*d^2*e
- 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(
g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*e^2*g^3 - (c*d^2*e^2 + 3
*b*d*e^3 - a*e^4)*f^2*g - (c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*f*g^2 + (2*c*e^
4*f^3 - 2*(c*d*e^3 + b*e^4)*f^2*g + (c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*f*g^2 -
 (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*
d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^
5*f^2*g^3 + 3*d^2*e^4*f*g^4 - d^3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g
^2 + 2*d^3*e^3*f*g^4 - d^4*e^2*g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16058, size = 381, normalized size = 2.31

$$\frac{\left(cd^2g - 4\,cdfe + bdge + 2\,bfe^2 - 3\,age^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{\left(d^2g^2e - 2\,dfge^2 + f^2e^3\right)\sqrt{dge - fe^2}} - \frac{\left(gx + f\right)cd^2g^2 + 2\,cdf^2ge - \left(gx + f\right)bdg^2e - 2\,bdfg^2e +}{\left(d^2g^3e - 2\,df\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] (c*d^2*g - 4*c*d*f*e + b*d*g*e + 2*b*f*e^2 - 3*a*g*e^2)*arctan(sqrt(g*x + f
)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e - 2*d*f*g*e^2 + f^2*e^3)*sqrt(d*g*e -
f*e^2)) - ((g*x + f)*c*d^2*g^2 + 2*c*d*f^2*g*e - (g*x + f)*b*d*g^2*e - 2*b*
d*f*g^2*e + 2*a*d*g^3*e + 2*(g*x + f)*c*f^2*e^2 - 2*c*f^3*e^2 - 2*(g*x + f)
*b*f*g*e^2 + 2*b*f^2*g*e^2 + 3*(g*x + f)*a*g^2*e^2 - 2*a*f*g^2*e^2)/((d^2*g
^3*e - 2*d*f*g^2*e^2 + f^2*g*e^3)*(sqrt(g*x + f)*d*g + (g*x + f)^(3/2)*e -
sqrt(g*x + f)*f*e))
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(3eg(5aeg-b(dg+4ef))+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)}{4e^{3/2}(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}+\frac{2\left(ag^2-bfg\right.}{\sqrt{f+gx}(ef}$$

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 - b*d*e
 + a*e^2)*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((c*d*(8*e*f - d
*g) - e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d
 + e*x)) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a*e*g - b*(4*e*
f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(3/2)*(e*
f - d*g)^(7/2))
```

Rubi [A] time = 0.628685, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {897, 1259, 456, 453, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)\left(3eg(5aeg-b(dg+4ef))+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)}{4e^{3/2}(ef-dg)^{7/2}}-\frac{\sqrt{f+gx}\left(ae^2-bde+cd^2\right)}{2e(d+ex)^2(ef-dg)^2}+\frac{2\left(ag^2-bfg\right.}{\sqrt{f+gx}(ef}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 - b*d*e
 + a*e^2)*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((c*d*(8*e*f - d
*g) - e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d
 + e*x)) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a*e*g - b*(4*e*
f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(3/2)*(e*
f - d*g)^(7/2))
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\,\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(3e(bd-ae)g^2+c\left(4e^2f^2-8defg+d^2g^2\right)\right)x^2}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\right.}{2e^2(ef-dg)^2}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{g^3\,\text{Sul}}{}$$

$$= \frac{2\left(cf^2-bfg+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7ae}{4e(ef-dg)^3(d+ex)}$$

$$= \frac{2\left(cf^2-bfg+ag^2\right)}{(ef-dg)^3\sqrt{f+gx}} - \frac{\left(cd^2-bde+ae^2\right)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7ae}{4e(ef-dg)^3(d+ex)}$$

Mathematica [A] time = 1.23514, size = 290, normalized size = 1.17

$$\frac{1}{4}\left(-\frac{2\sqrt{f+gx}\left(e(ae-bd)+cd^2\right)}{e(d+ex)^2(ef-dg)^2} + \frac{g\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-7aeg+3bdg+4bef)+cd(dg-8ef))}{e^{3/2}(ef-dg)^{7/2}} + \frac{8\left(g(ag-bf)+c\right.}{\sqrt{f+gx}(ef-d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] ((8*(c*f^2 + g*(-(b*f) + a*g)))/((e*f - d*g)^3*Sqrt[f + g*x]) - (2*(c*d^2 +
 e*(-(b*d) + a*e))*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)^2) - ((c*d*(-8
*e*f + d*g) + e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(e*(e*f - d*g
)^3*(d + e*x)) - (8*Sqrt[e]*(c*f^2 + g*(-(b*f) + a*g))*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/Sqrt[e*f - d*g]])/(e*f - d*g)^(7/2) + (g*(c*d*(-8*e*f + d*g) +
e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f -
 d*g]])/(e^(3/2)*(e*f - d*g)^(7/2)))/4
```

Maple [B] time = 0.246, size = 847, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x)
```

```
[Out] -2/(d*g-e*f)^3/(g*x+f)^(1/2)*a*g^2+2/(d*g-e*f)^3/(g*x+f)^(1/2)*b*f*g-2/(d*g
-e*f)^3/(g*x+f)^(1/2)*c*f^2-7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*a*e
^2*g^2+3/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*b*d*e*g^2+1/(d*g-e*f)^3/
(e*g*x+d*g)^2*(g*x+f)^(3/2)*b*e^2*f*g+1/4/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)
^(3/2)*c*d^2*g^2-2/(d*g-e*f)^3/(e*g*x+d*g)^2*(g*x+f)^(3/2)*c*d*e*f*g-9/4/(d
*g-e*f)^3/(e*g*x+d*g)^2*g^3*e*(g*x+f)^(1/2)*a*d+9/4/(d*g-e*f)^3/(e*g*x+d*g)
^2*g^2*e^2*(g*x+f)^(1/2)*a*f+5/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^3*(g*x+f)^(1/2
)*b*d^2-1/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*e*(g*x+f)^(1/2)*b*d*f-1/(d*g-e*f)
^3/(e*g*x+d*g)^2*g*e^2*(g*x+f)^(1/2)*b*f^2-1/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^
3/e*(g*x+f)^(1/2)*c*d^3-7/4/(d*g-e*f)^3/(e*g*x+d*g)^2*g^2*(g*x+f)^(1/2)*c*d
^2*f+2/(d*g-e*f)^3/(e*g*x+d*g)^2*g*e*(g*x+f)^(1/2)*c*d*f^2-15/4/(d*g-e*f)^3
*e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*a*g^2+3/
4/(d*g-e*f)^3/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2
))*b*d*g^2+3/(d*g-e*f)^3*e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g
-e*f)*e)^(1/2))*b*f*g+1/4/(d*g-e*f)^3/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f
)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d^2*g^2-2/(d*g-e*f)^3/((d*g-e*f)*e)^(1/2)*ar
ctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*d*f*g-2/(d*g-e*f)^3*e/((d*g-e*f
)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*c*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.8837, size = 3864, normalized size = 15.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g - (c*d^4 + 3*b*
d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 - 3*b*e^4)*f*g^
2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 12*(2*c*d*
e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g^2 - 2*(c*d^3
*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*(c*d^2*e^2 -
 b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2 - (c*d^4 + 3*
b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*
g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) + 2*(8*a*d^3*e^2*g^3 -
2*(7*c*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e^3 - 11*a
*d*e^4)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 -
 12*b*e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 + (c*d^3*e^2
+ 3*b*d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f^3 - (19*c*d
^2*e^3 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*e^3 - 5*a*d*e
^4)*f*g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(
d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6
*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f
*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*
d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d
^2*e^6*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^
6*e^2*g^5)*x), 1/4*((8*c*d^2*e^2*f^3 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f^2*g -
(c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g + 4*(2*c*d*e^3 -
3*b*e^4)*f*g^2 - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3
 + 12*(2*c*d*e^3 - b*e^4)*f^2*g + 3*(5*c*d^2*e^2 - 9*b*d*e^3 + 5*a*e^4)*f*g
^2 - 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24
*(c*d^2*e^2 - b*d*e^3)*f^2*g + 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*f*g^2
- (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sq
rt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (8*a*d^3*e^2*g^3 - 2*(7*c
*d^2*e^3 - b*d*e^4 - a*e^5)*f^3 + (13*c*d^3*e^2 + 11*b*d^2*e^3 - 11*a*d*e^4
)*f^2*g + (c*d^4*e - 13*b*d^3*e^2 + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 12*b*
e^5*f^2*g - 3*(3*c*d^2*e^3 - 3*b*d*e^4 - 5*a*e^5)*f*g^2 + (c*d^3*e^2 + 3*b*
d^2*e^3 - 15*a*d*e^4)*g^3)*x^2 - (4*(6*c*d*e^4 - b*e^5)*f^3 - (19*c*d^2*e^3
 + 17*b*d*e^4 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 4*b*d^2*e^3 - 5*a*d*e^4)*f*
g^2 - (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^
6*f^5 - 4*d^3*e^5*f^4*g + 6*d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f
*g^4 + (e^8*f^4*g - 4*d*e^7*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 +
 d^4*e^4*g^5)*x^3 + (e^8*f^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^
5*f^2*g^3 - 7*d^4*e^4*f*g^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6
*f^4*g + 8*d^3*e^5*f^3*g^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*
g^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18643, size = 624, normalized size = 2.52

$$\frac{\left(cd^2g^2 - 8\,cdfge + 3\,bdg^2e - 8\,cf^2e^2 + 12\,bfge^2 - 15\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{dge-fe^2}}\right)}{4\left(d^3g^3e - 3\,d^2fg^2e^2 + 3\,df^2ge^3 - f^3e^4\right)\sqrt{dge-fe^2}} - \frac{2\left(cf^2 - bfg + ag^2\right)}{\left(d^3g^3 - 3\,d^2fg^2e + 3\,df^2ge^2 - f^3e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^2*g^2 - 8*c*d*f*g*e + 3*b*d*g^2*e - 8*c*f^2*e^2 + 12*b*f*g*e^2 - 1
5*a*g^2*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d^3*g^3*e - 3*d^
2*f*g^2*e^2 + 3*d*f^2*g*e^3 - f^3*e^4)*sqrt(d*g*e - f*e^2)) - 2*(c*f^2 - b*
f*g + a*g^2)/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(g*x
+ f)) - 1/4*(sqrt(g*x + f)*c*d^3*g^3 - (g*x + f)^(3/2)*c*d^2*g^2*e + 7*sqrt
(g*x + f)*c*d^2*f*g^2*e - 5*sqrt(g*x + f)*b*d^2*g^3*e + 8*(g*x + f)^(3/2)*c
*d*f*g*e^2 - 8*sqrt(g*x + f)*c*d*f^2*g*e^2 - 3*(g*x + f)^(3/2)*b*d*g^2*e^2
+ sqrt(g*x + f)*b*d*f*g^2*e^2 + 9*sqrt(g*x + f)*a*d*g^3*e^2 - 4*(g*x + f)^(
3/2)*b*f*g*e^3 + 4*sqrt(g*x + f)*b*f^2*g*e^3 + 7*(g*x + f)^(3/2)*a*g^2*e^3
- 9*sqrt(g*x + f)*a*f*g^2*e^3)/((d^3*g^3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^
3 - f^3*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

3.833 $\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx$

Optimal. Leaf size=91

$$\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{\sqrt{5}-2}\sqrt{x-1}}\right)-\cosh^{-1}(x)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2+\sqrt{5}}\sqrt{x-1}}\right)$$

```
[Out] -ArcCosh[x] + Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[Sqrt[1 + x]/(Sqrt[-2 + Sqrt
[5]]*Sqrt[-1 + x])] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[1 + x]/(Sqrt[2
 + Sqrt[5]]*Sqrt[-1 + x])]
```

Rubi [B] time = 0.140556, antiderivative size = 191, normalized size of antiderivative = 2.1, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {901, 991, 217, 206, 1034, 725, 204}

$$\frac{\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\sqrt{x-1}\sqrt{x+1}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}-\frac{\sqrt{x-1}\sqrt{x+1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}-\frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\sqrt{x-1}\sqrt{x+1}\tan}{\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[5])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTan[(2 - (1 - Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2] - (Sqrt[-1 + x
]*Sqrt[1 + x]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^2] - (Sqrt[(1 + Sqrt[5
])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 +
Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2]
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 991

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2}\,dx &= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{\sqrt{-1+x^2}}{1+x-x^2}\,dx}{\sqrt{-1+x^2}}\\
&= -\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}}+\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{x}{\left(1+x-x^2\right)\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}}\\
&= -\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}+\frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{1}{\left(1-\sqrt{5}-2x\right)\sqrt{-1+x^2}}}{5\sqrt{-1+x^2}}\\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}-\frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{-4+\left(1-\sqrt{5}\right)^2-x^2}\,dx,x,\frac{2}{}\right.}{5\sqrt{-1+x^2}}\\
&= \frac{\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\sqrt{-1+x}\sqrt{1+x}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}-\frac{\sqrt{-1+x}\sqrt{1+x}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}-
\end{aligned}$$

Mathematica [A] time = 0.207948, size = 113, normalized size = 1.24

$$-\frac{1}{5}\sqrt{\sqrt{5}-2}\left(5+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\sqrt{5}-2}\sqrt{\frac{x-1}{x+1}}\right)-2\tanh^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{1}{5}\left(\sqrt{5}-5\right)\sqrt{2+\sqrt{5}}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\sqrt{\frac{x-1}{x+1}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] -(Sqrt[-2 + Sqrt[5]]*(5 + Sqrt[5])*ArcTan[Sqrt[-2 + Sqrt[5]]*Sqrt[(-1 + x)/
(1 + x)]])/5 - 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]] - ((-5 + Sqrt[5])*Sqrt[2 +
 Sqrt[5]]*ArcTanh[Sqrt[2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 + x)]])/5
```

Maple [B] time = 0.125, size = 231, normalized size = 2.5

$$-\frac{\sqrt{5}}{5\sqrt{2+2\sqrt{5}}\sqrt{-2+2\sqrt{5}}}\sqrt{-1+x}\sqrt{1+x}\left(\ln\left(x+\sqrt{x^2-1}\right)\sqrt{2+2\sqrt{5}}\sqrt{-2+2\sqrt{5}}\sqrt{5}-\operatorname{Artanh}\left(\frac{x\sqrt{5}+x-2}{\sqrt{2+2\sqrt{5}}}\frac{1}{\sqrt{x^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x)
```

```
[Out] -1/5*(-1+x)^(1/2)*(1+x)^(1/2)*5^(1/2)*(ln(x+(x^2-1)^(1/2))*(2+2*5^(1/2))^(1
/2)*(-2+2*5^(1/2))^(1/2)*5^(1/2)-arctanh((x*5^(1/2)+x-2)/(2+2*5^(1/2))^(1/2
)/(x^2-1)^(1/2))*(-2+2*5^(1/2))^(1/2)*5^(1/2)-arctan((x*5^(1/2)-x+2)/(-2+2*
5^(1/2))^(1/2)/(x^2-1)^(1/2))*(2+2*5^(1/2))^(1/2)*5^(1/2)-arctanh((x*5^(1/2
)+x-2)/(2+2*5^(1/2))^(1/2)/(x^2-1)^(1/2))*(-2+2*5^(1/2))^(1/2)+arctan((x*5^
(1/2)-x+2)/(-2+2*5^(1/2))^(1/2)/(x^2-1)^(1/2))*(2+2*5^(1/2))^(1/2))/(2+2*5^
(1/2))^(1/2)/(-2+2*5^(1/2))^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{x+1}\sqrt{x-1}}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x + 1)*sqrt(x - 1)/(x^2 - x - 1), x)
```

Fricas [B] time = 1.68578, size = 671, normalized size = 7.37

$$\frac{2}{5}\sqrt{5}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{1}{8}\sqrt{-4\left(2x+\sqrt{5}-1\right)\sqrt{x+1}\sqrt{x-1}+8x^2+4\sqrt{5}x-4x}\sqrt{2\sqrt{5}-2}\left(\sqrt{5}+1\right)-\frac{1}{4}\left(\sqrt{x+1}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(2*sqrt(5) - 2)*arctan(1/8*sqrt(-4*(2*x + sqrt(5) - 1)*sqrt
(x + 1)*sqrt(x - 1) + 8*x^2 + 4*sqrt(5)*x - 4*x)*sqrt(2*sqrt(5) - 2)*(sqrt(
5) + 1) - 1/4*(sqrt(x + 1)*sqrt(x - 1)*(sqrt(5) + 1) - sqrt(5)*x - x - 2)*s
qrt(2*sqrt(5) - 2)) + 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(2*sqrt(x + 1)*sq
rt(x - 1) - 2*x + sqrt(5) + sqrt(2*sqrt(5) + 2) + 1) - 1/10*sqrt(5)*sqrt(2*
sqrt(5) + 2)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x + sqrt(5) - sqrt(2*sqrt(5)
 + 2) + 1) + log(sqrt(x + 1)*sqrt(x - 1) - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{x-1}\sqrt{x+1}}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)*(1+x)**(1/2)/(-x**2+x+1),x)

[Out] -Integral(sqrt(x - 1)*sqrt(x + 1)/(x**2 - x - 1), x)
```

Giac [A] time = 1.16558, size = 22, normalized size = 0.24

$$\log\left(\left(\sqrt{x+1}-\sqrt{x-1}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="giac")

[Out] log((sqrt(x + 1) - sqrt(x - 1))^2)
```

3.834 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{4e^2g^2}+$$

```
[Out] -((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) +
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rubi [A] time = 0.179186, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {951, 80, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{4e^2g^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```


```
[Out] -((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) +
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```


Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg+3d^2g^2\right)\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg+3d^2g^2\right)\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg+3d^2g^2\right)\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg+3d^2g^2\right)\right.}{}$$

Mathematica [A] time = 0.822182, size = 173, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)+e\sqrt{g}\sqrt{d+ex}(f+gx)(4beg}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x))
 + Sqrt[e*f - d*g]*(c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g
- b*(e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d +
 e*x])/Sqrt[e*f - d*g]])/(4*e^3*g^(5/2)*Sqrt[f + g*x])
```

Maple [B] time = 0.307, size = 425, normalized size = 2.6

$$\frac{1}{8e^2g^2}\left(8\ln\left(1/2\,\frac{2\,egx+2\,\sqrt{(gx+f)(ex+d)}\sqrt{eg}+dg+ef}{\sqrt{eg}}\right)ae^2g^2-4\ln\left(1/2\,\frac{2\,egx+2\,\sqrt{(gx+f)(ex+d)}\sqrt{eg}+dg+}{\sqrt{eg}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)
^(1/2))*a*e^2*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^2*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e
*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2*e*g*x+
2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+3*ln(
1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*
e^2*f^2+4*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*x*c*e*g+8*(e*g)^(1/2)*((g*x+f
)*(e*x+d))^(1/2)*b*e*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d*g-6*(e*g)^
(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99462, size = 879, normalized size = 5.36

$$\left[\frac{\left(3\,ce^2f^2 + 2\left(cde - 2\,be^2\right)fg + \left(3\,cd^2 - 4\,bde + 8\,ae^2\right)g^2\right)\sqrt{eg}\log\left(8\,e^2g^2x^2 + e^2f^2 + 6\,defg + d^2g^2 + 4\left(2\,egx + ef + \right.\right.}{16\,e^3g^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^
2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*
e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e
*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sqrt(e
*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*
f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e
*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (
e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^
2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{\sqrt{d + ex}\sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [A] time = 1.21021, size = 242, normalized size = 1.48

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\sqrt{xe+d}\left(\frac{2\,(xe+d)ce^{(-3)}}{g}-\frac{\left(5\,cdg^2e^5+3\,cfge^6-4\,bg^2e^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3\,cd^2g^2+2\,cdfge-4\,bdg^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6 - 4*b*g^2*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2
*g^2 + 2*c*d*f*g*e - 4*b*d*g^2*e + 3*c*f^2*e^2 - 4*b*f*g*e^2 + 8*a*g^2*e^2)
*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g
*e + f*e^2)))/g^(5/2)
```

3.835 $\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=333

$$\frac{(d+ex)^{3/2}\sqrt{f+gx}\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)}{96e^2g^3}-\frac{\sqrt{d+ex}\sqrt{f+gx}(ef-dg)\left(8eg(6aeg\right.}{}$$

```
[Out] -((e*f - d*g)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b
*(5*e*f + d*g)))*Sqrt[d + e*x]*Sqrt[f + g*x])/(64*e^2*g^4) + ((c*(35*e^2*f^
2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*(d + e*x)^
(3/2)*Sqrt[f + g*x])/(96*e^2*g^3) - ((7*c*e*f + 9*c*d*g - 8*b*e*g)*(d + e*x
)^(5/2)*Sqrt[f + g*x])/(24*e^2*g^2) + (c*(d + e*x)^(7/2)*Sqrt[f + g*x])/(4*
e^2*g) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6
*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
+ g*x])])/(64*e^(5/2)*g^(9/2))
```

Rubi [A] time = 0.353488, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{(d+ex)^{3/2}\sqrt{f+gx}\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)}{96e^2g^3}-\frac{\sqrt{d+ex}\sqrt{f+gx}(ef-dg)\left(8eg(6aeg\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] -((e*f - d*g)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b
*(5*e*f + d*g)))*Sqrt[d + e*x]*Sqrt[f + g*x])/(64*e^2*g^4) + ((c*(35*e^2*f^
2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*(d + e*x)^
(3/2)*Sqrt[f + g*x])/(96*e^2*g^3) - ((7*c*e*f + 9*c*d*g - 8*b*e*g)*(d + e*x
)^(5/2)*Sqrt[f + g*x])/(24*e^2*g^2) + (c*(d + e*x)^(7/2)*Sqrt[f + g*x])/(4*
e^2*g) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6
*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
+ g*x])])/(64*e^(5/2)*g^(9/2))
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}dx = \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(8ae^2g-cd(7ef+dg)\right)-\frac{1}{2}e(7cef+9cdg-8beg)x\right)}{\sqrt{f+gx}}dx}{4e^2g}$$

$$= -\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2} + \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{\left(c\left(35e^2f^2+10de\right.\right.}{}$$

$$= \frac{\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f+gx}}{96e^2g^3} - \frac{(7cef}{}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c\left(35e^2f^2+10defg+3d^2g^2\right)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{64e^2g^4}$$

Mathematica [A] time = 1.67101, size = 313, normalized size = 0.94

$$3(ef-dg)^{5/2}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(8eg(6aeg-b(dg+5ef))+c\left(3d^2g^2+10defg+35e^2f^2\right)\right)-e\sqrt{g}\sqrt{d+ex}(f+$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-(e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(c*(9*d^3*g^3 + 3*d^2*e*g^2*(5*f - 2*g
*x) + d*e^2*g*(-145*f^2 + 92*f*g*x - 72*g^2*x^2) + e^3*(105*f^3 - 70*f^2*g*
x + 56*f*g^2*x^2 - 48*g^3*x^3)) - 8*e*g*(6*a*e*g*(-3*e*f + 5*d*g + 2*e*g*x)
 + b*(3*d^2*g^2 + 2*d*e*g*(-11*f + 7*g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*
x^2))))) + 3*(e*f - d*g)^(5/2)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8
*e*g*(6*a*e*g - b*(5*e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(
Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(192*e^3*g^(9/2)*Sqrt[f + g*x])
```

Maple [B] time = 0.286, size = 1207, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/384*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(144*x^2*c*d*e^2*g^3*((g*x+f)*(e*x+d))^(1
/2)*(e*g)^(1/2)-112*x^2*c*e^3*f*g^2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-352
*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*b*d*e^2*f*g^2+290*(e*g)^(1/2)*((g*x+f)
*(e*x+d))^(1/2)*c*d*e^2*f^2*g+224*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*x*b*d
*e^2*g^3-160*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*x*b*e^3*f*g^2+12*(e*g)^(1/
2)*((g*x+f)*(e*x+d))^(1/2)*x*c*d^2*e*g^3+140*(e*g)^(1/2)*((g*x+f)*(e*x+d))^
(1/2)*x*c*e^3*f^2*g-30*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d^2*e*f*g^2+14
4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2
))*a*d^2*e^2*g^4+105*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*c*e^4*f^4+9*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)
*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^4*g^4+144*ln(1/2*(2*e*g*x+2*((g*x+f)
*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^4*f^2*g^2-120*ln(1/2*
(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^4*
f^3*g-210*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e^3*f^3-24*ln(1/2*(2*e*g*x+
2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^3*e*g^4-18*
(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d^3*g^3-184*(e*g)^(1/2)*((g*x+f)*(e*x
+d))^(1/2)*x*c*d*e^2*f*g^2-288*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e
*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*d*e^3*f*g^3+240*(e*g)^(1/2)*((g*x+f)*(e*x
+d))^(1/2)*b*e^3*f^2*g+96*x^3*c*e^3*g^3*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)
+128*x^2*b*e^3*g^3*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+192*(e*g)^(1/2)*((g*
x+f)*(e*x+d))^(1/2)*x*a*e^3*g^3+48*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*b*d^
2*e*g^3+12*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(
e*g)^(1/2))*c*d^3*e*f*g^3-72*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g
)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e^2*f*g^3+216*ln(1/2*(2*e*g*x+2*((g*x+f
)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^3*f^2*g^2+54*ln(1/
2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^
2*e^2*f^2*g^2-180*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g
+e*f)/(e*g)^(1/2))*c*d*e^3*f^3*g+480*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*a*
d*e^2*g^3-288*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*a*e^3*f*g^2)/e^2/g^4/((g*
x+f)*(e*x+d))^(1/2)/(e*g)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.15194, size = 1912, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*c*e^4*f^4 - 20*(3*c*d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 +
 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g
^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*g^4)*sqrt(e*g)*log(8*e^2*g^2*x^2
+ e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*
x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(48*c*e^4*g^4*x^3 - 105
*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (15*c*d^2*e^2 + 176*b*d*
e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*g^4 - 8*(
7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(35*c*e^4*f^2*g^2 - 2*(2
3*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*g^4)*x)
*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5), -1/384*(3*(35*c*e^4*f^4 - 20*(3*c*
d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 + 12*b*d*e^3 + 8*a*e^4)*f^2*g^2 + 4
*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g^3 + (3*c*d^4 - 8*b*d^3*e + 48*a*d
^2*e^2)*g^4)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*
x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(
48*c*e^4*g^4*x^3 - 105*c*e^4*f^3*g + 5*(29*c*d*e^3 + 24*b*e^4)*f^2*g^2 - (1
5*c*d^2*e^2 + 176*b*d*e^3 + 144*a*e^4)*f*g^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 -
 80*a*d*e^3)*g^4 - 8*(7*c*e^4*f*g^3 - (9*c*d*e^3 + 8*b*e^4)*g^4)*x^2 + 2*(3
5*c*e^4*f^2*g^2 - 2*(23*c*d*e^3 + 20*b*e^4)*f*g^3 + (3*c*d^2*e^2 + 56*b*d*e
^3 + 48*a*e^4)*g^4)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28985, size = 605, normalized size = 1.82

$$\frac{1}{192}\sqrt{(xe+d)ge-dge+fe^2}\left(2\left(4\,(xe+d)\left(\frac{6\,(xe+d)ce^{(-3)}}{g}-\frac{\left(9\,cdg^6e^6+7\,cfg^5e^7-8\,bg^6e^7\right)e^{(-9)}}{g^7}\right)+\frac{\left(3\,cd^2g^6e^6+10\,c\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*(2*(4*(x*e + d)*(6*(x*e + d)*c*e^
(-3)/g - (9*c*d*g^6*e^6 + 7*c*f*g^5*e^7 - 8*b*g^6*e^7)*e^(-9)/g^7) + (3*c*d
^2*g^6*e^6 + 10*c*d*f*g^5*e^7 - 8*b*d*g^6*e^7 + 35*c*f^2*g^4*e^8 - 40*b*f*g
^5*e^8 + 48*a*g^6*e^8)*e^(-9)/g^7)*(x*e + d) + 3*(3*c*d^3*g^6*e^6 + 7*c*d^2
*f*g^5*e^7 - 8*b*d^2*g^6*e^7 + 25*c*d*f^2*g^4*e^8 - 32*b*d*f*g^5*e^8 + 48*a
*d*g^6*e^8 - 35*c*f^3*g^3*e^9 + 40*b*f^2*g^4*e^9 - 48*a*f*g^5*e^9)*e^(-9)/g
^7)*sqrt(x*e + d) - 1/64*(3*c*d^4*g^4 + 4*c*d^3*f*g^3*e - 8*b*d^3*g^4*e + 1
8*c*d^2*f^2*g^2*e^2 - 24*b*d^2*f*g^3*e^2 + 48*a*d^2*g^4*e^2 - 60*c*d*f^3*g*
e^3 + 72*b*d*f^2*g^2*e^3 - 96*a*d*f*g^3*e^3 + 35*c*f^4*e^4 - 40*b*f^3*g*e^4
 + 48*a*f^2*g^2*e^4)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt
((x*e + d)*g*e - d*g*e + f*e^2)))/g^(9/2)
```

3.836 $\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}} \, dx$

Optimal. Leaf size=246

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{8e^2g^3}-\frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(2eg(4aeg-b(dg\right.}{8e^{5/2}g^{7/2}}$$

```
[Out] ((c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*
Sqrt[d + e*x]*Sqrt[f + g*x])/(8*e^2*g^3) - ((5*c*e*f + 7*c*d*g - 6*b*e*g)*(
d + e*x)^(3/2)*Sqrt[f + g*x])/(12*e^2*g^2) + (c*(d + e*x)^(5/2)*Sqrt[f + g*
x])/(3*e^2*g) - ((e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(
4*a*e*g - b*(3*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
 + g*x])])/(8*e^(5/2)*g^(7/2))
```

Rubi [A] time = 0.255505, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\left(2eg(4aeg-b(dg+3ef))+c\left(d^2g^2+2defg+5e^2f^2\right)\right)}{8e^2g^3}-\frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(2eg(4aeg-b(dg\right.}{8e^{5/2}g^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```


```
[Out] ((c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*
Sqrt[d + e*x]*Sqrt[f + g*x])/(8*e^2*g^3) - ((5*c*e*f + 7*c*d*g - 6*b*e*g)*(
d + e*x)^(3/2)*Sqrt[f + g*x])/(12*e^2*g^2) + (c*(d + e*x)^(5/2)*Sqrt[f + g*
x])/(3*e^2*g) - ((e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(
4*a*e*g - b*(3*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
 + g*x])])/(8*e^(5/2)*g^(7/2))
```


Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\int \frac{\sqrt{d+ex}\left(\frac{1}{2}\left(6ae^2g-cd(5ef+dg)\right)-\frac{1}{2}e(5cef+7cdg-6beg)x\right)}{\sqrt{f+gx}}\,dx}{3e^2g}$$

$$= -\frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\left(c\left(5e^2f^2+2def\right.\right.}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5cef+7c}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5cef+7c}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5cef+7c}{}$$

$$= \frac{\left(c\left(5e^2f^2+2defg+d^2g^2\right)+2eg(4aeg-b(3ef+dg))\right)\sqrt{d+ex}\sqrt{f+gx}}{8e^2g^3} - \frac{(5cef+7c}{}$$

Mathematica [A] time = 1.08633, size = 225, normalized size = 0.91

$$\frac{-e\sqrt{g}\sqrt{d+ex}(f+gx)\left(c\left(3d^2g^2-2deg(gx-2f)+e^2\left(-15f^2+10fgx-8g^2x^2\right)\right)-6eg(4aeg+b(dg-3ef+2egx))\right)-}{24e^3g^{7/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-(e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(-6*e*g*(4*a*e*g + b*(-3*e*f + d*g + 2
*e*g*x)) + c*(3*d^2*g^2 - 2*d*e*g*(-2*f + g*x) + e^2*(-15*f^2 + 10*f*g*x -
8*g^2*x^2)))) - 3*(e*f - d*g)^(3/2)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) +
2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[
(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(24*e^3*g^(7/2)*Sqrt[f + g*x])
```

Maple [B] time = 0.301, size = 763, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(16*x^2*c*e^2*g^2*((g*x+f)*(e*x+d))^(1/2)*
(e*g)^(1/2)+24*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*
f)/(e*g)^(1/2))*a*d*e^2*g^3-24*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e
*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^3*f*g^2-6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e
*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^3-12*ln(1/2*(2*e*g
*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*f*g^
2+18*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(
1/2))*b*e^3*f^2*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*c*d^3*g^3+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e*f*g^2+9*ln(1/2*(2*e*g*x+2*((g*x+f
)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^2*f^2*g-15*ln(1/2*
(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^3*
f^3+24*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*x*b*e^2*g^2+4*(e*g)^(1/2)*((g*x+
f)*(e*x+d))^(1/2)*x*c*d*e*g^2-20*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*x*c*e^
2*f*g+48*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*a*e^2*g^2+12*(e*g)^(1/2)*((g*x
+f)*(e*x+d))^(1/2)*b*d*e*g^2-36*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*b*e^2*f
*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d^2*g^2-8*(e*g)^(1/2)*((g*x+f)*(
e*x+d))^(1/2)*c*d*e*f*g+30*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e^2*f^2)/g
^3/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*g)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12624, size = 1277, normalized size = 5.19

$$\left[-\frac{3\left(5\,ce^{3}f^{3}-3\left(cde^{2}+2\,be^{3}\right)f^{2}g-\left(cd^{2}e-4\,bde^{2}-8\,ae^{3}\right)fg^{2}-\left(cd^{3}-2\,bd^{2}e+8\,ade^{2}\right)g^{3}\right)\sqrt{eg}\log\left(8\,e^{2}g^{2}x^{2}+e^{2}f^{2}+6\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3)*f^2*g - (c*d^2*e - 4*b*d*e^2
 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*g^3)*sqrt(e*g)*log(8*e^
2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*sqrt(e*
g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) - 4*(8*c*e^3*g^3*
x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*e - 2*b*d*e
^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*x)*sqrt(e*x
 + d)*sqrt(g*x + f))/(e^3*g^4), 1/48*(3*(5*c*e^3*f^3 - 3*(c*d*e^2 + 2*b*e^3
)*f^2*g - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f*g^2 - (c*d^3 - 2*b*d^2*e + 8*a*
d*e^2)*g^3)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x
 + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) + 2*(8
*c*e^3*g^3*x^2 + 15*c*e^3*f^2*g - 2*(2*c*d*e^2 + 9*b*e^3)*f*g^2 - 3*(c*d^2*
e - 2*b*d*e^2 - 8*a*e^3)*g^3 - 2*(5*c*e^3*f*g^2 - (c*d*e^2 + 6*b*e^3)*g^3)*
x)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23661, size = 393, normalized size = 1.6

$$\frac{1}{24}\sqrt{(xe+d)ge-dge+fe^2}\left(2\,(xe+d)\left(\frac{4\,(xe+d)ce^{(-3)}}{g}-\frac{\left(7\,cdg^4e^6+5\,cfg^3e^7-6\,bg^4e^7\right)e^{(-9)}}{g^5}\right)+\frac{3\,\left(cd^2g^4e^6+2\,cdfg^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*(2*(x*e + d)*(4*(x*e + d)*c*e^(-3)
/g - (7*c*d*g^4*e^6 + 5*c*f*g^3*e^7 - 6*b*g^4*e^7)*e^(-9)/g^5) + 3*(c*d^2*g
^4*e^6 + 2*c*d*f*g^3*e^7 - 2*b*d*g^4*e^7 + 5*c*f^2*g^2*e^8 - 6*b*f*g^3*e^8
+ 8*a*g^4*e^8)*e^(-9)/g^5)*sqrt(x*e + d) - 1/8*(c*d^3*g^3 + c*d^2*f*g^2*e -
 2*b*d^2*g^3*e + 3*c*d*f^2*g*e^2 - 4*b*d*f*g^2*e^2 + 8*a*d*g^3*e^2 - 5*c*f^
3*e^3 + 6*b*f^2*g*e^3 - 8*a*f*g^2*e^3)*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt
(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g*e + f*e^2)))/g^(7/2)
```

3.837 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{4e^2g^2}+\frac{c(\ldots}{\ldots}$$

```
[Out] -((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) +
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rubi [A] time = 0.142798, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {951, 80, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{4e^2g^2}+\frac{c(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(4*e^2*g^2) +
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg+3d^2g\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg+3d^2g\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg+3d^2g\right.\right.}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\left(c\left(3e^2f^2+2defg+3d^2g\right.\right.}{}$$

Mathematica [A] time = 0.71818, size = 173, normalized size = 1.05

$$\frac{\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)\left(4eg(2aeg-b(dg+ef))+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)+e\sqrt{g}\sqrt{d+ex}(f+gx)(4b}{4e^3g^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (e*Sqrt[g]*Sqrt[d + e*x]*(f + g*x)*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x))
 + Sqrt[e*f - d*g]*(c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g
- b*(e*f + d*g)))*Sqrt[(e*(f + g*x))/(e*f - d*g)]*ArcSinh[(Sqrt[g]*Sqrt[d +
 e*x])/Sqrt[e*f - d*g]])/(4*e^3*g^(5/2)*Sqrt[f + g*x])
```

Maple [B] time = 0.336, size = 425, normalized size = 2.6

$$\frac{1}{8e^2g^2}\left(8\ln\left(1/2\,\frac{2\,egx+2\,\sqrt{(gx+f)(ex+d)}\sqrt{eg}+dg+ef}{\sqrt{eg}}\right)ae^2g^2-4\ln\left(1/2\,\frac{2\,egx+2\,\sqrt{(gx+f)(ex+d)}\sqrt{eg}+dg}{\sqrt{eg}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/8*(8*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)
^(1/2))*a*e^2*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^2*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e
*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2*e*g*x+
2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+3*ln(
1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*
e^2*f^2+4*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*x*c*e*g+8*(e*g)^(1/2)*((g*x+f
)*(e*x+d))^(1/2)*b*e*g-6*(e*g)^(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*d*g-6*(e*g)^
(1/2)*((g*x+f)*(e*x+d))^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/g^2/e^2/((g*x+f)*(e*x+d))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99842, size = 879, normalized size = 5.36

$$\left[\frac{\left(3\,ce^2f^2+2\left(cde-2\,be^2\right)fg+\left(3\,cd^2-4\,bde+8\,ae^2\right)g^2\right)\sqrt{eg}\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\left(2\,egx+ef+dg\right)\right.}{16\,e^3g^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^
2)*g^2)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*
e*g*x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e
*g^2)*x) + 4*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^2)*g^2)*sqrt(e
*x + d)*sqrt(g*x + f))/(e^3*g^3), -1/8*((3*c*e^2*f^2 + 2*(c*d*e - 2*b*e^2)*
f*g + (3*c*d^2 - 4*b*d*e + 8*a*e^2)*g^2)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e
*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (
e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e^2*f*g - (3*c*d*e - 4*b*e^
2)*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [A] time = 1.18298, size = 242, normalized size = 1.48

$$\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\sqrt{xe+d}\left(\frac{2(xe+d)ce^{(-3)}}{g}-\frac{\left(5cdg^2e^5+3cfge^6-4bg^2e^6\right)e^{(-8)}}{g^3}\right)-\frac{\left(3cd^2g^2+2cdfge-4bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*(2*(x*e + d)*c*e^(-3)
/g - (5*c*d*g^2*e^5 + 3*c*f*g*e^6 - 4*b*g^2*e^6)*e^(-8)/g^3) - 1/4*(3*c*d^2
*g^2 + 2*c*d*f*g*e - 4*b*d*g^2*e + 3*c*f^2*e^2 - 4*b*f*g*e^2 + 8*a*g^2*e^2)
*e^(-5/2)*log(abs(-sqrt(x*e + d)*sqrt(g)*e^(1/2) + sqrt((x*e + d)*g*e - d*g
*e + f*e^2)))/g^(5/2)
```

3.838 $\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=129

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)}-\frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}$$

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e*f - d*g)*Sqrt[d + e*x]) +
(c*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g) - ((c*e*f + 3*c*d*g - 2*b*e*g)*ArcT
anh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*g^(3/2))
```

Rubi [A] time = 0.134899, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {949, 80, 63, 217, 206}

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)}-\frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e*f - d*g)*Sqrt[d + e*x]) +
(c*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g) - ((c*e*f + 3*c*d*g - 2*b*e*g)*ArcT
anh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*g^(3/2))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx &= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}-\frac{2\int \frac{\frac{(cd-be)(ef-dg)}{2e^2}-\frac{c(ef-dg)x}{2e}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{ef-dg}\\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}-\frac{(cef+3cdg-2beg)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}\\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}-\frac{(cef+3cdg-2beg)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx}{e}}}\right.}{e^3g}\\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}-\frac{(cef+3cdg-2beg)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx,\right.}{e^3g}\\
&= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}-\frac{(cef+3cdg-2beg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.617401, size = 222, normalized size = 1.72

$$-\frac{2\sqrt{f+gx}\left(e\sqrt{ef-dg}\sqrt{\frac{e(f+gx)}{ef-dg}}\left(g^2(ae-bd)+cf(2dg-ef)\right)+e\sqrt{g}\sqrt{d+ex}(2cf-bg)(ef-dg)\sinh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)+c\right.}{e^2g^2\sqrt{d+ex}(ef-dg)^{3/2}\sqrt{\frac{e(f+gx)}{ef-dg}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(e*Sqrt[e*f - d*g]*((-(b*d) + a*e)*g^2 + c*f*(-(e*f) + 2*
d*g))*Sqrt[(e*(f + g*x))/(e*f - d*g)] + e*Sqrt[g]*(2*c*f - b*g)*(e*f - d*g)
*Sqrt[d + e*x]*ArcSinh[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]] + c*(e*f -
d*g)^(5/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (g*(d + e*x))/(-(e*f) + d*g)]
))/(e^2*g^2*(e*f - d*g)^(3/2)*Sqrt[d + e*x]*Sqrt[(e*(f + g*x))/(e*f - d*g)]
)
```

Maple [B] time = 0.347, size = 697, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*x*b*d*e^2*g^2-2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(
1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x*b*e^3*f*g-3*ln(1/2*(2*e*g*x+2*((g*
x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x*c*d^2*e*g^2+2*ln(1/
2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x*c*
d*e^2*f*g+ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e
*g)^(1/2))*x*c*e^3*f^2+2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1
/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^2-2*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*f*g-3*ln(1/2*(2*e*g*x+2*((
g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^3*g^2+2*ln(1/2*
(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*
e*f*g+ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^
(1/2))*c*d*e^2*f^2+2*x*c*d*e*g*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-2*x*c*e^
2*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+4*a*e^2*g*((g*x+f)*(e*x+d))^(1/2)*(
e*g)^(1/2)-4*b*d*e*g*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+6*c*d^2*g*((g*x+f)
*(e*x+d))^(1/2)*(e*g)^(1/2)-2*c*d*e*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2))/
g/(e*g)^(1/2)/(d*g-e*f)/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*x+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.91926, size = 1256, normalized size = 9.74

$$\left[-\frac{\left(cde^2f^2+2\left(cd^2e-bde^2\right)fg-\left(3\,cd^3-2\,bd^2e\right)g^2+\left(ce^3f^2+2\left(cde^2-be^3\right)fg-\left(3\,cd^2e-2\,bde^2\right)g^2\right)x\right)\sqrt{eg}\log\left(8\,e^2g^2x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - 2*b*d^2*e)*g^2
 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - 2*b*d*e^2)*g^2)*x)*s
qrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e
*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x)
- 4*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g - c*d
*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3 + (e^5
*f*g^2 - d*e^4*g^3)*x), 1/2*((c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*
c*d^3 - 2*b*d^2*e)*g^2 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e
- 2*b*d*e^2)*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)
*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x
)) + 2*(c*d*e^2*f*g - (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*g^2 + (c*e^3*f*g -
```

```
c*d*e^2*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d*e^4*f*g^2 - d^2*e^3*g^3 + (
e^5*f*g^2 - d*e^4*g^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(3/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Giac [A] time = 1.27301, size = 271, normalized size = 2.1

$$\frac{\sqrt{(xe+d)ge-dge+fe^2}\sqrt{xe+d}ce^{(-3)}}{g}+\frac{4\left(cd^2\sqrt{g}e^{\frac{1}{2}}-bd\sqrt{g}e^{\frac{3}{2}}+a\sqrt{g}e^{\frac{5}{2}}\right)e^{(-2)}}{dge+\left(\sqrt{xe+d}\sqrt{g}e^{\frac{1}{2}}-\sqrt{(xe+d)ge-dge+fe^2}\right)^2-fe^2}+\frac{\left(3\,cdg^{\frac{3}{2}}e^{\frac{1}{2}}+cf\sqrt{\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt((x*e + d)*g*e - d*g*e + f*e^2)*sqrt(x*e + d)*c*e^(-3)/g + 4*(c*d^2*sqr
t(g)*e^(1/2) - b*d*sqrt(g)*e^(3/2) + a*sqrt(g)*e^(5/2))*e^(-2)/(d*g*e + (sq
rt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2 - f*e^
2) + 1/2*(3*c*d*g^(3/2)*e^(1/2) + c*f*sqrt(g)*e^(3/2) - 2*b*g^(3/2)*e^(3/2)
)*e^(-3)*log((sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e +
f*e^2))^2)/g^2
```

3.839 $\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{f+gx}\left(c\left(6def-4d^2g\right)-e(-2aeg-bdg+3bef)\right)}{3e^2\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)
) + (2*(c*(6*d*e*f - 4*d^2*g) - e*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x
])/(3*e^2*(e*f - d*g)^2*Sqrt[d + e*x]) + (2*c*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*Sqrt[g])
```

Rubi [A] time = 0.180485, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {949, 78, 63, 217, 206}

$$\frac{2\sqrt{f+gx}\left(c\left(6def-4d^2g\right)-e(-2aeg-bdg+3bef)\right)}{3e^2\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)
) + (2*(c*(6*d*e*f - 4*d^2*g) - e*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x
])/(3*e^2*(e*f - d*g)^2*Sqrt[d + e*x]) + (2*c*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*Sqrt[g])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} - \frac{2\int \frac{\frac{cd(3ef-dg)-e(3bef-bdg-2aeg)}{2e^2}-\frac{3}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{3(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{c\int \cdots}{\cdots}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{(2c)\,\mathrm{S}\cdots}{\cdots}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{(2c)\,\mathrm{S}\cdots}{\cdots}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2\left(c\left(6def-4d^2g\right)-e(3bef-bdg-2aeg)\right)\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}} + \frac{2c\,\mathrm{ta}\cdots}{\cdots}$$

Mathematica [C] time = 0.2441, size = 173, normalized size = 1.08

$$\frac{2\sqrt{f+gx}\left(2g(d+ex)\left(g(ag-bf)+cf^2\right)-(ef-dg)\left(g(ag-bf)+cf^2\right)+(f+gx)(2cf-bg)(ef-dg)-\frac{c(ef-dg)^3\,{}_2F_1\left(-\cdots\right)}{e^2\sqrt{\cdots}}\right)}{3g^2(d+ex)^{3/2}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(-((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) + 2*g*(c*f^2 +
g*(-(b*f) + a*g))*(d + e*x) + (2*c*f - b*g)*(e*f - d*g)*(f + g*x) - (c*(e*f
 - d*g)^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (g*(d + e*x))/(-(e*f) + d*g)]
)/(e^2*Sqrt[(e*(f + g*x))/(e*f - d*g)])))/(3*g^2*(e*f - d*g)^2*(d + e*x)^(3
/2))
```

Maple [B] time = 0.327, size = 773, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/3*(g*x+f)^(1/2)*(3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*x^2*c*d^2*e^2*g^2-6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d
))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x^2*c*d*e^3*f*g+3*ln(1/2*(2*e*g*
x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x^2*c*e^4*f^2
+6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/
2))*x*c*d^3*e*g^2-12*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*x*c*d^2*e^2*f*g+6*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))
^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x*c*d*e^3*f^2+3*ln(1/2*(2*e*g*x+2*
((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^4*g^2-6*ln(1/
2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^
3*e*f*g+3*ln(1/2*(2*e*g*x+2*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e
*g)^(1/2))*c*d^2*e^2*f^2+4*x*a*e^3*g*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+2*
x*b*d*e^2*g*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-6*x*b*e^3*f*((g*x+f)*(e*x+d
))^(1/2)*(e*g)^(1/2)-8*x*c*d^2*e*g*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+12*x
*c*d*e^2*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)+6*a*d*e^2*g*((g*x+f)*(e*x+d)
)^(1/2)*(e*g)^(1/2)-2*a*e^3*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-4*b*d*e^2
*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2)-6*c*d^3*g*((g*x+f)*(e*x+d))^(1/2)*(e
*g)^(1/2)+10*c*d^2*e*f*((g*x+f)*(e*x+d))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2)/(d*
g-e*f)^2/((g*x+f)*(e*x+d))^(1/2)/e^2/(e*x+d)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 26.9215, size = 1671, normalized size = 10.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g^2 + (c*e^4*f^2 - 2*c*d*e^3
*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c*d^2*e^2*f*g + c*d^3*e*g^2)
*x)*sqrt(e*g)*log(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*
x + e*f + d*g)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2
)*x) + 4*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*e^3)*g^2
 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*e^3*g^3
+ (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g - 2*d^2*e^
```

```
5*f*g^2 + d^3*e^4*g^3)*x), -1/3*(3*(c*d^2*e^2*f^2 - 2*c*d^3*e*f*g + c*d^4*g
^2 + (c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x^2 + 2*(c*d*e^3*f^2 - 2*c
*d^2*e^2*f*g + c*d^3*e*g^2)*x)*sqrt(-e*g)*arctan(1/2*(2*e*g*x + e*f + d*g)*
sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(e^2*g^2*x^2 + d*e*f*g + (e^2*f*g +
d*e*g^2)*x)) - 2*((5*c*d^2*e^2 - 2*b*d*e^3 - a*e^4)*f*g - 3*(c*d^3*e - a*d*
e^3)*g^2 + (3*(2*c*d*e^3 - b*e^4)*f*g - (4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*g
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(d^2*e^5*f^2*g - 2*d^3*e^4*f*g^2 + d^4*
e^3*g^3 + (e^7*f^2*g - 2*d*e^6*f*g^2 + d^2*e^5*g^3)*x^2 + 2*(d*e^6*f^2*g -
2*d^2*e^5*f*g^2 + d^3*e^4*g^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+cx^2}{(d+ex)^{\frac{5}{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(5/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(5/2)*sqrt(f + g*x)), x)
```

Giac [B] time = 1.4474, size = 680, normalized size = 4.25

$$-\frac{ce^{\left(-\frac{5}{2}\right)}\log\left(\left(\sqrt{xe+d}\sqrt{g}e^{\frac{1}{2}}-\sqrt{(xe+d)ge-dge+fe^2}\right)^2\right)}{\sqrt{g}}-\frac{4\left(4\,cd^3g^{\frac{5}{2}}e^{\frac{5}{2}}+6\left(\sqrt{xe+d}\sqrt{g}e^{\frac{1}{2}}-\sqrt{(xe+d)ge-dge+f}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -c*e^(-5/2)*log((sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
 + f*e^2))^2)/sqrt(g) - 4/3*(4*c*d^3*g^(5/2)*e^(5/2) + 6*(sqrt(x*e + d)*sqr
t(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*g^(3/2)*e^(3/2)
 + 6*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
4*c*d*sqrt(g)*e^(1/2) - 10*c*d^2*f*g^(3/2)*e^(7/2) - b*d^2*g^(5/2)*e^(7/2)
- 12*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
2*c*d*f*sqrt(g)*e^(5/2) - 3*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)
*g*e - d*g*e + f*e^2))^4*b*sqrt(g)*e^(3/2) + 6*c*d*f^2*sqrt(g)*e^(9/2) + 4*
b*d*f*g^(3/2)*e^(9/2) - 2*a*d*g^(5/2)*e^(9/2) + 6*(sqrt(x*e + d)*sqrt(g)*e^
(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*f*sqrt(g)*e^(7/2) - 6*(sqr
t(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*g^(3/
2)*e^(7/2) - 3*b*f^2*sqrt(g)*e^(11/2) + 2*a*f*g^(3/2)*e^(11/2))*e^(-2)/(d*g
*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^
2 - f*e^2)^3
```

3.840 $\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=198

$$\frac{2\sqrt{f+gx}\left(2eg(-4aeg-bdg+5bef)-c\left(3d^2g^2-10defg+15e^2f^2\right)\right)}{15e^2\sqrt{d+ex}(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}+\frac{2\sqrt{f+gx}(2cd(5ef-3}{15e^2(d+}$$

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(5*(e*f - d*g)*(d + e*x)^(5/2)
) + (2*(2*c*d*(5*e*f - 3*d*g) - e*(5*b*e*f - b*d*g - 4*a*e*g))*Sqrt[f + g*x
])/(15*e^2*(e*f - d*g)^2*(d + e*x)^(3/2)) + (2*(2*e*g*(5*b*e*f - b*d*g - 4*
a*e*g) - c*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(15*e^2*(e
*f - d*g)^3*Sqrt[d + e*x])
```

Rubi [A] time = 0.212091, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {949, 78, 37}

$$\frac{2\sqrt{f+gx}\left(2eg(-4aeg-bdg+5bef)-c\left(3d^2g^2-10defg+15e^2f^2\right)\right)}{15e^2\sqrt{d+ex}(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}+\frac{2\sqrt{f+gx}(2cd(5ef-3}{15e^2(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(5*(e*f - d*g)*(d + e*x)^(5/2)
) + (2*(2*c*d*(5*e*f - 3*d*g) - e*(5*b*e*f - b*d*g - 4*a*e*g))*Sqrt[f + g*x
])/(15*e^2*(e*f - d*g)^2*(d + e*x)^(3/2)) + (2*(2*e*g*(5*b*e*f - b*d*g - 4*
a*e*g) - c*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(15*e^2*(e
*f - d*g)^3*Sqrt[d + e*x])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} - \frac{2\int \frac{\frac{cd(5ef-dg)-e(5bef-bdg-4aeg)}{2e^2}-\frac{5}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{5(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}} - \frac{(2eg(5b \cdots}{\cdots}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}} + \frac{2(2eg(\cdots}{\cdots}$$

Mathematica [A] time = 0.239307, size = 178, normalized size = 0.9

$$-\frac{2\sqrt{f+gx}\left(a\left(15d^2g^2-10deg(f-2gx)+e^2\left(3f^2-4fgx+8g^2x^2\right)\right)+b\left(5d^2g(gx-2f)+2de\left(f^2-13fgx+g^2x^2\right)+\cdots\right.\right.}{15(d+ex)^{5/2}(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(b*(5*e^2*f*x*(f - 2*g*x) + 5*d^2*g*(-2*f + g*x) + 2*d*e*
(f^2 - 13*f*g*x + g^2*x^2)) + c*(15*e^2*f^2*x^2 + 10*d*e*f*x*(2*f - g*x) +
d^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + a*(15*d^2*g^2 - 10*d*e*g*(f - 2*g*x) +
 e^2*(3*f^2 - 4*f*g*x + 8*g^2*x^2))))/(15*(e*f - d*g)^3*(d + e*x)^(5/2))
```

Maple [A] time = 0.057, size = 238, normalized size = 1.2

$$\frac{16\,ae^2g^2x^2+4\,bdeg^2x^2-20\,be^2fgx^2+6\,cd^2g^2x^2-20\,cdefgx^2+30\,ce^2f^2x^2+40\,adeg^2x-8\,ae^2fgx+10\,bd^2g^2x-52\cdots}{15\,g^3d^3-45\,d^2efg^2+45\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(8*a*e^2*g^2*x^2+2*b*d*e*g^2*x^2-10*b*e^2*f*g*x^2+3*c*d^
2*g^2*x^2-10*c*d*e*f*g*x^2+15*c*e^2*f^2*x^2+20*a*d*e*g^2*x-4*a*e^2*f*g*x+5*
b*d^2*g^2*x-26*b*d*e*f*g*x+5*b*e^2*f^2*x-4*c*d^2*f*g*x+20*c*d*e*f^2*x+15*a*
d^2*g^2-10*a*d*e*f*g+3*a*e^2*f^2-10*b*d^2*f*g+2*b*d*e*f^2+8*c*d^2*f^2)/(e*x
+d)^(5/2)/(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 76.2546, size = 740, normalized size = 3.74

$$-\frac{2\left(15\,ad^2g^2+\left(8\,cd^2+2\,bde+3\,ae^2\right)f^2-10\left(bd^2+ade\right)fg+\left(15\,ce^2f^2-10\left(cde+be^2\right)fg+\left(3\,cd^2+2\,bde+8\,ae^2\right)g^2\right)x\cdots}{15\left(d^3e^3f^3-3\,d^4e^2f^2g+3\,d^5efg^2-d^6g^3+\left(e^6f^3-3\,de^5f^2g+3\,d^2e^4fg^2-d^3e^3g^3\right)x^3+3\left(de^5f^3-3\,d^2e\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*a*d^2*g^2 + (8*c*d^2 + 2*b*d*e + 3*a*e^2)*f^2 - 10*(b*d^2 + a*d*e
)*f*g + (15*c*e^2*f^2 - 10*(c*d*e + b*e^2)*f*g + (3*c*d^2 + 2*b*d*e + 8*a*e
^2)*g^2)*x^2 + (5*(4*c*d*e + b*e^2)*f^2 - 2*(2*c*d^2 + 13*b*d*e + 2*a*e^2)*
f*g + 5*(b*d^2 + 4*a*d*e)*g^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(d^3*e^3*f^3
- 3*d^4*e^2*f^2*g + 3*d^5*e*f*g^2 - d^6*g^3 + (e^6*f^3 - 3*d*e^5*f^2*g + 3*
d^2*e^4*f*g^2 - d^3*e^3*g^3)*x^3 + 3*(d*e^5*f^3 - 3*d^2*e^4*f^2*g + 3*d^3*e
^3*f*g^2 - d^4*e^2*g^3)*x^2 + 3*(d^2*e^4*f^3 - 3*d^3*e^3*f^2*g + 3*d^4*e^2*
f*g^2 - d^5*e*g^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(7/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.58423, size = 1458, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(3*c*d^4*g^(9/2)*e^(9/2) + 30*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x
*e + d)*g*e - d*g*e + f*e^2))^4*c*d^2*g^(5/2)*e^(5/2) + 15*(sqrt(x*e + d)*s
qrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c*sqrt(g)*e^(1/2) -
 16*c*d^3*f*g^(7/2)*e^(11/2) + 2*b*d^3*g^(9/2)*e^(11/2) - 20*(sqrt(x*e + d)
*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*f*g^(5/2)*e
^(9/2) + 10*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^2*b*d^2*g^(7/2)*e^(9/2) - 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((
x*e + d)*g*e - d*g*e + f*e^2))^4*c*d*f*g^(3/2)*e^(7/2) - 10*(sqrt(x*e + d)*
sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*d*g^(5/2)*e^(7/2
) - 60*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2)
)^6*c*f*sqrt(g)*e^(5/2) + 30*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d
)*g*e - d*g*e + f*e^2))^6*b*g^(3/2)*e^(5/2) + 38*c*d^2*f^2*g^(5/2)*e^(13/2)
 - 14*b*d^2*f*g^(7/2)*e^(13/2) + 8*a*d^2*g^(9/2)*e^(13/2) + 80*(sqrt(x*e +
```

```
d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d*f^2*g^(3/2)
*e^(11/2) - 60*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e
+ f*e^2))^2*b*d*f*g^(5/2)*e^(11/2) + 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sq
rt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d*g^(7/2)*e^(11/2) + 90*(sqrt(x*e +
d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*f^2*sqrt(g)*e
^(9/2) - 70*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f
*e^2))^4*b*f*g^(3/2)*e^(9/2) + 80*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*
e + d)*g*e - d*g*e + f*e^2))^4*a*g^(5/2)*e^(9/2) - 40*c*d*f^3*g^(3/2)*e^(15
/2) + 22*b*d*f^2*g^(5/2)*e^(15/2) - 16*a*d*f*g^(7/2)*e^(15/2) - 60*(sqrt(x*
e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*f^3*sqrt(
g)*e^(13/2) + 50*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*
e + f*e^2))^2*b*f^2*g^(3/2)*e^(13/2) - 40*(sqrt(x*e + d)*sqrt(g)*e^(1/2) -
sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*f*g^(5/2)*e^(13/2) + 15*c*f^4*sqrt
(g)*e^(17/2) - 10*b*f^3*g^(3/2)*e^(17/2) + 8*a*f^2*g^(5/2)*e^(17/2))*e^(-2)
/(d*g*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e
^2))^2 - f*e^2)^5
```

3.841 $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=281

$$-\frac{4g\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)}{105e^2\sqrt{d+ex}(ef-dg)^4}+\frac{2\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g\right.\right.}{105e^2(d+ex)^{3/2}(ef-dg}$$

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(7*(e*f - d*g)*(d + e*x)^(7/2)
) + (2*(2*c*d*(7*e*f - 4*d*g) - e*(7*b*e*f - b*d*g - 6*a*e*g))*Sqrt[f + g*x
])/(35*e^2*(e*f - d*g)^2*(d + e*x)^(5/2)) + (2*(4*e*g*(7*b*e*f - b*d*g - 6*
a*e*g) - c*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(
e*f - d*g)^3*(d + e*x)^(3/2)) - (4*g*(4*e*g*(7*b*e*f - b*d*g - 6*a*e*g) - c
*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(e*f - d*g)
^4*Sqrt[d + e*x])
```

Rubi [A] time = 0.291033, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {949, 78, 45, 37}

$$-\frac{4g\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g^2-14defg+35e^2f^2\right)\right)}{105e^2\sqrt{d+ex}(ef-dg)^4}+\frac{2\sqrt{f+gx}\left(4eg(-6aeg-bdg+7bef)-c\left(3d^2g\right.\right.}{105e^2(d+ex)^{3/2}(ef-dg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(7*(e*f - d*g)*(d + e*x)^(7/2)
) + (2*(2*c*d*(7*e*f - 4*d*g) - e*(7*b*e*f - b*d*g - 6*a*e*g))*Sqrt[f + g*x
])/(35*e^2*(e*f - d*g)^2*(d + e*x)^(5/2)) + (2*(4*e*g*(7*b*e*f - b*d*g - 6*
a*e*g) - c*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(
e*f - d*g)^3*(d + e*x)^(3/2)) - (4*g*(4*e*g*(7*b*e*f - b*d*g - 6*a*e*g) - c
*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(e*f - d*g)
^4*Sqrt[d + e*x])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} - \frac{2\int \frac{\frac{cd(7ef-dg)-e(7bef-bdg-6aeg)}{2e^2}-\frac{7}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx}{7(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} - \frac{(4eg(7b}{}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} + \frac{2(4eg(}{}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}} + \frac{2(4eg(}{}$$

Mathematica [A] time = 0.365036, size = 332, normalized size = 1.18

$$2\sqrt{f+gx}\left(3a\left(-35d^2eg^2(f-2gx)+35d^3g^3+7de^2g\left(3f^2-4fgx+8g^2x^2\right)+e^3\left(6f^2gx-5f^3-8fg^2x^2+16g^3x^3\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*(-35*e^3*f^2*x^2*(f - 2*g*x) + 7*d^3*g*(8*f^2 - 4*f*g*x
 + 3*g^2*x^2) - 7*d*e^2*f*x*(4*f^2 - 37*f*g*x + 4*g^2*x^2) + d^2*e*(-8*f^3
+ 200*f^2*g*x - 101*f*g^2*x^2 + 6*g^3*x^3)) + b*(35*d^3*g^2*(-2*f + g*x) +
7*d^2*e*g*(4*f^2 - 37*f*g*x + 4*g^2*x^2) - 7*e^3*f*x*(3*f^2 - 4*f*g*x + 8*g
^2*x^2) + d*e^2*(-6*f^3 + 101*f^2*g*x - 200*f*g^2*x^2 + 8*g^3*x^3)) + 3*a*(
35*d^3*g^3 - 35*d^2*e*g^2*(f - 2*g*x) + 7*d*e^2*g*(3*f^2 - 4*f*g*x + 8*g^2*
x^2) + e^3*(-5*f^3 + 6*f^2*g*x - 8*f*g^2*x^2 + 16*g^3*x^3))))/(105*(e*f - d
*g)^4*(d + e*x)^(7/2))
```

Maple [A] time = 0.056, size = 468, normalized size = 1.7

$$96\,ae^3g^3x^3+16\,bde^2g^3x^3-112\,be^3fg^2x^3+12\,cd^2eg^3x^3-56\,cde^2fg^2x^3+140\,ce^3f^2gx^3+336\,ade^2g^3x^2-48\,ae^3fg^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x)
```

```
[Out] 2/105*(g*x+f)^(1/2)*(48*a*e^3*g^3*x^3+8*b*d*e^2*g^3*x^3-56*b*e^3*f*g^2*x^3+
6*c*d^2*e*g^3*x^3-28*c*d*e^2*f*g^2*x^3+70*c*e^3*f^2*g*x^3+168*a*d*e^2*g^3*x
^2-24*a*e^3*f*g^2*x^2+28*b*d^2*e*g^3*x^2-200*b*d*e^2*f*g^2*x^2+28*b*e^3*f^2
*g*x^2+21*c*d^3*g^3*x^2-101*c*d^2*e*f*g^2*x^2+259*c*d*e^2*f^2*g*x^2-35*c*e^
3*f^3*x^2+210*a*d^2*e*g^3*x-84*a*d*e^2*f*g^2*x+18*a*e^3*f^2*g*x+35*b*d^3*g^
3*x-259*b*d^2*e*f*g^2*x+101*b*d*e^2*f^2*g*x-21*b*e^3*f^3*x-28*c*d^3*f*g^2*x
+200*c*d^2*e*f^2*g*x-28*c*d*e^2*f^3*x+105*a*d^3*g^3-105*a*d^2*e*f*g^2+63*a*
d*e^2*f^2*g-15*a*e^3*f^3-70*b*d^3*f*g^2+28*b*d^2*e*f^2*g-6*b*d*e^2*f^3+56*c
*d^3*f^2*g-8*c*d^2*e*f^3)/(e*x+d)^(7/2)/(d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^
2*g^2-4*d*e^3*f^3*g+e^4*f^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(9/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.01607, size = 2522, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="giac")

[Out] 8/105*(3*c*d^5*g^(13/2)*e^(11/2) + 21*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^4*g^(11/2)*e^(9/2) - 42*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d^3*g^(9/2)*e^(7/2) + 210*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c*d^2*g^(7/2)*e^(5/2) - 105*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c*d*g^(5/2)*e^(3/2) + 105*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^10*c*g^(3/2)*e^(1/2) - 23*c*d^4*f*g^(11/2)*e^(13/2) + 4*b*d^4*g^(13/2)*e^(13/2) - 140*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^3*f*g^(9/2)*e^(11/2) + 28*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*d^3*g^(11/2)*e^(11/2) - 42*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d^2*f*g^(7/2)*e^(9/2) + 84*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*d^2*g^(9/2)*e^(9/2) - 140*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c*d*f*g^(5/2)*e^(7/2) - 140*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*b*d*g^(7/2)*e^(7/2) - 455*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*c*f*g^(3/2)*e^(5/2) + 280*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^8*b*g^(5/2)*e^(5/2) + 86*c*d^3*f^2*g^(9/2)*e^(15/2) - 40*b*d^3*f*g^(11/2)*e^(15/2) + 24*a*d^3*g^(13/2)*e^(15/2) + 462*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d^2*f^2*g^(7/2)*e^(13/2) - 252*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*d^2*f*g^(9/2)*e^(13/2) + 168*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d^2*g^(11/2)*e^(13/2) + 714*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*d*f^2*g^(5/2)*e^(11/2) - 672*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*d*f*g^(7/2)*e^(11/2) + 504*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*d*g^(9/2)*e^(11/2) + 770*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*c*f^2*g^(3/2)*e^(9/2) - 700*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*b*f*g^(5/2)*e^(9/2) + 840*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^6*a*g^(7/2)*e^(9/2) - 150*c*d^2*f^3*g^(7/2)*e^(17/2) + 96*b*d^2*f^2*g^(9/2)*e^(17/2) - 72*a*d^2*f*g^(11/2)*e^(17/2) - 588*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*d*f^3*g^(5/2)*e^(15/2) + 420*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*d*f^2*g^(7/2)*e^(15/2) - 336*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*d*f*g^(9/2)*e^(15/2) - 630*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*c*f^3*g^(3/2)*e^(13/2) + 588*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*b*f^2*g^(5/2)*e^(13/2) - 504*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^4*a*f*g^(7/2)*e^(13/2) + 119*c*d*f^4*g^(5/2)*e^(19/2) - 88*b*d*f^3*g^(7/2)*e^(19/2) + 72*a*d*f^2*g^(9/2)*e^(19/2) + 245*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*c*f^4*g^(3/2)*e^(17/2) - 196*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*b*f^3*g^(5/2)*e^(17/2) + 168*(sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2*a*f^2*g^(7/2)*e^(17/2) - 35*c*f^5*g^(3/2)*e^(21/2) + 28*b*f^4*g^(5/2)*e^(21/2) - 24*a*f^3*g^(7/2)*e^(21/2))*e^(-1)/(d*g*e + (sqrt(x*e + d)*sqrt(g)*e^(1/2) - sqrt((x*e + d)*g*e - d*g*e + f*e^2))^2 - f*e^2)^7

3.842 $\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx$

Optimal. Leaf size=249

$$\frac{\sqrt{d+ex}\sqrt{e+fx}\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{4ef^3\left(e^2-df\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)\left(4ef\left(-2aef-bdf+3be^2\right.\right.}{4e^{3/2}f^{7/2}}$$

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(3/2))/((e^2 - d*f)*Sqrt[e + f*x]) +
 ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sq
rt[d + e*x]*Sqrt[e + f*x])/(4*e*f^3*(e^2 - d*f)) + (c*(d + e*x)^(3/2)*Sqrt[
e + f*x])/(2*e*f^2) - ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d
*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[e + f*x])]
)/(4*e^(3/2)*f^(7/2))
```

Rubi [A] time = 0.280444, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {949, 80, 50, 63, 217, 206}

$$\frac{\sqrt{d+ex}\sqrt{e+fx}\left(4ef\left(-2aef-bdf+3be^2\right)-c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{4ef^3\left(e^2-df\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)\left(4ef\left(-2aef-bdf+3be^2\right.\right.}{4e^{3/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(3/2))/((e^2 - d*f)*Sqrt[e + f*x]) +
 ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sq
rt[d + e*x]*Sqrt[e + f*x])/(4*e*f^3*(e^2 - d*f)) + (c*(d + e*x)^(3/2)*Sqrt[
e + f*x])/(2*e*f^2) - ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d
*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[e + f*x])]
)/(4*e^(3/2)*f^(7/2))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{2\int \frac{\sqrt{d+ex}\left(\frac{f\left(3be^2-bdf-2aef\right)-c\left(3e^3-def\right)}{2f^2}-\frac{1}{2}c\left(d-\frac{e^2}{f}\right)x\right)}{\sqrt{e+fx}}\,dx}{e^2-df}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e\right.\right.}{4ef^2\left(e^2\right.}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2f^2\right)\right)\sqrt{d}}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2f^2\right)\right)\sqrt{d}}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2f^2\right)\right)\sqrt{d}}{4ef^3\left(e^2-df\right)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{\left(e^2-df\right)\sqrt{e+fx}} + \frac{\left(4ef\left(3be^2-bdf-2aef\right)-c\left(15e^4-6de^2f-d^2f^2\right)\right)\sqrt{d}}{4ef^3\left(e^2-df\right)}$$

Mathematica [A] time = 1.17084, size = 196, normalized size = 0.79

$$\frac{\frac{\sqrt{e^2-df}\sqrt{\frac{e(e+fx)}{e^2-df}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e^2-df}}\right)\left(4ef\left(2aef+bdf-3be^2\right)+c\left(-d^2f^2-6de^2f+15e^4\right)\right)}{e} + \sqrt{f}\sqrt{d+ex}\left(4ef(-2af+3be+bfx)+c\left(ef\left(d+2f\right.\right.\right.}{4ef^{7/2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (Sqrt[f]*Sqrt[d + e*x]*(4*e*f*(3*b*e - 2*a*f + b*f*x) + c*(-15*e^3 - 5*e^2*
f*x + d*f^2*x + e*f*(d + 2*f*x^2))) + (Sqrt[e^2 - d*f]*(4*e*f*(-3*b*e^2 + b
*d*f + 2*a*e*f) + c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sqrt[(e*(e + f*x))/(e^2
 - d*f)]*ArcSinh[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[e^2 - d*f]])/e)/(4*e*f^(7/2)*
Sqrt[e + f*x])
```

Maple [B] time = 0.333, size = 834, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

```
[Out] 1/8*(e*x+d)^(1/2)*(8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+
d*f+e^2)/(e*f)^(1/2))*x*a*e^2*f^3+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/
2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*b*d*e*f^3-12*ln(1/2*(2*e*f*x+2*((e*x
+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*b*e^3*f^2-ln(1/2*(2*
e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*x*c*d^2*f
^3-6*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(
1/2))*x*c*d*e^2*f^2+15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2
)+d*f+e^2)/(e*f)^(1/2))*x*c*e^4*f+4*x^2*c*e*f^2*(e*f)^(1/2)*((e*x+d)*(f*x+e
))^(1/2)+8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(
e*f)^(1/2))*a*e^3*f^2+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/
2)+d*f+e^2)/(e*f)^(1/2))*b*d*e^2*f^2-12*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))
^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^4*f-ln(1/2*(2*e*f*x+2*((e*x+d)
*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^2*e*f^2-6*ln(1/2*(2*e
*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d*e^3*f+
15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/
2))*c*e^5+8*x*b*e*f^2*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(1/2)+2*x*c*d*f^2*(e*f)
^(1/2)*((e*x+d)*(f*x+e))^(1/2)-10*x*c*e^2*f*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(
1/2)-16*a*e*f^2*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(1/2)+24*b*e^2*f*(e*f)^(1/2)*
((e*x+d)*(f*x+e))^(1/2)+2*c*d*e*f*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(1/2)-30*c*
e^3*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(1/2))/(e*f)^(1/2)/e/((e*x+d)*(f*x+e))^(1
/2)/f^3/(f*x+e)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.99904, size = 1268, normalized size = 5.09

$$\left[\frac{\left(15\,ce^5-\left(cd^2e-4\,bde^2-8\,ae^3\right)f^2-6\left(cde^3+2\,be^4\right)f+\left(15\,ce^4f-\left(cd^2-4\,bde-8\,ae^2\right)f^3-6\left(cde^2+2\,be^3\right)f^2\right)x\right)\sqrt{e}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)*f^2 - 6*(c*d*e^3 + 2*b*e
^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*f^3 - 6*(c*d*e^2 + 2*b*e^
3)*f^2)*x)*sqrt(e*f)*log(8*e^2*f^2*x^2 + e^4 + 6*d*e^2*f + d^2*f^2 + 4*(2*e
*f*x + e^2 + d*f)*sqrt(e*f)*sqrt(e*x + d)*sqrt(f*x + e) + 8*(e^3*f + d*e*f^
2)*x) + 4*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2 + 12*b*e^3
)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sqrt(f*x + e
))/(e^2*f^5*x + e^3*f^4), -1/8*((15*c*e^5 - (c*d^2*e - 4*b*d*e^2 - 8*a*e^3)
*f^2 - 6*(c*d*e^3 + 2*b*e^4)*f + (15*c*e^4*f - (c*d^2 - 4*b*d*e - 8*a*e^2)*
f^3 - 6*(c*d*e^2 + 2*b*e^3)*f^2)*x)*sqrt(-e*f)*arctan(1/2*(2*e*f*x + e^2 +
d*f)*sqrt(-e*f)*sqrt(e*x + d)*sqrt(f*x + e)/(e^2*f^2*x^2 + d*e^2*f + (e^3*f
 + d*e*f^2)*x)) - 2*(2*c*e^2*f^3*x^2 - 15*c*e^4*f - 8*a*e^2*f^3 + (c*d*e^2
+ 12*b*e^3)*f^2 - (5*c*e^3*f^2 - (c*d*e + 4*b*e^2)*f^3)*x)*sqrt(e*x + d)*sq
rt(f*x + e))/(e^2*f^5*x + e^3*f^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)}{\left(e+fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(a + b*x + c*x**2)/(e + f*x)**(3/2), x)
```

Giac [A] time = 1.24471, size = 320, normalized size = 1.29

$$\frac{\left((xe+d)\left(\frac{2\,(xe+d)ce^{(-1)}}{f}-\frac{\left(3\,cdf^4e^2-4\,bf^4e^3+5\,cf^3e^4\right)e^{(-3)}}{f^5}\right)+\frac{\left(cd^2f^4e^2-4\,bdf^4e^3+6\,cdf^3e^4-8\,af^4e^4+12\,bf^3e^5-15\,cf^2e^6\right)e^{(-3)}}{f^5}\right)\sqrt{xe+d}}{4\sqrt{(xe+d)fe-dfe+e^3}}+\frac{\left(cd^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((x*e + d)*(2*(x*e + d)*c*e^(-1)/f - (3*c*d*f^4*e^2 - 4*b*f^4*e^3 + 5*c
*f^3*e^4)*e^(-3)/f^5) + (c*d^2*f^4*e^2 - 4*b*d*f^4*e^3 + 6*c*d*f^3*e^4 - 8*
a*f^4*e^4 + 12*b*f^3*e^5 - 15*c*f^2*e^6)*e^(-3)/f^5)*sqrt(x*e + d)/sqrt((x*
e + d)*f*e - d*f*e + e^3) + 1/4*(c*d^2*f^2 - 4*b*d*f^2*e + 6*c*d*f*e^2 - 8*
a*f^2*e^2 + 12*b*f*e^3 - 15*c*e^4)*e^(-3/2)*log(abs(-sqrt(x*e + d)*sqrt(f)*
e^(1/2) + sqrt((x*e + d)*f*e - d*f*e + e^3)))/f^(7/2)
```

3.843 $\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{a+bx}(d+ex)^{3/2}\left(35a^2e^2-90abde+73b^2d^2\right)}{12b^3}+\frac{\sqrt{a+bx}\sqrt{d+ex}(bd-ae)\left(35a^2e^2-90abde+73b^2d^2\right)}{8b^4}+\frac{(bd-ae)^2\left(35\right.}{}$$

```
[Out] ((b*d - a*e)*(73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d +
e*x])/(8*b^4) + ((73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*(d +
e*x)^(3/2))/(12*b^3) + ((17*b*d - 13*a*e)*Sqrt[a + b*x]*(d + e*x)^(5/2))/(3
*b^2) + (2*e*(a + b*x)^(3/2)*(d + e*x)^(5/2))/b^2 + ((b*d - a*e)^2*(73*b^2*
d^2 - 90*a*b*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqr
t[d + e*x])])/(8*b^(9/2)*Sqrt[e])
```

Rubi [A] time = 0.225081, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx}(d+ex)^{3/2}\left(35a^2e^2-90abde+73b^2d^2\right)}{12b^3}+\frac{\sqrt{a+bx}\sqrt{d+ex}(bd-ae)\left(35a^2e^2-90abde+73b^2d^2\right)}{8b^4}+\frac{(bd-ae)^2\left(35\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((b*d - a*e)*(73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d +
e*x])/(8*b^4) + ((73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*(d +
e*x)^(3/2))/(12*b^3) + ((17*b*d - 13*a*e)*Sqrt[a + b*x]*(d + e*x)^(5/2))/(3
*b^2) + (2*e*(a + b*x)^(3/2)*(d + e*x)^(5/2))/b^2 + ((b*d - a*e)^2*(73*b^2*
d^2 - 90*a*b*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqr
t[d + e*x])])/(8*b^(9/2)*Sqrt[e])
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\int \frac{(d+ex)^{3/2}\left(4e\left(15b^2d^2-3abde-5a^2e^2\right)+4be^2(17bd-13ae)x\right)}{\sqrt{a+bx}}\,dx}{4b^2e}$$

$$= \frac{(17bd-13ae)\sqrt{a+bx}(d+ex)^{5/2}}{3b^2} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\left(73b^2d^2-9\right.}{}$$

$$= \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}(d+ex)^{3/2}}{12b^3} + \frac{(17bd-13ae)\sqrt{a+bx}(\cdots}{3b^2}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-90abd\cdots\right.}{}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-90abd\cdots\right.}{}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-90abd\cdots\right.}{}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{8b^4} + \frac{\left(73b^2d^2-90abd\cdots\right.}{}$$

Mathematica [A] time = 0.909865, size = 204, normalized size = 0.85

$$\frac{\sqrt{d+ex}\left(\sqrt{a+bx}\left(5a^2be^2(89d+14ex)-105a^3e^3-ab^2e\left(725d^2+292dex+56e^2x^2\right)+b^3\left(466d^2ex+501d^3+232de^2x\cdots\right.\right.\right.}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x
]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[a + b*x]*(-105*a^3*e^3 + 5*a^2*b*e^2*(89*d + 14*e*x) -
 a*b^2*e*(725*d^2 + 292*d*e*x + 56*e^2*x^2) + b^3*(501*d^3 + 466*d^2*e*x +
232*d*e^2*x^2 + 48*e^3*x^3)) + (3*(b*d - a*e)^(3/2)*(73*b^2*d^2 - 90*a*b*d*
e + 35*a^2*e^2)*ArcSinh[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(Sqrt[e]*
Sqrt[(b*(d + e*x))/(b*d - a*e)])))/(24*b^4)
```

Maple [B] time = 0.375, size = 571, normalized size = 2.4

$$\frac{1}{48\,b^4}\sqrt{ex+d}\sqrt{bx+a}\left(96\,x^3b^3e^3\sqrt{(bx+a)\,(ex+d)}\sqrt{be}-112\,x^2ab^2e^3\sqrt{(bx+a)\,(ex+d)}\sqrt{be}+464\,x^2b^3de^2\sqrt{(bx+a)\,(ex+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(96*x^3*b^3*e^3*((b*x+a)*(e*x+d))^(1/2)*(b
*e)^(1/2)-112*x^2*a*b^2*e^3*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+464*x^2*b^3
*d*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+105*ln(1/2*(2*b*x*e+2*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^4*e^4-480*ln(1/2*(2*b*x*e
+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^3*b*d*e^3+86
4*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2
))*a^2*b^2*d^2*e^2-708*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2
)+a*e+b*d)/(b*e)^(1/2))*a*b^3*d^3*e+219*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))
^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^4*d^4+140*((b*x+a)*(e*x+d))^(1/2
)*(b*e)^(1/2)*x*a^2*b*e^3-584*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*x*a*b^2*d
*e^2+932*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)*x*b^3*d^2*e-210*(b*e)^(1/2)*((
b*x+a)*(e*x+d))^(1/2)*a^3*e^3+890*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a^2*b
*d*e^2-1450*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*b^2*d^2*e+1002*(b*e)^(1/2
)*((b*x+a)*(e*x+d))^(1/2)*b^3*d^3)/b^4/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12577, size = 1250, normalized size = 5.21

$$\left[\frac{3\left(73\,b^4d^4-236\,ab^3d^3e+288\,a^2b^2d^2e^2-160\,a^3bde^3+35\,a^4e^4\right)\sqrt{be}\log\left(8\,b^2e^2x^2+b^2d^2+6\,abde+a^2e^2+4\,(2\,bex+bd\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/96*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 160*a^3*b*d*
e^3 + 35*a^4*e^4)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e
^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2
*d*e + a*b*e^2)*x) + 4*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*e^2
+ 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x^2 +
2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x + a)*sqr
t(e*x + d))/(b^5*e), -1/48*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d
^2*e^2 - 160*a^3*b*d*e^3 + 35*a^4*e^4)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d
 + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^
2*d*e + a*b*e^2)*x)) - 2*(48*b^4*e^4*x^3 + 501*b^4*d^3*e - 725*a*b^3*d^2*e^
2 + 445*a^2*b^2*d*e^3 - 105*a^3*b*e^4 + 8*(29*b^4*d*e^3 - 7*a*b^3*e^4)*x^2
+ 2*(233*b^4*d^2*e^2 - 146*a*b^3*d*e^3 + 35*a^2*b^2*e^4)*x)*sqrt(b*x + a)*s
qrt(e*x + d))/(b^5*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46333, size = 975, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/48*(720*((b^2*d - a*b*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2)
 + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) - sqrt(b^2*d + (b*x + a)*b
*e - a*b*e)*sqrt(b*x + a))*d^3*abs(b)/b^2 - 56*(sqrt(b^2*d + (b*x + a)*b*e
- a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^
5*e^4)*e^(-4)/b^7) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)*e^(-4
)/b^7) - 3*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*e^(-5/2)*log
(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))
/b^(3/2))*d*abs(b)*e^2/b^2 - 35*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b
*x + a)*(2*(b*x + a)*e^(-2)/b^4 + (b*d*e - 5*a*e^2)*e^(-4)/b^4) + (b^2*d^2
+ 2*a*b*d*e - 3*a^2*e^2)*e^(-7/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) +
sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/b^(7/2))*d^2*abs(b)*e/b^3 - 2*(sqrt(b
^2*d + (b*x + a)*b*e - a*b*e)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 +
(b^12*d*e^5 - 25*a*b^11*e^6)*e^(-6)/b^14) - (5*b^13*d^2*e^4 + 14*a*b^12*d*e
^5 - 163*a^2*b^11*e^6)*e^(-6)/b^14) + 3*(5*b^14*d^3*e^3 + 9*a*b^13*d^2*e^4
+ 15*a^2*b^12*d*e^5 - 93*a^3*b^11*e^6)*e^(-6)/b^14)*sqrt(b*x + a) + 3*(5*b^
4*d^4 + 4*a*b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 - 35*a^4*e^4)*e^
(-7/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e
- a*b*e)))/b^(5/2))*abs(b)*e^3/b^2)/b
```

3.844 $\int \frac{\sqrt{d+ex}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=176

$$\frac{\sqrt{a+bx}\sqrt{d+ex}\left(5a^2e^2-13abde+11b^2d^2\right)}{b^3}+\frac{(bd-ae)\left(5a^2e^2-13abde+11b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}+\frac{8e(a+bx)^{3/2}(d+}{3b^2}$$

```
[Out] ((11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^3 + (
2*(4*b*d - 3*a*e)*Sqrt[a + b*x]*(d + e*x)^(3/2))/b^2 + (8*e*(a + b*x)^(3/2)
*(d + e*x)^(3/2))/(3*b^2) + ((b*d - a*e)*(11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e
^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(7/2)*Sqrt
[e])
```

Rubi [A] time = 0.171327, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx}\sqrt{d+ex}\left(5a^2e^2-13abde+11b^2d^2\right)}{b^3}+\frac{(bd-ae)\left(5a^2e^2-13abde+11b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}+\frac{8e(a+bx)^{3/2}(d+}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```


```
[Out] ((11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^3 + (
2*(4*b*d - 3*a*e)*Sqrt[a + b*x]*(d + e*x)^(3/2))/b^2 + (8*e*(a + b*x)^(3/2)
*(d + e*x)^(3/2))/(3*b^2) + ((b*d - a*e)*(11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e
^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(7/2)*Sqrt
[e])
```


Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{\int \frac{\sqrt{d+ex}\left(3e(3bd-2ae)(5bd+2ae)+12be^2(4bd-3ae)x\right)}{\sqrt{a+bx}}\,dx}{3b^2e}$$

$$= \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{\left(11b^2d^2-13ab\right.}{}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^3}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^3}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^3}{b^2}$$

$$= \frac{\left(11b^2d^2-13abde+5a^2e^2\right)\sqrt{a+bx}\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a+bx}(d+ex)^3}{b^2}$$

Mathematica [A] time = 0.472013, size = 163, normalized size = 0.93

$$\frac{\sqrt{d+ex}\left(\sqrt{a+bx}\left(15a^2e^2-abe(49d+10ex)+b^2\left(57d^2+32dex+8e^2x^2\right)\right)+\frac{3\sqrt{bd-ae}\left(5a^2e^2-13abde+11b^2d^2\right)\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}\sqrt{\frac{b(d+ex)}{bd-ae}}}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[a + b*x]*(15*a^2*e^2 - a*b*e*(49*d + 10*e*x) + b^2*(57
*d^2 + 32*d*e*x + 8*e^2*x^2)) + (3*Sqrt[b*d - a*e]*(11*b^2*d^2 - 13*a*b*d*e
 + 5*a^2*e^2)*ArcSinh[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(Sqrt[e]*Sq
rt[(b*(d + e*x))/(b*d - a*e)])))/(3*b^3)
```

Maple [B] time = 0.318, size = 392, normalized size = 2.2

$$-\frac{1}{6b^3}\sqrt{ex+d}\sqrt{bx+a}\left(-16x^2b^2e^2\sqrt{(bx+a)(ex+d)}\sqrt{be}+15\ln\left(1/2\frac{2bxe+2\sqrt{(bx+a)(ex+d)}\sqrt{be}+ae+bd}{\sqrt{be}}\right)a^3e^3-5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x)
```

```
[Out] -1/6*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(-16*x^2*b^2*e^2*((b*x+a)*(e*x+d))^(1/2)*(
b*e)^(1/2)+15*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d
)/(b*e)^(1/2))*a^3*e^3-54*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(
1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*b*d*e^2+72*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d
))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b^2*d^2*e-33*ln(1/2*(2*b*x*e+2
*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^3*d^3+20*(b*e)
^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*a*b*e^2-64*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(
1/2)*x*b^2*d*e-30*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a^2*e^2+98*(b*e)^(1/2
)*((b*x+a)*(e*x+d))^(1/2)*a*b*d*e-114*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*b
^2*d^2)/b^3/((b*x+a)*(e*x+d))^(1/2)/(b*e)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7625, size = 948, normalized size = 5.39

$$\left[-\frac{3\left(11b^3d^3-24ab^2d^2e+18a^2bde^2-5a^3e^3\right)\sqrt{be}\log\left(8b^2e^2x^2+b^2d^2+6abde+a^2e^2-4(2bex+bd+ae)\sqrt{be}\sqrt{bx+a}\sqrt{\right.}{12\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-1/12*(3*(11*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(b
*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(2*b*e*x + b*d +
a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) - 4*(
```

```
8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*
e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^4*e), -1/6*(3*(11*b^3
*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(-b*e)*arctan(1/2*(
2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 +
a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(8*b^3*e^3*x^2 + 57*b^3*d^2*e - 49*a*
b^2*d*e^2 + 15*a^2*b*e^3 + 2*(16*b^3*d*e^2 - 5*a*b^2*e^3)*x)*sqrt(b*x + a)*
sqrt(e*x + d))/(b^4*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32129, size = 602, normalized size = 3.42

$$-\frac{\frac{180\left(\frac{(b^2d-abe)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\sqrt{be}^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{b}}-\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\right)d^2|b|}{b^2}-\frac{4\left(\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}+\frac{(b^6de^3-13}{}\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/12*(180*((b^2*d - a*b*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2)
 + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) - sqrt(b^2*d + (b*x + a)*b
*e - a*b*e)*sqrt(b*x + a))*d^2*abs(b)/b^2 - 4*(sqrt(b^2*d + (b*x + a)*b*e -
 a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^5
*e^4)*e^(-4)/b^7) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)*e^(-4)
/b^7) - 3*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*e^(-5/2)*log(
abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/
b^(3/2))*abs(b)*e^2/b^2 - 5*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x +
 a)*(2*(b*x + a)*e^(-2)/b^4 + (b*d*e - 5*a*e^2)*e^(-4)/b^4) + (b^2*d^2 + 2*
a*b*d*e - 3*a^2*e^2)*e^(-7/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt
(b^2*d + (b*x + a)*b*e - a*b*e)))/b^(7/2))*d*abs(b)*e/b^3)/b
```

3.845 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=122

$$\frac{2\left(3a^2e^2-8abde+8b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2\sqrt{a+bx}\sqrt{d+ex}(7bd-5ae)}{b^2}$$

```
[Out] (2*(7*b*d - 5*a*e)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^2 + (4*e*(a + b*x)^(3/2)*
Sqrt[d + e*x])/b^2 + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[e
]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(5/2)*Sqrt[e])
```

Rubi [A] time = 0.123517, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {951, 80, 63, 217, 206}

$$\frac{2\left(3a^2e^2-8abde+8b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2\sqrt{a+bx}\sqrt{d+ex}(7bd-5ae)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*(7*b*d - 5*a*e)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^2 + (4*e*(a + b*x)^(3/2)*
Sqrt[d + e*x])/b^2 + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[e
]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(5/2)*Sqrt[e])
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\int \frac{2e\left(15b^2d^2-6abde-2a^2e^2\right)+4be^2(7bd-5ae)x}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{2b^2e}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(8b^2d^2-8abde+3a^2e^2\right)\int \frac{}{\sqrt{a+}}}{b^2}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(2\left(8b^2d^2-8abde+3a^2e^2\right)\right)\mathrm{S}}{}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{\left(2\left(8b^2d^2-8abde+3a^2e^2\right)\right)\mathrm{S}}{b^3}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{2\left(8b^2d^2-8abde+3a^2e^2\right)\tan}{b^{5/2}\sqrt{e}}$$

Mathematica [A] time = 0.421291, size = 135, normalized size = 1.11

$$\frac{2\left(\frac{\sqrt{bd-ae}\left(3a^2e^2-8abde+8b^2d^2\right)\sqrt{\frac{b(d+ex)}{bd-ae}}\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}}+b\sqrt{a+bx}(d+ex)(-3ae+7bd+2bex)\right)}{b^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*(b*Sqrt[a + b*x]*(d + e*x)*(7*b*d - 3*a*e + 2*b*e*x) + (Sqrt[b*d - a*e]*
(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*Sqrt[(b*(d + e*x))/(b*d - a*e)]*ArcSinh
[(Sqrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/Sqrt[e]))/(b^3*Sqrt[d + e*x])
```

Maple [B] time = 0.318, size = 247, normalized size = 2.

$$\frac{1}{b^2}\left(3\ln\left(1/2\,\frac{2\,bxe+2\,\sqrt{(bx+a)\,(ex+d)}\sqrt{be}+ae+bd}{\sqrt{be}}\right)a^2e^2-8\ln\left(1/2\,\frac{2\,bxe+2\,\sqrt{(bx+a)\,(ex+d)}\sqrt{be}+ae+bd}{\sqrt{be}}\right)a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] (3*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/
2))*a^2*e^2-8*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d
)/(b*e)^(1/2))*a*b*d*e+8*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1
/2)+a*e+b*d)/(b*e)^(1/2))*b^2*d^2+4*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*x*b
*e-6*(b*e)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*e+14*(b*e)^(1/2)*((b*x+a)*(e*x+d
))^(1/2)*b*d)*(e*x+d)^(1/2)*(b*x+a)^(1/2)/(b*e)^(1/2)/b^2/((b*x+a)*(e*x+d))
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97896, size = 710, normalized size = 5.82

$$\left[\frac{\left(8\,b^2d^2 - 8\,abde + 3\,a^2e^2\right)\sqrt{be}\log\left(8\,b^2e^2x^2 + b^2d^2 + 6\,abde + a^2e^2 + 4\,(2\,bex + bd + ae)\sqrt{be}\sqrt{bx + a}\sqrt{ex + d} + 8\,\left(b^2de + \right.\right.}{2\,b^3e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/2*((8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2
*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a
)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + 4*(2*b^2*e^2*x + 7*b^2*d*e - 3
*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e), -((8*b^2*d^2 - 8*a*b*d*e +
3*a^2*e^2)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x
+ a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - 2*(2*
b^2*e^2*x + 7*b^2*d*e - 3*a*b*e^2)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*sqrt(d + e*x)),
x)
```

Giac [A] time = 1.19239, size = 196, normalized size = 1.61

$$\frac{2\left(\sqrt{b^2d+(bx+a)be-abe}\sqrt{bx+a}\left(\frac{2\,(bx+a)e}{b^3}+\frac{\left(7\,b^6de^2-5\,ab^5e^3\right)e^{(-2)}}{b^8}\right)-\frac{\left(8\,b^2d^2-8\,abde+3\,a^2e^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\sqrt{b}e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-}\right.\right.}{b^{\frac{5}{2}}}}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*e/b^3 + (
7*b^6*d*e^2 - 5*a*b^5*e^3)*e^(-2)/b^8) - (8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2
)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*
b*e - a*b*e)))/b^(5/2))*b/abs(b)
```

3.846 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd-ae)}+\frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}$$

```
[Out] (6*d^2*Sqrt[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) + (8*Sqrt[a + b*x]*Sqrt[d
 + e*x])/b + (8*(2*b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt
[d + e*x])])/(b^(3/2)*Sqrt[e])
```

Rubi [A] time = 0.119984, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {949, 80, 63, 217, 206}

$$\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd-ae)}+\frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) + (8*Sqrt[a + b*x]*Sqrt[d
 + e*x])/b + (8*(2*b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt
[d + e*x])])/(b^(3/2)*Sqrt[e])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx &= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{2\int \frac{6d(bd-ae)+4e(bd-ae)x}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{bd-ae}\\
&= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}+\frac{(4(2bd-ae))\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{b}\\
&= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}+\frac{(8(2bd-ae))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx,x,\sqrt{a+\right.}{b^2}\\
&= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}+\frac{(8(2bd-ae))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{b^2}\\
&= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}+\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.357536, size = 134, normalized size = 1.24

$$\frac{2\left(\frac{b\sqrt{a+bx}(bd(7d+4ex)-4ae(d+ex))}{bd-ae}+\frac{4\sqrt{bd-ae}(2bd-ae)\sqrt{\frac{b(d+ex)}{bd-ae}}\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{\sqrt{e}}\right)}{b^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x
]
```

```
[Out] (2*((b*Sqrt[a + b*x]*(-4*a*e*(d + e*x) + b*d*(7*d + 4*e*x)))/(b*d - a*e) +
(4*Sqrt[b*d - a*e]*(2*b*d - a*e)*Sqrt[(b*(d + e*x))/(b*d - a*e)]*ArcSinh[(S
qrt[e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/Sqrt[e]))/(b^2*Sqrt[d + e*x])
```

Maple [B] time = 0.388, size = 438, normalized size = 4.1

$$-2\,\frac{\sqrt{bx+a}}{b\sqrt{be}\,(ae-bd)\,\sqrt{(bx+a)\,(ex+d)}\sqrt{ex+d}}\left(2\,\ln\left(1/2\,\frac{2\,bxe+2\,\sqrt{(bx+a)\,(ex+d)}\sqrt{be}+ae+bd}{\sqrt{be}}\right)xa^2e^3-6\,\ln\left(1/2\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)*(2*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a
*e+b*d)/(b*e)^(1/2))*x*a^2*e^3-6*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*
(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*x*a*b*d*e^2+4*ln(1/2*(2*b*x*e+2*((b*x+a)*
(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*x*b^2*d^2*e+2*ln(1/2*(2*b*
x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*d*e^2-6
*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2)
)*a*b*d^2*e+4*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d
)/(b*e)^(1/2))*b^2*d^3-4*x*a*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+4*x*b*
d*e*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)-4*a*d*e*((b*x+a)*(e*x+d))^(1/2)*(b*
e)^(1/2)+7*b*d^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/b/(b*e)^(1/2)/(a*e-b*
d)/((b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35676, size = 1000, normalized size = 9.26

$$\left[-\frac{2\left(\left(2\,b^2d^3 - 3\,abd^2e + a^2de^2 + \left(2\,b^2d^2e - 3\,abde^2 + a^2e^3\right)x\right)\sqrt{be}\log\left(8\,b^2e^2x^2 + b^2d^2 + 6\,abde + a^2e^2 - 4\,(2\,bex + bd + a\right.\right.}{b^3d^2e - ab^2de^2 + \left(b^3de^2 -\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-2*((2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2 + a^
2*e^3)*x)*sqrt(b*e)*log(8*b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 - 4*(
2*b*e*x + b*d + a*e)*sqrt(b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a
*b*e^2)*x) - (7*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b
*x + a)*sqrt(e*x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x
), -2*(2*(2*b^2*d^3 - 3*a*b*d^2*e + a^2*d*e^2 + (2*b^2*d^2*e - 3*a*b*d*e^2
+ a^2*e^3)*x)*sqrt(-b*e)*arctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b
*x + a)*sqrt(e*x + d)/(b^2*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7
*b^2*d^2*e - 4*a*b*d*e^2 + 4*(b^2*d*e^2 - a*b*e^3)*x)*sqrt(b*x + a)*sqrt(e*
x + d))/(b^3*d^2*e - a*b^2*d*e^2 + (b^3*d*e^2 - a*b^2*e^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(3/2)
), x)
```

Giac [B] time = 1.21235, size = 261, normalized size = 2.42

$$-\frac{8\,(2\,bd-ae)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\sqrt{b}e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{b}|b|}+\frac{2\,\sqrt{bx+a}\left(\frac{4\left(b^3de^3-ab^2e^4\right)(bx+a)}{b^3d|b|e^2-ab^2|b|e^3}+\frac{7\,b^4d^2e^2-8\,ab^3de^3-}{b^3d|b|e^2-ab^2|}\right.}{\sqrt{b^2d+(bx+a)be-abe}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -8*(2*b*d - a*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2
*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b)*abs(b)) + 2*sqrt(b*x + a)*(4*(b^3*d*
e^3 - a*b^2*e^4)*(b*x + a)/(b^3*d*abs(b)*e^2 - a*b^2*abs(b)*e^3) + (7*b^4*d
^2*e^2 - 8*a*b^3*d*e^3 + 4*a^2*b^2*e^4)/(b^3*d*abs(b)*e^2 - a*b^2*abs(b)*e^
3))/sqrt(b^2*d + (b*x + a)*b*e - a*b*e)
```

3.847 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}+\frac{4d\sqrt{a+bx}(3bd-2ae)}{\sqrt{d+ex}(bd-ae)^2}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

```
[Out] (2*d^2*Sqrt[a + b*x])/((b*d - a*e)*(d + e*x)^(3/2)) + (4*d*(3*b*d - 2*a*e)*
Sqrt[a + b*x])/((b*d - a*e)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(Sqrt[b]*Sqrt[e])
```

Rubi [A] time = 0.121351, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {949, 78, 63, 217, 206}

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}+\frac{4d\sqrt{a+bx}(3bd-2ae)}{\sqrt{d+ex}(bd-ae)^2}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x]
```


```
[Out] (2*d^2*Sqrt[a + b*x])/((b*d - a*e)*(d + e*x)^(3/2)) + (4*d*(3*b*d - 2*a*e)*
Sqrt[a + b*x])/((b*d - a*e)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(Sqrt[b]*Sqrt[e])
```


Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx &= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{2\int \frac{3d(7bd-6ae)+12e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{3/2}}\,dx}{3(bd-ae)}\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+8\int \frac{1}{\sqrt{a+bx}\sqrt{d+ex}}\,dx\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\,\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+\frac{ex^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b}\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{d+ex}}\right)}{b}\\
&= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.304688, size = 128, normalized size = 1.1

$$\frac{2\left(\frac{8(bd-ae)^{3/2}\left(\frac{b(d+ex)}{bd-ae}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{bd-ae}}\right)}{b^2\sqrt{e}}+\frac{d\sqrt{a+bx}(bd(7d+6ex)-ae(5d+4ex))}{(bd-ae)^2}\right)}{(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x
]
```

```
[Out] (2*((d*Sqrt[a + b*x]*(-(a*e*(5*d + 4*e*x)) + b*d*(7*d + 6*e*x)))/(b*d - a*e
)^2 + (8*(b*d - a*e)^(3/2)*((b*(d + e*x))/(b*d - a*e))^(3/2)*ArcSinh[(Sqrt[
e]*Sqrt[a + b*x])/Sqrt[b*d - a*e]])/(b^2*Sqrt[e])))/(d + e*x)^(3/2)
```

Maple [B] time = 0.345, size = 601, normalized size = 5.2

$$2\,\frac{\sqrt{bx+a}}{\sqrt{be}\,(ae-bd)^2\,\sqrt{(bx+a)\,(ex+d)}\,(ex+d)^{3/2}}\left(4\,\ln\left(1/2\,\frac{2\,bxe+2\,\sqrt{(bx+a)\,(ex+d)}\sqrt{be}+ae+bd}{\sqrt{be}}\right)x^2a^2e^4-8\,\ln\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)*(4*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*
e+b*d)/(b*e)^(1/2))*x^2*a^2*e^4-8*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)
*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*x^2*a*b*d*e^3+4*ln(1/2*(2*b*x*e+2*((b*x+
a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*x^2*b^2*d^2*e^2+8*ln(1/
2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*x*a^
2*d*e^3-16*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(
b*e)^(1/2))*x*a*b*d^2*e^2+8*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)
^(1/2)+a*e+b*d)/(b*e)^(1/2))*x*b^2*d^3*e+4*ln(1/2*(2*b*x*e+2*((b*x+a)*(e*x+
d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a^2*d^2*e^2-8*ln(1/2*(2*b*x*e+2
*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*a*b*d^3*e+4*ln(1
/2*(2*b*x*e+2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+a*e+b*d)/(b*e)^(1/2))*b^2
*d^4-4*x*a*d*e^2*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+6*x*b*d^2*e*((b*x+a)*(
e*x+d))^(1/2)*(b*e)^(1/2)-5*a*d^2*e*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2)+7*b
*d^3*((b*x+a)*(e*x+d))^(1/2)*(b*e)^(1/2))/(b*e)^(1/2)/(a*e-b*d)^2/((b*x+a)*
(e*x+d))^(1/2)/(e*x+d)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.35777, size = 1407, normalized size = 12.13

$$\left[\frac{2\left(2\left(b^2d^4-2\,abd^3e+a^2d^2e^2+\left(b^2d^2e^2-2\,abde^3+a^2e^4\right)x^2+2\left(b^2d^3e-2\,abd^2e^2+a^2de^3\right)x\right)\sqrt{be}\log\left(8\,b^2e^2x^2+b^2d^2+\right.\right.}{b^3d^4e-2\,ab^2d^3e^2+a^2bd^2e^3+\left(b^3d^2e^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [2*(2*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3 + a
^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(b*e)*log(8*
b^2*e^2*x^2 + b^2*d^2 + 6*a*b*d*e + a^2*e^2 + 4*(2*b*e*x + b*d + a*e)*sqrt(
b*e)*sqrt(b*x + a)*sqrt(e*x + d) + 8*(b^2*d*e + a*b*e^2)*x) + (7*b^2*d^3*e
- 5*a*b*d^2*e^2 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x
 + d))/(b^3*d^4*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^
2*d*e^4 + a^2*b*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*
x), -2*(4*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d*e^3
 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*sqrt(-b*e)*a
rctan(1/2*(2*b*e*x + b*d + a*e)*sqrt(-b*e)*sqrt(b*x + a)*sqrt(e*x + d)/(b^2
*e^2*x^2 + a*b*d*e + (b^2*d*e + a*b*e^2)*x)) - (7*b^2*d^3*e - 5*a*b*d^2*e^2
 + 2*(3*b^2*d^2*e^2 - 2*a*b*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d))/(b^3*d^4
*e - 2*a*b^2*d^3*e^2 + a^2*b*d^2*e^3 + (b^3*d^2*e^3 - 2*a*b^2*d*e^4 + a^2*b
```

```
*e^5)*x^2 + 2*(b^3*d^3*e^2 - 2*a*b^2*d^2*e^3 + a^2*b*d*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,(d + ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(5/2)
), x)
```

Giac [B] time = 1.29966, size = 294, normalized size = 2.53

$$-\frac{16\sqrt{b}e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\sqrt{b}e^{\frac{1}{2}} + \sqrt{b^2d + (bx+a)be - abe}\right|\right)}{|b|} + \frac{2\sqrt{bx+a}\left(\frac{2\left(3b^6d^2e^2 - 2ab^5de^3\right)(bx+a)}{b^4d^2|b|e - 2ab^3d|b|e^2 + a^2b^2|b|e^3} + \frac{7b^7d^3e - 11ab^6d^2e^2 + \cdots}{b^4d^2|b|e - 2ab^3d|b|e^2 \cdots}\right.}{\left(b^2d + (bx+a)be - abe\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -16*sqrt(b)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d +
(b*x + a)*b*e - a*b*e)))/abs(b) + 2*sqrt(b*x + a)*(2*(3*b^6*d^2*e^2 - 2*a*b
^5*d*e^3)*(b*x + a)/(b^4*d^2*abs(b)*e - 2*a*b^3*d*abs(b)*e^2 + a^2*b^2*abs(
b)*e^3) + (7*b^7*d^3*e - 11*a*b^6*d^2*e^2 + 4*a^2*b^5*d*e^3)/(b^4*d^2*abs(b
)*e - 2*a*b^3*d*abs(b)*e^2 + a^2*b^2*abs(b)*e^3))/(b^2*d + (b*x + a)*b*e -
a*b*e)^(3/2)
```

3.848 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{16\sqrt{a+bx}\left(15a^2e^2-35abde+23b^2d^2\right)}{15\sqrt{d+ex}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}+\frac{8d\sqrt{a+bx}(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

```
[Out] (6*d^2*Sqrt[a + b*x])/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (8*d*(8*b*d - 5*a*e
)*Sqrt[a + b*x])/(15*(b*d - a*e)^2*(d + e*x)^(3/2)) + (16*(23*b^2*d^2 - 35*
a*b*d*e + 15*a^2*e^2)*Sqrt[a + b*x])/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

Rubi [A] time = 0.1296, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {949, 78, 37}

$$\frac{16\sqrt{a+bx}\left(15a^2e^2-35abde+23b^2d^2\right)}{15\sqrt{d+ex}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}+\frac{8d\sqrt{a+bx}(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (8*d*(8*b*d - 5*a*e
)*Sqrt[a + b*x])/(15*(b*d - a*e)^2*(d + e*x)^(3/2)) + (16*(23*b^2*d^2 - 35*
a*b*d*e + 15*a^2*e^2)*Sqrt[a + b*x])/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{2\int \frac{6d(6bd-5ae)+20e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{5/2}}\,dx}{5(bd-ae)}$$
$$= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{\left(8\left(23b^2d^2-35abde+15a^2e^2\right)\right)\int \frac{}{\sqrt{a}}}{15(bd-ae)^2}$$
$$= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{16\left(23b^2d^2-35abde+15a^2e^2\right)\sqrt{a+}}{15(bd-ae)^3\sqrt{d+ex}}$$

Mathematica [A] time = 0.0876009, size = 110, normalized size = 0.83

$$\frac{2\sqrt{a+bx}\left(a^2e^2\left(149d^2+260dex+120e^2x^2\right)-2abde\left(175d^2+306dex+140e^2x^2\right)+b^2d^2\left(225d^2+400dex+184e^2x^2\right)\right.}{15(d+ex)^{5/2}(bd-ae)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(a^2*e^2*(149*d^2 + 260*d*e*x + 120*e^2*x^2) - 2*a*b*d*e*(
175*d^2 + 306*d*e*x + 140*e^2*x^2) + b^2*d^2*(225*d^2 + 400*d*e*x + 184*e^2
*x^2)))/(15*(b*d - a*e)^3*(d + e*x)^(5/2))
```

Maple [A] time = 0.055, size = 150, normalized size = 1.1

$$-\frac{240\,a^2e^4x^2-560\,abde^3x^2+368\,b^2d^2e^2x^2+520\,a^2de^3x-1224\,abd^2e^2x+800\,b^2d^3ex+298\,a^2d^2e^2-700\,abd^3e+450}{15\,a^3e^3-45\,a^2bde^2+45\,ab^2d^2e-15\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(1/2)*(120*a^2*e^4*x^2-280*a*b*d*e^3*x^2+184*b^2*d^2*e^2*x^2+
260*a^2*d*e^3*x-612*a*b*d^2*e^2*x+400*b^2*d^3*e*x+149*a^2*d^2*e^2-350*a*b*d
^3*e+225*b^2*d^4)/(e*x+d)^(5/2)/(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^
3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.95919, size = 608, normalized size = 4.57

$$\frac{2\left(225\,b^2d^4-350\,abd^3e+149\,a^2d^2e^2+8\left(23\,b^2d^2e^2-35\,abde^3+15\,a^2e^4\right)x^2+4\left(100\,b^2d^3e-153\,ab\right.\right.}{15\left(b^3d^6-3\,ab^2d^5e+3\,a^2bd^4e^2-a^3d^3e^3+\left(b^3d^3e^3-3\,ab^2d^2e^4+3\,a^2bde^5-a^3e^6\right)x^3+3\left(b^3d^4e^2-3\,ab^2d^3e^3+3\,a^2bd^2e^4-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/15*(225*b^2*d^4 - 350*a*b*d^3*e + 149*a^2*d^2*e^2 + 8*(23*b^2*d^2*e^2 - 3
5*a*b*d*e^3 + 15*a^2*e^4)*x^2 + 4*(100*b^2*d^3*e - 153*a*b*d^2*e^2 + 65*a^2
*d*e^3)*x)*sqrt(b*x + a)*sqrt(e*x + d)/(b^3*d^6 - 3*a*b^2*d^5*e + 3*a^2*b*d
^4*e^2 - a^3*d^3*e^3 + (b^3*d^3*e^3 - 3*a*b^2*d^2*e^4 + 3*a^2*b*d*e^5 - a^3
*e^6)*x^3 + 3*(b^3*d^4*e^2 - 3*a*b^2*d^3*e^3 + 3*a^2*b*d^2*e^4 - a^3*d*e^5)
*x^2 + 3*(b^3*d^5*e - 3*a*b^2*d^4*e^2 + 3*a^2*b*d^3*e^3 - a^3*d^2*e^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29768, size = 455, normalized size = 3.42

$$\frac{2\left(4\,(bx+a)\left(\frac{2\left(23\,b^8d^2e^4-35\,ab^7de^5+15\,a^2b^6e^6\right)(bx+a)}{b^5d^3|b|e^2-3\,ab^4d^2|b|e^3+3\,a^2b^3d|b|e^4-a^3b^2|b|e^5}+\frac{5\left(20\,b^9d^3e^3-49\,ab^8d^2e^4+41\,a^2b^7de^5-12\,a^3b^6e^6\right)}{b^5d^3|b|e^2-3\,ab^4d^2|b|e^3+3\,a^2b^3d|b|e^4-a^3b^2|b|e^5}\right)+\frac{15\left(15\,b^{10}d^4e^2-50\,ab^9d^3e^3+63\,a^2b^8d^2e^4\right.}{b^5d^3|b|e^2-3\,ab^4d^2|b|e^3+3\,a^2b^3d\right.}{15\left(b^2d+(bx+a)be-abe\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2/15*(4*(b*x + a)*(2*(23*b^8*d^2*e^4 - 35*a*b^7*d*e^5 + 15*a^2*b^6*e^6)*(b*
x + a)/(b^5*d^3*abs(b)*e^2 - 3*a*b^4*d^2*abs(b)*e^3 + 3*a^2*b^3*d*abs(b)*e^
4 - a^3*b^2*abs(b)*e^5) + 5*(20*b^9*d^3*e^3 - 49*a*b^8*d^2*e^4 + 41*a^2*b^7
*d*e^5 - 12*a^3*b^6*e^6)/(b^5*d^3*abs(b)*e^2 - 3*a*b^4*d^2*abs(b)*e^3 + 3*a
^2*b^3*d*abs(b)*e^4 - a^3*b^2*abs(b)*e^5)) + 15*(15*b^10*d^4*e^2 - 50*a*b^9
*d^3*e^3 + 63*a^2*b^8*d^2*e^4 - 36*a^3*b^7*d*e^5 + 8*a^4*b^6*e^6)/(b^5*d^3*
abs(b)*e^2 - 3*a*b^4*d^2*abs(b)*e^3 + 3*a^2*b^3*d*abs(b)*e^4 - a^3*b^2*abs(
b)*e^5))*sqrt(b*x + a)/(b^2*d + (b*x + a)*b*e - a*b*e)^(5/2)
```

3.849 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}} dx$

Optimal. Leaf size=189

$$\frac{32b\sqrt{a+bx}\left(35a^2e^2-84abde+58b^2d^2\right)}{105\sqrt{d+ex}(bd-ae)^4}+\frac{16\sqrt{a+bx}\left(35a^2e^2-84abde+58b^2d^2\right)}{105(d+ex)^{3/2}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}+\frac{4d\sqrt{a+}}{35(d+}$$

```
[Out] (6*d^2*Sqrt[a + b*x])/(7*(b*d - a*e)*(d + e*x)^(7/2)) + (4*d*(23*b*d - 14*a
*e)*Sqrt[a + b*x])/(35*(b*d - a*e)^2*(d + e*x)^(5/2)) + (16*(58*b^2*d^2 - 8
4*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a*e)^3*(d + e*x)^(3/2))
+ (32*b*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a
*e)^4*Sqrt[d + e*x])
```

Rubi [A] time = 0.19222, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {949, 78, 45, 37}

$$\frac{32b\sqrt{a+bx}\left(35a^2e^2-84abde+58b^2d^2\right)}{105\sqrt{d+ex}(bd-ae)^4}+\frac{16\sqrt{a+bx}\left(35a^2e^2-84abde+58b^2d^2\right)}{105(d+ex)^{3/2}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}+\frac{4d\sqrt{a+}}{35(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(7*(b*d - a*e)*(d + e*x)^(7/2)) + (4*d*(23*b*d - 14*a
*e)*Sqrt[a + b*x])/(35*(b*d - a*e)^2*(d + e*x)^(5/2)) + (16*(58*b^2*d^2 - 8
4*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a*e)^3*(d + e*x)^(3/2))
+ (32*b*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a
*e)^4*Sqrt[d + e*x])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{2\int \frac{3d(17bd-14ae)+28e(bd-ae)x}{\sqrt{a+bx}(d+ex)^{7/2}}\,dx}{7(bd-ae)}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{\left(8\left(58b^2d^2-84abde+35a^2e^2\right)\right)\int \frac{}{\sqrt{a+}}}{35(bd-ae)^2}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16\left(58b^2d^2-84abde+35a^2e^2\right)\sqrt{a+b}}{105(bd-ae)^3(d+ex)^{3/2}}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16\left(58b^2d^2-84abde+35a^2e^2\right)\sqrt{a+b}}{105(bd-ae)^3(d+ex)^{3/2}}$$

Mathematica [A] time = 0.124592, size = 173, normalized size = 0.92

$$\frac{2\sqrt{a+bx}\left(a^2be^2\left(3890d^2ex+1953d^3+2632de^2x^2+560e^3x^3\right)-a^3e^3\left(409d^2+644dex+280e^2x^2\right)-ab^2de\left(6664d^2ex+2\right.\right.}{105(d+ex)^{7/2}(bd-ae)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(-(a^3*e^3*(409*d^2 + 644*d*e*x + 280*e^2*x^2)) + a^2*b*e^
2*(1953*d^3 + 3890*d^2*e*x + 2632*d*e^2*x^2 + 560*e^3*x^3) + b^3*d^2*(1575*
d^3 + 3850*d^2*e*x + 3248*d*e^2*x^2 + 928*e^3*x^3) - a*b^2*d*e*(2975*d^3 +
6664*d^2*e*x + 5168*d*e^2*x^2 + 1344*e^3*x^3)))/(105*(b*d - a*e)^4*(d + e*x
)^(7/2))
```

Maple [A] time = 0.057, size = 248, normalized size = 1.3

$$-\frac{-1120\,a^2be^5x^3+2688\,ab^2de^4x^3-1856\,b^3d^2e^3x^3+560\,a^3e^5x^2-5264\,a^2bde^4x^2+10336\,ab^2d^2e^3x^2-6496\,b^3d^3e^2x^2+12}{105\,e^4a^4-420\,be^3da^3+630\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/105*(b*x+a)^(1/2)*(-560*a^2*b*e^5*x^3+1344*a*b^2*d*e^4*x^3-928*b^3*d^2*e
^3*x^3+280*a^3*e^5*x^2-2632*a^2*b*d*e^4*x^2+5168*a*b^2*d^2*e^3*x^2-3248*b^3
```

```
*d^3*e^2*x^2+644*a^3*d*e^4*x-3890*a^2*b*d^2*e^3*x+6664*a*b^2*d^3*e^2*x-3850
*b^3*d^4*e*x+409*a^3*d^2*e^3-1953*a^2*b*d^3*e^2+2975*a*b^2*d^4*e-1575*b^3*d
^5)/(e*x+d)^(7/2)/(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^
4*d^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 14.2424, size = 1021, normalized size = 5.4

$$\frac{2\left(1575\,b^3d^5 - 2975\,ab^2d^4e + 1953\,a^2bd^3e^2 - 409\,a^3d^2e^3 + 16\left(58\,b^3d^2e^3 - 84\,ab^2de^4 + 35\right.\right.}{105\left(b^4d^8 - 4\,ab^3d^7e + 6\,a^2b^2d^6e^2 - 4\,a^3bd^5e^3 + a^4d^4e^4 + \left(b^4d^4e^4 - 4\,ab^3d^3e^5 + 6\,a^2b^2d^2e^6 - 4\,a^3bde^7 + a^4e^8\right)x^4 + 4\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/105*(1575*b^3*d^5 - 2975*a*b^2*d^4*e + 1953*a^2*b*d^3*e^2 - 409*a^3*d^2*e
^3 + 16*(58*b^3*d^2*e^3 - 84*a*b^2*d*e^4 + 35*a^2*b*e^5)*x^3 + 8*(406*b^3*d
^3*e^2 - 646*a*b^2*d^2*e^3 + 329*a^2*b*d*e^4 - 35*a^3*e^5)*x^2 + 2*(1925*b^
3*d^4*e - 3332*a*b^2*d^3*e^2 + 1945*a^2*b*d^2*e^3 - 322*a^3*d*e^4)*x)*sqrt(
b*x + a)*sqrt(e*x + d)/(b^4*d^8 - 4*a*b^3*d^7*e + 6*a^2*b^2*d^6*e^2 - 4*a^3
*b*d^5*e^3 + a^4*d^4*e^4 + (b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2*d^2*e
^6 - 4*a^3*b*d*e^7 + a^4*e^8)*x^4 + 4*(b^4*d^5*e^3 - 4*a*b^3*d^4*e^4 + 6*a^
2*b^2*d^3*e^5 - 4*a^3*b*d^2*e^6 + a^4*d*e^7)*x^3 + 6*(b^4*d^6*e^2 - 4*a*b^3
*d^5*e^3 + 6*a^2*b^2*d^4*e^4 - 4*a^3*b*d^3*e^5 + a^4*d^2*e^6)*x^2 + 4*(b^4*
d^7*e - 4*a*b^3*d^6*e^2 + 6*a^2*b^2*d^5*e^3 - 4*a^3*b*d^4*e^4 + a^4*d^3*e^5
)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45319, size = 707, normalized size = 3.74

$$\frac{2\left(2\left(4\,(bx+a)\left(\frac{2\left(58\,b^{10}d^2e^6-84\,ab^9de^7+35\,a^2b^8e^8\right)(bx+a)}{b^6d^4|b|e^3-4\,ab^5d^3|b|e^4+6\,a^2b^4d^2|b|e^5-4\,a^3b^3d|b|e^6+a^4b^2|b|e^7}+\frac{7\left(58\,b^{11}d^3e^5-142\,ab^{10}d^2e^6+119\,a^2b^9de^7-35\,a^3b^8e^8\right)}{b^6d^4|b|e^3-4\,ab^5d^3|b|e^4+6\,a^2b^4d^2|b|e^5-4\,a^3b^3d|b|e^6+a^4b^2|b|e^7}\right)+\frac{35\left(55\,b^{12}\cdots\right.}{b^6d^4|\cdots}\right.\right.}{105\left(b^2d+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2/105*(2*(4*(b*x + a)*(2*(58*b^10*d^2*e^6 - 84*a*b^9*d*e^7 + 35*a^2*b^8*e^8
)*(b*x + a)/(b^6*d^4*abs(b)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*ab
s(b)*e^5 - 4*a^3*b^3*d*abs(b)*e^6 + a^4*b^2*abs(b)*e^7) + 7*(58*b^11*d^3*e^
5 - 142*a*b^10*d^2*e^6 + 119*a^2*b^9*d*e^7 - 35*a^3*b^8*e^8)/(b^6*d^4*abs(b
)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*abs(b)*e^5 - 4*a^3*b^3*d*abs
(b)*e^6 + a^4*b^2*abs(b)*e^7)) + 35*(55*b^12*d^4*e^4 - 188*a*b^11*d^3*e^5 +
 243*a^2*b^10*d^2*e^6 - 142*a^3*b^9*d*e^7 + 32*a^4*b^8*e^8)/(b^6*d^4*abs(b)
*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*abs(b)*e^5 - 4*a^3*b^3*d*abs(
b)*e^6 + a^4*b^2*abs(b)*e^7))*(b*x + a) + 105*(15*b^13*d^5*e^3 - 65*a*b^12*
d^4*e^4 + 113*a^2*b^11*d^3*e^5 - 99*a^3*b^10*d^2*e^6 + 44*a^4*b^9*d*e^7 - 8
*a^5*b^8*e^8)/(b^6*d^4*abs(b)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*
abs(b)*e^5 - 4*a^3*b^3*d*abs(b)*e^6 + a^4*b^2*abs(b)*e^7))*sqrt(b*x + a)/(b
^2*d + (b*x + a)*b*e - a*b*e)^(7/2)
```

3.850 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=417

$$-\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tanh}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-}}$$

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) - (2*(e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqr
t[b^2 - 4*a*c])*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e
*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) - (
2*(e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rubi [A] time = 3.14052, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {909, 63, 217, 206, 6728, 93, 208}

$$-\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tanh}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) - (2*(e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqr
t[b^2 - 4*a*c])*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e
*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) - (
2*(e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 909

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}} + \frac{cd^2-ae^2+e(2cd-be)x}{c\sqrt{d+ex}\sqrt{f+gx}\left(a+bx+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\sqrt{f+gx}(a+bx+cx^2)}\,dx}{c} + \frac{e^2\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{e(2cd-be)+\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e(2cd-be)-\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{d}{e}}} \cdots\right.}{\cdots}$$

$$= \frac{(2e)\,\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} + \frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{\cdots}{\left(b+\sqrt{b^2-4ac}+2\cdots\right.}}{c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(2\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{\cdots}{-2cd+\left(b+\sqrt{\cdots}\right.}\right.}{c}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4\cdots}\right.}}{\sqrt{2cd-\left(b-\sqrt{b^2-4\cdots}\right.}}\right.}{c\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\cdots\right.}}$$

Mathematica [A] time = 1.76804, size = 404, normalized size = 0.97

$$\frac{\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)^{3/2}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\sqrt{b^2-4ac}-bg+2cf}}{\sqrt{f+gx}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)-\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)^{3/2}\cdots}{c\sqrt{b^2-4ac}\sqrt{g\left(\sqrt{b^2-4ac}-b\right)+2cf}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)^(3/2)*((e*(f + g*x))/(e*f - d*g))^(3/2)*ArcSinh[(Sqrt[g]*Sqr
t[d + e*x])/Sqrt[e*f - d*g]])/(c*Sqrt[g]*(f + g*x)^(3/2)) + ((-2*c*d + (b -
 Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*ArcTan
[(Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + b*e
 - Sqrt[b^2 - 4*a*c]*e]*Sqrt[f + g*x])] - (-2*c*d + (b + Sqrt[b^2 - 4*a*c])
*e)^(3/2)*Sqrt[2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g]*ArcTan[(Sqrt[2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c
])*e]*Sqrt[f + g*x])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f + (-b + Sqrt[b^2 - 4
*a*c])*g]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Maple [B] time = 0.695, size = 11688, normalized size = 28.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx+a)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.851 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=285

$$\frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] (-2*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b - Sqrt
[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*S
qrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g])
 + (2*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
])
```

Rubi [A] time = 0.52577, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {909, 93, 208}

$$\frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b - Sqrt
[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*S
qrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g])
 + (2*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
])
```

Rule 909

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e+\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{e-\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\right)$$

$$= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}\,dx + \left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{(b}$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b+\sqrt{b^2-4ac}\right)g\right)}\right.$$

$$= -\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{2\sqrt{2cd-\left(b+\sqrt{b^2-4a\right.}}}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 1.0404, size = 268, normalized size = 0.94

$$\frac{2\left(\frac{\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\sqrt{b^2-4ac}-bg+2cf}}{\sqrt{f+gx}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{\sqrt{g\left(\sqrt{b^2-4ac}-b\right)+2cf}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*(-((Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*f - b*g +
Sqrt[b^2 - 4*a*c]*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*
e]*Sqrt[f + g*x])])/Sqrt[2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g]) + (Sqrt[-2*c*
d + (b + Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])
])/Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]))/Sqrt[b^2 - 4*a*c]
```

Maple [B] time = 0.474, size = 5482, normalized size = 19.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\left(cx^2+bx+a\right)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.852 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=287

$$\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] (-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g])
```

Rubi [A] time = 0.414519, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {911, 93, 208}

$$\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g])
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\left(a+bx+cx^2\right)}dx = \int \left(\frac{2c}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}} - \frac{}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2\right.}}\right.$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\sqrt{f+gx}}dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(4c)\,\text{Subst}\left(\int \frac{1}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)x^2}dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}} - \frac{(4c)\,\text{Sub}}{}$$

$$= -\frac{4c\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}} + \frac{}{\sqrt{b^2-4a}}$$

Mathematica [A] time = 0.803607, size = 269, normalized size = 0.94

$$\frac{4c\left(\frac{\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\sqrt{b^2-4ac}-bg+2cf}}{\sqrt{f+gx}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{\sqrt{e\left(b-\sqrt{b^2-4ac}\right)-2cd}\sqrt{g\left(\sqrt{b^2-4ac}-b\right)+2cf}} - \frac{\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*(ArcTan[(Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g]*Sqrt[d + e*x])/(Sqrt[
-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]*Sqrt[f + g*x])]/(Sqrt[-2*c*d + (b - Sqr
t[b^2 - 4*a*c])*e]*Sqrt[2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g]) - ArcTan[(Sqrt
[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt
[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]
*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])))/Sqrt[b^2 - 4*a*c]
```

Maple [B] time = 0.553, size = 5507, normalized size = 19.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)\sqrt{ex + d}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex}\sqrt{f + gx}\,(a + bx + cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.853 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx$

Optimal. Leaf size=429

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}+\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)*(e*f - d*g)*Sqrt[d + e*x]) - (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(e*f - d*g)*Sqrt[d + e*x]) - (8*c^2*ArcTan
h[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) + (8*c^
2*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rubi [A] time = 1.35614, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {911, 96, 93, 208}

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}+\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)*(e*f - d*g)*Sqrt[d + e*x]) - (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(e*f - d*g)*Sqrt[d + e*x]) - (8*c^2*ArcTan
h[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) + (8*c^
2*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{2c}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}} - \frac{}{\sqrt{b^2-4ac}\left(b+\sqrt{b\right.}}\right.$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{}{\sqrt{b^2-4ac}\left(2cd-\left(b\right.\right.}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{}{\sqrt{b^2-4ac}\left(2cd-\left(b\right.\right.}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} - \frac{}{\sqrt{b^2-4ac}\left(2cd-\left(b\right.\right.}$$

Mathematica [A] time = 2.11857, size = 340, normalized size = 0.79

$$\frac{4c\left(\frac{e^2\sqrt{b^2-4ac}\sqrt{f+gx}}{2c\sqrt{d+ex}(dg-ef)\left(e(ae-bd)+cd^2\right)} - \frac{2c\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{g\sqrt{b^2-4ac}-bg+2cf}}{\sqrt{f+gx}\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right)}{\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)^{3/2}\sqrt{g\left(\sqrt{b^2-4ac}-b\right)+2cf}} + \frac{2c\tan^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}\right)}{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)^{3/2}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*((Sqrt[b^2 - 4*a*c]*e^2*Sqrt[f + g*x])/(2*c*(c*d^2 + e*(-(b*d) + a*e))
*(-(e*f) + d*g)*Sqrt[d + e*x]) - (2*c*ArcTan[(Sqrt[2*c*f - b*g + Sqrt[b^2 -
 4*a*c]*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]*Sqrt[f
+ g*x])]])/((-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f + (-b + Sq
rt[b^2 - 4*a*c])*g]) + (2*c*ArcTan[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]
*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/
((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g])))/Sqrt[b^2 - 4*a*c]
```

Maple [B] time = 0.916, size = 47351, normalized size = 110.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)(ex + d)^{\frac{3}{2}}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.854 $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=532

$$\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6}{64c^3e^4}$$

```
[Out] ((5*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g +
 a*e*g) + 16*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e
^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*
g + d^2*g^2))*x)*Sqrt[a + b*x + c*x^2])/(64*c^3*e^4) + (g^2*(24*c*e*f - 14*
c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^(3/2))/(24*c^2*e^2) + (g^3*(d + e*x)*(a
+ b*x + c*x^2)^(3/2))/(4*c*e^2) - ((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3 + 5
*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 - (b^2
*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g
+ a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*e^5) + (Sqrt[c*d^2 - b*d*e
+ a*e^2]*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rubi [A] time = 1.70409, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)-4bce^2g^2(aeg-2bdg+6}{64c^3e^4}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((5*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g +
 a*e*g) + 16*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e
^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*
g + d^2*g^2))*x)*Sqrt[a + b*x + c*x^2])/(64*c^3*e^4) + (g^2*(24*c*e*f - 14*
c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^(3/2))/(24*c^2*e^2) + (g^3*(d + e*x)*(a
+ b*x + c*x^2)^(3/2))/(4*c*e^2) - ((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3 + 5
*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 - (b^2
*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g
+ a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*e^5) + (Sqrt[c*d^2 - b*d*e
+ a*e^2]*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
```

```
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{1}{2}e\left(8ce^2f^3-d(3bd+2ae)g^3\right)-eg\left(e(4bd+ae)g^2-3c\left(4e^2f^2-\right.\right.\right.}{d+ex}}{4ce^3}$$

$$= \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\left(\frac{3}{4}e^3(1\right.}{}}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3defg+\right.\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3defg+\right.\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3defg+\right.\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg\left(3e^2f^2-3defg+\right.\right.}{}$$

Mathematica [A] time = 1.03056, size = 559, normalized size = 1.05

$$\frac{-\frac{24e^2g(bg-2cf)(ef-dg)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{5/2}} - \frac{48eg\left(b^2-4ac\right)(ef-dg)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{e^3g\left(3\left(-4cg(ag\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (384*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)] + (96*e*g*(e*f - d*g)^2*(b + 2*c*x
)*Sqrt[a + x*(b + c*x)])/c + (128*e^2*g^2*(e*f - d*g)*(a + x*(b + c*x))^(3/
2))/c + (96*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(3/2))/c - (48*(b^2 - 4*a*c
)*e*g*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
/c^(3/2) - (24*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt
[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))/c^(5/2) + (e^3*g*(80*c^(3/2)*g*(2*c*f - b*g)*(a + x*(b + c*
x))^(3/2) + 3*(16*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*(2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])])))/c^(7/2) + (192*(e*f - d*g)^3*((-2*c*d + b*e)*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*e))/(384*e^4)
```

Maple [B] time = 0.321, size = 3941, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] g^2/e*(c*x^2+b*x+a)^(3/2)/c*f+3/2*g/e*f^2*(c*x^2+b*x+a)^(1/2)*x-3/4*g^2/e*b
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f-3/2*g^2/e^2*d*f/c^
```

(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/8*g^2/e^2*d*f/c^(3/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/4*g^3/e^2*b/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d-3/4*g^2/e^2*d*f/c*(c*x^2+b*x+a)
^(1/2)*b+1/4*g^3/e^2*b/c*(c*x^2+b*x+a)^(1/2)*x*d+1/e*((d/e+x)^2*c+(b*e-2*c*
d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f^3-3/e^3/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(d/e+x))*a*d^2*f*g^2+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(d/e+x))*a*d*f^2*g+3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*
e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+
x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^3
*f*g^2-3/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+
(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^2*f^2*g-3/e^5/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
(d/e+x)+2((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+
x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d^4*f*g^2+3/e^4/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d^3*f^2*g-3/4*g^2/e*b/c*(c*x^2+b*x+a)^(1/2
)*x*f+3/2/e^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d^2*f*g^2-3/2/e^2*
ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d*f^2*g-1/e^4*((d/e+x)^2*c+(b*e-
2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^3*g^3+3/e^3*((d/e+x)^2*c+
(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^2*f*g^2-3/e^2*((d/e+
x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f^2*g+5/64*g^
3/e*b^3/c^3*(c*x^2+b*x+a)^(1/2)-5/128*g^3/e*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))+1/2*g^3/e^3*d^2*(c*x^2+b*x+a)^(1/2)*x-1/3*g^3/e^2*
(c*x^2+b*x+a)^(3/2)/c*d+1/2/e*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/
e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*f^
3+1/e^5*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^4*g^3-1/e^2*ln((1/2*(b*e
-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*
e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*f^3-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
d/e+x))*a*f^3-1/8*g^3/e*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))+1/4*g^3/e*x*(c*x^2+b*x+a)^(3/2)/c-5/24*g^3/e*b/c^2*(c*x^2+b*x+a)^(3/2)
+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2
*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/
e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d*f^3-3/8*g/e*f^2/c^(3
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/2*g^3/e^3*d^2/c^(1/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/16*g^3/e*b^2/c^(5/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/16*g^2/e*b^3/c^(5/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f+1/8*g^3/e^2*b^2/c^2*(c*x^2+b*x+a)^(1/2)*
d-3/8*g^2/e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*f-1/16*g^3/e^2*b^3/c^(5/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d-1/16*g^3/e*a/c^2*(c*x^2+b*x+a)^(1/2)*
b-1/8*g^3/e*a/c*(c*x^2+b*x+a)^(1/2)*x+3/4*g/e*f^2/c*(c*x^2+b*x+a)^(1/2)*b+3
/2*g/e*f^2/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*g^3/e^
3*d^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-3/2*g^2/e^2*d
f(c*x^2+b*x+a)^(1/2)*x+1/4*g^3/e^3*d^2/c*(c*x^2+b*x+a)^(1/2)*b+5/32*g^3/e
b^2/c^2(c*x^2+b*x+a)^(1/2)*x+1/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/
e+x))*c*d^5*g^3-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2

```
*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d^2*f^3
-1/2/e^4*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/
e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d^3*g^3-3/e^4*ln((1/2*(
b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^3*f*g^2+3/e^3*ln((1/2*(b*e-2*c*d)/e+(d/e+
x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))*c^(1/2)*d^2*f^2*g+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-
b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a
*d^3*g^3-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e
-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^4*g^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.855 $\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=325

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)}{16c^{5/2}e^4}-\frac{\sqrt{a-}}{}$$

```
[Out] -((b^2*e^2*g^2 - 8*c^2*(e*f - d*g)^2 - 2*b*c*e*g*(2*e*f - d*g) - 2*c*e*g*(4
*c*e*f - 2*c*d*g - b*e*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*e^3) + (g^2*(a +
 b*x + c*x^2)^(3/2))/(3*c*e) + (((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*
g*(4*c*e*f - 2*c*d*g - b*e*g) - 4*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*e^4) +
(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d -
b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^4
```

Rubi [A] time = 0.707723, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(g\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)(-beg-2cdg+4cef)-4ce(2cd-be)\left(2cef^2-bdg^2\right)\right)}{16c^{5/2}e^4}-\frac{\sqrt{a-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] -((b^2*e^2*g^2 - 8*c^2*(e*f - d*g)^2 - 2*b*c*e*g*(2*e*f - d*g) - 2*c*e*g*(4
*c*e*f - 2*c*d*g - b*e*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*e^3) + (g^2*(a +
 b*x + c*x^2)^(3/2))/(3*c*e) + (((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*
g*(4*c*e*f - 2*c*d*g - b*e*g) - 4*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*e^4) +
(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d -
b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
```

```
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{g^2\left(a+bx+cx^2\right)^{3/2}}{3ce} + \frac{\int \frac{\left(\frac{3}{2}e\left(2cef^2-bdg^2\right)+\frac{3}{2}eg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{3ce^2}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3}$$

$$= -\frac{\left(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{8c^2e^3}$$

Mathematica [A] time = 0.426362, size = 372, normalized size = 1.14

$$\frac{-\frac{6eg\left(b^2-4ac\right)(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}-\frac{3e^2g(bg-2cf)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{5/2}}+\frac{24(ef-dg)^2\left(2\sqrt{c}\sqrt{e(ae-bd)+cd}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (48*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)] + (12*e*g*(e*f - d*g)*(b + 2*c*x)*S
qrt[a + x*(b + c*x)])/c + (16*e^2*g^2*(a + x*(b + c*x))^(3/2))/c - (6*(b^2
- 4*a*c)*e*g*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])/c^(3/2) - (3*e^2*g*(-2*c*f + b*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(
b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*
x)])]))/c^(5/2) + (24*(e*f - d*g)^2*((-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*e))/(48*e^3)
```

Maple [B] time = 0.325, size = 2602, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^2
*g^2-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c
*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*d^2*g^2+1/2/e^3*ln
((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d^2*g^2+2/e^3*ln((1/2*(b*e-2*c*d)/
e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))*c^(1/2)*d^2*f*g-1/4*g^2/e*b/c*x*(c*x^2+b*x+a)^(1/2)-1/4*g^2/e*
b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/e/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(d/e+x))*a*f^2-2/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f*g+1/2/e*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c
)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/c^(1/2)*b*f^2+1/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*f^2-2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^2*f*g+
2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*d*f*g-1/8*g^2/e*b^2/c^2
*(c*x^2+b*x+a)^(1/2)+1/16*g^2/e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))-1/2*g^2/e^2*d*x*(c*x^2+b*x+a)^(1/2)+g/e*f*x*(c*x^2+b*x+a)^(1/2
)-1/e^4*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^3*g^2-1/e^2*ln((1/2*(b*e
-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*
e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*f^2+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(d/e+x))*c*d^3*f*g-1/e^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*d*f*g+
1/2*g/e*f/c*(c*x^2+b*x+a)^(1/2)*b-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
(2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(d/e+x))*c*d^4*g^2-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e
```

```
+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x
)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d^2*
f^2+1/8*g^2/e^2*d/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+g
/e*f/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/4*g/e*f/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-1/4*g^2/e^2*d/c*(c*x^2+b*
x+a)^(1/2)*b+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2
)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+
(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^3*g^2+1/
e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*
d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(
d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d*f^2-1/2*g^2/e^2*d/c^(1/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/3*g^2*(c*x^2+b*x+a)^(3/2
)/c/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f + gx\right)^2 \sqrt{a + bx + cx^2}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**2*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.856 $\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=219

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg)\right)}{8c^{3/2}e^3}+\frac{(ef-dg)\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x}{2\sqrt{a+bx+c}}\right.}{e^3}$$

```
[Out] ((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) -
 ((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*e^3) + (Sqrt[c*
d^2 - b*d*e + a*e^2]*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^3
```

Rubi [A] time = 0.324312, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {814, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg)\right)}{8c^{3/2}e^3}+\frac{(ef-dg)\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x}{2\sqrt{a+bx+c}}\right.}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```


```
[Out] ((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) -
 ((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*e^3) + (Sqrt[c*
d^2 - b*d*e + a*e^2]*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^3
```


Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\int \frac{\frac{1}{2}(4ce(bd-2ae)f+4acdeg-bd(4cd-be)g)+\frac{1}{2}\left(b^2e^2g+\right.}{(d+ex)\sqrt{a+bx+c}}}{4ce^2}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} + \frac{\left(\left(cd^2-bde+ae^2\right)(ef-dg)\right)\int \frac{1}{(d+ex)\sqrt{a-}}}{e^3}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)(ef-dg)\right)\text{Subst}\left(\right.}{e}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\left(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg\right.}{8c^{3/2}e^3}$$

Mathematica [A] time = 0.369378, size = 216, normalized size = 0.99

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4ce(aeg-bdg+bef)-b^2e^2g+8c^2d(dg-ef)\right)+2\sqrt{c}\left(4c(dg-ef)\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\right.\right.}{8c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((-(b^2*e^2*g) + 8*c^2*d*(-(e*f) + d*g) + 4*c*e*(b*e*f - b*d*g + a*e*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(e*Sqrt[a
+ x*(b + c*x)]*(b*e*g + 2*c*(2*e*f - 2*d*g + e*g*x)) + 4*c*Sqrt[c*d^2 + e*(
-(b*d) + a*e)]*(-(e*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2
*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]))/(8*c^(3/2)*e^3)
```

Maple [B] time = 0.299, size = 1559, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2/e*g*(c*x^2+b*x+a)^(1/2)*x+1/4/e*g/c*(c*x^2+b*x+a)^(1/2)*b+1/2/e*g/c^(1/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8/e*g/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2-1/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*g+1/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+
x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f-1/2/e^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c
)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/c^(1/2)*b*d*g+1/2/e*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b*f+1/e^3*ln
((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*g-1/e^2*ln((1/2*(b*e-2*c*d)/e+(d
/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))*c^(1/2)*d*f+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d
*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*d*
g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*f-1/e^3/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^2*g+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(d/e+x))*b*d*f+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+
c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d^3*g
-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2
*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/
e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*d^2*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)\sqrt{a + bx + cx^2}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.857 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

```
[Out] Sqrt[a + b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/e^2
```

Rubi [A] time = 0.15105, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {734, 843, 621, 206, 724}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/e^2
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx &= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e}\\ &= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(e(bd-2ae)-d(2cd-be))\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e^2}\\ &= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int\right.}{}\\ &= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd}{2\sqrt{cd^2-bde+ae^2}\sqrt{}}\right.}{e^2}\end{aligned}$$

Mathematica [A] time = 0.153325, size = 145, normalized size = 0.95

$$\frac{-2\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+\frac{(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+2e\sqrt{a+x(b+cx)}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] (2*e*Sqrt[a + x*(b + c*x)] + ((-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])])/Sqrt[c] - 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt
[a + x*(b + c*x)])])/(2*e^2)
```

Maple [B] time = 0.312, size = 715, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2/e
*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/e^2*ln((1/2*(b*e-2*c*d)/e+(d/
e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))*c^(1/2)*d-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
```

```
d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2
*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a+1/e^2/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(d/e+x))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.5229, size = 2244, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*sq
rt(c*d^2 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
- (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e +
a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2),
 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2
 - b*d*e + a*e^2)*c*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2
*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*s
qrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c*e^2), 1/4*(4*
sqrt(c*x^2 + b*x + a)*c*e + 4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sq
rt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*
e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b
*c*d^2 - b^2*d*e + a*b*e^2)*x)) - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/(c*e^2)
, 1/2*(2*sqrt(c*x^2 + b*x + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arcta
n(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2
*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)
*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)))/(c*e
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.858 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=228

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{eg}$$

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(e*g) + (S
qrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)) - (Sqrt[c*f
^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
 b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g))
```

Rubi [A] time = 0.330511, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {895, 724, 206, 843, 621}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{eg}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(e*g) + (S
qrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)) - (Sqrt[c*f
^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
 b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g))
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{cdf-bef+aeg-c(ef-dg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)}$$

$$= \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{eg} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} -$$

$$= \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} + \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{eg} +$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg} + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)} - \frac{\sqrt{cf^2-bf}}{}$$

Mathematica [A] time = 0.347088, size = 218, normalized size = 0.96

$$\frac{g\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+bd-bex+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{ae^2-bde+cd^2}}\right) + \sqrt{c}(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) - e\sqrt{ag^2-bfg+cf^2}\tanh^{-1}}{eg(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[c]*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
 + Sqrt[c*d^2 - b*d*e + a*e^2]*g*ArcTanh[(b*d - 2*a*e + 2*c*d*x - b*e*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + x*(b + c*x)])] - e*Sqrt[c*f^2 - b*f*g
 + a*g^2]*ArcTanh[(b*f - 2*a*g + 2*c*f*x - b*g*x)/(2*Sqrt[c*f^2 - b*f*g + a
*g^2]*Sqrt[a + x*(b + c*x)])])/(e*g*(e*f - d*g))
```

Maple [B] time = 0.517, size = 1529, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] -1/(d*g-e*f)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)-1/2/(d*g-e*f)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*
```

```
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/(d*g-e*f)/e*
ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d+1/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/(d/e+x))*a-1/(d*g-e*f)/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
(2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(d/e+x))*b*d+1/(d*g-e*f)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b
*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c*
d^2+1/(d*g-e*f)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)+1/2/(d*g-e*f)*ln((1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+
(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)*b-1/(d*g-e*f)
/g*ln((1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f
/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f-1/(d*g-e*f)/((a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^
2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2))/(x+f/g))*a+1/(d*g-e*f)/g/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
))/(x+f/g))*b*f-1/(d*g-e*f)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^
2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))
*c*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\left(f + gx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.859 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2}-\frac{e\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2}+\frac{(2cf-bg)\tan}{2g(ef-}$$

```
[Out] Sqrt[a + b*x + c*x^2]/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^2) + (e*
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[c]*g*(e*f - d*g)^2) - (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(g*(e*f - d*g)) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d
- 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*
x^2])])/(e*f - d*g)^2 + ((2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g
)*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g*(e*f - d*g)^2)
```

Rubi [A] time = 0.695025, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {960, 734, 843, 621, 206, 724, 732}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2}-\frac{e\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2}+\frac{(2cf-bg)\tan}{2g(ef-}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^2) + (e*
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[c]*g*(e*f - d*g)^2) - (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(g*(e*f - d*g)) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d
- 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*
x^2])])/(e*f - d*g)^2 + ((2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g
)*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g*(e*f - d*g)^2)
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx = \int\left(\frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^2(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^2} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)}\right)dx$$

$$= \frac{e^2\int\frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^2} - \frac{(eg)\int\frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^2} - \frac{g\int\frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{ef-dg}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{e\int\frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{e\int\frac{bf-2ag+(2cf-bg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} - \frac{\int\frac{b+2cx}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{\left(cd^2-bde+ae^2\right)\int\frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} + \frac{(e(2c}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\,\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} - \frac{\left(2\left(cd^2-bde+ae^2\right)\right)\text{Subst}}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2} + \frac{e(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^2} -$$

Mathematica [A] time = 0.527154, size = 222, normalized size = 0.45

$$\frac{2\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right) - \frac{(2aeg-b(dg+ef)+2cdf)\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\sqrt{g(ag-bf)+cf^2}} + \frac{2\sqrt{a+x(b+cx)}(ef-dg)}{f+gx}}{2(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] ((2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + 2*Sqrt[c*d^2 + e*(-(b*d)
 + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d)
 + a*e)]*Sqrt[a + x*(b + c*x)])] - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*Arc
Tanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqr
t[a + x*(b + c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)])/(2*(e*f - d*g)^2)
```

Maple [B] time = 0.428, size = 3162, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x)
```

```
[Out] e/(d*g-e*f)^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)+1/2*e/(d*g-e*f)^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/(d*g-e*
f)^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d
/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d-e/(d*g-e*f)^2/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a+1/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^
```

```
(1/2))/(d/e+x))*b*d-1/e/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e
+x))*c*d^2-e/(d*g-e*f)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*
f^2)/g^2)^(1/2)-1/2*e/(d*g-e*f)^2*ln((1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+
((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)*
b+e/(d*g-e*f)^2/g*ln((1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*
g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f+e/(d*g-e*f)^2/
((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g
*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a-e/(d*g-e*f)^2/g/((a*g^2-b*f*g
+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f
*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*f+e/(d*g-e*f)^2/g^2/((a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f
*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)
/g^2)^(1/2))/(x+f/g))*c*f^2-g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g
)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+g/(d*g-e*f)/(a*g
^2-b*f*g+c*f^2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)*b-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*c*f-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*ln((1/2
*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2
-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f*b+1/g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*ln(
(1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(3/2)*f^2-1/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^
2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f
)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a*b+1/(d*g-e*f)/(a*g^2-b*f*g
+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-
2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)
/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a*c*f+1/2/(d*g-e*f)/(a*
g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/
g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b
*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b^2*f-3/2/g/(d
*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*
f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+
f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*f
^2*c+1/g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln
((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^
2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))
/(x+f/g))*c^2*f^3+g/(d*g-e*f)*c/(a*g^2-b*f*g+c*f^2)*((x+f/g)^2*c+(b*g-2*c*f
)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x+g/(d*g-e*f)*c^(1/2)/(a*g^2-b*f
*g+c*f^2)*ln((1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)
/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)\left(gx + f\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.860 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx$

Optimal. Leaf size=673

$$\frac{g\left(b^2-4ac\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{e\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3}-\frac{e^2\sqrt{ag^2-bfg+\cdots}}{\cdots}$$

```
[Out] (e*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*(f + g*x)) - (g*(b*f - 2*a*g + (2*
c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)
*(f + g*x)^2) - (e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^3) + (e^2*(2*c*f - b*g)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^3) - (Sq
rt[c]*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d
*g)^2) + (e*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e
)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^3
 + ((b^2 - 4*a*c)*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g + a*g
^2)^(3/2)) + (e*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sq
rt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g)^2*Sqrt[
c*f^2 - b*f*g + a*g^2]) - (e^2*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2
*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2
])])/(g*(e*f - d*g)^3)
```

Rubi [A] time = 0.861806, antiderivative size = 673, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {960, 734, 843, 621, 206, 724, 720, 732}

$$\frac{g\left(b^2-4ac\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{e\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3}-\frac{e^2\sqrt{ag^2-bfg+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] (e*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*(f + g*x)) - (g*(b*f - 2*a*g + (2*
c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)
*(f + g*x)^2) - (e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^3) + (e^2*(2*c*f - b*g)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^3) - (Sq
rt[c]*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d
*g)^2) + (e*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e
)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^3
 + ((b^2 - 4*a*c)*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g + a*g
^2)^(3/2)) + (e*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sq
rt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g)^2*Sqrt[
c*f^2 - b*f*g + a*g^2]) - (e^2*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2
*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2
])])/(g*(e*f - d*g)^3)
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
```

```
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \left(\frac{e^3\sqrt{a+bx+cx^2}}{(ef-dg)^3(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^3} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)}\right)$$

$$= \frac{e^3\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^3}\,dx}{ef-dg}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e^2\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^3} + \cdots$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{(e(2cd-be))\int \frac{1}{\sqrt{a+bx+cx^2}}\cdots}{2(ef-dg)^3}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{\left(b^2-4ac\right)g\tanh^{-1}\left(\frac{\cdots}{2\sqrt{cf\cdots}}\right.}{8(ef-dg)\left(cf^2-\cdots\right.}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e(2cd-be)\tanh^{-1}\left(\frac{b\cdots}{2\sqrt{c}\sqrt{\cdots}}\right.}{2\sqrt{c}(ef-dg)^3}$$

Mathematica [A] time = 1.46861, size = 609, normalized size = 0.9

$$\frac{\frac{g\left(b^2-4ac\right)(ef-dg)^2\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{3/2}} + 8e\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right) + \frac{2g\sqrt{a+x(b+cx)}(ef-dg)^2\cdots}{(f+gx)^2\left(g(ag\cdots\right.}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*e*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (2*g*(e*f - d*g)^2*(-(
b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f)
+ a*g))*(f + g*x)^2) + (4*e*(-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + x*(b + c*x)])]])/Sqrt[c] + 8*e*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])] + ((b^2 - 4*a*c)*g*(e*f - d*g)^2*ArcTanh[(-2*a*g + 2*c*f*
x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])
/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (4*e*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*
g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b
+ c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)]))/g + (4*e^2*((2*c*f - b*g)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*g))/(8*(e*f - d*g)^3)
```

Maple [B] time = 0.355, size = 6714, normalized size = 10.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)(gx + f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 5.47724, size = 2489, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

[Out] -1/4*(b^2*d^2*g^3 - 4*a*c*d^2*g^3 - 8*c^2*d*f^3*e + 12*b*c*d*f^2*g*e - 6*b^2*d*f*g^2*e + 4*a*b*d*g^3*e + 4*b*c*f^3*e^2 - 3*b^2*f^2*g*e^2 - 12*a*c*f^2*g*e^2 + 12*a*b*f*g^2*e^2 - 8*a^2*g^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c*d^3*f^2*g^3 - b*d^3*f*g^4 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e + 3*b*d^2*f^2*g^3*e - 3*a*d^2*f*g^4*e + 3*c*d*f^4*g*e^2 - 3*b*d*f^3*g^2*e^2 + 3*a*d*f^2*g^3*e^2 - c*f^5*e^3 + b*f^4*g*e^3 - a*f^3*g^2*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) - 2*(c*d^2*e - b*d*e^2 + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f^2*g^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*f^3*g - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^3 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f^4*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f^3*g*e + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f^2*g^2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f^2*g^2*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^3*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^2 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*f*g^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f^4*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f^3*g*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3*g*e + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f^2*g^2*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*g^2*e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^3*e - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*g^4*e + 2*b^2*c^(3/2)*d*f^3*g - b^3*sqrt(c)*d*f^2*g^2 - 4*a*b*c^(3/2)*d*f^2*g^2 + a*b^2*sqrt(c)*d*f*g^3 + 4*a^2*c^(3/2)*d*f*g^3 + 2*b^2*c^(3/2)*f^4*e - 3*b^3*sqrt(c)*f^3*g*e - 8*a*b*c^(3/2)*f^3*g*e + 15*a*b^2*sqrt(c)*f^2*g^2*e + 4*a^2*c^(3/2)*f^2*g^2*e - 20*a^2*b*sqrt(c)*f*g^3*e + 8*a^3*sqrt(c)*g^4*e)/((c*d^2*f^2*g^3 - b*d^2*f*g^4 + a*d^2*g^5 - 2*c*d*f^3*g^2*e + 2*b*d*f^2*g^3*e - 2*a*d*f*g^4*e + c*f^4*g*e^2 - b*f^3*g^2*e^2 + a*f^2*g^3*e^2)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*f + b*f - a*g)^2)

3.861 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$

Optimal. Leaf size=933

$$\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{2\sqrt{c}g(ef-dg)^4}-\frac{\sqrt{cf^2-bgf+ag^2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^3}{g(ef-dg)^4}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{g(ef-dg)^3}$$

```
[Out] (e^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^3*(f + g*x)) - (g*(2*c*f - b*g)*(b
*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*(e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^2*(f + g*x)^2) - (e*g*(b*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt
[a + b*x + c*x^2])/(4*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^2) +
(g^2*(a + b*x + c*x^2)^(3/2))/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^3) - (e^2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*(e*f - d*g)^4) + (e^3*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^4) - (Sqrt[c]
*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)
^3) + (e^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)
*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^4 +
 ((b^2 - 4*a*c)*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*
Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(16*(e*f - d*g)*(c*f^2
 - b*f*g + a*g^2)^(5/2)) + ((b^2 - 4*a*c)*e*g*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) + (e^2*(2*c*f - b*g)*ArcTanh[(b*f -
 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x
^2])])/(2*g*(e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e^3*Sqrt[c*f^2 -
b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*
g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)^4)
```

Rubi [A] time = 1.22482, antiderivative size = 933, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {960, 734, 843, 621, 206, 724, 730, 720, 732}

$$\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^3}{2\sqrt{c}g(ef-dg)^4}-\frac{\sqrt{cf^2-bgf+ag^2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)e^3}{g(ef-dg)^4}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{g(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```


```
[Out] (e^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^3*(f + g*x)) - (g*(2*c*f - b*g)*(b
*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*(e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^2*(f + g*x)^2) - (e*g*(b*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt
[a + b*x + c*x^2])/(4*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^2) +
(g^2*(a + b*x + c*x^2)^(3/2))/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^3) - (e^2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*(e*f - d*g)^4) + (e^3*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^4) - (Sqrt[c]
*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)
^3) + (e^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)
*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^4 +
 ((b^2 - 4*a*c)*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*
Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(16*(e*f - d*g)*(c*f^2
 - b*f*g + a*g^2)^(5/2)) + ((b^2 - 4*a*c)*e*g*ArcTanh[(b*f - 2*a*g + (2*c*f
```

```
- b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) + (e^2*(2*c*f - b*g)*ArcTanh[(b*f -
 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x
^2])])/(2*g*(e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e^3*Sqrt[c*f^2 -
b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*
g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)^4)
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
```

```
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \int \left(\frac{e^4\sqrt{a+bx+cx^2}}{(ef-dg)^4(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^4} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^3} - \frac{e^2g\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)^2} - \frac{e^{\cdots}}{(e\cdots}\right.$$

$$= \frac{e^4\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^4} - \frac{\left(e^3g\right)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^4} - \frac{\left(e^2g\right)\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^3}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{\cdots}}{\cdots}}{e\cdots}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{g^2\left(a+bx+cx^2\right)^3\cdots}{3(ef-dg)\left(cf^2-bfg+ag^2\cdots\right.}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf-2ag+(2c\cdots}{4(ef-dg)^2\left(cf^2\cdots\right.}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf-2ag+(2c\cdots}{4(ef-dg)^2\left(cf^2\cdots\right.}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} - \frac{eg(bf-2ag+(2c\cdots}{4(ef-dg)^2\left(cf^2\cdots\right.}$$

Mathematica [A] time = 4.87107, size = 858, normalized size = 0.92

$$\frac{\frac{24\left((2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\sqrt{cf^2+g(ag-bf)}\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(ag-bf)}\sqrt{a+x(b+cx)}}\right)\right)e^3}{\sqrt{cg}} + 24\left(\frac{(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + 2\sqrt{cd^2+e(a\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] ((48*e^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (12*e*g*(e*f - d*g)
^2*(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-
(b*f) + a*g))*(f + g*x)^2) - (16*g^2*(-(e*f) + d*g)^3*(a + x*(b + c*x))^(3/
2))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^3) + 24*e^2*(((-2*c*d + b*e)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]) + (6*(b^2 - 4*a*c)*e*g*(e*f -
d*g)^2*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) +
 a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (3*g*(2*
c*f - b*g)*(e*f - d*g)^3*((2*Sqrt[a + x*(b + c*x)]*(-2*a*g + 2*c*f*x + b*(f
 - g*x)))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^2) + ((-b^2 + 4*a*c)*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2)))/(c*f^2 + g*(-(b*f) +
a*g)) - (24*e^2*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^2 + g
*(-(b*f) + a*g)]))/g + (24*e^3*((2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])]))/(Sqrt[c]*g))/(48*(e*f - d*g)^4)
```

Maple [B] time = 0.329, size = 11995, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)\left(gx + f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.862 $\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=1098

result too large to display

```
[Out] -((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e*f - d*g)^3 + 128*c^4*e*(5*b*d - 4*a*e)
*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b*c^2*e^
3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3*d*e*f*g
 + d^2*g^2)) - 32*b*c^3*e^2*(2*b*(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2 - 3*d*e
*f*g + d^2*g^2))) + 2*c*e*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*
g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (
3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a
*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*x)*Sqrt[a + b*x + c*x^2]
)/(1536*c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(
9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)
 + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3
*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3*e^4)
+ (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^(5/2))/(60*c^2*e^2
) + (g^3*(d + e*x)*(a + b*x + c*x^2)^(5/2))/(6*c*e^2) + ((4*c*e*(2*c*d - b*
e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*
f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^2*g^2 -
4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g
^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2*c*d - b*e
)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g))
- 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 -
4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g
^2))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(3072*c^(9/2
)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/
e^7
```

Rubi [A] time = 3.86374, antiderivative size = 1098, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{(d+ex)\left(cx^2+bx+a\right)^{5/2}g^3}{6ce^2}+\frac{(36cef-22cdg-7beg)\left(cx^2+bx+a\right)^{5/2}g^2}{60c^2e^2}+\frac{\left(7b^3e^3g^3-4bce^2(9bef-3bdg+aeg)g^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```


```
[Out] -((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e*f - d*g)^3 + 128*c^4*e*(5*b*d - 4*a*e)
*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b*c^2*e^
3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3*d*e*f*g
 + d^2*g^2)) - 32*b*c^3*e^2*(2*b*(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2 - 3*d*e
*f*g + d^2*g^2))) + 2*c*e*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*
g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (
3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a
*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*x)*Sqrt[a + b*x + c*x^2]
)/(1536*c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(
9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)
 + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3
*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3*e^4)
```

```
+ (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^(5/2))/(60*c^2*e^2
) + (g^3*(d + e*x)*(a + b*x + c*x^2)^(5/2))/(6*c*e^2) + ((4*c*e*(2*c*d - b*
e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*
f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^2*g^2 -
4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g
^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2*c*d - b*e
)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g))
- 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 -
4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g
^2))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(3072*c^(9/2
)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/
e^7
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} + \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(\frac{1}{2}e\left(12ce^2f^3-d(5bd+2ae)g^3\right)-eg\left(e(6bd+ae)g^2-c(1 \ldots}{d+ex}}{6ce^3}$$

$$= \frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} + \frac{\int \ldots}{\ldots}$$

$$= \frac{\left(7b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(9bef-3bdg+aeg)+24bc^2eg\left(3e^2f^2-3def \ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2(9bef \ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2(9bef \ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2(9bef \ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(3\left(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2(9bef \ldots\right.\right.}{\ldots}$$

Mathematica [A] time = 2.48474, size = 743, normalized size = 0.68

$$\frac{960(ef-dg)^3\left(-(2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(e\sqrt{a+x(b+cx)}\left(-2ce(4ae-5bd+bex)-b^2e^2+4c^2d(ex-2d)\right)+8c\left(e(ae-bd)+cd^2\right) \ldots\right.\right.}{c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (5120*(e*f - d*g)^3*(a + x*(b + c*x))^(3/2) + (1920*e*g*(e*f - d*g)^2*(b +
2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3072*e^2*g^2*(e*f - d*g)*(a + x*(b + c
*x))^(5/2))/c + (2560*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(5/2))/c + (360*(
b^2 - 4*a*c)*e*g*(e*f - d*g)^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)
] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/
c^(5/2) - (60*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[
a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)
^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(7/2) + (e^3*
g*(1792*g*(2*c*f - b*g)*(a + x*(b + c*x))^(5/2) + 5*(24*c^2*f^2 + 7*b^2*g^2
 - 4*c*g*(6*b*f + a*g))*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b
^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*A
```

```
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c^2 + (9
60*(e*f - d*g)^3*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*
a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*Sqrt[c]*(
e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*e*(-5*b*d
+ 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) +
2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
c*x)])])))/(c^(3/2)*e^3))/(15360*e^4)
```

Maple [B] time = 0.291, size = 10058, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.863 $\int \frac{(f+gx)^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=662

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16bc^2e^3\left(3a^2e^2g^2+3abeg(2ef-dg)+b^2(ef-dg)^2\right)+96c^3e^2\left(-a^2e^2g(2ef-dg)-2abe(ef-dg\right.\right.}{256c^{7/2}e}$$

```
[Out] ((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f
 - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e
*f - d*g)^2 + 3*a*e*g*(2*e*f - d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c
*e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2*
c*e*f^2 - b*d*g^2))*x)*Sqrt[a + b*x + c*x^2]])/(128*c^3*e^5) - ((3*b^2*e^2*g
^2 - 16*c^2*(e*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2*
c*d*g - b*e*g)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*e^3) + (g^2*(a + b*x + c
*x^2)^(5/2))/(5*c*e) - ((3*b^5*e^5*g^2 + 256*c^5*d^3*(e*f - d*g)^2 - 384*c^
4*d*e*(b*d - a*e)*(e*f - d*g)^2 - 6*b^3*c*e^4*g*(2*b*e*f - b*d*g + 4*a*e*g)
 + 16*b*c^2*e^3*(3*a^2*e^2*g^2 + b^2*(e*f - d*g)^2 + 3*a*b*e*g*(2*e*f - d*g
)) + 96*c^3*e^2*(b^2*d*(e*f - d*g)^2 - 2*a*b*e*(e*f - d*g)^2 - a^2*e^2*g*(2
*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*
c^(7/2)*e^6) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^2*ArcTanh[(b*d -
2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/e^6
```

Rubi [A] time = 1.55408, antiderivative size = 662, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1653, 814, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16bc^2e^3\left(3a^2e^2g^2+3abeg(2ef-dg)+b^2(ef-dg)^2\right)+96c^3e^2\left(-a^2e^2g(2ef-dg)-2abe(ef-dg\right.\right.}{256c^{7/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f
 - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e
*f - d*g)^2 + 3*a*e*g*(2*e*f - d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c
*e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2*
c*e*f^2 - b*d*g^2))*x)*Sqrt[a + b*x + c*x^2]])/(128*c^3*e^5) - ((3*b^2*e^2*g
^2 - 16*c^2*(e*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2*
c*d*g - b*e*g)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*e^3) + (g^2*(a + b*x + c
*x^2)^(5/2))/(5*c*e) - ((3*b^5*e^5*g^2 + 256*c^5*d^3*(e*f - d*g)^2 - 384*c^
4*d*e*(b*d - a*e)*(e*f - d*g)^2 - 6*b^3*c*e^4*g*(2*b*e*f - b*d*g + 4*a*e*g)
 + 16*b*c^2*e^3*(3*a^2*e^2*g^2 + b^2*(e*f - d*g)^2 + 3*a*b*e*g*(2*e*f - d*g
)) + 96*c^3*e^2*(b^2*d*(e*f - d*g)^2 - 2*a*b*e*(e*f - d*g)^2 - a^2*e^2*g*(2
*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*
c^(7/2)*e^6) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^2*ArcTanh[(b*d -
2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/e^6
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce} + \frac{\int \frac{\left(\frac{5}{2}e\left(2cef^2-bdg^2\right)+\frac{5}{2}eg(4cef-2cdg-beg)x\right)\left(a+bx+cx^2\right)^{3/2}}{d+ex}dx}{5ce^2}$$

$$= -\frac{\left(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-beg)x\right)\left(a+bx+\right.}{48c^2e^3}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-bdg+\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-bdg+\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-bdg+\right.}{}$$

$$= \frac{\left(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(2bef-bdg+\right.}{}$$

Mathematica [A] time = 1.32335, size = 536, normalized size = 0.81

$$\frac{240(ef-dg)^2\left(-(2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(e\sqrt{a+x(b+cx)}\left(-2ce(4ae-5bd+bex)-b^2e^2+4c^2d(ex-2d)\right)+8c\left(e(ae-bd)+cd^2\right)^{3/2}\tanh\ldots\right.\right.}{c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (1280*(e*f - d*g)^2*(a + x*(b + c*x))^(3/2) + (480*e*g*(e*f - d*g)*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))/c + (768*e^2*g^2*(a + x*(b + c*x))^(5/2))/c +
(90*(b^2 - 4*a*c)*e*g*(e*f - d*g)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2) + (15*e^2*g*(2*c*f - b*g)*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/
2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2
)))/c + (240*(e*f - d*g)^2*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-
2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2
*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*
e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[
(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])]))))/(c^(3/2)*e^3))/(3840*e^3)
```

Maple [B] time = 0.287, size = 6860, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f+gx)^2 \left(a+bx+cx^2\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**2*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.864 $\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=441

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2(-d)(ef-dg)\right)-8b^2ce^3(3aeg-bdg+bef)+192c^3de(bd-ae)\right.}{128c^{5/2}e^5}$$

```
[Out] -((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)*(e*f - d
*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g + 16*c^2
*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x)*Sqrt[a + b*x + c*x
^2])/(64*c^2*e^4) + ((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x + c
*x^2)^(3/2))/(24*c*e^2) + ((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*c^3
*d*e*(b*d - a*e)*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c
^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*e^5) + ((c*d^2 - b*
d*e + a*e^2)^(3/2)*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rubi [A] time = 0.853214, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2(-d)(ef-dg)\right)-8b^2ce^3(3aeg-bdg+bef)+192c^3de(bd-ae)\right.}{128c^{5/2}e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)*(e*f - d
*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g + 16*c^2
*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x)*Sqrt[a + b*x + c*x
^2])/(64*c^2*e^4) + ((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x + c
*x^2)^(3/2))/(24*c*e^2) + ((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*c^3
*d*e*(b*d - a*e)*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c
^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*e^5) + ((c*d^2 - b*
d*e + a*e^2)^(3/2)*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{(8cef-8cdg+3beg+6cegx)\left(a+bx+cx^2\right)^{3/2}}{24ce^2} - \int \frac{\left(\frac{1}{2}\left(8ce(bd-2ae)f+4acdeg-2bd\left(4cd-\frac{3be}{2}\right.\right.\right.}{}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg+3a\right.}{64c}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg+3a\right.}{64c}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg+3a\right.}{64c}$$

$$= -\frac{\left(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg+3a\right.}{64c}$$

Mathematica [A] time = 1.17622, size = 420, normalized size = 0.95

$$\frac{3\left(\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2d(dg-ef)\right)-8b^2ce^3(3aeg-bdg+bef)-192c^3de(bd-ae)(dg-ef)+3b^4e^4g+128c^4d^3(dg-ef)\right)+2\sqrt{c}e\sqrt{a+x}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*(3*b*e*g + c*(8*e*f - 8*d*g + 6*e*g*x)) + (3*(2*Sq
rt[c]*e*Sqrt[a + x*(b + c*x)]*(-3*b^3*e^3*g - 32*c^3*d*(e*f - d*g)*(-2*d +
e*x) + 2*b*c*e^2*(6*a*e*g + b*(4*e*f - 4*d*g - 3*e*g*x)) + 8*c^2*e*(2*b*(e*
f - d*g)*(-5*d + e*x) + a*e*(8*e*f - 8*d*g + 3*e*g*x))) + (3*b^4*e^4*g + 12
8*c^4*d^3*(-(e*f) + d*g) - 192*c^3*d*e*(b*d - a*e)*(-(e*f) + d*g) - 8*b^2*c
*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c^2*e^2*(a^2*e^2*g + 2*a*b*e*(e*f - d*g
) + b^2*d*(-(e*f) + d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c
*x)])] + 128*c^(5/2)*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*(-(e*f) + d*g)*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt
[a + x*(b + c*x)])]))/(16*c^(3/2)*e^3))/(24*c*e^2)
```

Maple [B] time = 0.276, size = 4188, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] 2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c*d^3*g-2/e^3/((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c*d^2*f-2/e^5/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(d/e+x))*b*d^4*c*g+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e
+x))*b*d^3*c*f-3/4/e^2/c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b*d*g-2/e
^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d
)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d
/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b*d^2*g+2/e^2/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b*d*f-1/16/e/c^(3/2)*ln((1/2*(b*e-2*c*d)/
e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))*b^3*f-1/e^4*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*c*d^3*g-5/4/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*b*d*f+5/4/e^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d^2*g+3/8/e*g/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*a^2+3/128/e*g/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*b^4+1/3/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(3/2)*f-1/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*a*d*g-1/e^4*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2
*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^3*f+1/e^
5*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+
x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^4*g-1/e/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(d/e+x))*a^2*f+1/e^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*c*d^2*f+1/4/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*f+3/8/e*g*(c*x^2+b*x+a)^(1/2)*x*a+1/8/e/c
*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f-1/
3/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)*d*g
```

```
+1/4/e*g*(c*x^2+b*x+a)^(3/2)*x+1/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*a*f+1/8/e*g/c*(c*x^2+b*x+a)^(3/2)*b-3/64/e*g/c^2*
(c*x^2+b*x+a)^(1/2)*b^3-3/32/e*g/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16/e*g/c*(c*
x^2+b*x+a)^(1/2)*b*a-3/16/e*g/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*b^2*a+3/2/e^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+
(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*b*f-3/2/e
^4*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^3*b*g-3/8/e^2*ln((1/2*(b*e-2*
c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d*f+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
(2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(d/e+x))*b^2*d^3*g-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e
+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x
)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^2*d^
2*f+1/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*
e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*
d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^5*g+3/8/e^3*ln(
(1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d^2*g+1/16/e^2/c^(3/2)*ln((1/2*(b
*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))*b^3*d*g-1/2/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c*d*f+1/2/e^3*((d/e+x)^2*c+(b*e-2*c*d)/e*(d
/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c*d^2*g+1/e^2/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(d/e+x))*a^2*d*g+3/4/e/c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c
)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)*a*b*f+3/2/e^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-
2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*a*g-1/4/e^2*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*d*g-1/8
/e^2/c*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^
2*d*g-3/2/e^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a*f-1/e^5/((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)
+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^4*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)(a + bx + cx^2)^{\frac{3}{2}}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.865 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=252

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh}{16c^{3/2}e^4}$$

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/e^4
```

Rubi [A] time = 0.345611, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh}{16c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
```

```
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} + \frac{\int \frac{1}{2}(4ce(\ldots}{\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} + \frac{(cd^2-\ldots}{\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{(2(cd^2\ldots}{\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{(2cd-\ldots}{\ldots}$$

Mathematica [A] time = 0.444152, size = 236, normalized size = 0.94

$$\frac{2\sqrt{c}\left(e\sqrt{a+x(b+cx)}\left(2ce(16ae-15bd+7bex)+3b^2e^2+4c^2\left(6d^2-3dex+2e^2x^2\right)\right)-24c\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\ldots\right.}{48c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (-3*(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(e*Sqrt[a + x*(b +
 c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x) + 4*c^2*(6*d^2 - 3*d
*e*x + 2*e^2*x^2)) - 24*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d)
+ 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b
+ c*x)])]))/(48*c^(3/2)*e^4)
```

Maple [B] time = 0.27, size = 1946, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d),x)
```

```
[Out] 1/3/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/4
/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b-1/
2/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c
*d+1/8/e/c*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b^2-5/4/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*b*d+3/4/e/c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b-3/2/e^2*ln((1/2
*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a-1/16/e/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+
(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))*b^3-3/8/e^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*
c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d+1/e*(
(d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a+1/e^3*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c*d^2+3/2/e
^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^2*b-1/e^4*ln((1/2*(b*e-2*c*d)
/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))*c^(3/2)*d^3-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-
b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a
^2+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b*d-2/e^3/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c*d^2-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(d/e+x))*b^2*d^2+2/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*
b*d^3*c-1/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2
+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-
2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.866 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=491

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)}{8\sqrt{c}eg^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}\,\left(ae^2-b\ldots\right.}{e^2(ef-dg)}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2])/(e^2*(e*f - d*g)) - ((4*c*e
*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*Sqrt[a + b*x +
c*x^2])/(4*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*e^3*(e*f - d
*g)) + ((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e*g) - 4*c*g*(3*b*e*f^2
 - a*g*(3*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(8*Sqrt[c]*e*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
 + c*x^2])])/(e^3*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g^3*(e*f - d*g))
```

Rubi [A] time = 0.841436, antiderivative size = 491, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {895, 734, 843, 621, 206, 724, 814}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg\left(3bef^2-ag(3ef-dg)\right)+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)}{8\sqrt{c}eg^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}\,\left(ae^2-b\ldots\right.}{e^2(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2])/(e^2*(e*f - d*g)) - ((4*c*e
*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*Sqrt[a + b*x +
c*x^2])/(4*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*e^3*(e*f - d
*g)) + ((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e*g) - 4*c*g*(3*b*e*f^2
 - a*g*(3*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(8*Sqrt[c]*e*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
 + c*x^2])])/(e^3*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g^3*(e*f - d*g))
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\sqrt{a+bx+cx^2}}{f+gx}\,dx}{e(ef-dg)} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{e(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{\left(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x\right)\sqrt{a+bx}}{4eg^2(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{\left(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x\right)\sqrt{a+bx}}{4eg^2(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{\left(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x\right)\sqrt{a+bx}}{4eg^2(ef-dg)}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{\left(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x\right)\sqrt{a+bx}}{4eg^2(ef-dg)}$$

Mathematica [A] time = 1.13982, size = 323, normalized size = 0.66

$$\frac{\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-12ceg(-aeg+bdg+bef)+3b^2e^2g^2+8c^2\left(d^2g^2+defg+e^2f^2\right)\right)}{\sqrt{c}} + \frac{2\left(-4g^3\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+eg\sqrt{a+}\right.}{}}{8e^3g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (((3*b^2*e^2*g^2 - 12*c*e*g*(b*e*f + b*d*g - a*e*g) + 8*c^2*(e^2*f^2 + d*e*
f*g + d^2*g^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqr
t[c] + (2*(e*g*(e*f - d*g)*Sqrt[a + x*(b + c*x)]*(5*b*e*g + c*(-4*e*f - 4*d
*g + 2*e*g*x)) - 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*g^3*ArcTanh[(-(b*d) + 2
*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c
*x)])] + 4*e^3*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g - 2
*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))
/(e*f - d*g))/(8*e^3*g^3)
```

Maple [B] time = 0.385, size = 4226, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] -1/(d*g-e*f)/g^3*ln((1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(3/2)*f^3+1/8/(d*g-e*f)
/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^2+5/
4/(d*g-e*f)/e*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*b*d-3/4/(d*g-e*f)/c^(1/2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b-1/(d*g-
e*f)/e^2*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
c*d^2+1/(d*g-e*f)/e^3*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^3-5/4/(d*g
```

```
-e*f)/g*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b
*f+3/4/(d*g-e*f)/c^(1/2)*ln((1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*a*b+1/(d*g-e*f)/g
^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*c*f^2+
1/3/(d*g-e*f)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(
3/2)-1/3/(d*g-e*f)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(3/2)-2/(d*g-e*f)/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*
e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+
x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b*d^3
*c+2/(d*g-e*f)/g/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/
g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b
*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a*b*f-2/(d*g-e
*f)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-
2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)
/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a*c*f^2-2/(d*g-e*f)/e/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*b*d+2/(d*g-e*f)/e^2/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*a*c*d^2+2/(d*g-e*f)/g^3/((a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^
2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2))/(x+f/g))*b*f^3*c-1/4/(d*g-e*f)*((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b-1/8/(d*g-e*f)/c*((d/e+x)^2*c+(b
*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+1/16/(d*g-e*f)/c^(3/
2)*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b^3+1/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2))/(d/e+x))*a^2-1/16/(d*g-e*f)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+(x+f/
g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(
1/2))*b^3-1/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*
f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*a^2+1/4/(
d*g-e*f)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
x*b+1/(d*g-e*f)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)*a-1/(d*g-e*f)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*a-3/2/(d*g-e*f)/e^2*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d^
2*b+1/(d*g-e*f)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*b^2*d^2+1/(
d*g-e*f)/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+
(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*c^2*d^4+3/2/(d*g-e
*f)/e*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(
d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a+1/2/(d*g-e*f)/e*((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c*d-1/(d*g-e*f)
/g^4/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c
*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*
(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c^2*f^4-1/2/(d*g-e*f)/g*((
x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c*f-3/2/(
d*g-e*f)/g*ln((1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f
)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))*c^(1/2)*f*a-3/8/(d*g-e*f)/g*ln(
(1/2*(b*g-2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)*b^2*f+3/2/(d*g-e*f)/g^2*ln((1/2*(b*g-
2*c*f)/g+(x+f/g)*c)/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g
+c*f^2)/g^2)^(1/2))*c^(1/2)*f^2*b-1/(d*g-e*f)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)
```

```
^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+
c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^
2)^(1/2))/(x+f/g))*b^2*f^2+3/8/(d*g-e*f)/e*ln((1/2*(b*e-2*c*d)/e+(d/e+x)*c)
/c^(1/2)+((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/c^(1/2)*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}}{(d+ex)\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.867 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx$

Optimal. Leaf size=787

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce(ef-dg)^2}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}(}{16c^{3/2}e^2(ef-dg)^2}$$

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e*(e*f - d*g)^2) + (3*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(
5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c*g^2*(e*
f - d*g)^2) + (a + b*x + c*x^2)^(3/2)/((e*f - d*g)*(f + g*x)) - ((2*c*d - b
*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(2*c*f -
 b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) - (3*(8*c^2
*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*g)*Sqrt[c*f^2
 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b
*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) - (e*(c*f^2 - b*
f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2)
```

Rubi [A] time = 1.3761, antiderivative size = 787, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {960, 734, 814, 843, 621, 206, 724, 732}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2\right)}{8ce(ef-dg)^2}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}(}{16c^{3/2}e^2(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```


```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e*(e*f - d*g)^2) + (3*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(
5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c*g^2*(e*
f - d*g)^2) + (a + b*x + c*x^2)^(3/2)/((e*f - d*g)*(f + g*x)) - ((2*c*d - b
*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(2*c*f -
 b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) - (3*(8*c^2
*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*g)*Sqrt[c*f^2
 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b
*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) - (e*(c*f^2 - b*
f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2)
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \int \left(\frac{e^2\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(d+ex)} - \frac{g\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)^2} - \frac{eg\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)}\right)dx$$

$$= \frac{e^2\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{f+gx}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^2}\,dx}{ef-dg}$$

$$= \frac{\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)} - \frac{e\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2(ef-dg)^2} + \frac{e\int \frac{(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{f+gx}\,dx}{2(ef-dg)^2} - \frac{3\int}{}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cgx)\sqrt{a+b}}{4g^2(ef-dg)}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cgx)\sqrt{a+b}}{4g^2(ef-dg)}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cgx)\sqrt{a+b}}{4g^2(ef-dg)}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg-2cgx)\sqrt{a+b}}{4g^2(ef-dg)}$$

Mathematica [A] time = 1.5675, size = 357, normalized size = 0.45

$$\frac{-2g^3(f+gx)\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)+e\left(2g\sqrt{a+x(b+cx)}(dg-ef)(eg(bf-ag)+cdg(f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] (-(Sqrt[c]*(e*f - d*g)^2*(4*c*e*f + 2*c*d*g - 3*b*e*g)*(f + g*x)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*(c*d^2 + e*(-(b*d) + a*e)
)^(3/2)*g^3*(f + g*x)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])] + e*(2*g*(-(e*f) + d*g)*Sqr
t[a + x*(b + c*x)]*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g*x))
- e*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*(2*c*f*(2*e*f - 3*d*g) + g*(-(b*e*f) + 3
*b*d*g - 2*a*e*g))*(f + g*x)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*
Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(2*e^2*g^3*(e*f -
d*g)^2*(f + g*x))
```

Maple [B] time = 0.408, size = 7959, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(ex + d)(gx + f)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.868 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

Optimal. Leaf size=1066

$$\frac{(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{16c^{3/2}g^3(ef-dg)^3}-\frac{\left(cf^2-bgf+ag^2\right)^{3/2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+b}}\right.}{g^3(ef-dg)^3}$$

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*Sqrt
[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^2*g^
2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(
8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/(2*(e*f - d*g)*(f + g*x)^2
) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d - b*e)*
(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])]])/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*Sqrt[c]*(2*c*f
 - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]])/(2*g^3*(e*f
 - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]])/(16*c^(3/2)*g^3*(e
*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]])/(8*Sqrt[c]*g^3*(e*f - d*g)^2)
+ ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])]])/(e*(e*f - d*g)^3) + (
3*e*(2*c*f - b*g)*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]])/(2*g^3*(
e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]])/(g^
3*(e*f - d*g)^3) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(
b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x
+ c*x^2])]])/(8*g^3*(e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rubi [A] time = 1.7059, antiderivative size = 1066, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {960, 734, 814, 843, 621, 206, 724, 732, 812}

$$\frac{(2cf-bg)\left(8c^2f^2-b^2g^2-4cg(2bf-3ag)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)e^2}{16c^{3/2}g^3(ef-dg)^3}-\frac{\left(cf^2-bgf+ag^2\right)^{3/2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+b}}\right.}{g^3(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*Sqrt
[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^2*g^
2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(
8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/(2*(e*f - d*g)*(f + g*x)^2
) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d - b*e)*
(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])]])/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*Sqrt[c]*(2*c*f
 - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]])/(2*g^3*(e*f
 - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))
```

```
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e
*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)^2)
+ ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)^3) + (
3*e*(2*c*f - b*g)*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(
e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^
3*(e*f - d*g)^3) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(
b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x
+ c*x^2])])/(8*g^3*(e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \int \left(\frac{e^3\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^3(d+ex)} - \frac{g\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)(f+gx)^3} - \frac{eg\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^3(f+gx)}\right.$$

$$= \frac{e^3\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{f+gx}dx}{(ef-dg)^3} - \frac{(eg)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^2}dx}{(ef-dg)^2} - \frac{g\int \frac{\left(a+bx+cx^2\right)^{3/2}}{(f+gx)^3}dx}{ef-dg}$$

$$= \frac{\left(a+bx+cx^2\right)^{3/2}}{2(ef-dg)(f+gx)^2} + \frac{e\left(a+bx+cx^2\right)^{3/2}}{(ef-dg)^2(f+gx)} - \frac{e^2\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}dx}{2(ef-dg)^3} + \frac{e^2\int \frac{(bf-2ag\ldots}{}}{}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cgx)\sqrt{a\ldots}}{4g^2(ef-dg)^2\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cgx)\sqrt{a\ldots}}{4g^2(ef-dg)^2\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cgx)\sqrt{a\ldots}}{4g^2(ef-dg)^2\ldots}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3bg-2cgx)\sqrt{a\ldots}}{4g^2(ef-dg)^2\ldots}$$

Mathematica [A] time = 3.76584, size = 1036, normalized size = 0.97

$$\frac{1}{4}\left(-\frac{\left((2cf-bg)\left(8c^2f^2-b^2g^2+4cg(3ag-2bf)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(8c\tanh^{-1}\left(\frac{-bf-2cxf+2ag+bgx}{2\sqrt{cf^2+g(ag-bf)}\sqrt{a+x(b+cx)}}\right)\right.\ldots}{4c^{3/2}g^3(dg-ef)^3}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((2*(a + x*(b + c*x))^(3/2))/((e*f - d*g)*(f + g*x)^2) + (4*e*(a + x*(b + c
*x))^(3/2))/((e*f - d*g)^2*(f + g*x)) + (-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e
^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
 c*x)])]) - 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d
+ e*x) - 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(
3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + x*(b + c*x)])]))/(4*c^(3/2)*e*(e*f - d*g)^3) - (3*e*((8*c^2
*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] + 2*Sqrt[c]*(g*(-4*c*f + 3*b*g + 2*c*g*x)*Sqrt[a + x*(b
+ c*x)] + 2*(2*c*f - b*g)*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTanh[(-(b*f) +
2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b +
c*x)])])))/(2*Sqrt[c]*g^3*(e*f - d*g)^2) + (3*(((-2*c*f + b*g)*(a + x*(b +
c*x))^(3/2))/(f + g*x) - (Sqrt[a + x*(b + c*x)]*(b^2*g^2 + 2*c^2*f*(2*f - g
*x) + c*g*(-5*b*f + 2*a*g + b*g*x)))/g^2 + (4*Sqrt[c]*(2*c*f - b*g)*(c*f^2
+ g*(-(b*f) + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
+ (8*c^2*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*Sqrt[c*f^2 + g*(-(b*f) + a*g
)]*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a
*g)]*Sqrt[a + x*(b + c*x)])])/(2*g^3)))/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a
```

```
*g))) - (e^2*((2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 + 4*c*g*(-2*b*f + 3*a*g))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(g*Sqrt[
a + x*(b + c*x)]*(-(b^2*g^2) + 4*c^2*f*(-2*f + g*x) - 2*c*g*(-5*b*f + 4*a*g
 + b*g*x)) + 8*c*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g -
 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]
)))/(4*c^(3/2)*g^3*(-(e*f) + d*g)^3))/4
```

Maple [B] time = 0.356, size = 15927, normalized size = 14.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{(ex + d)\left(gx + f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.869 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=886

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)\left(cd^2-bed+ae^2\right)^{5/2}}{e^5(ef-dg)}+\frac{\left(cx^2+bx+a\right)^{3/2}\left(cd^2-bed+ae^2\right)}{3e^2(ef-dg)}-\frac{(2cd-be)\left(8c^2d^2-b^2e^2-4c\right.}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c
*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*e^4*(e*f - d*g)) - ((64*c^3
*e*f^4 - 16*c^2*e*f^2*g*(9*b*f - 8*a*g) - b^2*g^3*(5*b*e*f + 3*b*d*g - 8*a*
e*g) + 4*c*g^2*(22*b^2*e*f^2 + 16*a^2*e*g^2 - 3*a*b*g*(13*e*f - d*g)) - 2*c
*g*(16*c^2*e*f^3 + b*g^2*(5*b*e*f + 3*b*d*g - 8*a*e*g) - 4*c*g*(6*b*e*f^2 -
 a*g*(7*e*f - 3*d*g)))*x)*Sqrt[a + b*x + c*x^2])/(64*c*e*g^4*(e*f - d*g)) +
 ((c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))/(3*e^2*(e*f - d*g)) - ((
8*c*e*f^2 - g*(11*b*e*f - 3*b*d*g - 8*a*e*g) - 6*c*g*(e*f - d*g)*x)*(a + b*
x + c*x^2)^(3/2))/(24*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e +
a*e^2)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^5*(e*f - d*g)) + ((128*c^4*
e*f^5 - 320*c^3*e*f^3*g*(b*f - a*g) - b^3*g^4*(5*b*e*f + 3*b*d*g - 8*a*e*g)
 + 48*c^2*g^2*(5*b^2*e*f^3 - 10*a*b*e*f^2*g + a^2*g^2*(5*e*f - d*g)) - 8*b*
c*g^3*(5*b^2*e*f^2 + 12*a^2*e*g^2 - 3*a*b*g*(5*e*f + d*g)))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(3/2)*e*g^5*(e*f - d*g)) +
((c*d^2 - b*d*e + a*e^2)^(5/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^5*(e*f - d*g)) - ((c
*f^2 - b*f*g + a*g^2)^(5/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt
[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^5*(e*f - d*g))
```

Rubi [A] time = 1.80028, antiderivative size = 886, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {895, 734, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)\left(cd^2-bed+ae^2\right)^{5/2}}{e^5(ef-dg)}+\frac{\left(cx^2+bx+a\right)^{3/2}\left(cd^2-bed+ae^2\right)}{3e^2(ef-dg)}-\frac{(2cd-be)\left(8c^2d^2-b^2e^2-4c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```



```
[Out] ((c*d^2 - b*d*e + a*e^2)*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c
*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*e^4*(e*f - d*g)) - ((64*c^3
*e*f^4 - 16*c^2*e*f^2*g*(9*b*f - 8*a*g) - b^2*g^3*(5*b*e*f + 3*b*d*g - 8*a*
e*g) + 4*c*g^2*(22*b^2*e*f^2 + 16*a^2*e*g^2 - 3*a*b*g*(13*e*f - d*g)) - 2*c
*g*(16*c^2*e*f^3 + b*g^2*(5*b*e*f + 3*b*d*g - 8*a*e*g) - 4*c*g*(6*b*e*f^2 -
 a*g*(7*e*f - 3*d*g)))*x)*Sqrt[a + b*x + c*x^2])/(64*c*e*g^4*(e*f - d*g)) +
 ((c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))/(3*e^2*(e*f - d*g)) - ((
8*c*e*f^2 - g*(11*b*e*f - 3*b*d*g - 8*a*e*g) - 6*c*g*(e*f - d*g)*x)*(a + b*
x + c*x^2)^(3/2))/(24*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e +
a*e^2)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^5*(e*f - d*g)) + ((128*c^4*
e*f^5 - 320*c^3*e*f^3*g*(b*f - a*g) - b^3*g^4*(5*b*e*f + 3*b*d*g - 8*a*e*g)
 + 48*c^2*g^2*(5*b^2*e*f^3 - 10*a*b*e*f^2*g + a^2*g^2*(5*e*f - d*g)) - 8*b*
c*g^3*(5*b^2*e*f^2 + 12*a^2*e*g^2 - 3*a*b*g*(5*e*f + d*g)))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(3/2)*e*g^5*(e*f - d*g)) +
```

```
((c*d^2 - b*d*e + a*e^2)^(5/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^5*(e*f - d*g)) - ((c
*f^2 - b*f*g + a*g^2)^(5/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt
[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^5*(e*f - d*g))
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{5/2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\left(a+bx+cx^2\right)^{3/2}}{f+gx}\,dx}{e(ef-dg)}+\frac{\left(cd^2-bde+ae^2\right)\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{e(ef-dg)}$$

$$=\frac{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)^{3/2}}{3e^2(ef-dg)}-\frac{\left(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x\right)\left(a+b\right.}{24eg^2(ef-dg)}$$

$$=\frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}-\frac{\left(64c^3ef^4\right.}{}$$

$$=\frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}-\frac{\left(64c^3ef^4\right.}{}$$

$$=\frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}-\frac{\left(64c^3ef^4\right.}{}$$

$$=\frac{\left(cd^2-bde+ae^2\right)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}-\frac{\left(64c^3ef^4\right.}{}$$

Mathematica [A] time = 2.69505, size = 647, normalized size = 0.73

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(240c^2e^2g^2(ef-dg)\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)-40b^2ce^3g^3(ef-dg)(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (3*(5*b^4*e^4*g^4*(-(e*f) + d*g) - 40*b^2*c*e^3*g^3*(e*f - d*g)*(b*e*f + b*
d*g - 3*a*e*g) + 320*c^3*e*g*(-(b*e^4*f^4) + a*e^4*f^3*g + b*d^4*g^4 - a*d^
3*e*g^4) + 128*c^4*(e^5*f^5 - d^5*g^5) + 240*c^2*e^2*g^2*(e*f - d*g)*(a^2*e
^2*g^2 - 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(-(e*g*(-(e*f)
 + d*g)*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 + 2*b*c*e^2*g^2*(278*a*e*g +
b*(-132*e*f - 132*d*g + 59*e*g*x)) - 16*c^3*(12*d^3*g^3 - 6*d^2*e*g^2*(-2*f
 + g*x) + 2*d*e^2*g*(6*f^2 - 3*f*g*x + 2*g^2*x^2) + e^3*(12*f^3 - 6*f^2*g*x
 + 4*f*g^2*x^2 - 3*g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-56*e*f - 56*d*g + 27*e*g*
x) + b*(54*d^2*g^2 + 2*d*e*g*(27*f - 13*g*x) + e^2*(54*f^2 - 26*f*g*x + 17*
g^2*x^2))))) - 192*c*(c*d^2 + e*(-(b*d) + a*e))^(5/2)*g^5*ArcTanh[(-(b*d) +
 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
 c*x)])] + 192*c*e^5*(c*f^2 + g*(-(b*f) + a*g))^(5/2)*ArcTanh[(-(b*f) + 2*a
*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x
)])]))/(384*c^(3/2)*e^5*g^5*(e*f - d*g))
```

Maple [B] time = 0.372, size = 9052, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.870 $\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=431

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4ceg(4aeg-7bdg+18bef)+15b^2e^2g^2+4c^2\left(11d^2g^2-36defg+36e^2f^2\right)\right)}{24c^3e^3}-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{}$$

```
[Out] (g^2*(15*b^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e
^2*f^2 - 36*d*e*f*g + 11*d^2*g^2))*Sqrt[a + b*x + c*x^2])/(24*c^3*e^3) + (g
^3*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(d + e*x)*Sqrt[a + b*x + c*x^2])/(12*c^2
*e^3) + (g^4*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*e^3) - (g*(5*b^3*e^3*g
^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^
2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*
e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(16*c^(7/2)*e^4) + ((e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^4*S
qrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 1.37034, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4ceg(4aeg-7bdg+18bef)+15b^2e^2g^2+4c^2\left(11d^2g^2-36defg+36e^2f^2\right)\right)}{24c^3e^3}-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*(15*b^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e
^2*f^2 - 36*d*e*f*g + 11*d^2*g^2))*Sqrt[a + b*x + c*x^2])/(24*c^3*e^3) + (g
^3*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(d + e*x)*Sqrt[a + b*x + c*x^2])/(12*c^2
*e^3) + (g^4*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*e^3) - (g*(5*b^3*e^3*g
^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^
2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*
e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(16*c^(7/2)*e^4) + ((e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^4*S
qrt[c*d^2 - b*d*e + a*e^2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{\int \frac{\frac{1}{2}e\left(6ce^3f^4-d^2(bd+4ae)g^4\right)-\frac{1}{2}eg\left(de(7bd+8ae)g^3-c\left(24e^3f^3-2d^3g^3\right)\right)x-\frac{1}{2}e^2 \cdots}{(d+ex)\sqrt{a \cdots}}}{3c \cdots}$$

$$= \frac{g^3(24cef-14cdg-5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{\int \frac{\frac{1}{4}e^4(\cdots}{\cdots}}{\cdots}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{a+b \cdots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{a+b \cdots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{a+b \cdots}}{24c^3e^3}$$

$$= \frac{g^2\left(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2\left(36e^2f^2-36defg+11d^2g^2\right)\right)\sqrt{a+b \cdots}}{24c^3e^3}$$

Mathematica [A] time = 0.893065, size = 553, normalized size = 1.28

$$\frac{\frac{6e^2g(ef-dg)\left(\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+6\sqrt{c}g\sqrt{a+x(b+cx)}(2cf-bg)\right)}{c^{5/2}} + \frac{e^3g\left(\frac{3(2cf-bg)\left(-4cg(3ag+2bf)+5b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{b \cdots}{2\sqrt{c}\sqrt{\cdots}}\right)}{c^{5/2}} \cdots\right)}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((48*e*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/c + (24*e^2*g^2*(e*f - d*g)
*(f + g*x)*Sqrt[a + x*(b + c*x)])/c + (16*e^3*g^2*(f + g*x)^2*Sqrt[a + x*(b
 + c*x)])/c + (24*e*g*(2*c*f - b*g)*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (48*g*(e*f - d*g)^3*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (6*e^2*g*(e*f - d*g)*(6
*Sqrt[c]*g*(2*c*f - b*g)*Sqrt[a + x*(b + c*x)] + (8*c^2*f^2 + 3*b^2*g^2 - 4
*c*g*(2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
)/c^(5/2) + (e^3*g*((2*g*Sqrt[a + x*(b + c*x)]*(15*b^2*g^2 + 4*c^2*f*(16*f
+ 5*g*x) - 2*c*g*(27*b*f + 8*a*g + 5*b*g*x)))/c^2 + (3*(2*c*f - b*g)*(8*c^2
*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])])/c^(5/2)))/c + (48*(e*f - d*g)^4*ArcTanh[(-2*a*e + 2*
c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(48*e^4)
```

Maple [B] time = 0.304, size = 1597, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*g^3/e^2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-1/e^5/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^4*g^4+4*g/e*f^3*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/3*g^4/e*x^2/c*(c*x^2+b*x+a)^(1/2)+5
/8*g^4/e*b^2/c^3*(c*x^2+b*x+a)^(1/2)-5/16*g^4/e*b^3/c^(7/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/3*g^4/e*a/c^2*(c*x^2+b*x+a)^(1/2)+g^4/e^3/c*
(c*x^2+b*x+a)^(1/2)*d^2+6*g^2/e/c*(c*x^2+b*x+a)^(1/2)*f^2-g^4/e^4*d^3*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/e/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(d/e+x))*f^4+1/2*g^4/e^2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*d-2*g^3/e*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*f-4*g^3/e^2/c*(c*x^2+b*x+a)^(1/2)*d*f-1/2*g^4/e^3*b/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d^2-5/12*g^4/e*b/c^2*x*(c*x^2+b*x+a)^(1/2)
+3/4*g^4/e*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g^4/
e^2*x/c*(c*x^2+b*x+a)^(1/2)*d-3*g^2/e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*f^2+4*g^3/e^3*d^2*f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))/c^(1/2)-6*g^2/e^2*d*f^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c
^(1/2)+4/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+
(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^3*f*g^3-6/e^3/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^2*f^2*g^2+4/e^2/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(d/e+x))*d*f^3*g+2*g^3/e*x/c*(c*x^2+b*x+a)^(1/2)*f+3/4*
g^4/e^2*b/c^2*(c*x^2+b*x+a)^(1/2)*d-3*g^3/e*b/c^2*(c*x^2+b*x+a)^(1/2)*f-3/8
*g^4/e^2*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d+3/2*g^3/
e*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**4/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.871 $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=270

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{8c^{5/2}e^3}+\frac{3g^2\sqrt{a+bx+cx^2}(-beg-}{4c^2e^2}$$

```
[Out] (3*g^2*(4*c*e*f - 2*c*d*g - b*e*g)*Sqrt[a + b*x + c*x^2])/(4*c^2*e^2) + (g^
3*(d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c*e^2) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*
(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*e^3) + ((e*f -
 d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(e^3*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.708168, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2\left(d^2g^2-3defg+3e^2f^2\right)\right)}{8c^{5/2}e^3}+\frac{3g^2\sqrt{a+bx+cx^2}(-beg-}{4c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] (3*g^2*(4*c*e*f - 2*c*d*g - b*e*g)*Sqrt[a + b*x + c*x^2])/(4*c^2*e^2) + (g^
3*(d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c*e^2) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*
(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*e^3) + ((e*f -
 d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(e^3*Sqrt[c*d^2 - b*d*e + a*e^2])
```


Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{2}e\left(4ce^2f^3-d(bd+2ae)g^3\right)-eg\left(e(2bd+ae)g^2-c\left(6e^2f^2-d^2g^2\right)\right)x+\frac{3}{2}e^2g^2(4cef-}{(d+ex)\sqrt{a+bx+cx^2}}}{2ce^3}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{4}e^3\left(8c^2e^2f^3+3b^2de\right.}{}}{}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{(ef-dg)^3\int \frac{}{(d+ex}}{e^3}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} - \frac{\left(2(ef-dg)^3\right)\mathrm{Sul}}{}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{g\left(3b^2e^2g^2-4ceg\right.}{}$$

Mathematica [A] time = 0.400689, size = 358, normalized size = 1.33

$$\frac{\frac{e^2g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}} + \frac{4eg(2cf-bg)(ef-dg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{6e^2g^2\sqrt{a+x(b+cx)}(2cf-bg)}{c^2} + \frac{8(ef-dg)^3\,\mathrm{ta}}{}}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((6*e^2*g^2*(2*c*f - b*g)*Sqrt[a + x*(b + c*x)])/c^2 + (8*e*g^2*(e*f - d*g)
*Sqrt[a + x*(b + c*x)])/c + (4*e^2*g^2*(f + g*x)*Sqrt[a + x*(b + c*x)])/c +
 (4*e*g*(2*c*f - b*g)*(e*f - d*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/c^(3/2) + (8*g*(e*f - d*g)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (e^2*g*(8*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(2
*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2
) + (8*(e*f - d*g)^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)
```

```
])/(8*e^3)
```

Maple [B] time = 0.301, size = 1007, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*g^3/e*x/c*(c*x^2+b*x+a)^(1/2)-3/4*g^3/e*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*g
^3/e*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g^3/e*a/c^
(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-g^3/e^2/c*(c*x^2+b*x+a)^(
1/2)*d+3*g^2/e/c*(c*x^2+b*x+a)^(1/2)*f+1/2*g^3/e^2*b/c^(3/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d-3/2*g^2/e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*f+g^3/e^3*d^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))/c^(1/2)-3*g^2/e^2*d*f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/
2)+3*g/e*f^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/e^4/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/
e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^3*g^3-3/e^3/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(d/e+x))*d^2*f*g^2+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
d/e+x))*d*f^2*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*f^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f+gx\right)^3}{\left(d+ex\right)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**3/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.872 $\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}} \, dx$

Optimal. Leaf size=176

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(c*e) + (g*(4*c*e*f - 2*c*d*g - b*e*g)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*e^2) + ((e*f - d
*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x + c*x^2])])/(e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.304916, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1653, 843, 621, 206, 724}

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```



```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(c*e) + (g*(4*c*e*f - 2*c*d*g - b*e*g)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*e^2) + ((e*f - d
*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x + c*x^2])])/(e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```


Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{\int \frac{\frac{1}{2}e\left(2cef^2-bdg^2\right)+\frac{1}{2}eg(4cef-2cdg-beg)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ce^2}\\ &= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^2}+\frac{(g(4cef-2cdg-beg))\int \frac{1}{\sqrt{a+bx+c}}}{2ce^2}\\ &= \frac{g^2\sqrt{a+bx+cx^2}}{ce}-\frac{\left(2(ef-dg)^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{e^2}+\\ &= \frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{g(4cef-2cdg-beg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\right.}{e^2\sqrt{cd^2}}\end{aligned}$$

Mathematica [A] time = 0.536556, size = 170, normalized size = 0.97

$$\frac{-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(beg+2cdg-4cef)}{c^{3/2}}+\frac{2(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{2eg^2\sqrt{a+x(b+cx)}}{c}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*e*g^2*Sqrt[a + x*(b + c*x)])/c - (g*(-4*c*e*f + 2*c*d*g + b*e*g)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (2*(e*f - d*g)^
2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)
]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(2*e^2)
```

Maple [B] time = 0.296, size = 613, normalized size = 3.5

$$\frac{g^2}{ce}\sqrt{cx^2+bx+a}-\frac{g^2b}{2e}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}}-\frac{g^2d}{e^2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}}+2\frac{fg}{e\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g^2*(c*x^2+b*x+a)^(1/2)/c/e-1/2*g^2/e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))-g^2/e^2*d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1
/2)+2*g/e*f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/e^3/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e
+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^2*g^2+2/e^2/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2))/(d/e+x))*d*f*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^
2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))
*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f+gx\right)^{2}}{\left(d+ex\right)\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**2/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.873 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=131

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce}}$$

```
[Out] (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + ((
e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.0932019, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {843, 621, 206, 724}

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + ((
e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e}$$
$$= \frac{(2g)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e} - \frac{(2(ef-dg))\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-l}{}\right.}{e}$$
$$= \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce}} + \frac{(ef-dg)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] time = 0.151974, size = 126, normalized size = 0.96

$$\frac{\frac{(dg-ef)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + ((-(e
*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(
b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/e
```

Maple [B] time = 0.271, size = 349, normalized size = 2.7

$$\frac{g}{e}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}}+\frac{dg}{e^2}\ln\left(\left(2\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/e*g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/e^2/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(d/e+x))*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.874 $\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Rubi [A] time = 0.0378462, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.0166029, size = 78, normalized size = 0.99

$$-\frac{\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*
e)]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d^2 + e*(-(b*d) + a*e)])
```

Maple [B] time = 0.273, size = 157, normalized size = 2.

$$-\frac{1}{e}\ln\left(\left(2\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2cd}{e}\left(\frac{d}{e}+x\right)+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2c+\frac{be-2cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2-bde+cd^2}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49591, size = 743, normalized size = 9.41

$$\left[\frac{\log\left(\frac{8\,abde-8\,a^2e^2-\left(b^2+4\,ac\right)d^2-\left(8\,c^2d^2-8\,bcde+\left(b^2+4\,ac\right)e^2\right)x^2-4\,\sqrt{cd^2-bde+ae^2}\sqrt{cx^2+bx+a}\left(bd-2\,ae+\left(2\,cd-be\right)x\right)-2\left(4\,bcd^2+4\,abe^2-\left(3\,b^2+4\,ac\right)de\right)x}{e^2x^2+2\,dex+d^2}\right)}{2\,\sqrt{cd^2-bde+ae^2}}, \frac{\sqrt{-}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x
 + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))/sqrt(c*d^2 - b*d*e + a*e^2), sqr
t(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x
^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2
+ (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x))/(c*
d^2 - b*d*e + a*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [A] time = 1.18533, size = 97, normalized size = 1.23

$$\frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 +
 b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

3.875 $\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{e\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}}-\frac{g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

```
[Out] (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)) - (g*ArcT
anh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a +
 b*x + c*x^2])])/((e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rubi [A] time = 0.218878, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {960, 724, 206}

$$\frac{e\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}}-\frac{g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)) - (g*ArcT
anh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a +
 b*x + c*x^2])])/((e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e}{(ef-dg)(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)\sqrt{a+bx+cx^2}}\right)dx$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$

$$= -\frac{(2e)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{ef-dg} + \frac{(2g)\,\text{Subst}\left(\int \frac{}{4cf^2-}\right.}{}$$

$$= \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)} - \frac{g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$

Mathematica [A] time = 0.260909, size = 169, normalized size = 0.93

$$\frac{\frac{g\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\sqrt{g(ag-bf)+cf^2}} - \frac{e\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}}{dg-ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((e*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) +
a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)]) + (g*ArcTanh
[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
+ x*(b + c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)])/(-(e*f) + d*g)
```

Maple [A] time = 0.387, size = 327, normalized size = 1.8

$$\frac{1}{dg-ef}\ln\left(\left(2\frac{ae^2-bde+cd^2}{e^2} + \frac{be-2cd}{e}\left(\frac{d}{e}+x\right) + 2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(\frac{d}{e}+x\right)^2 c + \frac{be-2cd}{e}\left(\frac{d}{e}+x\right) + \frac{ae^2-bd}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))-1/(d*g-e*f)/((a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g
)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(f+gx\right)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.876 $\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=340

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\frac{g(2}{}$$

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x))
 + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2) -
(g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*
f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2
)^(3/2)) - (e*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f
*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*Sqrt[c*f^2 - b*f*g + a*
g^2])
```

Rubi [A] time = 0.393571, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {960, 724, 206, 730}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\frac{g(2}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x))
 + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2) -
(g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*
f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2
)^(3/2)) - (e*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f
*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*Sqrt[c*f^2 - b*f*g + a*
g^2])
```


Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^2\sqrt{a+bx+cx^2}} - \frac{\quad}{(ef\cdots}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^2\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-b\cdots}{\quad}\right.}{(ef-dg)^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^2} - \frac{eg\cdots}{\quad}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^2} - \frac{g(2\cdots}{\quad}$$

Mathematica [A] time = 1.05932, size = 256, normalized size = 0.75

$$-\frac{-\frac{2e^2\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{2g^2\sqrt{a+x(b+cx)}(dg-ef)}{(f+gx)\left(g(ag-bf)+cf^2\right)} + \frac{g(g(2aeg+bdg-3bef)+2cf(2ef-dg))\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{3/2}}}{2(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((2*g^2*(-(e*f) + d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*
(f + g*x)) - (2*e^2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)]
 + (g*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(-2*a*
g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b
+ c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(2*(e*f - d*g)^2)
```

Maple [B] time = 0.419, size = 788, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -e/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e
-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))+e/(d*g-e*f)^2/((
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(
x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))-g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(
x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/
2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*
g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2
)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g
))*b-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*
(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(
1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+
f/g))*c*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2), x)
```

3.877 $\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=587

$$-\frac{g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g\tan\cdots}{(ef\cdots}$$

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x
)^2) + (3*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 -
b*f*g + a*g^2)^2*(f + g*x)) + (e*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)*(f + g*x)) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b
*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2
- b*d*e + a*e^2]*(e*f - d*g)^3) - (e*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g +
 (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(
2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (e^2*g*ArcTanh[(b*f - 2*a*
g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]
)/((e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (g*(8*c^2*f^2 + 3*b^2*g^2 -
 4*c*g*(2*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2
 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)^(5/2))
```

Rubi [A] time = 0.810142, antiderivative size = 587, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {960, 724, 206, 744, 806, 730}

$$-\frac{g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g\tan\cdots}{(ef\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x
)^2) + (3*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 -
b*f*g + a*g^2)^2*(f + g*x)) + (e*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)*(f + g*x)) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b
*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2
- b*d*e + a*e^2]*(e*f - d*g)^3) - (e*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g +
 (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(
2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (e^2*g*ArcTanh[(b*f - 2*a*
g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]
)/((e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (g*(8*c^2*f^2 + 3*b^2*g^2 -
 4*c*g*(2*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2
 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)^(5/2))
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
```

```
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}dx = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^3\sqrt{a+bx+cx^2}} - \cdots\right.$$

$$= \frac{e^3\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}dx}{(ef-dg)^3} - \frac{(eg)\int \frac{1}{(f+gx)^2\sqrt{a+bx+cx^2}}\cdots}{(ef-dg)^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+\cdots}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+\cdots}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+\cdots}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+\cdots}$$

Mathematica [A] time = 2.7358, size = 549, normalized size = 0.94

$$\frac{g(ef-dg)^2\left(\frac{6g\sqrt{a+x(b+cx)}(2cf-bg)}{(f+gx)\left(g(ag-bf)+cf^2\right)^2} - \frac{\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2cfx}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}}\right)}{\left(g(ag-bf)+cf^2\right)^{5/2}}\right) + \frac{8e^3\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}\cdots}{8(e\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f
 + g*x)^2) + (8*e*g^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f
) + a*g))*(f + g*x)) + (8*e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*S
qrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]])/Sqrt[c*d^2 + e*(-(b*
d) + a*e)] + (4*e*g*(-2*c*f + b*g)*(e*f - d*g)*ArcTanh[(-2*a*g + 2*c*f*x +
b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*
f^2 + g*(-(b*f) + a*g))^(3/2) - (8*e^2*g*ArcTanh[(-2*a*g + 2*c*f*x + b*(f -
 g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]])/Sqrt[c*f^
2 + g*(-(b*f) + a*g)] + g*(e*f - d*g)^2*((6*g*(2*c*f - b*g)*Sqrt[a + x*(b +
 c*x)])/((c*f^2 + g*(-(b*f) + a*g))^2*(f + g*x)) - ((8*c^2*f^2 + 3*b^2*g^2
- 4*c*g*(2*b*f + a*g))*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f
^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^
(5/2)))/(8*(e*f - d*g)^3)
```

Maple [B] time = 0.379, size = 1817, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] e^2/(d*g-e*f)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))-1/2/(d*g-e*f)/(
a*g^2-b*f*g+c*f^2)/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*
g+c*f^2)/g^2)^(1/2)+3/4*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(x+f/g)*((x+f/g
)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b-3/2*g/(d*g-e*f
)/(a*g^2-b*f*g+c*f^2)^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)*c*f-3/8*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)
+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^
2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b^2+3/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)
^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f
)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b*c*f-3/2/(d*g-e*f)/(a*g^2-b
*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2
+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2+1/2/(d*g-
e*f)*c/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f
*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f
/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))-e^2/
(d*g-e*f)^3/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(
b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+e*g/(d*g-e*f)^2/(a*
g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*
f^2)/g^2)^(1/2)-1/2*e*g/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2
)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))/(x+f/g))*b+e/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+
c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a
*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*
g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)\left(gx + f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.67581, size = 3046, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d^2*f^2*g^3 - 8*b*c*d^2*f*g^4 + 3*b^2*d^2*g^5 - 4*a*c*d^2*g^5 -
24*c^2*d*f^3*g^2*e + 28*b*c*d*f^2*g^3*e - 10*b^2*d*f*g^4*e + 4*a*b*d*g^5*e
+ 24*c^2*f^4*g*e^2 - 36*b*c*f^3*g^2*e^2 + 15*b^2*f^2*g^3*e^2 + 20*a*c*f^2*g
^3*e^2 - 20*a*b*f*g^4*e^2 + 8*a^2*g^5*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^2*d^3*f^4*g^3
 - 2*b*c*d^3*f^3*g^4 + b^2*d^3*f^2*g^5 + 2*a*c*d^3*f^2*g^5 - 2*a*b*d^3*f*g^
6 + a^2*d^3*g^7 - 3*c^2*d^2*f^5*g^2*e + 6*b*c*d^2*f^4*g^3*e - 3*b^2*d^2*f^3
*g^4*e - 6*a*c*d^2*f^3*g^4*e + 6*a*b*d^2*f^2*g^5*e - 3*a^2*d^2*f*g^6*e + 3*
c^2*d*f^6*g*e^2 - 6*b*c*d*f^5*g^2*e^2 + 3*b^2*d*f^4*g^3*e^2 + 6*a*c*d*f^4*g
^3*e^2 - 6*a*b*d*f^3*g^4*e^2 + 3*a^2*d*f^2*g^5*e^2 - c^2*f^7*e^3 + 2*b*c*f^
6*g*e^3 - b^2*f^5*g^2*e^3 - 2*a*c*f^5*g^2*e^3 + 2*a*b*f^4*g^3*e^3 - a^2*f^3
*g^4*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 2*arctan(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^3/((d^3*g^3 - 3
*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4
*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^3 - 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*b*c*d*f*g^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b^2*d*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^5 - 16*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f^3*g^2*e + 20*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b*c*f^2*g^3*e - 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f
*g^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^4*e + 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a*b*g^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*c^(5/2)*d*f^3*g^2 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2
)*d*f^2*g^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^4 -
 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^4 - 40*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f^4*g*e + 44*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*b*c^(3/2)*f^3*g^2*e - 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
b^2*sqrt(c)*f^2*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f
^2*g^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^4*e + 8*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*g^5*e + 24*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^2 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*b^2*c*d*f^2*g^3 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^
3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*f*g^4 + 28*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c*d*f*g^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
*b^2*d*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*g^5 - 40*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f^4*g*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*b^2*c*f^3*g^2*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3
*g^2*e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f^2*g^3*e - 72*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*g^3*e + 13*(sqrt(c)*x - sqrt(c*x^2 + b
```

```
*x + a))*a*b^2*f*g^4*e + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^4
*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*g^5*e + 6*b^2*c^(3/2)*d*f^
3*g^2 - 3*b^3*sqrt(c)*d*f^2*g^3 - 20*a*b*c^(3/2)*d*f^2*g^3 + 11*a*b^2*sqrt(
c)*d*f*g^4 + 12*a^2*c^(3/2)*d*f*g^4 - 8*a^2*b*sqrt(c)*d*g^5 - 10*b^2*c^(3/2
)*f^4*g*e + 7*b^3*sqrt(c)*f^3*g^2*e + 32*a*b*c^(3/2)*f^3*g^2*e - 27*a*b^2*s
qrt(c)*f^2*g^3*e - 20*a^2*c^(3/2)*f^2*g^3*e + 28*a^2*b*sqrt(c)*f*g^4*e - 8*
a^3*sqrt(c)*g^5*e)/((c^2*d^2*f^4*g^2 - 2*b*c*d^2*f^3*g^3 + b^2*d^2*f^2*g^4
+ 2*a*c*d^2*f^2*g^4 - 2*a*b*d^2*f*g^5 + a^2*d^2*g^6 - 2*c^2*d*f^5*g*e + 4*b
*c*d*f^4*g^2*e - 2*b^2*d*f^3*g^3*e - 4*a*c*d*f^3*g^3*e + 4*a*b*d*f^2*g^4*e
- 2*a^2*d*f*g^5*e + c^2*f^6*e^2 - 2*b*c*f^5*g*e^2 + b^2*f^4*g^2*e^2 + 2*a*c
*f^4*g^2*e^2 - 2*a*b*f^3*g^3*e^2 + a^2*f^2*g^4*e^2)*((sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*f + b*f -
 a*g)^2)
```

3.878 $\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=496

$$-\frac{2\left(x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab(ef-dg)+4b^2df\right)+b^3g^4(bd-ae)\right.\right.}{}$$

```
[Out] (-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^2
*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*d*
f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*f^4
 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f - d*g)
) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*g)) +
 c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x))/(c^2*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^4*Sqrt[a + b*x + c*x^
2])/(c^2*e) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*e^2) + ((e*f - d*g)^4*ArcTanh[(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c
*x^2])])/(e^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 1.19943, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1646, 1653, 843, 621, 206, 724}

$$-\frac{2\left(x\left(2c^2g^2\left(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2\right)-bcg^3\left(-3a^2eg-4ab(ef-dg)+4b^2df\right)+b^3g^4(bd-ae)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^2
*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*d*
f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*f^4
 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f - d*g)
) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*g)) +
 c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x))/(c^2*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^4*Sqrt[a + b*x + c*x^
2])/(c^2*e) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*e^2) + ((e*f - d*g)^4*ArcTanh[(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c
*x^2])])/(e^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
```

```
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2\right.\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2\right.\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2\right.\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2\right.\right.}{}$$

$$= -\frac{2\left(ab^3dg^4-b^2\left(c^2ef^4+4acdfg^3+a^2eg^4\right)+2ac\left(a^2eg^4+c^2f^3(ef-4dg)-2acfg^2\right.\right.}{}$$

Mathematica [A] time = 2.9499, size = 587, normalized size = 1.18

$$-\frac{2e\left(b^2\left(3a^2e^2g^4+acg^3\left(d^2g+4de(2f+3gx)+e^2x(gx-8f)\right)+c^2\left(d^2g^4x^2-12def^2g^2x+2e^2f^4\right)\right)-2bc\left(a^2eg^3(-5dg+4ef+5egx)+2acg\left(d^2g^3x+deg\left(3f^2+6fgx-g^2x^2\right)+e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*e*(-3*b^4*d*e*g^4*x + b^3*g^3*(3*a*e*g*(-d + e*x) + c*d*x*(8*e*f + d*g
 - e*g*x)) + b^2*(3*a^2*e^2*g^4 + c^2*(2*e^2*f^4 - 12*d*e*f^2*g^2*x + d^2*g
^4*x^2) + a*c*g^3*(d^2*g + e^2*x*(-8*f + g*x) + 4*d*e*(2*f + 3*g*x))) - 2*b
*c*(a^2*e*g^3*(4*e*f - 5*d*g + 5*e*g*x) + c^2*e*f^3*(-(e*f*x) + d*(f - 4*g*
x)) + 2*a*c*g*(d^2*g^3*x + e^2*f^2*(2*f - 3*g*x) + d*e*g*(3*f^2 + 6*f*g*x -
 g^2*x^2))) - 4*c*(2*a^3*e^2*g^4 + c^3*d*e*f^4*x + a*c^2*(d^2*g^4*x^2 - 2*d
*e*f^2*g*(2*f + 3*g*x) + e^2*f^3*(f + 4*g*x)) + a^2*c*g^2*(d^2*g^2 + d*e*g*
(4*f + g*x) + e^2*(-6*f^2 - 4*f*g*x + g^2*x^2)))))/(c^2*(b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*(e*f - d*g)^4*Log[d + e*x
])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*Lo
g[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2) - (2*(e*f - d*g)^4*
Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a
 + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/(2*e^2)
```

Maple [B] time = 0.277, size = 4453, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f^4+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/
```

```
e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f^4+4/(
a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*b^2*d*f^3*g-1/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^4*g^4+
4/e^2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*
e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+
x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^3*f
*g^3-6/e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b
*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^
2*f^2*g^2+8*g^3/e^3*d^2*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-12*g^2/e^2*d*f^
2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-2*g^4/e^3*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)*x*d^2-12*g^2/e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f^2-g^4/e^3*b^2/c/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d^2-6*g^2/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)*f^2+4/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^3*f*g^3+2/e^4/(a*e^2-b*d*e+c*d
^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*b*c*d^5*g^4-12/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d^2*f^2*g^2+8/e^2/(a*e
^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*x*b*c*d^3*f*g^3-g^4/e^3/c/(c*x^2+b*x+a)^(1/2)*d^2-6*g^2/
e/c/(c*x^2+b*x+a)^(1/2)*f^2-16/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2
*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*f^3*g-8/e
^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^4*f*g^3+12/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b
^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d
^3*f^2*g^2-8/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(
d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^2*f^3*g-2/e^3/(a*e^2-b*d*e+c*d^
2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*x*b*c*d^4*g^4-16/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^4*f*g^3+24/e^2/(a
*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*x*c^2*d^3*f^2*g^2+8/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d*f^3
*g-3/4*g^4/e*b^2/c^3/(c*x^2+b*x+a)^(1/2)+1/e^3/(a*e^2-b*d*e+c*d^2)/((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^4*g^4-e/(a*e^2-
b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+
(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*f^4-4/(a*e^2-b*d*e
+c*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d
*f^3*g-g^4/e^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d+4*g^3/
e/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f-3/2*g^4/e*b/c^(5/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*g^4/e*a/c^2/(c*x^2+b*x+a)^(1
/2)+g^4/e*x^2/c/(c*x^2+b*x+a)^(1/2)-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f^4-6/
e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^2*f^2*g^2+4/e^4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b
^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2
*d^5*g^4+8*g^3/e^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f+4*g^3/e^2*b^2/c/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*f+16*g^3/e^3*d^2*f/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2)*x*c-24*g^2/e^2*d*f^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c+4*g^3/e*
b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f+4*g^4/e*a/c*b/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2)*x-g^4/e^2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d+e/(a*e^2-b
*d*e+c*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*f^4-1/2*g^4/e^2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d+2*g^3/e*b^3/c^
2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f-3/2*g^4/e*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*
x+a)^(1/2)*x+2*g^4/e*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-4*g^4/e^4*d^
3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c+16*g/e*f^3/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)*x*c-1/2*g^4/e^2*b/c^2/(c*x^2+b*x+a)^(1/2)*d+2*g^3/e*b/c^2/(c*x^2+b*x+
```

```
a)^(1/2)*f+g^4/e^2*x/c/(c*x^2+b*x+a)^(1/2)*d-4*g^3/e*x/c/(c*x^2+b*x+a)^(1/2
)*f-2*g^4/e^4*d^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+4/(a*e^2-b*d*e+c*d^2)/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d*f^3*g+6/e/(a*e^2-b*d*e+c*d^2)/
((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^2*f^2*g
^2-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f^4-1/e^3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^4*g^4-4/e^2/(a*e^2-b*d*e+c*d^2)/((d/e+x)^2
*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^3*f*g^3+8*g/e*f^3
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+3/2*g^4/e*b/c^2*x/(c*x^2+b*x+a)^(1/2)-3/
4*g^4/e*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4*g^3/e^2/c/(c*x^2+b*x+a)^(
1/2)*d*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.879 $\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=357

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\left(a^2e\right.\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bd\right.}$$

```
[Out] (2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f - d
*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 - b
^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 3*a*b*e*f + 3*a*b*d*g - 2*a^2*e*g)
+ c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x))/(c*(b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^3*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*e) + ((e*f - d*g)^3*ArcTanh[(b*d - 2
*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2
])])/(e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.52805, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1646, 843, 621, 206, 724}

$$\frac{2\left(-x\left(cg^2\left(-2a^2eg+3abdg-3abef+3b^2df\right)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3\right)-b\left(a^2e\right.\right.}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bd\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f - d
*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 - b
^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 3*a*b*e*f + 3*a*b*d*g - 2*a^2*e*g)
+ c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x))/(c*(b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^3*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*e) + ((e*f - d*g)^3*ArcTanh[(b*d - 2
*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2
])])/(e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2eg^3+3acfg\right.\right.}{c\left(b^2-\right.}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2eg^3+3acfg\right.\right.}{c\left(b^2-\right.}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2eg^3+3acfg\right.\right.}{c\left(b^2-\right.}$$

$$= \frac{2\left(b^2\left(cef^3+adg^3\right)-2ac\left(cf^2(ef-3dg)-ag^2(3ef-dg)\right)-b\left(c^2df^3+a^2eg^3+3acfg\right.\right.}{c\left(b^2-\right.}$$

Mathematica [A] time = 1.17417, size = 373, normalized size = 1.04

$$\frac{2\left(b\left(a^2eg^3+3acg(dg(f+gx)+ef(f-gx))+c^2f^2(d(f-3gx)-efx)\right)+2c\left(a^2g^2(dg-e(3f+gx))+acf(ef(f+3gx)-\right.\right.}{c\left(4ac-b^2\right)\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(b^3*d*g^3*x) + b^2*(a*g^3*(-d + e*x) + c*(-(e*f^3) + 3*d*f*g^2*x)) +
b*(a^2*e*g^3 + c^2*f^2*(-(e*f*x) + d*(f - 3*g*x)) + 3*a*c*g*(e*f*(f - g*x)
+ d*g*(f + g*x))) + 2*c*(c^2*d*f^3*x + a^2*g^2*(d*g - e*(3*f + g*x)) + a*c*
f*(-3*d*g*(f + g*x) + e*f*(f + 3*g*x)))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b
*d) + a*e))*Sqrt[a + x*(b + c*x)]) + ((e*f - d*g)^3*Log[d + e*x])/(e*(c*d^2
```

```
 + e*(-(b*d) + a*e))^(3/2)) + (g^3*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)]])/(c^(3/2)*e) + ((-(e*f) + d*g)^3*Log[-(b*d) + 2*a*e - 2*c*d*x + b*
e*x + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + x*(b + c*x)]])/(e*(c*d^2 + e*(
-(b*d) + a*e))^(3/2))
```

Maple [B] time = 0.317, size = 3127, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*g^3/e^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d-6*g^2/e*b/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*x*f-6/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*
c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^2*f^2*g+2/e^2/(a*e^2-b*
d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*x*b*c*d^3*g^3+12/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^3*f*g^2-1
2/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*f^2*g+6/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c
-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c
*d^3*f*g^2-6/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(
d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d^2*f*g^2+e/(a*e^2-b*d*e+c*d^2)
/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f^3+4/(a
*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f^3+1/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/
e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^3*g^3+3
/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*b^2*d*f^2*g+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f^3-3/e/
(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c
+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^2*f*g^2+g
^3/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+6/(a*e^2-b*d*e+c*d^2)/(4*a*c-b
^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c
*d*f^2*g-3*g^2/e/c/(c*x^2+b*x+a)^(1/2)*f+1/2*g^3/e*b/c^2/(c*x^2+b*x+a)^(1/2
)-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f^3-4/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2
)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d
^4*g^3-3/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+
x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^2*f*g^2-2/e^3/(a*e^2-b*d*e+c*d^2)/(
4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b*c*d^4*g^3-12*g^2/e^2*d*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c+g^3/e/c^(3
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/(a*e^2-b*d*e+c*d^2)/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e
+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*f^3+g^3/e^2/c/(c*x^2+b*x+a)^(1/2)*d-
g^3/e*x/c/(c*x^2+b*x+a)^(1/2)-1/e^2/(a*e^2-b*d*e+c*d^2)/((d/e+x)^2*c+(b*e-2
*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^3*g^3-3/(a*e^2-b*d*e+c*d^2
)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*f^2*g
+g^3/e^2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d+12*g/e*f^2/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)*x*c-3*g^2/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f+4*g^3/
e^3*d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c-6*g^2/e^2*d*f/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*b+6*g/e*f^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+1/2*g^3/e*b^3/
c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+2*g^3/e^3*d^2/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)*b+3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
```

```
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
(d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*
d*f^2*g+3/e/(a*e^2-b*d*e+c*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*d^2*f*g^2-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f^3+1/e^2/(a*
e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/
e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b
*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d^3*g^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.880 $\int \frac{(f+gx)^2}{(d+ex)(a+bx+cx^2)^{3/2}} dx$

Optimal. Leaf size=240

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-2dg\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (2*(b^2*e*f^2 + 2*a*(a*e*g^2 - c*f*(e*f - 2*d*g)) - b*(c*d*f^2 + a*g*(2*e*f
 + d*g)) - (2*c^2*d*f^2 + b*(b*d - a*e)*g^2 + c*(2*a*g*(2*e*f - d*g) - b*f*
(e*f + 2*d*g)))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c
*x^2]) + ((e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d
^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2)
```

Rubi [A] time = 0.305313, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1646, 12, 724, 206}

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-2dg\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*e*f^2 + 2*a*(a*e*g^2 - c*f*(e*f - 2*d*g)) - b*(c*d*f^2 + a*g*(2*e*f
 + d*g)) - (2*c^2*d*f^2 + b*(b*d - a*e)*g^2 + c*(2*a*g*(2*e*f - d*g) - b*f*
(e*f + 2*d*g)))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c
*x^2]) + ((e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d
^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2)
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+b(bd-ae)\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+b(bd-ae)\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+b(bd-ae)\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2ef^2+2a\left(aeg^2-cf(ef-2dg)\right)-b\left(cdf^2+ag(2ef+dg)\right)-\left(2c^2df^2+b(bd-ae)\right.\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.693444, size = 265, normalized size = 1.1

$$\frac{2\left(-2a^2eg^2+abg(dg+2ef-egx)-2acdg(2f+gx)+2acef(f+2gx)+b^2\left(dg^2x-ef^2\right)+bcf(d(f-2gx)-efx)+2c^2d\right.}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-2*a^2*e*g^2 + 2*c^2*d*f^2*x - 2*a*c*d*g*(2*f + g*x) + 2*a*c*e*f*(f + 2
*g*x) + a*b*g*(2*e*f + d*g - e*g*x) + b^2*(-(e*f^2) + d*g^2*x) + b*c*f*(-(e
*f*x) + d*(f - 2*g*x))))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a +
 x*(b + c*x)]) + ((e*f - d*g)^2*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^(3
/2) - ((e*f - d*g)^2*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 -
b*d*e + a*e^2]*Sqrt[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2)
```

Maple [B] time = 0.263, size = 2123, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 4/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^3*g^2+e/(a*e^2-b*d*e+c*d^2)/((d/e+x)^2
*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*f^2-g^2/e/c/(c*x^2+
b*x+a)^(1/2)-1/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e
```

```
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*d^2*g^2+2/(a*e^2-b*d*e+c*d^2)/(
4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*b^2*d*f*g+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d
/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f^2-2/e/(a*e^2-b*d*e+c*d^2)/(4
*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*x*b*c*d^2*g^2-4/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)
/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^2*f*g+4/(a*e^2-b*d*e+c*d^2)
/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*x*b*c*d*f*g-8/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d
)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*f*g+1/e/(a*e^2-b*d*e+c
*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d^2
*g^2-e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d
*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*f^2+
2/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d^3*g^2-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2
)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f
^2-2/(a*e^2-b*d*e+c*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*d*f*g-4*g^2/e^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c+8*g/e
*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f^2+4
*g/e*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-g^2/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2)-2*g^2/e^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-2*g^2/e*b/(4*a*c-b^
2)/(c*x^2+b*x+a)^(1/2)*x-1/e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(d/e+x))*d^2*g^2+2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(d/e+x))*d*f*g-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-
2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 44.006, size = 4150, normalized size = 17.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4
*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g
+ (b^2*c - 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*
b*c)*d*e*f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log(
(8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2
```

```
+ 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*
d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d
*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e
 + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d^3 - 3*a*
b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3 + 3*a^2*b
*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3 - 3*b*c^2*d
^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3 + 3*a*b*c*d*
e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g^2)*x)*sqrt(c*
x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^
3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d
*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^
3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^
2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x), (((a
*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4*a^2*c)*d
^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g + (b^2*c
- 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*b*c)*d*e*
f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d -
b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e
 + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f^2 - 2*(2*a*c^2*d^3 - 3*a*
b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*f*g + (a*b*c*d^3 + 3*a^2*b
*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e)*g^2 + ((2*c^3*d^3 - 3*b*c^2*d
^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f^2 - 2*(b*c^2*d^3 + 3*a*b*c*d*
e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*f*g - (a^2*b*e^3 - (b^2*c - 2*
a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*g^2)*x)*sqrt(c*
x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^
3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d
*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^
3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^
2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17985, size = 1022, normalized size = 4.26

$$-\frac{2\left(\frac{(2c^3d^3f^2-2bc^2d^3fg+b^2cd^3g^2-2ac^2d^3g^2-3bc^2d^2f^2e+2b^2cd^2fge+4ac^2d^2fge-b^3d^2g^2e+abcd^2g^2e+b^2cdf^2e^2+2ac^2df^2e^2-6abcdfge^2+2ab^2dg^2e^2-2a^2cdg^2\cdots}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f^2 - 2*b*c^2*d^3*f*g + b^2*c*d^3*g^2 - 2*a*c^2*d^3*g^2 - 3*
b*c^2*d^2*f^2*e + 2*b^2*c*d^2*f*g*e + 4*a*c^2*d^2*f*g*e - b^3*d^2*g^2*e + a
*b*c*d^2*g^2*e + b^2*c*d*f^2*e^2 + 2*a*c^2*d*f^2*e^2 - 6*a*b*c*d*f*g*e^2 +
2*a*b^2*d*g^2*e^2 - 2*a^2*c*d*g^2*e^2 - a*b*c*f^2*e^3 + 4*a^2*c*f*g*e^3 - a
^2*b*g^2*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*
e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8
*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f^2 - 4*a*c^2*d^3*
f*g + a*b*c*d^3*g^2 - 2*b^2*c*d^2*f^2*e + 2*a*c^2*d^2*f^2*e + 6*a*b*c*d^2*f
*g*e - a*b^2*d^2*g^2*e - 2*a^2*c*d^2*g^2*e + b^3*d*f^2*e^2 - a*b*c*d*f^2*e^
2 - 2*a*b^2*d*f*g*e^2 - 4*a^2*c*d*f*g*e^2 + 3*a^2*b*d*g^2*e^2 - a*b^2*f^2*e
^3 + 2*a^2*c*f^2*e^3 + 2*a^2*b*f*g*e^3 - 2*a^3*g^2*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^2 + b*x + a) + 2*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*arctan(
-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e -
a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

3.881 $\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{e(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*a
*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x + c*x^2]) + (e*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^
(3/2)
```

Rubi [A] time = 0.134968, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {822, 12, 724, 206}

$$\frac{e(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*a
*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x + c*x^2]) + (e*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^
(3/2)
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int}{\left(b^2-\right.}$$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{(e(ef}{}$$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{(2e(ef}{}$$

$$= -\frac{2\left(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e(ef-}{}$$

Mathematica [A] time = 0.185996, size = 183, normalized size = 0.98

$$\frac{2b(aeg+cd(f-gx)-cefx)+4c(-adg+ae(f+gx)+cdfx)-2b^2ef}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)} + \frac{e(dg-ef)\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*b^2*e*f + 2*b*(a*e*g - c*e*f*x + c*d*(f - g*x)) + 4*c*(-(a*d*g) + c*d*f
*x + a*e*(f + g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(
b + c*x)]) + (e*(-(e*f) + d*g)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(
2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])])/(c*d^2 + e*(-(b*d
) + a*e))^(3/2)
```

Maple [B] time = 0.258, size = 1261, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2/e*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/(a*e^2-b*d*e+c*d^2)/((d/e
+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*d*g+e/(a*e^2-b
*d*e+c*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*f+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(
```

```
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*d*g-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)
/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c*f-
4/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d^2*g+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((
d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d*f+1
/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*b^2*d*g-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^
2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2*f-2/e/(a*e^2-b
*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*b*c*d^2*g+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*
e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d*f+1/(a*e^2-b*d*e+c*
d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*
(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*d*g-e/(a*e^2-b*d*e+c*d^2)/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+
x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 37.0972, size = 3411, normalized size = 18.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g + ((b^2*c - 4*a*c
^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c)*e^2*f - (b^3 -
4*a*b*c)*d*e*g)*x)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 -
 (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sq
rt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e
)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*
x + d^2)) + 4*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e +
 (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f - (2*a*c^2*d^3 - 3*a*b*c*d^
2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((2*c^3*d^3 - 3*b*c^2*d^2*e
- a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a
^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2
*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
- 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a
*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^
3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)
*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
 - 4*a^3*b*c)*e^4)*x), (((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g
+ ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c
```

```
)*e^2*f - (b^3 - 4*a*b*c)*d*e*g)*x)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/
2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d
- b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2
+ (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) - 2*sqrt(c*x^2 + b*x + a)*((b*c^2*d^3 -
 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*f -
 (2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2)*g + ((
2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*f - (b*c^2
*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*g)*x))/((a*b^
2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b
^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^
4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*
c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a
^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*
a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a
^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19965, size = 767, normalized size = 4.1

$$-\frac{2\left(\frac{\left(2c^3d^3f-bc^2d^3g-3bc^2d^2fe+b^2cd^2ge+2ac^2d^2ge+b^2cdfe^2+2ac^2dfe^2-3abcdge^2-abcfe^3+2a^2cge^3\right)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3f-2ac^2d^3g-2b^2cd^2fe+2ac^2d^2fe+3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+}\right.}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f - b*c^2*d^3*g - 3*b*c^2*d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*
d^2*g*e + b^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 - a*b*c*f*e^3 +
 2*a^2*c*g*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^
3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 +
 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f - 2*a*c^2*d^3*
g - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e + b^3*d*f*e^2 - a*b
*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3
+ a^2*b*g*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e
 + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*
a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a) - 2*(d*g*
e - f*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt
(-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*
e^2))
```

3.882 $\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=155

$$\frac{e^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 -
b*d*e + a*e^2)^(3/2)
```

Rubi [A] time = 0.10111, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {740, 12, 724, 206}

$$\frac{e^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 -
b*d*e + a*e^2)^(3/2)
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,\right.}{cd^2-bde+ae^2}$$

$$= -\frac{2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\left(cd^2-bde+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.161801, size = 177, normalized size = 1.14

$$\frac{2\sqrt{ae^2-bde+cd^2}\left(2c(ae+cdx)+b^2(-e)+bc(d-ex)\right)+e^2\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2ae-bd+bex-2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right.}{\left(4ac-b^2\right)\sqrt{a+x(b+cx)}\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*Sqrt[c*d^2 - b*d*e + a*e^2]*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*x
)) + (b^2 - 4*a*c)*e^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(-(b*d) + 2*a*e - 2*c*
d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/((-
b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.272, size = 603, normalized size = 3.9

$$\frac{e}{ae^2-bde+cd^2}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c+\frac{be-2cd}{e}\left(\frac{d}{e}+x\right)+\frac{ae^2-bde+cd^2}{e^2}}}-2\frac{bxec}{\left(ae^2-bde+cd^2\right)\left(4ac-b^2\right)}\frac{1}{\sqrt{\left(\frac{d}{e}+x\right)^2c+\frac{be-2cd}{e}\left(\frac{d}{e}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] e/(a*e^2-b*d*e+c*d^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)-2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e
*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-
b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^
2*d-e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)
^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d-e/(a*e^2-b*
```

```
d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e-2*c
*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.18464, size = 2768, normalized size = 17.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*c
)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*
c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b
*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) -
4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a
^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^
2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 -
 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*
(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2
*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^3*c
^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*
e^4)*x), (((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a
^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a
*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*
d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 -
 (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c +
2*a*c^2)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*
(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 -
 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*
c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3
)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*
x^2 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3
- 4*a^3*b*c)*e^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [B] time = 1.23315, size = 603, normalized size = 3.89

$$-\frac{2\left(\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{bc^2d^3-2\,b^2cd^2e+\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2\cdots}\cdots}{\sqrt{cx^2+bx+a}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^2 + b*x + a) + 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*e^2)
*sqrt(-c*d^2 + b*d*e - a*e^2))
```

3.883 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=352

$$-\frac{2e\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)\left(ae^2-bde+cd^2\right)}+\frac{2g\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)\left(ag^2-bfg+cf^2\right)}+\frac{e^3\tanh^{-1}}{(ef-}$$

```
[Out] (-2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g +
 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)*Sqrt[a + b*x + c*x^2]) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x
)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e +
 a*e^2)^(3/2)*(e*f - d*g)) - (g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(
2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^(3/2))
```

Rubi [A] time = 0.443355, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {960, 740, 12, 724, 206}

$$-\frac{2e\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)\left(ae^2-bde+cd^2\right)}+\frac{2g\left(2acg+b^2(-g)+cx(2cf-bg)+bcf\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)\left(ag^2-bfg+cf^2\right)}+\frac{e^3\tanh^{-1}}{(ef-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```


```
[Out] (-2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g +
 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)*Sqrt[a + b*x + c*x^2]) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x
)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e +
 a*e^2)^(3/2)*(e*f - d*g)) - (g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(
2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^(3/2))
```


Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
```

```
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \left(\frac{e}{(ef-dg)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)\left(a+bx+cx^2\right)^{3}}\right.$$

$$= \frac{e\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{ef-dg}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(bcf-\right.}{\left(b^2-4ac\right)(ef-d}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(bcf-\right.}{\left(b^2-4ac\right)(ef-d}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(bcf-\right.}{\left(b^2-4ac\right)(ef-d}$$

$$= -\frac{2e\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{2g\left(bcf-\right.}{\left(b^2-4ac\right)(ef-d}$$

Mathematica [A] time = 1.61699, size = 317, normalized size = 0.9

$$\frac{-\frac{2e\left(-2c(ae+cdx)+b^2e+bc(ex-d)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(e(bd-ae)-cd^2\right)} + \frac{2g\left(-2c(ag+cfx)+b^2g+bc(gx-f)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(g(bf-ag)-cf^2\right)} + \frac{e^3\tanh^{-1}\left(\frac{-2ae+b(d-ex)+2cdx}{2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}} - \frac{g^3\tanh^{-1}\left(\frac{-2ag+b(f-gx)+2c}{2\sqrt{a+x(b+cx)}\sqrt{g(ag-b}}\right.}{\left(g(ag-bf)+cf^2\right)^{3/2}}}{ef-dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*e*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d^
2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*g*(b^2*g - 2*c*(a*g + c*f*x
) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(
b + c*x)]) + (e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 +
e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e))^(3/2)
 - (g^3*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f)
+ a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(e*f - d
*g)
```

Maple [B] time = 0.453, size = 1343, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -1/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)*e^2/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)+2/(d*g-e*f)*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)
/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*b*c-4/
(d*g-e*f)*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e
+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*d+1/(d*g-e*f)*e^2/(a*e^2-b*d*e+c*d
^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*b^2-2/(d*g-e*f)*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-
2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d+1/(d*g-e*f)/(a*e^2-b*
d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((d/e+x)^2*c+(b*e
-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/e+x))+1/(d*g-e*f)/(a*g
^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)-2/(d*g-e*f)*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*
g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*b*c+4/(d*g-e*f)*g/(a*g^
2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+
c*f^2)/g^2)^(1/2)*x*c^2*f-1/(d*g-e*f)*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/(
(x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^2+2/(d*g
-e*f)*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b*c*f-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)*g^2/((
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(
x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}(ex + d)\left(gx + f\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(f+gx\right)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.884 $\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=642

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{\left(b^2-4ac\right)(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}-\frac{2e^2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)^2\left(ae^2-bde+cd^2\right)}+\frac{2eg}{\left(b^2-4ac\right)}$$

```
[Out] (-2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b
^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 -
b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c*(
2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*
x)*Sqrt[a + b*x + c*x^2]) + (g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*
g))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^(3/2)*
(e*f - d*g)^2) - (3*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*
x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(
c*f^2 - b*f*g + a*g^2)^(5/2)) - (e*g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)^(3/2))
```

Rubi [A] time = 0.910685, antiderivative size = 642, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {960, 740, 12, 724, 206, 806}

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{\left(b^2-4ac\right)(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}-\frac{2e^2\left(2ace+b^2(-e)+cx(2cd-be)+bcd\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}(ef-dg)^2\left(ae^2-bde+cd^2\right)}+\frac{2eg}{\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```


```
[Out] (-2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b
^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 -
b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c*(
2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*
x)*Sqrt[a + b*x + c*x^2]) + (g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*
g))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^(3/2)*
(e*f - d*g)^2) - (3*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*
x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(
c*f^2 - b*f*g + a*g^2)^(5/2)) - (e*g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)^(3/2))
```


Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int\left(\frac{e^2}{(ef-dg)^2(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)^2\left(a+bx+cx^2\right)^{3}}\right.$$

$$= \frac{e^2\int\frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int\frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^2} - \frac{g\int\frac{1}{(f+gx)^2\left(a+bx+cx^2\right)^{3/}}}{ef-dg}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg\left(bcf-\right.}{\left(b^2-4ac\right)(ef-dg\right.}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg\left(bcf-\right.}{\left(b^2-4ac\right)(ef-dg\right.}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg\left(bcf-\right.}{\left(b^2-4ac\right)(ef-dg\right.}$$

$$= -\frac{2e^2\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{2eg\left(bcf-\right.}{\left(b^2-4ac\right)(ef-dg\right.}$$

Mathematica [A] time = 6.21851, size = 868, normalized size = 1.35

$$\frac{\left(cx^2+bx+a\right)^{3/2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-e(bd-ae)}\sqrt{cx^2+bx+a}}\right)e^4}{\left(cd^2-e(bd-ae)\right)^{3/2}(ef-dg)^2(a+x(b+cx))^{3/2}} - \frac{2\left(-eb^2+cdb+2ace+c(2cd-be)x\right)\left(cx^2+bx+a\right)e^2}{\left(b^2-4ac\right)\left(cd^2-e(bd-ae)\right)(ef-dg)^2(a+x(b+cx))^{3/2}} - \frac{g^3\left(cx^2\right.}{(ef-}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2))/((
b^2 - 4*a*c)*(c*d^2 - e*(b*d - a*e))*(e*f - d*g)^2*(a + x*(b + c*x))^(3/2))
 + (2*e*g*(b*c*f - b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x)*(a + b*x + c*x^2))/
((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - g*(b*f - a*g))*(a + x*(b + c*x))^(3/2
)) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x)*(a + b*x + c*x^2))/
((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - g*(b*f - a*g))*(f + g*x)*(a + x*(b + c*
x))^(3/2)) + (e^4*(a + b*x + c*x^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b
*e)*x)/(2*Sqrt[c*d^2 - e*(b*d - a*e)]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - e*
(b*d - a*e))^(3/2)*(e*f - d*g)^2*(a + x*(b + c*x))^(3/2)) - (2*g*(a + b*x +
 c*x^2)^(3/2)*(-(((-(c*f*g*(2*c*f - b*g)) + (g^2*(2*b*c*f - 3*b^2*g + 8*a*c
*g))/2)*Sqrt[a + b*x + c*x^2])/((c*f^2 - b*f*g + a*g^2)*(f + g*x))) + (2*(-
2*(a*c*g^2*(2*c*f - b*g) + (c*f*g*(2*b*c*f - 3*b^2*g + 8*a*c*g))/2) + b*(c*
f*g*(2*c*f - b*g) + (g^2*(2*b*c*f - 3*b^2*g + 8*a*c*g))/2))*ArcTanh[(-(b*f)
 + 2*a*g - (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c
*x^2])])/(Sqrt[c*f^2 - b*f*g + a*g^2]*(4*c*f^2 - 4*b*f*g + 4*a*g^2))))/((b^
2 - 4*a*c)*(-(e*f) + d*g)*(c*f^2 - b*f*g + a*g^2)*(a + x*(b + c*x))^(3/2))
- (e*g^3*(a + b*x + c*x^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2
*Sqrt[c*f^2 - g*(b*f - a*g)]*Sqrt[a + b*x + c*x^2]])])/((e*f - d*g)^2*(c*f^2
 - g*(b*f - a*g))^(3/2)*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.416, size = 2807, normalized size = 4.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -e^3/(d*g-e*f)^2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(d/e+x)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(d/
e+x))-g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+2*e/(d*g-e*f)^2*g^2/(a*g^2-b*f*g+c*f^2
)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(
1/2)*x*b*c-4*e/(d*g-e*f)^2*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(
b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c^2*f-12*g^2/(d*g-e*f
)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)*x*b*c^2*f+6*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4
*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
*b*c^2*f^2+2*e^2/(d*g-e*f)^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(
b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*c*d+3*g^3/(d*g-e*f)/(
a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-
b*f*g+c*f^2)/g^2)^(1/2)*x*b^2*c+12*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c
-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*x*c
^3*f^2-6*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^2*c*f-2*e^3/(d*g-e*f)^2/(
a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*x*b*c+4*e^2/(d*g-e*f)^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^
2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c^2*
d-3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g
)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b-e/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)*g^2/((
x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-2*e/(d*g-e*
f)^2*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b*c*f+e^3/(d*g-e*f)^2/(a*e^2-b*d*e+c*d^2)/((d
/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+e/(d*g-e*f)^
2*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b^2-3*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+
f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f-8*g/(d*g-e*f)*c^2/(a*g^2-b*f*g+
c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)*x-4*g/(d*g-e*f)*c/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b
*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*b-e^3/(d*g-e*f)^2/(a*e^2
-b*d*e+c*d^2)/(4*a*c-b^2)/((d/e+x)^2*c+(b*e-2*c*d)/e*(d/e+x)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*b^2+3*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((x+f/g)^2*c+(b*
g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*c*f+3/2*g^3/(d*g-e*f)/(a*
g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*b^3+3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+
2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2
-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*b+e/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)*g^2/
((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g
*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2), x)
```

3.885 $\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=1064

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e^5}{\left(cd^2-bed+ae^2\right)^{3/2}(ef-dg)^3}-\frac{2\left(-eb^2+cdb+2ace+c(2cd-be)x\right)e^3}{\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)(ef-dg)^3\sqrt{cx^2+bx+a}}-\frac{g^3\tanh^{-1}\left(\frac{bf-2ag+(2c}{2\sqrt{cf^2-bgf+ag^2}}\right.}{(ef-dg)^3\left(cf^2-bgf\right.}$$

```
[Out] (-2*e^3*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2]) + (2*e^2*g*(b*c*f -
 b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^3*(c*f^2
- b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c
*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f +
g*x)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b^2*g + 2*a*c*g + c*(2*c*f
- b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)*S
qrt[a + b*x + c*x^2]) + (g^2*(8*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g)
)*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)^2) + (e*g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*g))*Sq
rt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^2
*(f + g*x)) + (g^2*(2*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 1
3*a*g))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^3*(f + g*x)) + (e^5*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqr
t[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^
(3/2)*(e*f - d*g)^3) - (3*e*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(5/2)) - (e^2*g^3*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e
*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (3*g^3*(16*c^2*f^2 + 5*b^2*g^2
 - 4*c*g*(4*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f
^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^(7/2))
```

Rubi [A] time = 1.89795, antiderivative size = 1064, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {960, 740, 12, 724, 206, 834, 806}

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)e^5}{\left(cd^2-bed+ae^2\right)^{3/2}(ef-dg)^3}-\frac{2\left(-eb^2+cdb+2ace+c(2cd-be)x\right)e^3}{\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)(ef-dg)^3\sqrt{cx^2+bx+a}}-\frac{g^3\tanh^{-1}\left(\frac{bf-2ag+(2c}{2\sqrt{cf^2-bgf+ag^2}}\right.}{(ef-dg)^3\left(cf^2-bgf\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```


```
[Out] (-2*e^3*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2]) + (2*e^2*g*(b*c*f -
 b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^3*(c*f^2
- b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c
*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f +
g*x)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b^2*g + 2*a*c*g + c*(2*c*f
- b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)*S
qrt[a + b*x + c*x^2]) + (g^2*(8*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g)
)*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)^2) + (e*g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*g))*Sq
```

```
rt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^2
*(f + g*x)) + (g^2*(2*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 1
3*a*g))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^3*(f + g*x)) + (e^5*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqr
t[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^
(3/2)*(e*f - d*g)^3) - (3*e*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(5/2)) - (e^2*g^3*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e
*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (3*g^3*(16*c^2*f^2 + 5*b^2*g^2
 - 4*c*g*(4*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f
^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^(7/2))
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
```

```
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \int\left(\frac{e^3}{(ef-dg)^3(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)^3\left(a+bx+cx\right.}\right.$$

$$= \frac{e^3\int\frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^3} - \frac{\left(e^2g\right)\int\frac{1}{(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx}{(ef-dg)^3} - \frac{(eg)\int\frac{1}{(f+gx)^2(a+}}{(ef-d}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2g\left(bc\right.}{\left(b^2-4ac\right)(ef-}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2g\left(bc\right.}{\left(b^2-4ac\right)(ef-}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2g\left(bc\right.}{\left(b^2-4ac\right)(ef-}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2g\left(bc\right.}{\left(b^2-4ac\right)(ef-}$$

$$= -\frac{2e^3\left(bcd-b^2e+2ace+c(2cd-be)x\right)}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{2e^2g\left(bc\right.}{\left(b^2-4ac\right)(ef-}$$

Mathematica [A] time = 5.98072, size = 1013, normalized size = 0.95

$$-\frac{\tanh^{-1}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(ae-bd)}\sqrt{a+x(b+cx)}}\right)e^5}{\left(cd^2+e(ae-bd)\right)^{3/2}(dg-ef)^3} - \frac{2\left(eb^2+c(ex-d)b-2c(ae+cdx)\right)e^3}{\left(b^2-4ac\right)\left(e(bd-ae)-cd^2\right)(ef-dg)^3\sqrt{a+x(b+cx)}} - \frac{2g\left(gb^2+c(g\right.}{\left(b^2-4ac\right)(dg-ef)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)]) - (2*e^2*g*(b^2*g
```

```
 - 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)^3*(-(
c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c
*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f -
 a*g))*(f + g*x)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g - 2*c*(a*g + c*f*
x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2) + g*(b*f - a*g
))*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (e*g^2*((2*(4*c^2*f^2 + 3*b^2*g^2 - 4
*c*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f
) + a*g))^2*(f + g*x)) + (3*g*(2*c*f - b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f
*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f
^2 + g*(-(b*f) + a*g))^(5/2)))/(2*(e*f - d*g)^2) - (g^2*((4*(8*c^2*f^2 + 5*
b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*Sqrt[a + x*(b + c*x)])/(f + g*x)^2 + (2*(2
*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 13*a*g))*Sqrt[a + x*(b
 + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)) + (3*(b^2 - 4*a*c)*g*(16*c
^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x
 + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2
 + g*(-(b*f) + a*g))^(3/2)))/(8*(b^2 - 4*a*c)*(-(e*f) + d*g)*(c*f^2 + g*(-(
b*f) + a*g))^2) - (e^5*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/((c*d^2 + e*(-(b*d) + a*e))
^(3/2)*(-(e*f) + d*g)^3) - (e^2*g^3*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/((e*f - d*g)^3
*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

Maple [B] time = 0.484, size = 5459, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(ex+d)\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 22.5388, size = 19887, normalized size = 18.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^9*d^3*f^9 - 9*b*c^8*d^3*f^8*g + 18*b^2*c^7*d^3*f^7*g^2 - 21*b^3*c^
6*d^3*f^6*g^3 + 15*b^4*c^5*d^3*f^5*g^4 + 6*a*b^2*c^6*d^3*f^5*g^4 - 12*a^2*c
^7*d^3*f^5*g^4 - 6*b^5*c^4*d^3*f^4*g^5 - 15*a*b^3*c^5*d^3*f^4*g^5 + 30*a^2*
b*c^6*d^3*f^4*g^5 + b^6*c^3*d^3*f^3*g^6 + 12*a*b^4*c^4*d^3*f^3*g^6 - 18*a^2
*b^2*c^5*d^3*f^3*g^6 - 16*a^3*c^6*d^3*f^3*g^6 - 3*a*b^5*c^3*d^3*f^2*g^7 - 3
*a^2*b^3*c^4*d^3*f^2*g^7 + 24*a^3*b*c^5*d^3*f^2*g^7 + 3*a^2*b^4*c^3*d^3*f*g
^8 - 6*a^3*b^2*c^4*d^3*f*g^8 - 6*a^4*c^5*d^3*f*g^8 - a^3*b^3*c^3*d^3*g^9 +
3*a^4*b*c^4*d^3*g^9 - 3*b*c^8*d^2*f^9*e + 15*b^2*c^7*d^2*f^8*g*e - 6*a*c^8*
d^2*f^8*g*e - 33*b^3*c^6*d^2*f^7*g^2*e + 24*a*b*c^7*d^2*f^7*g^2*e + 41*b^4*
c^5*d^2*f^6*g^3*e - 34*a*b^2*c^6*d^2*f^6*g^3*e - 16*a^2*c^7*d^2*f^6*g^3*e -
 30*b^5*c^4*d^2*f^5*g^4*e + 9*a*b^3*c^5*d^2*f^5*g^4*e + 66*a^2*b*c^6*d^2*f^
5*g^4*e + 12*b^6*c^3*d^2*f^4*g^5*e + 24*a*b^4*c^4*d^2*f^4*g^5*e - 96*a^2*b^
2*c^5*d^2*f^4*g^5*e - 12*a^3*c^6*d^2*f^4*g^5*e - 2*b^7*c^2*d^2*f^3*g^6*e -
23*a*b^5*c^3*d^2*f^3*g^6*e + 49*a^2*b^3*c^4*d^2*f^3*g^6*e + 48*a^3*b*c^5*d^
2*f^3*g^6*e + 6*a*b^6*c^2*d^2*f^2*g^7*e + 3*a^2*b^4*c^3*d^2*f^2*g^7*e - 54*
a^3*b^2*c^4*d^2*f^2*g^7*e - 6*a^2*b^5*c^2*d^2*f*g^8*e + 15*a^3*b^3*c^3*d^2*
f*g^8*e + 9*a^4*b*c^4*d^2*f*g^8*e + 2*a^3*b^4*c^2*d^2*g^9*e - 7*a^4*b^2*c^3
*d^2*g^9*e + 2*a^5*c^4*d^2*g^9*e + b^2*c^7*d*f^9*e^2 + 2*a*c^8*d*f^9*e^2 -
6*b^3*c^6*d*f^8*g*e^2 - 3*a*b*c^7*d*f^8*g*e^2 + 15*b^4*c^5*d*f^7*g^2*e^2 -
6*a*b^2*c^6*d*f^7*g^2*e^2 - 20*b^5*c^4*d*f^6*g^3*e^2 + 13*a*b^3*c^5*d*f^6*g
^3*e^2 + 16*a^2*b*c^6*d*f^6*g^3*e^2 + 15*b^6*c^3*d*f^5*g^4*e^2 - 48*a^2*b^2
*c^5*d*f^5*g^4*e^2 - 12*a^3*c^6*d*f^5*g^4*e^2 - 6*b^7*c^2*d*f^4*g^5*e^2 - 1
5*a*b^5*c^3*d*f^4*g^5*e^2 + 51*a^2*b^3*c^4*d*f^4*g^5*e^2 + 42*a^3*b*c^5*d*f
^4*g^5*e^2 + b^8*c*d*f^3*g^6*e^2 + 12*a*b^6*c^2*d*f^3*g^6*e^2 - 19*a^2*b^4*
c^3*d*f^3*g^6*e^2 - 50*a^3*b^2*c^4*d*f^3*g^6*e^2 - 16*a^4*c^5*d*f^3*g^6*e^2
 - 3*a*b^7*c*d*f^2*g^7*e^2 - 3*a^2*b^5*c^2*d*f^2*g^7*e^2 + 27*a^3*b^3*c^3*d
*f^2*g^7*e^2 + 24*a^4*b*c^4*d*f^2*g^7*e^2 + 3*a^2*b^6*c*d*f*g^8*e^2 - 6*a^3
*b^4*c^2*d*f*g^8*e^2 - 9*a^4*b^2*c^3*d*f*g^8*e^2 - 6*a^5*c^4*d*f*g^8*e^2 -
a^3*b^5*c*d*g^9*e^2 + 3*a^4*b^3*c^2*d*g^9*e^2 + a^5*b*c^3*d*g^9*e^2 - a*b*c
^7*f^9*e^3 + 6*a*b^2*c^6*f^8*g*e^3 - 6*a^2*c^7*f^8*g*e^3 - 15*a*b^3*c^5*f^7
*g^2*e^3 + 24*a^2*b*c^6*f^7*g^2*e^3 + 20*a*b^4*c^4*f^6*g^3*e^3 - 34*a^2*b^2
*c^5*f^6*g^3*e^3 - 16*a^3*c^6*f^6*g^3*e^3 - 15*a*b^5*c^3*f^5*g^4*e^3 + 15*a
^2*b^3*c^4*f^5*g^4*e^3 + 54*a^3*b*c^5*f^5*g^4*e^3 + 6*a*b^6*c^2*f^4*g^5*e^3
 + 9*a^2*b^4*c^3*f^4*g^5*e^3 - 66*a^3*b^2*c^4*f^4*g^5*e^3 - 12*a^4*c^5*f^4*
```

g^5*e^3 - a*b^7*c*f^3*g^6*e^3 - 11*a^2*b^5*c^2*f^3*g^6*e^3 + 31*a^3*b^3*c^3
*f^3*g^6*e^3 + 32*a^4*b*c^4*f^3*g^6*e^3 + 3*a^2*b^6*c*f^2*g^7*e^3 - 30*a^4*
b^2*c^3*f^2*g^7*e^3 - 3*a^3*b^5*c*f*g^8*e^3 + 9*a^4*b^3*c^2*f*g^8*e^3 + 3*a
^5*b*c^3*f*g^8*e^3 + a^4*b^4*c*g^9*e^3 - 4*a^5*b^2*c^2*g^9*e^3 + 2*a^6*c^3*
g^9*e^3)*x/(b^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 6*b^3*c^7*d^4*f^11*g + 24
*a*b*c^8*d^4*f^11*g + 15*b^4*c^6*d^4*f^10*g^2 - 54*a*b^2*c^7*d^4*f^10*g^2 -
 24*a^2*c^8*d^4*f^10*g^2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^
3 + 120*a^2*b*c^7*d^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^
4*f^8*g^4 - 60*a^3*c^7*d^4*f^8*g^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d
^4*f^7*g^5 + 180*a^2*b^3*c^5*d^4*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + b^8*
c^2*d^4*f^6*g^6 + 26*a*b^6*c^3*d^4*f^6*g^6 - 30*a^2*b^4*c^4*d^4*f^6*g^6 - 3
40*a^3*b^2*c^5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g
^7 - 36*a^2*b^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c
^5*d^4*f^5*g^7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 -
 60*a^5*c^5*d^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f
^3*g^9 + 120*a^5*b*c^4*d^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b
^2*c^3*d^4*f^2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 6*a^5*b^3*c^2*d^4*f*g^11 +
24*a^6*b*c^3*d^4*f*g^11 + a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*b^3
*c^7*d^3*f^12*e + 8*a*b*c^8*d^3*f^12*e + 12*b^4*c^6*d^3*f^11*g*e - 48*a*b^2
*c^7*d^3*f^11*g*e - 30*b^5*c^5*d^3*f^10*g^2*e + 108*a*b^3*c^6*d^3*f^10*g^2*
e + 48*a^2*b*c^7*d^3*f^10*g^2*e + 40*b^6*c^4*d^3*f^9*g^3*e - 100*a*b^4*c^5*
d^3*f^9*g^3*e - 240*a^2*b^2*c^6*d^3*f^9*g^3*e - 30*b^7*c^3*d^3*f^8*g^4*e +
450*a^2*b^3*c^5*d^3*f^8*g^4*e + 120*a^3*b*c^6*d^3*f^8*g^4*e + 12*b^8*c^2*d^
3*f^7*g^5*e + 72*a*b^6*c^3*d^3*f^7*g^5*e - 360*a^2*b^4*c^4*d^3*f^7*g^5*e -
480*a^3*b^2*c^5*d^3*f^7*g^5*e - 2*b^9*c*d^3*f^6*g^6*e - 52*a*b^7*c^2*d^3*f^
6*g^6*e + 60*a^2*b^5*c^3*d^3*f^6*g^6*e + 680*a^3*b^3*c^4*d^3*f^6*g^6*e + 16
0*a^4*b*c^5*d^3*f^6*g^6*e + 12*a*b^8*c*d^3*f^5*g^7*e + 72*a^2*b^6*c^2*d^3*f
^5*g^7*e - 360*a^3*b^4*c^3*d^3*f^5*g^7*e - 480*a^4*b^2*c^4*d^3*f^5*g^7*e -
30*a^2*b^7*c*d^3*f^4*g^8*e + 450*a^4*b^3*c^3*d^3*f^4*g^8*e + 120*a^5*b*c^4*
d^3*f^4*g^8*e + 40*a^3*b^6*c*d^3*f^3*g^9*e - 100*a^4*b^4*c^2*d^3*f^3*g^9*e
- 240*a^5*b^2*c^3*d^3*f^3*g^9*e - 30*a^4*b^5*c*d^3*f^2*g^10*e + 108*a^5*b^3
*c^2*d^3*f^2*g^10*e + 48*a^6*b*c^3*d^3*f^2*g^10*e + 12*a^5*b^4*c*d^3*f*g^11
*e - 48*a^6*b^2*c^2*d^3*f*g^11*e - 2*a^6*b^3*c*d^3*g^12*e + 8*a^7*b*c^2*d^3
*g^12*e + b^4*c^6*d^2*f^12*e^2 - 2*a*b^2*c^7*d^2*f^12*e^2 - 8*a^2*c^8*d^2*f
^12*e^2 - 6*b^5*c^5*d^2*f^11*g*e^2 + 12*a*b^3*c^6*d^2*f^11*g*e^2 + 48*a^2*b
*c^7*d^2*f^11*g*e^2 + 15*b^6*c^4*d^2*f^10*g^2*e^2 - 24*a*b^4*c^5*d^2*f^10*g
^2*e^2 - 132*a^2*b^2*c^6*d^2*f^10*g^2*e^2 - 48*a^3*c^7*d^2*f^10*g^2*e^2 - 2
0*b^7*c^3*d^2*f^9*g^3*e^2 + 10*a*b^5*c^4*d^2*f^9*g^3*e^2 + 220*a^2*b^3*c^5*
d^2*f^9*g^3*e^2 + 240*a^3*b*c^6*d^2*f^9*g^3*e^2 + 15*b^8*c^2*d^2*f^8*g^4*e^
2 + 30*a*b^6*c^3*d^2*f^8*g^4*e^2 - 225*a^2*b^4*c^4*d^2*f^8*g^4*e^2 - 510*a^
3*b^2*c^5*d^2*f^8*g^4*e^2 - 120*a^4*c^6*d^2*f^8*g^4*e^2 - 6*b^9*c*d^2*f^7*g
^5*e^2 - 48*a*b^7*c^2*d^2*f^7*g^5*e^2 + 108*a^2*b^5*c^3*d^2*f^7*g^5*e^2 + 6
00*a^3*b^3*c^4*d^2*f^7*g^5*e^2 + 480*a^4*b*c^5*d^2*f^7*g^5*e^2 + b^10*d^2*f
^6*g^6*e^2 + 28*a*b^8*c*d^2*f^6*g^6*e^2 + 22*a^2*b^6*c^2*d^2*f^6*g^6*e^2 -
400*a^3*b^4*c^3*d^2*f^6*g^6*e^2 - 760*a^4*b^2*c^4*d^2*f^6*g^6*e^2 - 160*a^5
*c^5*d^2*f^6*g^6*e^2 - 6*a*b^9*d^2*f^5*g^7*e^2 - 48*a^2*b^7*c*d^2*f^5*g^7*e
^2 + 108*a^3*b^5*c^2*d^2*f^5*g^7*e^2 + 600*a^4*b^3*c^3*d^2*f^5*g^7*e^2 + 48
0*a^5*b*c^4*d^2*f^5*g^7*e^2 + 15*a^2*b^8*d^2*f^4*g^8*e^2 + 30*a^3*b^6*c*d^2
*f^4*g^8*e^2 - 225*a^4*b^4*c^2*d^2*f^4*g^8*e^2 - 510*a^5*b^2*c^3*d^2*f^4*g^
8*e^2 - 120*a^6*c^4*d^2*f^4*g^8*e^2 - 20*a^3*b^7*d^2*f^3*g^9*e^2 + 10*a^4*b
^5*c*d^2*f^3*g^9*e^2 + 220*a^5*b^3*c^2*d^2*f^3*g^9*e^2 + 240*a^6*b*c^3*d^2*
f^3*g^9*e^2 + 15*a^4*b^6*d^2*f^2*g^10*e^2 - 24*a^5*b^4*c*d^2*f^2*g^10*e^2 -
 132*a^6*b^2*c^2*d^2*f^2*g^10*e^2 - 48*a^7*c^3*d^2*f^2*g^10*e^2 - 6*a^5*b^5
*d^2*f*g^11*e^2 + 12*a^6*b^3*c*d^2*f*g^11*e^2 + 48*a^7*b*c^2*d^2*f*g^11*e^2
 + a^6*b^4*d^2*g^12*e^2 - 2*a^7*b^2*c*d^2*g^12*e^2 - 8*a^8*c^2*d^2*g^12*e^2
 - 2*a*b^3*c^6*d*f^12*e^3 + 8*a^2*b*c^7*d*f^12*e^3 + 12*a*b^4*c^5*d*f^11*g*
e^3 - 48*a^2*b^2*c^6*d*f^11*g*e^3 - 30*a*b^5*c^4*d*f^10*g^2*e^3 + 108*a^2*b
^3*c^5*d*f^10*g^2*e^3 + 48*a^3*b*c^6*d*f^10*g^2*e^3 + 40*a*b^6*c^3*d*f^9*g^
3*e^3 - 100*a^2*b^4*c^4*d*f^9*g^3*e^3 - 240*a^3*b^2*c^5*d*f^9*g^3*e^3 - 30*

a*b^7*c^2*d*f^8*g^4*e^3 + 450*a^3*b^3*c^4*d*f^8*g^4*e^3 + 120*a^4*b*c^5*d*f ^8*g^4*e^3 + 12*a*b^8*c*d*f^7*g^5*e^3 + 72*a^2*b^6*c^2*d*f^7*g^5*e^3 - 360* a^3*b^4*c^3*d*f^7*g^5*e^3 - 480*a^4*b^2*c^4*d*f^7*g^5*e^3 - 2*a*b^9*d*f^6*g ^6*e^3 - 52*a^2*b^7*c*d*f^6*g^6*e^3 + 60*a^3*b^5*c^2*d*f^6*g^6*e^3 + 680*a^ 4*b^3*c^3*d*f^6*g^6*e^3 + 160*a^5*b*c^4*d*f^6*g^6*e^3 + 12*a^2*b^8*d*f^5*g^ 7*e^3 + 72*a^3*b^6*c*d*f^5*g^7*e^3 - 360*a^4*b^4*c^2*d*f^5*g^7*e^3 - 480*a^ 5*b^2*c^3*d*f^5*g^7*e^3 - 30*a^3*b^7*d*f^4*g^8*e^3 + 450*a^5*b^3*c^2*d*f^4* g^8*e^3 + 120*a^6*b*c^3*d*f^4*g^8*e^3 + 40*a^4*b^6*d*f^3*g^9*e^3 - 100*a^5* b^4*c*d*f^3*g^9*e^3 - 240*a^6*b^2*c^2*d*f^3*g^9*e^3 - 30*a^5*b^5*d*f^2*g^10 *e^3 + 108*a^6*b^3*c*d*f^2*g^10*e^3 + 48*a^7*b*c^2*d*f^2*g^10*e^3 + 12*a^6* b^4*d*f*g^11*e^3 - 48*a^7*b^2*c*d*f*g^11*e^3 - 2*a^7*b^3*d*g^12*e^3 + 8*a^8 *b*c*d*g^12*e^3 + a^2*b^2*c^6*f^12*e^4 - 4*a^3*c^7*f^12*e^4 - 6*a^2*b^3*c^5 *f^11*g*e^4 + 24*a^3*b*c^6*f^11*g*e^4 + 15*a^2*b^4*c^4*f^10*g^2*e^4 - 54*a^ 3*b^2*c^5*f^10*g^2*e^4 - 24*a^4*c^6*f^10*g^2*e^4 - 20*a^2*b^5*c^3*f^9*g^3*e ^4 + 50*a^3*b^3*c^4*f^9*g^3*e^4 + 120*a^4*b*c^5*f^9*g^3*e^4 + 15*a^2*b^6*c^ 2*f^8*g^4*e^4 - 225*a^4*b^2*c^4*f^8*g^4*e^4 - 60*a^5*c^5*f^8*g^4*e^4 - 6*a^ 2*b^7*c*f^7*g^5*e^4 - 36*a^3*b^5*c^2*f^7*g^5*e^4 + 180*a^4*b^3*c^3*f^7*g^5* e^4 + 240*a^5*b*c^4*f^7*g^5*e^4 + a^2*b^8*f^6*g^6*e^4 + 26*a^3*b^6*c*f^6*g^ 6*e^4 - 30*a^4*b^4*c^2*f^6*g^6*e^4 - 340*a^5*b^2*c^3*f^6*g^6*e^4 - 80*a^6*c ^4*f^6*g^6*e^4 - 6*a^3*b^7*f^5*g^7*e^4 - 36*a^4*b^5*c*f^5*g^7*e^4 + 180*a^5 *b^3*c^2*f^5*g^7*e^4 + 240*a^6*b*c^3*f^5*g^7*e^4 + 15*a^4*b^6*f^4*g^8*e^4 - 225*a^6*b^2*c^2*f^4*g^8*e^4 - 60*a^7*c^3*f^4*g^8*e^4 - 20*a^5*b^5*f^3*g^9* e^4 + 50*a^6*b^3*c*f^3*g^9*e^4 + 120*a^7*b*c^2*f^3*g^9*e^4 + 15*a^6*b^4*f^2 *g^10*e^4 - 54*a^7*b^2*c*f^2*g^10*e^4 - 24*a^8*c^2*f^2*g^10*e^4 - 6*a^7*b^3 *f*g^11*e^4 + 24*a^8*b*c*f*g^11*e^4 + a^8*b^2*g^12*e^4 - 4*a^9*c*g^12*e^4) + (b*c^8*d^3*f^9 - 6*b^2*c^7*d^3*f^8*g + 6*a*c^8*d^3*f^8*g + 15*b^3*c^6*d^3 *f^7*g^2 - 24*a*b*c^7*d^3*f^7*g^2 - 20*b^4*c^5*d^3*f^6*g^3 + 34*a*b^2*c^6*d ^3*f^6*g^3 + 16*a^2*c^7*d^3*f^6*g^3 + 15*b^5*c^4*d^3*f^5*g^4 - 15*a*b^3*c^5 *d^3*f^5*g^4 - 54*a^2*b*c^6*d^3*f^5*g^4 - 6*b^6*c^3*d^3*f^4*g^5 - 9*a*b^4*c ^4*d^3*f^4*g^5 + 66*a^2*b^2*c^5*d^3*f^4*g^5 + 12*a^3*c^6*d^3*f^4*g^5 + b^7* c^2*d^3*f^3*g^6 + 11*a*b^5*c^3*d^3*f^3*g^6 - 31*a^2*b^3*c^4*d^3*f^3*g^6 - 3 2*a^3*b*c^5*d^3*f^3*g^6 - 3*a*b^6*c^2*d^3*f^2*g^7 + 30*a^3*b^2*c^4*d^3*f^2* g^7 + 3*a^2*b^5*c^2*d^3*f*g^8 - 9*a^3*b^3*c^3*d^3*f*g^8 - 3*a^4*b*c^4*d^3*f *g^8 - a^3*b^4*c^2*d^3*g^9 + 4*a^4*b^2*c^3*d^3*g^9 - 2*a^5*c^4*d^3*g^9 - 2* b^2*c^7*d^2*f^9*e + 2*a*c^8*d^2*f^9*e + 12*b^3*c^6*d^2*f^8*g*e - 21*a*b*c^7 *d^2*f^8*g*e - 30*b^4*c^5*d^2*f^7*g^2*e + 66*a*b^2*c^6*d^2*f^7*g^2*e + 40*b ^5*c^4*d^2*f^6*g^3*e - 89*a*b^3*c^5*d^2*f^6*g^3*e - 32*a^2*b*c^6*d^2*f^6*g^ 3*e - 30*b^6*c^3*d^2*f^5*g^4*e + 45*a*b^4*c^4*d^2*f^5*g^4*e + 114*a^2*b^2*c ^5*d^2*f^5*g^4*e - 12*a^3*c^6*d^2*f^5*g^4*e + 12*b^7*c^2*d^2*f^4*g^5*e + 12 *a*b^5*c^3*d^2*f^4*g^5*e - 147*a^2*b^3*c^4*d^2*f^4*g^5*e + 6*a^3*b*c^5*d^2* f^4*g^5*e - 2*b^8*c*d^2*f^3*g^6*e - 21*a*b^6*c^2*d^2*f^3*g^6*e + 74*a^2*b^4 *c^3*d^2*f^3*g^6*e + 46*a^3*b^2*c^4*d^2*f^3*g^6*e - 16*a^4*c^5*d^2*f^3*g^6* e + 6*a*b^7*c*d^2*f^2*g^7*e - 3*a^2*b^5*c^2*d^2*f^2*g^7*e - 63*a^3*b^3*c^3* d^2*f^2*g^7*e + 24*a^4*b*c^4*d^2*f^2*g^7*e - 6*a^2*b^6*c*d^2*f*g^8*e + 21*a ^3*b^4*c^2*d^2*f*g^8*e - 6*a^5*c^4*d^2*f*g^8*e + 2*a^3*b^5*c*d^2*g^9*e - 9* a^4*b^3*c^2*d^2*g^9*e + 7*a^5*b*c^3*d^2*g^9*e + b^3*c^6*d*f^9*e^2 - a*b*c^7 *d*f^9*e^2 - 6*b^4*c^5*d*f^8*g*e^2 + 9*a*b^2*c^6*d*f^8*g*e^2 + 6*a^2*c^7*d* f^8*g*e^2 + 15*b^5*c^4*d*f^7*g^2*e^2 - 27*a*b^3*c^5*d*f^7*g^2*e^2 - 24*a^2* b*c^6*d*f^7*g^2*e^2 - 20*b^6*c^3*d*f^6*g^3*e^2 + 35*a*b^4*c^4*d*f^6*g^3*e^2 + 50*a^2*b^2*c^5*d*f^6*g^3*e^2 + 16*a^3*c^6*d*f^6*g^3*e^2 + 15*b^7*c^2*d*f ^5*g^4*e^2 - 15*a*b^5*c^3*d*f^5*g^4*e^2 - 75*a^2*b^3*c^4*d*f^5*g^4*e^2 - 42 *a^3*b*c^5*d*f^5*g^4*e^2 - 6*b^8*c*d*f^4*g^5*e^2 - 9*a*b^6*c^2*d*f^4*g^5*e^ 2 + 72*a^2*b^4*c^3*d*f^4*g^5*e^2 + 48*a^3*b^2*c^4*d*f^4*g^5*e^2 + 12*a^4*c^ 5*d*f^4*g^5*e^2 + b^9*d*f^3*g^6*e^2 + 11*a*b^7*c*d*f^3*g^6*e^2 - 32*a^2*b^5 *c^2*d*f^3*g^6*e^2 - 45*a^3*b^3*c^3*d*f^3*g^6*e^2 - 16*a^4*b*c^4*d*f^3*g^6* e^2 - 3*a*b^8*d*f^2*g^7*e^2 + 33*a^3*b^4*c^2*d*f^2*g^7*e^2 + 6*a^4*b^2*c^3* d*f^2*g^7*e^2 + 3*a^2*b^7*d*f*g^8*e^2 - 9*a^3*b^5*c*d*f*g^8*e^2 - 6*a^4*b^3 *c^2*d*f*g^8*e^2 + 3*a^5*b*c^3*d*f*g^8*e^2 - a^3*b^6*d*g^9*e^2 + 4*a^4*b^4* c*d*g^9*e^2 - a^5*b^2*c^2*d*g^9*e^2 - 2*a^6*c^3*d*g^9*e^2 - a*b^2*c^6*f^9*e

^3 + 2*a^2*c^7*f^9*e^3 + 6*a*b^3*c^5*f^8*g*e^3 - 15*a^2*b*c^6*f^8*g*e^3 - 1
5*a*b^4*c^4*f^7*g^2*e^3 + 42*a^2*b^2*c^5*f^7*g^2*e^3 + 20*a*b^5*c^3*f^6*g^3
*e^3 - 55*a^2*b^3*c^4*f^6*g^3*e^3 - 16*a^3*b*c^5*f^6*g^3*e^3 - 15*a*b^6*c^2
*f^5*g^4*e^3 + 30*a^2*b^4*c^3*f^5*g^4*e^3 + 60*a^3*b^2*c^4*f^5*g^4*e^3 - 12
*a^4*c^5*f^5*g^4*e^3 + 6*a*b^7*c*f^4*g^5*e^3 + 3*a^2*b^5*c^2*f^4*g^5*e^3 -
81*a^3*b^3*c^3*f^4*g^5*e^3 + 18*a^4*b*c^4*f^4*g^5*e^3 - a*b^8*f^3*g^6*e^3 -
10*a^2*b^6*c*f^3*g^6*e^3 + 43*a^3*b^4*c^2*f^3*g^6*e^3 + 14*a^4*b^2*c^3*f^3
*g^6*e^3 - 16*a^5*c^4*f^3*g^6*e^3 + 3*a^2*b^7*f^2*g^7*e^3 - 3*a^3*b^5*c*f^2
*g^7*e^3 - 33*a^4*b^3*c^2*f^2*g^7*e^3 + 24*a^5*b*c^3*f^2*g^7*e^3 - 3*a^3*b^
6*f*g^8*e^3 + 12*a^4*b^4*c*f*g^8*e^3 - 3*a^5*b^2*c^2*f*g^8*e^3 - 6*a^6*c^3*
f*g^8*e^3 + a^4*b^5*g^9*e^3 - 5*a^5*b^3*c*g^9*e^3 + 5*a^6*b*c^2*g^9*e^3)/(b
^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 6*b^3*c^7*d^4*f^11*g + 24*a*b*c^8*d^4*
f^11*g + 15*b^4*c^6*d^4*f^10*g^2 - 54*a*b^2*c^7*d^4*f^10*g^2 - 24*a^2*c^8*d
^4*f^10*g^2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^3 + 120*a^2*b
*c^7*d^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^4*f^8*g^4 - 6
0*a^3*c^7*d^4*f^8*g^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d^4*f^7*g^5 +
180*a^2*b^3*c^5*d^4*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + b^8*c^2*d^4*f^6*g
^6 + 26*a*b^6*c^3*d^4*f^6*g^6 - 30*a^2*b^4*c^4*d^4*f^6*g^6 - 340*a^3*b^2*c^
5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g^7 - 36*a^2*b
^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c^5*d^4*f^5*g^
7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 - 60*a^5*c^5*d
^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f^3*g^9 + 120*
a^5*b*c^4*d^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b^2*c^3*d^4*f^
2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 6*a^5*b^3*c^2*d^4*f*g^11 + 24*a^6*b*c^3*
d^4*f*g^11 + a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*b^3*c^7*d^3*f^12
*e + 8*a*b*c^8*d^3*f^12*e + 12*b^4*c^6*d^3*f^11*g*e - 48*a*b^2*c^7*d^3*f^11
*g*e - 30*b^5*c^5*d^3*f^10*g^2*e + 108*a*b^3*c^6*d^3*f^10*g^2*e + 48*a^2*b*
c^7*d^3*f^10*g^2*e + 40*b^6*c^4*d^3*f^9*g^3*e - 100*a*b^4*c^5*d^3*f^9*g^3*e
- 240*a^2*b^2*c^6*d^3*f^9*g^3*e - 30*b^7*c^3*d^3*f^8*g^4*e + 450*a^2*b^3*c
^5*d^3*f^8*g^4*e + 120*a^3*b*c^6*d^3*f^8*g^4*e + 12*b^8*c^2*d^3*f^7*g^5*e +
72*a*b^6*c^3*d^3*f^7*g^5*e - 360*a^2*b^4*c^4*d^3*f^7*g^5*e - 480*a^3*b^2*c
^5*d^3*f^7*g^5*e - 2*b^9*c*d^3*f^6*g^6*e - 52*a*b^7*c^2*d^3*f^6*g^6*e + 60*
a^2*b^5*c^3*d^3*f^6*g^6*e + 680*a^3*b^3*c^4*d^3*f^6*g^6*e + 160*a^4*b*c^5*d
^3*f^6*g^6*e + 12*a*b^8*c*d^3*f^5*g^7*e + 72*a^2*b^6*c^2*d^3*f^5*g^7*e - 36
0*a^3*b^4*c^3*d^3*f^5*g^7*e - 480*a^4*b^2*c^4*d^3*f^5*g^7*e - 30*a^2*b^7*c*
d^3*f^4*g^8*e + 450*a^4*b^3*c^3*d^3*f^4*g^8*e + 120*a^5*b*c^4*d^3*f^4*g^8*e
+ 40*a^3*b^6*c*d^3*f^3*g^9*e - 100*a^4*b^4*c^2*d^3*f^3*g^9*e - 240*a^5*b^2
*c^3*d^3*f^3*g^9*e - 30*a^4*b^5*c*d^3*f^2*g^10*e + 108*a^5*b^3*c^2*d^3*f^2*
g^10*e + 48*a^6*b*c^3*d^3*f^2*g^10*e + 12*a^5*b^4*c*d^3*f*g^11*e - 48*a^6*b
^2*c^2*d^3*f*g^11*e - 2*a^6*b^3*c*d^3*g^12*e + 8*a^7*b*c^2*d^3*g^12*e + b^4
*c^6*d^2*f^12*e^2 - 2*a*b^2*c^7*d^2*f^12*e^2 - 8*a^2*c^8*d^2*f^12*e^2 - 6*b
^5*c^5*d^2*f^11*g*e^2 + 12*a*b^3*c^6*d^2*f^11*g*e^2 + 48*a^2*b*c^7*d^2*f^11
*g*e^2 + 15*b^6*c^4*d^2*f^10*g^2*e^2 - 24*a*b^4*c^5*d^2*f^10*g^2*e^2 - 132*
a^2*b^2*c^6*d^2*f^10*g^2*e^2 - 48*a^3*c^7*d^2*f^10*g^2*e^2 - 20*b^7*c^3*d^2
*f^9*g^3*e^2 + 10*a*b^5*c^4*d^2*f^9*g^3*e^2 + 220*a^2*b^3*c^5*d^2*f^9*g^3*e
^2 + 240*a^3*b*c^6*d^2*f^9*g^3*e^2 + 15*b^8*c^2*d^2*f^8*g^4*e^2 + 30*a*b^6*
c^3*d^2*f^8*g^4*e^2 - 225*a^2*b^4*c^4*d^2*f^8*g^4*e^2 - 510*a^3*b^2*c^5*d^2
*f^8*g^4*e^2 - 120*a^4*c^6*d^2*f^8*g^4*e^2 - 6*b^9*c*d^2*f^7*g^5*e^2 - 48*a
*b^7*c^2*d^2*f^7*g^5*e^2 + 108*a^2*b^5*c^3*d^2*f^7*g^5*e^2 + 600*a^3*b^3*c^
4*d^2*f^7*g^5*e^2 + 480*a^4*b*c^5*d^2*f^7*g^5*e^2 + b^10*d^2*f^6*g^6*e^2 +
28*a*b^8*c*d^2*f^6*g^6*e^2 + 22*a^2*b^6*c^2*d^2*f^6*g^6*e^2 - 400*a^3*b^4*c
^3*d^2*f^6*g^6*e^2 - 760*a^4*b^2*c^4*d^2*f^6*g^6*e^2 - 160*a^5*c^5*d^2*f^6*
g^6*e^2 - 6*a*b^9*d^2*f^5*g^7*e^2 - 48*a^2*b^7*c*d^2*f^5*g^7*e^2 + 108*a^3*
b^5*c^2*d^2*f^5*g^7*e^2 + 600*a^4*b^3*c^3*d^2*f^5*g^7*e^2 + 480*a^5*b*c^4*d
^2*f^5*g^7*e^2 + 15*a^2*b^8*d^2*f^4*g^8*e^2 + 30*a^3*b^6*c*d^2*f^4*g^8*e^2
- 225*a^4*b^4*c^2*d^2*f^4*g^8*e^2 - 510*a^5*b^2*c^3*d^2*f^4*g^8*e^2 - 120*a
^6*c^4*d^2*f^4*g^8*e^2 - 20*a^3*b^7*d^2*f^3*g^9*e^2 + 10*a^4*b^5*c*d^2*f^3*
g^9*e^2 + 220*a^5*b^3*c^2*d^2*f^3*g^9*e^2 + 240*a^6*b*c^3*d^2*f^3*g^9*e^2 +
15*a^4*b^6*d^2*f^2*g^10*e^2 - 24*a^5*b^4*c*d^2*f^2*g^10*e^2 - 132*a^6*b^2*

c^2*d^2*f^2*g^10*e^2 - 48*a^7*c^3*d^2*f^2*g^10*e^2 - 6*a^5*b^5*d^2*f*g^11*e^2 + 12*a^6*b^3*c*d^2*f*g^11*e^2 + 48*a^7*b*c^2*d^2*f*g^11*e^2 + a^6*b^4*d^2*g^12*e^2 - 2*a^7*b^2*c*d^2*g^12*e^2 - 8*a^8*c^2*d^2*g^12*e^2 - 2*a*b^3*c^6*d*f^12*e^3 + 8*a^2*b*c^7*d*f^12*e^3 + 12*a*b^4*c^5*d*f^11*g*e^3 - 48*a^2*b^2*c^6*d*f^11*g*e^3 - 30*a*b^5*c^4*d*f^10*g^2*e^3 + 108*a^2*b^3*c^5*d*f^10*g^2*e^3 + 48*a^3*b*c^6*d*f^10*g^2*e^3 + 40*a*b^6*c^3*d*f^9*g^3*e^3 - 100*a^2*b^4*c^4*d*f^9*g^3*e^3 - 240*a^3*b^2*c^5*d*f^9*g^3*e^3 - 30*a*b^7*c^2*d*f^8*g^4*e^3 + 450*a^3*b^3*c^4*d*f^8*g^4*e^3 + 120*a^4*b*c^5*d*f^8*g^4*e^3 + 12*a*b^8*c*d*f^7*g^5*e^3 + 72*a^2*b^6*c^2*d*f^7*g^5*e^3 - 360*a^3*b^4*c^3*d*f^7*g^5*e^3 - 480*a^4*b^2*c^4*d*f^7*g^5*e^3 - 2*a*b^9*d*f^6*g^6*e^3 - 52*a^2*b^7*c*d*f^6*g^6*e^3 + 60*a^3*b^5*c^2*d*f^6*g^6*e^3 + 680*a^4*b^3*c^3*d*f^6*g^6*e^3 + 160*a^5*b*c^4*d*f^6*g^6*e^3 + 12*a^2*b^8*d*f^5*g^7*e^3 + 72*a^3*b^6*c*d*f^5*g^7*e^3 - 360*a^4*b^4*c^2*d*f^5*g^7*e^3 - 480*a^5*b^2*c^3*d*f^5*g^7*e^3 - 30*a^3*b^7*d*f^4*g^8*e^3 + 450*a^5*b^3*c^2*d*f^4*g^8*e^3 + 120*a^6*b*c^3*d*f^4*g^8*e^3 + 40*a^4*b^6*d*f^3*g^9*e^3 - 100*a^5*b^4*c*d*f^3*g^9*e^3 - 240*a^6*b^2*c^2*d*f^3*g^9*e^3 - 30*a^5*b^5*d*f^2*g^10*e^3 + 108*a^6*b^3*c*d*f^2*g^10*e^3 + 48*a^7*b*c^2*d*f^2*g^10*e^3 + 12*a^6*b^4*d*f*g^11*e^3 - 48*a^7*b^2*c*d*f*g^11*e^3 - 2*a^7*b^3*d*g^12*e^3 + 8*a^8*b*c*d*g^12*e^3 + a^2*b^2*c^6*f^12*e^4 - 4*a^3*c^7*f^12*e^4 - 6*a^2*b^3*c^5*f^11*g*e^4 + 24*a^3*b*c^6*f^11*g*e^4 + 15*a^2*b^4*c^4*f^10*g^2*e^4 - 54*a^3*b^2*c^5*f^10*g^2*e^4 - 24*a^4*c^6*f^10*g^2*e^4 - 20*a^2*b^5*c^3*f^9*g^3*e^4 + 50*a^3*b^3*c^4*f^9*g^3*e^4 + 120*a^4*b*c^5*f^9*g^3*e^4 + 15*a^2*b^6*c^2*f^8*g^4*e^4 - 225*a^4*b^2*c^4*f^8*g^4*e^4 - 60*a^5*c^5*f^8*g^4*e^4 - 6*a^2*b^7*c*f^7*g^5*e^4 - 36*a^3*b^5*c^2*f^7*g^5*e^4 + 180*a^4*b^3*c^3*f^7*g^5*e^4 + 240*a^5*b*c^4*f^7*g^5*e^4 + a^2*b^8*f^6*g^6*e^4 + 26*a^3*b^6*c*f^6*g^6*e^4 - 30*a^4*b^4*c^2*f^6*g^6*e^4 - 340*a^5*b^2*c^3*f^6*g^6*e^4 - 80*a^6*c^4*f^6*g^6*e^4 - 6*a^3*b^7*f^5*g^7*e^4 - 36*a^4*b^5*c*f^5*g^7*e^4 + 180*a^5*b^3*c^2*f^5*g^7*e^4 + 240*a^6*b*c^3*f^5*g^7*e^4 + 15*a^4*b^6*f^4*g^8*e^4 - 225*a^6*b^2*c^2*f^4*g^8*e^4 - 60*a^7*c^3*f^4*g^8*e^4 - 20*a^5*b^5*f^3*g^9*e^4 + 50*a^6*b^3*c*f^3*g^9*e^4 + 120*a^7*b*c^2*f^3*g^9*e^4 + 15*a^6*b^4*f^2*g^10*e^4 - 54*a^7*b^2*c*f^2*g^10*e^4 - 24*a^8*c^2*f^2*g^10*e^4 - 6*a^7*b^3*f*g^11*e^4 + 24*a^8*b*c*f*g^11*e^4 + a^8*b^2*g^12*e^4 - 4*a^9*c*g^12*e^4))/sqrt(c*x^2 + b*x + a) + 1/4*(48*c^2*d^2*f^2*g^5 - 48*b*c*d^2*f*g^6 + 15*b^2*d^2*g^7 - 12*a*c*d^2*g^7 - 120*c^2*d*f^3*g^4*e + 132*b*c*d*f^2*g^5*e - 42*b^2*d*f*g^6*e + 12*a*b*d*g^7*e + 80*c^2*f^4*g^3*e^2 - 100*b*c*f^3*g^4*e^2 + 35*b^2*f^2*g^5*e^2 + 28*a*c*f^2*g^5*e^2 - 28*a*b*f*g^6*e^2 + 8*a^2*g^7*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^3*d^3*f^6*g^3 - 3*b*c^2*d^3*f^5*g^4 + 3*b^2*c*d^3*f^4*g^5 + 3*a*c^2*d^3*f^4*g^5 - b^3*d^3*f^3*g^6 - 6*a*b*c*d^3*f^3*g^6 + 3*a*b^2*d^3*f^2*g^7 + 3*a^2*c*d^3*f^2*g^7 - 3*a^2*b*d^3*f*g^8 + a^3*d^3*g^9 - 3*c^3*d^2*f^7*g^2*e + 9*b*c^2*d^2*f^6*g^3*e - 9*b^2*c*d^2*f^5*g^4*e - 9*a*c^2*d^2*f^5*g^4*e + 3*b^3*d^2*f^4*g^5*e + 18*a*b*c*d^2*f^4*g^5*e - 9*a*b^2*d^2*f^3*g^6*e - 9*a^2*c*d^2*f^3*g^6*e + 9*a^2*b*d^2*f^2*g^7*e - 3*a^3*d^2*f*g^8*e + 3*c^3*d*f^8*g*e^2 - 9*b*c^2*d*f^7*g^2*e^2 + 9*b^2*c*d*f^6*g^3*e^2 + 9*a*c^2*d*f^6*g^3*e^2 - 3*b^3*d*f^5*g^4*e^2 - 18*a*b*c*d*f^5*g^4*e^2 + 9*a*b^2*d*f^4*g^5*e^2 + 9*a^2*c*d*f^4*g^5*e^2 - 9*a^2*b*d*f^3*g^6*e^2 + 3*a^3*d*f^2*g^7*e^2 - c^3*f^9*e^3 + 3*b*c^2*f^8*g*e^3 - 3*b^2*c*f^7*g^2*e^3 - 3*a*c^2*f^7*g^2*e^3 + b^3*f^6*g^3*e^3 + 6*a*b*c*f^6*g^3*e^3 - 3*a*b^2*f^5*g^4*e^3 - 3*a^2*c*f^5*g^4*e^3 + 3*a^2*b*f^4*g^5*e^3 - a^3*f^3*g^6*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) - 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^5/((c*d^5*g^3 - 3*c*d^4*f*g^2*e - b*d^4*g^3*e + 3*c*d^3*f^2*g*e^2 + 3*b*d^3*f*g^2*e^2 + a*d^3*g^3*e^2 - c*d^2*f^3*e^3 - 3*b*d^2*f^2*g*e^3 - 3*a*d^2*f*g^2*e^3 + b*d*f^3*e^4 + 3*a*d*f^2*g*e^4 - a*f^3*e^5)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4*(24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^6 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^7 - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f^3*g^4*e + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f^2*g^5*e - 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^6*e - 4*(sqrt(c)*x - sqrt(c*x

^2 + b*x + a))^3*a*c*f*g^6*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*
g^7*e + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*f^3*g^4 - 48*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*f^2*g^5 + 13*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^6 - 28*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*a*c^(3/2)*d*f*g^6 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*
sqrt(c)*d*g^7 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f^4*g^3*e
+ 68*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f^3*g^4*e - 17*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f^2*g^5*e + 20*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^(3/2)*f^2*g^5*e - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*a*b*sqrt(c)*f*g^6*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*
sqrt(c)*g^7*e + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^4 - 44
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^5 - 88*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*c^2*d*f^2*g^5 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^3*d*f*g^6 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^6 - 9*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^2*c*d*g^7 - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f^4*g^3
e + 64(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f^3*g^4*e + 112*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3*g^4*e - 13*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*b^3*f^2*g^5*e - 104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*
g^5*e + 17*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^6*e + 28*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^6*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a^2*b*g^7*e + 14*b^2*c^(3/2)*d*f^3*g^4 - 7*b^3*sqrt(c)*d*f^2*g^5 - 44
*a*b*c^(3/2)*d*f^2*g^5 + 23*a*b^2*sqrt(c)*d*f*g^6 + 28*a^2*c^(3/2)*d*f*g^6
- 16*a^2*b*sqrt(c)*d*g^7 - 18*b^2*c^(3/2)*f^4*g^3*e + 11*b^3*sqrt(c)*f^3*g^
4*e + 56*a*b*c^(3/2)*f^3*g^4*e - 39*a*b^2*sqrt(c)*f^2*g^5*e - 36*a^2*c^(3/2
)*f^2*g^5*e + 36*a^2*b*sqrt(c)*f*g^6*e - 8*a^3*sqrt(c)*g^7*e)/((c^3*d^2*f^6
*g^2 - 3*b*c^2*d^2*f^5*g^3 + 3*b^2*c*d^2*f^4*g^4 + 3*a*c^2*d^2*f^4*g^4 - b^
3*d^2*f^3*g^5 - 6*a*b*c*d^2*f^3*g^5 + 3*a*b^2*d^2*f^2*g^6 + 3*a^2*c*d^2*f^2
*g^6 - 3*a^2*b*d^2*f*g^7 + a^3*d^2*g^8 - 2*c^3*d*f^7*g*e + 6*b*c^2*d*f^6*g^
2*e - 6*b^2*c*d*f^5*g^3*e - 6*a*c^2*d*f^5*g^3*e + 2*b^3*d*f^4*g^4*e + 12*a*
b*c*d*f^4*g^4*e - 6*a*b^2*d*f^3*g^5*e - 6*a^2*c*d*f^3*g^5*e + 6*a^2*b*d*f^2
*g^6*e - 2*a^3*d*f*g^7*e + c^3*f^8*e^2 - 3*b*c^2*f^7*g*e^2 + 3*b^2*c*f^6*g^
2*e^2 + 3*a*c^2*f^6*g^2*e^2 - b^3*f^5*g^3*e^2 - 6*a*b*c*f^5*g^3*e^2 + 3*a*b
^2*f^4*g^4*e^2 + 3*a^2*c*f^4*g^4*e^2 - 3*a^2*b*f^3*g^5*e^2 + a^3*f^2*g^6*e^
2)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*sqrt(c)*f + b*f - a*g)^2)

3.886 $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=1551

result too large to display

```
[Out] (-2*(64*b^4*e^4*g^4 + 4*b^2*c*e^3*g^3*(7*b*e*f - 66*b*d*g - 69*a*e*g) + c^4
*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*g^3 +
315*d^4*g^4) + 3*c^2*e^2*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) +
 3*b^2*(e^2*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*e*g*(6*a*e*g*(2*e^2*f^2 -
 33*d*e*f*g + 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))
*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3465*c^4*e*g^4) + (2*(d + e*x)^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])/(11*e) + (2*(48*b^3*e^3*g^3 + b*c*e^2*g^2
*(67*b*e*f - 198*b*d*g - 157*a*e*g) + c^3*(233*e^3*f^3 - 843*d*e^2*f^2*g +
1107*d^2*e*f*g^2 - 567*d^3*g^3) - c^2*e*g*(2*a*e*g*(74*e*f - 231*d*g) - 3*b
*(24*e^2*f^2 - 88*d*e*f*g + 99*d^2*g^2)))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*
x^2])/(3465*c^3*g^4) - (2*e*(8*b^2*e^2*g^2 + c*e*g*(19*b*e*f - 33*b*d*g - 1
8*a*e*g) + c^2*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt
[a + b*x + c*x^2])/(693*c^2*g^4) + (2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(f + g*
x)^(7/2)*Sqrt[a + b*x + c*x^2])/(99*c*g^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(12
8*b^5*e^3*g^5 - 8*b^3*c*e^2*g^4*(7*b*e*f + 66*b*d*g + 87*a*e*g) + 2*c^5*f^2
*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + b*c^2*e*g
^3*(771*a^2*e^2*g^2 + 6*a*b*e*g*(43*e*f + 396*d*g) - b^2*(37*e^2*f^2 - 264*
d*e*f*g - 792*d^2*g^2)) - c^4*g*(b*f*(56*e^3*f^3 - 264*d*e^2*f^2*g + 495*d^
2*e*f*g^2 - 462*d^3*g^3) - 18*a*g*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*
g^2 + 77*d^3*g^3)) - c^3*g^2*(6*a^2*e^2*g^2*(26*e*f + 231*d*g) - 9*a*b*e*g*
(15*e^2*f^2 - 110*d*e*f*g - 319*d^2*g^2) + b^2*(37*e^3*f^3 - 198*d*e^2*f^2*
g + 495*d^2*e*f*g^2 + 462*d^3*g^3)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(3465*c^5*g^5*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 -
b*f*g + a*g^2)*(64*b^4*e^3*g^4 + 4*b^2*c*e^2*g^3*(7*b*e*f - 66*b*d*g - 69*a
*e*g) - 2*c^4*f*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g
^3) + 3*c^2*e*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) + 3*b^2*(e^2
*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*g*(6*a*e*g*(2*e^2*f^2 - 33*d*e*f*g +
 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3465*c^5*g^5*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 8.09324, antiderivative size = 1551, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {918, 1653, 843, 718, 424, 419}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^4}{11e}+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(2f^2\left(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d^3g^3\right)c^5-g\left(bf\left(56e^3\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(64*b^4*e^4*g^4 + 4*b^2*c*e^3*g^3*(7*b*e*f - 66*b*d*g - 69*a*e*g) + c^4
*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*g^3 +
315*d^4*g^4) + 3*c^2*e^2*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) +
 3*b^2*(e^2*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*e*g*(6*a*e*g*(2*e^2*f^2 -
 33*d*e*f*g + 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))
*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3465*c^4*e*g^4) + (2*(d + e*x)^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])/(11*e) + (2*(48*b^3*e^3*g^3 + b*c*e^2*g^2
*(67*b*e*f - 198*b*d*g - 157*a*e*g) + c^3*(233*e^3*f^3 - 843*d*e^2*f^2*g +
1107*d^2*e*f*g^2 - 567*d^3*g^3) - c^2*e*g*(2*a*e*g*(74*e*f - 231*d*g) - 3*b
*(24*e^2*f^2 - 88*d*e*f*g + 99*d^2*g^2)))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*
x^2])/(3465*c^3*g^4) - (2*e*(8*b^2*e^2*g^2 + c*e*g*(19*b*e*f - 33*b*d*g - 1
8*a*e*g) + c^2*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt
[a + b*x + c*x^2])/(693*c^2*g^4) + (2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(f + g*
x)^(7/2)*Sqrt[a + b*x + c*x^2])/(99*c*g^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(12
8*b^5*e^3*g^5 - 8*b^3*c*e^2*g^4*(7*b*e*f + 66*b*d*g + 87*a*e*g) + 2*c^5*f^2
*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + b*c^2*e*g
^3*(771*a^2*e^2*g^2 + 6*a*b*e*g*(43*e*f + 396*d*g) - b^2*(37*e^2*f^2 - 264*
d*e*f*g - 792*d^2*g^2)) - c^4*g*(b*f*(56*e^3*f^3 - 264*d*e^2*f^2*g + 495*d^
2*e*f*g^2 - 462*d^3*g^3) - 18*a*g*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*
g^2 + 77*d^3*g^3)) - c^3*g^2*(6*a^2*e^2*g^2*(26*e*f + 231*d*g) - 9*a*b*e*g*
(15*e^2*f^2 - 110*d*e*f*g - 319*d^2*g^2) + b^2*(37*e^3*f^3 - 198*d*e^2*f^2*
g + 495*d^2*e*f*g^2 + 462*d^3*g^3)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(3465*c^5*g^5*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 -
b*f*g + a*g^2)*(64*b^4*e^3*g^4 + 4*b^2*c*e^2*g^3*(7*b*e*f - 66*b*d*g - 69*a
*e*g) - 2*c^4*f*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g
^3) + 3*c^2*e*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) + 3*b^2*(e^2
*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*g*(6*a*e*g*(2*e^2*f^2 - 33*d*e*f*g +
 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3465*c^5*g^5*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} - \frac{\int \frac{(d+ex)^3(bdf-3aef+adg+2(cdf-bef+bdg-aeg)x-(cef-3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{11e}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} + \frac{2e^2(cef-3cdg+beg)(f+gx)^{7/2}\sqrt{a+bx+}}{99cg^4}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} - \frac{2e\left(8b^2e^2g^2+ceg(19bef-33bdg-18aeg)+\right.}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} + \frac{2\left(48b^3e^3g^3+bce^2g^2(67bef-198bdg-157\right.}{}$$

$$= -\frac{2\left(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4\left(187e^4f^4-732de^3f^3g+10\right.\right.}{}$$

$$= -\frac{2\left(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4\left(187e^4f^4-732de^3f^3g+10\right.\right.}{}$$

$$= -\frac{2\left(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4\left(187e^4f^4-732de^3f^3g+10\right.\right.}{}$$

$$= -\frac{2\left(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4\left(187e^4f^4-732de^3f^3g+10\right.\right.}{}$$

Mathematica [C] time = 18.2638, size = 26600, normalized size = 17.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.599, size = 32647, normalized size = 21.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(ex + d)^3 \sqrt{gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\sqrt{cx^2 + bx + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x + a)*sq
rt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^3 \sqrt{f + gx}\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.887 $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=1015

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^3}{9e}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(f^2\left(8e^2f^2-24degf+21d^2g^2\right)c^4+g\left(3ag\left(3e^2f^2-16degf-21d^2g^2\right)\right.\right.}{}$$

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(b*e*f - 8*b*d*g - 9*a*e*g) + c^3*(19*e^3
*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3) - 3*c^2*e*g^2*(2*a*e*(
e*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/
(315*c^3*e*g^3) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*e)
 - (4*(3*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 9*b*d*g - 7*a*e*g) + c^2*(8*e^2*f^2
 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(315*c^
2*g^3) + (2*e*(c*e*f - 3*c*d*g + b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^
2])/(63*c*g^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^4*e^2*g^4 - 4*b^2*c*e*g^
3*(b*e*f + 6*b*d*g + 9*a*e*g) + c^4*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^
2) + 3*c^2*g^2*(7*a^2*e^2*g^2 + a*b*e*g*(5*e*f + 29*d*g) - b^2*(e^2*f^2 - 5
*d*e*f*g - 7*d^2*g^2)) + c^3*g*(3*a*g*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2)
 - b*f*(4*e^2*f^2 - 15*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^2*g^3 + 3*b*c*e*g^2*(b*e*f - 8*b*d*g - 9*a
*e*g) - 2*c^3*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*c^2*g^2*(2*a*e*(e
*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 3.77533, antiderivative size = 1015, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {918, 1653, 843, 718, 424, 419}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^3}{9e}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(f^2\left(8e^2f^2-24degf+21d^2g^2\right)c^4+g\left(3ag\left(3e^2f^2-16degf-21d^2g^2\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(b*e*f - 8*b*d*g - 9*a*e*g) + c^3*(19*e^3
*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3) - 3*c^2*e*g^2*(2*a*e*(
e*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/
(315*c^3*e*g^3) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*e)
 - (4*(3*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 9*b*d*g - 7*a*e*g) + c^2*(8*e^2*f^2
 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(315*c^
```

```
2*g^3) + (2*e*(c*e*f - 3*c*d*g + b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^
2])/(63*c*g^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^4*e^2*g^4 - 4*b^2*c*e*g^
3*(b*e*f + 6*b*d*g + 9*a*e*g) + c^4*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^
2) + 3*c^2*g^2*(7*a^2*e^2*g^2 + a*b*e*g*(5*e*f + 29*d*g) - b^2*(e^2*f^2 - 5
*d*e*f*g - 7*d^2*g^2)) + c^3*g*(3*a*g*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2)
 - b*f*(4*e^2*f^2 - 15*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^2*g^3 + 3*b*c*e*g^2*(b*e*f - 8*b*d*g - 9*a
*e*g) - 2*c^3*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*c^2*g^2*(2*a*e*(e
*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
```

```
b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} - \frac{\int \frac{(d+ex)^2\left(bdf-3aef+adg+2(cdf-bef+bdg-aeg)x-(cef-3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{9e}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} + \frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{a+bx+c}}{63cg^3}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9e} - \frac{4\left(3b^2e^2g^2+ceg(4bef-9bdg-7aeg)+c^2\right(}{}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3\left(19e^3f^3-57de^2f^2g+63d^2efg^2-\right.\right.}{315c^3e}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3\left(19e^3f^3-57de^2f^2g+63d^2efg^2-\right.\right.}{315c^3e}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3\left(19e^3f^3-57de^2f^2g+63d^2efg^2-\right.\right.}{315c^3e}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3\left(19e^3f^3-57de^2f^2g+63d^2efg^2-\right.\right.}{315c^3e}$$

Mathematica [C] time = 15.8663, size = 15781, normalized size = 15.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.393, size = 20224, normalized size = 19.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+bx+a}\sqrt{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2\,\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.888 $\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=652

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(cg(-10aeg-7bdg+bef)+4b^2eg^2-2c^2f(4ef-7\right.}{105c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*Sqrt[f + g*x]*(4*b^2*e*g^2 + c^2*f*(4*e*f - 7*d*g) - c*g*(2*b*e*f + 7*b
*d*g - 5*a*e*g) - 3*c*g*(c*e*f + 7*c*d*g - 4*b*e*g)*x)*Sqrt[a + b*x + c*x^2
])/(105*c^2*g^2) + (2*e*Sqrt[f + g*x]*(a + b*x + c*x^2)^(3/2))/(7*c) + (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*((c*e*f + 7*c*d*g - 4*b*e*g)*(8*c^2*f^2 - 2*b^2*g^2
- 3*c*g*(b*f - 2*a*g)) - 5*c*g*(2*c*f - b*g)*(7*c*d*f - e*(3*b*f + a*g)))*S
qrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^3*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2
*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(4*b^2*e*g^2 - 2*c^2*f*(
4*e*f - 7*d*g) + c*g*(b*e*f - 7*b*d*g - 10*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(1
05*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.11233, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {832, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(cg(-10aeg-7bdg+bef)+4b^2eg^2-2c^2f(4ef-7\right.}{105c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*b^2*e*g^2 + c^2*f*(4*e*f - 7*d*g) - c*g*(2*b*e*f + 7*b
*d*g - 5*a*e*g) - 3*c*g*(c*e*f + 7*c*d*g - 4*b*e*g)*x)*Sqrt[a + b*x + c*x^2
])/(105*c^2*g^2) + (2*e*Sqrt[f + g*x]*(a + b*x + c*x^2)^(3/2))/(7*c) + (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*((c*e*f + 7*c*d*g - 4*b*e*g)*(8*c^2*f^2 - 2*b^2*g^2
- 3*c*g*(b*f - 2*a*g)) - 5*c*g*(2*c*f - b*g)*(7*c*d*f - e*(3*b*f + a*g)))*S
qrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^3*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2
*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(4*b^2*e*g^2 - 2*c^2*f*(
4*e*f - 7*d*g) + c*g*(b*e*f - 7*b*d*g - 10*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(1
```

```
05*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2e\sqrt{f+gx}\left(a+bx+cx^2\right)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}(7cdf-3bef-aeg)+\frac{1}{2}(cef+7cdg-4beg)x\right)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,d}{7c}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\left(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7}{105c^2g^2}$$

Mathematica [C] time = 13.9134, size = 8432, normalized size = 12.93

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.371, size = 10711, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}(ex + d)\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)\sqrt{f + gx}\sqrt{a + bx + cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.889 $\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=513

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2g\sqrt{\cdots}}{2cf-g(\cdots}\right.}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c*g) + (2*(f + g
*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*g) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*
f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)])/(15*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*f - b*g)
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^2*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])
```

Rubi [A] time = 0.535524, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {734, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)\cdots}\right.}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (-2*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c*g) + (2*(f + g
*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*g) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*
f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)])/(15*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*f - b*g)
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^2*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])
```


Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{\int \frac{\sqrt{f+gx}(bf-2ag+(2cf-bg)x)}{\sqrt{a+bx+cx^2}}dx}{5g}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\int \frac{\frac{1}{2}(bcf^2+b^2fg-8a}{}}{}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} + \frac{((2cf-bg)(cf^2}{}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\,(\right.}{}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\,(c}{}$$

Mathematica [C] time = 10.8536, size = 697, normalized size = 1.36

$$\frac{i(f+gx)\sqrt{1-\frac{2(g(ag-bf)+cf^2)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}-bg+2cf\right)}}\sqrt{\frac{4(g(ag-bf)+cf^2)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}+bg-2cf\right)}+2}\left(\left(c\left(ag^2\left(3\sqrt{g^2(b^2-4ac)}+8cf\right)-cf^2\sqrt{g^2(b^2-4ac)}\right)-b^2g^2\left(\sqrt{g^2(b^2-4ac)}+2cf\right)+bcg\left(f\sqrt{\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((-4*g^2*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*(a + x*(b + c*x)))/Sqrt[f
+ g*x] + 2*c*g^2*Sqrt[f + g*x]*(a + x*(b + c*x))*(b*g + c*(f + 3*g*x)) + (I
*(f + g*x)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^
2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c
*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 -
 4*a*c)*g^2])*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*EllipticE[I*ArcSinh[(
Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*
g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (b^3*g^3 - b^2*g^2*(2*c*f + Sqrt[(b^2 - 4*
a*c)*g^2]) + b*c*g*(-4*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-(c*f^2*Sqrt
[(b^2 - 4*a*c)*g^2]) + a*g^2*(8*c*f + 3*Sqrt[(b^2 - 4*a*c)*g^2])))*Elliptic
F[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2
 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/Sqrt[(c*f^2 + g*(-(b*f) + a*g
))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/(15*c^2*g^3*Sqrt[a + x*(b + c
*x)])
```

Maple [B] time = 0.352, size = 4356, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/15*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2*(2*a*b*c*f*g^3-2*2^(1/2)*(-(g*x+
f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2
))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*
f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*c*f*g^3+8*2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^
(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*f*g^3+3*2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b
*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*c*f^2*g^2-12*2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^
(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*f*g^3+6*x^4*c^3*g^4+2*x*b*
c^2*f^2*g^2+2*a*c^2*f^2*g^2+2*x*b^2*c*f*g^3+10*x^2*b*c^2*f*g^3+2*x*a*b*c*g^
4+8*x*a*c^2*f*g^3+3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(
1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/
2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*
EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^3*
f*g^3-2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*
c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)
*b^2*f*g^3-8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g
*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(
b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Ellipti
cE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*f^2*g
^2+8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+
(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/
2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*f^2*g^2-8*2^(
1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+
b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*f^3*g-3*2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1
/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*
```

```
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*f^2*g^2+2^(1/2)*(-(g*x+f)*c/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*
f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*
(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*g^4-2*2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b
*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c^2*f^3*g+12*2^(1/2)*(-
(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1
/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*f^2*g^2+12*2^(1/2)*(-(g*x+f)
*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*c*g^4+4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*
c+b^2)^(1/2)))^(1/2))*a*b^2*g^4-4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)
))^(1/2))*b^3*f*g^3-12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
)^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^
(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a
^2*c*g^4-3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+
2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF
(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*g^4+6*x
^2*a*c^2*g^4+2*x^2*b^2*c*g^4+2*x^2*c^3*f^2*g^2+8*x^3*b*c^2*g^4+8*x^3*c^3*f*
g^3+4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*
c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^3*f^4)/(c*g*x^3+
b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)/g^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}\sqrt{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{f + gx}\sqrt{a + bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.890 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=764

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(eg(2aeg-3bdg+bef)+c\left(3d^2g^2-e^2f^2\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ce^3g\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(3*c*e^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sq
rt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*g*(b*e*f - 3*b*d*g +
 2*a*e*g) + c*(-(e^2*f^2) + 3*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*
Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^3*g*Sqrt
[f + g*x]*Sqrt[a + x*(b + c*x)]) - (Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[2*
c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g))
, ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c
])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f
)/g)])/(Sqrt[c]*e^3*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 4.09875, antiderivative size = 969, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {918, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(3*c*e^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sq
rt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*(c*e*f - 3*c*d*g + b*
e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
- (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(3*c*d*(e*f - d*g) - e*(2*b*e*f - 3*b*d*g +
```

```
2*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2
]) - (Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f -
b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt
[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b
*x + c*x^2])
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} - \frac{\int \frac{bdf-3aef+adg+2(cdf-bef+bdg-aeg)x-(cef-3cdg+beg)x^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} - \frac{\int \left(\frac{3cd(ef-dg)-e(2bef-3bdg+2aeg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{(cef-3cdg+beg)x}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{3\left(cd^2-bde+ae\right.}{e^2(d+ex)\sqrt{f+gx}}\right.}{3e}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\left(\left(cd^2-bde+ae^2\right)(ef-dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^3} + \frac{(ce}{}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{(cef-3cdg+beg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3e^2g} - \frac{(f(cef-3cdg+beg))\int}{3e^2g}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} - \frac{2\sqrt{2}\sqrt{b^2-4ac}(3cd(ef-dg)-e(2bef-3bdg+2aeg))\sqrt{\frac{}{2cf-(}}}{3ce^3\sqrt{f+}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+b}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+b}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+b}}$$

Mathematica [C] time = 15.368, size = 35245, normalized size = 46.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.398, size = 6812, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}\sqrt{a + bx + cx^2}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.891 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=743

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(2beg-c(3dg+ef))\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2g\sqrt{b^2-4ac}}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}\right)}{ce^3\sqrt{f+gx}\sqrt{a+x(b+cx)}}+\cdots$$

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x))) + (3*Sqrt[b^2 - 4*a*
c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^2*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2])
+ (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b*e*g - c*(e*f + 3*d*g))*Sqrt[(c*(f + g*x))
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*
a*c)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*
c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]
)/(c*e^3*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)]) + (Sqrt[2*c*f - (b - Sqrt[b^2
 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 -
(2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x
))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b
^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/S
qrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g
)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e^3*(e*f - d*g)*Sq
rt[a + b*x + c*x^2])
```

Rubi [A] time = 3.31641, antiderivative size = 934, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {916, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$\frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{3\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\cdots}{e^2\sqrt{f\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x))) + (3*Sqrt[b^2 - 4*a*
c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^2*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2])
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*e*f - 3*c*d*g + 2*b
*e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
```

```
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticP
i[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2
]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqr
t[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*
Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\int \frac{bf+ag+2(cf+bg)x+3cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\int \left(\frac{2cef-3cdg+2beg}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{3cgx}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)+e(bef-}{e^2(d+ex)\sqrt{f+gx}\sqrt{a}}\right.}{2e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{(3cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^2} + \frac{(2cef-3cdg+2beg)\int \frac{}{\sqrt{f+g}}}{2e^3}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{(3c)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(3cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{\left((cd(2\right.}{}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}(2cef-3cdg+2beg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c}{}}}{ce^3\sqrt{f+gx}\sqrt{a}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 13.9878, size = 16573, normalized size = 22.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.402, size = 16696, normalized size = 22.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}\sqrt{a + bx + cx^2}}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.892 $\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3} dx$

Optimal. Leaf size=1034

$$-\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)(cd(2ef-3dg)-e(bef-2bdg+aeg))}{4\sqrt{2}e^2\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*e*(d + e*x)^2) + ((c*d*(2*e*f - 3
*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(
4*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)) - (Sqrt[b^2 - 4*a*c]*(c*
d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(e*f -
d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x +
 c*x^2]) - (Sqrt[b^2 - 4*a*c]*(-(c*d*(2*e*f + 3*d*g)) + e*(b*e*f + 4*b*d*g
- 5*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c
*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f
 + (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^3*(c*d^2 + e*(-(b*d) + a*e))*S
qrt[f + g*x]*Sqrt[a + x*(b + c*x)]) + (Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a*c]
*g]*(b^2*e^4*f^2 + a^2*e^4*g^2 + c^2*d^3*g*(4*e*f - 3*d*g) - 2*a*c*e^2*(2*e
^2*f^2 - 6*d*e*f*g + 3*d^2*g^2) - 2*b*e*g*(a*e^3*f + c*d^2*(3*e*f - 2*d*g))
)*Sqrt[(g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c
])*g)]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4
*a*c])*g)]*EllipticPi[(2*c*e*f - b*e*g + Sqrt[b^2 - 4*a*c]*e*g)/(2*c*e*f -
2*c*d*g), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)])/(4*Sqrt[2]*Sqrt[c]*e^3*(c*d^2 + e*(-(b*d) + a*e))*(e*f - d*
g)^2*Sqrt[a + x*(b + c*x)])
```

Rubi [A] time = 8.09083, antiderivative size = 1705, normalized size of antiderivative = 1.65, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {916, 6742, 718, 419, 939, 843, 424, 934, 169, 538, 537}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*e*(d + e*x)^2) + ((c*d*(2*e*f - 3
*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(
4*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)) - (Sqrt[b^2 - 4*a*c]*(c*
d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(e*f -
d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x +
 c*x^2]) + (3*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
```

```
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^3*Sqrt[f
 + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*f*(c*d*(2*e*f - 3*d*g)
- e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b
^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c
*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)])/(2*Sqrt[2]*e^3*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e
*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a
*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Elli
pticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(S
qrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b
- Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqr
t[2]*Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))^2*S
qrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g
+ Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f +
 g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (
2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e^3*(c*d
^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

```
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{\int \frac{bf+ag+2(cf+bg)x+3cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{\int \left(\frac{3cg}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef-3dg)+e(bef-2bdg+aeg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{2(c}{e^2(d+ex)}\right.}{4e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(3cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e^3} + \frac{(cef-3cdg+beg)\int \frac{}{(d+ex)\sqrt{f}}}{2e^3}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\sqrt{a+bx+}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}$$

Mathematica [C] time = 16.874, size = 33765, normalized size = 32.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.503, size = 55360, normalized size = 53.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.893 $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1098

result too large to display

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) - c^3*(152
*e^3*f^3 - 408*d*e^2*f^2*g + 336*d^2*e*f*g^2 - 70*d^3*g^3) - 3*c^2*e*g*(6*a
*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2])/(315*c^3*g^4) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2])/(9*g) - (2*e*(6*b^2*e^2*g^2 + c*e*g*(17*b*e*f - 27*b*d*
g - 14*a*e*g) - 2*c^2*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3
/2)*Sqrt[a + b*x + c*x^2])/(315*c^2*g^4) - (2*e^2*(8*c*e*f - 6*c*d*g - b*e*
g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(63*c*g^4) - (Sqrt[2]*Sqrt[b^2 -
4*a*c]*(16*b^4*e^3*g^4 + 8*b^2*c*e^2*g^3*(2*b*e*f - 9*b*d*g - 9*a*e*g) - 2*
c^4*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) + 3*c^
2*e*g^2*(14*a^2*e^2*g^2 - a*b*e*g*(19*e*f - 87*d*g) + b^2*(7*e^2*f^2 - 27*d
*e*f*g + 42*d^2*g^2)) - c^3*g*(6*a*e*g*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^
2) - b*(40*e^3*f^3 - 144*d*e^2*f^2*g + 189*d^2*e*f*g^2 - 105*d^3*g^3)))*Sqr
t[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sqrt[(c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*S
qrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^3*g^3 + 3*b*c*e^2
*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) + 2*c^3*(64*e^3*f^3 - 216*d*e^2*f^2*g +
 252*d^2*e*f*g^2 - 105*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8
*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2
*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sq
rt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 5.76396, antiderivative size = 1098, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {920, 1653, 843, 718, 424, 419}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^3}{9g} - \frac{\sqrt{2}\sqrt{b^2-4ac}\left(-2f\left(64e^3f^3-216de^2gf^2+252d^2eg^2f-105d^3g^3\right)c^4-g\left(6aeg\left(10e^2f\right.\right.\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) - c^3*(152
*e^3*f^3 - 408*d*e^2*f^2*g + 336*d^2*e*f*g^2 - 70*d^3*g^3) - 3*c^2*e*g*(6*a
*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2])/(315*c^3*g^4) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2])/(9*g) - (2*e*(6*b^2*e^2*g^2 + c*e*g*(17*b*e*f - 27*b*d*
g - 14*a*e*g) - 2*c^2*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3
/2)*Sqrt[a + b*x + c*x^2])/(315*c^2*g^4) - (2*e^2*(8*c*e*f - 6*c*d*g - b*e*
g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(63*c*g^4) - (Sqrt[2]*Sqrt[b^2 -
```

```
4*a*c]*(16*b^4*e^3*g^4 + 8*b^2*c*e^2*g^3*(2*b*e*f - 9*b*d*g - 9*a*e*g) - 2*
c^4*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) + 3*c^
2*e*g^2*(14*a^2*e^2*g^2 - a*b*e*g*(19*e*f - 87*d*g) + b^2*(7*e^2*f^2 - 27*d
*e*f*g + 42*d^2*g^2)) - c^3*g*(6*a*e*g*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^
2) - b*(40*e^3*f^3 - 144*d*e^2*f^2*g + 189*d^2*e*f*g^2 - 105*d^3*g^3)))*Sqr
t[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sqrt[(c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*S
qrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^3*g^3 + 3*b*c*e^2
*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) + 2*c^3*(64*e^3*f^3 - 216*d*e^2*f^2*g +
 252*d^2*e*f*g^2 - 105*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8
*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2
*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sq
rt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(2*(d + e*x)^m*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2])/(g*(2*m + 3)), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)
^(m - 1)*Simp[b*d*f + 2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e
*f - d*g)*(2*m + 1))*x - (b*e*g + 2*c*(d*g*m - e*f*(m + 1)))*x^2, x])/(Sqrt
[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
```

```
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} - \frac{\int \frac{(d+ex)^2\left(bdf+6aef-8adg+(2cdf+7bef-7bdg-2aeg)x+(8cef-6cdg \cdots\right.}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{9g} \cdots$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} - \frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}{63cg^4} - \cdots$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{9g} - \frac{2e\left(6b^2e^2g^2+ceg(17bef-27bdg-14aeg)-2c^2\left(64 \cdots\right.\right.}{315 \cdots}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336d^2efg^2-7 \cdots\right.\right.}{315 \cdots}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336d^2efg^2-7 \cdots\right.\right.}{315 \cdots}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336d^2efg^2-7 \cdots\right.\right.}{315 \cdots}$$

$$= \frac{2\left(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3\left(152e^3f^3-408de^2f^2g+336d^2efg^2-7 \cdots\right.\right.}{315 \cdots}$$

Mathematica [C] time = 16.3749, size = 17771, normalized size = 16.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.395, size = 22215, normalized size = 20.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^2+bx+a}}{\sqrt{gx+f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x + a)/sq
rt(g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.894 $\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=755

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2+c^2\left(35d^2g^2\right.\right.}{105c^3g^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (-4*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) - c^2*(21*e^2*f^2
- 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c^2*g
^3) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*g) - (2*e*(6*c
*e*f - 4*c*d*g - b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c*g^3) +
 (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^3*e^2*g^3 + b*c*e*g^2*(9*b*e*f - 28*b*d*g
- 29*a*e*g) - 2*c^3*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2) - c^2*g*(2*a*e
*g*(13*e*f - 42*d*g) - b*(16*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)))*Sqrt[f +
g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^4*Sqrt[(c*(f + g*x
))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f
- 7*b*d*g - 5*a*e*g) + c^2*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g)])/(105*c^3*g^4*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.93068, antiderivative size = 755, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {920, 1653, 843, 718, 424, 419}

$$-\frac{4\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-5aeg-7bdg+4bef)+2b^2e^2g^2+c^2\left(-\left(10d^2g^2-34defg+21e^2f^2\right)\right)\right)}{105c^2g^3}+\frac{4\sqrt{2}\sqrt{b^2-4\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```


```
[Out] (-4*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) - c^2*(21*e^2*f^2
- 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c^2*g
^3) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*g) - (2*e*(6*c
*e*f - 4*c*d*g - b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c*g^3) +
 (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^3*e^2*g^3 + b*c*e*g^2*(9*b*e*f - 28*b*d*g
- 29*a*e*g) - 2*c^3*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2) - c^2*g*(2*a*e
*g*(13*e*f - 42*d*g) - b*(16*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)))*Sqrt[f +
g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^4*Sqrt[(c*(f + g*x
))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]
```

```
*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f
- 7*b*d*g - 5*a*e*g) + c^2*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g)])/(105*c^3*g^4*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(2*(d + e*x)^m*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2])/(g*(2*m + 3)), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)
^(m - 1)*Simp[b*d*f + 2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e
*f - d*g)*(2*m + 1))*x - (b*e*g + 2*c*(d*g*m - e*f*(m + 1)))*x^2, x])/(Sqrt
[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
```

```
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g} - \frac{\int \frac{(d+ex)\left(bdf+4aef-6adg+(2cdf+5bef-5bdg-2aeg)x+(6cef-4\right.}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{7g}$$

$$= \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g} - \frac{2e(6cef-4cdg-beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35cg^3}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{}}{105c^2g^3}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{}}{105c^2g^3}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{}}{105c^2g^3}$$

$$= -\frac{4\left(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2\left(21e^2f^2-34defg+10d^2g^2\right)\right)\sqrt{f+gx}\sqrt{}}{105c^2g^3}$$

Mathematica [C] time = 14.3634, size = 10030, normalized size = 13.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.426, size = 12923, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+bx+a}}{\sqrt{gx+f}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.895 $\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=519

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(beg-10cdg+8cef)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2\ldots}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*Sqrt[f + g*x]*(4*c*e*f - 5*c*d*g - b*e*g - 3*c*e*g*x)*Sqrt[a + b*x + c*
x^2])/(15*c*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b^2*e*g^2 - 2*c^2*f*(4*e*f
 - 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*c*e*f - 10*c*d*g + b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(
15*c^2*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.535008, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {814, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(beg-10cdg+8cef)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{\ldots}{2c\ldots}\right.}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```


```
[Out] (-2*Sqrt[f + g*x]*(4*c*e*f - 5*c*d*g - b*e*g - 3*c*e*g*x)*Sqrt[a + b*x + c*
x^2])/(15*c*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b^2*e*g^2 - 2*c^2*f*(4*e*f
 - 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*c*e*f - 10*c*d*g + b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(
15*c^2*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```


Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{2\int \frac{\frac{1}{2}\left(5cdg(bf-2ag)-bef(4cf-bg)+2\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\left((8cef-10cdg+beg)\left(cf^2-\cdots\right.\right.}{1\cdots}$$

$$= -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(2b^2eg^2-2c^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{f+gx}(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\sqrt{2}\sqrt{b^2-4ac}\left(2b^2eg^2-2c^2f\cdots\right.}{\cdots}$$

<u>Mathematica</u> [C] time = 10.6082, size = 911, normalized size = 1.76

$$\frac{2\sqrt{a+x(b+cx)}\left(\left(2f(4ef-5dg)c^2+g(-3bef+5bdg+6aeg)c-2b^2eg^2\right)\left(c\left(\frac{f}{f+gx}-1\right)^2+\frac{g\left(-\frac{fb}{f+gx}+b+\frac{ag}{f+gx}\right)}{f+gx}\right)+\frac{i\sqrt{1-\frac{\cdots}{(2cf\cdots}}\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] ((2*(-4*c*e*f + 5*c*d*g + b*e*g))/(15*c*g^2) + (2*e*x)/(5*g))*Sqrt[f + g*x]
*Sqrt[a + x*(b + c*x)] + (2*(f + g*x)^(3/2)*Sqrt[a + x*(b + c*x)]*((-2*b^2*
e*g^2 + 2*c^2*f*(4*e*f - 5*d*g) + c*g*(-3*b*e*f + 5*b*d*g + 6*a*e*g))*(c*(-
1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x))
 + ((I/2)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2
 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*
f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 -
4*a*c)*g^2])*(2*b^2*e*g^2 + 2*c^2*f*(-4*e*f + 5*d*g) + c*g*(3*b*e*f - 5*b*d
*g - 6*a*e*g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-
2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g +
Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (2*b^3
*e*g^3 - b^2*g^2*(-(c*e*f) + 5*c*d*g + 2*e*Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*g
*(-8*a*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(-3*e*f + 5*d*g)) + 2*c*(c*f*Sqrt[(b
^2 - 4*a*c)*g^2]*(4*e*f - 5*d*g) + a*g^2*(-2*c*e*f + 10*c*d*g + 3*e*Sqrt[(b
^2 - 4*a*c)*g^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^
2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b
*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(
Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*
```

```
g^2]))]*Sqrt[f + g*x])))/(15*c^2*g^4*Sqrt[a + b*x + c*x^2]*Sqrt[((f + g*x)^2
*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f +
 g*x)))/g^2])
```

Maple [B] time = 0.37, size = 6207, normalized size = 12.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}(ex + d)}{\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(ex + d)}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)\sqrt{a + bx + cx^2}}{\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

<u>**Giac**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")

[Out] Timed out
```

3.896 $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=444

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2g\sqrt{b^2-4ac}}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\right)}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\ldots$$

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*g) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*g
^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c
*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*c*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.321993, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {734, 843, 718, 424, 419}

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{b^2}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*g) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*g
^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c
*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*c*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
```

```
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{\int \frac{bf-2ag+(2cf-bg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3g}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{(2cf-bg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3g^2} + \frac{\left(2\left(cf^2-bfg+ag^2\right)\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+c}}}{3g^2}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2}{2cf-}}}{\sqrt{}}\right.}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 8.70738, size = 936, normalized size = 2.11

$$\sqrt{f+gx}\left(4(a+x(b+cx))g^2+\frac{(f+gx)\left(\frac{4(bg-2cf)\sqrt{\frac{cf^2+g(ag-bf)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))g^2}{(f+gx)^2}+\frac{i\sqrt{2}(2cf-bg)\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\sqrt{\frac{-2ag^2+2cfxg+\sqrt{(b^2-4ac)g^2}xg+b(f-\cdots}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*(4*g^2*(a + x*(b + c*x)) + ((f + g*x)*((4*g^2*(-2*c*f + b*g)
*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*(2*c*f - b*g)*(2*c*f - b*g + Sq
rt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g
*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2
 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c
*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqr
t[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
- b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]],
 -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*(b^2*g^2 - 4*a*c*g^2 + 2*c*f*Sqrt[(b
^2 - 4*a*c)*g^2] - b*g*Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^
2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/
((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt
[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g
*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c
)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f
 - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(c*Sqrt[(c*f^2 + g*(-(
b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])))/(6*g^3*Sqrt[a + x
*(b + c*x)])
```

Maple [B] time = 0.347, size = 1854, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
```

```
[Out] -2/3*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(1/2)/c*(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
```

```
*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*g^3-2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(
-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*f*g^2+2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*
c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*f^2*g+2^(1/2)*(-(g*x+f)*c/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b
*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4
*a*c+b^2)^(1/2)))^(1/2))*a*b*g^3-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
)))^(1/2))*a*c*f*g^2-2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2
*f*g^2+3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2
^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*f^2*g-2*2^(
1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+
b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*f^3-x^3*c^2*g^3-x^2*b*c
*g^3-x^2*c^2*f*g^2-x*a*c*g^3-x*b*c*f*g^2-a*c*f*g^2)/(c*g*x^3+b*g*x^2+c*f*x^
2+a*g*x+b*f*x+a*f)/g^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.897 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=700

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2g\sqrt{b^2-4ac}}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\right.}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(e*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
+ b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + c*d*g - b*e*g)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2
*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2
 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/
(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^2*(e*f - d*g)*Sqrt[a + b*x
 + c*x^2])
```

Rubi [A] time = 1.95284, antiderivative size = 700, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {922, 934, 169, 538, 537, 843, 718, 424, 419}

$$-\frac{\sqrt{2}\left(ae^2-bde+cd^2\right)\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right)}{2c(ef-dg)};\sin\right.}{\sqrt{c}e^2\sqrt{a+bx+cx^2}(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(e*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
+ b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + c*d*g - b*e*g)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2
*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2
```

```
 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/
(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^2*(e*f - d*g)*Sqrt[a + b*x
 + c*x^2])
```

Rule 922

```
Int[Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.
) + (g_.)*(x_)]), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[1/((d
+ e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d
- b*e - c*e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d
^2 - b*d*e + a*e^2, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
```

```
)/(b^2 - 4*a*c))]])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = -\frac{\int \frac{cd-be-cex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2}$$

$$= \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{eg} - \frac{(cef+cdg-beg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2 g} + \frac{\left(\left(cd^2-bde+ae^2\right)\sqrt{b-\sqrt{b^2-\right.\right.}{}$$

$$= -\frac{\left(2\left(cd^2-bde+ae^2\right)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\mathrm{Subst}\left(\int \frac{}{\left(ef-dg-ex^2\right)\sqrt{b-}}\right.}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2}}{}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2}}{}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2}}{}$$

Mathematica [C] time = 13.9868, size = 16471, normalized size = 23.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.4, size = 3126, normalized size = 4.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] (2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c
^2*e^2*f^3+EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^
(1/2))*(-4*a*c+b^2)^(1/2)*c*d^2*g^3-2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e^2*f^2*g-EllipticF(2^(1/2)*(-(g*x+f)
*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e*g^3+EllipticF(2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*e^2*f*g^2+EllipticF
(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^2*g^3-2
*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2
*d^2*f*g^2-EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)
^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/
2)*a*e^2*g^3-EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(
1/2)*c*d^2*g^3-EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e^2*g^3+
EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e*g^3-EllipticPi(2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)
^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b
*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^2*g^3+2*EllipticPi(2^(1/2)*(-(g*x+f)
*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2)))^(1/2))*c^2*d^2*f*g^2+2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*e*f*g^2-2*EllipticE(2^(1/2)*(-(g*x+f)
*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d*e*f^2*g-2*EllipticE(2^(1/
2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1
```

```
/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e*g^3+2*Ellip
ticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^2*f
*g^2-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),
(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))
*(-4*a*c+b^2)^(1/2)*b*d*e*g^3+EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)
^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*e^2*f*g^2-EllipticF(2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e
^2*f^2*g-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1
/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2))*b*c*e^2*f^2*g+2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2
)^(1/2)))^(1/2))*c^2*d*e*f^2*g+2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g
*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e*g^3-2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2
*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^2*f*g^2+EllipticPi(2^(1/2)*(-(
g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e*g^3+2*EllipticPi(2^(1/2)
*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^2*f*g^2-2*EllipticPi(2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2))*b*c*d*e*f*g^2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f))^(1/2)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g^2/e^2/(d*g-e*f)/(c*g*x^
3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.898 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=736

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2g\sqrt{b^2-4ac}}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}\right)}{e^2\sqrt{f+gx}\sqrt{a+x(b+cx)}}-\frac{\sqrt{2cf-g\left(b-\sqrt{b^2-\right.}}}{}$$

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x))) + (Sqrt[b^
2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*
e*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*Elli
pticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*Sqr
t[f + g*x]*Sqrt[a + x*(b + c*x)]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]*(e^2*(b*f - a*g) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (
b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f
- d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2
- 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] -
 (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 3.23004, antiderivative size = 957, normalized size of antiderivative = 1.3, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {924, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}+\frac{\sqrt{2}\sqrt{b^2-4ac}(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-\right.}}}}{e^2(e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x))) + (Sqrt[b^
2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*
e*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sqrt[(c*(f + g*x))/(2*c*f
- (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(
e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(2*e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b
```

```
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g
) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Ell
ipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(
Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b
 - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sq
rt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 924

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)), x] - Dist[1/(2*(m + 1)*(e*f -
 d*g)), Int[((d + e*x)^(m + 1)*Simp[b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m +
 2))*x + c*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \frac{bf-ag+2cfx+cgx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \left(\frac{c(2ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{cgx}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{e^2(bf-ag)-cd(2ef-dg)}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\right) dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e(ef-dg)} + \frac{(c(2ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e^2(ef-dg)} + \cdots$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2e(ef-dg)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e(ef-dg)} + \frac{\left(\left(bf-ag-\frac{cd(2ef-\cdots}{e^2}\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\cdots\right.\right.}{e^2(ef-dg)\sqrt{f+gx}\sqrt{a+bx+cx\cdots}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\cdots}{2cf-(\cdots}\right.}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\cdots}{2cf-(\cdots}\right.}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\cdots}{2cf-(\cdots}\right.}{\sqrt{2}e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 13.476, size = 6911, normalized size = 9.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.423, size = 13874, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^2 \sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.899 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1049

$$-\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)(cd(2ef+dg)-e(bef+2bdg-3aeg))}{4\sqrt{2}e\left(cd^2-bed+ae^2\right)(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((c*d*
(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x +
 c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)) - (Sqrt[b^2 -
4*a*c]*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sq
rt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)
*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt
[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*(e^2*(b*f - a*g) + c*d*(-2*e*f + d*
g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b +
Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^2 + e*(-(b*d) + a*e))*(e*f - d*
g)*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)]) - (Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*
a*c]*g]*(3*a^2*e^4*g^2 + c^2*d^3*g*(4*e*f - d*g) + b^2*e^3*f*(-(e*f) + 4*d*
g) + 2*a*c*e^2*(2*e^2*f^2 - 2*d*e*f*g + 3*d^2*g^2) - 2*b*e^2*g*(3*c*d^2*f +
 a*e*(e*f + 2*d*g)))*Sqrt[(g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f
+ (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(2*c*e*f - b*e*g + Sqrt[b^2 - 4*a*
c]*e*g)/(2*c*e*f - 2*c*d*g), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*
c*f - b*g + Sqrt[b^2 - 4*a*c]*g]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*
c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*Sqrt[c]*e^2*(c*d^2 + e*(-(b*d
) + a*e))*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)])
```

Rubi [A] time = 8.18444, antiderivative size = 1747, normalized size of antiderivative = 1.67, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {924, 6742, 718, 419, 939, 843, 424, 934, 169, 538, 537}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((c*d*
(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x +
 c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)) - (Sqrt[b^2 -
4*a*c]*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sq
rt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)
*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt
[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
```

```
cF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^
2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*f*(
c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2
*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2]) - (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g
 - 3*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*
c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(
e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt
[b^2 - 4*a*c])*g]*(c*e*f + c*d*g - b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
 (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*
f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^
2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) +
 (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f + d*g) - e*(b*e*f + 2
*b*d*g - 3*a*e*g))*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt
[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + S
qrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*
x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c
*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e^2*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2])
```

Rule 924

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)), x] - Dist[1/(2*(m + 1)*(e*f -
 d*g)), Int[((d + e*x)^(m + 1)*Simp[b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m +
 2))*x + c*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \frac{bf-3ag+2(cf-bg)x-cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \left(-\frac{cg}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(2ef+dg)+e(bef+2bdg-3aeg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{2(cef+c\cdots}{e^2(d+ex)\sqrt{f+g\cdots}}\right.}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e^2(ef-dg)} + \frac{(cef+cdg-beg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+b\cdots}}}{2e^2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \cdots$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \cdots$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \cdots$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \cdots$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \cdots$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \cdots$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \cdots$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)} - \cdots$$

Mathematica [C] time = 16.9141, size = 36617, normalized size = 34.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.538, size = 57841, normalized size = 55.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^3 \sqrt{gx + f}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^3 \sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.900 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=774

$$-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2+c^2\left(105\right.\right.}{105c^4g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*e*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) - c^2*(7*e^2*
f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c
^3*g^2) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*c) + (2*
e^2*(c*e*f + 11*c*d*g - 6*b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35
*c^2*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b
*e*f + 21*b*d*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f
*g^2 + 105*d^3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) - b*(9*e^2*f^2 - 63
*d*e*f*g - 210*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(105*c^4*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) + c^2*(8*e^2*
f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^4*g^3*Sqrt[f
+ g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 2.10553, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {941, 1653, 843, 718, 424, 419}

$$\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}\left(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2+c^2\left(-\left(-90d^2g^2+12defg+7e^2f^2\right)\right)\right)}{105c^3g^2}-\frac{2\sqrt{2}e\sqrt{b^2-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*e*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) - c^2*(7*e^2*
f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c
^3*g^2) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*c) + (2*
e^2*(c*e*f + 11*c*d*g - 6*b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35
*c^2*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b
*e*f + 21*b*d*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f
*g^2 + 105*d^3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) - b*(9*e^2*f^2 - 63
*d*e*f*g - 210*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
```

```
*g)])/(105*c^4*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) + c^2*(8*e^2*
f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^4*g^3*Sqrt[f
+ g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 941

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d
 + e*x)^(m - 2)*Simp[e*(b*d*f + a*(d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1
) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(
2*m - 1)))*x + e*(2*b*e*g*m - c*(e*f + d*g*(4*m - 1)))*x^2, x])/(Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Ne
Q[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c} - \frac{\int \frac{(d+ex)\left(-7cd^2f+e(bdf+4aef+adg)-(cd(12ef+7dg)-e(5bef+2bdg+5aeg)\right.}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{7c}$$

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c} + \frac{2e^2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{35c^2g^2} - \frac{2\int}{}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+bx}}{105c^3g^2}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+bx}}{105c^3g^2}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+bx}}{105c^3g^2}$$

$$= \frac{2e\left(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2\left(7e^2f^2+12defg-90d^2g^2\right)\right)\sqrt{f+gx}\sqrt{a+bx}}{105c^3g^2}$$

Mathematica [C] time = 14.9232, size = 10649, normalized size = 13.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.395, size = 14978, normalized size = 19.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(g*x + f)/sqrt(c*x^2
 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.901 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=567

$$\frac{4\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(2beg-5cdg+cef)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\right.}{15c^3g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*e*(c*e*f + 7*c*d*g - 4*b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c
^2*g) + (2*e*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(8*b^2*e^2*g^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) -
 c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c
*e*f - 5*c*d*g + 2*b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15
*c^3*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.93054, antiderivative size = 567, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {941, 1653, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2+c^2\left(-\left(-15d^2g^2-10defg+2e^2f^2\right)\right)\right)E\left(\sin\right.}{15c^3g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (2*e*(c*e*f + 7*c*d*g - 4*b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c
^2*g) + (2*e*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(8*b^2*e^2*g^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) -
 c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c
*e*f - 5*c*d*g + 2*b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15
*c^3*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 941

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d
 + e*x)^(m - 2)*Simp[e*(b*d*f + a*(d*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1
) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(
2*m - 1)))*x + e*(2*b*e*g*m - c*(e*f + d*g*(4*m - 1)))*x^2, x])/(Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Ne
Q[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} - \frac{\int \frac{-5cd^2f+e(bdf+2aef+adg)-(cd(8ef+5dg)-e(3bef+2bdg+3aeg))x-e(cef+\cdots}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} - \frac{2\int \frac{-\frac{1}{2}g(4\cdots}{\cdots}}{\cdots}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} + \frac{\left(2e(cef-\cdots\right.}{\cdots}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} + \frac{\left(\sqrt{2}\sqrt{b^2\cdots}\right.}{\cdots}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c} + \frac{\sqrt{2}\sqrt{b^2-\cdots}}{\cdots}$$

<u>**Mathematica**</u> **[C]** time = 11.5826, size = 1002, normalized size = 1.77

$$\frac{\left(\frac{2e^2x}{5c}-\frac{2e(-cef-10cdg+4beg)}{15c^2g}\right)\sqrt{f+gx}\left(cx^2+bx+a\right)}{\sqrt{a+x(b+cx)}} - \frac{2(f+gx)^{3/2}\sqrt{cx^2+bx+a}\left(\left(\left(2e^2f^2-10degf-15d^2g^2\right)c^2+eg(3be\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (((-2*e*(-(c*e*f) - 10*c*d*g + 4*b*e*g))/(15*c^2*g) + (2*e^2*x)/(5*c))*Sqrt
[f + g*x]*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(f + g*x)^(3/2)*Sqr
t[a + b*x + c*x^2]*((-8*b^2*e^2*g^2 + c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g)
+ c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(c*(-1 + f/(f + g*x))^2 + (g*(
b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + ((I/2)*Sqrt[1 - (2*(c*
f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x)
)]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a
*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(8*b^2*e^2*g
^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) + c^2*(-2*e^2*f^2 + 10*d*e*f*g +
15*d^2*g^2))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (-30*c^3
*d^2*f*g^2 + 8*b^2*e^2*g^2*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*e*g*(-17*a*b
*e*g^2 + 9*a*e*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*Sqrt[(b^2 - 4*a*c)*g^2]*(3*e*f
 + 20*d*g) - b^2*g*(11*e*f + 20*d*g)) - c^2*(-15*b*d*g^2*(2*e*f + d*g) - 2*
```

```
a*e*g^2*(7*e*f + 10*d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(-2*e^2*f^2 + 10*d*e*f*g
 + 15*d^2*g^2)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(Sqr
t[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])]*Sqrt[f + g*x])))/(15*c^3*g^3*Sqrt[a + x*(b + c*x)]*Sqrt[((f + g*x)^2*(c
*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*
x)))/g^2])
```

Maple [B] time = 0.348, size = 8248, normalized size = 14.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.902 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=452

$$-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2g\sqrt{b^2-4ac}}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\right)}{3c^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3*c^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.436786, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {832, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-2beg+3cdg+cef)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3*c^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```


Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}(3cdf-e(bf+ag))+\frac{1}{2}(cef+3cdg-2beg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c} + \frac{(cef+3cdg-2beg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3cg} - \frac{\left(e\left(cf^2-bfg+ag^2\right)\right)\int \frac{\dots}{\sqrt{f+g\dots}}}{3cg}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c} + \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(cef+3cdg-2beg)\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\dots\right.}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+\dots}}$$

$$= \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c} + \frac{\sqrt{2}\sqrt{b^2-4ac}(cef+3cdg-2beg)\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\dots\right.\right.}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+c\dots}}$$

Mathematica [C] time = 7.50768, size = 638, normalized size = 1.41

$$2\sqrt{f+gx}\left(ce(a+x(b+cx))+\frac{(f+gx)\left(\frac{g^2(a+x(b+cx))(-2beg+3cdg+cef)}{(f+gx)^2}+\frac{i\sqrt{1-\frac{2\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}-bg+2cf\right)}}\sqrt{\frac{2\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}+bg-2cf\right)}+1}\left(\left(c\left(\sqrt{g^2(b^2-4ac\cdots\right.\right.\right.}{\cdots}\right)}{\cdots}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e*(a + x*(b + c*x)) + ((f + g*x)*((g^2*(c*e*f + 3*c*d*g
 - 2*b*e*g)*(a + x*(b + c*x)))/(f + g*x)^2 + ((I/2)*Sqrt[1 - (2*(c*f^2 + g*
(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[
1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(2*b*e*g - c*(e*f +
3*d*g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f +
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b
^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (6*c^2*d*f*g
+ 2*b*e*g*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-2*a*e*g^2 - 3*b*g*(e*f + d*
g) + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f + 3*d*g)))*EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt
[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[
(b^2 - 4*a*c)*g^2]))]))/(Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x]))/g^2))/(3*c^2*Sqrt[a + x*(b
 + c*x)])
```

Maple [B] time = 0.324, size = 3805, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*(3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*a*b*e*g^3-6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(
1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(
1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(
```

```
1/2))*a*c*d*g^3-2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(
g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Elli
pticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+
b^2)^(1/2)*a*e*g^3-3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2
*e*f*g^2+6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+
2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF
(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*f*g^2+3
*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(
-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*
a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e*f^2*g+2^(1/2)*(-(
g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^
(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/
2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(
2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*e*f*g^2-6*2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*
c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b
^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x
+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d*f^2*g-2^(1/2)*(-(g*x+f
)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2)
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f
-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e*f^2*g-4*2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(
1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e*g^3+6*2^(1/2)*(-(g*x+f)*c/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*g^3+2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*a*c*e*f*g^2+4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)
^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))
^(1/2))*b^2*e*f*g^2-6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(
1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2
)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*
```

```
c*d*f*g^2-6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b
+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Elliptic
E(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e*f^2*g+
6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+
(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4
*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d*f^2*g+2*2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^
(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*e*f^3-2*x^3*c^2*e*g^3-2*x^2*
b*c*e*g^3-2*x^2*c^2*e*f*g^2-2*x*a*c*e*g^3-2*x*b*c*e*f*g^2-2*a*c*e*f*g^2)/c^
2/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)/g^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.903 $\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(c*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a +
b*x + c*x^2])
```

Rubi [A] time = 0.064988, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {718, 424}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(c*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a +
b*x + c*x^2])
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)}{c\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.815616, size = 365, normalized size = 1.94

$$\frac{i\left(g\left(\sqrt{b^2-4ac}-b\right)+2cf\right)\sqrt{\frac{g\left(\sqrt{b^2-4ac}+b+2cx\right)}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}\sqrt{1-\frac{2c(f+gx)}{g\left(\sqrt{b^2-4ac}-b\right)+2cf}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\left(b+\sqrt{b^2-4ac}\right)g-2cf}}\sqrt{f+gx}\right)\middle|\frac{2cf-\left(b+\sqrt{\right.}}{2cf+\left(\sqrt{b^2\right.}}\right.}{\sqrt{2}cg\sqrt{a+x(b+cx)}\sqrt{\frac{c}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (I*(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 + (-b + Sqrt[b^2 - 4*a*c])*g)]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f
 + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)] - EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]))/(Sqrt[2]*c*g
*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.311, size = 747, normalized size = 4.

$$\frac{\sqrt{2}}{2g\left(cgx^3+bgx^2+cfx^2+agx+bfx+af\right)c^2}\sqrt{gx+f}\sqrt{cx^2+bx+a}\left(g\sqrt{-4ac+b^2}+bg-2cf\right)\sqrt{-c\left(gx+f\right)\left(g\sqrt{-4ac\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*
c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b
^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(EllipticF(2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*g*b-2*f*EllipticF(2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+
```

```
b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c-EllipticF(2^(1/2)*(-(
g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*g*(-4*a*c+b^2)^(1/2)-Elli
pticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*g+2*El
lipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*f+(-
4*a*c+b^2)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2)))^(1/2))*g)/g/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

3.904 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=467

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2g\sqrt{b^2-4ac}}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{2cf-g\left(\right.}}{}$$

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt
[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + S
qrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*
x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c
*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e*Sqrt[a + b*x + c*x^
2])
```

Rubi [A] time = 1.60246, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {943, 718, 419, 934, 169, 538, 537}

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt
[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + S
qrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*
x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c
*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e*Sqrt[a + b*x + c*x^
2])
```

Rule 943

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{\left((ef-dg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2c}}}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4a}}{2cf-\left(b+\sqrt{b^2-}\right.}\right.}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4a}}{2cf-\left(b+\sqrt{b^2-}\right.}\right.}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4a}}{2cf-\left(b+\sqrt{b^2-}\right.}\right.}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4a}}{2cf-\left(b+\sqrt{b^2-}\right.}\right.}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 1.64604, size = 379, normalized size = 0.81

$$-\frac{i\sqrt{2}\sqrt{\frac{g\left(\sqrt{b^2-4ac}+b+2cx\right)}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}\sqrt{1-\frac{2c(f+gx)}{g\left(\sqrt{b^2-4ac}-b\right)+2cf}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{f+gx}\sqrt{\frac{c}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}\right),\frac{2cf-g\left(\sqrt{b^2-4ac}+b\right)}{g\left(\sqrt{b^2-4ac}-b\right)+2cf}\right)-\right.}{e\sqrt{a+x(b+cx)}\sqrt{\frac{c}{g\left(\sqrt{b^2-4ac}+b\right)-2cf}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c]
)*g)]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqr
t[b^2 - 4*a*c])*g)] - EllipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(2
*c*(e*f - d*g)), I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqr
t[b^2 - 4*a*c])*g)]))/(e*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + x*(b + c*x)])
```

Maple [B] time = 0.329, size = 834, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*g*
(-4*a*c+b^2)^(1/2)-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^
(1/2)))^(1/2))*g*b+2*f*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2))*c+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*
c+b^2)^(1/2)*g+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*g-2*Ellipt
icPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^
(1/2)/e*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-
2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c
*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)/c/(c*g*x^3+b
*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

3.905 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=994

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e}{\left(cd^2-bed+ae^2\right)(d+ex)}+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\left(cd^2-bed+ae^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{\sqrt{2}\sqrt{b^2-\cdots}}{\cdots}$$

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d + e*x
))) + (Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sq
rt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c
*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(c*d^2 - b
*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f +
 g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sq
rt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)*(
e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 3.70064, antiderivative size = 994, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {945, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e}{\left(cd^2-bed+ae^2\right)(d+ex)}+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\left(cd^2-bed+ae^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}-\frac{\sqrt{2}\sqrt{b^2-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d + e*x
))) + (Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sq
```

```
rt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c
*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(c*d^2 - b
*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f +
 g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sq
rt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)*(
e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rule 945

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*Sqrt[f + g*x]
*Sqrt[a + b*x + c*x^2])/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*c*d*f*(m + 1
) - e*(a*g + b*f*(2*m + 3)) - 2*(b*e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m +
2)))*x - c*e*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\int \frac{-2cdf+bef-aeg-2cdgx-cegx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\int \left(-\frac{cdg}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{cgx}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{e^2(bf-ag)-cd(2\cdots}{e(d+ex)\sqrt{f+gx}\sqrt{a\cdots}}\right.}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2e\left(cd^2-bde+ae^2\right)} \cdots$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2\left(cd^2-bde+ae^2\right)} - \frac{\left(\left(e^2(\cdots\right.\right.}{\cdots}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}F\left(\sin^{\cdots}\right.}{e\left(cd^2-bde+ae^2\right)\sqrt{f+gx}\sqrt{\cdots}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2c\cdots}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a\cdots}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2c\cdots}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a\cdots}}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2c\cdots}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a\cdots}}$$

Mathematica [C] time = 13.8658, size = 18563, normalized size = 18.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.37, size = 13017, normalized size = 13.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.906 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1786

result too large to display

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*
x)^2) - (e*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*
x)) + (Sqrt[b^2 - 4*a*c]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*
g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*(c*d^
2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*
x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^
2]) - (Sqrt[b^2 - 4*a*c]*f*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*
e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(6*e*f
 - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipt
icF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]
, (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]
*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2
]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqr
t[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g +
Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g
*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*
c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b
*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[
b^2 - 4*a*c])*g]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*(c*d
*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2
*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2
*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - S
qrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f -
 d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 8.39578, antiderivative size = 1786, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {945, 6742, 718, 419, 939, 843, 424, 934, 169, 538, 537}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*
x)^2) - (e*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*
x)) + (Sqrt[b^2 - 4*a*c]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*
g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*(c*d^
2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*
x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^
2]) - (Sqrt[b^2 - 4*a*c]*f*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*
e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(6*e*f
 - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipt
icF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]
, (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]
*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2
]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqr
t[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g +
Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g
*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*
c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b
*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[
b^2 - 4*a*c])*g]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*(c*d
*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2
*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2
*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - S
qrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f -
 d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 945

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*Sqrt[f + g*x]
*Sqrt[a + b*x + c*x^2])/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*c*d*f*(m + 1
) - e*(a*g + b*f*(2*m + 3)) - 2*(b*e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m +
2)))*x - c*e*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
```

```
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \frac{-4cdf+3bef-aeg+2(cef-2cdg+beg)x+cegx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \left(\frac{cg}{e\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{-cd(6ef-5dg)+e(3bef-2bdg-aeg)}{e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{2 \ldots}{e(d+e \ldots}\right.}{4\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4e\left(cd^2-bde+ae^2\right)} - \frac{(cef-3cdg+beg)\int \frac{\ldots}{(d+ex)\sqrt{\ldots}}}{2e\left(cd^2-bde+\ldots\right.}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+b\ldots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+b\ldots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+b\ldots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+b\ldots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+b\ldots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+b\ldots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+b\ldots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\sqrt{a+b\ldots}}{4\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)}$$

Mathematica [C] time = 17.304, size = 36634, normalized size = 20.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.514, size = 59522, normalized size = 33.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)
```

3.907 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=675

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2g\sqrt{b^2-4ac}}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\right)}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}(e\ldots}{\ldots}$$

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(c*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
 (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*
f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^
2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)])/(Sqrt[c]*e^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.65465, antiderivative size = 675, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {957, 718, 419, 934, 169, 538, 537, 424}

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}(ef-dg)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(c*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
 (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*
f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^
```

```
2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)])/(Sqrt[c]*e^2*Sqrt[a + b*x + c*x^2])
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{g(ef-dg)}{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{g\sqrt{f+gx}}{e\sqrt{a+bx+cx^2}}\right)$$

$$= \frac{g\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+c}}}{e^2}$$

$$= \frac{\left((ef-dg)^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+}}}{e^2\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \text{-} \quad 2$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \text{-} \quad 2$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \text{-} \quad 2$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} + \text{-} \quad 2$$

Mathematica [B] time = 3.92238, size = 1385, normalized size = 2.05

$$\sqrt{2}\sqrt{\frac{c(f+gx)}{2cf+\left(\sqrt{b^2-4ac}-b\right)g}}\left(-\frac{4\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\Pi\left(\frac{2\sqrt{b^2-4ac}e}{2cd-be+\sqrt{b^2-4ac}e};\sin^{-1}\left(\frac{\sqrt{\frac{-b-2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}g}{2cf-bg+\sqrt{b^2-4ac}g}\right)f^2}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}+\frac{2g\left(b+2cx-\sqrt{b^2-4ac}\right)\sqrt{\frac{b+2cx+\sqrt{b^2}}{\sqrt{b^2-4a}}}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*((2*f*g*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g)])/(c*e*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]) - (d
*g^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2
- 4*a*c]*g)])/(c*e^2*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c
]]) + (g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] +
2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*((-2*c*f + (b + Sqrt[b^2 - 4*
a*c])*g)*EllipticE[ArcSin[Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)] - (b + Sqrt[b^2 - 4*a*c])*g*EllipticF[ArcSin[Sqrt[2]*Sqrt[(
c*(f + g*x))/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2
 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]))/(2*c^2*e*Sqrt[(g*(-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]) - (4*Sq
rt[b^2 - 4*a*c]*f^2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticPi[(
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e), ArcSin[Sqrt[(-b
 + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*
a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)])/(2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e) + (8*Sqrt[b^2 - 4*a*c]*d*f*g*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a
*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)
, ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)])/(e*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)) - (4*Sqrt[b^2 - 4*a*c]*d^2*g^2*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e
 + Sqrt[b^2 - 4*a*c]*e), ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4
*a*c]*g)])/(e^2*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))))/(Sqrt[f + g*x]*Sqrt
[a + x*(b + c*x)])
```

Maple [B] time = 0.325, size = 1879, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^
```

```
2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f))^(1/2)/c*(EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*d*g^2-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*e*f*g+2*EllipticF(2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*e*g^2+EllipticF(
2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*d*g^2-3*Elli
pticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e*f*g-
2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*
d*f*g+4*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/
2))*c*e*f^2-2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)
))^(1/2))*a*e*g^2+2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)
^(1/2)))^(1/2))*b*e*f*g-2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*
c+b^2)^(1/2)))^(1/2))*c*e*f^2-EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f
),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
))*(-4*a*c+b^2)^(1/2)*d*g^2+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),
(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))
*(-4*a*c+b^2)^(1/2)*e*f*g-EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b
*d*g^2+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e*f*g+2*EllipticPi
(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*d*f*g-2*EllipticPi(2^(1/2)*(-(g*x+f
)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c
+b^2)^(1/2)))^(1/2))*c*e*f^2)/e^2/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.908 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1138

result too large to display

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) + (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(3*c^2*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
 + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[f + g*x]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^2*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a +
b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(c*f^2 - b*f*g + a*g^2)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(3*c^2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)^2*Sqrt[1 - (2*c*(f + g*
x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*
g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f -
(b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[
b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 2.13819, antiderivative size = 1138, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {957, 718, 419, 934, 169, 538, 537, 424, 742, 843}

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}g^2}{3ce}+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4a}\right.}\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) + (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(3*c^2*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
 + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[f + g*x]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g
```

```
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^2*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a +
b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(c*f^2 - b*f*g + a*g^2)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(3*c^2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)^2*Sqrt[1 - (2*c*(f + g*
x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*
g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f -
(b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[
b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b*x + c*x^2])
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{g(ef-dg)\sqrt{f+gx}}{e^2\sqrt{a+bx+cx^2}} + \cdots \right.$$

$$= \frac{g\int \frac{(f+gx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{\left(g(ef-dg)^2\right)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^3} + \cdots$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce} + \frac{(2g)\int \frac{\frac{1}{2}\left(3cf^2-g(bf+ag)\right)+g(2cf-bg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{3ce} + \frac{\left((ef-dg)^3\sqrt{b-\sqrt{b^2\cdots}}\right.}{\cdots}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+\cdots}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{ce^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+\cdots}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce} + \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx\cdots}}$$

$$= \frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce} + \frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx\cdots}}$$

Mathematica [C] time = 16.2133, size = 37137, normalized size = 32.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.326, size = 7464, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+bx+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.909 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=631

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce^2g(ag(7ef-15dg)-3bf(ef-5dg))+4be^3g^2(bf-ag)+c^2\left(45d^2ef\right.\right.}{15c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (-8*e^2*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*g^2) + (2*e^2*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c*g) +
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9
*a*e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*
a*c]*(4*b*e^3*g^2*(b*f - a*g) + c^2*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*
f*g^2 - 15*d^3*g^3) - c*e^2*g*(a*g*(7*e*f - 15*d*g) - 3*b*f*(e*f - 5*d*g)))
*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.09021, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {930, 1653, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce^2g(ag(7ef-15dg)-3bf(ef-5dg))+4be^3g^2(bf-ag)+c^2\left(45d^2ef\right.\right.}{15c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-8*e^2*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*g^2) + (2*e^2*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c*g) +
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9
*a*e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*
a*c]*(4*b*e^3*g^2*(b*f - a*g) + c^2*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*
f*g^2 - 15*d^3*g^3) - c*e^2*g*(a*g*(7*e*f - 15*d*g) - 3*b*f*(e*f - 5*d*g)))
*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + S
```

qrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), I
nt[((d + e*x)^(m - 3)*Simp[b*d*e^2*f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*
g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g
*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(m - 1)*x^2, x])/(Sqrt[f +
 g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} - \frac{\int \frac{bde^2f-5cd^3g+ae^2(2ef+dg)+e(cd(2ef-15dg)+e(3bef+2bdg+3aeg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{5cg}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} -$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} +$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} +$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} +$$

Mathematica [C] time = 13.7705, size = 12746, normalized size = 20.2

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.351, size = 8755, normalized size = 13.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{cgx^3 + \left(cf + bg\right)x^2 + af + \left(bf + ag\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + b*x + a)*sq
rt(g*x + f)/(c*g*x^3 + (c*f + b*g)*x^2 + a*f + (b*f + a*g)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{\sqrt{f + gx}\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.910 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=479

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(e^2g(bf-ag)+c\left(3d^2g^2-6defg+2e^2f^2\right)\right)\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2c}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*g) - (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)])/(3*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e^2*g*(b*f -
 a*g) + c*(2*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2
*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.553347, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {930, 24, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(e^2g(bf-ag)+c\left(3d^2g^2-6defg+2e^2f^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{}{2c}\right.}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*g) - (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2
 - 4*a*c])*g)])/(3*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e^2*g*(b*f -
 a*g) + c*(2*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El
lipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2
*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```


Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), I
```

```
nt[((d + e*x)^(m - 3)*Simp[b*d*e^2*f + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*
g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*f + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g
*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(m - 1)*x^2, x])/(Sqrt[f +
 g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[2*m] && GeQ[m, 2]
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\int \frac{d\left(be^2f-3cd^2g+ae^2g\right)+e(cd(2ef-9dg)+e(bef+2bdg+aeg))x+2e^2(cef-3cdg+b}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{3cg}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\int \frac{e^2\left(be^2f-3cd^2g+ae^2g\right)+2e^3(cef-3cdg+beg)x}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx}{3ce^2g}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{(2e(cef-3cdg+beg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}dx}{3cg^2} + \frac{\left(e^2g(bf-ag)+c\right(}{}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}e(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)}{3c^2g^2\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{}}$$

$$= \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg} - \frac{2\sqrt{2}\sqrt{b^2-4ac}e(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E}{3c^2g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a}}$$

Mathematica [C] time = 12.7737, size = 1080, normalized size = 2.25

$$\frac{2\sqrt{f+gx}\left(cx^2+bx+a\right)e^2}{3cg\sqrt{a+x(b+cx)}} + \frac{(f+gx)^{3/2}\sqrt{cx^2+bx+a}\left(-4e(cef-3cdg+beg)\sqrt{\frac{cf^2+g(ag-bf)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\left(c\left(\frac{f}{f+gx}-1\right)^2+\frac{g(-}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x]*(a + b*x + c*x^2))/(3*c*g*Sqrt[a + x*(b + c*x)]) + ((f
 + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[(c
*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 +
 f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) +
(I*Sqrt[2]*e*(c*e*f - 3*c*d*g + b*e*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^
2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f
+ g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2
])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f
+ g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g
 + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 -
 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - (I
*Sqrt[2]*(-3*c^2*d^2*g^2 + b*e^2*g*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]) + c*e
*(3*b*d*g^2 + a*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f - 3*d*g)))*Sqrt[(Sqrt[
```

```
(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(
-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[
(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g
- (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I
*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 -
4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(
2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(3*c^2*g^3*Sqrt[(
c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a
+ x*(b + c*x)]*Sqrt[((f + g*x)^2*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f
 + g*x) + (a*g)/(f + g*x)))/(f + g*x)))/g^2])
```

Maple [B] time = 0.329, size = 4295, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3*(-4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b
-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2
^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*e^2*f^3-2*a
*c*e^2*f*g^2-2*x^2*c^2*e^2*f*g^2+6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
)))^(1/2))*a*c*e^2*f*g^2+2*(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*
c+b^2)^(1/2)))^(1/2))*c*e^2*f^2*g-2*x^3*c^2*e^2*g^3+4*2^(1/2)*(-(g*x+f)*c/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*e^2*f*g^2+3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*
c+b^2)^(1/2)))^(1/2))*b*c*d^2*g^3-6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/
2)))^(1/2))*c^2*d^2*f*g^2-4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/
2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/
2))*a*b*e^2*g^3-3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)
*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*El
```

```
lipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*e^
2*f*g^2-(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2
),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
))*a*e^2*g^3+3*(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)
))^(1/2))*c*d^2*g^3-2*x*a*c*e^2*g^3-2*x^2*b*c*e^2*g^3-6*2^(1/2)*(-(g*x+f)*c
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(
2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*d*e*f*g^2+6*2^(1/2)*(-
(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)
^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1
/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*e*f*g^2+3*2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b
*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e^2*g^3-12*2^(1/2)*(-(g*x+f)*c/(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*
a*c+b^2)^(1/2)))^(1/2))*b*c*d*e*f*g^2-2*x*b*c*e^2*f*g^2-12*2^(1/2)*(-(g*x+f
)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2)
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f
-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e*g^3+12*2^(1/2)*(-(g*x+f)*c/(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)
^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e*g^3+(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b
*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e^2*f*g^2-4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g
+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a
*c+b^2)^(1/2)))^(1/2))*a*c*e^2*f*g^2+12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)
^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*
a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)
^(1/2)))^(1/2))*c^2*d*e*f^2*g)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3/(c
*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e^2x^2+2\,dex+d^2\right)\sqrt{cx^2+bx+a}\sqrt{gx+f}}{cgx^3+\left(cf+bg\right)x^2+af+\left(bf+ag\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(c*g
*x^3 + (c*f + b*g)*x^2 + a*f + (b*f + a*g)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.911 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=393

$$\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-d\ldots}{}$$

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[(c*(f + g
*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.209592, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {843, 718, 424, 419}

$$\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-d\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```


```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[(c*(f + g
*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```


Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{e\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{g} + \frac{(-ef+dg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{g}$$

$$= \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}\,gx^2}{2cf-bg-\sqrt{b^2-4ac}\,g}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}} \cdots\right.}{cg\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\sqrt{a+bx+cx^2}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}} - \cdots$$

Mathematica [C] time = 5.69684, size = 814, normalized size = 2.07

$$-\frac{(f+gx)^{3/2}\left(-\frac{4e\sqrt{\frac{cf^2+g(ag-bf)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))g^2}{(f+gx)^2} + \frac{i\sqrt{2}e\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\sqrt{\frac{-2ag^2+2cfxg+b(f-gx)g+\sqrt{(b^2-4ac)g^2}(f+gx)}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{\frac{2ag^2-2cfxg+b(gx-f\cdots}{\left(-2cf+bg+\sqrt{(\cdots}\right.}}\cdots}{\sqrt{f+gx}\cdots}\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((f + g*x)^(3/2)*((-4*e*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*e*(
2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f
- g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a
*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x + b*g*(-f + g*x) + Sqrt[(b^
2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g
*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b
*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2
 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] -
(I*Sqrt[2]*(2*c*d*g + e*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]))*Sqrt[(-2*a*g^2
+ 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*f -
b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x + b*g*
(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(b^2
- 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g
 + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
))])/Sqrt[f + g*x]))/(2*c*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g
 + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.317, size = 1014, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a
*e*g^2-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2
),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
))*b*d*g^2-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^
(1/2))*b*e*f*g+2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2)))^(1/2))*c*d*f*g-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2
)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*d*g^2+EllipticF(2^(1/2)*(-(g*x+f)*c/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*e*f*g-2*EllipticE(
2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*e*g^2+2*Elli
pticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e*f*g-
2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*
e*f^2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1
/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*x+f)^(1/2)
*(c*x^2+b*x+a)^(1/2)/c/g^2/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}}{cgx^3+\left(cf+bg\right)x^2+af+\left(bf+ag\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)/(c*g*x^3 + (c*f + b*
g)*x^2 + a*f + (b*f + a*g)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.912 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2g\sqrt{b^2-4ac}}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*Sqrt[f + g*x]*Sqrt[a
+ b*x + c*x^2])
```

Rubi [A] time = 0.0722348, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {718, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*Sqrt[f + g*x]*Sqrt[a
+ b*x + c*x^2])
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}gx^2}{2cf-bg-\sqrt{b^2-4ac}g}}}\,dx, x\ldots\right.}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle| -\frac{2\sqrt{b^2-4ac}\ldots}{2cf-\left(b+\sqrt{b^2-\ldots}\right.}\right.}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Mathematica [C] time = 0.683923, size = 308, normalized size = 1.63

$$\frac{i(f+gx)\sqrt{2-\frac{4(g(ag-bf)+cf^2)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}-bg+2cf\right)}}\sqrt{\frac{2(g(ag-bf)+cf^2)}{(f+gx)\left(\sqrt{g^2(b^2-4ac)}+bg-2cf\right)}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{ag^2-bfg+cf^2}{\sqrt{g^2(b^2-4ac)}+bg-2cf}}}{\sqrt{f+gx}}\right), -\frac{\sqrt{g^2(\ldots}}{\sqrt{g^2(\ldots}}\right.}{g\sqrt{a+x(b+cx)}\sqrt{\frac{g(ag-bf)+cf^2}{\sqrt{g^2(b^2-4ac)}+bg-2cf}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])
/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2]))])/(g*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*
g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.311, size = 287, normalized size = 1.5

$$\frac{\sqrt{2}}{cg\left(cgx^3+bgx^2+cfx^2+agx+bfx+af\right)}\left(-g\sqrt{-4\,ac+b^2}-bg+2\,cf\right)\text{EllipticF}\left(\sqrt{2}\sqrt{-c\left(gx+f\right)\left(g\sqrt{-4\,ac+b^2}\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (-g*(-4*a*c+b^2)^(1/2)-b*g+2*c*f)/c*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*
(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2)/g*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*
g*x+b*f*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{cgx^3 + \left(cf + bg\right)x^2 + af + \left(bf + ag\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(c*g*x^3 + (c*f + b*g)*x^2 + a
*f + (b*f + a*g)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{f + gx}\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.913 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=280

$$-\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4a\right.}}}\right.\right.}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)}$$

```
[Out] -((Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))
/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b
- Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2
 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]))
```

Rubi [A] time = 1.24698, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {934, 169, 538, 537}

$$-\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4a\right.}}}\right.\right.}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))
/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b
- Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2
 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]))
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{1}{\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}(d+}}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}\left(\int \frac{}{\left(ef-dg-ex^2\right)\sqrt{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{}{\left(ef-dg-ex^2\right)\sqrt{b+\sqrt{b^2-4ac}-}}\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\sqrt{1+\frac{2c(f+gx)}{\left(b+\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right)\text{Subst}\left(\int \frac{}{\left(ef-dg-ex^2\right)\sqrt{1+\frac{2cx^2}{\left(b-\sqrt{b^2-4a}\right.}}}\right.}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\Pi\left(\frac{e(2}{}\right.}{\sqrt{c}(ef-dg)\sqrt{a+bx+cx}}$$

Mathematica [C] time = 1.82843, size = 499, normalized size = 1.78

$$\frac{i(f+gx)\sqrt{2-\frac{4\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2\left(b^2-4ac\right)}-bg+2cf\right)}}\sqrt{\frac{2\left(g(ag-bf)+cf^2\right)}{(f+gx)\left(\sqrt{g^2\left(b^2-4ac\right)}+bg-2cf\right)}+1}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{ag^2-bfg+cf^2}{\sqrt{g^2\left(b^2-4ac\right)}+bg-2cf}}}{\sqrt{f+gx}}\right),-\frac{\sqrt{g^2(b^2}}{\sqrt{g^2(b^2}}\right.\right.}{\sqrt{a+x(b+cx)}(dg-ef)\sqrt{\frac{g(ag}{\sqrt{g^2(b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*(EllipticF[I*ArcSinh[(Sqr
```

```
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))] - EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(
b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sq
rt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[
f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2]))]))/((-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.386, size = 330, normalized size = 1.2

$$\frac{\sqrt{2}}{\left(dg-ef\right)c\left(cgx^{3}+bgx^{2}+cfx^{2}+agx+bfx+af\right)}\left(-g\sqrt{-4\,ac+b^{2}}-bg+2\,cf\right)\text{EllipticPi}\left(\sqrt{2}\sqrt{-c\left(gx+f\right)\left(g\sqrt{-4\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (-g*(-4*a*c+b^2)^(1/2)-b*g+2*c*f)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g
-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^
(1/2))*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1
/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)/c*(c*x^2+b*x+
a)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)
```

3.914 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1037

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)}+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*f -
 d*g)*(d + e*x))) + (Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[(c*(
f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sq
rt[2]*Sqrt[b^2 - 4*a*c]*e*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*
(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*
c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d
*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2
*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2
*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - S
qrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^
2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 3.52709, antiderivative size = 1037, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {939, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}e^2}{\left(cd^2-bed+ae^2\right)(ef-dg)(d+ex)}+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{2}\left(cd^2-bed+ae^2\right)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*f -
 d*g)*(d + e*x))) + (Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[(c*(
```

```
f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sq
rt[2]*Sqrt[b^2 - 4*a*c]*e*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*
(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*
c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d
*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2
*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2
*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - S
qrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^
2*Sqrt[a + b*x + c*x^2])
```

<u>Rule 939</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

<u>Rule 6742</u>

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

<u>Rule 718</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

<u>Rule 419</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

<u>Rule 843</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} - \frac{\int \frac{-2cd(ef-dg)+e(bef-2bdg+aeg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,d}{2(cd^2-bde+ae^2)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} - \frac{\int\left(-\frac{cdg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{cegx}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \right.}{2(cd^2-bde+ae^2)}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)} + \frac{(ceg)\int}{2(cd^2-}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{(ce)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)(ef-dg)} - \frac{(cef)\int}{2(cd^2-}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c(}{}}}{(cd^2-bde+ae^2)(}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\text{sin}\right.}{\sqrt{2}(cd^2-bde+ae^2)(ef-dg)\sqrt{}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\text{sin}\right.}{\sqrt{2}(cd^2-bde+ae^2)(ef-dg)\sqrt{}}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\text{sin}\right.}{\sqrt{2}(cd^2-bde+ae^2)(ef-dg)\sqrt{}}$$

Mathematica [C] time = 14.1065, size = 10881, normalized size = 10.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.374, size = 14048, normalized size = 13.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

3.915 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1114

result too large to display

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(e*f
- d*g)*(d + e*x)^2) - (3*e^2*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*
e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*
f - d*g)^2*(d + e*x)) + (3*Sqrt[b^2 - 4*a*c]*e*(c*d*(2*e*f - 3*d*g) - e*(b*
e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*
a*c]*(c*d*(-6*e*f + 7*d*g) + e*(3*b*e*f - 4*b*d*g + a*e*g))*Sqrt[(c*(f + g*
x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 +
 4*a*c)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*
g)])/(2*Sqrt[2]*(c*d^2 + e*(-(b*d) + a*e))^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt
[a + x*(b + c*x)]) + (Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g]*(c^2*d^2*(8*e
^2*f^2 - 20*d*e*f*g + 15*d^2*g^2) + 2*c*e*(b*d*(-4*e^2*f^2 + 11*d*e*f*g - 1
0*d^2*g^2) + a*e*(-2*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2)) + e^2*(3*a^2*e^2*g^2
 + 2*a*b*e*g*(e*f - 4*d*g) + b^2*(3*e^2*f^2 - 8*d*e*f*g + 8*d^2*g^2)))*Sqrt
[(g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]
*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*EllipticPi[(2*c*e*f - b*e*g + Sqrt[b^2 - 4*a*c]*e*g)/(2*c*e*f - 2*c*d*
g), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*
a*c]*g]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 + e*(-(b*d) + a*e))^2*(-(e*f) + d*g)^3*
Sqrt[a + x*(b + c*x)])
```

Rubi [A] time = 8.01181, antiderivative size = 1762, normalized size of antiderivative = 1.58, number of steps used = 25, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {939, 6742, 718, 419, 843, 424, 934, 169, 538, 537}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(e*f
- d*g)*(d + e*x)^2) - (3*e^2*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*
e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*
f - d*g)^2*(d + e*x)) + (3*Sqrt[b^2 - 4*a*c]*e*(c*d*(2*e*f - 3*d*g) - e*(b*
e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*
a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*S
qrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (3*Sqrt[b^2 - 4*a*c]*e*f*(c*d*(2*e*f
```

```
- 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c
*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
+ (3*Sqrt[b^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*
g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^
2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*
c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g))
, ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c
])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f
)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x
+ c*x^2]) - (3*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g)
 - e*(b*e*f - 2*b*d*g + a*e*g))^2*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)
), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*
c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*
f)/g)])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^3*Sqrt[a +
 b*x + c*x^2])
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

```
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \frac{3e^2(bf+ag)-4d(cef-cdg+beg)+2e(cef-2cdg+b}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{4\left(cd^2-bde+ae^2\right)(ef-d}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \left(\frac{cg}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{3(-cd(2ef-3dg)+e(be}{(d+ex)^2\sqrt{f+gx}\sqrt{}}\right.}{4\left(cd^2-bde+\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{4\left(cd^2-bde+ae^2\right)(ef-dg)} - \frac{(cef-}{}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+a}{4\left(cd^2-bde+ae^2\right)^2(ef}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+a}{4\left(cd^2-bde+ae^2\right)^2(ef}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+a}{4\left(cd^2-bde+ae^2\right)^2(ef}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+a}{4\left(cd^2-bde+ae^2\right)^2(ef}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+a}{4\left(cd^2-bde+ae^2\right)^2(ef}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+a}{4\left(cd^2-bde+ae^2\right)^2(ef}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+a}{4\left(cd^2-bde+ae^2\right)^2(ef}$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+a}{4\left(cd^2-bde+ae^2\right)^2(ef}$$

Mathematica [C] time = 18.2261, size = 40396, normalized size = 36.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.616, size = 64947, normalized size = 58.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

3.916 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=553

$$-\frac{\sqrt{2}g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{a+bx+cx^2}(ef-dg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{\sqrt{2}e\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{}$$

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f +
 g*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*e*Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b -
 Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d
*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4
*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2
*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 1.613, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {957, 744, 21, 718, 424, 934, 169, 538, 537}

$$-\frac{\sqrt{2}g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{a+bx+cx^2}(ef-dg)\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{\sqrt{2}e\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f +
 g*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*e*Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b -
 Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d
*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4
*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2
*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
```

```
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int\left(-\frac{g}{(ef-dg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}+\frac{e}{(ef-dg)(d+ex)\sqrt{f+gx}\sqrt{a+b}}\right.$$

$$=\frac{e\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{ef-dg}-\frac{g\int\frac{1}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$

$$=\frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}+\frac{(2g)\int\frac{-\frac{cf}{2}-\frac{cgx}{2}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)\left(cf^2-bfg+ag^2\right)}+\frac{\left(e\sqrt{\right.}{}$$

$$=\frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}-\frac{(cg)\int\frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)\left(cf^2-bfg+ag^2\right)}-\frac{\left(2e\sqrt{\right.}{}$$

$$=\frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}-\frac{\left(\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+}{b^2-4a}}\right.}{(ef-dg)\left(cf^2-bfg\right.}$$

$$=\frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+c}{b^2-4ac}}}{(ef-dg)\left(cf^2-bfg+a\right.}$$

$$=\frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+c}{b^2-4ac}}}{(ef-dg)\left(cf^2-bfg+a\right.}$$

Mathematica [C] time = 6.88032, size = 950, normalized size = 1.72

$$2\left(\frac{(a+x(b+cx))g^2}{ef-dg}+\frac{(f+gx)^2\left(\frac{cf^2}{(f+gx)^2}-\frac{2cf}{f+gx}-\frac{bgf}{(f+gx)^2}+c-\frac{i\sqrt{1-\frac{2(cf^2+g(ag-bf))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{\frac{4(cf^2+g(ag-bf))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}+2}\left((ef-dg)\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\left(\left(E\left(i\sinh^{-1}\cdots\right.\right.\right.\right.}{\cdots}\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*((g^2*(a + x*(b + c*x)))/(e*f - d*g) + ((f + g*x)^2*(c + (c*f^2)/(f + g*
x)^2 - (b*f*g)/(f + g*x)^2 + (a*g^2)/(f + g*x)^2 - (2*c*f)/(f + g*x) + (b*g
)/(f + g*x) - ((I/4)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) + a*g
)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((e*f - d*g)*(2*c
*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f
^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x
]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4
*a*c)*g^2]))] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]) - 2*e*
(c*f^2 + g*(-(b*f) + a*g))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g
 + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
))] + 2*e*(c*f^2 + g*(-(b*f) + a*g))*EllipticPi[((e*f - d*g)*(2*c*f - b*g -
 Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqr
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))]))/((e*f - d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x])))/(-(e*f) + d*g)))
/((c*f^2 + g*(-(b*f) + a*g))*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.444, size = 4757, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] (2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x
+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-
4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)
*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2
```

)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*g^3+2*2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2
)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(
1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d*f^2*g-2*2^(1/2)*(-(g*x+f)*c
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(
2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*g^3-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*
(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2)))^(1/2))*c^2*d*f^2*g-2*x*b*c*d*g^3+2*2^(1/2)*(-(g*x+f)*c/(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*
a*c+b^2)^(1/2)))^(1/2))*c^2*e*f^3-2*a*c*d*g^3+2*x^2*c^2*e*f*g^2+2*a*c*e*f*g
^2-2*x^2*c^2*d*g^3-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
EllipticF(2^(1/2)(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c
*d*f*g^2+2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+
2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF
(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e*f^2*g-2
EllipticPi(2^(1/2)(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2
(g(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^2*e*f^3*(-(g*x+f
)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2)
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)-2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c
*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^2*e*f^3*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g
+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(
-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*
2^(1/2)*a*b*e*g^3*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-
b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2
*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)+EllipticPi
(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*a*e*g^3*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2)*(-4*a*c+b^2)^(1/2)-EllipticPi(2^(1/2)*(-(g*x+f)*c
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*2^(1/2)*b^2*e*f*g^2*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(
1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2)+2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*

```
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b
+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Elliptic
E(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e*f*g^2+
2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+
(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4
*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*f*g^2-2*2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^
2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^
(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e*f^2*g+2*x*b*c*e*f*g^2+3*El
lipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*b*c*e*f^2*g*(-(g*x+f)
*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)-EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/
(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
)))^(1/2))*2^(1/2)*b*e*f*g^2*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
*(-4*a*c+b^2)^(1/2)+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)
*c*e*f^2*g*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x
+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-
4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(-4*a*c+b^2)^(1/2
)-2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(
-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*
2^(1/2)*a*c*e*f*g^2*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*
(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b
+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)-2*Ellipt
icPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*a*c*e*f*g^2*(-(g*x+f)*c/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2))*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g
-e*f)^2/(a*g^2-b*f*g+c*f^2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(f+gx\right)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)\left(gx+f\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

3.917 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1125

result too large to display

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^(3/2)) + (4*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*
f^2 - b*f*g + a*g^2)^2*Sqrt[f + g*x]) + (2*e*g^2*Sqrt[a + b*x + c*x^2])/((e
*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*
a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*e*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((e*f - d*g)^2
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]) - (Sqrt[2]*e^2*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x)
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sq
rt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)
/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^3*Sqrt[a + b*x
+ c*x^2])
```

Rubi [A] time = 2.31048, antiderivative size = 1125, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {957, 744, 834, 843, 718, 424, 419, 21, 934, 169, 538, 537}

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}g\right)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right)\right.}{\sqrt{c}(ef-dg)^3\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^(3/2)) + (4*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*
f^2 - b*f*g + a*g^2)^2*Sqrt[f + g*x]) + (2*e*g^2*Sqrt[a + b*x + c*x^2])/((e
*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*
a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b
```

```
+ Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*e*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((e*f - d*g)^2
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]) - (Sqrt[2]*e^2*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x)
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sq
rt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)
/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^3*Sqrt[a + b*x
+ c*x^2])
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
```

```
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)^{3/2}\sqrt{a+bx+}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{1}{(f+gx)^{5/2}\sqrt{a+l}}}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{2eg^2\sqrt{a+bx+cx^2}}{(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{.}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a+bx+c}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a+bx+c}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a+bx+c}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{}}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-bg)\sqrt{a+bx+c}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{}}$$

Mathematica [C] time = 15.8711, size = 14762, normalized size = 13.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.46, size = 27597, normalized size = 24.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.918 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=475

$$\frac{\sqrt{2}(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\sqrt{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(d+ex)\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)}}\sqrt{\frac{\left(x\left(\sqrt{b^2-4ac}+b\right)+2a\right)(ef-dg)}{(d+ex)\left(f\sqrt{b^2-4ac}-2ag+bf\right)}}\Pi\left(\frac{e(2c\cdots}{(2c\cdots}\right.}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}\sqrt{a+bx+cx^2}\sqrt{2cd-e\left(\sqrt{b^2\cdots}\right.}}$$

```
[Out] (Sqrt[2]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[b - Sqrt[b^2 - 4*a*c]
 + 2*c*x]*Sqrt[((e*f - d*g)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)*(d + e*x))]*Sqrt[((e*f - d*g)*(2*a + (b + Sqrt[b^2 -
 4*a*c])*x))/((b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g)*(d + e*x))]*(d + e*x)*Ell
ipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)*g), ArcSin[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x]
)/(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])], ((b*d + Sqrt[b^
2 - 4*a*c]*d - 2*a*e)*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)*(b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g))])/(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*g*Sqrt[(2*a*c)/(b + Sqrt[b^2 - 4*a*c]) + c*x]*Sqr
t[a + b*x + c*x^2])
```

Rubi [A] time = 0.419329, antiderivative size = 475, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {926}

$$\frac{\sqrt{2}(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\sqrt{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(d+ex)\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)}}\sqrt{\frac{\left(x\left(\sqrt{b^2-4ac}+b\right)+2a\right)(ef-dg)}{(d+ex)\left(f\sqrt{b^2-4ac}-2ag+bf\right)}}\Pi\left(\frac{e(2c\cdots}{(2c\cdots}\right.}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}\sqrt{a+bx+cx^2}\sqrt{2cd-e\left(\sqrt{b^2\cdots}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[b - Sqrt[b^2 - 4*a*c]
 + 2*c*x]*Sqrt[((e*f - d*g)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)*(d + e*x))]*Sqrt[((e*f - d*g)*(2*a + (b + Sqrt[b^2 -
 4*a*c])*x))/((b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g)*(d + e*x))]*(d + e*x)*Ell
ipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)*g), ArcSin[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x]
)/(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])], ((b*d + Sqrt[b^
2 - 4*a*c]*d - 2*a*e)*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)*(b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g))])/(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*g*Sqrt[(2*a*c)/(b + Sqrt[b^2 - 4*a*c]) + c*x]*Sqr
t[a + b*x + c*x^2])
```

Rule 926

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(Sqr
t[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[((e*f - d*g
)*(b + q + 2*c*x))/((2*c*f - g*(b + q))*(d + e*x))]*Sqrt[((e*f - d*g)*(2*a
+ (b + q)*x))/((b*f + q*f - 2*a*g)*(d + e*x))]*EllipticPi[(e*(2*c*f - g*(b
```

```
+ q)))/(g*(2*c*d - e*(b + q))), ArcSin[(Sqrt[2*c*d - e*(b + q)]*Sqrt[f + g*
x])/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x])], ((b*d + q*d - 2*a*e)*(2*c*f -
 g*(b + q)))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q)))])/(g*Sqrt[2*c*d - e*
(b + q)]*Sqrt[(2*a*c)/(b + q) + c*x]*Sqrt[a + b*x + c*x^2]), x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a\ldots\right.}{\left(bf+\sqrt{b^2-4\ldots\right.}}}\ldots}{\sqrt{2cd-\left(b+\sqrt{b^2-\ldots\right.}}\ldots}$$

Mathematica [B] time = 9.95143, size = 1118, normalized size = 2.35

$$-\frac{\sqrt{2}\sqrt{-\frac{g\left(cf^2+g(ag-bf)\right)(d+ex)}{\left(-2aeg^2-2cdfg+b(ef+dg)g-d\sqrt{\left(b^2-4ac\right)g^2}g+ef\sqrt{\left(b^2-4ac\right)g^2}\right)(f+gx)}}(f+gx)^{3/2}\left(\frac{2ef\sqrt{\left(b^2-4ac\right)g^2}\sqrt{-\frac{\left(cf^2+g(ag-bf)\right)(a+x(b+cx))}{\left(b^2-4ac\right)(f+gx)^2}}\,\text{EllipticF}\left(\sin^{-1}\ldots\right.}{\ldots}\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[-((g*(c*f^2 + g*(-(b*f) + a*g))*(d + e*x))/((-2*c*d*f*g - 2
*a*e*g^2 + e*f*Sqrt[(b^2 - 4*a*c)*g^2] - d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*
(e*f + d*g))*(f + g*x)))]*(f + g*x)^(3/2)*((2*e*f*Sqrt[(b^2 - 4*a*c)*g^2]*S
qrt[-(((c*f^2 + g*(-(b*f) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*
x)^2))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[
(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sqrt[2]]
, (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2 - e*f*S
qrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))])/(
c*f^2 + g*(-(b*f) + a*g)) + (d*g*(2*a*g^2 - f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c
*f*g*x - g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))*Sqrt[(2*a*g^2 - 2*c*
f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*
a*c)*g^2]*(f + g*x))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f
- g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g
*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a
*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e
*f + d*g))])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)*Sqrt[(-2*a*g^2 + 2*c*f*g
*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)
*g^2]*(f + g*x))]) - (4*e*Sqrt[(b^2 - 4*a*c)*g^2]*Sqrt[-(((c*f^2 + g*(-(b*f
) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*x)^2))]*EllipticPi[(2*Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]), ArcSin[Sqrt
[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))
/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*
(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*
Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))])/(2*c*f - b*g + Sqrt[(b^2 - 4*a
```

```
*c)*g^2])))/(g^2*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]))
```

Maple [A] time = 0.637, size = 645, normalized size = 1.4

$$4\,\frac{\sqrt{ex+d}\sqrt{gx+f}\sqrt{cx^2+bx+a}\left(\sqrt{-4\,ac+b^2}x^2e^2g+be^2gx^2-2\,ce^2fx^2+2\,\sqrt{-4\,ac+b^2}xdeg+2\,xbdeg-4\,xcdef+\right.}{g\left(e\sqrt{-4\,ac+b^2}+be-2\,cd\right)\sqrt{cegx^4+begx^3+cdgx^3+cefx^3+aegx^2+bdgx^2+befx^2+cdfx^2+adgx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 4*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g*((e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)*(g*x+f)/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(e*x+d))^(1/2)*((d*g-e*
f)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/(e*x+d))^
(1/2)*((d*g-e*f)*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f)/(e*x+d))^(1/2)*EllipticPi(((e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)*(g*x+f)/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(e*x+d))^(1/2),(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f)*e/g/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d),((2*c*d-b*e+e*(-4*a*c+b^2)^(1/2))*(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d))^(1/2))*((-4*a*c+b^2)^(1/2)*x^2*e^2*g+b*e^2*g*x^2-2*
c*e^2*f*x^2+2*(-4*a*c+b^2)^(1/2)*x*d*e*g+2*x*b*d*e*g-4*x*c*d*e*f+(-4*a*c+b^
2)^(1/2)*d^2*g+b*d^2*g-2*c*d^2*f)/(-1/c*(g*x+f)*(e*x+d)*(-b-2*c*x+(-4*a*c+b
^2)^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d)/(c*e*g*x^4+b*e*g*x^3+c*d*g*x^3+c*e*f*x^3+a*e*g*x^2+b*d*g*x^2+b*e*f*x^2
+c*d*f*x^2+a*d*g*x+a*e*f*x+b*d*f*x+a*d*f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

3.919 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=588

$$-\frac{(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg-b(dg+ef)+2cdf)}{(d+ex)(ag^2-bfg+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)^2}}\,\mathrm{Elli}\cdots}{\sqrt{a+bx+cx^2}(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+g\cdots}{(d\cdots}}}$$

```
[Out] -(((c*f^2 - g*(b*f - a*g))^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + b*x + c
*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 - b*d*e + a*
e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*g)]*(d + e*x)))*Sqrt[(1 - ((2*c*d*
f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g + a*g^2)*(d + e*x))
 + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f - a*g))*(d + e*x)
^2))/(1 + (Sqrt[c*d^2 - b*d*e + a*e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*
g)]*(d + e*x)))^2]*EllipticF[2*ArcTan[((c*d^2 - b*d*e + a*e^2)^(1/4)*Sqrt[f
 + g*x])/((c*f^2 - b*f*g + a*g^2)^(1/4)*Sqrt[d + e*x])], (2 + (2*c*d*f + 2*
a*e*g - b*(e*f + d*g))/(Sqrt[c*d^2 - e*(b*d - a*e)]*Sqrt[c*f^2 - g*(b*f - a
*g)]))/4])/((c*d^2 - b*d*e + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]
*Sqrt[1 - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g +
 a*g^2)*(d + e*x)) + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f
 - a*g))*(d + e*x)^2)]))
```

Rubi [A] time = 1.16692, antiderivative size = 588, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {935, 1103}

$$-\frac{(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg-b(dg+ef)+2cdf)}{(d+ex)(ag^2-bfg+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)^2}}\,F\Big(2\cdots}{\sqrt{a+bx+cx^2}(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg\cdots}{(d+ex)(\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((c*f^2 - g*(b*f - a*g))^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + b*x + c
*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 - b*d*e + a*
e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*g)]*(d + e*x)))*Sqrt[(1 - ((2*c*d*
f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g + a*g^2)*(d + e*x))
 + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f - a*g))*(d + e*x)
^2))/(1 + (Sqrt[c*d^2 - b*d*e + a*e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*
g)]*(d + e*x)))^2]*EllipticF[2*ArcTan[((c*d^2 - b*d*e + a*e^2)^(1/4)*Sqrt[f
 + g*x])/((c*f^2 - b*f*g + a*g^2)^(1/4)*Sqrt[d + e*x])], (2 + (2*c*d*f + 2*
a*e*g - b*(e*f + d*g))/(Sqrt[c*d^2 - e*(b*d - a*e)]*Sqrt[c*f^2 - g*(b*f - a
*g)]))/4])/((c*d^2 - b*d*e + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]
*Sqrt[1 - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g +
 a*g^2)*(d + e*x)) + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f
 - a*g))*(d + e*x)^2)]))
```

Rule 935

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2
*(a + b*x + c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sq
rt[a + b*x + c*x^2]), Subst[Int[1/Sqrt[1 - ((2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*x^2)/(c*f^2 - b*f*g + a*g^2) + ((c*d^2 - b*d*e + a*e^2)*x^4)/(c*f^2 -
b*f*g + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(2(d+ex)\sqrt{\frac{(ef-dg)^2\left(a+bx+cx^2\right)}{\left(cf^2-bfg+ag^2\right)(d+ex)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(2cdf-bef-bdg+2aeg)x^2}{cf^2-bfg+ag^2}+\frac{\left(cd^2-bde+ae^2\right)x^4}{cf^2-bfg+ag^2}}}\,dx, \ldots\right.}{(ef-dg)\sqrt{a+bx+cx^2}}$$

$$= -\frac{\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2\left(a+bx+cx^2\right)}{\left(cf^2-bfg+ag^2\right)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2-bde+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}\right)\sqrt{\frac{1-\frac{(2cd\ldots}{\ldots}}{\ldots}}\ldots}{\sqrt[4]{cd^2-bde+ae^2}(ef-dg)\sqrt{a+bx+\ldots}}$$

Mathematica [A] time = 3.51726, size = 375, normalized size = 0.64

$$\frac{2\sqrt{2}e\sqrt{a+x(b+cx)}\sqrt{-\frac{e(f+gx)\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(-dg\sqrt{e^2\left(b^2-4ac\right)}+ef\sqrt{e^2\left(b^2-4ac\right)}-2ae^2g+be(dg+ef)-2cdef\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{(d+ex)\sqrt{e^2\left(b^2-4ac\right)}+2ae^2+be(ex-d)-2\ldots}{(d+ex)\sqrt{e^2\left(b^2-4ac\right)}}}}{\sqrt{2}}\right)\ldots\right)}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{e^2\left(b^2-4ac\right)}\sqrt{-\frac{(a+x(b+cx))\left(e(ae-bd)+cd^2\right)}{\left(b^2-4ac\right)(d+ex)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*e*Sqrt[-((e*(c*d^2 + e*(-(b*d) + a*e))*(f + g*x))/((-2*c*d*e*f +
 e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a*c)*e^2]*g + b*
e*(e*f + d*g))*(d + e*x)))]*Sqrt[a + x*(b + c*x)]*EllipticF[ArcSin[Sqrt[(2*
a*e^2 - 2*c*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/(Sq
rt[(b^2 - 4*a*c)*e^2]*(d + e*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*e^2]*(e*f
 - d*g))/(-2*c*d*e*f + e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^
2 - 4*a*c)*e^2]*g + b*e*(e*f + d*g))])/(Sqrt[(b^2 - 4*a*c)*e^2]*Sqrt[d + e*
x]*Sqrt[f + g*x]*Sqrt[-(((c*d^2 + e*(-(b*d) + a*e))*(a + x*(b + c*x)))/((b^
2 - 4*a*c)*(d + e*x)^2))])
```

Maple [A] time = 0.471, size = 605, normalized size = 1.

$$4\frac{\left(\sqrt{-4ac+b^2}x^2e^2g+be^2gx^2-2ce^2fx^2+2\sqrt{-4ac+b^2}xdeg+2xbdeg-4xcdef+\sqrt{-4ac+b^2}d^2g+bd^2g-2cd^2f\right.}{\left(dg-ef\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)\sqrt{cegx^4+begx^3+cdgx^3+cefx^3+aegx^2+bdgx^2+befx^2+cdfx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 4*((-4*a*c+b^2)^(1/2)*x^2*e^2*g+b*e^2*g*x^2-2*c*e^2*f*x^2+2*(-4*a*c+b^2)^(1
/2)*x*d*e*g+2*x*b*d*e*g-4*x*c*d*e*f+(-4*a*c+b^2)^(1/2)*d^2*g+b*d^2*g-2*c*d^
2*f)*EllipticF(((e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)*(g*x+f)/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f)/(e*x+d))^(1/2),((2*c*d-b*e+e*(-4*a*c+b^2)^(1/2))*(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d))^(1/2))*((d*g-e*f)*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f)/(e*x+d))^(1/2)*((d*g-e*f)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/(e*x+d))^(1/2)*((e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d)*(g*x+f)/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(e*x+d))^(1/2)*(e*x+d)^(1/2
)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-1/c*(g*x+f)*(e*x+d)*(-b-2*c*x+(-4*a*c
+b^2)^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)/(d*g-e*f)/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d)/(c*e*g*x^4+b*e*g*x^3+c*d*g*x^3+c*e*f*x^3+a*e*g*x^2+b*d*g*x
^2+b*e*f*x^2+c*d*f*x^2+a*d*g*x+a*e*f*x+b*d*f*x+a*d*f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}{cegx^4+\left(cef+(cd+be)g\right)x^3+adf+\left((cd+be)f+(bd+ae)g\right)x^2+\left(adg+(bd+ae)f\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c*
e*f + (c*d + b*e)*g)*x^3 + a*d*f + ((c*d + b*e)*f + (b*d + a*e)*g)*x^2 + (a
*d*g + (b*d + a*e)*f)*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

3.920 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=220

$$\frac{(d+ex)^{m+3}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(ef-d\ldots}{}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (
(e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*x)
^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 -
6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f - 4*
c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5 + m))
/(e^5*(5 + m))
```

Rubi [A] time = 0.215489, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {947}

$$\frac{(d+ex)^{m+3}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(ef-d\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (
(e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*x)
^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 -
6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f - 4*
c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5 + m))
/(e^5*(5 + m))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^m}{e^4}+\frac{(ef-dg)(-2cd(ef-2dg)+e(b\ldots}{e\ldots}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^{1+m}}{e^5(1+m)}-\frac{(ef-dg)(2cd(ef-2dg)-e(bef\ldots}{e^5(2+\ldots}$$

Mathematica [A] time = 0.309542, size = 198, normalized size = 0.9

$$\frac{(d+ex)^{m+1}\left(\frac{(d+ex)^2\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{m+3}+\frac{(ef-dg)^2\left(e(ae-bd)+cd^2\right)}{m+1}+\frac{(d+ex)(ef-dg)(e(2aeg-3bdg+bef)+2cd(2dg-ef))}{m+2}+\ldots\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(((c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)^2)/(1 + m) + ((
e*f - d*g)*(2*c*d*(-(e*f) + 2*d*g) + e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*
x))/(2 + m) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 - 6*d*e*f*g +
6*d^2*g^2))*(d + e*x)^2)/(3 + m) + (g*(2*c*e*f - 4*c*d*g + b*e*g)*(d + e*x)
^3)/(4 + m) + (c*g^2*(d + e*x)^4)/(5 + m)))/e^5
```

Maple [B] time = 0.062, size = 1347, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x)
```

```
[Out] (e*x+d)^(1+m)*(c*e^4*g^2*m^4*x^4+b*e^4*g^2*m^4*x^3+2*c*e^4*f*g*m^4*x^3+10*c
*e^4*g^2*m^3*x^4+a*e^4*g^2*m^4*x^2+2*b*e^4*f*g*m^4*x^2+11*b*e^4*g^2*m^3*x^3
-4*c*d*e^3*g^2*m^3*x^3+c*e^4*f^2*m^4*x^2+22*c*e^4*f*g*m^3*x^3+35*c*e^4*g^2*
m^2*x^4+2*a*e^4*f*g*m^4*x+12*a*e^4*g^2*m^3*x^2-3*b*d*e^3*g^2*m^3*x^2+b*e^4*
f^2*m^4*x+24*b*e^4*f*g*m^3*x^2+41*b*e^4*g^2*m^2*x^3-6*c*d*e^3*f*g*m^3*x^2-2
4*c*d*e^3*g^2*m^2*x^3+12*c*e^4*f^2*m^3*x^2+82*c*e^4*f*g*m^2*x^3+50*c*e^4*g^
2*m*x^4-2*a*d*e^3*g^2*m^3*x+a*e^4*f^2*m^4+26*a*e^4*f*g*m^3*x+49*a*e^4*g^2*m
^2*x^2-4*b*d*e^3*f*g*m^3*x-24*b*d*e^3*g^2*m^2*x^2+13*b*e^4*f^2*m^3*x+98*b*e
^4*f*g*m^2*x^2+61*b*e^4*g^2*m*x^3+12*c*d^2*e^2*g^2*m^2*x^2-2*c*d*e^3*f^2*m^
3*x-48*c*d*e^3*f*g*m^2*x^2-44*c*d*e^3*g^2*m*x^3+49*c*e^4*f^2*m^2*x^2+122*c*
e^4*f*g*m*x^3+24*c*e^4*g^2*x^4-2*a*d*e^3*f*g*m^3-20*a*d*e^3*g^2*m^2*x+14*a*
e^4*f^2*m^3+118*a*e^4*f*g*m^2*x+78*a*e^4*g^2*m*x^2+6*b*d^2*e^2*g^2*m^2*x-b*
d*e^3*f^2*m^3-40*b*d*e^3*f*g*m^2*x-51*b*d*e^3*g^2*m*x^2+59*b*e^4*f^2*m^2*x+
156*b*e^4*f*g*m*x^2+30*b*e^4*g^2*x^3+12*c*d^2*e^2*f*g*m^2*x+36*c*d^2*e^2*g^
2*m*x^2-20*c*d*e^3*f^2*m^2*x-102*c*d*e^3*f*g*m*x^2-24*c*d*e^3*g^2*x^3+78*c*
e^4*f^2*m*x^2+60*c*e^4*f*g*x^3+2*a*d^2*e^2*g^2*m^2-24*a*d*e^3*f*g*m^2-58*a*
d*e^3*g^2*m*x+71*a*e^4*f^2*m^2+214*a*e^4*f*g*m*x+40*a*e^4*g^2*x^2+4*b*d^2*e
^2*f*g*m^2+36*b*d^2*e^2*g^2*m*x-12*b*d*e^3*f^2*m^2-116*b*d*e^3*f*g*m*x-30*b
*d*e^3*g^2*x^2+107*b*e^4*f^2*m*x+80*b*e^4*f*g*x^2-24*c*d^3*e*g^2*m*x+2*c*d^
2*e^2*f^2*m^2+72*c*d^2*e^2*f*g*m*x+24*c*d^2*e^2*g^2*x^2-58*c*d*e^3*f^2*m*x-
60*c*d*e^3*f*g*x^2+40*c*e^4*f^2*x^2+18*a*d^2*e^2*g^2*m-94*a*d*e^3*f*g*m-40*
a*d*e^3*g^2*x+154*a*e^4*f^2*m+120*a*e^4*f*g*x-6*b*d^3*e*g^2*m+36*b*d^2*e^2*
f*g*m+30*b*d^2*e^2*g^2*x-47*b*d*e^3*f^2*m-80*b*d*e^3*f*g*x+60*b*e^4*f^2*x-1
2*c*d^3*e*f*g*m-24*c*d^3*e*g^2*x+18*c*d^2*e^2*f^2*m+60*c*d^2*e^2*f*g*x-40*c
*d*e^3*f^2*x+40*a*d^2*e^2*g^2-120*a*d*e^3*f*g+120*a*e^4*f^2-30*b*d^3*e*g^2+
80*b*d^2*e^2*f*g-60*b*d*e^3*f^2+24*c*d^4*g^2-60*c*d^3*e*f*g+40*c*d^2*e^2*f^
2)/e^5/(m^5+15*m^4+85*m^3+225*m^2+274*m+120)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35803, size = 2974, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^4*f^2*m^4 + (c*e^5*g^2*m^4 + 10*c*e^5*g^2*m^3 + 35*c*e^5*g^2*m^2 + 5
0*c*e^5*g^2*m + 24*c*e^5*g^2)*x^5 + (60*c*e^5*f*g + 30*b*e^5*g^2 + (2*c*e^5
*f*g + (c*d*e^4 + b*e^5)*g^2)*m^4 + (22*c*e^5*f*g + (6*c*d*e^4 + 11*b*e^5)*
g^2)*m^3 + (82*c*e^5*f*g + (11*c*d*e^4 + 41*b*e^5)*g^2)*m^2 + (122*c*e^5*f*
g + (6*c*d*e^4 + 61*b*e^5)*g^2)*m)*x^4 - (2*a*d^2*e^3*f*g + (b*d^2*e^3 - 14
*a*d*e^4)*f^2)*m^3 + (40*c*e^5*f^2 + 80*b*e^5*f*g + 40*a*e^5*g^2 + (c*e^5*f
^2 + 2*(c*d*e^4 + b*e^5)*f*g + (b*d*e^4 + a*e^5)*g^2)*m^4 + 4*(3*c*e^5*f^2
+ 2*(2*c*d*e^4 + 3*b*e^5)*f*g - (c*d^2*e^3 - 2*b*d*e^4 - 3*a*e^5)*g^2)*m^3
+ (49*c*e^5*f^2 + 2*(17*c*d*e^4 + 49*b*e^5)*f*g - (12*c*d^2*e^3 - 17*b*d*e^
4 - 49*a*e^5)*g^2)*m^2 + 2*(39*c*e^5*f^2 + 2*(5*c*d*e^4 + 39*b*e^5)*f*g - (
4*c*d^2*e^3 - 5*b*d*e^4 - 39*a*e^5)*g^2)*m)*x^3 + 20*(2*c*d^3*e^2 - 3*b*d^2
*e^3 + 6*a*d*e^4)*f^2 - 20*(3*c*d^4*e - 4*b*d^3*e^2 + 6*a*d^2*e^3)*f*g + 2*
(12*c*d^5 - 15*b*d^4*e + 20*a*d^3*e^2)*g^2 + (2*a*d^3*e^2*g^2 + (2*c*d^3*e^
2 - 12*b*d^2*e^3 + 71*a*d*e^4)*f^2 + 4*(b*d^3*e^2 - 6*a*d^2*e^3)*f*g)*m^2 +
 (60*b*e^5*f^2 + 120*a*e^5*f*g + (a*d*e^4*g^2 + (c*d*e^4 + b*e^5)*f^2 + 2*(
b*d*e^4 + a*e^5)*f*g)*m^4 + ((10*c*d*e^4 + 13*b*e^5)*f^2 - 2*(3*c*d^2*e^3 -
 10*b*d*e^4 - 13*a*e^5)*f*g - (3*b*d^2*e^3 - 10*a*d*e^4)*g^2)*m^3 + ((29*c*
d*e^4 + 59*b*e^5)*f^2 - 2*(18*c*d^2*e^3 - 29*b*d*e^4 - 59*a*e^5)*f*g + (12*
c*d^3*e^2 - 18*b*d^2*e^3 + 29*a*d*e^4)*g^2)*m^2 + ((20*c*d*e^4 + 107*b*e^5)
*f^2 - 2*(15*c*d^2*e^3 - 20*b*d*e^4 - 107*a*e^5)*f*g + (12*c*d^3*e^2 - 15*b
*d^2*e^3 + 20*a*d*e^4)*g^2)*m)*x^2 + ((18*c*d^3*e^2 - 47*b*d^2*e^3 + 154*a*
d*e^4)*f^2 - 2*(6*c*d^4*e - 18*b*d^3*e^2 + 47*a*d^2*e^3)*f*g - 6*(b*d^4*e -
 3*a*d^3*e^2)*g^2)*m + (120*a*e^5*f^2 + (2*a*d*e^4*f*g + (b*d*e^4 + a*e^5)*
f^2)*m^4 - 2*(a*d^2*e^3*g^2 + (c*d^2*e^3 - 6*b*d*e^4 - 7*a*e^5)*f^2 + 2*(b*
d^2*e^3 - 6*a*d*e^4)*f*g)*m^3 - ((18*c*d^2*e^3 - 47*b*d*e^4 - 71*a*e^5)*f^2
 - 2*(6*c*d^3*e^2 - 18*b*d^2*e^3 + 47*a*d*e^4)*f*g - 6*(b*d^3*e^2 - 3*a*d^2
*e^3)*g^2)*m^2 - 2*((20*c*d^2*e^3 - 30*b*d*e^4 - 77*a*e^5)*f^2 - 10*(3*c*d^
3*e^2 - 4*b*d^2*e^3 + 6*a*d*e^4)*f*g + (12*c*d^4*e - 15*b*d^3*e^2 + 20*a*d^
2*e^3)*g^2)*m)*x)*(e*x + d)^m/(e^5*m^5 + 15*e^5*m^4 + 85*e^5*m^3 + 225*e^5*
m^2 + 274*e^5*m + 120*e^5)
```

Sympy [A] time = 17.642, size = 15144, normalized size = 68.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + b*f**2*x**2/2 + 2*
b*f*g*x**3/3 + b*g**2*x**4/4 + c*f**2*x**3/3 + c*f*g*x**4/2 + c*g**2*x**5/5
), Eq(e, 0)), (-2*a*d**3*e**3*f*g/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*
e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) - 3*a*d**2*e**4*f**2/(12
*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d*
```

```
*2*e**9*x**4) - 8*a*d**2*e**4*f*g*x/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**
4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 4*a*d*e**5*g**2*x**3
/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 1
2*d**2*e**9*x**4) + a*e**6*g**2*x**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d*
*4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) - b*d**3*e**3*f**2/(1
2*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d
**2*e**9*x**4) - 4*b*d**2*e**4*f**2*x/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d
**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 8*b*d*e**5*f*g*x**
3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 +
12*d**2*e**9*x**4) + 3*b*d*e**5*g**2*x**4/(12*d**6*e**5 + 48*d**5*e**6*x +
72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 2*b*e**6*f*g*x
**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3
+ 12*d**2*e**9*x**4) + 12*c*d**6*g**2*log(d/e + x)/(12*d**6*e**5 + 48*d**5*
e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 7*c*d
**6*g**2/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*
x**3 + 12*d**2*e**9*x**4) + 48*c*d**5*e*g**2*x*log(d/e + x)/(12*d**6*e**5 +
 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4
) + 16*c*d**5*e*g**2*x/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 +
 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 72*c*d**4*e**2*g**2*x**2*log(d/e
+ x)/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3
 + 12*d**2*e**9*x**4) + 48*c*d**3*e**3*g**2*x**3*log(d/e + x)/(12*d**6*e**5
 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x*
*4) - 24*c*d**3*e**3*g**2*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**
7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 12*c*d**2*e**4*g**2*x**4*
log(d/e + x)/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e
**8*x**3 + 12*d**2*e**9*x**4) - 18*c*d**2*e**4*g**2*x**4/(12*d**6*e**5 + 48
*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) +
 4*c*d*e**5*f**2*x**3/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 +
48*d**3*e**8*x**3 + 12*d**2*e**9*x**4) + 6*c*d*e**5*f*g*x**4/(12*d**6*e**5
+ 48*d**5*e**6*x + 72*d**4*e**7*x**2 + 48*d**3*e**8*x**3 + 12*d**2*e**9*x**
4) + c*e**6*f**2*x**4/(12*d**6*e**5 + 48*d**5*e**6*x + 72*d**4*e**7*x**2 +
48*d**3*e**8*x**3 + 12*d**2*e**9*x**4), Eq(m, -5)), (-2*a*d**2*e**3*f*g/(6*
d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) - 2*a*d*e**
4*f**2/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) -
 6*a*d*e**4*f*g*x/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e
**8*x**3) + 2*a*e**5*g**2*x**3/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7
*x**2 + 6*d*e**8*x**3) + 6*b*d**4*e*g**2*log(d/e + x)/(6*d**4*e**5 + 18*d**
3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) + 5*b*d**4*e*g**2/(6*d**4*e**
5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) + 18*b*d**3*e**2*g*
*2*x*log(d/e + x)/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e
**8*x**3) + 9*b*d**3*e**2*g**2*x/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e*
*7*x**2 + 6*d*e**8*x**3) - b*d**2*e**3*f**2/(6*d**4*e**5 + 18*d**3*e**6*x +
 18*d**2*e**7*x**2 + 6*d*e**8*x**3) + 18*b*d**2*e**3*g**2*x**2*log(d/e + x)
/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) - 3*b*d
*e**4*f**2*x/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x
**3) + 6*b*d*e**4*g**2*x**3*log(d/e + x)/(6*d**4*e**5 + 18*d**3*e**6*x + 18
*d**2*e**7*x**2 + 6*d*e**8*x**3) - 6*b*d*e**4*g**2*x**3/(6*d**4*e**5 + 18*d
**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) + 4*b*e**5*f*g*x**3/(6*d**4
*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) - 24*c*d**5*g**
2*log(d/e + x)/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8
*x**3) - 20*c*d**5*g**2/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 +
 6*d*e**8*x**3) + 12*c*d**4*e*f*g*log(d/e + x)/(6*d**4*e**5 + 18*d**3*e**6*
x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) + 10*c*d**4*e*f*g/(6*d**4*e**5 + 18*
d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) - 72*c*d**4*e*g**2*x*log(d
/e + x)/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3)
- 36*c*d**4*e*g**2*x/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*
d*e**8*x**3) + 36*c*d**3*e**2*f*g*x*log(d/e + x)/(6*d**4*e**5 + 18*d**3*e**
6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) + 18*c*d**3*e**2*f*g*x/(6*d**4*e**
5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) - 72*c*d**3*e**2*g*
```

```
*2*x**2*log(d/e + x)/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*
d*e**8*x**3) + 36*c*d**2*e**3*f*g*x**2*log(d/e + x)/(6*d**4*e**5 + 18*d**3*
e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) - 24*c*d**2*e**3*g**2*x**3*log(
d/e + x)/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3)
 + 24*c*d**2*e**3*g**2*x**3/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x*
*2 + 6*d*e**8*x**3) + 12*c*d*e**4*f*g*x**3*log(d/e + x)/(6*d**4*e**5 + 18*d
**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) - 12*c*d*e**4*f*g*x**3/(6*d
**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**3) + 6*c*d*e**4
*g**2*x**4/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 + 6*d*e**8*x**
3) + 2*c*e**5*f**2*x**3/(6*d**4*e**5 + 18*d**3*e**6*x + 18*d**2*e**7*x**2 +
 6*d*e**8*x**3), Eq(m, -4)), (2*a*d**2*e**2*g**2*log(d/e + x)/(2*d**2*e**5
+ 4*d*e**6*x + 2*e**7*x**2) + 3*a*d**2*e**2*g**2/(2*d**2*e**5 + 4*d*e**6*x
+ 2*e**7*x**2) - 2*a*d*e**3*f*g/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) +
4*a*d*e**3*g**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4
*a*d*e**3*g**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - a*e**4*f**2/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*a*e**4*f*g*x/(2*d**2*e**5 + 4*d*e
**6*x + 2*e**7*x**2) + 2*a*e**4*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e
**6*x + 2*e**7*x**2) - 6*b*d**3*e*g**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6
*x + 2*e**7*x**2) - 9*b*d**3*e*g**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2
) + 4*b*d**2*e**2*f*g*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 + 6*b*d**2*e**2*f*g/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*d**2*e
**2*g**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*b*d**
2*e**2*g**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - b*d*e**3*f**2/(2*d
**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*b*d*e**3*f*g*x*log(d/e + x)/(2*d**
2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 8*b*d*e**3*f*g*x/(2*d**2*e**5 + 4*d*e*
*6*x + 2*e**7*x**2) - 6*b*d*e**3*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*
e**6*x + 2*e**7*x**2) - 2*b*e**4*f**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*
x**2) + 4*b*e**4*f*g*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x
**2) + 2*b*e**4*g**2*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*c*d
**4*g**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 18*c*d**4*
g**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*c*d**3*e*f*g*log(d/e + x
)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 18*c*d**3*e*f*g/(2*d**2*e**5 +
 4*d*e**6*x + 2*e**7*x**2) + 24*c*d**3*e*g**2*x*log(d/e + x)/(2*d**2*e**5 +
 4*d*e**6*x + 2*e**7*x**2) + 24*c*d**3*e*g**2*x/(2*d**2*e**5 + 4*d*e**6*x +
 2*e**7*x**2) + 2*c*d**2*e**2*f**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x +
 2*e**7*x**2) + 3*c*d**2*e**2*f**2/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2)
 - 24*c*d**2*e**2*f*g*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**
2) - 24*c*d**2*e**2*f*g*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 12*c*d
**2*e**2*g**2*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) +
4*c*d*e**3*f**2*x*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4
*c*d*e**3*f**2*x/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 12*c*d*e**3*f*g
*x**2*log(d/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) - 4*c*d*e**3*g*
*2*x**3/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 2*c*e**4*f**2*x**2*log(d
/e + x)/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + 4*c*e**4*f*g*x**3/(2*d**
2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + c*e**4*g**2*x**4/(2*d**2*e**5 + 4*d*e*
*6*x + 2*e**7*x**2), Eq(m, -3)), (-12*a*d**2*e**2*g**2*log(d/e + x)/(6*d*e*
*5 + 6*e**6*x) - 12*a*d**2*e**2*g**2/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3*f*
g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3*f*g/(6*d*e**5 + 6*e**6*x
) - 12*a*d*e**3*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 6*a*e**4*f**2/(
6*d*e**5 + 6*e**6*x) + 12*a*e**4*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) +
 6*a*e**4*g**2*x**2/(6*d*e**5 + 6*e**6*x) + 18*b*d**3*e*g**2*log(d/e + x)/(
6*d*e**5 + 6*e**6*x) + 18*b*d**3*e*g**2/(6*d*e**5 + 6*e**6*x) - 24*b*d**2*e
**2*f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 24*b*d**2*e**2*f*g/(6*d*e**5 +
 6*e**6*x) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 6*b
*d*e**3*f**2*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 6*b*d*e**3*f**2/(6*d*e**5
 + 6*e**6*x) - 24*b*d*e**3*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 9*b*d
*e**3*g**2*x**2/(6*d*e**5 + 6*e**6*x) + 6*b*e**4*f**2*x*log(d/e + x)/(6*d*e
**5 + 6*e**6*x) + 12*b*e**4*f*g*x**2/(6*d*e**5 + 6*e**6*x) + 3*b*e**4*g**2*
x**3/(6*d*e**5 + 6*e**6*x) - 24*c*d**4*g**2*log(d/e + x)/(6*d*e**5 + 6*e**6
```

```
*x) - 24*c*d**4*g**2/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g*log(d/e + x)/(
6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g/(6*d*e**5 + 6*e**6*x) - 24*c*d**3*e*
g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 12*c*d**2*e**2*f**2*log(d/e + x
)/(6*d*e**5 + 6*e**6*x) - 12*c*d**2*e**2*f**2/(6*d*e**5 + 6*e**6*x) + 36*c*
d**2*e**2*f*g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*c*d**2*e**2*g**2*x*
*2/(6*d*e**5 + 6*e**6*x) - 12*c*d*e**3*f**2*x*log(d/e + x)/(6*d*e**5 + 6*e*
*6*x) - 18*c*d*e**3*f*g*x**2/(6*d*e**5 + 6*e**6*x) - 4*c*d*e**3*g**2*x**3/(
6*d*e**5 + 6*e**6*x) + 6*c*e**4*f**2*x**2/(6*d*e**5 + 6*e**6*x) + 6*c*e**4*
f*g*x**3/(6*d*e**5 + 6*e**6*x) + 2*c*e**4*g**2*x**4/(6*d*e**5 + 6*e**6*x),
Eq(m, -2)), (a*d**2*g**2*log(d/e + x)/e**3 - 2*a*d*f*g*log(d/e + x)/e**2 -
a*d*g**2*x/e**2 + a*f**2*log(d/e + x)/e + 2*a*f*g*x/e + a*g**2*x**2/(2*e) -
 b*d**3*g**2*log(d/e + x)/e**4 + 2*b*d**2*f*g*log(d/e + x)/e**3 + b*d**2*g*
*2*x/e**3 - b*d*f**2*log(d/e + x)/e**2 - 2*b*d*f*g*x/e**2 - b*d*g**2*x**2/(
2*e**2) + b*f**2*x/e + b*f*g*x**2/e + b*g**2*x**3/(3*e) + c*d**4*g**2*log(d
/e + x)/e**5 - 2*c*d**3*f*g*log(d/e + x)/e**4 - c*d**3*g**2*x/e**4 + c*d**2
*f**2*log(d/e + x)/e**3 + 2*c*d**2*f*g*x/e**3 + c*d**2*g**2*x**2/(2*e**3) -
 c*d*f**2*x/e**2 - c*d*f*g*x**2/e**2 - c*d*g**2*x**3/(3*e**2) + c*f**2*x**2
/(2*e) + 2*c*f*g*x**3/(3*e) + c*g**2*x**4/(4*e), Eq(m, -1)), (2*a*d**3*e**2
*g**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 18*a*d**3*e**2*g**2*m*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
+ 40*a*d**3*e**2*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 2*a*d**2*e**3*f*g*m**3*(d + e*x
)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
+ 120*e**5) - 24*a*d**2*e**3*f*g*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 94*a*d**2*e**3*
f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
 + 274*e**5*m + 120*e**5) - 120*a*d**2*e**3*f*g*(d + e*x)**m/(e**5*m**5 + 1
5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 2*a*d
**2*e**3*g**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 18*a*d**2*e**3*g**2*m**2*x*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) - 40*a*d**2*e**3*g**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + a*d*e**4*f**
2*m**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**
2 + 274*e**5*m + 120*e**5) + 14*a*d*e**4*f**2*m**3*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71
*a*d*e**4*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 154*a*d*e**4*f**2*m*(d + e*x)**m/
(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120
*e**5) + 120*a*d*e**4*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*a*d*e**4*f*g*m**4*x*(d +
 e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**
5*m + 120*e**5) + 24*a*d*e**4*f*g*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 94*a*d*e**4*
f*g*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5
*m**2 + 274*e**5*m + 120*e**5) + 120*a*d*e**4*f*g*m*x*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 a*d*e**4*g**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m
**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*a*d*e**4*g**2*m**3*x**2*(
d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*
e**5*m + 120*e**5) + 29*a*d*e**4*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 1
5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*a*
d*e**4*g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
225*e**5*m**2 + 274*e**5*m + 120*e**5) + a*e**5*f**2*m**4*x*(d + e*x)**m/(e
**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e
**5) + 14*a*e**5*f**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e*
*5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71*a*e**5*f**2*m**2*x*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
```

```
**5*m + 120*e**5) + 154*a*e**5*f**2*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m
**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*e**5*f*
*2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) + 2*a*e**5*f*g*m**4*x**2*(d + e*x)**m/(e**5*m**5 +
 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 26*
a*e**5*f*g*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 118*a*e**5*f*g*m**2*x**2*(d + e*
x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
 + 120*e**5) + 214*a*e**5*f*g*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 120*a*e**5*f*g*x
**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
 274*e**5*m + 120*e**5) + a*e**5*g**2*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 1
5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*a*
e**5*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 49*a*e**5*g**2*m**2*x**3*(d + e*x
)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
+ 120*e**5) + 78*a*e**5*g**2*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
+ 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*a*e**5*g**2*x*
*3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
274*e**5*m + 120*e**5) - 6*b*d**4*e*g**2*m*(d + e*x)**m/(e**5*m**5 + 15*e**
5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*b*d**4*
e*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**
2 + 274*e**5*m + 120*e**5) + 4*b*d**3*e**2*f*g*m**2*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 3
6*b*d**3*e**2*f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 80*b*d**3*e**2*f*g*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 6*b*d**3*e**2*g**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 30*b*d**3*e**2*g**2
*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) - b*d**2*e**3*f**2*m**3*(d + e*x)**m/(e**5*m**5 +
15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 12*b
*d**2*e**3*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
+ 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 47*b*d**2*e**3*f**2*m*(d + e*x)*
*m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m +
120*e**5) - 60*b*d**2*e**3*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85
*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 4*b*d**2*e**3*f*g*m**
3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 +
 274*e**5*m + 120*e**5) - 36*b*d**2*e**3*f*g*m**2*x*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 8
0*b*d**2*e**3*f*g*m*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 3*b*d**2*e**3*g**2*m**3*x**2*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) - 18*b*d**2*e**3*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 +
 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 15*
b*d**2*e**3*g**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b*d*e**4*f**2*m**4*x*(d + e*x
)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
+ 120*e**5) + 12*b*d*e**4*f**2*m**3*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 47*b*d*e**4*f**
2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
**2 + 274*e**5*m + 120*e**5) + 60*b*d*e**4*f**2*m*x*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2
*b*d*e**4*f*g*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*b*d*e**4*f*g*m**3*x**2*(d
+ e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e*
*5*m + 120*e**5) + 58*b*d*e**4*f*g*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e
**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 40*b*d*e
**4*f*g*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
```

```
e**5*m**2 + 274*e**5*m + 120*e**5) + b*d*e**4*g**2*m**4*x**3*(d + e*x)**m/(
e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*
e**5) + 8*b*d*e**4*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 17*b*d*e**4*g**2*m*
*2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 10*b*d*e**4*g**2*m*x**3*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 b*e**5*f**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**
3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 13*b*e**5*f**2*m**3*x**2*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 59*b*e**5*f**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 107*b*e**5*
f**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) + 60*b*e**5*f**2*x**2*(d + e*x)**m/(e**5*m*
*5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) +
 2*b*e**5*f*g*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*b*e**5*f*g*m**3*x**3*(d +
e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5
*m + 120*e**5) + 98*b*e**5*f*g*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*
m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 156*b*e**5*f
*g*m*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*
m**2 + 274*e**5*m + 120*e**5) + 80*b*e**5*f*g*x**3*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + b*
e**5*g**2*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 +
 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*b*e**5*g**2*m**3*x**4*(d + e*x
)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m
+ 120*e**5) + 41*b*e**5*g**2*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m*
*4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 61*b*e**5*g**2
*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 30*b*e**5*g**2*x**4*(d + e*x)**m/(e**5*m**5 +
 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 24*
c*d**5*g**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) - 12*c*d**4*e*f*g*m*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 6
0*c*d**4*e*f*g*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*
e**5*m**2 + 274*e**5*m + 120*e**5) - 24*c*d**4*e*g**2*m*x*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 2*c*d**3*e**2*f**2*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e*
*5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 18*c*d**3*e**2*f**2*m*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) + 40*c*d**3*e**2*f**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m
**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*d**3*e**
2*f*g*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e*
*5*m**2 + 274*e**5*m + 120*e**5) + 60*c*d**3*e**2*f*g*m*x*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 12*c*d**3*e**2*g**2*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 12*c*d**3*e**2*g**
2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m
**2 + 274*e**5*m + 120*e**5) - 2*c*d**2*e**3*f**2*m**3*x*(d + e*x)**m/(e**5
*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5
) - 18*c*d**2*e**3*f**2*m**2*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 40*c*d**2*e**3*f**2*m*
x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 2
74*e**5*m + 120*e**5) - 6*c*d**2*e**3*f*g*m**3*x**2*(d + e*x)**m/(e**5*m**5
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 3
6*c*d**2*e**3*f*g*m**2*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**
5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 30*c*d**2*e**3*f*g*m*x**2
*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 27
4*e**5*m + 120*e**5) - 4*c*d**2*e**3*g**2*m**3*x**3*(d + e*x)**m/(e**5*m**5
```

```
 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 1
2*c*d**2*e**3*g**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e*
*5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 8*c*d**2*e**3*g**2*m*x**
3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 2
74*e**5*m + 120*e**5) + c*d*e**4*f**2*m**4*x**2*(d + e*x)**m/(e**5*m**5 + 1
5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 10*c*
d*e**4*f**2*m**3*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3
 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 29*c*d*e**4*f**2*m**2*x**2*(d +
 e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**
5*m + 120*e**5) + 20*c*d*e**4*f**2*m*x**2*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*c*d*e**4*
f*g*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e
**5*m**2 + 274*e**5*m + 120*e**5) + 16*c*d*e**4*f*g*m**3*x**3*(d + e*x)**m/
(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120
*e**5) + 34*c*d*e**4*f*g*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 +
 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 20*c*d*e**4*f*g*m*
x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2
+ 274*e**5*m + 120*e**5) + c*d*e**4*g**2*m**4*x**4*(d + e*x)**m/(e**5*m**5
+ 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 6*
c*d*e**4*g**2*m**3*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m*
*3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 11*c*d*e**4*g**2*m**2*x**4*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) + 6*c*d*e**4*g**2*m*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**
5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c*e**5*f**
2*m**4*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) + 12*c*e**5*f**2*m**3*x**3*(d + e*x)**m/(e*
*5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e*
*5) + 49*c*e**5*f**2*m**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*
e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 78*c*e**5*f**2*m*x**3*
(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274
*e**5*m + 120*e**5) + 40*c*e**5*f**2*x**3*(d + e*x)**m/(e**5*m**5 + 15*e**5
*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 2*c*e**5*f*
g*m**4*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**
5*m**2 + 274*e**5*m + 120*e**5) + 22*c*e**5*f*g*m**3*x**4*(d + e*x)**m/(e**
5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**
5) + 82*c*e**5*f*g*m**2*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e*
*5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 122*c*e**5*f*g*m*x**4*(d
 + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e
**5*m + 120*e**5) + 60*c*e**5*f*g*x**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m*
*4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + c*e**5*g**2*m*
*4*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m*
*2 + 274*e**5*m + 120*e**5) + 10*c*e**5*g**2*m**3*x**5*(d + e*x)**m/(e**5*m
**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5)
+ 35*c*e**5*g**2*m**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5
*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 50*c*e**5*g**2*m*x**5*(d +
 e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**
5*m + 120*e**5) + 24*c*e**5*g**2*x**5*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**
4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5), True))
```

Giac [B] time = 1.23944, size = 3699, normalized size = 16.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="giac")
```

[Out] ((x*e + d)^m*c*g^2*m^4*x^5*e^5 + (x*e + d)^m*c*d*g^2*m^4*x^4*e^4 + 2*(x*e + d)^m*c*f*g*m^4*x^4*e^5 + (x*e + d)^m*b*g^2*m^4*x^4*e^5 + 10*(x*e + d)^m*c*g^2*m^3*x^5*e^5 + 2*(x*e + d)^m*c*d*f*g*m^4*x^3*e^4 + (x*e + d)^m*b*d*g^2*m^4*x^3*e^4 + 6*(x*e + d)^m*c*d*g^2*m^3*x^4*e^4 - 4*(x*e + d)^m*c*d^2*g^2*m^3*x^3*e^3 + (x*e + d)^m*c*f^2*m^4*x^3*e^5 + 2*(x*e + d)^m*b*f*g*m^4*x^3*e^5 + (x*e + d)^m*a*g^2*m^4*x^3*e^5 + 22*(x*e + d)^m*c*f*g*m^3*x^4*e^5 + 11*(x*e + d)^m*b*g^2*m^3*x^4*e^5 + 35*(x*e + d)^m*c*g^2*m^2*x^5*e^5 + (x*e + d)^m*c*d*f^2*m^4*x^2*e^4 + 2*(x*e + d)^m*b*d*f*g*m^4*x^2*e^4 + (x*e + d)^m*a*d*g^2*m^4*x^2*e^4 + 16*(x*e + d)^m*c*d*f*g*m^3*x^3*e^4 + 8*(x*e + d)^m*b*d*g^2*m^3*x^3*e^4 + 11*(x*e + d)^m*c*d*g^2*m^2*x^4*e^4 - 6*(x*e + d)^m*c*d^2*f*g*m^3*x^2*e^3 - 3*(x*e + d)^m*b*d^2*g^2*m^3*x^2*e^3 - 12*(x*e + d)^m*c*d^2*g^2*m^2*x^3*e^3 + 12*(x*e + d)^m*c*d^3*g^2*m^2*x^2*e^2 + (x*e + d)^m*b*f^2*m^4*x^2*e^5 + 2*(x*e + d)^m*a*f*g*m^4*x^2*e^5 + 12*(x*e + d)^m*c*f^2*m^3*x^3*e^5 + 24*(x*e + d)^m*b*f*g*m^3*x^3*e^5 + 12*(x*e + d)^m*a*g^2*m^3*x^3*e^5 + 82*(x*e + d)^m*c*f*g*m^2*x^4*e^5 + 41*(x*e + d)^m*b*g^2*m^2*x^4*e^5 + 50*(x*e + d)^m*c*g^2*m*x^5*e^5 + (x*e + d)^m*b*d*f^2*m^4*x*e^4 + 2*(x*e + d)^m*a*d*f*g*m^4*x*e^4 + 10*(x*e + d)^m*c*d*f^2*m^3*x^2*e^4 + 20*(x*e + d)^m*b*d*f*g*m^3*x^2*e^4 + 10*(x*e + d)^m*a*d*g^2*m^3*x^2*e^4 + 34*(x*e + d)^m*c*d*f*g*m^2*x^3*e^4 + 17*(x*e + d)^m*b*d*g^2*m^2*x^3*e^4 + 6*(x*e + d)^m*c*d*g^2*m*x^4*e^4 - 2*(x*e + d)^m*c*d^2*f^2*m^3*x*e^3 - 4*(x*e + d)^m*b*d^2*f*g*m^3*x*e^3 - 2*(x*e + d)^m*a*d^2*g^2*m^3*x*e^3 - 36*(x*e + d)^m*c*d^2*f*g*m^2*x^2*e^3 - 18*(x*e + d)^m*b*d^2*g^2*m^2*x^2*e^3 - 8*(x*e + d)^m*c*d^2*g^2*m*x^3*e^3 + 12*(x*e + d)^m*c*d^3*f*g*m^2*x*e^2 + 6*(x*e + d)^m*b*d^3*g^2*m^2*x*e^2 + 12*(x*e + d)^m*c*d^3*g^2*m*x^2*e^2 - 24*(x*e + d)^m*c*d^4*g^2*m*x*e + (x*e + d)^m*a*f^2*m^4*x*e^5 + 13*(x*e + d)^m*b*f^2*m^3*x^2*e^5 + 26*(x*e + d)^m*a*f*g*m^3*x^2*e^5 + 49*(x*e + d)^m*c*f^2*m^2*x^3*e^5 + 98*(x*e + d)^m*b*f*g*m^2*x^3*e^5 + 49*(x*e + d)^m*a*g^2*m^2*x^3*e^5 + 122*(x*e + d)^m*c*f*g*m*x^4*e^5 + 61*(x*e + d)^m*b*g^2*m*x^4*e^5 + 24*(x*e + d)^m*c*g^2*x^5*e^5 + (x*e + d)^m*a*d*f^2*m^4*e^4 + 12*(x*e + d)^m*b*d*f^2*m^3*x*e^4 + 24*(x*e + d)^m*a*d*f*g*m^3*x*e^4 + 29*(x*e + d)^m*c*d*f^2*m^2*x^2*e^4 + 58*(x*e + d)^m*b*d*f*g*m^2*x^2*e^4 + 29*(x*e + d)^m*a*d*g^2*m^2*x^2*e^4 + 20*(x*e + d)^m*c*d*f*g*m*x^3*e^4 + 10*(x*e + d)^m*b*d*g^2*m*x^3*e^4 - (x*e + d)^m*b*d^2*f^2*m^3*e^3 - 2*(x*e + d)^m*a*d^2*f*g*m^3*e^3 - 18*(x*e + d)^m*c*d^2*f^2*m^2*x*e^3 - 36*(x*e + d)^m*b*d^2*f*g*m^2*x*e^3 - 18*(x*e + d)^m*a*d^2*g^2*m^2*x*e^3 - 30*(x*e + d)^m*c*d^2*f*g*m*x^2*e^3 - 15*(x*e + d)^m*b*d^2*g^2*m*x^2*e^3 + 2*(x*e + d)^m*c*d^3*f^2*m^2*e^2 + 4*(x*e + d)^m*b*d^3*f*g*m^2*e^2 + 2*(x*e + d)^m*a*d^3*g^2*m^2*e^2 + 60*(x*e + d)^m*c*d^3*f*g*m*x*e^2 + 30*(x*e + d)^m*b*d^3*g^2*m*x*e^2 - 12*(x*e + d)^m*c*d^4*f*g*m*e - 6*(x*e + d)^m*b*d^4*g^2*m*e + 24*(x*e + d)^m*c*d^5*g^2 + 14*(x*e + d)^m*a*f^2*m^3*x*e^5 + 59*(x*e + d)^m*b*f^2*m^2*x^2*e^5 + 118*(x*e + d)^m*a*f*g*m^2*x^2*e^5 + 78*(x*e + d)^m*c*f^2*m*x^3*e^5 + 156*(x*e + d)^m*b*f*g*m*x^3*e^5 + 78*(x*e + d)^m*a*g^2*m*x^3*e^5 + 60*(x*e + d)^m*c*f*g*x^4*e^5 + 30*(x*e + d)^m*b*g^2*x^4*e^5 + 14*(x*e + d)^m*a*d*f^2*m^3*e^4 + 47*(x*e + d)^m*b*d*f^2*m^2*x*e^4 + 94*(x*e + d)^m*a*d*f*g*m^2*x*e^4 + 20*(x*e + d)^m*c*d*f^2*m*x^2*e^4 + 40*(x*e + d)^m*b*d*f*g*m*x^2*e^4 + 20*(x*e + d)^m*a*d*g^2*m*x^2*e^4 - 12*(x*e + d)^m*b*d^2*f^2*m^2*e^3 - 24*(x*e + d)^m*a*d^2*f*g*m^2*e^3 - 40*(x*e + d)^m*c*d^2*f^2*m*x*e^3 - 80*(x*e + d)^m*b*d^2*f*g*m*x*e^3 - 40*(x*e + d)^m*a*d^2*g^2*m*x*e^3 + 18*(x*e + d)^m*c*d^3*f^2*m*e^2 + 36*(x*e + d)^m*b*d^3*f*g*m*e^2 + 18*(x*e + d)^m*a*d^3*g^2*m*e^2 - 60*(x*e + d)^m*c*d^4*f*g*e - 30*(x*e + d)^m*b*d^4*g^2*e + 71*(x*e + d)^m*a*f^2*m^2*x*e^5 + 107*(x*e + d)^m*b*f^2*m*x^2*e^5 + 214*(x*e + d)^m*a*f*g*m*x^2*e^5 + 40*(x*e + d)^m*c*f^2*x^3*e^5 + 80*(x*e + d)^m*b*f*g*x^3*e^5 + 40*(x*e + d)^m*a*g^2*x^3*e^5 + 71*(x*e + d)^m*a*d*f^2*m^2*e^4 + 60*(x*e + d)^m*b*d*f^2*m*x*e^4 + 120*(x*e + d)^m*a*d*f*g*m*x*e^4 - 47*(x*e + d)^m*b*d^2*f^2*m*e^3 - 94*(x*e + d)^m*a*d^2*f*g*m*e^3 + 40*(x*e + d)^m*c*d^3*f^2*e^2 + 80*(x*e + d)^m*b*d^3*f*g*e^2 + 40*(x*e + d)^m*a*d^3*g^2*e^2 + 154*(x*e + d)^m*a*f^2*m*x*e^5 + 60*(x*e + d)^m*b*f^2*x^2*e^5 + 120*(x*e + d)^m*a*f*g*x^2*e^5 + 154*(x*e + d)^m*a*d*f^2*m*e^4 - 60*(x*e + d)^m*b*d^2*f^2*e^3 - 120*(x*e + d)^m*a*d^2*f*g*e^3 + 120*(x*e + d)^m*a*f^2*x*e^5 + 120*(x*e + d)^m*a*d*f^2*e^4)/(m^5*e^5 + 15*m

```
^4*e^5 + 85*m^3*e^5 + 225*m^2*e^5 + 274*m*e^5 + 120*e^5)
```

3.921 $\int (d+ex)^m (f+gx) \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=144

$$\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}-\frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)}+\frac{(d+ex)^{m+3}(beg-3cdg+c}{e^4(m+3)}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)^(1 + m))/(e^4*(1 + m)) - ((c
*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^4*(
2 + m)) + ((c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (c*
g*(d + e*x)^(4 + m))/(e^4*(4 + m))
```

Rubi [A] time = 0.113269, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {771}

$$\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^4(m+1)}-\frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)}+\frac{(d+ex)^{m+3}(beg-3cdg+c}{e^4(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)^(1 + m))/(e^4*(1 + m)) - ((c
*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^4*(
2 + m)) + ((c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (c*
g*(d + e*x)^(4 + m))/(e^4*(4 + m))
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m (f+gx)\left(a+bx+cx^2\right) dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^m}{e^3}+\frac{(-cd(2ef-3dg)+e(bef-2bdg+a}{e^3}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)}-\frac{(cd(2ef-3dg)-e(bef-2bdg+aeg)}{e^4(2+m)}$$

Mathematica [A] time = 0.352825, size = 180, normalized size = 1.25

$$\frac{(d+ex)^{m+1}\left(\frac{(d+ex)\left(ce(2aeg(m+3)+bdg(m-2)+bef(m+4))-b^2e^2g(m+2)+2c^2d(3dg-ef(m+4))\right)}{e^2(m+2)}-\frac{\left(e(ae-bd)+cd^2\right)(beg(m+1)+6cdg-2cef(m+4))}{e^2(m+1)}+(a+x(\right.}{ce^2(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(((c*d^2 + e*(-(b*d) + a*e))*(6*c*d*g + b*e*g*(1 + m)
- 2*c*e*f*(4 + m)))/(e^2*(1 + m))) + ((-(b^2*e^2*g*(2 + m)) + 2*c^2*d*(3*d*
g - e*f*(4 + m)) + c*e*(b*d*g*(-2 + m) + 2*a*e*g*(3 + m) + b*e*f*(4 + m)))*
(d + e*x))/(e^2*(2 + m)) + (a + x*(b + c*x))*(b*e*g + c*(-3*d*g + e*f*(4 +
m) + e*g*(3 + m)*x))))/(c*e^2*(3 + m)*(4 + m))
```

Maple [B] time = 0.052, size = 503, normalized size = 3.5

$$-\frac{(ex+d)^{1+m}\left(-ce^3gm^3x^3 - be^3gm^3x^2 - ce^3fm^3x^2 - 6\,ce^3gm^2x^3 - ae^3gm^3x - be^3fm^3x - 7\,be^3gm^2x^2 + 3\,cde^2gm^2x^2 - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x)
```

```
[Out] -(e*x+d)^(1+m)*(-c*e^3*g*m^3*x^3-b*e^3*g*m^3*x^2-c*e^3*f*m^3*x^2-6*c*e^3*g*
m^2*x^3-a*e^3*g*m^3*x-b*e^3*f*m^3*x-7*b*e^3*g*m^2*x^2+3*c*d*e^2*g*m^2*x^2-7
*c*e^3*f*m^2*x^2-11*c*e^3*g*m*x^3-a*e^3*f*m^3-8*a*e^3*g*m^2*x+2*b*d*e^2*g*m
^2*x-8*b*e^3*f*m^2*x-14*b*e^3*g*m*x^2+2*c*d*e^2*f*m^2*x+9*c*d*e^2*g*m*x^2-1
4*c*e^3*f*m*x^2-6*c*e^3*g*x^3+a*d*e^2*g*m^2-9*a*e^3*f*m^2-19*a*e^3*g*m*x+b*
d*e^2*f*m^2+10*b*d*e^2*g*m*x-19*b*e^3*f*m*x-8*b*e^3*g*x^2-6*c*d^2*e*g*m*x+1
0*c*d*e^2*f*m*x+6*c*d*e^2*g*x^2-8*c*e^3*f*x^2+7*a*d*e^2*g*m-26*a*e^3*f*m-12
*a*e^3*g*x-2*b*d^2*e*g*m+7*b*d*e^2*f*m+8*b*d*e^2*g*x-12*b*e^3*f*x-2*c*d^2*e
*f*m-6*c*d^2*e*g*x+8*c*d*e^2*f*x+12*a*d*e^2*g-24*a*e^3*f-8*b*d^2*e*g+12*b*d
*e^2*f+6*c*d^3*g-8*c*d^2*e*f)/e^4/(m^4+10*m^3+35*m^2+50*m+24)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01193, size = 1297, normalized size = 9.01

$$\frac{\left(ade^3fm^3 + \left(ce^4gm^3 + 6\,ce^4gm^2 + 11\,ce^4gm + 6\,ce^4g\right)x^4 + \left(8\,ce^4f + 8\,be^4g + \left(ce^4f + \left(cde^3 + be^4\right)g\right)m^3 + \left(7\,ce^4f + \left(3\cdots\right.\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*e^3*f*m^3 + (c*e^4*g*m^3 + 6*c*e^4*g*m^2 + 11*c*e^4*g*m + 6*c*e^4*g)*x
^4 + (8*c*e^4*f + 8*b*e^4*g + (c*e^4*f + (c*d*e^3 + b*e^4)*g)*m^3 + (7*c*e^
4*f + (3*c*d*e^3 + 7*b*e^4)*g)*m^2 + 2*(7*c*e^4*f + (c*d*e^3 + 7*b*e^4)*g)*
m)*x^3 - (a*d^2*e^2*g + (b*d^2*e^2 - 9*a*d*e^3)*f)*m^2 + (12*b*e^4*f + 12*a
*e^4*g + ((c*d*e^3 + b*e^4)*f + (b*d*e^3 + a*e^4)*g)*m^3 + ((5*c*d*e^3 + 8*
b*e^4)*f - (3*c*d^2*e^2 - 5*b*d*e^3 - 8*a*e^4)*g)*m^2 + ((4*c*d*e^3 + 19*b*
```

```
e^4)*f - (3*c*d^2*e^2 - 4*b*d*e^3 - 19*a*e^4)*g)*m)*x^2 + 4*(2*c*d^3*e - 3*
b*d^2*e^2 + 6*a*d*e^3)*f - 2*(3*c*d^4 - 4*b*d^3*e + 6*a*d^2*e^2)*g + ((2*c*
d^3*e - 7*b*d^2*e^2 + 26*a*d*e^3)*f + (2*b*d^3*e - 7*a*d^2*e^2)*g)*m + (24*
a*e^4*f + (a*d*e^3*g + (b*d*e^3 + a*e^4)*f)*m^3 - ((2*c*d^2*e^2 - 7*b*d*e^3
 - 9*a*e^4)*f + (2*b*d^2*e^2 - 7*a*d*e^3)*g)*m^2 - 2*((4*c*d^2*e^2 - 6*b*d*
e^3 - 13*a*e^4)*f - (3*c*d^3*e - 4*b*d^2*e^2 + 6*a*d*e^3)*g)*m)*x)*(e*x + d
)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)
```

Sympy [A] time = 6.69103, size = 5795, normalized size = 40.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f*x + a*g*x**2/2 + b*f*x**2/2 + b*g*x**3/3 + c*f*x**3/3
+ c*g*x**4/4), Eq(e, 0)), (-a*d**2*e**2*g/(6*d**4*e**4 + 18*d**3*e**5*x + 1
8*d**2*e**6*x**2 + 6*d*e**7*x**3) - 2*a*d*e**3*f/(6*d**4*e**4 + 18*d**3*e**
5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3) - 3*a*d*e**3*g*x/(6*d**4*e**4 + 18
*d**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3) - b*d**2*e**2*f/(6*d**4*e
**4 + 18*d**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3) - 3*b*d*e**3*f*x/
(6*d**4*e**4 + 18*d**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3) + 2*b*e*
*4*g*x**3/(6*d**4*e**4 + 18*d**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3
) + 6*c*d**4*g*log(d/e + x)/(6*d**4*e**4 + 18*d**3*e**5*x + 18*d**2*e**6*x*
*2 + 6*d*e**7*x**3) + 5*c*d**4*g/(6*d**4*e**4 + 18*d**3*e**5*x + 18*d**2*e*
*6*x**2 + 6*d*e**7*x**3) + 18*c*d**3*e*g*x*log(d/e + x)/(6*d**4*e**4 + 18*d
**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3) + 9*c*d**3*e*g*x/(6*d**4*e*
*4 + 18*d**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3) + 18*c*d**2*e**2*g
*x**2*log(d/e + x)/(6*d**4*e**4 + 18*d**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*
e**7*x**3) + 6*c*d*e**3*g*x**3*log(d/e + x)/(6*d**4*e**4 + 18*d**3*e**5*x +
 18*d**2*e**6*x**2 + 6*d*e**7*x**3) - 6*c*d*e**3*g*x**3/(6*d**4*e**4 + 18*d
**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3) + 2*c*e**4*f*x**3/(6*d**4*e
**4 + 18*d**3*e**5*x + 18*d**2*e**6*x**2 + 6*d*e**7*x**3), Eq(m, -4)), (-a*
d*e**2*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - a*e**3*f/(2*d**2*e**4 +
 4*d*e**5*x + 2*e**6*x**2) - 2*a*e**3*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**
6*x**2) + 2*b*d**2*e*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2
) + 3*b*d**2*e*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - b*d*e**2*f/(2*d
**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x*log(d/e + x)/(2*d**2*
e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x/(2*d**2*e**4 + 4*d*e**5*x
 + 2*e**6*x**2) - 2*b*e**3*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2
*b*e**3*g*x**2*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*
d**3*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 9*c*d**3*g/(
2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*d**2*e*f*log(d/e + x)/(2*d**2
*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 3*c*d**2*e*f/(2*d**2*e**4 + 4*d*e**5*x
+ 2*e**6*x**2) - 12*c*d**2*e*g*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2
*e**6*x**2) - 12*c*d**2*e*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*
c*d*e**2*f*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*c*d*
e**2*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*d*e**2*g*x**2*log(d
/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*f*x**2*log(d/e
+ x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*g*x**3/(2*d**2*e**
4 + 4*d*e**5*x + 2*e**6*x**2), Eq(m, -3)), (2*a*d*e**2*g*log(d/e + x)/(2*d*
e**4 + 2*e**5*x) + 2*a*d*e**2*g/(2*d*e**4 + 2*e**5*x) - 2*a*e**3*f/(2*d*e**
4 + 2*e**5*x) + 2*a*e**3*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d**2*
e*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d**2*e*g/(2*d*e**4 + 2*e**5*x)
 + 2*b*d*e**2*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*d*e**2*f/(2*d*e**4
 + 2*e**5*x) - 4*b*d*e**2*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e**3
```

*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e**3*g*x**2/(2*d*e**4 + 2*e**5*x) + 6*c*d**3*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 6*c*d**3*g/(2*d*e**4 + 2*e**5*x) - 4*c*d**2*e*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*c*d**2*e*f/(2*d*e**4 + 2*e**5*x) + 6*c*d**2*e*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*c*d*e**2*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 3*c*d*e**2*g*x**2/(2*d*e**4 + 2*e**5*x) + 2*c*e**3*f*x**2/(2*d*e**4 + 2*e**5*x) + c*e**3*g*x**3/(2*d*e**4 + 2*e**5*x), Eq(m, -2)), (-a*d*g*log(d/e + x)/e**2 + a*f*log(d/e + x)/e + a*g*x/e + b*d**2*g*log(d/e + x)/e**3 - b*d*f*log(d/e + x)/e**2 - b*d*g*x/e**2 + b*f*x/e + b*g*x**2/(2*e) - c*d**3*g*log(d/e + x)/e**4 + c*d**2*f*log(d/e + x)/e**3 + c*d**2*g*x/e**3 - c*d*f*x/e**2 - c*d*g*x**2/(2*e**2) + c*f*x**2/(2*e) + c*g*x**3/(3*e), Eq(m, -1)), (-a*d**2*e**2*g*m**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 7*a*d**2*e**2*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 12*a*d**2*e**2*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*f*m**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*d*e**3*f*m**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 26*a*d*e**3*f*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 24*a*d*e**3*f*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*g*m**3*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7*a*d*e**3*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*a*d*e**3*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*e**4*f*m**3*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*e**4*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 26*a*e**4*f*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 24*a*e**4*f*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*e**4*g*m**3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*a*e**4*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 19*a*e**4*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*a*e**4*g*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 2*b*d**3*e*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*b*d**3*e*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - b*d**2*e**2*f*m**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 7*b*d**2*e**2*f*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 12*b*d**2*e**2*f*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 2*b*d**2*e**2*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 8*b*d**2*e**2*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*d*e**3*f*m**3*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7*b*d*e**3*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*b*d*e**3*f*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*d*e**3*g*m**3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 5*b*d*e**3*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 4*b*d*e**3*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*e**4*f*m**3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*b*e**4*f*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 19*b*e**4*f*m*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 12*b*e**4*f*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + b*e**4*g*m**3*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7*b*e**4*g*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 14*b*e**4*g*m*x**3*(d +

```
e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8
*b*e**4*g*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e
**4*m + 24*e**4) - 6*c*d**4*g*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e
**4*m**2 + 50*e**4*m + 24*e**4) + 2*c*d**3*e*f*m*(d + e*x)**m/(e**4*m**4 +
10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*c*d**3*e*f*(d + e*x)
**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 6*c*d
**3*e*g*m*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4
*m + 24*e**4) - 2*c*d**2*e**2*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m*
*3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 8*c*d**2*e**2*f*m*x*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 3*c*d**2
*e**2*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 5
0*e**4*m + 24*e**4) - 3*c*d**2*e**2*g*m*x**2*(d + e*x)**m/(e**4*m**4 + 10*e
**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*f*m**3*x**2*(d +
e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 5
*c*d*e**3*f*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2
 + 50*e**4*m + 24*e**4) + 4*c*d*e**3*f*m*x**2*(d + e*x)**m/(e**4*m**4 + 10*
e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*g*m**3*x**3*(d +
 e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) +
3*c*d*e**3*g*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**
2 + 50*e**4*m + 24*e**4) + 2*c*d*e**3*g*m*x**3*(d + e*x)**m/(e**4*m**4 + 10
*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*e**4*f*m**3*x**3*(d +
e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7
*c*e**4*f*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 +
 50*e**4*m + 24*e**4) + 14*c*e**4*f*m*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**
4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 8*c*e**4*f*x**3*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*e**4*g
*m**3*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*
m + 24*e**4) + 6*c*e**4*g*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3
+ 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 11*c*e**4*g*m*x**4*(d + e*x)**m/(e*
*4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 6*c*e**4*g*x
**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*
e**4), True))
```

Giac [B] time = 1.16435, size = 1569, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*g*m^3*x^4*e^4 + (x*e + d)^m*c*d*g*m^3*x^3*e^3 + (x*e + d)^m*
c*f*m^3*x^3*e^4 + (x*e + d)^m*b*g*m^3*x^3*e^4 + 6*(x*e + d)^m*c*g*m^2*x^4*e
^4 + (x*e + d)^m*c*d*f*m^3*x^2*e^3 + (x*e + d)^m*b*d*g*m^3*x^2*e^3 + 3*(x*e
 + d)^m*c*d*g*m^2*x^3*e^3 - 3*(x*e + d)^m*c*d^2*g*m^2*x^2*e^2 + (x*e + d)^m
*b*f*m^3*x^2*e^4 + (x*e + d)^m*a*g*m^3*x^2*e^4 + 7*(x*e + d)^m*c*f*m^2*x^3*
e^4 + 7*(x*e + d)^m*b*g*m^2*x^3*e^4 + 11*(x*e + d)^m*c*g*m*x^4*e^4 + (x*e +
 d)^m*b*d*f*m^3*x*e^3 + (x*e + d)^m*a*d*g*m^3*x*e^3 + 5*(x*e + d)^m*c*d*f*m
^2*x^2*e^3 + 5*(x*e + d)^m*b*d*g*m^2*x^2*e^3 + 2*(x*e + d)^m*c*d*g*m*x^3*e^
3 - 2*(x*e + d)^m*c*d^2*f*m^2*x*e^2 - 2*(x*e + d)^m*b*d^2*g*m^2*x*e^2 - 3*(
x*e + d)^m*c*d^2*g*m*x^2*e^2 + 6*(x*e + d)^m*c*d^3*g*m*x*e + (x*e + d)^m*a*
f*m^3*x*e^4 + 8*(x*e + d)^m*b*f*m^2*x^2*e^4 + 8*(x*e + d)^m*a*g*m^2*x^2*e^4
 + 14*(x*e + d)^m*c*f*m*x^3*e^4 + 14*(x*e + d)^m*b*g*m*x^3*e^4 + 6*(x*e + d
)^m*c*g*x^4*e^4 + (x*e + d)^m*a*d*f*m^3*e^3 + 7*(x*e + d)^m*b*d*f*m^2*x*e^3
 + 7*(x*e + d)^m*a*d*g*m^2*x*e^3 + 4*(x*e + d)^m*c*d*f*m*x^2*e^3 + 4*(x*e +
 d)^m*b*d*g*m*x^2*e^3 - (x*e + d)^m*b*d^2*f*m^2*e^2 - (x*e + d)^m*a*d^2*g*m
^2*e^2 - 8*(x*e + d)^m*c*d^2*f*m*x*e^2 - 8*(x*e + d)^m*b*d^2*g*m*x*e^2 + 2*
```

```
(x*e + d)^m*c*d^3*f*m*e + 2*(x*e + d)^m*b*d^3*g*m*e - 6*(x*e + d)^m*c*d^4*g
 + 9*(x*e + d)^m*a*f*m^2*x*e^4 + 19*(x*e + d)^m*b*f*m*x^2*e^4 + 19*(x*e + d
)^m*a*g*m*x^2*e^4 + 8*(x*e + d)^m*c*f*x^3*e^4 + 8*(x*e + d)^m*b*g*x^3*e^4 +
 9*(x*e + d)^m*a*d*f*m^2*e^3 + 12*(x*e + d)^m*b*d*f*m*x*e^3 + 12*(x*e + d)^
m*a*d*g*m*x*e^3 - 7*(x*e + d)^m*b*d^2*f*m*e^2 - 7*(x*e + d)^m*a*d^2*g*m*e^2
 + 8*(x*e + d)^m*c*d^3*f*e + 8*(x*e + d)^m*b*d^3*g*e + 26*(x*e + d)^m*a*f*m
*x*e^4 + 12*(x*e + d)^m*b*f*x^2*e^4 + 12*(x*e + d)^m*a*g*x^2*e^4 + 26*(x*e
+ d)^m*a*d*f*m*e^3 - 12*(x*e + d)^m*b*d^2*f*e^2 - 12*(x*e + d)^m*a*d^2*g*e^
2 + 24*(x*e + d)^m*a*f*x*e^4 + 24*(x*e + d)^m*a*d*f*e^3)/(m^4*e^4 + 10*m^3*
e^4 + 35*m^2*e^4 + 50*m*e^4 + 24*e^4)
```

3.922 $\int \frac{(d+ex)^m(a+bx+cx^2)}{f+gx}\,dx$

Optimal. Leaf size=129

$$\frac{(d+ex)^{m+1}(ag^2-bfg+cf^2)\ {}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)}-\frac{(d+ex)^{m+1}(-beg+cdg+cef)}{e^2g^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

```
[Out] -(((c*e*f + c*d*g - b*e*g)*(d + e*x)^(1 + m))/(e^2*g^2*(1 + m))) + (c*(d +
e*x)^(2 + m))/(e^2*g*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f
 - d*g)*(1 + m))
```

Rubi [A] time = 0.159453, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {951, 80, 68}

$$\frac{(d+ex)^{m+1}(ag^2-bfg+cf^2)\ {}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)}-\frac{(d+ex)^{m+1}(-beg+cdg+cef)}{e^2g^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

```
[Out] -(((c*e*f + c*d*g - b*e*g)*(d + e*x)^(1 + m))/(e^2*g^2*(1 + m))) + (c*(d +
e*x)^(2 + m))/(e^2*g*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f
 - d*g)*(1 + m))
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)}{f+gx}\,dx = \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{\int \frac{(d+ex)^m(-e(cdf-aeg)(2+m)-e(cef+cdg-beg)(2+m)x)}{f+gx}\,dx}{e^2g(2+m)}$$

$$= -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)} + \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)\int \frac{(d+ex)^m}{f+gx}\,dx}{g^2}$$

$$= -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)} + \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)(d+ex)^{1+m}\,{}_2F_1}{g^2(ef-dg)(1}$$

Mathematica [A] time = 0.146815, size = 111, normalized size = 0.86

$$\frac{(d+ex)^{m+1}\left(\frac{\left(g(ag-bf)+cf^2\right){}_2F_1\left(1,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)}+\frac{beg-c(dg+ef)}{e^2(m+1)}+\frac{cg(d+ex)}{e^2(m+2)}\right)}{g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

```
[Out] ((d + e*x)^(1 + m)*((b*e*g - c*(e*f + d*g))/(e^2*(1 + m)) + (c*g*(d + e*x))
/(e^2*(2 + m)) + ((c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1, 1 + m, 2
+ m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m))))/g^2
```

Maple [F] time = 0.662, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)(ex+d)^m}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f),x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

3.923 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^2}\,dx$

Optimal. Leaf size=157

$$\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)(cf(2dg-ef(m+2))-g(aegm+b(dg-ef(m+1))))}{g^2(m+1)(ef-dg)^2}+\frac{(d+ex)^{m+1}\left(a+\frac{f}{\ldots}\right.}{(f+gx)(ef-\ldots}$$

```
[Out] (c*(d + e*x)^(1 + m))/(e*g^2*(1 + m)) + ((a + (f*(c*f - b*g))/g^2)*(d + e*x
)^(1 + m))/((e*f - d*g)*(f + g*x)) + ((c*f*(2*d*g - e*f*(2 + m)) - g*(a*e*g
*m + b*(d*g - e*f*(1 + m))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f - d*g)^2*(1 + m))
```

Rubi [A] time = 0.204089, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {949, 80, 68}

$$-\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)(g(aegm+bdg-bef(m+1))-cf(2dg-ef(m+2)))}{g^2(m+1)(ef-dg)^2}+\frac{(d+ex)^{m+1}\left(a+\ldots\right.}{(f+gx)(ef-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

```
[Out] (c*(d + e*x)^(1 + m))/(e*g^2*(1 + m)) + ((a + (f*(c*f - b*g))/g^2)*(d + e*x
)^(1 + m))/((e*f - d*g)*(f + g*x)) - ((g*(b*d*g + a*e*g*m - b*e*f*(1 + m))
- c*f*(2*d*g - e*f*(2 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f - d*g)^2*(1 + m))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{(f+gx)^2}\,dx = \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)}+\frac{\int \frac{(d+ex)^m\left(\frac{cdfg-aeg^2m-cef^2(1+m)-bg(dg-ef(1+m))}{g^2}-c\left(d-\frac{ef}{g}\right)x\right)}{f+gx}\,dx}{ef-dg}$$

$$=\frac{c(d+ex)^{1+m}}{eg^2(1+m)}+\frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)}-\frac{(g(bdg+aegm-bef(1+m))-cf(2dg-e\ldots}{g^2(ef-dg)}$$

$$=\frac{c(d+ex)^{1+m}}{eg^2(1+m)}+\frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)}-\frac{(g(bdg+aegm-bef(1+m))-cf(2dg-e\ldots}{g^2(ef\ldots}$$

Mathematica [A] time = 0.160932, size = 134, normalized size = 0.85

$$\frac{(d+ex)^{m+1}\left(e^2\left(g(ag-bf)+cf^2\right){}_2F_1\left(2,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)-e(2cf-bg)(ef-dg)\,{}_2F_1\left(1,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)+c(e\ldots\right.}{eg^2(m+1)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(c*(e*f - d*g)^2 - e*(2*c*f - b*g)*(e*f - d*g)*Hypergeom
etric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e^2*(c*f^2 + g*(-
(b*f) + a*g))*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*
g)]))/(e*g^2*(e*f - d*g)^2*(1 + m))
```

Maple [F] time = 0.737, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)(ex+d)^m}{g^2x^2+2\,fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**2,x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^2, x)
```

3.924 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^3} dx$

Optimal. Leaf size=245

$$\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(c\left(2d^2g^2-4defg(m+1)+e^2f^2\left(m^2+3m+2\right)\right)-egm(aeg(1-m)-b(2dg-\right.}{2g^2(m+1)(ef-dg)^3}$$

```
[Out] ((a + (f*(c*f - b*g))/g^2)*(d + e*x)^(1 + m))/(2*(e*f - d*g)*(f + g*x)^2) +
 ((c*f*(4*d*g - e*f*(3 + m)) + g*(a*e*g*(1 - m) - b*(2*d*g - e*f*(1 + m))))
*(d + e*x)^(1 + m))/(2*g^2*(e*f - d*g)^2*(f + g*x)) + ((c*(2*d^2*g^2 - 4*d*
e*f*g*(1 + m) + e^2*f^2*(2 + 3*m + m^2)) - e*g*m*(a*e*g*(1 - m) - b*(2*d*g
- e*f*(1 + m))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*
(d + e*x))/(e*f - d*g))])/(2*g^2*(e*f - d*g)^3*(1 + m))
```

Rubi [A] time = 0.317472, antiderivative size = 243, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {949, 78, 68}

$$\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(egm(-aeg(1-m)+2bdg-bef(m+1))+c\left(2d^2g^2-4defg(m+1)+e^2f^2\left(m^2\right.\right.\right.}{2g^2(m+1)(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

```
[Out] ((a + (f*(c*f - b*g))/g^2)*(d + e*x)^(1 + m))/(2*(e*f - d*g)*(f + g*x)^2) -
 ((g*(2*b*d*g - a*e*g*(1 - m) - b*e*f*(1 + m)) - c*f*(4*d*g - e*f*(3 + m)))
*(d + e*x)^(1 + m))/(2*g^2*(e*f - d*g)^2*(f + g*x)) + ((e*g*m*(2*b*d*g - a*
e*g*(1 - m) - b*e*f*(1 + m)) + c*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(
2 + 3*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*
(d + e*x))/(e*f - d*g))])/(2*g^2*(e*f - d*g)^3*(1 + m))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{(f+gx)^3}\,dx = \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} + \frac{\int \frac{(d+ex)^m\left(\frac{cf(2dg-ef(1+m))-g(2bdg-aeg(1-m)-bef(1+m))}{g^2}-2c\left(d-\frac{ef}{g}\right)x\right)}{(f+gx)^2}\,dx}{2(ef-dg)}$$

$$= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} - \frac{(g(2bdg-aeg(1-m)-bef(1+m))-cf(4dg-ef(3+}{2g^2(ef-dg)^2(f+gx)}$$

$$= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} - \frac{(g(2bdg-aeg(1-m)-bef(1+m))-cf(4dg-ef(3+}{2g^2(ef-dg)^2(f+gx)}$$

Mathematica [A] time = 0.16796, size = 157, normalized size = 0.64

$$-\frac{(d+ex)^{m+1}\left(e\left(e\left(g(ag-bf)+cf^2\right)\,{}_2F_1\left(3,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)-(2cf-bg)(ef-dg)\,{}_2F_1\left(2,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)\right.\right.}{g^2(m+1)(dg-ef)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

```
[Out] -(((d + e*x)^(1 + m)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + m, 2 + m, (g
*(d + e*x))/(-(e*f) + d*g)] + e*(-((2*c*f - b*g)*(e*f - d*g)*Hypergeometric
2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]) + e*(c*f^2 + g*(-(b*f)
+ a*g))*Hypergeometric2F1[3, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])))
/(g^2*(-(e*f) + d*g)^3*(1 + m)))
```

Maple [F] time = 0.691, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)(ex+d)^m}{g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x +
 f^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex)^m\left(a+bx+cx^2\right)}{\left(f+gx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**3,x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2+bx+a\right)(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f)^3, x)
```

3.925 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=525

$$\frac{(d+ex)^{m+3}\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)-bd\left(10d^2\right.\right.\right.}{e^7(m+3)}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^7*(1 + m)) -
 (2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((c^2*d^2*(6*e^2*f^2 -
20*d*e*f*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b
^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(a*e*(e^2*f^2 - 6*d*e*f*g + 6
*d^2*g^2) - b*d*(3*e^2*f^2 - 12*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(3 + m))/
(e^7*(3 + m)) + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 -
5*d*e*f*g + 5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*
g + 10*d^2*g^2)))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + ((b^2*e^2*g^2 + 2*c*e*
g*(2*b*e*f - 5*b*d*g + a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d
 + e*x)^(5 + m))/(e^7*(5 + m)) + (2*c*g*(c*e*f - 3*c*d*g + b*e*g)*(d + e*x)
^(6 + m))/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rubi [A] time = 0.61498, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {947}

$$\frac{(d+ex)^{m+3}\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)-bd\left(10d^2\right.\right.\right.}{e^7(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```


```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^7*(1 + m)) -
 (2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((c^2*d^2*(6*e^2*f^2 -
20*d*e*f*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b
^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(a*e*(e^2*f^2 - 6*d*e*f*g + 6
*d^2*g^2) - b*d*(3*e^2*f^2 - 12*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(3 + m))/
(e^7*(3 + m)) + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 -
5*d*e*f*g + 5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*
g + 10*d^2*g^2)))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + ((b^2*e^2*g^2 + 2*c*e*
g*(2*b*e*f - 5*b*d*g + a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d
 + e*x)^(5 + m))/(e^7*(5 + m)) + (2*c*g*(c*e*f - 3*c*d*g + b*e*g)*(d + e*x)
^(6 + m))/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m))/(e^7*(7 + m))
```


Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^2 (ef-dg)^2 (d+ex)^m}{e^6} + \frac{2\left(cd^2-bde+ae^2\right)(ef-dg)(-c\cdots}{\cdots} \right.$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 (ef-dg)^2 (d+ex)^{1+m}}{e^7(1+m)} - \frac{2\left(cd^2-bde+ae^2\right)(ef-dg)(cd(2\cdots}{\cdots}$$

Mathematica [A] time = 0.847475, size = 492, normalized size = 0.94

$$\frac{(d+ex)^{m+1}\left(\frac{(d+ex)^2\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)+bd\left(-10d^2g^2+12defg-3e^2f^2\right)\right)+c^2d^2\left(15d^2g^2-20de\cdots\right.\right.}{m+3}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((((c*d^2 + e*(-(b*d) + a*e))^2*(e*f - d*g)^2)/(1 + m) -
(2*(c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*(c*d*(-2*e*f + 3*d*g) + e*(b*e
*f - 2*b*d*g + a*e*g))*(d + e*x))/(2 + m) + ((c^2*d^2*(6*e^2*f^2 - 20*d*e*f
*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b^2*(e^2*
f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(b*d*(-3*e^2*f^2 + 12*d*e*f*g - 10*d^
2*g^2) + a*e*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)))*(d + e*x)^2)/(3 + m) + (2*
(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 - 5*d*e*f*g + 5*d^2*g
^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*g + 10*d^2*g^2)))*(
d + e*x)^3)/(4 + m) + ((b^2*e^2*g^2 + 2*c*e*g*(2*b*e*f - 5*b*d*g + a*e*g) +
 c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d + e*x)^4)/(5 + m) + (2*c*g*(c*
e*f - 3*c*d*g + b*e*g)*(d + e*x)^5)/(6 + m) + (c^2*g^2*(d + e*x)^6)/(7 + m)
))/e^7
```

Maple [B] time = 0.06, size = 5890, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.41609, size = 10647, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^6*f^2*m^6 + (c^2*e^7*g^2*m^6 + 21*c^2*e^7*g^2*m^5 + 175*c^2*e^7*g^
2*m^4 + 735*c^2*e^7*g^2*m^3 + 1624*c^2*e^7*g^2*m^2 + 1764*c^2*e^7*g^2*m + 7
20*c^2*e^7*g^2)*x^7 + (1680*c^2*e^7*f*g + 1680*b*c*e^7*g^2 + (2*c^2*e^7*f*g
 + (c^2*d*e^6 + 2*b*c*e^7)*g^2)*m^6 + (44*c^2*e^7*f*g + (15*c^2*d*e^6 + 44*
b*c*e^7)*g^2)*m^5 + 5*(76*c^2*e^7*f*g + (17*c^2*d*e^6 + 76*b*c*e^7)*g^2)*m^
4 + 5*(328*c^2*e^7*f*g + (45*c^2*d*e^6 + 328*b*c*e^7)*g^2)*m^3 + 2*(1849*c^
2*e^7*f*g + (137*c^2*d*e^6 + 1849*b*c*e^7)*g^2)*m^2 + 4*(1019*c^2*e^7*f*g +
 (30*c^2*d*e^6 + 1019*b*c*e^7)*g^2)*m)*x^6 - (2*a^2*d^2*e^5*f*g + (2*a*b*d^
2*e^5 - 27*a^2*d*e^6)*f^2)*m^5 + (1008*c^2*e^7*f^2 + 4032*b*c*e^7*f*g + 100
8*(b^2 + 2*a*c)*e^7*g^2 + (c^2*e^7*f^2 + 2*(c^2*d*e^6 + 2*b*c*e^7)*f*g + (2
*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*g^2)*m^6 + (23*c^2*e^7*f^2 + 2*(17*c^2*d*e^
6 + 46*b*c*e^7)*f*g - (6*c^2*d^2*e^5 - 34*b*c*d*e^6 - 23*(b^2 + 2*a*c)*e^7)
*g^2)*m^5 + 3*(69*c^2*e^7*f^2 + 2*(35*c^2*d*e^6 + 138*b*c*e^7)*f*g - (20*c^
2*d^2*e^5 - 70*b*c*d*e^6 - 69*(b^2 + 2*a*c)*e^7)*g^2)*m^4 + 5*(185*c^2*e^7*
f^2 + 2*(59*c^2*d*e^6 + 370*b*c*e^7)*f*g - (42*c^2*d^2*e^5 - 118*b*c*d*e^6
- 185*(b^2 + 2*a*c)*e^7)*g^2)*m^3 + 4*(536*c^2*e^7*f^2 + (187*c^2*d*e^6 + 2
144*b*c*e^7)*f*g - (75*c^2*d^2*e^5 - 187*b*c*d*e^6 - 536*(b^2 + 2*a*c)*e^7)
*g^2)*m^2 + 12*(201*c^2*e^7*f^2 + 4*(7*c^2*d*e^6 + 201*b*c*e^7)*f*g - (12*c
^2*d^2*e^5 - 28*b*c*d*e^6 - 201*(b^2 + 2*a*c)*e^7)*g^2)*m)*x^5 + (2*a^2*d^3
*e^4*g^2 - (50*a*b*d^2*e^5 - 295*a^2*d*e^6 - 2*(b^2 + 2*a*c)*d^3*e^4)*f^2 +
 2*(4*a*b*d^3*e^4 - 25*a^2*d^2*e^5)*f*g)*m^4 + (2520*b*c*e^7*f^2 + 2520*a*b
*e^7*g^2 + 2520*(b^2 + 2*a*c)*e^7*f*g + ((c^2*d*e^6 + 2*b*c*e^7)*f^2 + 2*(2
*b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*f*g + (2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*g^2
)*m^6 + ((19*c^2*d*e^6 + 48*b*c*e^7)*f^2 - 2*(5*c^2*d^2*e^5 - 38*b*c*d*e^6
- 24*(b^2 + 2*a*c)*e^7)*f*g - (10*b*c*d^2*e^5 - 48*a*b*e^7 - 19*(b^2 + 2*a*
c)*d*e^6)*g^2)*m^5 + ((131*c^2*d*e^6 + 452*b*c*e^7)*f^2 - 2*(65*c^2*d^2*e^5
 - 262*b*c*d*e^6 - 226*(b^2 + 2*a*c)*e^7)*f*g + (30*c^2*d^3*e^4 - 130*b*c*d
^2*e^5 + 452*a*b*e^7 + 131*(b^2 + 2*a*c)*d*e^6)*g^2)*m^4 + ((401*c^2*d*e^6
+ 2112*b*c*e^7)*f^2 - 2*(265*c^2*d^2*e^5 - 802*b*c*d*e^6 - 1056*(b^2 + 2*a*
c)*e^7)*f*g + (180*c^2*d^3*e^4 - 530*b*c*d^2*e^5 + 2112*a*b*e^7 + 401*(b^2
+ 2*a*c)*d*e^6)*g^2)*m^3 + 10*((54*c^2*d*e^6 + 509*b*c*e^7)*f^2 - (83*c^2*d
^2*e^5 - 216*b*c*d*e^6 - 509*(b^2 + 2*a*c)*e^7)*f*g + (33*c^2*d^3*e^4 - 83*
b*c*d^2*e^5 + 509*a*b*e^7 + 54*(b^2 + 2*a*c)*d*e^6)*g^2)*m^2 + 12*(3*(7*c^2
*d*e^6 + 164*b*c*e^7)*f^2 - (35*c^2*d^2*e^5 - 84*b*c*d*e^6 - 492*(b^2 + 2*a
*c)*e^7)*f*g + (15*c^2*d^3*e^4 - 35*b*c*d^2*e^5 + 492*a*b*e^7 + 21*(b^2 + 2
*a*c)*d*e^6)*g^2)*m)*x^4 - ((12*b*c*d^4*e^3 + 490*a*b*d^2*e^5 - 1665*a^2*d*
e^6 - 44*(b^2 + 2*a*c)*d^3*e^4)*f^2 - 2*(88*a*b*d^3*e^4 - 245*a^2*d^2*e^5 -
 6*(b^2 + 2*a*c)*d^4*e^3)*f*g + 4*(3*a*b*d^4*e^3 - 11*a^2*d^3*e^4)*g^2)*m^3
 + (6720*a*b*e^7*f*g + 1680*a^2*e^7*g^2 + 1680*(b^2 + 2*a*c)*e^7*f^2 + ((2*
b*c*d*e^6 + (b^2 + 2*a*c)*e^7)*f^2 + 2*(2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*f*
g + (2*a*b*d*e^6 + a^2*e^7)*g^2)*m^6 - ((4*c^2*d^2*e^5 - 42*b*c*d*e^6 - 25*
(b^2 + 2*a*c)*e^7)*f^2 + 2*(8*b*c*d^2*e^5 - 50*a*b*e^7 - 21*(b^2 + 2*a*c)*d
*e^6)*f*g - (42*a*b*d*e^6 + 25*a^2*e^7 - 4*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^5
- ((64*c^2*d^2*e^5 - 326*b*c*d*e^6 - 247*(b^2 + 2*a*c)*e^7)*f^2 - 2*(20*c^2
*d^3*e^4 - 128*b*c*d^2*e^5 + 494*a*b*e^7 + 163*(b^2 + 2*a*c)*d*e^6)*f*g - (
40*b*c*d^3*e^4 + 326*a*b*d*e^6 + 247*a^2*e^7 - 64*(b^2 + 2*a*c)*d^2*e^5)*g^
2)*m^4 - ((332*c^2*d^2*e^5 - 1134*b*c*d*e^6 - 1219*(b^2 + 2*a*c)*e^7)*f^2 -
 2*(200*c^2*d^3*e^4 - 664*b*c*d^2*e^5 + 2438*a*b*e^7 + 567*(b^2 + 2*a*c)*d*
e^6)*f*g + (120*c^2*d^4*e^3 - 400*b*c*d^3*e^4 - 1134*a*b*d*e^6 - 1219*a^2*e
^7 + 332*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^3 - 8*((76*c^2*d^2*e^5 - 211*b*c*d*e
^6 - 389*(b^2 + 2*a*c)*e^7)*f^2 - (115*c^2*d^3*e^4 - 304*b*c*d^2*e^5 + 1556
```

*a*b*e^7 + 211*(b^2 + 2*a*c)*d*e^6)*f*g + (45*c^2*d^4*e^3 - 115*b*c*d^3*e^4 - 211*a*b*d*e^6 - 389*a^2*e^7 + 76*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m^2 - 4*((84*c^2*d^2*e^5 - 210*b*c*d*e^6 - 949*(b^2 + 2*a*c)*e^7)*f^2 - 2*(70*c^2*d^3*e^4 - 168*b*c*d^2*e^5 + 1898*a*b*e^7 + 105*(b^2 + 2*a*c)*d*e^6)*f*g + (60*c^2*d^4*e^3 - 140*b*c*d^3*e^4 - 210*a*b*d*e^6 - 949*a^2*e^7 + 84*(b^2 + 2*a*c)*d^2*e^5)*g^2)*m)*x^3 + 168*(6*c^2*d^5*e^2 - 15*b*c*d^4*e^3 - 30*a*b*d^2*e^5 + 30*a^2*d*e^6 + 10*(b^2 + 2*a*c)*d^3*e^4)*f^2 - 168*(10*c^2*d^6*e - 24*b*c*d^5*e^2 - 40*a*b*d^3*e^4 + 30*a^2*d^2*e^5 + 15*(b^2 + 2*a*c)*d^4*e^3)*f*g + 24*(30*c^2*d^7 - 70*b*c*d^6*e - 105*a*b*d^4*e^3 + 70*a^2*d^3*e^4 + 42*(b^2 + 2*a*c)*d^5*e^2)*g^2 + 2*((12*c^2*d^5*e^2 - 108*b*c*d^4*e^3 - 1175*a*b*d^2*e^5 + 2552*a^2*d*e^6 + 179*(b^2 + 2*a*c)*d^3*e^4)*f^2 + (48*b*c*d^5*e^2 + 716*a*b*d^3*e^4 - 1175*a^2*d^2*e^5 - 108*(b^2 + 2*a*c)*d^4*e^3)*f*g - (108*a*b*d^4*e^3 - 179*a^2*d^3*e^4 - 12*(b^2 + 2*a*c)*d^5*e^2)*g^2)*m^2 + (5040*a*b*e^7*f^2 + 5040*a^2*e^7*f*g + (a^2*d*e^6*g^2 + (2*a*b*e^7 + (b^2 + 2*a*c)*d*e^6)*f^2 + 2*(2*a*b*d*e^6 + a^2*e^7)*f*g)*m^6 - ((6*b*c*d^2*e^5 - 52*a*b*e^7 - 23*(b^2 + 2*a*c)*d*e^6)*f^2 - 2*(46*a*b*d*e^6 + 26*a^2*e^7 - 3*(b^2 + 2*a*c)*d^2*e^5)*f*g + (6*a*b*d^2*e^5 - 23*a^2*d*e^6)*g^2)*m^5 + 3*((4*c^2*d^3*e^4 - 38*b*c*d^2*e^5 + 180*a*b*e^7 + 67*(b^2 + 2*a*c)*d*e^6)*f^2 + 2*(8*b*c*d^3*e^4 + 134*a*b*d*e^6 + 90*a^2*e^7 - 19*(b^2 + 2*a*c)*d^2*e^5)*f*g - (38*a*b*d^2*e^5 - 67*a^2*d*e^6 - 4*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^4 + ((168*c^2*d^3*e^4 - 750*b*c*d^2*e^5 + 2840*a*b*e^7 + 817*(b^2 + 2*a*c)*d*e^6)*f^2 - 2*(60*c^2*d^4*e^3 - 336*b*c*d^3*e^4 - 1634*a*b*d*e^6 - 1420*a^2*e^7 + 375*(b^2 + 2*a*c)*d^2*e^5)*f*g - (120*b*c*d^4*e^3 + 750*a*b*d^2*e^5 - 817*a^2*d*e^6 - 168*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^3 + 2*((330*c^2*d^3*e^4 - 951*b*c*d^2*e^5 + 3929*a*b*e^7 + 739*(b^2 + 2*a*c)*d*e^6)*f^2 - (480*c^2*d^4*e^3 - 1320*b*c*d^3*e^4 - 2956*a*b*d*e^6 - 3929*a^2*e^7 + 951*(b^2 + 2*a*c)*d^2*e^5)*f*g + (180*c^2*d^5*e^2 - 480*b*c*d^4*e^3 - 951*a*b*d^2*e^5 + 739*a^2*d*e^6 + 330*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m^2 + 12*((42*c^2*d^3*e^4 - 105*b*c*d^2*e^5 + 879*a*b*e^7 + 70*(b^2 + 2*a*c)*d*e^6)*f^2 - (70*c^2*d^4*e^3 - 168*b*c*d^3*e^4 - 280*a*b*d*e^6 - 879*a^2*e^7 + 105*(b^2 + 2*a*c)*d^2*e^5)*f*g + (30*c^2*d^5*e^2 - 70*b*c*d^4*e^3 - 105*a*b*d^2*e^5 + 70*a^2*d*e^6 + 42*(b^2 + 2*a*c)*d^3*e^4)*g^2)*m)*x^2 + 4*((78*c^2*d^5*e^2 - 321*b*c*d^4*e^3 - 1377*a*b*d^2*e^5 + 2007*a^2*d*e^6 + 319*(b^2 + 2*a*c)*d^3*e^4)*f^2 - (60*c^2*d^6*e - 312*b*c*d^5*e^2 - 1276*a*b*d^3*e^4 + 1377*a^2*d^2*e^5 + 321*(b^2 + 2*a*c)*d^4*e^3)*f*g - (60*b*c*d^6*e + 321*a*b*d^4*e^3 - 319*a^2*d^3*e^4 - 78*(b^2 + 2*a*c)*d^5*e^2)*g^2)*m + (5040*a^2*e^7*f^2 + (2*a^2*d*e^6*f*g + (2*a*b*d*e^6 + a^2*e^7)*f^2)*m^6 - (2*a^2*d^2*e^5*g^2 - (50*a*b*d*e^6 + 27*a^2*e^7 - 2*(b^2 + 2*a*c)*d^2*e^5)*f^2 + 2*(4*a*b*d^2*e^5 - 25*a^2*d*e^6)*f*g)*m^5 + ((12*b*c*d^3*e^4 + 490*a*b*d*e^6 + 295*a^2*e^7 - 44*(b^2 + 2*a*c)*d^2*e^5)*f^2 - 2*(88*a*b*d^2*e^5 - 245*a^2*d*e^6 - 6*(b^2 + 2*a*c)*d^3*e^4)*f*g + 4*(3*a*b*d^3*e^4 - 11*a^2*d^2*e^5)*g^2)*m^4 - ((24*c^2*d^4*e^3 - 216*b*c*d^3*e^4 - 2350*a*b*d*e^6 - 1665*a^2*e^7 + 358*(b^2 + 2*a*c)*d^2*e^5)*f^2 + 2*(48*b*c*d^4*e^3 + 716*a*b*d^2*e^5 - 1175*a^2*d*e^6 - 108*(b^2 + 2*a*c)*d^3*e^4)*f*g - 2*(108*a*b*d^3*e^4 - 179*a^2*d^2*e^5 - 12*(b^2 + 2*a*c)*d^4*e^3)*g^2)*m^3 - 4*((78*c^2*d^4*e^3 - 321*b*c*d^3*e^4 - 1377*a*b*d*e^6 - 1276*a^2*e^7 + 319*(b^2 + 2*a*c)*d^2*e^5)*f^2 - (60*c^2*d^5*e^2 - 312*b*c*d^4*e^3 - 1276*a*b*d^2*e^5 + 1377*a^2*d*e^6 + 321*(b^2 + 2*a*c)*d^3*e^4)*f*g - (60*b*c*d^5*e^2 + 321*a*b*d^3*e^4 - 319*a^2*d^2*e^5 - 78*(b^2 + 2*a*c)*d^4*e^3)*g^2)*m^2 - 12*((84*c^2*d^4*e^3 - 210*b*c*d^3*e^4 - 420*a*b*d*e^6 - 669*a^2*e^7 + 140*(b^2 + 2*a*c)*d^2*e^5)*f^2 - 14*(10*c^2*d^5*e^2 - 24*b*c*d^4*e^3 - 40*a*b*d^2*e^5 + 30*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^4)*f*g + 2*(30*c^2*d^6*e - 70*b*c*d^5*e^2 - 105*a*b*d^3*e^4 + 70*a^2*d^2*e^5 + 42*(b^2 + 2*a*c)*d^4*e^3)*g^2)*m)*x)*(e*x + d)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 13132*e^7*m^2 + 13068*e^7*m + 5040*e^7)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40195, size = 14160, normalized size = 26.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*g^2*m^6*x^7*e^7 + (x*e + d)^m*c^2*d*g^2*m^6*x^6*e^6 + 2*(x
*e + d)^m*c^2*f*g*m^6*x^6*e^7 + 2*(x*e + d)^m*b*c*g^2*m^6*x^6*e^7 + 21*(x*e
 + d)^m*c^2*g^2*m^5*x^7*e^7 + 2*(x*e + d)^m*c^2*d*f*g*m^6*x^5*e^6 + 2*(x*e
+ d)^m*b*c*d*g^2*m^6*x^5*e^6 + 15*(x*e + d)^m*c^2*d*g^2*m^5*x^6*e^6 - 6*(x*
e + d)^m*c^2*d^2*g^2*m^5*x^5*e^5 + (x*e + d)^m*c^2*f^2*m^6*x^5*e^7 + 4*(x*e
 + d)^m*b*c*f*g*m^6*x^5*e^7 + (x*e + d)^m*b^2*g^2*m^6*x^5*e^7 + 2*(x*e + d)
^m*a*c*g^2*m^6*x^5*e^7 + 44*(x*e + d)^m*c^2*f*g*m^5*x^6*e^7 + 44*(x*e + d)^
m*b*c*g^2*m^5*x^6*e^7 + 175*(x*e + d)^m*c^2*g^2*m^4*x^7*e^7 + (x*e + d)^m*c
^2*d*f^2*m^6*x^4*e^6 + 4*(x*e + d)^m*b*c*d*f*g*m^6*x^4*e^6 + (x*e + d)^m*b^
2*d*g^2*m^6*x^4*e^6 + 2*(x*e + d)^m*a*c*d*g^2*m^6*x^4*e^6 + 34*(x*e + d)^m*
c^2*d*f*g*m^5*x^5*e^6 + 34*(x*e + d)^m*b*c*d*g^2*m^5*x^5*e^6 + 85*(x*e + d)
^m*c^2*d*g^2*m^4*x^6*e^6 - 10*(x*e + d)^m*c^2*d^2*f*g*m^5*x^4*e^5 - 10*(x*e
 + d)^m*b*c*d^2*g^2*m^5*x^4*e^5 - 60*(x*e + d)^m*c^2*d^2*g^2*m^4*x^5*e^5 +
30*(x*e + d)^m*c^2*d^3*g^2*m^4*x^4*e^4 + 2*(x*e + d)^m*b*c*f^2*m^6*x^4*e^7
+ 2*(x*e + d)^m*b^2*f*g*m^6*x^4*e^7 + 4*(x*e + d)^m*a*c*f*g*m^6*x^4*e^7 + 2
*(x*e + d)^m*a*b*g^2*m^6*x^4*e^7 + 23*(x*e + d)^m*c^2*f^2*m^5*x^5*e^7 + 92*
(x*e + d)^m*b*c*f*g*m^5*x^5*e^7 + 23*(x*e + d)^m*b^2*g^2*m^5*x^5*e^7 + 46*(
x*e + d)^m*a*c*g^2*m^5*x^5*e^7 + 380*(x*e + d)^m*c^2*f*g*m^4*x^6*e^7 + 380*
(x*e + d)^m*b*c*g^2*m^4*x^6*e^7 + 735*(x*e + d)^m*c^2*g^2*m^3*x^7*e^7 + 2*(
x*e + d)^m*b*c*d*f^2*m^6*x^3*e^6 + 2*(x*e + d)^m*b^2*d*f*g*m^6*x^3*e^6 + 4*
(x*e + d)^m*a*c*d*f*g*m^6*x^3*e^6 + 2*(x*e + d)^m*a*b*d*g^2*m^6*x^3*e^6 + 1
9*(x*e + d)^m*c^2*d*f^2*m^5*x^4*e^6 + 76*(x*e + d)^m*b*c*d*f*g*m^5*x^4*e^6
+ 19*(x*e + d)^m*b^2*d*g^2*m^5*x^4*e^6 + 38*(x*e + d)^m*a*c*d*g^2*m^5*x^4*e
^6 + 210*(x*e + d)^m*c^2*d*f*g*m^4*x^5*e^6 + 210*(x*e + d)^m*b*c*d*g^2*m^4*
x^5*e^6 + 225*(x*e + d)^m*c^2*d*g^2*m^3*x^6*e^6 - 4*(x*e + d)^m*c^2*d^2*f^2
*m^5*x^3*e^5 - 16*(x*e + d)^m*b*c*d^2*f*g*m^5*x^3*e^5 - 4*(x*e + d)^m*b^2*d
^2*g^2*m^5*x^3*e^5 - 8*(x*e + d)^m*a*c*d^2*g^2*m^5*x^3*e^5 - 130*(x*e + d)^
m*c^2*d^2*f*g*m^4*x^4*e^5 - 130*(x*e + d)^m*b*c*d^2*g^2*m^4*x^4*e^5 - 210*(
x*e + d)^m*c^2*d^2*g^2*m^3*x^5*e^5 + 40*(x*e + d)^m*c^2*d^3*f*g*m^4*x^3*e^4
 + 40*(x*e + d)^m*b*c*d^3*g^2*m^4*x^3*e^4 + 180*(x*e + d)^m*c^2*d^3*g^2*m^3
*x^4*e^4 - 120*(x*e + d)^m*c^2*d^4*g^2*m^3*x^3*e^3 + (x*e + d)^m*b^2*f^2*m^
6*x^3*e^7 + 2*(x*e + d)^m*a*c*f^2*m^6*x^3*e^7 + 4*(x*e + d)^m*a*b*f*g*m^6*x
^3*e^7 + (x*e + d)^m*a^2*g^2*m^6*x^3*e^7 + 48*(x*e + d)^m*b*c*f^2*m^5*x^4*e
^7 + 48*(x*e + d)^m*b^2*f*g*m^5*x^4*e^7 + 96*(x*e + d)^m*a*c*f*g*m^5*x^4*e^
7 + 48*(x*e + d)^m*a*b*g^2*m^5*x^4*e^7 + 207*(x*e + d)^m*c^2*f^2*m^4*x^5*e^
7 + 828*(x*e + d)^m*b*c*f*g*m^4*x^5*e^7 + 207*(x*e + d)^m*b^2*g^2*m^4*x^5*e
^7 + 414*(x*e + d)^m*a*c*g^2*m^4*x^5*e^7 + 1640*(x*e + d)^m*c^2*f*g*m^3*x^6
*e^7 + 1640*(x*e + d)^m*b*c*g^2*m^3*x^6*e^7 + 1624*(x*e + d)^m*c^2*g^2*m^2*
```

x^7*e^7 + (x*e + d)^m*b^2*d*f^2*m^6*x^2*e^6 + 2*(x*e + d)^m*a*c*d*f^2*m^6*x^2*e^6 + 4*(x*e + d)^m*a*b*d*f*g*m^6*x^2*e^6 + (x*e + d)^m*a^2*d*g^2*m^6*x^2*e^6 + 42*(x*e + d)^m*b*c*d*f^2*m^5*x^3*e^6 + 42*(x*e + d)^m*b^2*d*f*g*m^5*x^3*e^6 + 84*(x*e + d)^m*a*c*d*f*g*m^5*x^3*e^6 + 42*(x*e + d)^m*a*b*d*g^2*m^5*x^3*e^6 + 131*(x*e + d)^m*c^2*d*f^2*m^4*x^4*e^6 + 524*(x*e + d)^m*b*c*d*f*g*m^4*x^4*e^6 + 131*(x*e + d)^m*b^2*d*g^2*m^4*x^4*e^6 + 262*(x*e + d)^m*a*c*d*g^2*m^4*x^4*e^6 + 590*(x*e + d)^m*c^2*d*f*g*m^3*x^5*e^6 + 590*(x*e + d)^m*b*c*d*g^2*m^3*x^5*e^6 + 274*(x*e + d)^m*c^2*d*g^2*m^2*x^6*e^6 - 6*(x*e + d)^m*b*c*d^2*f^2*m^5*x^2*e^5 - 6*(x*e + d)^m*b^2*d^2*f*g*m^5*x^2*e^5 - 12*(x*e + d)^m*a*c*d^2*f*g*m^5*x^2*e^5 - 6*(x*e + d)^m*a*b*d^2*g^2*m^5*x^2*e^5 - 64*(x*e + d)^m*c^2*d^2*f^2*m^4*x^3*e^5 - 256*(x*e + d)^m*b*c*d^2*f*g*m^4*x^3*e^5 - 64*(x*e + d)^m*b^2*d^2*g^2*m^4*x^3*e^5 - 128*(x*e + d)^m*a*c*d^2*g^2*m^4*x^3*e^5 - 530*(x*e + d)^m*c^2*d^2*f*g*m^3*x^4*e^5 - 530*(x*e + d)^m*b*c*d^2*g^2*m^3*x^4*e^5 - 300*(x*e + d)^m*c^2*d^2*g^2*m^2*x^5*e^5 + 12*(x*e + d)^m*c^2*d^3*f^2*m^4*x^2*e^4 + 48*(x*e + d)^m*b*c*d^3*f*g*m^4*x^2*e^4 + 12*(x*e + d)^m*b^2*d^3*g^2*m^4*x^2*e^4 + 24*(x*e + d)^m*a*c*d^3*g^2*m^4*x^2*e^4 + 400*(x*e + d)^m*c^2*d^3*f*g*m^3*x^3*e^4 + 400*(x*e + d)^m*b*c*d^3*g^2*m^3*x^3*e^4 + 330*(x*e + d)^m*c^2*d^3*g^2*m^2*x^4*e^4 - 120*(x*e + d)^m*c^2*d^4*f*g*m^3*x^2*e^3 - 120*(x*e + d)^m*b*c*d^4*g^2*m^3*x^2*e^3 - 360*(x*e + d)^m*c^2*d^4*g^2*m^2*x^3*e^3 + 360*(x*e + d)^m*c^2*d^5*g^2*m^2*x^2*e^2 + 2*(x*e + d)^m*a*b*f^2*m^6*x^2*e^7 + 2*(x*e + d)^m*a^2*f*g*m^6*x^2*e^7 + 25*(x*e + d)^m*b^2*f^2*m^5*x^3*e^7 + 50*(x*e + d)^m*a*c*f^2*m^5*x^3*e^7 + 100*(x*e + d)^m*a*b*f*g*m^5*x^3*e^7 + 25*(x*e + d)^m*a^2*g^2*m^5*x^3*e^7 + 452*(x*e + d)^m*b*c*f^2*m^4*x^4*e^7 + 452*(x*e + d)^m*b^2*f*g*m^4*x^4*e^7 + 904*(x*e + d)^m*a*c*f*g*m^4*x^4*e^7 + 452*(x*e + d)^m*a*b*g^2*m^4*x^4*e^7 + 925*(x*e + d)^m*c^2*f^2*m^3*x^5*e^7 + 3700*(x*e + d)^m*b*c*f*g*m^3*x^5*e^7 + 925*(x*e + d)^m*b^2*g^2*m^3*x^5*e^7 + 1850*(x*e + d)^m*a*c*g^2*m^3*x^5*e^7 + 3698*(x*e + d)^m*c^2*f*g*m^2*x^6*e^7 + 3698*(x*e + d)^m*b*c*g^2*m^2*x^6*e^7 + 1764*(x*e + d)^m*c^2*g^2*m*x^7*e^7 + 2*(x*e + d)^m*a*b*d*f^2*m^6*x*e^6 + 2*(x*e + d)^m*a^2*d*f*g*m^6*x*e^6 + 23*(x*e + d)^m*b^2*d*f^2*m^5*x^2*e^6 + 46*(x*e + d)^m*a*c*d*f^2*m^5*x^2*e^6 + 92*(x*e + d)^m*a*b*d*f*g*m^5*x^2*e^6 + 23*(x*e + d)^m*a^2*d*g^2*m^5*x^2*e^6 + 326*(x*e + d)^m*b*c*d*f^2*m^4*x^3*e^6 + 326*(x*e + d)^m*b^2*d*f*g*m^4*x^3*e^6 + 652*(x*e + d)^m*a*c*d*f*g*m^4*x^3*e^6 + 326*(x*e + d)^m*a*b*d*g^2*m^4*x^3*e^6 + 401*(x*e + d)^m*c^2*d*f^2*m^3*x^4*e^6 + 1604*(x*e + d)^m*b*c*d*f*g*m^3*x^4*e^6 + 401*(x*e + d)^m*b^2*d*g^2*m^3*x^4*e^6 + 802*(x*e + d)^m*a*c*d*g^2*m^3*x^4*e^6 + 748*(x*e + d)^m*c^2*d*f*g*m^2*x^5*e^6 + 748*(x*e + d)^m*b*c*d*g^2*m^2*x^5*e^6 + 120*(x*e + d)^m*c^2*d*g^2*m*x^6*e^6 - 2*(x*e + d)^m*b^2*d^2*f^2*m^5*x*e^5 - 4*(x*e + d)^m*a*c*d^2*f^2*m^5*x*e^5 - 8*(x*e + d)^m*a*b*d^2*f*g*m^5*x*e^5 - 2*(x*e + d)^m*a^2*d^2*g^2*m^5*x*e^5 - 114*(x*e + d)^m*b*c*d^2*f^2*m^4*x^2*e^5 - 114*(x*e + d)^m*b^2*d^2*f*g*m^4*x^2*e^5 - 228*(x*e + d)^m*a*c*d^2*f*g*m^4*x^2*e^5 - 114*(x*e + d)^m*a*b*d^2*g^2*m^4*x^2*e^5 - 332*(x*e + d)^m*c^2*d^2*f^2*m^3*x^3*e^5 - 1328*(x*e + d)^m*b*c*d^2*f*g*m^3*x^3*e^5 - 332*(x*e + d)^m*b^2*d^2*g^2*m^3*x^3*e^5 - 664*(x*e + d)^m*a*c*d^2*g^2*m^3*x^3*e^5 - 830*(x*e + d)^m*c^2*d^2*f*g*m^2*x^4*e^5 - 830*(x*e + d)^m*b*c*d^2*g^2*m^2*x^4*e^5 - 144*(x*e + d)^m*c^2*d^2*g^2*m*x^5*e^5 + 12*(x*e + d)^m*b*c*d^3*f^2*m^4*x*e^4 + 12*(x*e + d)^m*b^2*d^3*f*g*m^4*x*e^4 + 24*(x*e + d)^m*a*c*d^3*f*g*m^4*x*e^4 + 12*(x*e + d)^m*a*b*d^3*g^2*m^4*x*e^4 + 168*(x*e + d)^m*c^2*d^3*f^2*m^3*x^2*e^4 + 672*(x*e + d)^m*b*c*d^3*f*g*m^3*x^2*e^4 + 168*(x*e + d)^m*b^2*d^3*g^2*m^3*x^2*e^4 + 336*(x*e + d)^m*a*c*d^3*g^2*m^3*x^2*e^4 + 920*(x*e + d)^m*c^2*d^3*f*g*m^2*x^3*e^4 + 920*(x*e + d)^m*b*c*d^3*g^2*m^2*x^3*e^4 + 180*(x*e + d)^m*c^2*d^3*g^2*m*x^4*e^4 - 24*(x*e + d)^m*c^2*d^4*f^2*m^3*x*e^3 - 96*(x*e + d)^m*b*c*d^4*f*g*m^3*x*e^3 - 24*(x*e + d)^m*b^2*d^4*g^2*m^3*x*e^3 - 48*(x*e + d)^m*a*c*d^4*g^2*m^3*x*e^3 - 960*(x*e + d)^m*c^2*d^4*f*g*m^2*x^2*e^3 - 960*(x*e + d)^m*b*c*d^4*g^2*m^2*x^2*e^3 - 240*(x*e + d)^m*c^2*d^4*g^2*m*x^3*e^3 + 240*(x*e + d)^m*c^2*d^5*f*g*m^2*x*e^2 + 240*(x*e + d)^m*b*c*d^5*g^2*m^2*x*e^2 + 360*(x*e + d)^m*c^2*d^5*g^2*m*x^2*e^2 - 720*(x*e + d)^m*c^2*d^6*g^2*m*x*e + (x*e + d)^m*a^2*f^2*m^6*x*e^7 + 52*(x*e + d)^m*a*b*f^2*m^5*x^2*e^7 + 52*(x*e + d)^m*a^2*f*g*m^5*x^2*e^7 + 247*(x*e

+ d)^m*b^2*f^2*m^4*x^3*e^7 + 494*(x*e + d)^m*a*c*f^2*m^4*x^3*e^7 + 988*(x*e + d)^m*a*b*f*g*m^4*x^3*e^7 + 247*(x*e + d)^m*a^2*g^2*m^4*x^3*e^7 + 2112*(x*e + d)^m*b*c*f^2*m^3*x^4*e^7 + 2112*(x*e + d)^m*b^2*f*g*m^3*x^4*e^7 + 4224*(x*e + d)^m*a*c*f*g*m^3*x^4*e^7 + 2112*(x*e + d)^m*a*b*g^2*m^3*x^4*e^7 + 2144*(x*e + d)^m*c^2*f^2*m^2*x^5*e^7 + 8576*(x*e + d)^m*b*c*f*g*m^2*x^5*e^7 + 2144*(x*e + d)^m*b^2*g^2*m^2*x^5*e^7 + 4288*(x*e + d)^m*a*c*g^2*m^2*x^5*e^7 + 4076*(x*e + d)^m*c^2*f*g*m*x^6*e^7 + 4076*(x*e + d)^m*b*c*g^2*m*x^6*e^7 + 720*(x*e + d)^m*c^2*g^2*x^7*e^7 + (x*e + d)^m*a^2*d*f^2*m^6*e^6 + 50*(x*e + d)^m*a*b*d*f^2*m^5*x*e^6 + 50*(x*e + d)^m*a^2*d*f*g*m^5*x*e^6 + 201*(x*e + d)^m*b^2*d*f^2*m^4*x^2*e^6 + 402*(x*e + d)^m*a*c*d*f^2*m^4*x^2*e^6 + 804*(x*e + d)^m*a*b*d*f*g*m^4*x^2*e^6 + 201*(x*e + d)^m*a^2*d*g^2*m^4*x^2*e^6 + 1134*(x*e + d)^m*b*c*d*f^2*m^3*x^3*e^6 + 1134*(x*e + d)^m*b^2*d*f*g*m^3*x^3*e^6 + 2268*(x*e + d)^m*a*c*d*f*g*m^3*x^3*e^6 + 1134*(x*e + d)^m*a*b*d*g^2*m^3*x^3*e^6 + 540*(x*e + d)^m*c^2*d*f^2*m^2*x^4*e^6 + 2160*(x*e + d)^m*b*c*d*f*g*m^2*x^4*e^6 + 540*(x*e + d)^m*b^2*d*g^2*m^2*x^4*e^6 + 1080*(x*e + d)^m*a*c*d*g^2*m^2*x^4*e^6 + 336*(x*e + d)^m*c^2*d*f*g*m*x^5*e^6 + 336*(x*e + d)^m*b*c*d*g^2*m*x^5*e^6 - 2*(x*e + d)^m*a*b*d^2*f^2*m^5*e^5 - 2*(x*e + d)^m*a^2*d^2*f*g*m^5*e^5 - 44*(x*e + d)^m*b^2*d^2*f^2*m^4*x*e^5 - 88*(x*e + d)^m*a*c*d^2*f^2*m^4*x*e^5 - 176*(x*e + d)^m*a*b*d^2*f*g*m^4*x*e^5 - 44*(x*e + d)^m*a^2*d^2*g^2*m^4*x*e^5 - 750*(x*e + d)^m*b*c*d^2*f^2*m^3*x^2*e^5 - 750*(x*e + d)^m*b^2*d^2*f*g*m^3*x^2*e^5 - 1500*(x*e + d)^m*a*c*d^2*f*g*m^3*x^2*e^5 - 750*(x*e + d)^m*a*b*d^2*g^2*m^3*x^2*e^5 - 608*(x*e + d)^m*c^2*d^2*f^2*m^2*x^3*e^5 - 2432*(x*e + d)^m*b*c*d^2*f*g*m^2*x^3*e^5 - 608*(x*e + d)^m*b^2*d^2*g^2*m^2*x^3*e^5 - 1216*(x*e + d)^m*a*c*d^2*g^2*m^2*x^3*e^5 - 420*(x*e + d)^m*c^2*d^2*f*g*m*x^4*e^5 - 420*(x*e + d)^m*b*c*d^2*g^2*m*x^4*e^5 + 2*(x*e + d)^m*b^2*d^3*f^2*m^4*e^4 + 4*(x*e + d)^m*a*c*d^3*f^2*m^4*e^4 + 8*(x*e + d)^m*a*b*d^3*f*g*m^4*e^4 + 2*(x*e + d)^m*a^2*d^3*g^2*m^4*e^4 + 216*(x*e + d)^m*b*c*d^3*f^2*m^3*x*e^4 + 216*(x*e + d)^m*b^2*d^3*f*g*m^3*x*e^4 + 432*(x*e + d)^m*a*c*d^3*f*g*m^3*x*e^4 + 216*(x*e + d)^m*a*b*d^3*g^2*m^3*x*e^4 + 660*(x*e + d)^m*c^2*d^3*f^2*m^2*x^2*e^4 + 2640*(x*e + d)^m*b*c*d^3*f*g*m^2*x^2*e^4 + 660*(x*e + d)^m*b^2*d^3*g^2*m^2*x^2*e^4 + 1320*(x*e + d)^m*a*c*d^3*g^2*m^2*x^2*e^4 + 560*(x*e + d)^m*c^2*d^3*f*g*m*x^3*e^4 + 560*(x*e + d)^m*b*c*d^3*g^2*m*x^3*e^4 - 12*(x*e + d)^m*b*c*d^4*f^2*m^3*e^3 - 12*(x*e + d)^m*b^2*d^4*f*g*m^3*e^3 - 24*(x*e + d)^m*a*c*d^4*f*g*m^3*e^3 - 12*(x*e + d)^m*a*b*d^4*g^2*m^3*e^3 - 312*(x*e + d)^m*c^2*d^4*f^2*m^2*x*e^3 - 1248*(x*e + d)^m*b*c*d^4*f*g*m^2*x*e^3 - 312*(x*e + d)^m*b^2*d^4*g^2*m^2*x*e^3 - 624*(x*e + d)^m*a*c*d^4*g^2*m^2*x*e^3 - 840*(x*e + d)^m*c^2*d^4*f*g*m*x^2*e^3 - 840*(x*e + d)^m*b*c*d^4*g^2*m*x^2*e^3 + 24*(x*e + d)^m*c^2*d^5*f^2*m^2*e^2 + 96*(x*e + d)^m*b*c*d^5*f*g*m^2*e^2 + 24*(x*e + d)^m*b^2*d^5*g^2*m^2*e^2 + 48*(x*e + d)^m*a*c*d^5*g^2*m^2*e^2 + 1680*(x*e + d)^m*c^2*d^5*f*g*m*x*e^2 + 1680*(x*e + d)^m*b*c*d^5*g^2*m*x*e^2 - 240*(x*e + d)^m*c^2*d^6*f*g*m*e - 240*(x*e + d)^m*b*c*d^6*g^2*m*e + 720*(x*e + d)^m*c^2*d^7*g^2 + 27*(x*e + d)^m*a^2*f^2*m^5*x*e^7 + 540*(x*e + d)^m*a*b*f^2*m^4*x^2*e^7 + 540*(x*e + d)^m*a^2*f*g*m^4*x^2*e^7 + 1219*(x*e + d)^m*b^2*f^2*m^3*x^3*e^7 + 2438*(x*e + d)^m*a*c*f^2*m^3*x^3*e^7 + 4876*(x*e + d)^m*a*b*f*g*m^3*x^3*e^7 + 1219*(x*e + d)^m*a^2*g^2*m^3*x^3*e^7 + 5090*(x*e + d)^m*b*c*f^2*m^2*x^4*e^7 + 5090*(x*e + d)^m*b^2*f*g*m^2*x^4*e^7 + 10180*(x*e + d)^m*a*c*f*g*m^2*x^4*e^7 + 5090*(x*e + d)^m*a*b*g^2*m^2*x^4*e^7 + 2412*(x*e + d)^m*c^2*f^2*m*x^5*e^7 + 9648*(x*e + d)^m*b*c*f*g*m*x^5*e^7 + 2412*(x*e + d)^m*b^2*g^2*m*x^5*e^7 + 4824*(x*e + d)^m*a*c*g^2*m*x^5*e^7 + 1680*(x*e + d)^m*c^2*f*g*x^6*e^7 + 1680*(x*e + d)^m*b*c*g^2*x^6*e^7 + 27*(x*e + d)^m*a^2*d*f^2*m^5*e^6 + 490*(x*e + d)^m*a*b*d*f^2*m^4*x*e^6 + 490*(x*e + d)^m*a^2*d*f*g*m^4*x*e^6 + 817*(x*e + d)^m*b^2*d*f^2*m^3*x^2*e^6 + 1634*(x*e + d)^m*a*c*d*f^2*m^3*x^2*e^6 + 3268*(x*e + d)^m*a*b*d*f*g*m^3*x^2*e^6 + 817*(x*e + d)^m*a^2*d*g^2*m^3*x^2*e^6 + 1688*(x*e + d)^m*b*c*d*f^2*m^2*x^3*e^6 + 1688*(x*e + d)^m*b^2*d*f*g*m^2*x^3*e^6 + 3376*(x*e + d)^m*a*c*d*f*g*m^2*x^3*e^6 + 1688*(x*e + d)^m*a*b*d*g^2*m^2*x^3*e^6 + 252*(x*e + d)^m*c^2*d*f^2*m*x^4*e^6 + 1008*(x*e + d)^m*b*c*d*f*g*m*x^4*e^6 + 252*(x*e + d)^m*b^2*d*g^2*m*x^4*e^6 + 504*(x*e + d)^m*a*c*d*g^2*m*x^4*e^6 - 50*(x*e + d)^m*a*b*d^2*f^2*m^4*e^5 - 50*(x*e +

d)^m*a^2*d^2*f*g*m^4*e^5 - 358*(x*e + d)^m*b^2*d^2*f^2*m^3*x*e^5 - 716*(x*e + d)^m*a*c*d^2*f^2*m^3*x*e^5 - 1432*(x*e + d)^m*a*b*d^2*f*g*m^3*x*e^5 - 358*(x*e + d)^m*a^2*d^2*g^2*m^3*x*e^5 - 1902*(x*e + d)^m*b*c*d^2*f^2*m^2*x^2*e^5 - 1902*(x*e + d)^m*b^2*d^2*f*g*m^2*x^2*e^5 - 3804*(x*e + d)^m*a*c*d^2*f*g*m^2*x^2*e^5 - 1902*(x*e + d)^m*a*b*d^2*g^2*m^2*x^2*e^5 - 336*(x*e + d)^m*c^2*d^2*f^2*m*x^3*e^5 - 1344*(x*e + d)^m*b*c*d^2*f*g*m*x^3*e^5 - 336*(x*e + d)^m*b^2*d^2*g^2*m*x^3*e^5 - 672*(x*e + d)^m*a*c*d^2*g^2*m*x^3*e^5 + 44*(x*e + d)^m*b^2*d^3*f^2*m^3*e^4 + 88*(x*e + d)^m*a*c*d^3*f^2*m^3*e^4 + 176*(x*e + d)^m*a*b*d^3*f*g*m^3*e^4 + 44*(x*e + d)^m*a^2*d^3*g^2*m^3*e^4 + 1284*(x*e + d)^m*b*c*d^3*f^2*m^2*x*e^4 + 1284*(x*e + d)^m*b^2*d^3*f*g*m^2*x*e^4 + 2568*(x*e + d)^m*a*c*d^3*f*g*m^2*x*e^4 + 1284*(x*e + d)^m*a*b*d^3*g^2*m^2*x*e^4 + 504*(x*e + d)^m*c^2*d^3*f^2*m*x^2*e^4 + 2016*(x*e + d)^m*b*c*d^3*f*g*m*x^2*e^4 + 504*(x*e + d)^m*b^2*d^3*g^2*m*x^2*e^4 + 1008*(x*e + d)^m*a*c*d^3*g^2*m*x^2*e^4 - 216*(x*e + d)^m*b*c*d^4*f^2*m^2*e^3 - 216*(x*e + d)^m*b^2*d^4*f*g*m^2*e^3 - 432*(x*e + d)^m*a*c*d^4*f*g*m^2*e^3 - 216*(x*e + d)^m*a*b*d^4*g^2*m^2*e^3 - 1008*(x*e + d)^m*c^2*d^4*f^2*m*x*e^3 - 4032*(x*e + d)^m*b*c*d^4*f*g*m*x*e^3 - 1008*(x*e + d)^m*b^2*d^4*g^2*m*x*e^3 - 2016*(x*e + d)^m*a*c*d^4*g^2*m*x*e^3 + 312*(x*e + d)^m*c^2*d^5*f^2*m*e^2 + 1248*(x*e + d)^m*b*c*d^5*f*g*m*e^2 + 312*(x*e + d)^m*b^2*d^5*g^2*m*e^2 + 624*(x*e + d)^m*a*c*d^5*g^2*m*e^2 - 1680*(x*e + d)^m*c^2*d^6*f*g*e - 1680*(x*e + d)^m*b*c*d^6*g^2*e + 295*(x*e + d)^m*a^2*f^2*m^4*x*e^7 + 2840*(x*e + d)^m*a*b*f^2*m^3*x^2*e^7 + 2840*(x*e + d)^m*a^2*f*g*m^3*x^2*e^7 + 3112*(x*e + d)^m*b^2*f^2*m^2*x^3*e^7 + 6224*(x*e + d)^m*a*c*f^2*m^2*x^3*e^7 + 12448*(x*e + d)^m*a*b*f*g*m^2*x^3*e^7 + 3112*(x*e + d)^m*a^2*g^2*m^2*x^3*e^7 + 5904*(x*e + d)^m*b*c*f^2*m*x^4*e^7 + 5904*(x*e + d)^m*b^2*f*g*m*x^4*e^7 + 11808*(x*e + d)^m*a*c*f*g*m*x^4*e^7 + 5904*(x*e + d)^m*a*b*g^2*m*x^4*e^7 + 1008*(x*e + d)^m*c^2*f^2*x^5*e^7 + 4032*(x*e + d)^m*b*c*f*g*x^5*e^7 + 1008*(x*e + d)^m*b^2*g^2*x^5*e^7 + 2016*(x*e + d)^m*a*c*g^2*x^5*e^7 + 295*(x*e + d)^m*a^2*d*f^2*m^4*e^6 + 2350*(x*e + d)^m*a*b*d*f^2*m^3*x*e^6 + 2350*(x*e + d)^m*a^2*d*f*g*m^3*x*e^6 + 1478*(x*e + d)^m*b^2*d*f^2*m^2*x^2*e^6 + 2956*(x*e + d)^m*a*c*d*f^2*m^2*x^2*e^6 + 5912*(x*e + d)^m*a*b*d*f*g*m^2*x^2*e^6 + 1478*(x*e + d)^m*a^2*d*g^2*m^2*x^2*e^6 + 840*(x*e + d)^m*b*c*d*f^2*m*x^3*e^6 + 840*(x*e + d)^m*b^2*d*f*g*m*x^3*e^6 + 1680*(x*e + d)^m*a*c*d*f*g*m*x^3*e^6 + 840*(x*e + d)^m*a*b*d*g^2*m*x^3*e^6 - 490*(x*e + d)^m*a*b*d^2*f^2*m^3*e^5 - 490*(x*e + d)^m*a^2*d^2*f*g*m^3*e^5 - 1276*(x*e + d)^m*b^2*d^2*f^2*m^2*x*e^5 - 2552*(x*e + d)^m*a*c*d^2*f^2*m^2*x*e^5 - 5104*(x*e + d)^m*a*b*d^2*f*g*m^2*x*e^5 - 1276*(x*e + d)^m*a^2*d^2*g^2*m^2*x*e^5 - 1260*(x*e + d)^m*b*c*d^2*f^2*m*x^2*e^5 - 1260*(x*e + d)^m*b^2*d^2*f*g*m*x^2*e^5 - 2520*(x*e + d)^m*a*c*d^2*f*g*m*x^2*e^5 - 1260*(x*e + d)^m*a*b*d^2*g^2*m*x^2*e^5 + 358*(x*e + d)^m*b^2*d^3*f^2*m^2*e^4 + 716*(x*e + d)^m*a*c*d^3*f^2*m^2*e^4 + 1432*(x*e + d)^m*a*b*d^3*f*g*m^2*e^4 + 358*(x*e + d)^m*a^2*d^3*g^2*m^2*e^4 + 2520*(x*e + d)^m*b*c*d^3*f^2*m*x*e^4 + 2520*(x*e + d)^m*b^2*d^3*f*g*m*x*e^4 + 5040*(x*e + d)^m*a*c*d^3*f*g*m*x*e^4 + 2520*(x*e + d)^m*a*b*d^3*g^2*m*x*e^4 - 1284*(x*e + d)^m*b*c*d^4*f^2*m*e^3 - 1284*(x*e + d)^m*b^2*d^4*f*g*m*e^3 - 2568*(x*e + d)^m*a*c*d^4*f*g*m*e^3 - 1284*(x*e + d)^m*a*b*d^4*g^2*m*e^3 + 1008*(x*e + d)^m*c^2*d^5*f^2*e^2 + 4032*(x*e + d)^m*b*c*d^5*f*g*e^2 + 1008*(x*e + d)^m*b^2*d^5*g^2*e^2 + 2016*(x*e + d)^m*a*c*d^5*g^2*e^2 + 1665*(x*e + d)^m*a^2*f^2*m^3*x*e^7 + 7858*(x*e + d)^m*a*b*f^2*m^2*x^2*e^7 + 7858*(x*e + d)^m*a^2*f*g*m^2*x^2*e^7 + 3796*(x*e + d)^m*b^2*f^2*m*x^3*e^7 + 7592*(x*e + d)^m*a*c*f^2*m*x^3*e^7 + 15184*(x*e + d)^m*a*b*f*g*m*x^3*e^7 + 3796*(x*e + d)^m*a^2*g^2*m*x^3*e^7 + 2520*(x*e + d)^m*b*c*f^2*x^4*e^7 + 2520*(x*e + d)^m*b^2*f*g*x^4*e^7 + 5040*(x*e + d)^m*a*c*f*g*x^4*e^7 + 2520*(x*e + d)^m*a*b*g^2*x^4*e^7 + 1665*(x*e + d)^m*a^2*d*f^2*m^3*e^6 + 5508*(x*e + d)^m*a*b*d*f^2*m^2*x*e^6 + 5508*(x*e + d)^m*a^2*d*f*g*m^2*x*e^6 + 840*(x*e + d)^m*b^2*d*f^2*m*x^2*e^6 + 1680*(x*e + d)^m*a*c*d*f^2*m*x^2*e^6 + 3360*(x*e + d)^m*a*b*d*f*g*m*x^2*e^6 + 840*(x*e + d)^m*a^2*d*g^2*m*x^2*e^6 - 2350*(x*e + d)^m*a*b*d^2*f^2*m^2*e^5 - 2350*(x*e + d)^m*a^2*d^2*f*g*m^2*e^5 - 1680*(x*e + d)^m*b^2*d^2*f^2*m*x*e^5 - 3360*(x*e + d)^m*a*c*d^2*f^2*m*x*e^5 - 6720*(x*e + d)^m*a*b*d^2*f*g*m*x*e^5 - 1680*(x*e + d)^m*a^2*d^2*g^2*m*x*e^5 + 1276*(x*e

+ d)^m*b^2*d^3*f^2*m*e^4 + 2552*(x*e + d)^m*a*c*d^3*f^2*m*e^4 + 5104*(x*e + d)^m*a*b*d^3*f*g*m*e^4 + 1276*(x*e + d)^m*a^2*d^3*g^2*m*e^4 - 2520*(x*e + d)^m*b*c*d^4*f^2*e^3 - 2520*(x*e + d)^m*b^2*d^4*f*g*e^3 - 5040*(x*e + d)^m*a*c*d^4*f*g*e^3 - 2520*(x*e + d)^m*a*b*d^4*g^2*e^3 + 5104*(x*e + d)^m*a^2*f^2*m^2*x*e^7 + 10548*(x*e + d)^m*a*b*f^2*m*x^2*e^7 + 10548*(x*e + d)^m*a^2*f*g*m*x^2*e^7 + 1680*(x*e + d)^m*b^2*f^2*x^3*e^7 + 3360*(x*e + d)^m*a*c*f^2*x^3*e^7 + 6720*(x*e + d)^m*a*b*f*g*x^3*e^7 + 1680*(x*e + d)^m*a^2*g^2*x^3*e^7 + 5104*(x*e + d)^m*a^2*d*f^2*m^2*e^6 + 5040*(x*e + d)^m*a*b*d*f^2*m*x*e^6 + 5040*(x*e + d)^m*a^2*d*f*g*m*x*e^6 - 5508*(x*e + d)^m*a*b*d^2*f^2*m*e^5 - 5508*(x*e + d)^m*a^2*d^2*f*g*m*e^5 + 1680*(x*e + d)^m*b^2*d^3*f^2*e^4 + 3360*(x*e + d)^m*a*c*d^3*f^2*e^4 + 6720*(x*e + d)^m*a*b*d^3*f*g*e^4 + 1680*(x*e + d)^m*a^2*d^3*g^2*e^4 + 8028*(x*e + d)^m*a^2*f^2*m*x*e^7 + 5040*(x*e + d)^m*a*b*f^2*x^2*e^7 + 5040*(x*e + d)^m*a^2*f*g*x^2*e^7 + 8028*(x*e + d)^m*a^2*d*f^2*m*e^6 - 5040*(x*e + d)^m*a*b*d^2*f^2*e^5 - 5040*(x*e + d)^m*a^2*d^2*f*g*e^5 + 5040*(x*e + d)^m*a^2*f^2*x*e^7 + 5040*(x*e + d)^m*a^2*d*f^2*e^6)/(m^7*e^7 + 28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^7 + 13132*m^2*e^7 + 13068*m*e^7 + 5040*e^7)

3.926 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=311

$$\frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))+be^2(\ldots\right.}{e^6(m+3)}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) - (
(c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e*g
))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^2*(
b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*d*g)
))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5*d*g)
 + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m)) + (c*(
c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(d + e
*x)^(6 + m))/(e^6*(6 + m))
```

Rubi [A] time = 0.392187, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {771}

$$\frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))+be^2(\ldots\right.}{e^6(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) - (
(c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e*g
))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^2*(
b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*d*g)
))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5*d*g)
 + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m)) + (c*(
c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(d + e
*x)^(6 + m))/(e^6*(6 + m))
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^m}{e^5}+\frac{\left(cd^2-bde+ae^2\right)(-cd(4ef-5dg)\ldots}{\ldots}\right.$$

$$=\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^{1+m}}{e^6(1+m)}-\frac{\left(cd^2-bde+ae^2\right)(cd(4ef-5dg)-e\ldots}{e^6(2+\ldots}$$

Mathematica [B] time = 1.53982, size = 655, normalized size = 2.11

$$\frac{(d+ex)^{m+1}\left(\frac{2\left(\frac{(d+ex)\left(c^2e^2\left(4a^2e^2g\left(m^2+8m+15\right)+2abe\left(dg\left(4m^2+11m-18\right)+ef\left(2m^2+19m+42\right)\right)+b^2d\left(dg\left(m^2-13m+6\right)+2ef\left(m^2+5m-6\right)\right)\right)-b^2ce^3(m+2)(aeg(5m+21)+bdg(2m-3)+be\cdots}{e^2(m+2)}\cdots}{\cdots}\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + x*(b + c*x))^2*(2*b*e*g + c*(-5*d*g + e*f*(6 + m)
+ e*g*(5 + m)*x)) + (2*(((c*d^2 + e*(-(b*d) + a*e))*(b^3*e^3*g*(3 + 4*m + m
^2) + 12*c^3*d^2*(-5*d*g + e*f*(6 + m)) - b*c*e^2*(1 + m)*(b*d*g*(-6 + m) +
 b*e*f*(6 + m) + 2*a*e*g*(9 + 2*m)) + 2*c^2*e*(-3*b*d*(d*g*(-9 + m) + 2*e*f
*(6 + m)) + 2*a*e*(d*g*(-15 + m + m^2) + e*f*(24 + 10*m + m^2)))))/(e^2*(1
+ m)) + ((b^4*e^4*g*(6 + 5*m + m^2) + 12*c^4*d^3*(5*d*g - e*f*(6 + m)) - b^
2*c*e^3*(2 + m)*(b*e*f*(6 + m) + b*d*g*(-3 + 2*m) + a*e*g*(21 + 5*m)) + 2*c
^3*d*e*(3*b*d*(d*g*(-14 + m) + 3*e*f*(6 + m)) - 2*a*e*(d*g*(-30 - 4*m + m^2
) + e*f*(42 + 19*m + 2*m^2))) + c^2*e^2*(4*a^2*e^2*g*(15 + 8*m + m^2) + b^2
*d*(d*g*(6 - 13*m + m^2) + 2*e*f*(-6 + 5*m + m^2)) + 2*a*b*e*(e*f*(42 + 19*
m + 2*m^2) + d*g*(-18 + 11*m + 4*m^2))))*(d + e*x))/(e^2*(2 + m)) - (c*e*(4
 + m)*(b*d*(-5*c*d + 2*b*e)*g - 2*a*c*d*e*g*m + a*b*e^2*g*(1 + m) + c*e*(b*
d - 2*a*e)*f*(6 + m)) - (3*c*d - b*e)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g
+ e*f*(6 + m)) - c*e*(b*d*g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m))) +
c*e*(3 + m)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g + e*f*(6 + m)) - c*e*(b*d*
g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m)))*x)*(a + x*(b + c*x))))/(c*e^
2*(3 + m)*(4 + m))))/(c*e^2*(5 + m)*(6 + m))
```

Maple [B] time = 0.054, size = 2563, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x)
```

```
[Out] -(e*x+d)^(1+m)*(-c^2*e^5*g*m^5*x^5-2*b*c*e^5*g*m^5*x^4-c^2*e^5*f*m^5*x^4-15
*c^2*e^5*g*m^4*x^5-2*a*c*e^5*g*m^5*x^3-b^2*e^5*g*m^5*x^3-2*b*c*e^5*f*m^5*x^
3-32*b*c*e^5*g*m^4*x^4+5*c^2*d*e^4*g*m^4*x^4-16*c^2*e^5*f*m^4*x^4-85*c^2*e^
5*g*m^3*x^5-2*a*b*e^5*g*m^5*x^2-2*a*c*e^5*f*m^5*x^2-34*a*c*e^5*g*m^4*x^3-b^
2*e^5*f*m^5*x^2-17*b^2*e^5*g*m^4*x^3+8*b*c*d*e^4*g*m^4*x^3-34*b*c*e^5*f*m^4
*x^3-190*b*c*e^5*g*m^3*x^4+4*c^2*d*e^4*f*m^4*x^3+50*c^2*d*e^4*g*m^3*x^4-95*
c^2*e^5*f*m^3*x^4-225*c^2*e^5*g*m^2*x^5-a^2*e^5*g*m^5*x-2*a*b*e^5*f*m^5*x-3
6*a*b*e^5*g*m^4*x^2+6*a*c*d*e^4*g*m^4*x^2-36*a*c*e^5*f*m^4*x^2-214*a*c*e^5*
g*m^3*x^3+3*b^2*d*e^4*g*m^4*x^2-18*b^2*e^5*f*m^4*x^2-107*b^2*e^5*g*m^3*x^3+
6*b*c*d*e^4*f*m^4*x^2+96*b*c*d*e^4*g*m^3*x^3-214*b*c*e^5*f*m^3*x^3-520*b*c*
e^5*g*m^2*x^4-20*c^2*d^2*e^3*g*m^3*x^3+48*c^2*d*e^4*f*m^3*x^3+175*c^2*d*e^4
*g*m^2*x^4-260*c^2*e^5*f*m^2*x^4-274*c^2*e^5*g*m*x^5-a^2*e^5*f*m^5-19*a^2*e
^5*g*m^4*x+4*a*b*d*e^4*g*m^4*x-38*a*b*e^5*f*m^4*x-242*a*b*e^5*g*m^3*x^2+4*a
*c*d*e^4*f*m^4*x+84*a*c*d*e^4*g*m^3*x^2-242*a*c*e^5*f*m^3*x^2-614*a*c*e^5*g
*m^2*x^3+2*b^2*d*e^4*f*m^4*x+42*b^2*d*e^4*g*m^3*x^2-121*b^2*e^5*f*m^3*x^2-3
07*b^2*e^5*g*m^2*x^3-24*b*c*d^2*e^3*g*m^3*x^2+84*b*c*d*e^4*f*m^3*x^2+376*b*
c*d*e^4*g*m^2*x^3-614*b*c*e^5*f*m^2*x^3-648*b*c*e^5*g*m*x^4-12*c^2*d^2*e^3*
f*m^3*x^2-120*c^2*d^2*e^3*g*m^2*x^3+188*c^2*d*e^4*f*m^2*x^3+250*c^2*d*e^4*g
*m*x^4-324*c^2*e^5*f*m*x^4-120*c^2*e^5*g*x^5+a^2*d*e^4*g*m^4-20*a^2*e^5*f*m
```

```
^4-137*a^2*e^5*g*m^3*x+2*a*b*d*e^4*f*m^4+64*a*b*d*e^4*g*m^3*x-274*a*b*e^5*f
*m^3*x-744*a*b*e^5*g*m^2*x^2-12*a*c*d^2*e^3*g*m^3*x+64*a*c*d*e^4*f*m^3*x+39
0*a*c*d*e^4*g*m^2*x^2-744*a*c*e^5*f*m^2*x^2-792*a*c*e^5*g*m*x^3-6*b^2*d^2*e
^3*g*m^3*x+32*b^2*d*e^4*f*m^3*x+195*b^2*d*e^4*g*m^2*x^2-372*b^2*e^5*f*m^2*x
^2-396*b^2*e^5*g*m*x^3-12*b*c*d^2*e^3*f*m^3*x-216*b*c*d^2*e^3*g*m^2*x^2+390
*b*c*d*e^4*f*m^2*x^2+576*b*c*d*e^4*g*m*x^3-792*b*c*e^5*f*m*x^3-288*b*c*e^5*
g*x^4+60*c^2*d^3*e^2*g*m^2*x^2-108*c^2*d^2*e^3*f*m^2*x^2-220*c^2*d^2*e^3*g*
m*x^3+288*c^2*d*e^4*f*m*x^3+120*c^2*d*e^4*g*x^4-144*c^2*e^5*f*x^4+18*a^2*d*
e^4*g*m^3-155*a^2*e^5*f*m^3-461*a^2*e^5*g*m^2*x-4*a*b*d^2*e^3*g*m^3+36*a*b*
d*e^4*f*m^3+356*a*b*d*e^4*g*m^2*x-922*a*b*e^5*f*m^2*x-1016*a*b*e^5*g*m*x^2-
4*a*c*d^2*e^3*f*m^3-144*a*c*d^2*e^3*g*m^2*x+356*a*c*d*e^4*f*m^2*x+672*a*c*d
*e^4*g*m*x^2-1016*a*c*e^5*f*m*x^2-360*a*c*e^5*g*x^3-2*b^2*d^2*e^3*f*m^3-72*
b^2*d^2*e^3*g*m^2*x+178*b^2*d*e^4*f*m^2*x+336*b^2*d*e^4*g*m*x^2-508*b^2*e^5
*f*m*x^2-180*b^2*e^5*g*x^3+48*b*c*d^3*e^2*g*m^2*x-144*b*c*d^2*e^3*f*m^2*x-4
80*b*c*d^2*e^3*g*m*x^2+672*b*c*d*e^4*f*m*x^2+288*b*c*d*e^4*g*x^3-360*b*c*e^
5*f*x^3+24*c^2*d^3*e^2*f*m^2*x+180*c^2*d^3*e^2*g*m*x^2-240*c^2*d^2*e^3*f*m*
x^2-120*c^2*d^2*e^3*g*x^3+144*c^2*d*e^4*f*x^3+119*a^2*d*e^4*g*m^2-580*a^2*e
^5*f*m^2-702*a^2*e^5*g*m*x-60*a*b*d^2*e^3*g*m^2+238*a*b*d*e^4*f*m^2+776*a*b
*d*e^4*g*m*x-1404*a*b*e^5*f*m*x-480*a*b*e^5*g*x^2+12*a*c*d^3*e^2*g*m^2-60*a
*c*d^2*e^3*f*m^2-492*a*c*d^2*e^3*g*m*x+776*a*c*d*e^4*f*m*x+360*a*c*d*e^4*g*
x^2-480*a*c*e^5*f*x^2+6*b^2*d^3*e^2*g*m^2-30*b^2*d^2*e^3*f*m^2-246*b^2*d^2*
e^3*g*m*x+388*b^2*d*e^4*f*m*x+180*b^2*d*e^4*g*x^2-240*b^2*e^5*f*x^2+12*b*c*
d^3*e^2*f*m^2+336*b*c*d^3*e^2*g*m*x-492*b*c*d^2*e^3*f*m*x-288*b*c*d^2*e^3*g
*x^2+360*b*c*d*e^4*f*x^2-120*c^2*d^4*e*g*m*x+168*c^2*d^3*e^2*f*m*x+120*c^2*
d^3*e^2*g*x^2-144*c^2*d^2*e^3*f*x^2+342*a^2*d*e^4*g*m-1044*a^2*e^5*f*m-360*
a^2*e^5*g*x-296*a*b*d^2*e^3*g*m+684*a*b*d*e^4*f*m+480*a*b*d*e^4*g*x-720*a*b
*e^5*f*x+132*a*c*d^3*e^2*g*m-296*a*c*d^2*e^3*f*m-360*a*c*d^2*e^3*g*x+480*a*
c*d*e^4*f*x+66*b^2*d^3*e^2*g*m-148*b^2*d^2*e^3*f*m-180*b^2*d^2*e^3*g*x+240*
b^2*d*e^4*f*x-48*b*c*d^4*e*g*m+132*b*c*d^3*e^2*f*m+288*b*c*d^3*e^2*g*x-360*
b*c*d^2*e^3*f*x-24*c^2*d^4*e*f*m-120*c^2*d^4*e*g*x+144*c^2*d^3*e^2*f*x+360*
a^2*d*e^4*g-720*a^2*e^5*f-480*a*b*d^2*e^3*g+720*a*b*d*e^4*f+360*a*c*d^3*e^2
*g-480*a*c*d^2*e^3*f+180*b^2*d^3*e^2*g-240*b^2*d^2*e^3*f-288*b*c*d^4*e*g+36
0*b*c*d^3*e^2*f+120*c^2*d^5*g-144*c^2*d^4*e*f)/e^6/(m^6+21*m^5+175*m^4+735*
m^3+1624*m^2+1764*m+720)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95663, size = 5231, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="fricas")

[Out] (a^2*d*e^5*f*m^5 + (c^2*e^6*g*m^5 + 15*c^2*e^6*g*m^4 + 85*c^2*e^6*g*m^3 + 2
25*c^2*e^6*g*m^2 + 274*c^2*e^6*g*m + 120*c^2*e^6*g)*x^6 + (144*c^2*e^6*f +
```

```
288*b*c*e^6*g + (c^2*e^6*f + (c^2*d*e^5 + 2*b*c*e^6)*g)*m^5 + 2*(8*c^2*e^6*
f + (5*c^2*d*e^5 + 16*b*c*e^6)*g)*m^4 + 5*(19*c^2*e^6*f + (7*c^2*d*e^5 + 38
*b*c*e^6)*g)*m^3 + 10*(26*c^2*e^6*f + (5*c^2*d*e^5 + 52*b*c*e^6)*g)*m^2 + 1
2*(27*c^2*e^6*f + 2*(c^2*d*e^5 + 27*b*c*e^6)*g)*m)*x^5 - (a^2*d^2*e^4*g + 2
*(a*b*d^2*e^4 - 10*a^2*d*e^5)*f)*m^4 + (360*b*c*e^6*f + 180*(b^2 + 2*a*c)*e
^6*g + ((c^2*d*e^5 + 2*b*c*e^6)*f + (2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*g)*m^
5 + (2*(6*c^2*d*e^5 + 17*b*c*e^6)*f - (5*c^2*d^2*e^4 - 24*b*c*d*e^5 - 17*(b
^2 + 2*a*c)*e^6)*g)*m^4 + ((47*c^2*d*e^5 + 214*b*c*e^6)*f - (30*c^2*d^2*e^4
 - 94*b*c*d*e^5 - 107*(b^2 + 2*a*c)*e^6)*g)*m^3 + (2*(36*c^2*d*e^5 + 307*b*
c*e^6)*f - (55*c^2*d^2*e^4 - 144*b*c*d*e^5 - 307*(b^2 + 2*a*c)*e^6)*g)*m^2
+ 6*(6*(c^2*d*e^5 + 22*b*c*e^6)*f - (5*c^2*d^2*e^4 - 12*b*c*d*e^5 - 66*(b^2
 + 2*a*c)*e^6)*g)*m)*x^4 - ((36*a*b*d^2*e^4 - 155*a^2*d*e^5 - 2*(b^2 + 2*a*
c)*d^3*e^3)*f - 2*(2*a*b*d^3*e^3 - 9*a^2*d^2*e^4)*g)*m^3 + (480*a*b*e^6*g +
 240*(b^2 + 2*a*c)*e^6*f + ((2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*f + (2*a*b*e^
6 + (b^2 + 2*a*c)*d*e^5)*g)*m^5 - 2*((2*c^2*d^2*e^4 - 14*b*c*d*e^5 - 9*(b^2
 + 2*a*c)*e^6)*f + (4*b*c*d^2*e^4 - 18*a*b*e^6 - 7*(b^2 + 2*a*c)*d*e^5)*g)*
m^4 - ((36*c^2*d^2*e^4 - 130*b*c*d*e^5 - 121*(b^2 + 2*a*c)*e^6)*f - (20*c^2
*d^3*e^3 - 72*b*c*d^2*e^4 + 242*a*b*e^6 + 65*(b^2 + 2*a*c)*d*e^5)*g)*m^3 -
4*((20*c^2*d^2*e^4 - 56*b*c*d*e^5 - 93*(b^2 + 2*a*c)*e^6)*f - (15*c^2*d^3*e
^3 - 40*b*c*d^2*e^4 + 186*a*b*e^6 + 28*(b^2 + 2*a*c)*d*e^5)*g)*m^2 - 4*((12
*c^2*d^2*e^4 - 30*b*c*d*e^5 - 127*(b^2 + 2*a*c)*e^6)*f - (10*c^2*d^3*e^3 -
24*b*c*d^2*e^4 + 254*a*b*e^6 + 15*(b^2 + 2*a*c)*d*e^5)*g)*m)*x^3 - (2*(6*b*
c*d^4*e^2 + 119*a*b*d^2*e^4 - 290*a^2*d*e^5 - 15*(b^2 + 2*a*c)*d^3*e^3)*f -
 (60*a*b*d^3*e^3 - 119*a^2*d^2*e^4 - 6*(b^2 + 2*a*c)*d^4*e^2)*g)*m^2 + (720
*a*b*e^6*f + 360*a^2*e^6*g + ((2*a*b*e^6 + (b^2 + 2*a*c)*d*e^5)*f + (2*a*b*
d*e^5 + a^2*e^6)*g)*m^5 - (2*(3*b*c*d^2*e^4 - 19*a*b*e^6 - 8*(b^2 + 2*a*c)*
d*e^5)*f - (32*a*b*d*e^5 + 19*a^2*e^6 - 3*(b^2 + 2*a*c)*d^2*e^4)*g)*m^4 + (
(12*c^2*d^3*e^3 - 72*b*c*d^2*e^4 + 274*a*b*e^6 + 89*(b^2 + 2*a*c)*d*e^5)*f
+ (24*b*c*d^3*e^3 + 178*a*b*d*e^5 + 137*a^2*e^6 - 36*(b^2 + 2*a*c)*d^2*e^4)
*g)*m^3 + (2*(42*c^2*d^3*e^3 - 123*b*c*d^2*e^4 + 461*a*b*e^6 + 97*(b^2 + 2*
a*c)*d*e^5)*f - (60*c^2*d^4*e^2 - 168*b*c*d^3*e^3 - 388*a*b*d*e^5 - 461*a^2
*e^6 + 123*(b^2 + 2*a*c)*d^2*e^4)*g)*m^2 + 6*(2*(6*c^2*d^3*e^3 - 15*b*c*d^2
*e^4 + 117*a*b*e^6 + 10*(b^2 + 2*a*c)*d*e^5)*f - (10*c^2*d^4*e^2 - 24*b*c*d
^3*e^3 - 40*a*b*d*e^5 - 117*a^2*e^6 + 15*(b^2 + 2*a*c)*d^2*e^4)*g)*m)*x^2 +
 24*(6*c^2*d^5*e - 15*b*c*d^4*e^2 - 30*a*b*d^2*e^4 + 30*a^2*d*e^5 + 10*(b^2
 + 2*a*c)*d^3*e^3)*f - 12*(10*c^2*d^6 - 24*b*c*d^5*e - 40*a*b*d^3*e^3 + 30*
a^2*d^2*e^4 + 15*(b^2 + 2*a*c)*d^4*e^2)*g + 2*(2*(6*c^2*d^5*e - 33*b*c*d^4*
e^2 - 171*a*b*d^2*e^4 + 261*a^2*d*e^5 + 37*(b^2 + 2*a*c)*d^3*e^3)*f + (24*b
*c*d^5*e + 148*a*b*d^3*e^3 - 171*a^2*d^2*e^4 - 33*(b^2 + 2*a*c)*d^4*e^2)*g)
*m + (720*a^2*e^6*f + (a^2*d*e^5*g + (2*a*b*d*e^5 + a^2*e^6)*f)*m^5 + 2*((1
8*a*b*d*e^5 + 10*a^2*e^6 - (b^2 + 2*a*c)*d^2*e^4)*f - (2*a*b*d^2*e^4 - 9*a^
2*d*e^5)*g)*m^4 + ((12*b*c*d^3*e^3 + 238*a*b*d*e^5 + 155*a^2*e^6 - 30*(b^2
+ 2*a*c)*d^2*e^4)*f - (60*a*b*d^2*e^4 - 119*a^2*d*e^5 - 6*(b^2 + 2*a*c)*d^3
*e^3)*g)*m^3 - 2*(2*(6*c^2*d^4*e^2 - 33*b*c*d^3*e^3 - 171*a*b*d*e^5 - 145*a
^2*e^6 + 37*(b^2 + 2*a*c)*d^2*e^4)*f + (24*b*c*d^4*e^2 + 148*a*b*d^2*e^4 -
171*a^2*d*e^5 - 33*(b^2 + 2*a*c)*d^3*e^3)*g)*m^2 - 12*((12*c^2*d^4*e^2 - 30
*b*c*d^3*e^3 - 60*a*b*d*e^5 - 87*a^2*e^6 + 20*(b^2 + 2*a*c)*d^2*e^4)*f - (1
0*c^2*d^5*e - 24*b*c*d^4*e^2 - 40*a*b*d^2*e^4 + 30*a^2*d*e^5 + 15*(b^2 + 2*
a*c)*d^3*e^3)*g)*m)*x)*(e*x + d)^m/(e^6*m^6 + 21*e^6*m^5 + 175*e^6*m^4 + 73
5*e^6*m^3 + 1624*e^6*m^2 + 1764*e^6*m + 720*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**2,x)
```

[Out] Timed out

Giac [B] time = 1.32535, size = 6669, normalized size = 21.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*g*m^5*x^6*e^6 + (x*e + d)^m*c^2*d*g*m^5*x^5*e^5 + (x*e + d
)^m*c^2*f*m^5*x^5*e^6 + 2*(x*e + d)^m*b*c*g*m^5*x^5*e^6 + 15*(x*e + d)^m*c^
2*g*m^4*x^6*e^6 + (x*e + d)^m*c^2*d*f*m^5*x^4*e^5 + 2*(x*e + d)^m*b*c*d*g*m
^5*x^4*e^5 + 10*(x*e + d)^m*c^2*d*g*m^4*x^5*e^5 - 5*(x*e + d)^m*c^2*d^2*g*m
^4*x^4*e^4 + 2*(x*e + d)^m*b*c*f*m^5*x^4*e^6 + (x*e + d)^m*b^2*g*m^5*x^4*e^
6 + 2*(x*e + d)^m*a*c*g*m^5*x^4*e^6 + 16*(x*e + d)^m*c^2*f*m^4*x^5*e^6 + 32
*(x*e + d)^m*b*c*g*m^4*x^5*e^6 + 85*(x*e + d)^m*c^2*g*m^3*x^6*e^6 + 2*(x*e
+ d)^m*b*c*d*f*m^5*x^3*e^5 + (x*e + d)^m*b^2*d*g*m^5*x^3*e^5 + 2*(x*e + d)^
m*a*c*d*g*m^5*x^3*e^5 + 12*(x*e + d)^m*c^2*d*f*m^4*x^4*e^5 + 24*(x*e + d)^m
*b*c*d*g*m^4*x^4*e^5 + 35*(x*e + d)^m*c^2*d*g*m^3*x^5*e^5 - 4*(x*e + d)^m*c
^2*d^2*f*m^4*x^3*e^4 - 8*(x*e + d)^m*b*c*d^2*g*m^4*x^3*e^4 - 30*(x*e + d)^m
*c^2*d^2*g*m^3*x^4*e^4 + 20*(x*e + d)^m*c^2*d^3*g*m^3*x^3*e^3 + (x*e + d)^m
*b^2*f*m^5*x^3*e^6 + 2*(x*e + d)^m*a*c*f*m^5*x^3*e^6 + 2*(x*e + d)^m*a*b*g*
m^5*x^3*e^6 + 34*(x*e + d)^m*b*c*f*m^4*x^4*e^6 + 17*(x*e + d)^m*b^2*g*m^4*x
^4*e^6 + 34*(x*e + d)^m*a*c*g*m^4*x^4*e^6 + 95*(x*e + d)^m*c^2*f*m^3*x^5*e^
6 + 190*(x*e + d)^m*b*c*g*m^3*x^5*e^6 + 225*(x*e + d)^m*c^2*g*m^2*x^6*e^6 +
 (x*e + d)^m*b^2*d*f*m^5*x^2*e^5 + 2*(x*e + d)^m*a*c*d*f*m^5*x^2*e^5 + 2*(x
*e + d)^m*a*b*d*g*m^5*x^2*e^5 + 28*(x*e + d)^m*b*c*d*f*m^4*x^3*e^5 + 14*(x*
e + d)^m*b^2*d*g*m^4*x^3*e^5 + 28*(x*e + d)^m*a*c*d*g*m^4*x^3*e^5 + 47*(x*e
 + d)^m*c^2*d*f*m^3*x^4*e^5 + 94*(x*e + d)^m*b*c*d*g*m^3*x^4*e^5 + 50*(x*e
+ d)^m*c^2*d*g*m^2*x^5*e^5 - 6*(x*e + d)^m*b*c*d^2*f*m^4*x^2*e^4 - 3*(x*e +
 d)^m*b^2*d^2*g*m^4*x^2*e^4 - 6*(x*e + d)^m*a*c*d^2*g*m^4*x^2*e^4 - 36*(x*e
 + d)^m*c^2*d^2*f*m^3*x^3*e^4 - 72*(x*e + d)^m*b*c*d^2*g*m^3*x^3*e^4 - 55*(
x*e + d)^m*c^2*d^2*g*m^2*x^4*e^4 + 12*(x*e + d)^m*c^2*d^3*f*m^3*x^2*e^3 + 2
4*(x*e + d)^m*b*c*d^3*g*m^3*x^2*e^3 + 60*(x*e + d)^m*c^2*d^3*g*m^2*x^3*e^3
- 60*(x*e + d)^m*c^2*d^4*g*m^2*x^2*e^2 + 2*(x*e + d)^m*a*b*f*m^5*x^2*e^6 +
(x*e + d)^m*a^2*g*m^5*x^2*e^6 + 18*(x*e + d)^m*b^2*f*m^4*x^3*e^6 + 36*(x*e
+ d)^m*a*c*f*m^4*x^3*e^6 + 36*(x*e + d)^m*a*b*g*m^4*x^3*e^6 + 214*(x*e + d)
^m*b*c*f*m^3*x^4*e^6 + 107*(x*e + d)^m*b^2*g*m^3*x^4*e^6 + 214*(x*e + d)^m*
a*c*g*m^3*x^4*e^6 + 260*(x*e + d)^m*c^2*f*m^2*x^5*e^6 + 520*(x*e + d)^m*b*c
*g*m^2*x^5*e^6 + 274*(x*e + d)^m*c^2*g*m*x^6*e^6 + 2*(x*e + d)^m*a*b*d*f*m^
5*x*e^5 + (x*e + d)^m*a^2*d*g*m^5*x*e^5 + 16*(x*e + d)^m*b^2*d*f*m^4*x^2*e^
5 + 32*(x*e + d)^m*a*c*d*f*m^4*x^2*e^5 + 32*(x*e + d)^m*a*b*d*g*m^4*x^2*e^5
 + 130*(x*e + d)^m*b*c*d*f*m^3*x^3*e^5 + 65*(x*e + d)^m*b^2*d*g*m^3*x^3*e^5
 + 130*(x*e + d)^m*a*c*d*g*m^3*x^3*e^5 + 72*(x*e + d)^m*c^2*d*f*m^2*x^4*e^5
 + 144*(x*e + d)^m*b*c*d*g*m^2*x^4*e^5 + 24*(x*e + d)^m*c^2*d*g*m*x^5*e^5 -
 2*(x*e + d)^m*b^2*d^2*f*m^4*x*e^4 - 4*(x*e + d)^m*a*c*d^2*f*m^4*x*e^4 - 4*
(x*e + d)^m*a*b*d^2*g*m^4*x*e^4 - 72*(x*e + d)^m*b*c*d^2*f*m^3*x^2*e^4 - 36
*(x*e + d)^m*b^2*d^2*g*m^3*x^2*e^4 - 72*(x*e + d)^m*a*c*d^2*g*m^3*x^2*e^4 -
 80*(x*e + d)^m*c^2*d^2*f*m^2*x^3*e^4 - 160*(x*e + d)^m*b*c*d^2*g*m^2*x^3*e
^4 - 30*(x*e + d)^m*c^2*d^2*g*m*x^4*e^4 + 12*(x*e + d)^m*b*c*d^3*f*m^3*x*e^
3 + 6*(x*e + d)^m*b^2*d^3*g*m^3*x*e^3 + 12*(x*e + d)^m*a*c*d^3*g*m^3*x*e^3
+ 84*(x*e + d)^m*c^2*d^3*f*m^2*x^2*e^3 + 168*(x*e + d)^m*b*c*d^3*g*m^2*x^2*
e^3 + 40*(x*e + d)^m*c^2*d^3*g*m*x^3*e^3 - 24*(x*e + d)^m*c^2*d^4*f*m^2*x*e
^2 - 48*(x*e + d)^m*b*c*d^4*g*m^2*x*e^2 - 60*(x*e + d)^m*c^2*d^4*g*m*x^2*e^
2 + 120*(x*e + d)^m*c^2*d^5*g*m*x*e + (x*e + d)^m*a^2*f*m^5*x*e^6 + 38*(x*e
```

+ d)^m*a*b*f*m^4*x^2*e^6 + 19*(x*e + d)^m*a^2*g*m^4*x^2*e^6 + 121*(x*e + d
)^m*b^2*f*m^3*x^3*e^6 + 242*(x*e + d)^m*a*c*f*m^3*x^3*e^6 + 242*(x*e + d)^m
*a*b*g*m^3*x^3*e^6 + 614*(x*e + d)^m*b*c*f*m^2*x^4*e^6 + 307*(x*e + d)^m*b^
2*g*m^2*x^4*e^6 + 614*(x*e + d)^m*a*c*g*m^2*x^4*e^6 + 324*(x*e + d)^m*c^2*f
*m*x^5*e^6 + 648*(x*e + d)^m*b*c*g*m*x^5*e^6 + 120*(x*e + d)^m*c^2*g*x^6*e^
6 + (x*e + d)^m*a^2*d*f*m^5*e^5 + 36*(x*e + d)^m*a*b*d*f*m^4*x*e^5 + 18*(x*
e + d)^m*a^2*d*g*m^4*x*e^5 + 89*(x*e + d)^m*b^2*d*f*m^3*x^2*e^5 + 178*(x*e
+ d)^m*a*c*d*f*m^3*x^2*e^5 + 178*(x*e + d)^m*a*b*d*g*m^3*x^2*e^5 + 224*(x*e
+ d)^m*b*c*d*f*m^2*x^3*e^5 + 112*(x*e + d)^m*b^2*d*g*m^2*x^3*e^5 + 224*(x*
e + d)^m*a*c*d*g*m^2*x^3*e^5 + 36*(x*e + d)^m*c^2*d*f*m*x^4*e^5 + 72*(x*e +
d)^m*b*c*d*g*m*x^4*e^5 - 2*(x*e + d)^m*a*b*d^2*f*m^4*e^4 - (x*e + d)^m*a^2
*d^2*g*m^4*e^4 - 30*(x*e + d)^m*b^2*d^2*f*m^3*x*e^4 - 60*(x*e + d)^m*a*c*d^
2*f*m^3*x*e^4 - 60*(x*e + d)^m*a*b*d^2*g*m^3*x*e^4 - 246*(x*e + d)^m*b*c*d^
2*f*m^2*x^2*e^4 - 123*(x*e + d)^m*b^2*d^2*g*m^2*x^2*e^4 - 246*(x*e + d)^m*a
*c*d^2*g*m^2*x^2*e^4 - 48*(x*e + d)^m*c^2*d^2*f*m*x^3*e^4 - 96*(x*e + d)^m*
b*c*d^2*g*m*x^3*e^4 + 2*(x*e + d)^m*b^2*d^3*f*m^3*e^3 + 4*(x*e + d)^m*a*c*d
^3*f*m^3*e^3 + 4*(x*e + d)^m*a*b*d^3*g*m^3*e^3 + 132*(x*e + d)^m*b*c*d^3*f*
m^2*x*e^3 + 66*(x*e + d)^m*b^2*d^3*g*m^2*x*e^3 + 132*(x*e + d)^m*a*c*d^3*g*
m^2*x*e^3 + 72*(x*e + d)^m*c^2*d^3*f*m*x^2*e^3 + 144*(x*e + d)^m*b*c*d^3*g*
m*x^2*e^3 - 12*(x*e + d)^m*b*c*d^4*f*m^2*e^2 - 6*(x*e + d)^m*b^2*d^4*g*m^2*
e^2 - 12*(x*e + d)^m*a*c*d^4*g*m^2*e^2 - 144*(x*e + d)^m*c^2*d^4*f*m*x*e^2
- 288*(x*e + d)^m*b*c*d^4*g*m*x*e^2 + 24*(x*e + d)^m*c^2*d^5*f*m*e + 48*(x*
e + d)^m*b*c*d^5*g*m*e - 120*(x*e + d)^m*c^2*d^6*g + 20*(x*e + d)^m*a^2*f*m
^4*x*e^6 + 274*(x*e + d)^m*a*b*f*m^3*x^2*e^6 + 137*(x*e + d)^m*a^2*g*m^3*x^
2*e^6 + 372*(x*e + d)^m*b^2*f*m^2*x^3*e^6 + 744*(x*e + d)^m*a*c*f*m^2*x^3*e
^6 + 744*(x*e + d)^m*a*b*g*m^2*x^3*e^6 + 792*(x*e + d)^m*b*c*f*m*x^4*e^6 +
396*(x*e + d)^m*b^2*g*m*x^4*e^6 + 792*(x*e + d)^m*a*c*g*m*x^4*e^6 + 144*(x*
e + d)^m*c^2*f*x^5*e^6 + 288*(x*e + d)^m*b*c*g*x^5*e^6 + 20*(x*e + d)^m*a^2
*d*f*m^4*e^5 + 238*(x*e + d)^m*a*b*d*f*m^3*x*e^5 + 119*(x*e + d)^m*a^2*d*g*
m^3*x*e^5 + 194*(x*e + d)^m*b^2*d*f*m^2*x^2*e^5 + 388*(x*e + d)^m*a*c*d*f*m
^2*x^2*e^5 + 388*(x*e + d)^m*a*b*d*g*m^2*x^2*e^5 + 120*(x*e + d)^m*b*c*d*f*
m*x^3*e^5 + 60*(x*e + d)^m*b^2*d*g*m*x^3*e^5 + 120*(x*e + d)^m*a*c*d*g*m*x^
3*e^5 - 36*(x*e + d)^m*a*b*d^2*f*m^3*e^4 - 18*(x*e + d)^m*a^2*d^2*g*m^3*e^4
- 148*(x*e + d)^m*b^2*d^2*f*m^2*x*e^4 - 296*(x*e + d)^m*a*c*d^2*f*m^2*x*e^
4 - 296*(x*e + d)^m*a*b*d^2*g*m^2*x*e^4 - 180*(x*e + d)^m*b*c*d^2*f*m*x^2*e
^4 - 90*(x*e + d)^m*b^2*d^2*g*m*x^2*e^4 - 180*(x*e + d)^m*a*c*d^2*g*m*x^2*e
^4 + 30*(x*e + d)^m*b^2*d^3*f*m^2*e^3 + 60*(x*e + d)^m*a*c*d^3*f*m^2*e^3 +
60*(x*e + d)^m*a*b*d^3*g*m^2*e^3 + 360*(x*e + d)^m*b*c*d^3*f*m*x*e^3 + 180*
(x*e + d)^m*b^2*d^3*g*m*x*e^3 + 360*(x*e + d)^m*a*c*d^3*g*m*x*e^3 - 132*(x*
e + d)^m*b*c*d^4*f*m*e^2 - 66*(x*e + d)^m*b^2*d^4*g*m*e^2 - 132*(x*e + d)^m
*a*c*d^4*g*m*e^2 + 144*(x*e + d)^m*c^2*d^5*f*e + 288*(x*e + d)^m*b*c*d^5*g*
e + 155*(x*e + d)^m*a^2*f*m^3*x*e^6 + 922*(x*e + d)^m*a*b*f*m^2*x^2*e^6 + 4
61*(x*e + d)^m*a^2*g*m^2*x^2*e^6 + 508*(x*e + d)^m*b^2*f*m*x^3*e^6 + 1016*(
x*e + d)^m*a*c*f*m*x^3*e^6 + 1016*(x*e + d)^m*a*b*g*m*x^3*e^6 + 360*(x*e +
d)^m*b*c*f*x^4*e^6 + 180*(x*e + d)^m*b^2*g*x^4*e^6 + 360*(x*e + d)^m*a*c*g*
x^4*e^6 + 155*(x*e + d)^m*a^2*d*f*m^3*e^5 + 684*(x*e + d)^m*a*b*d*f*m^2*x*e
^5 + 342*(x*e + d)^m*a^2*d*g*m^2*x*e^5 + 120*(x*e + d)^m*b^2*d*f*m*x^2*e^5
+ 240*(x*e + d)^m*a*c*d*f*m*x^2*e^5 + 240*(x*e + d)^m*a*b*d*g*m*x^2*e^5 - 2
38*(x*e + d)^m*a*b*d^2*f*m^2*e^4 - 119*(x*e + d)^m*a^2*d^2*g*m^2*e^4 - 240*
(x*e + d)^m*b^2*d^2*f*m*x*e^4 - 480*(x*e + d)^m*a*c*d^2*f*m*x*e^4 - 480*(x*
e + d)^m*a*b*d^2*g*m*x*e^4 + 148*(x*e + d)^m*b^2*d^3*f*m*e^3 + 296*(x*e + d
)^m*a*c*d^3*f*m*e^3 + 296*(x*e + d)^m*a*b*d^3*g*m*e^3 - 360*(x*e + d)^m*b*c
*d^4*f*e^2 - 180*(x*e + d)^m*b^2*d^4*g*e^2 - 360*(x*e + d)^m*a*c*d^4*g*e^2
+ 580*(x*e + d)^m*a^2*f*m^2*x*e^6 + 1404*(x*e + d)^m*a*b*f*m*x^2*e^6 + 702*
(x*e + d)^m*a^2*g*m*x^2*e^6 + 240*(x*e + d)^m*b^2*f*x^3*e^6 + 480*(x*e + d)
^m*a*c*f*x^3*e^6 + 480*(x*e + d)^m*a*b*g*x^3*e^6 + 580*(x*e + d)^m*a^2*d*f*
m^2*e^5 + 720*(x*e + d)^m*a*b*d*f*m*x*e^5 + 360*(x*e + d)^m*a^2*d*g*m*x*e^5
- 684*(x*e + d)^m*a*b*d^2*f*m*e^4 - 342*(x*e + d)^m*a^2*d^2*g*m*e^4 + 240*
(x*e + d)^m*b^2*d^3*f*e^3 + 480*(x*e + d)^m*a*c*d^3*f*e^3 + 480*(x*e + d)^m

```
*a*b*d^3*g*e^3 + 1044*(x*e + d)^m*a^2*f*m*x*e^6 + 720*(x*e + d)^m*a*b*f*x^2
*e^6 + 360*(x*e + d)^m*a^2*g*x^2*e^6 + 1044*(x*e + d)^m*a^2*d*f*m*e^5 - 720
*(x*e + d)^m*a*b*d^2*f*e^4 - 360*(x*e + d)^m*a^2*d^2*g*e^4 + 720*(x*e + d)^
m*a^2*f*x*e^6 + 720*(x*e + d)^m*a^2*d*f*e^5)/(m^6*e^6 + 21*m^5*e^6 + 175*m^
4*e^6 + 735*m^3*e^6 + 1624*m^2*e^6 + 1764*m*e^6 + 720*e^6)
```

3.927 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{f+gx}\,dx$

Optimal. Leaf size=287

$$\frac{(d+ex)^{m+2}\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4g^3(m+2)}+\frac{(d+ex)^{m+1}(beg-c(dg+ef))\left(eg(2aeg\right.}{e^4g^4(m}$$

```
[Out] ((b*e*g - c*(e*f + d*g))*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(e*f + d
*g)))*(d + e*x)^(1 + m))/(e^4*g^4*(1 + m)) + ((b^2*e^2*g^2 + c^2*(e^2*f^2 +
 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x)^(2 +
 m))/(e^4*g^3*(2 + m)) - (c*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^(3 + m))/
(e^4*g^2*(3 + m)) + (c^2*(d + e*x)^(4 + m))/(e^4*g*(4 + m)) + ((c*f^2 - b*f
*g + a*g^2)^2*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d
+ e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m))
```

Rubi [A] time = 0.865186, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {951, 1620, 68}

$$\frac{(d+ex)^{m+2}\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4g^3(m+2)}+\frac{(d+ex)^{m+1}(beg-c(dg+ef))\left(eg(2aeg\right.}{e^4g^4(m}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

```
[Out] ((b*e*g - c*(e*f + d*g))*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(e*f + d
*g)))*(d + e*x)^(1 + m))/(e^4*g^4*(1 + m)) + ((b^2*e^2*g^2 + c^2*(e^2*f^2 +
 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x)^(2 +
 m))/(e^4*g^3*(2 + m)) - (c*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^(3 + m))/
(e^4*g^2*(3 + m)) + (c^2*(d + e*x)^(4 + m))/(e^4*g*(4 + m)) + ((c*f^2 - b*f
*g + a*g^2)^2*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d
+ e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m))
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{f+gx}\,dx = \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{\int \frac{(d+ex)^m\left(-e\left(c^2d^3f-a^2e^3g\right)(4+m)+e\left(2abe^3g-c^2d^2(3ef+dg)\right)(4+m)x+e^2\left(b^2e^2g+2ace^2g-3c\right.}{f+gx}}{e^4g(4+m)}$$

$$= \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{\int\left(\frac{e(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(4+m)(d+ex)^m}{g^3} + \frac{e\left(b^2e^2g^2+c^2\left(e^2f\right.\right.}{}\right.}{}$$

$$= \frac{(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(d+ex)^{1+m}}{e^4g^4(1+m)} + \frac{\left(b^2e^2g^2+c\right.}{}$$

$$= \frac{(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(d+ex)^{1+m}}{e^4g^4(1+m)} + \frac{\left(b^2e^2g^2+c\right.}{}$$

Mathematica [A] time = 0.447522, size = 265, normalized size = 0.92

$$\frac{(d+ex)^{m+1}\left(\frac{g(d+ex)\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4(m+2)} - \frac{(-beg+cdg+cef)\left(eg(2aeg-b(dg+ef))+c\left(d^2g^2+e^2f^2\right)\right)}{e^4(m+1)} + \frac{\left(g(ag-bf)+cf^2\right)^2}{(m-}\right.}{g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g
*(2*a*e*g - b*(e*f + d*g))))/(e^4*(1 + m))) + (g*(b^2*e^2*g^2 + c^2*(e^2*f^
2 + 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x))/
(e^4*(2 + m)) - (c*g^2*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^2)/(e^4*(3 + m
)) + (c^2*g^3*(d + e*x)^3)/(e^4*(4 + m)) + ((c*f^2 + g*(-(b*f) + a*g))^2*Hy
pergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g
)*(1 + m))))/g^4
```

Maple [F] time = 1.641, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)(ex+d)^m}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g*x + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f),x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)
```

3.928 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx$

Optimal. Leaf size=298

$$\frac{(d+ex)^{m+1}\left(2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2\left(d^2g^2+2defg+3e^2f^2\right)\right)}{e^3g^4(m+1)}+\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)\,{}_2F_1\left(1,m-\right.}{}$$

```
[Out] ((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*c*e*g*(a*e*g - b*
(2*e*f + d*g)))*(d + e*x)^(1 + m))/(e^3*g^4*(1 + m)) - (2*c*(c*e*f + c*d*g
- b*e*g)*(d + e*x)^(2 + m))/(e^3*g^3*(2 + m)) + (c^2*(d + e*x)^(3 + m))/(e^
3*g^2*(3 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^(1 + m))/(g^4*(e*f -
d*g)*(f + g*x)) + ((c*f^2 - b*f*g + a*g^2)*(c*f*(4*d*g - e*f*(4 + m)) - g*(
a*e*g*m + b*(2*d*g - e*f*(2 + m))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)^2*(1 + m))
```

Rubi [A] time = 1.14088, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {949, 1620, 68}

$$\frac{(d+ex)^{m+1}\left(2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2\left(d^2g^2+2defg+3e^2f^2\right)\right)}{e^3g^4(m+1)}-\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)\,{}_2F_1\left(1,m+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

```
[Out] ((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*c*e*g*(a*e*g - b*
(2*e*f + d*g)))*(d + e*x)^(1 + m))/(e^3*g^4*(1 + m)) - (2*c*(c*e*f + c*d*g
- b*e*g)*(d + e*x)^(2 + m))/(e^3*g^3*(2 + m)) + (c^2*(d + e*x)^(3 + m))/(e^
3*g^2*(3 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^(1 + m))/(g^4*(e*f -
d*g)*(f + g*x)) - ((c*f^2 - b*f*g + a*g^2)*(g*(2*b*d*g + a*e*g*m - b*e*f*(2
 + m)) - c*f*(4*d*g - e*f*(4 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)^2*(1 + m))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx = \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \int \frac{(d+ex)^m\left(\frac{c^2f^3(dg-ef(1+m))-2cfg(bf-ag)(dg-ef(1+m))-g^2\left(a^2eg^2m-b\right.}{g^4}\right.}{}$$

$$= \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \int \left(\frac{(ef-dg)\left(b^2e^2g^2+c^2\left(3e^2f^2+2defg+d^2g^2\right)+2ceg(aeg-b(2ef+\right.}{e^2g^4}\right.$$

$$= \frac{\left(b^2e^2g^2+c^2\left(3e^2f^2+2defg+d^2g^2\right)+2ceg(aeg-b(2ef+dg))\right)(d+ex)^{1+m}}{e^3g^4(1+m)} - \frac{2c(}{}$$

$$= \frac{\left(b^2e^2g^2+c^2\left(3e^2f^2+2defg+d^2g^2\right)+2ceg(aeg-b(2ef+dg))\right)(d+ex)^{1+m}}{e^3g^4(1+m)} - \frac{2c(}{}$$

Mathematica [A] time = 0.394679, size = 261, normalized size = 0.88

$$\frac{(d+ex)^{m+1}\left(\frac{2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2\left(d^2g^2+2defg+3e^2f^2\right)}{e^3(m+1)} + \frac{e\left(g(ag-bf)+cf^2\right)^2{}_2F_1\left(2,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)^2} - \frac{2(2cf-bg)\left(g(ag-bf)+cf^2\right){}_2F_1(}{(m+1)(ef-dg}\right.}{g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) +
2*c*e*g*(a*e*g - b*(2*e*f + d*g)))/(e^3*(1 + m)) - (2*c*g*(c*e*f + c*d*g -
b*e*g)*(d + e*x))/(e^3*(2 + m)) + (c^2*g^2*(d + e*x)^2)/(e^3*(3 + m)) - (2*
(2*c*f - b*g)*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1, 1 + m, 2 + m,
 (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m)) + (e*(c*f^2 + g*(-(b*
f) + a*g))^2*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g
)])/((e*f - d*g)^2*(1 + m))))/g^4
```

Maple [F] time = 1.626, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)(ex+d)^m}{g^2x^2+2\,fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**2,x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{\left(gx+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

3.929 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx$

Optimal. Leaf size=461

$(d+ex)^{m+1}{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(-g^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(m+1))+b^2\left(-\left(2d^2g^2-4defg(\right.\right.\right.$

```
[Out] -((c*(3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)^(1 + m))/(e^2*g^4*(1 + m))) + (c
^2*(d + e*x)^(2 + m))/(e^2*g^3*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e
*x)^(1 + m))/(2*g^4*(e*f - d*g)*(f + g*x)^2) + ((c*f^2 - b*f*g + a*g^2)*(c*
f*(8*d*g - e*f*(7 + m)) + g*(a*e*g*(1 - m) - b*(4*d*g - e*f*(3 + m))))*(d +
 e*x)^(1 + m))/(2*g^4*(e*f - d*g)^2*(f + g*x)) + ((c^2*f^2*(12*d^2*g^2 - 8*
d*e*f*g*(3 + m) + e^2*f^2*(12 + 7*m + m^2)) - g^2*(a^2*e^2*g^2*(1 - m)*m -
2*a*b*e*g*m*(2*d*g - e*f*(1 + m)) - b^2*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^
2*f^2*(2 + 3*m + m^2))) + 2*c*g*(a*g*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f
^2*(2 + 3*m + m^2)) - b*f*(6*d^2*g^2 - 6*d*e*f*g*(2 + m) + e^2*f^2*(6 + 5*m
 + m^2))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e
*x))/(e*f - d*g))])/(2*g^4*(e*f - d*g)^3*(1 + m))
```

Rubi [A] time = 1.48709, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {949, 1621, 951, 80, 68}

$(d+ex)^{m+1}{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(-g^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(m+1))+b^2\left(-\left(2d^2g^2-4defg(\right.\right.\right.$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

```
[Out] -((c*(3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)^(1 + m))/(e^2*g^4*(1 + m))) + (c
^2*(d + e*x)^(2 + m))/(e^2*g^3*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e
*x)^(1 + m))/(2*g^4*(e*f - d*g)*(f + g*x)^2) - ((c*f^2 - b*f*g + a*g^2)*(g*
(4*b*d*g - a*e*g*(1 - m) - b*e*f*(3 + m)) - c*f*(8*d*g - e*f*(7 + m)))*(d +
 e*x)^(1 + m))/(2*g^4*(e*f - d*g)^2*(f + g*x)) + ((c^2*f^2*(12*d^2*g^2 - 8*
d*e*f*g*(3 + m) + e^2*f^2*(12 + 7*m + m^2)) - g^2*(a^2*e^2*g^2*(1 - m)*m -
2*a*b*e*g*m*(2*d*g - e*f*(1 + m)) - b^2*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^
2*f^2*(2 + 3*m + m^2))) + 2*c*g*(a*g*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f
^2*(2 + 3*m + m^2)) - b*f*(6*d^2*g^2 - 6*d*e*f*g*(2 + m) + e^2*f^2*(6 + 5*m
 + m^2))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e
*x))/(e*f - d*g))])/(2*g^4*(e*f - d*g)^3*(1 + m))
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx = \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} + \int \frac{(d+ex)^m\left(\frac{c^2f^3(2dg-ef(1+m))-2cfg(bf-ag)(2dg-ef(1+m))+g^2\left(a^2eg^2(1-m)\cdots\right.}{g^4}\right.}{\cdots}$$

$$= \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{\left(cf^2-bfg+ag^2\right)\left(g(4bdg-aeg(1-m)-bef(3+\cdots\right.}{2g^4(ef-dg)^2(f+\cdots}$$

$$= \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{\left(cf^2-bfg+ag^2\right)\left(g(4bdg-aeg(1\cdots\right.}{2\cdots}$$

$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2}\cdots$$

$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2}\cdots$$

Mathematica [A] time = 0.411693, size = 257, normalized size = 0.56

$$\frac{(d+ex)^{m+1}\left(\frac{\left(2cg(ag-3bf)+b^2g^2+6c^2f^2\right){}_2F_1\left(1,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)}+\frac{e^2\left(g(ag-bf)+cf^2\right)^2{}_2F_1\left(3,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)^3}-\frac{2e(2cf-bg)\left(g(ag-bf)+cf^2\right){}_2F_1}{(m+1)(ef-dg}\right.}{g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*(-((c*(3*c*e*f + c*d*g - 2*b*e*g))/(e^2*(1 + m))) + (c^2
*g*(d + e*x))/(e^2*(2 + m)) + ((6*c^2*f^2 + b^2*g^2 + 2*c*g*(-3*b*f + a*g))
*Hypergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f -
d*g)*(1 + m)) - (2*e*(2*c*f - b*g)*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometri
c2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^2*(1 + m)
) + (e^2*(c*f^2 + g*(-(b*f) + a*g))^2*Hypergeometric2F1[3, 1 + m, 2 + m, (g
*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^3*(1 + m))))/g^4
```

Maple [F] time = 1.459, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2(ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+2\,bcx^3+2\,abx+\left(b^2+2\,ac\right)x^2+a^2\right)(ex+d)^m}{g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d
)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{\left(f+gx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**3,x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^2 (ex+d)^m}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

3.930 $\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2} dx$

Optimal. Leaf size=183

$$-\frac{3\left(5499-1631\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(5499+1631\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+\right.}{26\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] (3687*(1 + 4*x)^(1 + m))/(64*(1 + m)) + (207*(1 + 4*x)^(2 + m))/(32*(2 + m)
) + (27*(1 + 4*x)^(3 + m))/(64*(3 + m)) - (3*(5499 - 1631*Sqrt[13])*(1 + 4*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13
])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(5499 + 1631*Sqrt[13])*(1 + 4*x)^(
1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])
/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.23311, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1628, 68}

$$-\frac{3\left(5499-1631\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(5499+1631\sqrt{13}\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+\right.}{26\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```


```
[Out] (3687*(1 + 4*x)^(1 + m))/(64*(1 + m)) + (207*(1 + 4*x)^(2 + m))/(32*(2 + m)
) + (27*(1 + 4*x)^(3 + m))/(64*(3 + m)) - (3*(5499 - 1631*Sqrt[13])*(1 + 4*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13
])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(5499 + 1631*Sqrt[13])*(1 + 4*x)^(
1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])
/(26*(13 + 2*Sqrt[13])*(1 + m))
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \left(\frac{3687}{16}(1+4x)^m + \frac{207}{8}(1+4x)^{1+m} + \frac{27}{16}(1+4x)^{2+m} + \frac{\left(1269+\frac{4893}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(1269-\right.}{-5}\right.$$

$$= \frac{3687(1+4x)^{1+m}}{64(1+m)} + \frac{207(1+4x)^{2+m}}{32(2+m)} + \frac{27(1+4x)^{3+m}}{64(3+m)} + \frac{1}{13}\left(3\left(5499-1631\sqrt{13}\right)\right)\int \frac{(1+}{-5+\sqrt{}}$$

$$= \frac{3687(1+4x)^{1+m}}{64(1+m)} + \frac{207(1+4x)^{2+m}}{32(2+m)} + \frac{27(1+4x)^{3+m}}{64(3+m)} - \frac{3\left(5499-1631\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1}{26\left(13-2\sqrt{13}\right)(}$$

Mathematica [A] time = 0.284896, size = 151, normalized size = 0.83

$$\frac{3}{832}(4x+1)^{m+1}\left(-\frac{32\left(1631\sqrt{13}-5499\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} - \frac{32\left(5499+1631\sqrt{13}\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12}{13+}\right.}{\left(13+2\sqrt{13}\right)(m+1)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*(15977/(1 + m) + (1794*(1 + 4*x))/(2 + m) + (117*(1 +
4*x)^2)/(3 + m) - (32*(-5499 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (32*(54
99 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*S
qrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/832
```

Maple [F] time = 1.43, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(2+3x)^4}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^4*(4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)^4*(4*x+1)^m/(3*x^2-5*x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^4}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(81\,x^{4}+216\,x^{3}+216\,x^{2}+96\,x+16\right)(4\,x+1)^{m}}{3\,x^{2}-5\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(3*x^2 - 5*x
+ 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4\,x+1)^{m}(3\,x+2)^{4}}{3\,x^{2}-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

3.931 $\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=165

$$-\frac{3\left(416-135\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(416+135\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] (123*(1 + 4*x)^(1 + m))/(16*(1 + m)) + (9*(1 + 4*x)^(2 + m))/(16*(2 + m)) -
 (3*(416 - 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(13*(13 - 2*Sqrt[13])*(1 + m)) - (3*(4
16 + 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*(13 + 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.152348, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1628, 68}

$$-\frac{3\left(416-135\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(416+135\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```


```
[Out] (123*(1 + 4*x)^(1 + m))/(16*(1 + m)) + (9*(1 + 4*x)^(2 + m))/(16*(2 + m)) -
 (3*(416 - 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(13*(13 - 2*Sqrt[13])*(1 + m)) - (3*(4
16 + 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*(13 + 2*Sqrt[13])*(1 + m))
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \left(\frac{123}{4}(1+4x)^m + \frac{9}{4}(1+4x)^{1+m} + \frac{\left(192+\frac{810}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(192-\frac{810}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right) dx$$

$$= \frac{123(1+4x)^{1+m}}{16(1+m)} + \frac{9(1+4x)^{2+m}}{16(2+m)} + \frac{1}{13}\left(6\left(416-135\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx + \frac{1}{13}\left(6\left(\right.\right.$$

$$= \frac{123(1+4x)^{1+m}}{16(1+m)} + \frac{9(1+4x)^{2+m}}{16(2+m)} - \frac{3\left(416-135\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1}{13-}\right.}{13\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A] time = 0.115976, size = 117, normalized size = 0.71

$$\frac{(4x+1)^{m+1}\left(16\left(71\sqrt{13}-146\right)(m+2)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)-16\left(146+71\sqrt{13}\right)(m+2)\,{}_2F_1\left(1,m+1;m+2\right.\right.}{624\left(m^2+3m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(117*(85 + 12*x + 4*m*(11 + 3*x)) + 16*(-146 + 71*Sqrt[1
3])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])
] - 16*(146 + 71*Sqrt[13])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 +
12*x)/(13 + 2*Sqrt[13])]))/(624*(2 + 3*m + m^2))
```

Maple [F] time = 1.305, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m (2+3x)^3}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^3*(4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)^3*(4*x+1)^m/(3*x^2-5*x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^3}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(27x^3+54x^2+36x+8\right)(4x+1)^m}{3x^2-5x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^3 (4x+1)^m}{3x^2-5x+1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m (3x+2)^3}{3x^2-5x+1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

3.932 $\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=147

$$-\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] (3*(1 + 4*x)^(1 + m))/(4*(1 + m)) - (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m
)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*
(13 - 2*Sqrt[13])*(1 + m)) - (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2
*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.152146, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1628, 68}

$$-\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m))/(4*(1 + m)) - (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m
)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*
(13 - 2*Sqrt[13])*(1 + m)) - (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2
*Sqrt[13])*(1 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \left(3(1+4x)^m + \frac{\left(27+\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(27-\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx$$

$$= \frac{3(1+4x)^{1+m}}{4(1+m)} + \frac{1}{13}\left(3\left(117-47\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx + \frac{1}{13}\left(3\left(117+47\sqrt{13}\right)\right)\int \frac{(1}{-5-}$$

$$= \frac{3(1+4x)^{1+m}}{4(1+m)} - \frac{3\left(117-47\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)} - \frac{3\left(117+47\sqrt{1}\right.}{}$$

Mathematica [A] time = 0.0903008, size = 91, normalized size = 0.62

$$\frac{(4x+1)^{m+1}\left(\left(58\sqrt{13}-46\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)-2\left(23+29\sqrt{13}\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)+117\right)}{156(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(117 + (-46 + 58*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] - 2*(23 + 29*Sqrt[13])*Hypergeometric2F
1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(156*(1 + m))
```

Maple [F] time = 1.404, size = 0, normalized size = 0.

$$\int \frac{(4\,x+1)^m\,(2+3\,x)^2}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*(4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)^2*(4*x+1)^m/(3*x^2-5*x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4\,x+1)^m(3\,x+2)^2}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(9\,x^2+12\,x+4\right)(4\,x+1)^m}{3\,x^2-5\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^2 (4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m (3x+2)^2}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

3.933 $\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx$

Optimal. Leaf size=129

$$-\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\,_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] (-3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(13 +
 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4
*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.108537, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {830, 68}

$$-\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\,_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (-3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(13 +
 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4
*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx &= \int \left(\frac{\left(3+\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x}+\frac{\left(3-\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right)dx\\
&= \frac{1}{13}\left(3\left(13-9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx+\frac{1}{13}\left(3\left(13+9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx\\
&= -\frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)}-\frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,_2F_1\left(1\right.}{26\left(13+2\sqrt{13}\right)}
\end{aligned}$$

Mathematica [A] time = 0.0808766, size = 89, normalized size = 0.69

$$\frac{(4x+1)^{m+1}\left(\left(5+7\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)+\left(5-7\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)\right)}{78(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((5 + 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
+ 12*x)/(13 - 2*Sqrt[13])] + (5 - 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(78*(1 + m))
```

Maple [F] time = 1.129, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m (2+3x)}{3x^2-5x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*(4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)*(4*x+1)^m/(3*x^2-5*x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x+1)^m(3x+2)}{3x^2-5x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)(4x+1)^m}{3x^2-5x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

3.934 $\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=117

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}-\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 -
 2*Sqrt[13])])/(Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) - (3*(1 + 4*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sqrt[
13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.113681, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {711, 68}

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}-\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 -
 2*Sqrt[13])])/(Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) - (3*(1 + 4*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sqrt[
13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 711

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx &= \int \left(-\frac{6(1+4x)^m}{\sqrt{13}\left(5+\sqrt{13}-6x\right)}-\frac{6(1+4x)^m}{\sqrt{13}\left(-5+\sqrt{13}+6x\right)}\right)dx\\ &= -\frac{6\int \frac{(1+4x)^m}{5+\sqrt{13}-6x}\,dx}{\sqrt{13}}-\frac{6\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{\sqrt{13}}\\ &= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}-\frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}\end{aligned}$$

Mathematica [A] time = 0.107085, size = 94, normalized size = 0.8

$$\frac{(4x+1)^{m+1}\left(\left(13+2\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)+\left(2\sqrt{13}-13\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)\right)}{39\sqrt{13}(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])] + (-13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m
, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(39*Sqrt[13]*(1 + m))
```

Maple [F] time = 1.245, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((4*x+1)^m/(3*x^2-5*x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x+1)^m}{3x^2-5x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.935 $\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)}\,dx$

Optimal. Leaf size=164

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{85(m+1)}+\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)}+\frac{3\left(13-9\sqrt{1\right.}}{}$$

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/
(85*(1 + m)) + (3*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(442*(13 - 2*Sqrt[13])*(1 +
 m)) + (3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(442*(13 + 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.200582, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {960, 68, 830}

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{85(m+1)}+\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)}+\frac{3\left(13-9\sqrt{1\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/
(85*(1 + m)) + (3*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(442*(13 - 2*Sqrt[13])*(1 +
 m)) + (3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(442*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)}\,dx = \int\left(\frac{3(1+4x)^m}{17(2+3x)} + \frac{(7-3x)(1+4x)^m}{17\left(1-5x+3x^2\right)}\right)dx$$
$$= \frac{1}{17}\int \frac{(7-3x)(1+4x)^m}{1-5x+3x^2}\,dx + \frac{3}{17}\int \frac{(1+4x)^m}{2+3x}\,dx$$
$$= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)} + \frac{1}{17}\int\left(\frac{\left(-3+\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{(-3\right.}{}$$
$$= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)} - \frac{1}{221}\left(3\left(13-9\sqrt{13}\right)\right)\int \frac{(1+4x)^n}{-5-\sqrt{13}+}$$
$$= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)} + \frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1\right.}{442\left(13-2\sqrt{13}\right)(1}$$

Mathematica [A] time = 0.166683, size = 110, normalized size = 0.67

$$\frac{(4x+1)^{m+1}\left(234\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)+5\left(31+11\sqrt{13}\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)+5\left(31-11\sqrt{13}\right)\right.}{6630(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(234*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5
] + 5*(31 + 11*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
- 2*Sqrt[13])] + 5*(31 - 11*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 + 2*Sqrt[13])]))/(6630*(1 + m))
```

Maple [F] time = 1.312, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(2+3x)\left(3x^2-5x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(2+3*x)/(3*x^2-5*x+1),x)
```

```
[Out] int((4*x+1)^m/(2+3*x)/(3*x^2-5*x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)(3x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x+1)^m}{9x^3-9x^2-7x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(9*x^3 - 9*x^2 - 7*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x+2)\left(3x^2-5x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1),x)
```

```
[Out] Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)(3x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

3.936 $\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx$

Optimal. Leaf size=199

$$\frac{27(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}+\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}+\frac{3\left(11\right.}{}$$

```
[Out] (27*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])
/(1445*(1 + m)) + (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13]
)*(1 + m)) + (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1
+ m)) + (12*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5])/(425*(1 + m))
```

Rubi [A] time = 0.221183, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {960, 68, 830}

$$\frac{27(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}+\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}+\frac{3\left(11\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```


```
[Out] (27*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])
/(1445*(1 + m)) + (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13]
)*(1 + m)) + (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1
+ m)) + (12*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5])/(425*(1 + m))
```


Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)}\,dx = \int \left(\frac{3(1+4x)^m}{17(2+3x)^2}+\frac{27(1+4x)^m}{289(2+3x)}+\frac{(46-27x)(1+4x)^m}{289\left(1-5x+3x^2\right)}\right)dx$$

$$=\frac{1}{289}\int \frac{(46-27x)(1+4x)^m}{1-5x+3x^2}\,dx+\frac{27}{289}\int \frac{(1+4x)^m}{2+3x}\,dx+\frac{3}{17}\int \frac{(1+4x)^m}{(2+3x)^2}\,dx$$

$$=\frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{12(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\right.}{425(1+m)}$$

$$=\frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{12(1+4x)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\right.}{425(1+m)}$$

$$=\frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{3\left(117+47\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,\right.}{7514\left(13-2\sqrt{13}\right)(1}$$

Mathematica [A] time = 0.134042, size = 152, normalized size = 0.76

$$\frac{(4x+1)^{m+1}\left(10530\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)+25\left(211+65\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)-1625\sqrt{13}\,{}_2F_1\left(\right.\right.}{563550(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]

[Out] ((1 + 4*x)^(1 + m)*(10530*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))
/5] + 25*(211 + 65*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/
(13 - 2*Sqrt[13])] + 5275*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
 + 2*Sqrt[13])] - 1625*Sqrt[13]*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*
x)/(13 + 2*Sqrt[13])] + 15912*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5]))/(563550*(1 + m))
```

Maple [F] time = 1.34, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(2+3x)^2\left(3x^2-5x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(2+3*x)^2/(3*x^2-5*x+1),x)

[Out] int((4*x+1)^m/(2+3*x)^2/(3*x^2-5*x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)(3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(4\,x+1)^m}{27\,x^4-9\,x^3-39\,x^2-8\,x+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(27*x^4 - 9*x^3 - 39*x^2 - 8*x + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x+2)^2\left(3x^2-5x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1),x)
```

```
[Out] Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)(3\,x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

3.937 $\int \frac{(2+3x)^4(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{\left(13689-\sqrt{13}\left(-1570\sqrt{13}m+4474m+297\right)\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(m+1)}-\frac{\left(\sqrt{13}\left(1570\sqrt{13}m+4474m\right.\right.}{}$$

```
[Out] (9*(1 + 4*x)^(1 + m))/(4*(1 + m)) + ((844 - 2355*x)*(1 + 4*x)^(1 + m))/(39*
(1 - 5*x + 3*x^2)) - ((13689 - Sqrt[13]*(297 + 4474*m - 1570*Sqrt[13]*m))*(
1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*S
qrt[13])])/(169*(13 - 2*Sqrt[13])*(1 + m)) - ((13689 + Sqrt[13]*(297 + 4474
*m + 1570*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(169*(13 + 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.294886, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1648, 1628, 68}

$$-\frac{\left(13689-\sqrt{13}\left(-1570\sqrt{13}m+4474m+297\right)\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(m+1)}-\frac{\left(\sqrt{13}\left(1570\sqrt{13}m+4474m\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] (9*(1 + 4*x)^(1 + m))/(4*(1 + m)) + ((844 - 2355*x)*(1 + 4*x)^(1 + m))/(39*
(1 - 5*x + 3*x^2)) - ((13689 - Sqrt[13]*(297 + 4474*m - 1570*Sqrt[13]*m))*(
1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*S
qrt[13])])/(169*(13 - 2*Sqrt[13])*(1 + m)) - ((13689 + Sqrt[13]*(297 + 4474
*m + 1570*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(169*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(2+3x)^4(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m\left(13(4617+3376m)-39(1521+3140m)x\right.}{1-5x+3x^2}$$

$$= \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int\left(-4563(1+4x)^m + \frac{\left(-82134-122460m-6\sqrt{13}(297\right.}{-5-\sqrt{13}+6}\right.$$

$$= \frac{9(1+4x)^{1+m}}{4(1+m)} + \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\left(-82134-122460m+6\sqrt{13}(297+447\right.$$

$$= \frac{9(1+4x)^{1+m}}{4(1+m)} + \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(13689-\sqrt{13}\left(297+4474m-1570\sqrt{13}m\right)\right.}{169\left(13-2\sqrt{\right.}}$$

Mathematica [A] time = 0.308564, size = 251, normalized size = 1.24

$$\frac{(4x+1)^{m+1}\left(-\frac{1053\left(128\sqrt{13}-117\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} - \frac{1053\left(117+128\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(m+1)} - \frac{\left(2\left(5731+667\sqrt{13}\right)m-14679\left(\sqrt{13}-\right.\right.}{}\right.}{1521}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(13689/(4 + 4*m) + (39*(844 - 2355*x))/(1 - 5*x + 3*x^2)
 - (1053*(-117 + 128*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x
)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (1053*(117 + 128*Sqrt[
13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13
 + 2*Sqrt[13])*(1 + m)) - (-((-14679*(2 + Sqrt[13]) + 2*(-5731 + 667*Sqrt[1
3])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])]) +
(-14679*(-2 + Sqrt[13]) + 2*(5731 + 667*Sqrt[13])*m)*Hypergeometric2F1[1, 1
 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(1 + m)))/1521
```

Maple [F] time = 1.332, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(2+3x)^4}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^4*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)^4*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^4}{(3x^2-5x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(81x^4+216x^3+216x^2+96x+16)(4x+1)^m}{9x^4-30x^3+31x^2-10x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(9*x^4 - 30*x
^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^4}{(3x^2-5x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

3.938 $\int \frac{(2+3x)^3(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=181

$$-\frac{\left(\sqrt{13}\left(568\sqrt{13}m-1168m+1701\right)+1521\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\sqrt{13}(1701-1168m)-13(5\right.}{}$$

```
[Out] ((209 - 426*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((1521 + Sqrt[13
]*(1701 - 1168*m + 568*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(338*(13 - 2*Sqrt[13])*(1 +
 m)) + ((Sqrt[13]*(1701 - 1168*m) - 13*(117 + 568*m))*(1 + 4*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(338*(13
+ 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.280688, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1648, 830, 68}

$$-\frac{\left(\sqrt{13}\left(568\sqrt{13}m-1168m+1701\right)+1521\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\sqrt{13}(1701-1168m)-13(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((209 - 426*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((1521 + Sqrt[13
]*(1701 - 1168*m + 568*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(338*(13 - 2*Sqrt[13])*(1 +
 m)) + ((Sqrt[13]*(1701 - 1168*m) - 13*(117 + 568*m))*(1 + 4*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(338*(13
+ 2*Sqrt[13])*(1 + m))
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
```

```
b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(2+3x)^3(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(13(1143+836m)-39(117+568m)x)}{1-5x+3x^2}\,dx$$
$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(\frac{\left(-39(117+568m)-3\sqrt{13}(-1701+1168m)\right)(1+4x)^m}{-5-\sqrt{13}+6x}\right.$$
$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{169}\left(\sqrt{13}(1701-1168m)-13(117+568m)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}$$
$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(\sqrt{13}(1701-1168m)+13(117+568m)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+\right.}{338\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A] time = 0.299516, size = 252, normalized size = 1.39

$$\frac{(4x+1)^{m+1}\left(-\frac{12\left(\sqrt{13}(1215-292m)+1846m\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(m+1)} - \frac{351\left(27\sqrt{13}-13\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} + \frac{12\left(\sqrt{13}(1215-292m)-1846m\right){}_2}{\left(13+2\sqrt{13}\right)(}\right.}{1014}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((5434 - 11076*x)/(1 - 5*x + 3*x^2) - (351*(-13 + 27*Sqr
t[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((
-13 + 2*Sqrt[13])*(1 + m)) - (12*(Sqrt[13]*(1215 - 292*m) + 1846*m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 - 2*Sqrt[13
])*(1 + m)) - (351*(13 + 27*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (12*(Sqrt[13]*(1
215 - 292*m) - 1846*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 +
2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/1014
```

Maple [F] time = 1.35, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(2+3x)^3}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^3*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)^3*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^3}{(3x^2-5x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(27x^3+54x^2+36x+8)(4x+1)^m}{9x^4-30x^3+31x^2-10x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2
- 10*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^3}{(3x^2-5x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

3.939 $\int \frac{(2+3x)^2(1+4x)^m}{\left(1-5x+3x^2\right)^2} \, dx$

Optimal. Leaf size=179

$$-\frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(153-\left(23+29\sqrt{13}\right)m\right)(4x+1)^{m+1}{}_2F_1\left(1,m\right.}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] ((61 - 87*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(153 - (23 - 29
*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
4*x))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(153
 - (23 + 29*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rubi [A] time = 0.248798, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1648, 830, 68}

$$-\frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(153-\left(23+29\sqrt{13}\right)m\right)(4x+1)^{m+1}{}_2F_1\left(1,m\right.}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((61 - 87*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(153 - (23 - 29
*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
4*x))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(153
 - (23 + 29*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
```

```
b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x)^2(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx &= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(26(153+122m)-4524mx)}{1-5x+3x^2}\,dx\\
&= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(\frac{\left(-4524m-12\sqrt{13}(-153+23m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-4524m\right.}{}\right.\\
&= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} + \frac{\left(4\left(153-\left(23-29\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{13\sqrt{13}} - \frac{\left(4\left(153-\left(23+2\right.\right.\right.}{}\\
&= \frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+}{13-2}\right.}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}\end{aligned}$$

Mathematica [A] time = 0.294192, size = 156, normalized size = 0.87

$$\frac{1}{507}(4x+1)^{m+1}\left(-\frac{6\left(\left(23\sqrt{13}-377\right)m-153\sqrt{13}\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} - \frac{6\left(\left(377+23\sqrt{13}\right)m-153\sqrt{13}\right)\,{}_2}{\left(13+2\sqrt{13}\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((793 - 1131*x)/(1 - 5*x + 3*x^2) - (6*(-153*Sqrt[13] +
(-377 + 23*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 -
 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (6*(-153*Sqrt[13] + (377 + 23
*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13
])])/((13 + 2*Sqrt[13])*(1 + m))))/507
```

Maple [F] time = 1.329, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(2+3x)^2}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)^2*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^2}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(9x^2+12x+4\right)(4x+1)^m}{9x^4-30x^3+31x^2-10x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^2(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^2}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)
```

3.940 $\int \frac{(2+3x)(1+4x)^m}{\left(1-5x+3x^2\right)^2} dx$

Optimal. Leaf size=179

$$-\frac{\left(2\left(5+7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1\right.}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] ((20 - 21*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((81 + 2*(5 + 7*Sq
rt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x
))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + ((81 + 2*(
5 - 7*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.212097, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {822, 830, 68}

$$-\frac{\left(2\left(5+7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1\right.}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((20 - 21*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((81 + 2*(5 + 7*Sq
rt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x
))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + ((81 + 2*(
5 - 7*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(2+3x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(13(81+80m)-1092mx)}{1-5x+3x^2}\,dx$$

$$= \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \left(\frac{\left(-1092m+6\sqrt{13}(81+10m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-1092m-6\sqrt{13}\right.}{-5+}\right.$$

$$= \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} + \frac{1}{169}\left(2\left(182m+\sqrt{13}(81+10m)\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx - \frac{1}{507}\left(-10\right.$$

$$= \frac{(20-21x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(182m+\sqrt{13}(81+10m)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A] time = 0.177787, size = 149, normalized size = 0.83

$$\frac{1}{507}(4x+1)^{m+1}\left(\frac{3\left(182m+\sqrt{13}(10m+81)\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} - \frac{3\left(182m-\sqrt{13}(10m+81)\right)\,{}_2F_1\left(1,m+1;\right.}{\left(13+2\sqrt{13}\right)(m+1)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((260 - 273*x)/(1 - 5*x + 3*x^2) + (3*(182*m + Sqrt[13]*
(81 + 10*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13]
)])/((-13 + 2*Sqrt[13])*(1 + m)) - (3*(182*m - Sqrt[13]*(81 + 10*m))*Hyperg
eometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[1
3])*(1 + m))))/507
```

Maple [F] time = 1.311, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(2+3x)}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)*(4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4\,x+1)^m(3\,x+2)}{9\,x^4-30\,x^3+31\,x^2-10\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m*(3*x + 2)/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(3x+2)\,(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(4\,x+1)^m(3\,x+2)}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

3.941 $\int \frac{(1+4x)^m}{\left(1-5x+3x^2\right)^2} dx$

Optimal. Leaf size=177

$$-\frac{2\left(2\left(2+\sqrt{13}\right)m+9\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(2\left(2-\sqrt{13}\right)m+9\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+\ldots\right.}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

```
[Out] ((7 - 6*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(9 + 2*(2 + Sqrt[
13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/
(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(9 + 2*(2
- Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rubi [A] time = 0.199295, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {740, 830, 68}

$$-\frac{2\left(2\left(2+\sqrt{13}\right)m+9\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(2\left(2-\sqrt{13}\right)m+9\right)(4x+1)^{m+1}{}_2F_1\left(1,m+1;m+\ldots\right.}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```


```
[Out] ((7 - 6*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(9 + 2*(2 + Sqrt[
13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/
(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(9 + 2*(2
- Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```


Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(26(9+14m)-312mx)}{1-5x+3x^2}\,dx$$

$$= \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int\left(\frac{\left(-312m+12\sqrt{13}(9+4m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-312m-12\sqrt{13}(9\right.}{-5+\sqrt{13}}\right.$$

$$= \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(4\left(9+2\left(2-\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{13\sqrt{13}} + \frac{\left(4\left(9+2\left(2+\sqrt{13}\right)m\right)\right)\int}{13\sqrt{13}}$$

$$= \frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{2\left(9+2\left(2+\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} + \frac{2(}{}$$

Mathematica [A] time = 0.162479, size = 150, normalized size = 0.85

$$\frac{1}{507}(4x+1)^{m+1}\left(\frac{6\left(26m+\sqrt{13}(4m+9)\right)\,{}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(2\sqrt{13}-13\right)(m+1)} + \frac{6\sqrt{13}\left(9-2\left(\sqrt{13}-2\right)m\right)\,{}_2F_1\left(1,m+1;m\right.}{\left(13+2\sqrt{13}\right)(m+1)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((91 - 78*x)/(1 - 5*x + 3*x^2) + (6*(26*m + Sqrt[13]*(9
+ 4*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/(
(-13 + 2*Sqrt[13])*(1 + m)) + (6*Sqrt[13]*(9 - 2*(-2 + Sqrt[13])*m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13
])*(1 + m))))/507
```

Maple [F] time = 1.174, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((4*x+1)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4\,x+1)^m}{9\,x^4-30\,x^3+31\,x^2-10\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4\,x+1)^m}{\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

3.942 $\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=340

$$\frac{9(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}-\frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+$$

```
[Out] ((43 - 33*x)*(1 + 4*x)^(1 + m))/(663*(1 - 5*x + 3*x^2)) + (9*(1 + 4*x)^(1 +
 m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1445*(1 + m)) +
(9*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (
3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13])*(1 + m)) - ((81 +
(62 + 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(221*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m))
+ (9*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1 + m)) + ((81
+ (62 - 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(221*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rubi [A] time = 0.50447, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {960, 68, 822, 830}

$$\frac{9(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}-\frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```


```
[Out] ((43 - 33*x)*(1 + 4*x)^(1 + m))/(663*(1 - 5*x + 3*x^2)) + (9*(1 + 4*x)^(1 +
 m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1445*(1 + m)) +
(9*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (
3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13])*(1 + m)) - ((81 +
(62 + 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(221*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m))
+ (9*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1 + m)) + ((81
+ (62 - 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(221*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```


Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx = \int \left(\frac{9(1+4x)^m}{289(2+3x)} + \frac{(7-3x)(1+4x)^m}{17\left(1-5x+3x^2\right)^2} - \frac{3(-7+3x)(1+4x)^m}{289\left(1-5x+3x^2\right)}\right)dx$$

$$= -\left(\frac{3}{289}\int \frac{(-7+3x)(1+4x)^m}{1-5x+3x^2}\,dx\right) + \frac{9}{289}\int \frac{(1+4x)^m}{2+3x}\,dx + \frac{1}{17}\int \frac{(7-3x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} - \int \frac{(1+4x)^m(13(\cdots}{1\cdots}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} - \int \left(\frac{-1716m+6\cdots}{\cdots}\right.$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} + \frac{9\left(13+9\sqrt{1\cdots}\right.}{\cdots}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)} + \frac{9\left(13+9\sqrt{1\cdots}\right.}{\cdots}$$

Mathematica [A] time = 0.547949, size = 274, normalized size = 0.81

$$\frac{(4x+1)^{m+1}\left(\frac{9126\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{m+1} + \frac{1755\left(13+9\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(m+1)} + \frac{1755\left(13-9\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(m+1)} + \frac{510\sqrt{13}\cdots}{\cdots}\right.}{1465230}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((2210*(43 - 33*x))/(1 - 5*x + 3*x^2) + (9126*Hypergeome
tric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (1755*(13 + 9*Sqrt[13
])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 -
 2*Sqrt[13])*(1 + m)) + (1755*(13 - 9*Sqrt[13])*Hypergeometric2F1[1, 1 + m,
 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (510*S
qrt[13]*(((81 + (62 + 22*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])])/(-13 + 2*Sqrt[13]) + ((81 + (62 - 22*Sqrt[13])
*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(13 +
 2*Sqrt[13])))/(1 + m)))/1465230
```

Maple [F] time = 1.341, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(2+3x)\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(2+3*x)/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((4*x+1)^m/(2+3*x)/(3*x^2-5*x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2(3x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(4x+1)^m}{27x^5-72x^4+33x^3+32x^2-17x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(27*x^5 - 72*x^4 + 33*x^3 + 32*x^2 - 17*x + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x+2)\left(3x^2-5x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)^2(3x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

3.943 $\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=376

$$\frac{162(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{24565(m+1)}-\frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+}{13-2}\right.}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$

```
[Out] ((268 - 195*x)*(1 + 4*x)^(1 + m))/(11271*(1 - 5*x + 3*x^2)) + (162*(1 + 4*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(24565*(1 +
 m)) + (9*(117 + 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(63869*(13 - 2*Sqrt[13])*(1 + m))
 - ((423 + 2*(211 + 65*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(3757*Sqrt[13]*(13 - 2*Sqrt
[13])*(1 + m)) + (9*(117 - 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(63869*(13 + 2*Sqrt[13]
)*(1 + m)) + ((423 + (422 - 130*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(3757*Sqrt[13]*(13
 + 2*Sqrt[13])*(1 + m)) + (36*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m,
 2 + m, (-3*(1 + 4*x))/5])/(7225*(1 + m))
```

Rubi [A] time = 0.493232, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {960, 68, 822, 830}

$$\frac{162(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{24565(m+1)}-\frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+}{13-2}\right.}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```


```
[Out] ((268 - 195*x)*(1 + 4*x)^(1 + m))/(11271*(1 - 5*x + 3*x^2)) + (162*(1 + 4*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(24565*(1 +
 m)) + (9*(117 + 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(63869*(13 - 2*Sqrt[13])*(1 + m))
 - ((423 + 2*(211 + 65*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(3757*Sqrt[13]*(13 - 2*Sqrt
[13])*(1 + m)) + (9*(117 - 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(63869*(13 + 2*Sqrt[13]
)*(1 + m)) + ((423 + (422 - 130*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(3757*Sqrt[13]*(13
 + 2*Sqrt[13])*(1 + m)) + (36*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m,
 2 + m, (-3*(1 + 4*x))/5])/(7225*(1 + m))
```


Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2}\,dx = \int \left(\frac{9(1+4x)^m}{289(2+3x)^2}+\frac{162(1+4x)^m}{4913(2+3x)}+\frac{(46-27x)(1+4x)^m}{289\left(1-5x+3x^2\right)^2}-\frac{3(1+4x)^m(-109+54x)}{4913\left(1-5x+3x^2\right)}\right)$$

$$= -\frac{3\int \frac{(1+4x)^m(-109+54x)}{1-5x+3x^2}\,dx}{4913}+\frac{1}{289}\int \frac{(46-27x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx+\frac{9}{289}\int \frac{(1+4x)^m}{(2+3x)^2}\,dx+\frac{1}{}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)}+\frac{36(1+}{}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)}+\frac{36(1+}{}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)}+\frac{9(117}{}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)}+\frac{9(117}{}$$

Mathematica [A] time = 0.575377, size = 287, normalized size = 0.76

$$(4x+1)^{m+1}\left(\frac{1232010\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{m+1}+\frac{26325\left(117+64\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(m+1)}+\frac{26325\left(117-64\sqrt{13}\right){}_2F_1\left(1,m+1;m+2;\frac{12x+3}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(m+1)}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((16575*(268 - 195*x))/(1 - 5*x + 3*x^2) + (1232010*Hype
rgeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (26325*(117 + 6
4*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])
])/((13 - 2*Sqrt[13])*(1 + m)) + (26325*(117 - 64*Sqrt[13])*Hypergeometric2
F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 +
m)) - (425*((423*(2 + Sqrt[13]) + (2534 + 682*Sqrt[13])*m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] + (-423*(-2 + Sqrt[13]) +
(2534 - 682*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
+ 2*Sqrt[13])]))/(1 + m) + (930852*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(
1 + 4*x))/5])/(1 + m)))/186816825
```

Maple [F] time = 1.35, size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(2+3x)^2 \left(3x^2-5x+1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((4*x+1)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{\left(3x^2-5x+1\right)^2 (3x+2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(4x+1)^m}{81x^6-162x^5-45x^4+162x^3+13x^2-28x+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(81*x^6 - 162*x^5 - 45*x^4 + 162*x^3 + 13*x^2 - 28*x +
 4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)^2(3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

3.944 $\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} \, dx$

Optimal. Leaf size=237

$$\frac{2\sqrt{e+fx}(d+ex)^m \left(-\frac{f(d+ex)}{e^2-df}\right)^{-m} {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{e(e+fx)}{e^2-df}\right)\left(c\left(d^2f^2+4de^2f(m+1)-4e^4\left(m^2+3m+2\right)\right)-ef(2m+3)\left(\right.\right.}{ef^3(2m+3)\left(e^2-df\right)}$$

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(1 + m))/((e^2 - d*f)*Sqrt[e + f*x])
 + (2*c*(d + e*x)^(1 + m)*Sqrt[e + f*x])/(e*f^2*(3 + 2*m)) + (2*(c*(d^2*f^2
 + 4*d*e^2*f*(1 + m) - 4*e^4*(2 + 3*m + m^2)) - e*f*(3 + 2*m)*(a*e*f*(1 + 2
*m) + b*(d*f - 2*e^2*(1 + m))))*(d + e*x)^m*Sqrt[e + f*x]*Hypergeometric2F1
[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)])/(e*f^3*(e^2 - d*f)*(3 + 2*m)*(-(
(f*(d + e*x))/(e^2 - d*f)))^m)
```

Rubi [A] time = 0.348564, antiderivative size = 230, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {949, 80, 70, 69}

$$-\frac{2\sqrt{e+fx}(d+ex)^m \left(-\frac{f(d+ex)}{e^2-df}\right)^{-m} {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{e(e+fx)}{e^2-df}\right)\left(f\left(aef(2m+1)+bdf-2be^2(m+1)\right)-\frac{c\left(d^2f^2+4de^2f(m+1)-4e^4\left(\right.\right.}{e(2m+3)}\right.}{f^3\left(e^2-df\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(1 + m))/((e^2 - d*f)*Sqrt[e + f*x])
 + (2*c*(d + e*x)^(1 + m)*Sqrt[e + f*x])/(e*f^2*(3 + 2*m)) - (2*(f*(b*d*f -
 2*b*e^2*(1 + m) + a*e*f*(1 + 2*m)) - (c*(d^2*f^2 + 4*d*e^2*f*(1 + m) - 4*e
^4*(2 + 3*m + m^2)))/(e*(3 + 2*m)))*(d + e*x)^m*Sqrt[e + f*x]*Hypergeometri
c2F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)])/(f^3*(e^2 - d*f)*(-((f*(d +
e*x))/(e^2 - d*f)))^m)
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{\left(e^2-df\right)\sqrt{e+fx}}+\frac{2\int\frac{(d+ex)^m\left(\frac{c\left(def-2e^3(1+m)\right)-f\left(bdf-2be^2(1+m)+aef(1+2m)\right)}{2f^2}-\frac{1}{2}c\left(d-\frac{e^2}{f}\right)x\right)}{\sqrt{e+fx}}\,dx}{e^2-df}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{\left(e^2-df\right)\sqrt{e+fx}}+\frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)}-\frac{\left(f\left(bdf-2be^2(1+m)+aef(1-\right.\right.}{\ldots}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{\left(e^2-df\right)\sqrt{e+fx}}+\frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)}-\frac{\left(\left(f\left(bdf-2be^2(1+m)+aef(1\right.\right.\right.}{\ldots}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{\left(e^2-df\right)\sqrt{e+fx}}+\frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)}-\frac{2\left(f\left(bdf-2be^2(1+m)+aef(1\right.\right.}{\ldots}$$

Mathematica [A] time = 0.291858, size = 171, normalized size = 0.72

$$\frac{2(d+ex)^m\left(\frac{f(d+ex)}{df-e^2}\right)^{-m}\left(-3\left(f(af-be)+ce^2\right){}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{e(e+fx)}{e^2-df}\right)-(e+fx)\left((6ce-3bf)\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{e(e+fx)}{e^2-df}\right)-c(e\ldots\right.\right.}{3f^3\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(d + e*x)^m*(-3*(c*e^2 + f*(-(b*e) + a*f))*Hypergeometric2F1[-1/2, -m, 1
/2, (e*(e + f*x))/(e^2 - d*f)] - (e + f*x)*((6*c*e - 3*b*f)*Hypergeometric2
F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)] - c*(e + f*x)*Hypergeometric2F1
[3/2, -m, 5/2, (e*(e + f*x))/(e^2 - d*f)])))/(3*f^3*((f*(d + e*x))/(-e^2 +
d*f))^m*Sqrt[e + f*x])
```

Maple [F] time = 0.628, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(cx^2+bx+a\right)\left(fx+e\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{(fx + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + bx + a)\sqrt{fx + e}(ex + d)^m}{f^2x^2 + 2efx + e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*sqrt(f*x + e)*(e*x + d)^m/(f^2*x^2 + 2*e*f*x + e
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{(fx + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

3.945 $\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=509

$$\frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(eg^2(m+1)(bd-ae)+c\left(3d^2g^2-2d\right.\right.}{ce^3(m+1)(m+4)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*e*(4 + m)) + ((e*(b*d -
a*e)*g^2*(1 + m) + c*(3*d^2*g^2 + e^2*f^2*(4 + m) - 2*d*e*f*g*(4 + m)))*(d
+ e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*
c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^3*(1 + m)*(4 + m)*Sqrt[1 - (2*c*(d + e*x)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)]) - (g*(b*e*g*(5 + 2*m) + 2*c*(3*d*g - 2*e*f*(4 +
m)))*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3
+ m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c
*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rubi [A] time = 0.84172, antiderivative size = 506, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1653, 843, 759, 133}

$$\frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(g^2(bd-ae)+\frac{c\left(3d^2g^2-2defg(m+4)+e^2f^2\right.}{e(m+1)}\right.}{ce^2(m+4)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*e*(4 + m)) + (((b*d - a*
e)*g^2 + (c*(3*d^2*g^2 + e^2*f^2*(4 + m) - 2*d*e*f*g*(4 + m)))/(e*(1 + m)))
*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(4 + m)*Sqrt[1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]) - (g*(6*c*d*g - 4*c*e*f*(4 + m) + b*e*g*(5 + 2*m))*(
d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (
2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(d +
e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
```

```
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} + \frac{\int (d+ex)^m\left(\frac{1}{2}e\left(2cef^2(4+m)-g^2(3bd+\right.\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} - \frac{(g(6cdg-4cef(4+m)+beg(5+2m)))\int}{2ce^2(4+m)}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} - \frac{\left(g(6cdg-4cef(4+m)+beg(5+2m))\sqrt{a}\right.}{2ce}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} + \frac{\left(e(bd-ae)g^2(1+m)+c\left(3d^2g^2+e^2f^2(4\right.\right.}{}$$

Mathematica [F] time = 1.45589, size = 0, normalized size = 0.

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]

[Out] Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2], x]
```

Maple [F] time = 1.539, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(gx+f\right)^2 \sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)

[Out] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}\left(gx+f\right)^2(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g^2x^2+2\,fgx+f^2\right)\sqrt{cx^2+bx+a}(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}\left(gx + f\right)^2 (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)
```

3.946 $\int (d+ex)^m (f+gx)\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=388

$$\frac{\sqrt{a+bx+cx^2}(ef-dg)(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{g\sqrt{a+bx+cx^2}(d+ex)^{n}}{e^2(m+}$$

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2,
-1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*Sqrt[1 - (2*c*(d
+ e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)]) + (g*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^
2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rubi [A] time = 0.359971, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {843, 759, 133}

$$\frac{\sqrt{a+bx+cx^2}(ef-dg)(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{g\sqrt{a+bx+cx^2}(d+ex)^{n}}{e^2(m+}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2,
-1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*Sqrt[1 - (2*c*(d
+ e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)]) + (g*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^
2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
```

```
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \frac{g\int (d+ex)^{1+m}\sqrt{a+bx+cx^2}\,dx}{e} + \frac{(ef-dg)\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx}{e}$$

$$= \frac{\left(g\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int x^{1+m}\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,d\right.}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{a+bx+cx^2}F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\right.}{e^2(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

Mathematica [F] time = 0.749146, size = 0, normalized size = 0.

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2], x]
```

Maple [F] time = 1.436, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(gx+f\right)\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}\left(gx+f\right)(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2+bx+a}\left(gx+f\right)(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}\left(gx+f\right)(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)
```

3.947 $\int (d+ex)^m \sqrt{a+bx+cx^2}\, dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])
```

Rubi [A] time = 0.124477, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{\sqrt{a+bx+cx^2}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{a+bx+cx^2}\,\mathrm{Subst}\left(\int x^m\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}dx,x,d+ex\right)}{e\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{a+bx+cx^2}F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

Mathematica [A] time = 0.0195565, size = 207, normalized size = 1.1

$$\frac{\sqrt{a+x(b+cx)}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e(m+1)\sqrt{\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + x*(b + c*x)]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (ex+d)^m\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2+bx+a}(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^m \sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)
```

3.948 $\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\sqrt{a+bx+cx^2}(d+ex)^m}{f+gx},x\right)$$

```
[Out] Defer[Int][((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

Rubi [A] time = 0.0344326, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
```

```
[Out] Defer[Int][((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

Rubi steps

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx=\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Mathematica [A] time = 0.109119, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
```

```
[Out] Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

Maple [A] time = 1.922, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{gx+f}\sqrt{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}(ex + d)^m}{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(ex + d)^m}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m \sqrt{a + bx + cx^2}}{f + gx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2)/(g*x+f),x)
```

```
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2)/(f + g*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}(ex + d)^m}{gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)
```

3.949 $\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=502

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(eg^2(m+1\right.}{ce^3(m+1)(m+2)\sqrt{a+bx+cx^2}}$$

```
[Out] (g^2*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2])/(c*e*(2 + m)) + ((e*(b*d - a*
e)*g^2*(1 + m) + c*(d^2*g^2 + e^2*f^2*(2 + m) - 2*d*e*f*g*(2 + m)))*(d + e*
x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[1 + m,
 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^3*(1 + m)*(2 + m)*Sq
rt[a + b*x + c*x^2]) - (g*(b*e*g*(3 + 2*m) + c*(2*d*g - 4*e*f*(2 + m)))*(d
+ e*x)^(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2
+ m, 1/2, 1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)^2*Sq
rt[a + b*x + c*x^2])
```

Rubi [A] time = 0.681, antiderivative size = 500, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1653, 843, 759, 133}

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(g^2(bd-ae\right.}{ce^2(m+2)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2])/(c*e*(2 + m)) + (((b*d - a*e)
*g^2 + (c*(d^2*g^2 + e^2*f^2*(2 + m) - 2*d*e*f*g*(2 + m)))/(e*(1 + m)))*(d
+ e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[1
+ m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(2 + m)*Sqrt[a
 + b*x + c*x^2]) - (g*(2*c*d*g - 4*c*e*f*(2 + m) + b*e*g*(3 + 2*m))*(d + e*
x)^(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m,
 1/2, 1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)^2*Sqrt[a
 + b*x + c*x^2])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
```

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} + \frac{\int \frac{(d+ex)^m\left(\frac{1}{2}e\left(2cef^2(2+m)-g^2(bd+2ae(1+m))\right)-\frac{1}{2}eg(2cdg-4cef(2+m)+beg(\cdots}{\sqrt{a+bx+cx^2}}\cdots}{ce^2(2+m)}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} - \frac{(g(2cdg-4cef(2+m)+beg(3+2m)))\int \frac{(d+ex)^{1+m}}{\sqrt{a+bx+cx^2}}\,dx}{2ce^2(2+m)} + \cdots$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} - \frac{\left(g(2cdg-4cef(2+m)+beg(3+2m))\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\cdots\right.}{2ce^3\cdots}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} + \frac{\left(e(bd-ae)g^2(1+m)+c\left(d^2g^2+e^2f^2(2+m)-2defg(2+\cdots\right.\right.}{\cdots}$$

Mathematica [F] time = 1.27056, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2], x]
```

Maple [F] time = 1.701, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(gx+f\right)^2 \frac{1}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^2 (ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(g^2x^2+2\,fgx+f^2\right)(ex+d)^m}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)^2(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.950 $\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=388

$$\frac{(ef-dg)(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{a+bx+cx^2}}+$$

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(1 + m)*Sqrt[a + b*x + c*x^2]) + (g*(d + e*x)^(2 + m)*Sqrt[1 - (2*c*(d + e*
x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2*c*(d + e*x)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(e^2*(2 + m)*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.346535, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {843, 759, 133}

$$\frac{(ef-dg)(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{a+bx+cx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(1 + m)*Sqrt[a + b*x + c*x^2]) + (g*(d + e*x)^(2 + m)*Sqrt[1 - (2*c*(d + e*
x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2*c*(d + e*x)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(e^2*(2 + m)*Sqrt[a + b*x + c*x^2])
```


Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
```

```
(e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{(d+ex)^{1+m}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{\left(g\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{x^{1+m}}{\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}\,dx, x,$$

$$e^2\sqrt{a+bx+cx^2}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{}{2cd-(}\right.}{e^2(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [F] time = 0.742372, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]

[Out] Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2], x]
```

Maple [F] time = 1.243, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(gx+f\right)\frac{1}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)

[Out] int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx+f\right)(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m (f+gx)}{\sqrt{a+bx+cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m*(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.951 $\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a
 + b*x + c*x^2])
```

Rubi [A] time = 0.126014, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759, 133}

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a
 + b*x + c*x^2])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{x^m}{\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2cx}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}dx, x, d+ex\right)}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{}{2cd-}\right.}{e(1+m)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.0229803, size = 207, normalized size = 1.1

$$\frac{(d+ex)^{m+1}\sqrt{\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\sqrt{\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e(m+1)\sqrt{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*
a*c])*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a + x*(b + c*x)])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (ex+d)^m \frac{1}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^m}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/sqrt(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/sqrt(c*x^2 + b*x + a), x)
```

3.952 $\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}},x\right)$$

```
[Out] Defer[Int][(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

Rubi [A] time = 0.0355541, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Defer[Int][(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Mathematica [A] time = 0.122449, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

Maple [A] time = 1.486, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{gx+f}\frac{1}{\sqrt{cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}(ex+d)^m}{cgx^3+(cf+bg)x^2+af+(bf+ag)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(c*g*x^3 + (c*f + b*g)*x^2 + a*f
 + (b*f + a*g)*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/((f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{\sqrt{cx^2+bx+a}(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

3.953 $\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=265

$$\frac{(d+ex)^{m+1}(f+gx)^n \left(\frac{e(f+gx)}{ef-dg}\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(g(m+n+2)\left(ae^2g(m+n+3)-cd(dg(n+1)+ef(m\right.\right.}{e^3g^2(m+1)(m+n+2)(m+n+3)}$$

```
[Out] ((b*e*g*(3 + m + n) - c*(e*f*(2 + m) + d*g*(4 + m + 2*n)))*(d + e*x)^(1 + m
)*(f + g*x)^(1 + n))/(e^2*g^2*(2 + m + n)*(3 + m + n)) + (c*(d + e*x)^(2 +
m)*(f + g*x)^(1 + n))/(e^2*g*(3 + m + n)) + ((g*(2 + m + n)*(a*e^2*g*(3 + m
 + n) - c*d*(e*f*(2 + m) + d*g*(1 + n))) - (e*f*(1 + m) + d*g*(1 + n))*(b*e
*g*(3 + m + n) - c*(e*f*(2 + m) + d*g*(4 + m + 2*n))))*(d + e*x)^(1 + m)*(f
 + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g))]
)/(e^3*g^2*(1 + m)*(2 + m + n)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n)
```

Rubi [A] time = 0.350857, antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {951, 80, 70, 69}

$$\frac{(d+ex)^{m+1}(f+gx)^n \left(\frac{e(f+gx)}{ef-dg}\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(g(m+n+2)\left(ae^2g(m+n+3)-cd(dg(n+1)+ef(m\right.\right.}{e^3g^2(m+1)(m+n+2)(m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

```
[Out] -((((c*e*f*(2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n))*(d + e*x)^(1 +
 m)*(f + g*x)^(1 + n))/(e^2*g^2*(2 + m + n)*(3 + m + n))) + (c*(d + e*x)^(2
 + m)*(f + g*x)^(1 + n))/(e^2*g*(3 + m + n)) + (((e*f*(1 + m) + d*g*(1 + n)
)*(c*e*f*(2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n)) + g*(2 + m + n)
*(a*e^2*g*(3 + m + n) - c*d*(e*f*(2 + m) + d*g*(1 + n))))*(d + e*x)^(1 + m)
*(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g
))])/(e^3*g^2*(1 + m)*(2 + m + n)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n
)
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (d+ex)^m (f+gx)^n \left(a+bx+cx^2\right) dx = \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2 g(3+m+n)} + \frac{\int (d+ex)^m (f+gx)^n \left(ae^2 g(3+m+n) - cd(ef(\right.}{}$$

$$= -\frac{(cef(2+m) - beg(3+m+n) + cdg(4+m+2n))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2 g^2 (2+m+n)(3+m+n)}$$

$$= -\frac{(cef(2+m) - beg(3+m+n) + cdg(4+m+2n))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2 g^2 (2+m+n)(3+m+n)}$$

$$= -\frac{(cef(2+m) - beg(3+m+n) + cdg(4+m+2n))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2 g^2 (2+m+n)(3+m+n)}$$

Mathematica [A] time = 0.208153, size = 187, normalized size = 0.71

$$\frac{(d+ex)^{m+1}(f+gx)^n \left(\frac{e(f+gx)}{ef-dg}\right)^{-n} \left(e\left(e\left(g(ag-bf)+cf^2\right) {}_2F_1\left(m+1,-n;m+2;\frac{g(d+ex)}{dg-ef}\right) - (2cf-bg)(ef-dg)\,{}_2F_1\left(m\right.\right.\right.}{e^3 g^2 (m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -2
 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(-((2*c*f - b*g)*(e*f - d*g)
*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]) + e
*(c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d + e*x
))/(-(e*f) + d*g)])))/(e^3*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

Maple [F] time = 0.623, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(gx+f\right)^n \left(cx^2+bx+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)(ex + d)^m \left(gx + f\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx + a\right)(ex + d)^m \left(gx + f\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n*(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)(ex + d)^m \left(gx + f\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)
```

3.954 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=525

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\ldots\right.}{ce^3(m+1)(m+2p+\ldots}$$

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*e*(3 + m + 2*p)) + ((e
*(b*d - a*e)*g^2*(1 + m) + c*(2*d^2*g^2*(1 + p) + e^2*f^2*(3 + m + 2*p) - 2
*d*e*f*g*(3 + m + 2*p)))*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 +
 m, -p, -p, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*
c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^3*(1 + m)*(3 + m +
2*p)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p) - (g*(b*e*g*(2 + m + p)
+ 2*c*(d*g*(1 + p) - e*f*(3 + m + 2*p)))*(d + e*x)^(2 + m)*(a + b*x + c*x^2
)^p*AppellF1[2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^3*(
2 + m)*(3 + m + 2*p)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rubi [A] time = 0.701753, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1653, 843, 759, 133}

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\ldots\right.}{ce^2(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*e*(3 + m + 2*p)) + (((
b*d - a*e)*g^2 + (c*(2*d^2*g^2*(1 + p) + e^2*f^2*(3 + m + 2*p) - 2*d*e*f*g*
(3 + m + 2*p)))/(e*(1 + m)))*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1
[1 + m, -p, -p, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(3 + m + 2*p)
*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p) - (g*(2*c*d*g*(1 + p) + b*e*
g*(2 + m + p) - 2*c*e*f*(3 + m + 2*p))*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^
p*AppellF1[2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^3*(2
+ m)*(3 + m + 2*p)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
)^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} + \frac{\int (d+ex)^m\left(e\left(cef^2(3+m+2p)-g^2(ae(1\right.\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} - \frac{(g(2cdg(1+p)+beg(2+m+p)-2cef(3}{ce^2(3+}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} - \frac{\left(g(2cdg(1+p)+beg(2+m+p)-2cef(3\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} + \frac{\left(e(bd-ae)g^2(1+m)+c\left(2d^2g^2(1+p)+e\right.\right.}{}$$

Mathematica [F] time = 2.2038, size = 0, normalized size = 0.

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
```

```
[Out] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p, x]
```

Maple [F] time = 1.473, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(gx+f\right)^2 \left(cx^2+bx+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx+f\right)^2 \left(cx^2+bx+a\right)^p (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(g^2x^2+2\,fgx+f^2\right)\left(cx^2+bx+a\right)^p (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx+f\right)^2 \left(cx^2+bx+a\right)^p (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="giac")

[Out] integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.955 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=384

$$\frac{(ef-dg)(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+}{2cd-\left(b-\sqrt{}\right.}\right.}{e^2(m+1)}$$

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p,
2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))
/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e))^p) + (g*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[
2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(1 - (2*
c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rubi [A] time = 0.337478, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {843, 759, 133}

$$\frac{(ef-dg)(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+}{2cd-\left(b-\sqrt{}\right.}\right.}{e^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p,
2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))
/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e))^p) + (g*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[
2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(1 - (2*
c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
```

```
p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] && !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \frac{g\int (d+ex)^{1+m}\left(a+bx+cx^2\right)^p\,dx}{e} + \frac{(ef-dg)\int (d+ex)^m\left(a+bx+cx^2\right)^p\,dx}{e}$$

$$= \frac{\left(g\left(a+bx+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right)\operatorname{Subst}\left(\int x^{1+m}\left(1\right.\right.}{e^2}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right.}\right.}{e^2(1+m)}$$

Mathematica [F] time = 1.22558, size = 0, normalized size = 0.

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x]
```

Maple [F] time = 1.454, size = 0, normalized size = 0.

$$\int (ex+d)^m\left(gx+f\right)\left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx+f\right)\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx+f\right)\left(cx^2+bx+a\right)^p(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(gx+f\right)\left(cx^2+bx+a\right)^p(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.956 $\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=187

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{}{2cd-(}\right.}{e(m+1)}$$

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e))^p)
```

Rubi [A] time = 0.127272, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {759, 133}

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{}{2cd-(}\right.}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e))^p)
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] && !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p dx = \frac{\left(\left(a+bx+cx^2\right)^p \left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p} \left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\right) \text{Subst}\left(\int x^m \left(1-\frac{}{2cd-\left(b-\right.}\right.\right.}{e}$$

$$= \frac{(d+ex)^{1+m} \left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} F_1\left(1+r\right.}{e(1+m)}$$

Mathematica [A] time = 0.104033, size = 205, normalized size = 1.1

$$\frac{(d+ex)^{m+1}(a+x(b+cx))^p \left(\frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)^{-p} \left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2cd}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+}{2cd+\left(\sqrt{b^2-}\right.}\right.}{e(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)
)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int (ex+d)^m \left(cx^2+bx+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p (ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^p (ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)^p (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)
```

3.957 $\int \frac{(d+ex)^m(a+bx+cx^2)^p}{f+gx}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx},x\right)$$

```
[Out] Defer[Int][((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

Rubi [A] time = 0.0269876, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
```

```
[Out] Defer[Int][((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx=\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Mathematica [A] time = 0.173333, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
```

```
[Out] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

Maple [A] time = 1.63, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m\left(cx^2+bx+a\right)^p}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^2+bx+a\right)^p(ex+d)^m}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)^p(ex+d)^m}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)
```

3.958 $\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[Out] (-2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rubi [A] time = 0.226979, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1574, 933, 168, 538, 537}

$$-\frac{2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

```
[Out] (-2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx &= \frac{\sqrt{-\frac{1}{c^2}+x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{\sqrt{1-\frac{1}{c^2x^2}}x}\\
&= \frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{\sqrt{1-\frac{1}{c^2x^2}}x}\\
&= -\frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x}\\
&= -\frac{\left(2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\
&= -\frac{2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] time = 0.636164, size = 188, normalized size = 2.11

$$
-\frac{2i(d+ex)\sqrt{\frac{e(cx-1)}{c(d+ex)}}\sqrt{\frac{cex+e}{cd+cex}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right),\frac{cd-e}{cd+e}\right)-\Pi\left(\frac{cd}{cd+e};i\sinh^{-1}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right)|\frac{cd-e}{cd+e}\right)\right)}{dx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{cd+e}{c}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

```
[Out] ((-2*I)*Sqrt[(e*(-1 + c*x))/(c*(d + e*x))]*(d + e*x)*Sqrt[(e + c*e*x)/(c*d
+ c*e*x)]*(EllipticF[I*ArcSinh[Sqrt[-((c*d + e)/c)]/Sqrt[d + e*x]], (c*d -
e)/(c*d + e)] - EllipticPi[(c*d)/(c*d + e), I*ArcSinh[Sqrt[-((c*d + e)/c)]/
Sqrt[d + e*x]], (c*d - e)/(c*d + e)]))/(d*Sqrt[-((c*d + e)/c)]*Sqrt[1 - 1/(
c^2*x^2)]*x)
```

Maple [A] time = 0.706, size = 148, normalized size = 1.7

$$-2\frac{cd-e}{x\sqrt{ex+dcd}}\text{EllipticPi}\left(\sqrt{\frac{(ex+d)c}{cd-e}},\frac{cd-e}{cd},\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{-\frac{(cx+1)e}{cd-e}}\sqrt{-\frac{(cx-1)e}{cd+e}}\sqrt{\frac{(ex+d)c}{cd-e}}\frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -2/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c*d-e)*EllipticPi(((e*x+d)*c/(c*d-e))^(1/
2),(c*d-e)/d/c,((c*d-e)/(c*d+e))^(1/2))*(-(c*x+1)*e/(c*d-e))^(1/2)*(-(c*x-1
)*e/(c*d+e))^(1/2)*((e*x+d)*c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/c/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+d}x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{-\left(-1+\frac{1}{cx}\right)\left(1+\frac{1}{cx}\right)}\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1-1/c**2/x**2)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-1 + 1/(c*x))*(1 + 1/(c*x)))*sqrt(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+d}\,x^2\sqrt{-\frac{1}{c^2x^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                    #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```